Robust extraction of specific signals with temporal structure

Zhi-Lin Zhang, Zhang Yi

Computational Intelligence Laboratory, School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 610054, People's Republic of China.

Abstract

There is a trend to develop blind or semi-blind source extraction algorithms based on second-order statistics, due to its low computation load and fast processing speed. An important and primary work is done by Barros and Cichocki, who propose an extraction algorithm based on a time delay. The algorithm is simple and fast, but its performance is not satisfying. The paper extends their work and proposes a robust algorithm based on eigenvalue decomposition of several delayed covariance matrices. It is faster and has better performance, which is confirmed by theoretical analysis and computer simulations.

Key words: Temporal structure; Eigenvalue decomposition; Source extraction; Blind source separation

1 Introduction

Blind source extraction (BSE) [1] is a type of powerful technique that is closely related to blind source separation (BSS) [1,11,12]. The basic task of BSE is estimating part of source signals that are linearly combined in observations. Compared to BSS, BSE has many advantages, and has received wide attention in various fields such as biomedical signal processing [3,4,6] and speech processing [1].

Email address: `zlzhang@uestc.edu.cn, zhangyi@uestc.edu.cn` (Zhi-Lin Zhang, Zhang Yi).

[1] This work was supported by the National Science Foundation of China under Grant 60471055

In BSE, one observes an n-dimentional stochastic signal vector $\mathbf{x}$ that is regarded as the linear transformation of an n-dimensional zero-mean and unit-variance source vector $\mathbf{s}$, i.e., $\mathbf{x} = \mathbf{A}\mathbf{s}$, where $\mathbf{A}$ is an unknown mixing matrix. The goal of BSE is to find a vector $\mathbf{w}$ such that $y = \mathbf{w}^T\mathbf{x} = \mathbf{w}^T\mathbf{A}\mathbf{s}$ is an estimated source signal up to a scalar. To cope with ill-conditioned cases and to make algorithms simpler and faster, a linear transformation called prewhitening is often used to transform the observed signals $\mathbf{x}$ to $\tilde{\mathbf{x}} = \mathbf{V}\mathbf{x}$ such that $E\{\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T\} = \mathbf{I}$, where $\mathbf{V}$ is a prewhitening matrix. For convenience, in the following we assume that $\mathbf{x}$ are the prewhitened observed signals.

Many source extraction algorithms [1] extract a specific signal as the first output, by using some *a priori* information, such as sparseness [2] and high-order statistics [3]. But the algorithms exploiting sparseness or high-order statistics often have relatively high computation load. Thus a trend is to develop second-order statistics based extraction algorithms using *a priori* knowledge about signals' time structure. An important work is done by Barros and Cichocki [6]. They propose a fast and simple algorithm, which requires the precise estimation of an optimal time delay that corresponds to some time structure of the desired source signal. Unfortunately, the algorithm's performance strongly depends on the time delay; small estimation errors of the time delay often lead to poor performance.

In this paper we propose a robust extraction algorithm based on eigenvalue decomposition, furthering the primary work of Barros and Cichocki. Our algorithm is very fast, and its performance is not affected by the estimation errors of the time delay as long as the errors are not large, which is confirmed by theoretical analysis and computer simulations.

2 Proposed algorithm

Assume that the desired source signal s_i is temporally correlated, satisfying the following relations for a specific time delay τ^*:

$$\begin{cases} E\left\{s_i(k)s_i(k-\tau^*)\right\} > 0 \\ E\left\{s_i(k)s_j(k-\tau^*)\right\} = 0 \\ E\left\{s_j(k)s_l(k-\tau^*)\right\} = 0 \quad \forall j \neq i, l \neq i, \end{cases} \tag{1}$$

where k is the time index, and τ^* is an integer delay, which can be positive or negative. Without losing generality, in the following the time delay is assumed to be positive. Note that (1) is very similar to the assumption in [6]. Under the constraint $\|\mathbf{w}\| = 1$, maximizing

