Available online at www.sciencedirect.com
ScienceDirect
Neurocomputing 71 (2008) 1669–1679
NEUROCOMPUTING
www.elsevier.com/locate/neucom

Morphologically constrained ICA for extracting weak temporally correlated signals

Zhi-Lin Zhang[a,b,*]

[a]School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China
[b]Department of Computer Science and Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

Received 10 March 2006; received in revised form 9 October 2006; accepted 11 April 2007
Communicated by H. Valpola
Available online 24 April 2007

Abstract

Recently the constrained ICA (cICA) algorithm has been widely applied to many applications. But a crucial problem to the algorithm is how to design a reference signal in advance, which should be closely related to the desired source signal. If the desired source signal is very weak in mixed signals and there is no enough *a priori* information about it, the reference signal is difficult to design. With some detailed discussions on the cICA algorithm, the paper proposes a second-order statistics based approach to reliably find suitable reference signals for weak temporally correlated source signals. Simulations on synthetic data and real-world data have shown its validity and usefulness.

Keywords: Constrained independent component analysis (cICA); Blind source extraction (BSE); Blind source separation (BSS); Reference signal; Temporally correlated signal

1. Introduction

Temporally correlated signals widely exist in various fields, such as biomedical engineering [3,6,8,9,19,21] and financial time series analysis [7]. These signals are often "interesting" and important to us. For example, in abnormal EEG analysis the quasi-periodic complex resulted from periodic synchronous discharge is of importance to determine whether or not the subject suffers from subacute sclerosing panencephalitis (SSPE) or other diseases. In the non-invasive extraction of fetal electrocardiogram (FECG) [8,19,21], the FECG, which provides information about fetal maturity, position of the fetus and multiple pregnancies, also can be regarded as a quasi-periodic signal.

Unfortunately, these valuable temporally correlated source signals and other unwanted source signals are often mixed in observed signals and often contaminated by noise. To obtain the desired source signals, one powerful technique is the blind source separation (BSS) [4,7], which simultaneously separates all of the source signals. However, in many applications the number of sensors is often large, which may result in heavy computational load and cost lots of time, while the "interesting" source signals (the desired ones) are few. For example, in EEG or MEG we obtain typically more than 64 sensor signals but only several source signals (e.g. the periodically evoked brain potentials) are considered interesting, and the rest are considered to be interfering noise. For such applications it is essential to develop reliable, robust and effective learning algorithms which enable us to extract only a small number of temporally correlated source signals that are potentially interesting and contain useful information [4,9,20–22].

The constrained ICA (cICA) (also called ICA with reference) [10–14] is a good candidate for extracting several source signals from a large number of observed signals.

*Corresponding author at: School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China.
E-mail address: zlzhang@uestc.edu.cn.

0925-2312/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.neucom.2007.04.004

It needs to first construct a reference signal that should be closely related to the desired source signal. The reference signal is the one and only the one that is closest to the desired source signal in terms of a closeness measure. Therefore, the reference signal is of vital importance to cICA. It has been pointed out [10,22] that both the shape and the phase of the reference signal may strongly affect the extraction results. Thus in some applications the reference signal is difficult to construct, at least in an automatic manner. For example, in the non-invasive extraction of FECG, the desired FECG is very weak so that the accurate occurrence time and shape of each complex of FECG are often not easy to obtain, especially when the fetus is in early phase. Besides, an improperly selected threshold will result in the failure of cICA [14]. A feasible threshold depends on both the designed reference signal and the closeness measure. Given the same data set, different reference signals require different suitable ranges of the threshold value. Similarly, different closeness measures also determine different suitable ranges. Lu and Rajapakse [14] suggest to use a small threshold initially, and then gradually increase the threshold. However, this method is not always feasible in practice. Due to the above reasons, the applications of cICA are limited to the ones where the reference signals are easy to design or find, such as the artifact removal of EEG [10,13].

In this paper, we propose a second-order statistics based approach for designing suitable reference signals for reliably extracting weak temporally correlated source signals, which the original cICA often fails to extract. Another advantage of the approach is that the threshold is easy to set, ensuring the global convergence of cICA, since the reference signal is morphologically close to the desired signal. The rest of the paper is organized as follows. In Section 2 the cICA algorithm is introduced, and some important related issues are discussed in Section 3. In the next section, an approach to obtain feasible reference signals for extracting weak temporally correlated source signals is proposed. Computer simulations are presented in Section 5 and conclusions are drawn in Section 6.

2. The constrained independent component analysis

Let us consider unknown stochastic source signals s_i ($i = 1,\ldots,n$) which are mutually independent, zero-mean and unit-variance. It is assumed that the source signals are non-Gaussian (at most one is Gaussian). The model for the observed signals is

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k), \tag{1}$$

where k is the time index, $\mathbf{A}$ is an unknown non-singular mixing matrix, $\mathbf{s}(k) = [s_1(k),\ldots,s_n(k)]^{\mathrm{T}}$ and $\mathbf{x}(k) = [x_1(k),\ldots,x_n(k)]^{\mathrm{T}}$. Without knowing the source signals and the mixing matrix, we want to recover a source signal from the observed signals $\mathbf{x}(k)$ by the following linear transform:

$$y(k) = \mathbf{w}^{\mathrm{T}}\mathbf{x}(k) = \mathbf{w}^{\mathrm{T}}\mathbf{A}\mathbf{s}(k), \tag{2}$$

where $\mathbf{w}$ is a column vector and $y(k)$ is a recovered source signal up to a scalar. To cope with ill-conditioned cases and to make algorithms simpler and faster, before running algorithms the whitening [4] is often used to transform the observed signals $\mathbf{x}(k)$ to

$$\mathbf{z}(k) = \mathbf{V}\mathbf{x}(k), \tag{3}$$

such that $E\{\mathbf{z}(k)\mathbf{z}(k)^{\mathrm{T}}\} = \mathbf{I}$, where $\mathbf{V}$ is a whitening matrix. A typical solution is given by

$$\mathbf{V} = \mathbf{D}^{-1/2}\mathbf{E}^{\mathrm{T}}, \tag{4}$$

where $\mathbf{D}$ is the diagonal matrix of the eigenvalues of the matrix $E\{\mathbf{x}(k)\mathbf{x}(k)^{\mathrm{T}}\}$, and $\mathbf{E}$ is the matrix whose columns are the corresponding unit-norm eigenvectors.

