Comparison of Sparse Signal Recovery Algorithms with Highly Coherent Dictionary Matrices: The Advantage of T-MSBL

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***Abstract*— This note reports a comparison result of twelve typical sparse signal recovery algorithms when the dictionary matrix is highly coherent. The dictionary matrix is a simplified real-world lead-field matrix in EEG source localization. The comparison result shows the superiority of T-MSBL [1] in this case.**

***Index Terms*— Sparse Signal Recovery, Compressed Sensing, Sparse Bayesian Learning (SBL), T-MSBL**

I. Model used in the comparison

The basic model of sparse signal recovery is

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{v}, \tag{1}$$

where $\mathbf{\Phi} \in \mathbb{R}^{N \times M}(N \ll M)$ is a known dictionary matrix, $\mathbf{y} \in \mathbb{R}^{N \times 1}$ is an available measurement vector, and $\mathbf{v}$ is an unknown noise vector. The task is to estimate the source vector $\mathbf{x}$, which has only K nonzero elements (K is a very small number). Sparse signal recovery has wide applications in source localization, radar detection, target tracking, and power spectrum estimation, etc. In these applications, the dictionary matrix is highly coherent.

In this note we report an experiment result, in which twelve typical algorithms were compared when the dictionary matrix $\mathbf{\Phi}$ was highly correlated. The dictionary matrix was a simplified real-world lead-field matrix used in EEG source localization (see Fig.1), whose size was 80×390. The maximum coherence of the columns of $\mathbf{\Phi}$ was 0.9983.

The twelve algorithms were:

- T-MSBL [1], downloaded at `http://dsp.ucsd.edu/~zhilin/TMSBL.html`. Although T-MSBL is developed for the multiple measurement vector model [1], it can also be used in this basic model (1);
- EM-SBL [2], downloaded at `http://dsp.ucsd.edu/~zhilin/Software.html`;
- ExCov [3], downloaded at `http://home.eng.iastate.edu/~ald/ExCoV.htm`;
- CoSaMP [4], downloaded at `http://igorcarron.googlepages.com/cosamp.m`;
- Subspace Pursuit [5], downloaded at `http://igorcarron.googlepages.com/CSRec_SP.m`;
- Approximate Message Passing (AMP) [6], downloaded at `http://documents.epfl.ch/users/k/ka/kamilov/www/ApproximateMessagePassing_v1.tar.gz`;
- Bayesian Compressive Sensing (BCS) [7], downloaded at `http://www.ece.duke.edu/~shji/code/bcs_ver0.1.zip`;
- Magic ℓ_1 [8], downloaded at `http://users.ece.gatech.edu/~justin/l1magic/`;
- Hard Thresholding Pursuit (HTP) [9], downloaded at `http://www.math.drexel.edu/~foucart/HTP.zip`;
- Fast Bayesian Matching Pursuit (FBMP) [10], downloaded at `http://www2.ece.ohio-state.edu/~zinielj/fbmp/download.html`;
- FOCUSS [11], downloaded at `http://dsp.ucsd.edu/~zhilin/Software.html`;
- Smooth ℓ_0 [12], downloaded at `http://ee.sharif.ir/~SLzero/`.

The experiment was repeated 1000 trials. In each trial, the number of nonzero elements in the source vector $\mathbf{x}$ was 3, i.e. $K = 3$. These nonzero elements had the unit amplitude. Their indexes in $\mathbf{x}$ were randomly chosen. The noise vector $\mathbf{v}$ was generated as a Gaussian vector such that the SNR was 25dB. The SNR is defined as $\mathrm{SNR(dB)} \triangleq 20\log_{10}(\|\mathbf{\Phi}\mathbf{x}\|_2/\|\mathbf{v}\|_2)$.

We used two performance measures. One was the *Failure Rate* defined in [13], which indicated the percentage of failed trials in the total trials. A failed trial was recognized if the indexes of estimated $\widehat{\mathbf{x}}$ with the K largest amplitude were not the same as the true indexes. Another measurement was the *mean square error* (MSE), defined by $\|\widehat{\mathbf{x}} - \mathbf{x}\|_2^2/\|\mathbf{x}\|_2^2$.