$$J(\mathbf{w}) = E\{y(k)y(k-\tau^*)\} = \mathbf{w}^T E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\}\mathbf{w} \tag{2}$$

leads to the desired source signal. Here, $y(k) = \mathbf{w}^T\mathbf{x}(k)$ is the output signal. When $J(\mathbf{w})$ reaches a maxima, $y(k)$ estimates the desired signal s_i up to a scalar. The reason for this proposal is that for the desired source signal, this autocorrelation should have a high value, while for other source signals this value should be very small. For convenience, we assume the desired source signal is periodic. But this does not imply that the proposed algorithm in this paper is limited to the extraction of periodic signals. In fact, other non-periodic signals can also be extracted, providing that they satisfy the assumption (1) and the corresponding time delay can be estimated.

Following the idea of Barros and Cichocki [6] and based on the assumption (1), we can easily obtain the Barros's algorithm:

$$\begin{cases} \mathbf{w}^+ = E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\}\mathbf{w} \\ \mathbf{w} = \mathbf{w}^+/\|\mathbf{w}^+\| \end{cases} \tag{3}$$

If the desired signal is periodic with fundamental period τ_0, then τ^* can be set as $\tau^* = r\tau_0$, where r is a non-zero integer.

Now consider the objective function (2) again. We have

$$\begin{aligned} J(\mathbf{w}) &= \frac{1}{2}J(\mathbf{w}) + \frac{1}{2}J(\mathbf{w})^T \\ &= \frac{1}{2}\mathbf{w}^T E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\}\mathbf{w} + \frac{1}{2}\mathbf{w}^T E\{\mathbf{x}(k-\tau^*)\mathbf{x}(k)^T\}\mathbf{w} \\ &= \frac{1}{2}\mathbf{w}^T\Big(\mathbf{R}_x(\tau^*) + \mathbf{R}_x(\tau^*)^T\Big)\mathbf{w}, \end{aligned} \tag{4}$$

implying that maximization of (2) under the constraint $\|\mathbf{w}\| = 1$ is equivalent to finding the eigenvector corresponding to the maximal eigenvalue of the real symmetric matrix $\mathbf{R} = \mathbf{R}_x(\tau^*)+\mathbf{R}_x(\tau^*)^T$, where $\mathbf{R}_x(\tau^*) = E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\}$. Thus we have the following algorithm:

$$\begin{cases} \mathbf{R}_x(\tau^*) = E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\} \\ \mathbf{w} = EIG\Big(\mathbf{R}_x(\tau^*) + \mathbf{R}_x(\tau^*)^T\Big), \end{cases} \tag{5}$$

where $EIG(\mathbf{T})$ is the operator that calculates the normalized eigenvector corresponding to the maximal eigenvalue of the real symmetric matrix $\mathbf{T}$. Compared to Barros's algorithm (3), the algorithm (5) has faster speed due to efficient eigenvalue decomposition techniques [8].

Some practical issues, however, should be considered. An important issue is the effect of finite samples. Although the signals are mutually uncorrelated, in fact the cross-correlation values of source signals calculated over the finite samples are generally non-zero, due to the fact that the expectation operator $E\{z(k)\}$ is replaced by the mathematical average $\sum_{k=1}^{N} z(k)/N$ (N is the data length). Even the number of available samples is very large, in practice they are often divided into consecutive blocks, because of the processing type of batch algorithms. So in this case the cross-correlation values are still non-zero. Another crucial issue is the estimation errors of the time delay. In many cases the errors cannot be avoided. As we will see later, the limit of available samples and estimation errors greatly influence the performance of the algorithms (3) and (5).