For extracting one independent source signal, a reliable and flexible contrast function is the one based on negentropy [7], defined by

$$J(y) \approx \rho[E\{G(y)\} - E\{G(v)\}]^2, \tag{5}$$

where $y = \mathbf{w}^{\mathrm{T}}\mathbf{z}$ is the algorithm output, ρ is a positive constant, v is a Gaussian variable with zero mean and unit variance, and $G(\cdot)$ can be any non-quadratic function, such as

$$G_1(y) = \log(\cosh(a_1 y)/a_1), \tag{6}$$

$$G_2(y) = \exp(-a_2 y^2/2)/a_2, \tag{7}$$

$$G_3(y) = y^4/4, \tag{8}$$

where $1 \leqslant a_1 \leqslant 2$ and $a_2 \approx 1$. The algorithm that maximizes the contrast function (5) is called one-unit ICA [7]. However, maximization of (5) will give any of source signals. That is to say, any source signal may be the output. When one desires a specific source signal, he needs to use some *a priori* information of the desired signal to modify the one-unit algorithm, ensuring the output is necessarily the desired one. To achieve this goal, the cICA is derived [14] by first defining the following constrained contrast function:

$$\max \quad J(\mathbf{w}) \approx \rho[E\{G(\mathbf{w}^{\mathrm{T}}\mathbf{z}\} - E\{G(v)\}]^2 \tag{9}$$

$$\text{s.t.} \quad g(\mathbf{w}) = \varepsilon(y,r) - \xi \leqslant 0, \quad h(\mathbf{w}) = E\{y^2\} - 1 = 0, \tag{10}$$

where the equality constraint $h(\mathbf{w})$ ensures that the contrast function $J(y)$ and the weight vector $\mathbf{w}$ are bounded. $\varepsilon(y,r)$ is the closeness measure between the extracted signal y and the reference signal r. Note that the desired source signal is the one and only the one closest to the reference signal r, satisfying the following inequality relationship:

$$\varepsilon(\mathbf{w}^{*\mathrm{T}}\mathbf{z}, r) < \varepsilon(\mathbf{w}_1^{\mathrm{T}}\mathbf{z}, r) \leqslant \cdots \leqslant \varepsilon(\mathbf{w}_{n-1}^{\mathrm{T}}\mathbf{z}, r), \tag{11}$$

where the optimum vector $\mathbf{w}^*$ corresponds to the desired output and $\mathbf{w}_i$ ($i = 1,\ldots,n-1$) corresponds to other unwanted source signals. The value of the threshold ξ in (10) lies in $[\varepsilon(\mathbf{w}^{*\mathrm{T}}\mathbf{z}, r), \varepsilon(\mathbf{w}_1^{\mathrm{T}}\mathbf{z}, r))$.

From the contrast function (9)–(10) Lu and Rajapakse derived the cICA algorithm [12–14]:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta\mathbf{R}_z^{-1}\Gamma_1/\Gamma_2, \tag{12}$$

$$\Gamma_1 = \overline{\rho} E\{\mathbf{z}G'_y(y)\} - \tfrac{1}{2}\mu E\{\mathbf{z}g'_y(y)\} - \lambda E\{\mathbf{z}y\}, \tag{13}$$

$$\Gamma_2 = \overline{\rho} E\{G''_{y^2}(y)\} - \tfrac{1}{2}\mu E\{g''_{y^2}(y)\} - \lambda, \tag{14}$$

where t is the iteration count, $\mathbf{R}_z = E\{\mathbf{z}\mathbf{z}^{\mathrm{T}}\}$, $\overline{\rho} = \rho \cdot \mathrm{sign}\ (E\{G(y)\} - E\{G(v)\})$, $G'_y(y)$ and $g'_y(y)$ are the first derivatives of $G(y)$ and $g(y)$ with respect to y, and $G''_{y^2}(y)$ and $g''_{y^2}(y)$ are the second derivatives. The optimum multipliers μ^* and λ^* are found by iteratively updating them based on a gradient-ascent method:

$$\mu_t = \max\{0, \mu_{t-1} + \gamma g(\mathbf{w}_{t-1})\}, \tag{15}$$

$$\lambda_t = \lambda_{t-1} + \gamma h(\mathbf{w}_{t-1}). \tag{16}$$

3. Some important issues of cICA

There are several important issues that should be noticed. One issue is the value of the threshold ξ. If the threshold value is too large, the cICA may converge to other source signals since there may exist several source signals whose closeness measures are also less than the threshold. In contrast, if the threshold is too small, the cICA cannot converge. So the threshold value should be carefully selected. But the value is influenced by some factors. One is the choice of the closeness $\varepsilon(y,r)$. A common choice is the mean square error (MSE) given by $\varepsilon(y,r) = E\{(y-r)^2\}$, and another is the correlation $\varepsilon(y,r) = -E\{yr\}$. Clearly, different choices require different ranges of the threshold value. Another factor affecting the threshold value is the choice of the reference signal r. If the reference signal is very similar to the desired source signal, then the threshold value should be very small so that the algorithm can globally converge. Otherwise, the value should be larger. Obviously, the suitable value in some degree depends on the *a priori* knowledge about the quantization of the closeness between the reference signal and the closest source signal (i.e. the desired signal), and of the closeness between the reference signal and the second closest source signal. Unfortunately, according to our knowledge, there is no guide on how to set a suitable value.

The second issue is the design of the reference signal. James and Gibson [10] pointed out that the shape of the reference signal may influence the output result in the sense that source signals of slightly different morphology may be extracted for different reference morphology. They also pointed out that the phase of the reference signal must be closely matched to that of the desired source signal, or the occurrence time of each impulse of the reference signal is consistent with that of the desired signal. Until now, in most literature the main method is constructing a simple impulse signal by observing the waveform of sensor signals and/or by exploiting strong *a priori* information about the desired source signal [10,12–14]. This method requires that the desired source signal is strong enough to be observed in a sensor signal and/or there is sufficient *a priori* information available (e.g. the morphology, the phase and the occurrence time of the desired signal), which cannot be always satisfied. In addition, sometimes one should use many similar reference signals, each shifted by one sample to cover one expected period of the desired source signal [10]. Thus, the method sometimes may be trivial and results in increased computational load and time.

In the next section we will propose a valid approach to find a reference signal, whose phase and waveform is well matched to the desired source signal. Thus a suitable threshold value is easy to set, regardless of which closeness measure is adopted.

4. The proposed approach for designing reference signals

In this section we propose an approach to design suitable reference signals for extracting temporally correlated weak source signals. Our idea is first finding several time delays at which the autocorrelation of the desired source signal is maximized, and then using the time delays to roughly extract the desired source signal. The roughly extracted signal serves as the reference signal for the cICA algorithm.

Suppose the desired source signal is non-Gaussian and exhibits periodic behavior with period τ_0.[1] Without loss of generality, we assume s_1 is the desired source signal, satisfying the following relations:

$$E\left\{\sum_{p=1}^{P} s_1(k)s_1(k-l_p\tau_0)\right\} > 0, \tag{17}$$

$$E\left\{\sum_{p=1}^{P} s_j(k)s_j(k-l_p\tau_0)\right\} = 0 \quad \forall j \neq 1, \tag{18}$$

where s_j are other source signals, l_p $(p=1,\ldots,P)$ are positive integers and P is the number of the time delays (how to find these time delays will be addressed later). Then we propose the following objective function for coarsely extracting s_1:

$$\begin{aligned} \max \quad J(\mathbf{w}) &= 2E\left\{\sum_{p=1}^{P} y(k)y(k-l_p\tau_0)\right\} \\ &= \mathbf{w}^{\mathrm{T}}\left\{\sum_{p=1}^{P}(\mathbf{R}_z(l_p\tau_0) + \mathbf{R}_z(l_p\tau_0)^{\mathrm{T}})\right\}\mathbf{w} \end{aligned} \tag{19}$$

$$\text{s.t.} \quad \|\mathbf{w}\| = 1, \tag{20}$$

where $y(k) = \mathbf{w}^{\mathrm{T}}\mathbf{z}(k) = \mathbf{w}^{\mathrm{T}}\mathbf{V}\mathbf{x}(k)$ and $\mathbf{R}_z(l_p\tau_0) = E\{\mathbf{z}(k)\,\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\}$. The reason for this formulation is that for the desired signal s_1, this averaged delayed autocorrelation has a large positive value, while for other source signals this

