Master the Usage of T-MSBL in 3 Minutes

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Version: 1.2
May 24, 2011

0. What is T-MSBL?

The algorithm was developed for the multiple measurement vector (MMV) model, i.e.

$$Y = \Phi X + V,$$

where Phi is the known dictionary matrix. T-MSBL exploits the correlation exists in each nonzero row of X (namely, temporal correlation). For details, see Ref.[1].

1. When Shall I use T-MSBL?

✓ MMV model with temporally correlated source vectors

Of course, the main goal of T-MSBL is to solve this problem. However, when temporal correlation is very small or there is no temporal correlation, T-MSBL still has good performance due to its relatively robust learning rule of the regularization parameter *lambda*. Besides, T-MSBL and T-SBL can deal with the difficult cases when X has many nonzero rows, and the cases when the under-determinacy ratio is very high.

✓ Single measurement vector (SMV) model

T-MSBL can be used for SMV models. In this case, it is essentially the same to the basic SBL algorithm. But in most cases it has better performance due to the learning rule of the regularization parameter *lambda*.

✓ Time-varying sparsity model, or dynamic compressed sensing model

The support of each source vector X(:,i) is slowly time-varying, we can use the concatenate of MMV models to approximate this scenario. See Ref [2] and the demo file: **demo_time_varying.m**.

✓ MMV model with identical source vectors

This model sometimes appears in the theoretical work or is combined with Kalman filtering model (random walk model). See the demo file **demo_identicalVector.m** for the usage.

But note: for different models, some input arguments should be adjusted (see demo files and the case study below).

2. The most convenient way to use T-MSBL

- You can simply use the command to call T-MSBL for most cases (MMV model with temporal correlated source vectors, when SNR varies from 6 dB to 22 dB)

```
X_est = TMSBL(Phi, Y);
```

- In most cases you can roughly know or estimate the noise level (of course you need not to know the exact value). For example, when solving practical problems, you may say: "the noise is large/mild/small". In this case, you can choose these commands for a better performance:
  - When noise is large (SNR <=6 dB)

```
X_est = TMSBL(Phi, Y, 'noise', 'large')
```

  - When noise is mild (7 dB <= SNR <=22 dB)

```
X_est = TMSBL(Phi, Y, 'noise', 'mild')
```

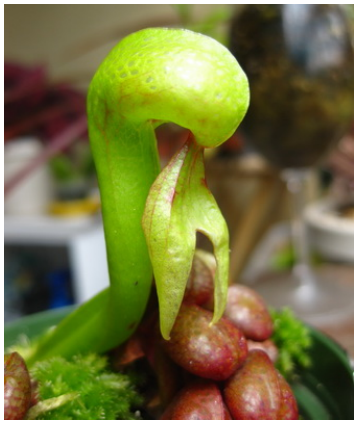
  - When noise is small (SNR >23 dB)

```
X_est = TMSBL(Phi, Y, 'noise', 'small')
```

  - When no noise

```
X_est = TMSBL(Phi, Y, 'noise', 'no')
```

Note: Each of the above five commands uses a set of pre-defined parameter values. See the TMSBL code for details about these pre-defined values. Since I don't know your specific problems, **these pre-defined values are only sub-optimal. When the L2-norms of rows in X are too large or too small (far from 1), you may want to tune these parameters for better performance**.



Now you've spent 3 minutes. As I said, you may want to select your values for the parameters of TMSBL for better performance. Then please continue to read the following if you want to spend another 15 minutes.

First, let's see the full command to call TMSBL:

```
[X, gamma_ind, gamma_est, count, B_est] = TMSBL(Phi, Y, 'Prune_Gamma', 1e-5, 'Learn_Lambda',1, 'Enhance_Lambda',1,
'Matrix_Reg', 2*eye(L), 'lambda', 0.015, 'MAX_ITERS', 500, 'Fix_B', eye(L), 'EPSILON', 1e-8, 'PRINT', 1);
```

You can see the m-file for descriptions about these parameters. When tuning these parameters, you only need to pay attention to four input parameters, indicated by red color. Fortunately, tuning them is not a tough job. See the following golden rules.

3. Four golden rules when you tune parameters

- **'Prune_Gamma': Larger noise level means larger threshold to prune out small γi**

  Theoretically, when the i-th row of X is zero, γi should also be zero. However, due to noise, γi won't be zero. Larger noise means larger γi. So if your threshold is too small, there would be many nonzero γi that should be pruned out but are not. So you need to adjust the threshold according to the noise level. But you need not to know exactly the noise level. Empirically,

  - At strongly noisy cases: e.g. set 'prune_gamma' = 1e-2 or 1e-3;
  - At most noisy cases: e.g. set 'prune_gamma' = 1e-3 or 1e-4.
  - At noiseless cases: set it to any small number (e.g. 1e-5) but not too small (for quickly pruning out γi)

  Warning: Too large or too small 'prune_gamma' value could lead to poorer performance.

- **'Learn_Lambda': In most noisy cases use the λ learning rule (i.e. set 'Learn_Lambda'=1); in strongly noisy case (e.g. SNR < 7dB), the λ rule may not be robust; in noiseless case, you need not to use the λ rule.**

- **'Enhance_Lambda': In mildly/strongly noisy case (e.g. SNR <= 22 dB) use the modified λ learning rule (set 'Enhance_Lambda' = 1).**

- **'Matrix_Reg': In mildly/strongly noisy cases (<22dB), regulate the estimated covariance matrix B (set 'Matrix_Reg' = c*eye(L)).**

  Note that in the development of T-MSBL we used an approximation. The approximation is exact only when there is no noise or there is no temporal correlation. This approximation requires regulating the estimated covariance matrix B in each iteration in noisy cases. The higher noisy level requires stronger regularization, i.e. larger c in c*eye(L). For example,

  - In mildly noisy case (7-22dB): c = 2
  - In strongly noisy case (<7 dB): c = 4

4. Some Warnings

- **The default value of 'MAX_ITERS' is 2000. In some cases (e.g. large-scale data set, strongly noisy cases), 2000 iterations may not be enough.**

- **The argument 'MAX_ITERS', 'EPSILON', and 'PRUNE_GAMMA' all determine the speed. For fast speed, you must tune these parameters for a good tradeoff between performance and speed.**

5. Case Study

Note: You can set the input argument 'print' = 1, so you can check the parameters you used.