Thus we modify the algorithm (5) to (the reason will be given in Section 3)

$$\begin{cases} \mathbf{R}_x(\tau^*) = E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\} \\ \mathbf{w} = EIG\Big(\sum_{i=1}^{P} \big(\mathbf{R}_x(i\tau^*) + \mathbf{R}_x(i\tau^*)^T\big)\Big), \end{cases} \tag{6}$$

which maximizes the new objective function

$$J(\mathbf{w}) = \sum_{i=1}^{P} E\{y(k)y(k-i\tau^*)\} = \mathbf{w}^T\Big(\sum_{i=1}^{P} E\{\mathbf{x}(k)\mathbf{x}(k-i\tau^*)\}\Big)\mathbf{w} \tag{7}$$

under the constraint $\|\mathbf{w}\| = 1$, where P is a positive integer, and τ^* is the fundamental period of the desired source signal.

3 Theoretical analysis

In this section we consider the effect of finite samples. The issue is primarily discussed from the perspective of high-order statistics by Bermejo [9]. Here we discuss the issue from the perspective of second-order statistics in the case of two source signals.

Denote by $\mathbf{V}$ the prewhitening matrix and by $\mathbf{A}$ the unknown mixing matrix, then $\mathbf{VA}$ is orthogonal. Therefore, the function (2) becomes

$$J(\mathbf{w}) = \mathbf{w}^T E\{(\mathbf{VA})\mathbf{s}(k)\mathbf{s}(k-\tau^*)^T(\mathbf{VA})^T\}\mathbf{w} = \mathbf{q}^T\mathbf{R}_s(\tau^*)\mathbf{q}, \tag{8}$$

where $\mathbf{q} = \mathbf{w}^T\mathbf{V}\mathbf{A}$ and $\mathbf{R}_s(\tau^*) = E\{\mathbf{s}(k)\mathbf{s}(k-\tau^*)^T\}$. Thus maximizing (2) is equivalent to maximizing (8) under the constraint $\|\mathbf{q}\|^2 = 1$. Due to the effect of finite samples mentioned above, $\mathbf{R}_s(\tau^*) \neq \mathbf{I}$.

In the case of two source signals, maximization of (8) can be expressed as maximization of

$$J(q_1, q_2) = aq_1^2 + bq_2^2 + cq_1q_2, \tag{9}$$

under the constraint $q_1^2 + q_2^2 = 1$, where $\mathbf{q} = [q_1, q_2]^T$, $a = E\{s_1(k)s_1(k-\tau^*)\}$, $b = E\{s_2(k)s_2(k-\tau^*)\}$, and $c = E\{s_1(k)s_2(k-\tau^*)\} + E\{s_2(k)s_1(k-\tau^*)\}$. s_1 is the desired signal with period τ^*. Generally we have $a > 0$ and $a > b$.

If $c = 0$, implying the calculated cross-correlation value of s_1 and s_2 is zero (this is the ideal case), the optimal solution to (9) is $q_1 = \pm 1, q_2 = 0$, and the extracted signal $y(k) = \mathbf{w}^T\mathbf{x}(k) = \mathbf{q}^T\mathbf{s}(k) = q_1s_1(k) + q_2s_2(k) = \pm s_1(k)$. Obviously, in this case the desired source signal is perfectly extracted.

But in practice, $c \neq 0$. For $c > 0$, the solution to (9) is given by

$$\begin{cases} q_1 = \pm\frac{h+\sqrt{h^2+1}}{\sqrt{1+(h+\sqrt{h^2+1})^2}} \\ q_2 = \pm\frac{1}{\sqrt{1+(h+\sqrt{h^2+1})^2}} \end{cases} \tag{10}$$

where $h = (a-b)/c$. For $c < 0$, the solution is given by

$$\begin{cases} q_1 = \pm\frac{h-\sqrt{h^2+1}}{\sqrt{1+(h-\sqrt{h^2+1})^2}} \\ q_2 = \pm\frac{1}{\sqrt{1+(h-\sqrt{h^2+1})^2}} \end{cases} \tag{11}$$