[1] Here we mean that its waveform in a period is similar to its waveform in another period. Thus, it is not a strictly periodic signal. The reason why we make this assumption is that in practice the strictly periodic signal is rare.

value is zero. It is easy to see [20] that maximization of (19) under the constraint (20) is equivalent to finding the normalized eigenvector corresponding to the maximal eigenvalue of $\sum_{p=1}^{P}(\mathbf{R}_z(l_p\tau_0)+\mathbf{R}_z(l_p\tau_0)^{\mathrm{T}})$. Thus we directly have the following algorithm:

$$\mathbf{w}=EIG\left(\sum_{p=1}^{P}(\mathbf{R}_z(l_p\tau_0)+\mathbf{R}_z(l_p\tau_0)^{\mathrm{T}})\right), \tag{21}$$

where $EIG(\sum_{p=1}^{P}(\mathbf{R}_z(l_p\tau_0)+\mathbf{R}_z(l_p\tau_0)^{\mathrm{T}}))$ is the operator that calculates the normalized eigenvector corresponding to the maximal eigenvalue of $\sum_{p=1}^{P}(\mathbf{R}_z(l_p\tau_0)+\mathbf{R}_z(l_p\tau_0)^{\mathrm{T}})$. Denote by $\widehat{\mathbf{w}}$ the converged solution of the algorithm (21). Then the reference signal is given by $\widehat{y}=\widehat{\mathbf{w}}^{\mathrm{T}}\mathbf{z}$.

If several desired source signals exhibit periodic behavior with the same period τ_0, we still can obtain suitable reference signals for each desired source signal. To see this, suppose among the n source signals there are q ones, say $s_1,s_2,\ldots,s_q$, that have the same period τ_0. Let $r_\beta=E\{\sum_{p=1}^{P}s_\beta(k)s_\beta(k-l_p\tau_0)\}$ $(\beta=1,\ldots,n)$. Without loss of generality, suppose $r_1>r_2>\cdots>r_q>r_j$ $(j=q+1,\ldots,n)$.[2] In the following we will show that we can use the normalized eigenvector $\mathbf{w}_i$ associated with the ith largest eigenvalue of $E\{\sum_{p=1}^{P}\mathbf{z}(k)\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\}$ to obtain the reference signal for s_i $(i=1,\ldots,q)$.

First, note that $\mathbf{VA}$ is an orthogonal matrix, since $\mathbf{I}=E\{\mathbf{zz}^{\mathrm{T}}\}=\mathbf{VA}E\{\mathbf{ss}^{\mathrm{T}}\}\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$ and $E\{\mathbf{ss}^{\mathrm{T}}\}=\mathbf{I}$. Then consider the following relation:

$$E\left\{\sum_{p=1}^{P}\mathbf{z}(k)\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\right\} = \mathbf{VA}E\left\{\sum_{p=1}^{P}\mathbf{s}(k)\mathbf{s}(k-l_p\tau_0)^{\mathrm{T}}\right\}\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}, \tag{22}$$

which implies that all the eigenvalues of $E\{\sum_{p=1}^{P}\mathbf{z}(k)\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\}$ are those of $E\{\sum_{p=1}^{P}\mathbf{s}(k)\mathbf{s}(k-l_p\tau_0)^{\mathrm{T}}\}$. Denote by $\mathbf{w}_i$ the normalized eigenvector associated with the ith largest eigenvalue of $E\{\sum_{p=1}^{P}\mathbf{z}(k)\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\}$. We have

$$E\left\{\sum_{p=1}^{P}\mathbf{z}(k)\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\right\}\mathbf{w}_i=\lambda_i\mathbf{w}_i,\quad i=1,\ldots,q, \tag{23}$$

where λ_i is the ith largest eigenvalue. Since $\mathbf{z}(k)=\mathbf{VAs}(k)$ and $\mathbf{VA}$ is an orthogonal matrix, (23) is further reduced to

$$E\left\{\sum_{p=1}^{P}\mathbf{s}(k)\mathbf{s}(k-l_p\tau_0)^{\mathrm{T}}\right\}(\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\mathbf{w}_i) = \lambda_i\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\mathbf{w}_i,\quad i=1,\ldots,q, \tag{24}$$

implying that $\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\mathbf{w}_i$ is the normalized eigenvector associated with the ith largest eigenvalue λ_i of $E\{\sum_{p=1}^{P}\mathbf{s}(k)\mathbf{s}(k-l_p\tau_0)^{\mathrm{T}}\}$. Suppose $E\{\sum_{p=1}^{P}s_\alpha(k)s_\beta(k-l_p\tau_0)^{\mathrm{T}}\}=0$ $(\alpha\neq\beta$ and $\alpha,\beta=1,\ldots,n)$, then $E\{\sum_{p=1}^{P}\mathbf{s}(k)\mathbf{s}(k-l_p\tau_0)^{\mathrm{T}}\}$ is a diagonal matrix. Therefore $\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\mathbf{w}_i=\mathbf{e}_i$, whose the ith element is one while other elements are zero. Then we have $y=\mathbf{w}_i^{\mathrm{T}}\mathbf{z}=\mathbf{w}_i^{\mathrm{T}}\mathbf{VAs}=\mathbf{e}_i^{\mathrm{T}}\mathbf{s}=s_i$, which means that s_i is extracted by the vector $\mathbf{w}_i$. However, in practice the assumption $E\{\sum_{p=1}^{P}s_i(k)s_j(k-l_p\tau_0)^{\mathrm{T}}\}=0$ $(i\neq j$ and $i,j=1,\ldots,n)$ cannot be met. One reason is that in real world two source signals may be weakly correlated. Another reason is concerned with the numerical errors. Practically we use the empirical average over limited samples, instead of the statistical expectation. As a result, even if two signals satisfy the assumption, the calculated cross-correlation over limited samples is generally non-zero (although we have shown [20] that with sufficient time delays the numerical errors tend to zero, in practice we may not find so many suitable time delays). Due to these reasons, $\mathbf{A}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\mathbf{w}_i\approx\mathbf{e}_i$ and thus $y\approx s_i$. This means that the extracted signal by $\mathbf{w}_i$ is only a good reference signal for further extracting s_i by the cICA algorithm.[3]

So, according to the analysis, if we find the matrix $E\{\sum_{p=1}^{P}\mathbf{z}(k)\mathbf{z}(k-l_p\tau_0)^{\mathrm{T}}\}$ has several large eigenvalues, say q eigenvalues that are obviously larger than zero, then we can conclude that there may be q source signals with the same period, and calculate the corresponding eigenvectors to obtain reference signals for each source signal.