Case Study 1: Noiseless Cases

- The default value of 'prune_gamma' is 1e-3. In noiseless cases, you can use smaller values, e.g. 1e-5.
- No need to use the λ learning rule ('learn_lambda' = 0). Freeze 'lambda' to a very small value, eg. 1e-10 ('lambda' = 1e-10)
- No need to regulate B, so set 'matrix_reg'= zeros(L) (note: **the default value is 2*eye(L)**).

[Example] X_est = TMSBL (Phi, Y, 'prune_gamma',1e-4, 'lambda',1e-10, 'learn_lambda', 0, 'matrix_reg', zeros(L));
[Note 1] For the SMV model, the above commands are the same.
[Note 2] You can use smaller 'epsilon' (its default value is 1e-8), e.g. 'epsilon'=1e-10.

Case Study 2: Small Noise Level (>22 dB)

- Use the default value for 'prune_gamma' (1e-3) or set a smaller value (e.g. 1e-4, 1e-5)
- Use the λ learning rule ('learn_lambda' = 1), but not use the enhance strategy ('enhance_lambda' = 0).
- Randomly guess an initial value for 'lambda' or use the default value (1e-3)
- No need to regulate B since the noise is very small ('matrix_reg'= zeros(L))

[Example] X_est = TMSBL (Phi, Y, 'prune_gamma' ,1e-4, 'learn_lambda', 1, 'enhance_lambda', 0, 'matrix_reg', zeros(L));

Case Study 3: Mild Noise Levels (6-22 dB)

- The default values in TMSBL aim to this scenario
- Use the default value for 'prune_gamma' (1e-3) or set a smaller one (e.g 1e-4)
- Use the λ learning rule ('learn_lambda' = 1), AND use the enhance strategy ('enhance_lambda' = 1).
- Randomly guess an initial value for 'lambda' or use the default value (1e-3)
- Regulate the estimate of B: 'matrix_reg'= c * eye(L), where c can be chosen from 1 to 3

[Example] X_est = TMSBL (Phi, Y, 'learn_lambda', 1, 'enhance_lambda', 1, 'matrix_reg', 2.5 *eye(L));

Case Study 3: Large Noise Levels (e.g. SNR < 6 dB)

- Use the default value for 'prune_gamma' (1e-3) or a larger one (e.g. 1e-2)
- I do not suggest using the λ learning rules. But if you use them, the algorithm may still obtain better performance than many other algorithms; thus set 'learn_lambda' = 1, 'enhance_lambda' = 1
- If you do not use the λ learning rule, then estimate a good value for λ and freeze it. In many cases, a good value is about **2-4** times of the true noise variance (note: for another algorithm, T-SBL, a good value is around the true noise variance).
- Use a stronger regularization to B, e.g. : 'matrix_reg'= c * eye(L), where c > 3

[Example] (Assume the estimated noise variance is 0.02)
X_est = TMSBL (Phi, Y, 'prune_gamma' , 1e-2, 'lambda', 0.06, 'learn_lambda', 0, 'matrix_reg', 4*eye(L));

Case Study 4: Source Vectors are Identical (i.e. Temporal Correlation is exactly 1)

- Basically, T-MSBL is suitable for the cases when the temporal correlation is large **but not exactly 1**. When the correlation is 1, the learning of B could have problems. However, you can

freeze B to an identical matrix ('Fix_B' = eye(L)). Although the resulting algorithm is basically the same as MSBL, the performance is still better than MSBL due to the superiority of the λ learning rule in T-MSBL.

- When there is no temporal correlation **AND** the noisy is very large, you can also set 'Fix_B' = eye(L) and other suitable input argument values to run T-MSBL.
- See the demo file **demo_identicalVector.m**

[Example] (Assume a mildly noisy case)

```
X_est = TMSBL ( Phi, Y,  'noise', 'mild',  'fix_B', eye(L) );            % use the default values in the code
X_est = TMSBL ( Phi, Y,  'prune_gamma', 1e-4,  'learn_lambda', 1,  'enhance_lambda', 1,  'fix_B', eye(L) );
```

Case Study 5: Time-Varying Sparsity Model (Dynamic Compressed Sensing)

- Set a suitable sliding time-window (i.e. choosing L, which determines your input argument Y), in which the data can be approximately modeled as an MMV model.
- Then according to previous examples, choose suitable input arguments.
- See Ref [2] and the demo file: **demo_time_varying.m**.

References

[1] Zhilin Zhang, Bhaskar D. Rao, Sparse Signal Recovery with Temporally Correlated Source Vectors Using Sparse Bayesian Learning, IEEE Journal of Selected Topics in Signal Processing, Special Issue on Adaptive Sparse Representation of Data and Applications in Signal and Image Processing, 2011

[2] Zhilin Zhang, Bhaskar D. Rao, Exploiting Correlation in Sparse Signal Recovery Problems: Multiple Measurement Vectors, Block Sparsity, and Time-Varying Sparsity, [online] http://arxiv.org/abs/1105.0725

Feel free to contact me if you have any questions (send email to z4zhang@ucsd.edu)