Since $\mathbf{q} = \mathbf{w}^T\mathbf{V}\mathbf{A}$ is a global vector, a good extraction performance measure is given by

$$PI_1 = \frac{1}{N-1}\left(\sum_{i=1}^{N}\frac{q_i^2}{\max_i q_i^2} - 1\right), \tag{12}$$

whose value lies in $[0, 1]$ for any vector $\mathbf{q} = [q_1, \cdots, q_N]^T$. The smaller PI_1 is, the better the extraction performance is. In the case of two source signals, without losing generality, we consider the case of $c > 0$, in which

$$PI_1 = \left(\frac{a-b}{c} + \sqrt{\left(\frac{a-b}{c}\right)^2 + 1}\right)^{-2}. \tag{13}$$

So, in order to improve the extraction performance, we should increase the value of a, and decrease the values of b and $|c|$ (incorporating results in the case of $c < 0$).

Now consider the modified objective function (7), which is equivalent to

$$\tilde{J}(\mathbf{w}) = \frac{1}{P}\mathbf{q}^T\Big(\sum_{i=1}^{P} E\{\mathbf{s}(k)\mathbf{s}(k - i\tau^*)\}\Big)\mathbf{q}, \tag{14}$$

whose $\tilde{a} = \sum_{i=1}^{P} E\{s_1(k)s_1(k - i\tau^*)\}/P$, $\tilde{b} = \sum_{i=1}^{P} E\{s_2(k)s_2(k - i\tau^*)\}/P$ and $\tilde{c} = \sum_{i=1}^{P} E\{s_1(k)s_2(k - i\tau^*) + s_2(k)s_1(k - i\tau^*)\}/P$. Remember that τ^* is the fundamental period of s_1. So, compared to a, b and c in (9), $\tilde{b}$ and $|\tilde{c}|$ generally tend to decrease rapidly with P increasing, while $\tilde{a}$ tends to be invariant (or tends to decrease with relatively slow speed). As a result, the PI_1 of the proposed algorithm (6) tends to be smaller than that of the algorithm (5), implying the extraction quality is improved, which will be confirmed by simulations below.

4 Computer simulations

4.1 Effect of the non-zero cross-correlation values of source signals

In this simulation we select two source signals with zero mean and unit variance (one is a fetal electrocardiogram (FECG), the other is a noise signal), shown in Fig.1. The desired source signal is the FECG, whose fundamental period is 112. The source signals are randomly mixed, followed by the prewhitening. We set τ^* as 112, 224, 336, 448, respectively. For each value, we run the algorithm (5) and the Barros's algorithm (3). The simulation is repeated 100 times, and their averaged performance indices are calculated and shown in Table 1.

It is clear to see that with the decline of the values of b and $|c|$, the performance of (5) is improved. Furthermore, one can find that for both algorithms different time delay leads to different performance. Some delays (for example, $\tau^* = 224$) result in good extraction performance, while some delays result in bad performance. In addition, for the same τ^*, the performance of (5) is better than that of the Barros's algorithm.

Then, we set $\tau^* = 112$ and $P = 1, 2, 3, 4$, respectively, for the algorithm (6). The theoretical and the averaged performance indices are shown in Table 2. Comparing Table 1 with Table 2, we can see that the algorithm (6) provides satisfying and stable performance, in contrast to the Barros's algorithm (3)

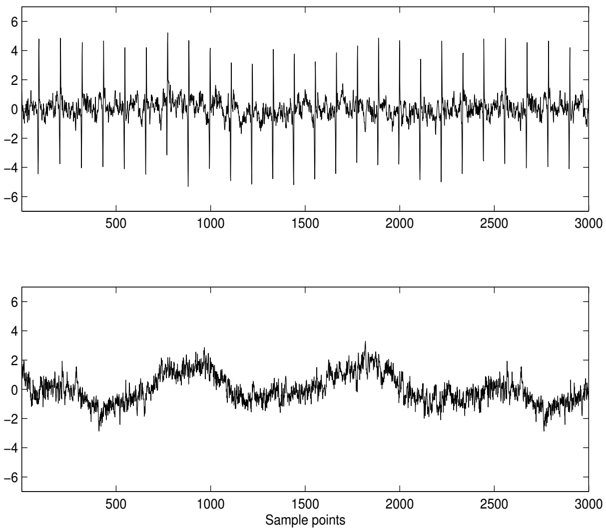

Fig. 1. Two source signals. The top is an FECG ; the bottom is a noise signal.