If the desired source signal does not exhibit periodic behavior, but only a general temporally correlated signal, the above results should be slightly modified. Denote by s_1 the desired source signal and suppose it satisfies:

$$E\left\{\sum_{p=1}^{P}s_1(k)s_1(k-\tau_p)\right\}>0, \tag{25}$$

$$E\left\{\sum_{p=1}^{P}s_j(k)s_j(k-\tau_p)\right\}=0\quad\forall j\neq 1, \tag{26}$$

where $\tau_1,\tau_2,\ldots,\tau_P$ are time delays and s_j are other source signals. Similarly, the objective function is

$$\max\quad J(\mathbf{w})=2E\left\{\sum_{p=1}^{P}y(k)y(k-\tau_p)\right\} = \mathbf{w}^{\mathrm{T}}\left\{\sum_{p=1}^{P}(\mathbf{R}_z(\tau_p)+\mathbf{R}_z(\tau_p)^{\mathrm{T}})\right\}\mathbf{w} \tag{27}$$

$$\text{s.t.}\quad \|\mathbf{w}\|=1. \tag{28}$$

And the corresponding algorithm is given by

$$\mathbf{w}=EIG\left(\sum_{p=1}^{P}(\mathbf{R}_z(\tau_p)+\mathbf{R}_z(\tau_p)^{\mathrm{T}})\right). \tag{29}$$

Now we have an important issue that has not been addressed, i.e. how to find the suitable time delays, which

[2] Since $s_1,s_2,\ldots,s_q$ are not strictly periodic signals, we have $E\{s_g(k)s_g(k-l_p\tau_0)\}<1$ and it is probable that $E\{s_g(k)s_g(k-l_p\tau_0)\}\neq E\{s_h(k)s_h(k-l_p\tau_0)\}$ $(g\neq h)$. Therefore, the assumption that $r_1>r_2>\cdots>r_q>r_j$ $(j=q+1,\ldots,n)$ is held in most cases.

[3] We would like to draw the readers' attention to some similar work [3,14] that also showed second-order statistics based reference construction methods are insufficient to recover independent source signals.

are embodied as the peaks in correlation functions. However, it is not a difficult problem. There are numerous methods to estimate them, such as the autocorrelation method [1], the heart instantaneous frequency (HIF) estimation technique [2] and the cepstrally transformed discrete cosine transform (CTDCT) [16]. Note that lots of pitch estimation methods [17,18] in the speech and audio processing field can also be directly used to find the time delays. In addition, in some cases the time delays are readily available [1,14]. For example, in MEG analysis the periods of some oscillatory artifacts, such as the power supply interference (50–60 Hz), can be easily obtained. For some EEG experiments the periods of stimuli to which event-related brain potentials respond are readily available as well.

Note that although the proposed approach needs to first estimate the time delays, its performance is non-sensitive to small estimate errors of the time delays, which is reported in our previous work [20,22]. Furthermore, one can employ other signal processing techniques, such as the band-pass filtering, low-pass filtering and wavelet transform, to the roughly extracted signal $\widehat{y}$ to obtain a better reference signal, according to the problem in hand. Another advantage of the proposed approach is that it is non-sensitive to additive white sensor noise and suppresses color sensor noise to a great degree.

5. Simulations and experiments

5.1. Synthetic data set

We generated seven zero-mean and unit-variance source signals, shown in Fig. 1. Each signal had 2500 samples. s_1 was a quasi-periodic signal, while s_2 and s_3 behaved periodically with the same period: 500 sampling periods. Our goal was to extract the desired temporally correlated source signals s_1, s_2 and s_3 using the cICA algorithm with suitable reference signals obtained by the proposed approach.

The source signals were randomly mixed, shown in Fig. 2. Denote by $\mathbf{A}$ the mixing matrix. Obviously, the waveform of any source signal was not visible in the mixed signals. Thus, it was very difficult to design reference signals by the conventional methods, i.e. constructing reference signals only via observing the waveform of mixed signals. Here we used the proposed approach to find suitable reference signals for successfully extracting each desired source signal.

First, we should find the time delays. They correspond to the peaks in autocorrelation functions of mixed signals. There are many algorithms for finding these peaks [1,2,16–18]. Here we used our recently proposed lag-finding method [23], which can accurately find suitable time delays

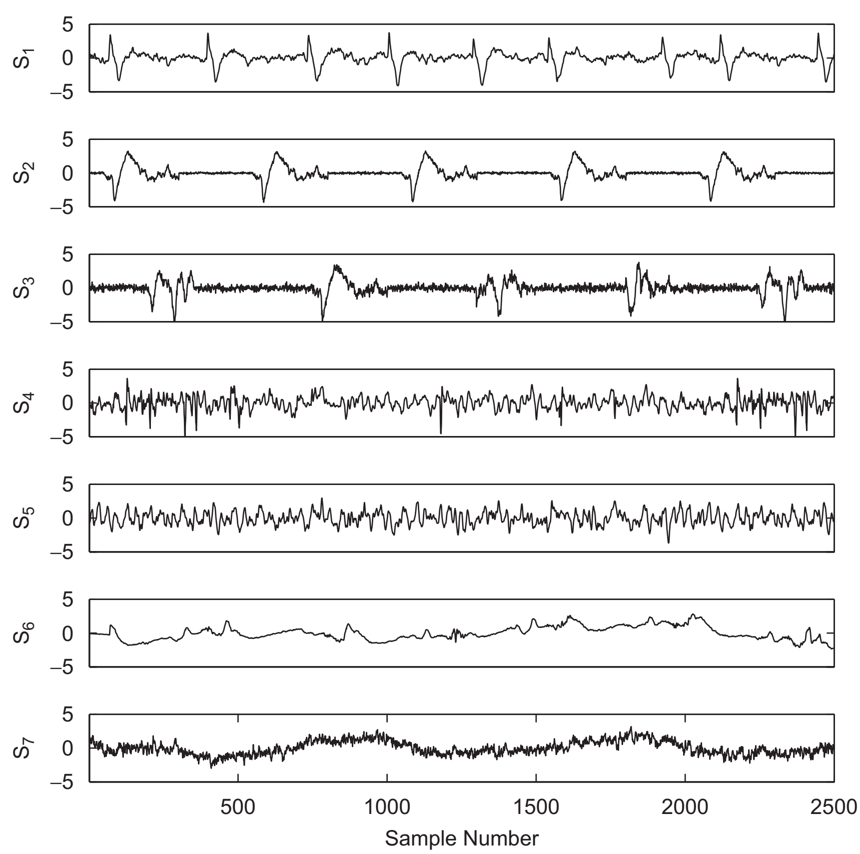

Fig. 1. The seven synthetic source signals. s_1 was a quasi-periodic signal, while s_2 and s_3 behaved periodically with the same period: 500 sampling periods.

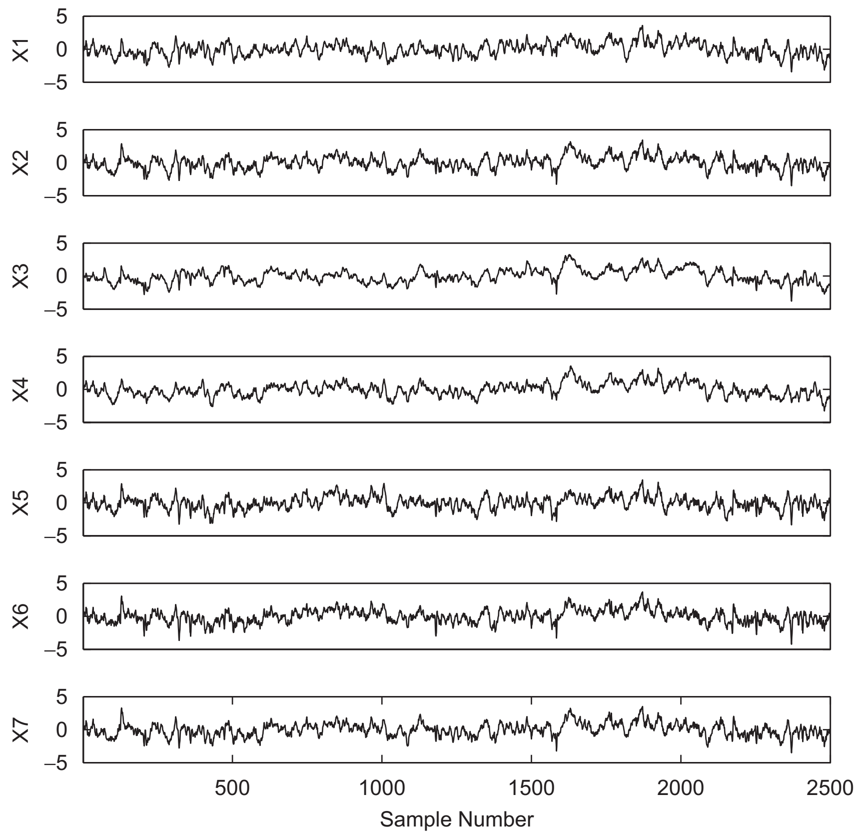

Fig. 2. The mixed signals, from which the waveforms of the original source signals were not visible.

