Bootstrap statistics

Robert Oostenveld

r.oostenveld@fcdonders.ru.nl

Arno Delorme *(SCCN, San Diego, USA)*

Eric Maris *(NICI, Nijmegen, The Netherlands)*

Alternative title:

Parametric and non-parametric methods for statistical inference: bootstrapping and randomization

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Outline

- Background on statistical inference
- Motivation for bootstrapping and randomization
- Theory behind randomization
- Bootstrapping and randomization applied in EEGLAB

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What is statistics about?

- Statistics is not about the truth
- Statistics is about dealing with uncertainty
  - Decision making under uncertainty (inferential statistics, statistical testing)
  - Data analysis: methods that reveal patterns in the data which cannot be identified by eyeballing

Statistical decision making

- Based on rational principles
- You make the decision…
  - … that convinces ***you***
  - … that convinces the reviewer
  - … that convinces the audience
- What are the reasons for not being certain?

Implications for Neuroscience

- Inferential statistics is about principles of rational decision making and *not* about the nature of the biological system
- Therefore it must be boring for a neuroscientist
- However, because there is uncertainty in neuroscience data, inferential statistics cannot be ignored

Statistical inference

- Formulate a so-called null-hypothesis
  - H0: there is no effect
- Formulate an alternative hypothesis
  - H1: there is an effect, describe it

- Determine the likelihood (p-value) of H0
- Reject H0 if it is too unlikely
  e.g. smaller than 5% -> accept H1

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Conventional statistics

- Parametric statistics
  - t-test
  - ANOVA
  - Correlation
- Parameter
  - t
  - F
  - R

Behaviour of these parameters is known under the null-hypothesis

Parametric statistics
t-test

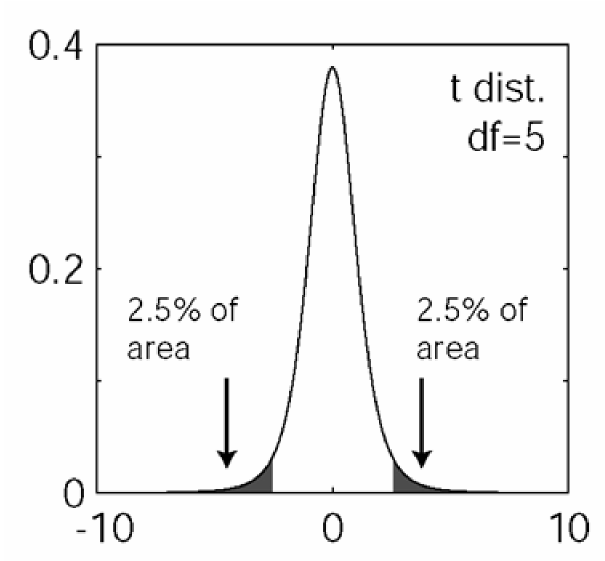
T-test: Compare paired/unpaired samples for continuous data.

Paired

$$t = \frac{Mean\_difference}{Standard\_deviation}\sqrt{N-1}$$

Unpaired

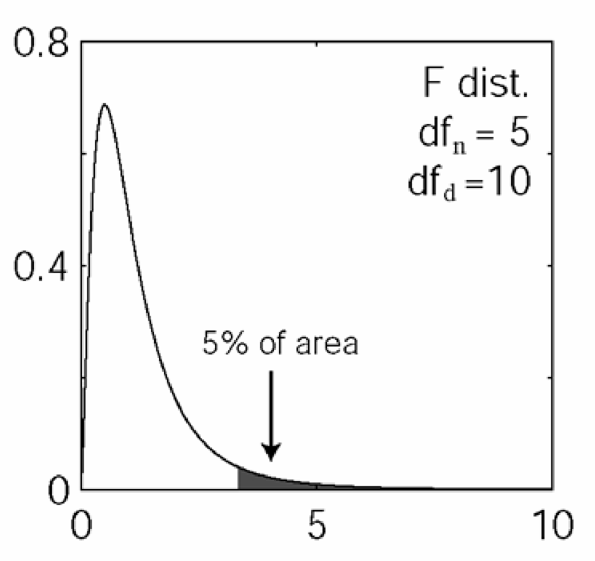
$$t = \sqrt{N}\frac{Mean_A - Mean_B}{\sqrt{(SD_A)^2 - (SD_B)^2}}$$



Parametric statistics ANOVA

ANOVA: compare several groups (can test interaction between two factors for the repeated measure ANOVA)

$$F = \frac{Variance_{interGroup} / (N_{Group} - 1)}{Variance_{WithinGroup} / (N - N_{Group})}$$



Why something else than parametric statistics?