Table 1
The relationship between the cross-correlation matrix $\mathbf{R}_x(\tau^*)$ and the averaged performance indices over 100 independent trials in Section 4.1. In addition, the corresponding theoretical performance index is given as well. PI_1, $\widehat{PI}_1$ and $\widetilde{PI}_1$ are, respectively, the theoretical performance index, the averaged one of (5), and the averaged one of (3). Note that here we multiply the $\mathbf{R}_s(\tau^*)$ by a scalar that makes the element in the first row and the first column of $\mathbf{R}_s(\tau^*)$ unity.

τ^*	112	224	336	448
a	1.000	1.000	1.000	1.000
b	0.902	0.188	-1.507	-5.426
c	0.200	-0.045	-0.781	-2.076
PI_1	0.390	0.000	0.023	0.025
$\widehat{PI_1}$	0.447	0.000	0.021	0.024
$\widetilde{PI_1}$	0.450	0.064	0.100	0.052

and the algorithm (5), whose performance is greatly affected by the selection of τ^*. Through lots of experiments we find a good result will be obtained when $\tau^* P < N/5$, where N is the length of each data block.

4.2 Effect of the estimation errors of the time delay

The estimation of the time delay is crucial to the algorithms (3), (5) and (6). For estimating it, there are several methods, such as calculating the autocorrelation of the mixtures [6], and the heart instantaneous frequency (HIF)

Table 2

The relationship between the cross-correlation matrix $\mathbf{R}_x(\tau^*)$ and the averaged performance index over 100 independent trials in Section 4.1. In addition, the corresponding theoretical performance index is given as well. PI_1 and $\widehat{PI}_1$ are, respectively, the theoretical performance index and the averaged one of (6). Note that here we multiply the $\mathbf{R}_s(\tau^*)$ by a scalar that makes the element in the first row and the first column of $\mathbf{R}_s(\tau^*)$ unity.

P	1	2	3	4
a	1.000	1.000	1.000	1.000
b	0.902	0.635	0.274	-0.079
c	0.200	0.108	-0.041	-0.167
PI_1	0.390	0.021	0.000	0.005
$\widehat{PI}_1$	0.447	0.035	0.000	0.004

estimation technique [7]. In many applications the task of estimating τ^* is not difficult, but estimation errors are inevitable. For many extraction algorithms that use one time delay, such as the Barros's algorithm (3), their performance is greatly affected by the estimation errors of the time delay. We will see, however, that the new algorithm (6) is relatively robust to the errors, if some techniques are taken.

To verify this, we carry out another simulation using the same source signals (Fig.1). Assume we only know the value of the fundamental period of the FECG lies in the range [109, 115] (the case often appears when the estimation errors cannot be avoided). We set $\tau^* = 109, 110, 111, 112, 113, 114, 115$, respectively, for the Barros's algorithm (3). The performance is measured by

$$PI_2 = -10E\{lg(s(k) - \tilde{s}(k))^2\}, \quad (dB) \tag{15}$$

where $s(k)$ is the desired source signal (i.e., the FECG), and $\tilde{s}(k)$ is the extracted signal (both of them are normalized to have zero mean and unit variance). The higher PI_2 is, the better the performance is. We have found that only for $\tau^* = 112$ (i.e., the estimated time delay is accurately equal to the period of the FECG), the Barros's algorithm can extract the FECG with the averaged performance index $\widehat{PI}_2 = 6.2$ dB (averaged over 100 independent trials), while for other values it fails.