(of course, other lag-finding methods [16–18] can obtain similar results). The lag-finding method first employs the linear prediction operator to a mixed signal, and obtains a prediction error signal. Then it employs the ordinary autocorrelation operator to the error signal. Next, the result is transformed by a parameterized nonlinear function, and the final result is plotted. In this simulation we employed the lag-finding method to the third mixed signal x_3. The result is plotted in Fig. 3, from which we can see there were a number of peaks, each peak corresponding to a source signal. Of course, several peaks might correspond to a same source signal. For example, the peaks locating at time delay 500 and time delay 1000 held the harmonic relation, and thus the two peaks were believed to correspond to the same source signal. We selected the time delay set $\tau_1 = \{272\}$ and the time delay set $\tau_2 = \{500, 1000\}$ for our proposed approach.

Then the mixed signals were whitened. Denote the whitening matrix by $\mathbf{V}$ and the whitened signals by $\mathbf{z}(k)$. Using the time delay set τ_1, the proposed approach obtained a reference signal, shown in Fig. 4 (see r_1). Compared with the original source signal s_1, the reference signal matched its phase and morphology to some degree. Finally, using the reference signal the cICA algorithm successfully extracted the source signal s_1, shown in Fig. 4 (see y_1).

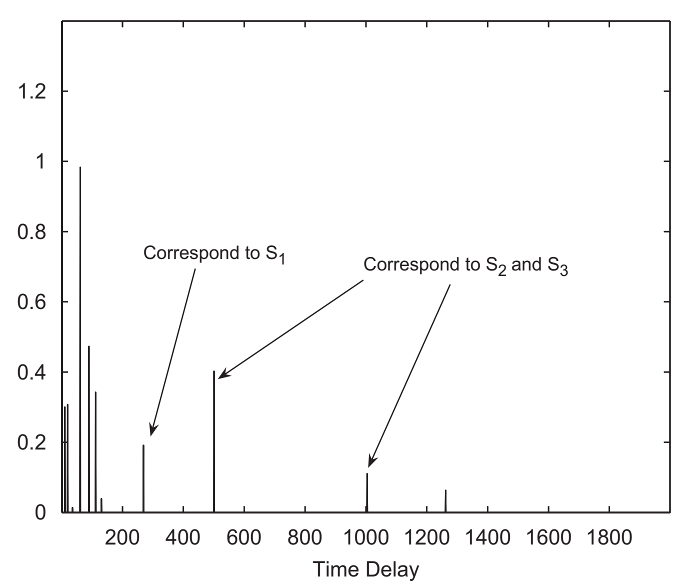

Fig. 3. Finding the time delays. The result was calculated using the mixed signal x_3 in Fig. 2. Each peak corresponded to a source signal.

Using the time delay set τ_2 the proposed approach found that there were two very large eigenvalues in $E\{\sum_{p=1}^{2}\mathbf{z}(k)\mathbf{z}(k-p*500)^{\mathrm{T}}\}$, while other eigenvalues were close to zero. According to the results in Section 4, the

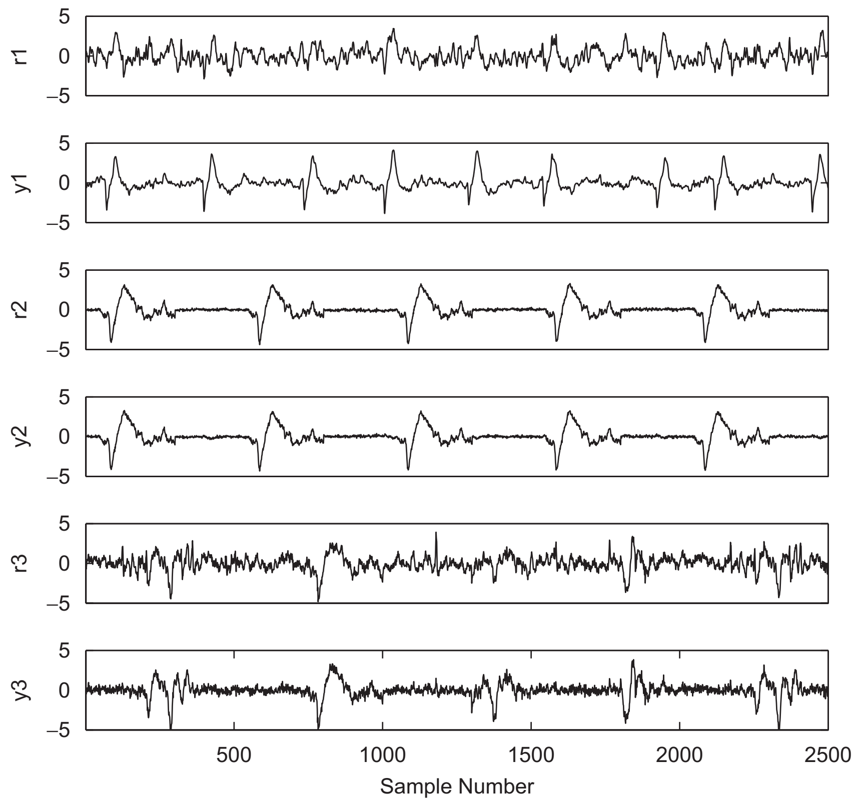

Fig. 4. Final extraction results by the reference signals obtained by the proposed approach. y_i was the final extracted signal using the reference signal r_i, $i = 1, 2, 3$.

proposed approach calculated the eigenvectors associated with the two large eigenvalues and then obtained two reference signals, shown in Fig. 4 (see r_2 and r_3). Using these reference signals the cICA algorithm also successfully extracted the corresponding source signals (see y_2 and y_3 in Fig. 4).

To measure the final extraction quality, we used the following cross-talk index:

$$PI = -10E\{\lg(s(k) - \tilde{s}(k))^2\} \quad \text{(dB)}, \tag{30}$$

where $s(k)$ is a desired source signal and $\tilde{s}(k)$ is a extracted signal (both of them should be normalized to be zero-mean and unit-variance). This index measures the similarity between the source signal and the extracted signal. The larger the PI is, the better the extraction quality. Generally, the value larger than 20 dB indicates a good extraction quality. In the simulation, the averaged cross-talk indexes over 100 independent trials are listed in Table 1, from which we can see that by using the reference signals obtained by our proposed approach the cICA algorithm can finally extract the weak source signals with good performance.