Note that some of the algorithms needed to know some a priori information, and we fed these algorithms with the required a priori information. Details are given in the following list:

- T-MSBL: did not require any a priori information
- EM-SBL: did not require any a priori information
- ExCov: did not require any a priori information
- CoSaMP: fed with the number of nonzero elements
- Subspace Pursuit: fed with the number of nonzero elements
- AMP: did not require any a priori information
- BCS: did not require any a priori information
- Magic ℓ_1: needed to know the SNR to calculate the regularization parameter
- FBMP: fed with the true SNR value, and the number of nonzero elements (used to calculate the activity probabil-

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ity of elements)

- FOCUSS: fed with the true SNR value
- HTP: **noise was removed**, since it can only be used in noiseless cases; in the noisy case it completely failed
- Smooth ℓ_0: **noise was removed**, since it can only be used in noiseless cases; in the noisy case it completely failed

The comparison results are shown in Fig.2 (measured by the Failure Rate) and Fig.3 (measured by the MSE). We can clearly see T-MSBL has the best performance in both measurement indexes. It is worth emphasizing that T-MSBL does not require any a priori information; all the parameters of the algorithm (such as the regularization parameter) are automatically estimated.

All the codes and demos can be downloaded at `http://dsp.ucsd.edu/~zhilin/papers/Experiment.rar`.

In many applications such as neuroelectromagnetic source localization, Direction-of-Arrival estimation, radar detection, under-water sonar processing, power spectrum estimation, the ability of algorithms to handle the cases when dictionary matrices are highly coherent is very important (especially in the presence of noise). The simple experiment shows the superiority of T-MSBL for these tasks.

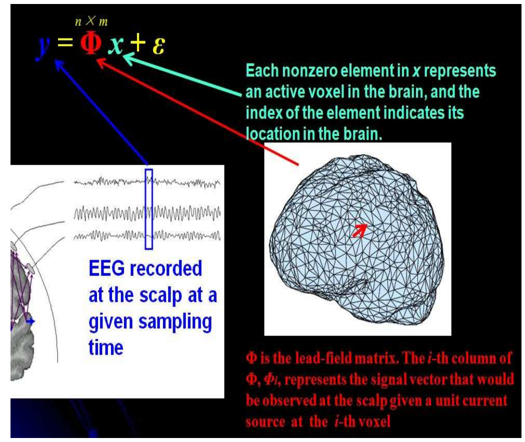

Fig. 1. EEG source localization.

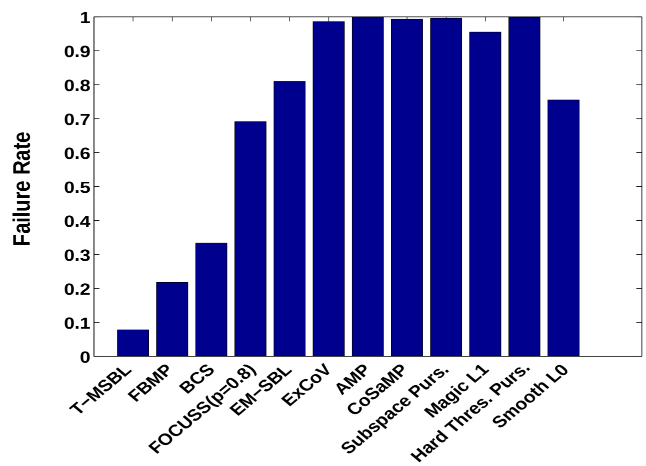

Fig. 2. Algorithm comparison in terms of the Failure Rate.

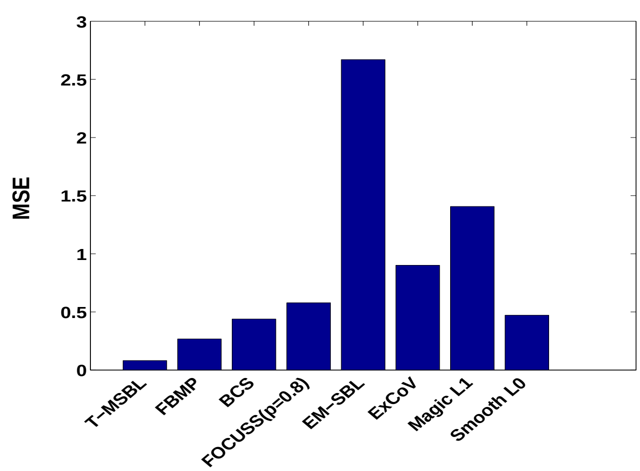

Fig. 3. Algorithm comparison in terms of MSE. Here we only show the MSE's of 8 algorithms, since other algorithms completely failed as shown in Fig.2.

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