Similarly, we run the proposed algorithm (6). Since we do not know the true value of the fundamental period, we cannot directly perform it. However, we can modify it as follows,

$$
\begin{aligned}
\mathbf{w} = EIG\bigg(\sum_{i=1}^{3}\Big(&\mathbf{R}_x(109i) + \mathbf{R}_x(109i)^T + \mathbf{R}_x(110i) + \mathbf{R}_x(110i)^T \\
&+ \mathbf{R}_x(111i) + \mathbf{R}_x(111i)^T + \mathbf{R}_x(112i) + \mathbf{R}_x(112i)^T + \mathbf{R}_x(113i) \\
&+ \mathbf{R}_x(113i)^T + \mathbf{R}_x(114i) + \mathbf{R}_x(114i)^T + \mathbf{R}_x(115i) + \mathbf{R}_x(115i)^T\Big)\bigg). \quad (16)
\end{aligned}
$$

The technique is feasible as long as the estimation errors are not too large (generally the condition is satisfied in practice). The averaged performance index over 100 independent trials is 21.5 dB. The results show that even if we cannot obtain the accurate fundamental period of the desired source signal, we still can get satisfying results by the new algorithm (6).

4.3 Experiment on real-world ECG data

Now we test our algorithm for the well-known ECG data (Fig.2) measured from a pregnant woman and distributed by De Moor [10]. Sampling rate is 250 Hz. From it one can see the strong and slow heart beating of the mother and the weak and fast one of the fetus. The task is to extract the FECG. The estimated fundamental period is 112 sampling period [6]. Then three algorithms are performed. One is the famous second-order statistics based SOBI algorithm [2] [11], for which the number of delayed covariance matrices is 150. Another algorithm is the Barros's algorithm (3), for which $\tau^* = 112$. The third algorithm is our algorithm (6), for which $P = 3$ and $\tau^* = 112$. The results are shown in Fig.3 (Note that SOBI algorithm is a blind source separation algorithm, and it separates all the source signals at one time). Obviously, the extracted FECG by our algorithm (6) is the clearest, while the one by Barros's algorithm is mixed by noise. The result by the SOBI algorithm is the worst, which is mixed by the heart beating of the mother and lots of noise.

5 Conclusions

In this paper we propose a fast and robust source extraction algorithm based on eigenvalue decomposition of several delayed covariance matrices. It provides stable and good performance, confirmed by theoretical analysis and simulations. It is clear to see that the proposed algorithm relates to the principle component analysis (PCA). Thus many results [1,5,8] on PCA can be used to improve the algorithm, which is our future work.