Next, we illustrated how the phase and morphology of reference signals affect the final extraction. We noticed that there was a large wave from 1550 sample points to 1650 sample points in each mixed signal, indicating it might be a part of some source signal. So, like the method in [10], we designed several simple reference signals according to the location and width of the wave, shown in Fig. 5. In each simple reference signal there was only a square impulse. The width and the occurrence time of the impulse in each reference signal had slight differences. The width of the impulse in c_1, c_2, c_3 and c_4 was, respectively, 20, 50, 20 and 50 sample points. The occurrence time of the impulse in c_1, c_2, c_3 and c_4 was, respectively, 1575, 1575, 1625 and 1625 sample points. The corresponding extracted signals u_1, u_2, u_3 and u_4 using these reference signals are shown in Fig. 6. Obviously, the original source signals were not recovered well. To see this, we calculated the global vector, defined by

$$\mathbf{g} = \mathbf{w}^{\mathrm{T}}\mathbf{V}\mathbf{A}, \tag{31}$$

Table 1
The averaged cross-talk indexes over 100 independent trials of the reference signals and the final extracted signals

	s_1	s_2	s_3
Reference signal	7.1	30.7	9.7
Final extraction	27.0	31.8	21.0

The 7.1 in the second column means that the index measuring the similarity between the reference signal r_1 and the source signal s_1 is 7.1 dB and the 27.0 means that the index measuring the similarity between the final extracted signal y_1 and s_1 is 27.0 dB. The same is with the other four data.

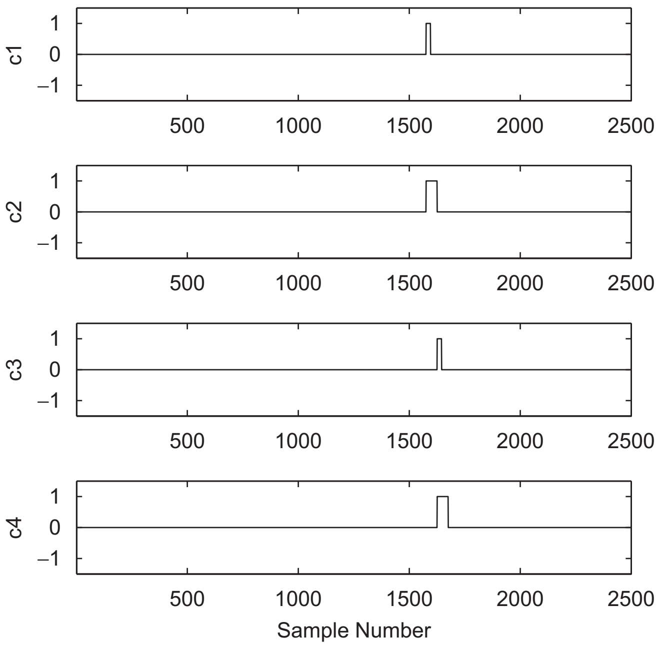

Fig. 5. Four simple reference signals. The width of the impulse in c_1, c_2, c_3 and c_4 was, respectively, 20, 50, 20 and 50 sample points. The occurrence time of the impulse in c_1, c_2, c_3 and c_4 was, respectively, 1575, 1575, 1625 and 1625 sample points.

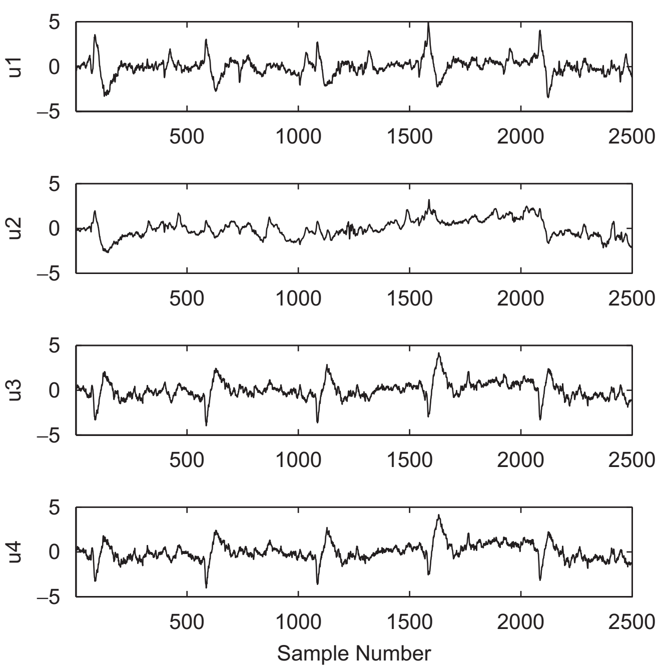

Fig. 6. Extracted signals by the simple reference signals in Fig. 5. u_i was the final extracted signal using the simple reference signal c_i, $i = 1, 2, 3, 4$.

where $\mathbf{w}$ is the demixing vector obtained by the cICA algorithm, and $\mathbf{V}$ and $\mathbf{A}$ are, respectively, the whitening matrix and the mixing matrix. If a source signal is perfectly extracted, the global vector has only one non-zero element. The calculated global vectors associated with the finally extracted signals u_1, u_2, u_3 and u_4 are given below:

$$\begin{aligned}\mathbf{g}_1 &= \mathbf{w}_1^{\mathrm{T}}\mathbf{VA}\\ &= [-\mathbf{0.48}\ \ -\mathbf{0.77}\ \ 0.05\ \ -0.13\ \ 0.17\ \ \mathbf{0.35}\ \ -0.02],\end{aligned}$$

$$\begin{aligned}\mathbf{g}_2 &= \mathbf{w}_2^{\mathrm{T}}\mathbf{VA}\\ &= [-\mathbf{0.20}\ \ -\mathbf{0.29}\ \ 0.01\ \ 0.06\ \ 0.03\ \ \mathbf{0.93}\ \ 0.01],\end{aligned}$$

$$\begin{aligned}\mathbf{g}_3 &= \mathbf{w}_3^{\mathrm{T}}\mathbf{VA}\\ &= [\mathbf{0.20}\ \ \mathbf{0.82}\ \ -0.01\ \ -0.04\ \ 0.15\ \ \mathbf{0.45}\ \ 0.18],\end{aligned}$$

$$\begin{aligned}\mathbf{g}_4 &= \mathbf{w}_4^{\mathrm{T}}\mathbf{VA}\\ &= [0.11\ \ \mathbf{0.79}\ \ 0.00\ \ -0.08\ \ 0.09\ \ \mathbf{0.51}\ \ \mathbf{0.24}].\end{aligned}$$

The results showed that each extracted signal was a mixture of several original source signals. Further, from the global vectors we could find that the extracted signal $u_1(=\mathbf{w}_1^{\mathrm{T}}\mathbf{VAs})$ was more similar to the second source signal s_2, and so were $u_3(=\mathbf{w}_3^{\mathrm{T}}\mathbf{VAs})$ and $u_4(=\mathbf{w}_4^{\mathrm{T}}\mathbf{VAs})$, while $u_2(=\mathbf{w}_2^{\mathrm{T}}\mathbf{VAs})$ was more similar to the sixth source signal s_6. Thus we calculated the cross-talk indexes of u_1, u_3 and u_4 with the source signal s_2, and the cross-talk index of u_2 with the source signal s_6. The cross-talk indexes of u_1, u_2, u_3 and u_4 were, respectively, 10.3, 15.9, 8.9 and 8.3 dB, indicating bad extraction quality. In addition, we can see that the reference signal c_1 and c_2 had the same occurrence time, but slightly different impulse width. However, the results were greatly different: the extracted signal by using c_1 was more similar to s_2, while the extracted signal by using c_2 was more similar to s_6. We repeated the simulation by different randomly generated mixing matrixes and different simple reference signals, and obtained similar results.

Thus, we can draw the conclusion that the simple reference signals sometimes may not lead to good extraction quality, even may not lead to the desired source signals, especially in the cases where the desired source signals are very weak in mixed signals. Besides, the phase and morphology of the reference signals affect the final extraction results; slightly different phase or morphology may result in different source signals. Therefore, the simple reference signals are not always safe.

5.2. ECG data

Next we used real-world ECG data to verify our approach. The ECG data set used in this experiment was distributed by De Moor [15], which was measured from a pregnant woman by eight electrodes placed at different positions on her body (Fig. 7). Signal x_1–x_5 were the recordings by five electrodes placed on the woman's abdomen. Thus the FECG, respiratory motion artifacts as well as the maternal ECG (MECG) were visible in these recordings. Signal x_6–x_8 were the recordings by three electrodes placed on the woman's thorax. In these thoracic measurements the FECG was invisible because of the distance between the fetus and the chest leads. It has been shown [8,19] that the recordings are the linear mixtures of

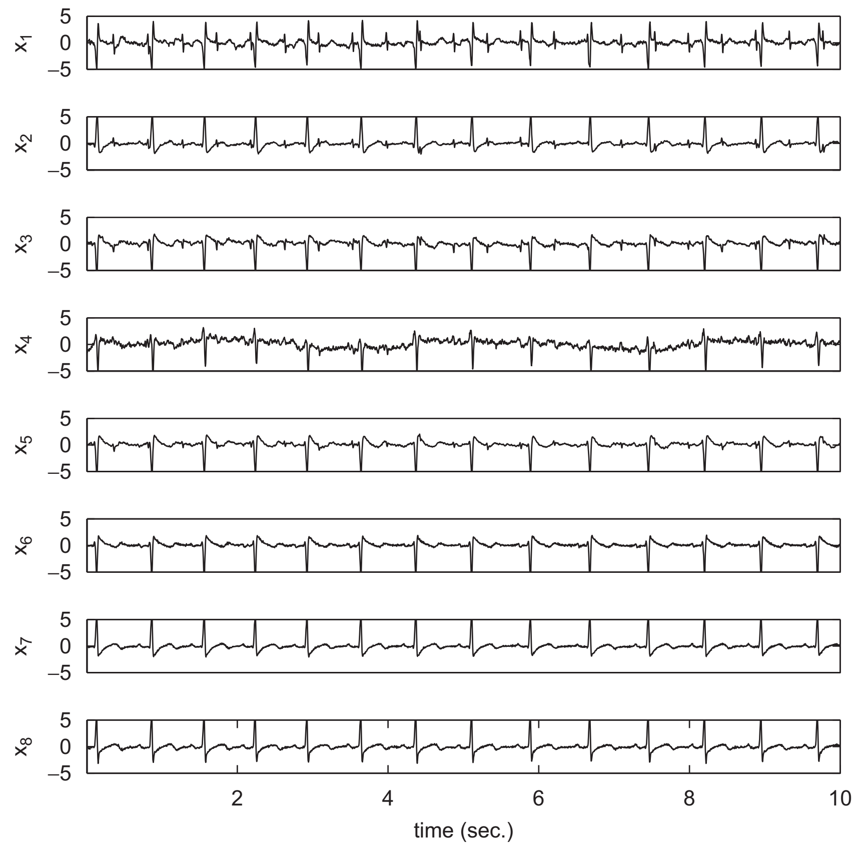

Fig. 7. The ECG data measured from a pregnant woman. Signal x_1–x_5 were the recordings by five electrodes placed on the woman's abdomen. And x_6–x_8 were the recordings by three electrodes placed on the woman's thorax.

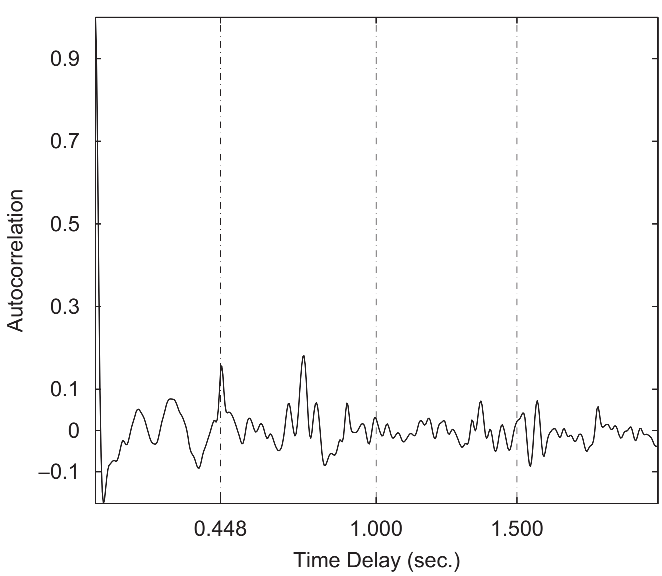

Fig. 8. The autocorrelation of the recording x_1 in Fig. 7.

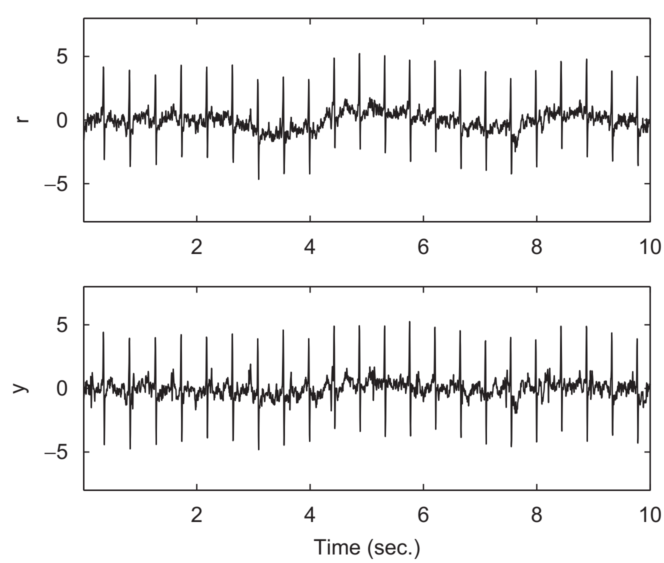

Fig. 9. Extraction results. r was the reference signal obtained by the proposed approach. y was the final extracted signal using the cICA algorithm and the reference signal r.

the FECG and MECG contributions, as well as the respiration noise, and that the separation of MECG and FECG components can be formulated as a BSS problem. The ECG measurements were recorded over 10 s, and were sampled at 250 Hz.

Since the FECG was visible in x_1, we calculated its autocorrelation. By carefully examining the autocorrelation, and using *a priori* knowledge that the fetal heart should strike every 0.5 s or so, we found that a peak obviously lay at 0.448 s (corresponding to 112 sampling periods), and we believed it was just the peak for extracting the FECG (Fig. 8). After whitening the recordings, we ran the algorithm (29) with parameter $\tau_1 = 112$ and parameter $P = 1$. The result is shown in Fig. 9. Next, using the extracted signal as the reference signal and setting $\xi = 0.2$, we ran the cICA algorithm for extracting the desired source signal. The result is also shown in Fig. 9. From the figure we can see that the desired FECG was perfectly extracted, and the respiratory noise was removed, which was contained in the reference signal r.