[2] The Matlab files come from http://www.bsp.brain.riken.go.jp/ICALAB/

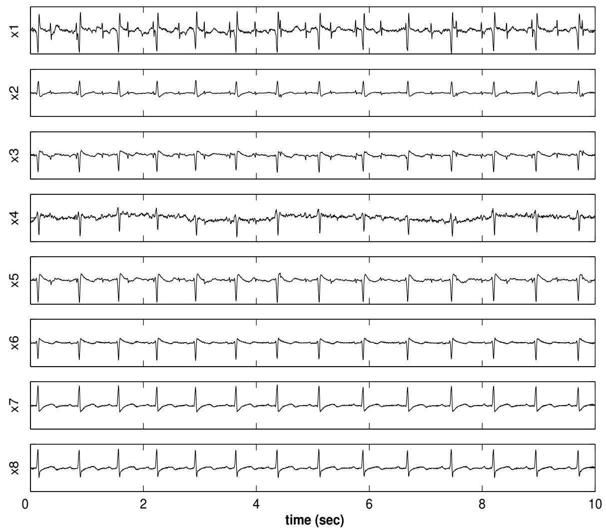

Fig. 2. ECG data from a pregnant woman

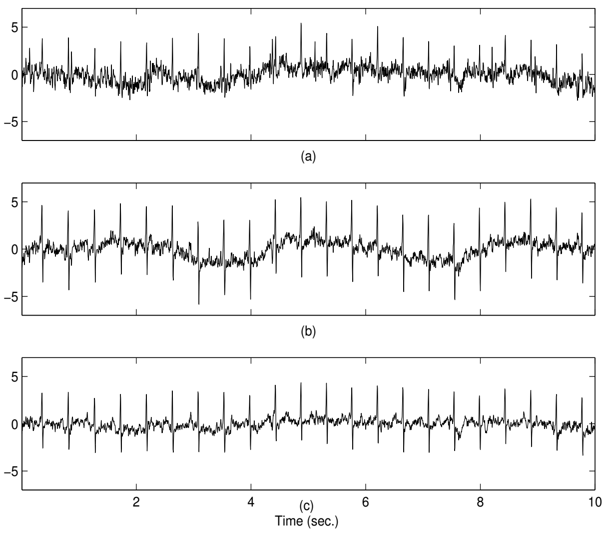

Fig. 3. Extracted FECG. (a) is separated by SOBI algorithm [11]; (b) is by Barros's algorithm (3); (c) is by our algorithm (6).

Note that developing algorithms by eigenvalue decomposition is one of the trends in BSE and BSS. To achieve satisfying results, the previous algorithms [13] use lots of delayed covariance matrices of observations, say, 500 matrices, thus reducing algorithms' efficiency. The situation also appears in the joint diagonalization based algorithms [11,12]. This paper, however, indicates that it is possible to greatly reduce the number of used matrices by exploiting *a priori* information. Another work on this topic refers to [4,14].

References

[1] A. Cichocki, S. Amari, Adaptive blind signal and image processing, New York: John Wiley & Sons, 2002

[2] M. Zibulevsky, Y.Y. Zeevi, Extraction of a source from multichannel data using sparse decomposition, Neurocomputing 49 (1-4) (December 2002) 163-173

[3] Z.-L. Zhang, Z. Yi, Extraction of a source signal whose kurtosis value lies in a specific range, to appear in Neurocomputing

[4] Z.-L. Zhang, Z. Yi, Extraction of temporally correlated sources with its application to non-invasive fetal electrocardiogram extraction, to appear in Neurocomputing

[5] Z. Yi, et al., Neural network based approach for computing eigenvectors and eigenvalues of symmetric matrix, Computer Mathematics with Applications 47 (8-9) (April/May, 2004) 1155-1164

[6] A.K. Barros, A. Cichocki, Extraction of specific signals with temporal structure, Neural Computation 13 (9) (September 2001) 1995-2003

[7] A.K. Barros, N. Ohnishi, Heart instantaneous frequency (HIF): an alternative approach to extract heart rate variability, IEEE Trans. On Biomedical Engineering 48 (8) (August 2001) 850-855

[8] G.H. Golub, C.F. Van Loan, Matrix Computation, 3rd ed. Baltimore: The John Hopkins University Press, 1996

[9] S. Bermejo, Finite sample effects in higher order statistics contrast functions for sequential blind source separation, IEEE Signal Processing Letters 12 (6) (June 2005) 481-484

[10] D. De Moor(Ed.), Daisy: Database for the identification of systems. Available online at: http://www.esat.kuleuven.ac.be/sista/daisy

[11] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso, et al., A blind source separation technique using second-order statistics, IEEE Trans. On Signal Processing 45 (2) (February 1997) 434-444

[12] J.-F. Cardoso, A. Souloumiac, Blind beamforming for non-Gaussian signals, IEE Proceedings-F 140 (6) (December 1993) 362-370

[13] L. Parra, P. Sajda, Blind source separation via generalized eigenvalue decomposition, Journal of Machine Learning Research 4 (7-8) (October 2003) 1261-1269

[14] M.G. Jafari, et al., Sequential blind source separation based exclusively on second order statistics developed for a class of periodic signals, to appear in IEEE Trans. On Signal Processing