6. Conclusions

In this paper we propose a novel approach for designing feasible reference signals for the cICA algorithm. Based on the maximization of the averaged delayed autocorrelation of the desired source signal at several time delays, the approach roughly extract the desired source signal. And the extracted signal serves as the reference signal for the cICA algorithm.

It is worth comparing the proposed approach with the conventional reference signal construction methods. In literature the conventional construction methods are designing various simple impulse signals that are matched to desired source signals to some extent. If the desired source signal is strong enough to appear in some mixed signals, then the conventional methods often work well. In contrast, if the desired source signal is very weak so that its any waveform information cannot be obtained from the waveforms of mixed signals, then we have to exploit strong *a priori* knowledge to carefully design the impulse signals [11]. For example, in some fMRI experiments the input stimuli are available, and the desired task-related fMRI responses usually follow the stimuli. Thus, we can directly use the input stimuli as the reference signals [14]. However, the task-related responses do not necessarily synchronize the input stimuli, as Funase et al. [5] reported. In our simulations we have shown that the slight different morphology or occurrence time of the impulses of reference signals may result in different source signals. Therefore, it is not always a safe way to use the input stimuli as the reference signals.

However, the proposed approach in this paper has solved the problem greatly. It does not need to know the occurrence time of such responses and their shapes. In fact it only requires that the desired source signals are temporally correlated signals, which is satisfied in most cases. Therefore the approach is convenient when the desired source signals are very weak or extra *a priori* information (e.g. the input stimuli) is not available. Although the proposed approach needs to first estimate several time delays, it is not a difficult problem since there are numerous algorithms to estimate such time delays. In addition, the proposed approach is non-sensitive to small estimate errors of the time delays [20] and the sensor white noise. Since the approach can obtain a reference signal that is similar to the desired source signal, the task of selecting the value of the threshold ξ of the cICA algorithm becomes easy. It is interesting to notice that the FICAR algorithm [3,5] also needs to construct a reference signal to extract the desired source signal, and our proposed approach can be used for the algorithm as well.

References

[1] A.K. Barros, A. Cichocki, Extraction of specific signals with temporal structure, Neural Comput. 13 (9) (2001) 1995–2003.
[2] A.K. Barros, N. Ohnishi, Heart instantaneous frequency (HIF): an alternative approach to extract heart rate variability, IEEE Trans. Biomed. Eng. 48 (8) (2001) C850–C855.
[3] A.K. Barros, R. Vigário, V. Jousmäki, et al., Extraction of event-related signals from multichannel bioelectrical measurements, IEEE Trans. Biomed. Eng. 47 (5) (2000) 583–588.
[4] A. Cichocki, S. Amari, Adaptive Blind Signal and Image Processing, Wiley, New York, 2002.
[5] A. Funase, Y. Tohru, M. Mouri, et al., Analysis on EEG signals in visually and auditorily guided saccade task by FICAR, in: Proceedings of the 6th International Conference on Independent Component Analysis and Blind Signal Separation (ICA 2006), South Carolina, USA, Lecture Notes in Computer Science, vol. 3889, Springer, Berlin, 2006, pp. 438–445.
[6] L.K. Hansen, F.Å. Nielsen, J. Larsen, Exploring FMRI data for periodic signal components, Artif. Intell. Med. 25 (1) (2002) 35–44.
[7] A. Hyvärinen, J. Karhunen, E. Oja, Independent Component Analysis, Wiley, New York, 2001.
[8] M.G. Jafari, J.A. Chambers, Fetal electrocardiogram extraction by sequential source separation in the wavelet domain, IEEE Trans. Biomed. Eng. 52 (3) (2005) 390–400.
[9] M.G. Jafari, W. Wang, J.A. Chambers, et al., Sequential blind source separation based exclusively on second-order statistics developed for a class of periodic signals, IEEE Trans. Signal Process. 54 (3) (2006) 1028–1040.
[10] C.J. James, O.J. Gibson, Temporally constrained ICA: an application to artifact rejection in electromagnetic brain signal analysis, IEEE Trans. Biomed. Eng. 50 (9) (2003) 1108–1116.
[11] L.K.L. Joshua, J.C. Rajapakse, Extraction of event-related potentials from EEG signals using ICA with reference, in: Proceedings of International Joint Conference on Neural Networks (IJCNN 2005), Montreal, Canada, July 31–August 4, 2005, pp. 2526–2531.
[12] W. Lu, J.C. Rajapakse, Constrained independent component analysis, in: Advances in Neural Information Processing Systems, vol. 13 (NIPS 2000), MIT Press, Cambridge, MA, 2000, pp. 570–576.
[13] W. Lu, J.C. Rajapakse, Approach and applications of constrained ICA, IEEE Trans. Neural Networks 16 (1) (2005) 203–212.
[14] W. Lu, J.C. Rajapakse, ICA with reference, Neurocomputing 69 (16–18) (2006) 2244–2257.
[15] D. De Moor (Ed.), Daisy: Database for the Identification of Systems. Available online at: ⟨http://www.esat.kuleuven.ac.be/sista/daisy⟩.
[16] J.S. Paul, M.R. Reddy, V.J. Kumar, A cepstral transformation technique for dissociation of wide QRS-type ECG signals using DCT, Signal Process. 75 (1999) 29–39.
[17] L.R. Rabiner, On the use of autocorrelation analysis for pitch detection, IEEE Trans. Acoustics Speech Signal Process. ASSP-25 (1) (1977) 24–33.
[18] T. Tolonen, M. Karjalainen, A computationally efficient multipitch analysis model, IEEE Trans. Speech Audio Process. 8 (6) (2000) 708–716.
[19] V. Zarzoso, A.K. Nandi, Noninvasive fetal electrocardiogram extraction blind separation versus adaptive noise cancellation, IEEE Trans. Biomed. Eng. 48 (1) (2001) 12–18.
[20] Z.-L. Zhang, Z. Yi, Robust extraction of specific signals with temporal structure, Neurocomputing 69 (7–9) (2006) 888–893.

[21] Z.-L. Zhang, Z. Yi, Extraction of temporally correlated sources with its application to non-invasive fetal electrocardiogram extraction, Neurocomputing 69 (7–9) (2006) 894–899.
[22] Z.-L. Zhang, L. Zhang, A two-stage based approach for extracting periodic signals, in: Proceedings of the 6th International Conference on Independent Component Analysis and Blind Signal Separation (ICA 2006), South Carolina, USA, Lecture Notes in Computer Science, vol. 3889, Springer, Berlin, 2006, pp. 303–310.
[23] Z.-L. Zhang, L. Zhang, Extraction of temporally correlated signals using a two-stage method with applications in biomedical engineering, IEEE Trans. Biomed. Eng., submitted.

Zhi-Lin Zhang was born in Hunan, China, in 1980. He received his B.S. degree in Automatics in 2002, and his M.S. degree in Signal and Information Processing in 2005, respectively, both from the University of Electronic Science and Technology of China, Chengdu, China. He is currently a visiting student at the Department of Computer Science and Engineering, Shanghai Jiao Tong University, Shanghai, China.

He is a student member of the Chinese Institute of Electronics. His research interests include blind source separation, biomedical signal processing, machine learning and statistical signal processing. He has authored more than 20 research publications in refereed international journals and international conference proceedings.