- Problems
  - Assumes normal distribution of the value of interest
  - Assumes that error terms (deviation from the group mean) are normally distributed
  - Assumes known behaviour of the parameters

Why something else than parametric statistics?

- More problems
  - Difficult to solve the multiple comparison problem
- New opportunities
  - Use intuitive explanation of the data -> express parameters the way we want
  - Increased sensitivity for our ‘new parameters’

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New approaches for statistics

- Bootstrap
- Jackknife
- Resampling
- Randomization
- Permutation
- Pseudo data
- Surrogate data

New approaches for statistics

- Resampling framework
  - Systematic resampling
  - Random resampling
- Randomization framework
  - Randomization
  - Permutation

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Baron von Münchhausen

Baron von Münchhausen

New approaches for statistics

- Stuck in the swamp?
- Pull yourself out by your bootstraps!

General principle

- Observed data
  - Some subjects are observed in one condition
  - Some subjects are observed in another contition

  - Independent variable: condition
  - Dependent variable: data

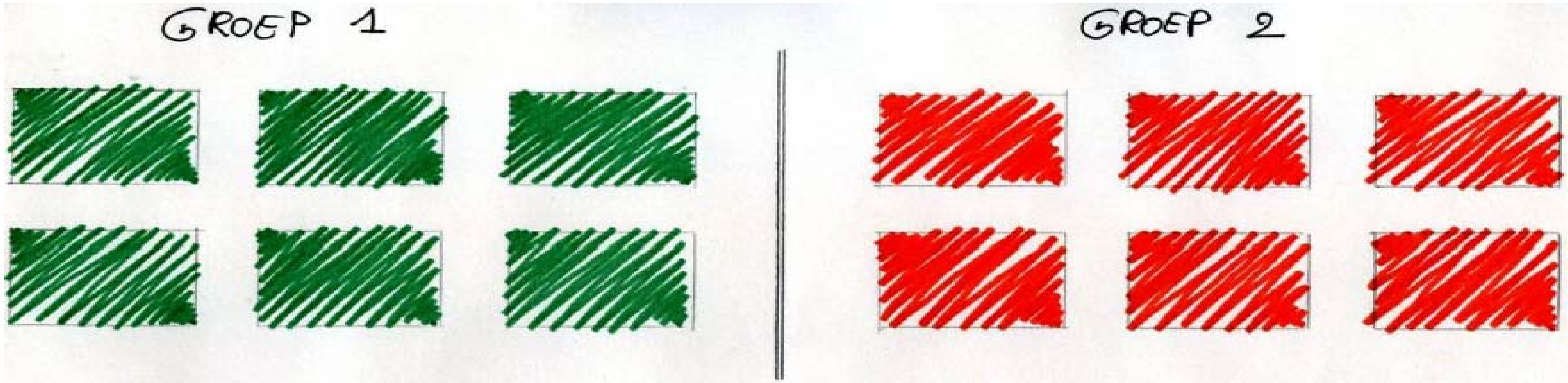
General principle

- Independent variable: condition
- Dependent variable: data

H0: the data is independent from the condition in which it was observed

The data in the two conditions is **not** different

Randomization approach



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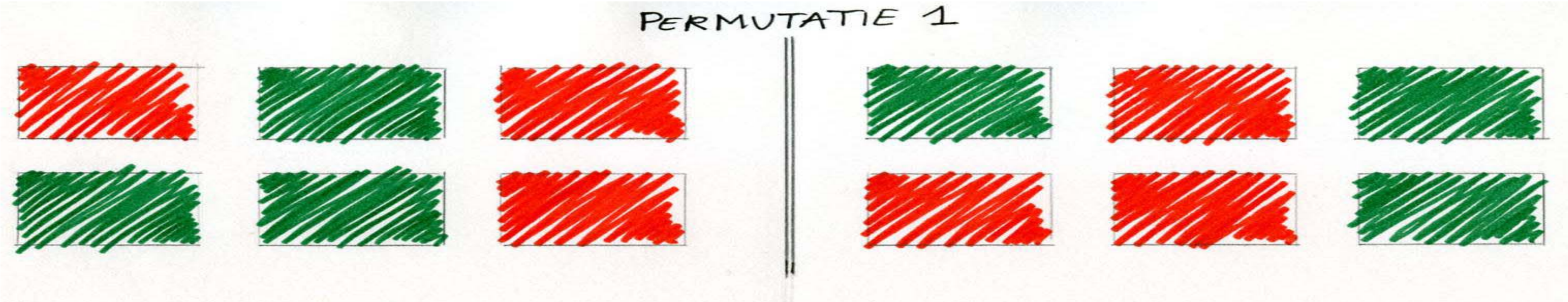
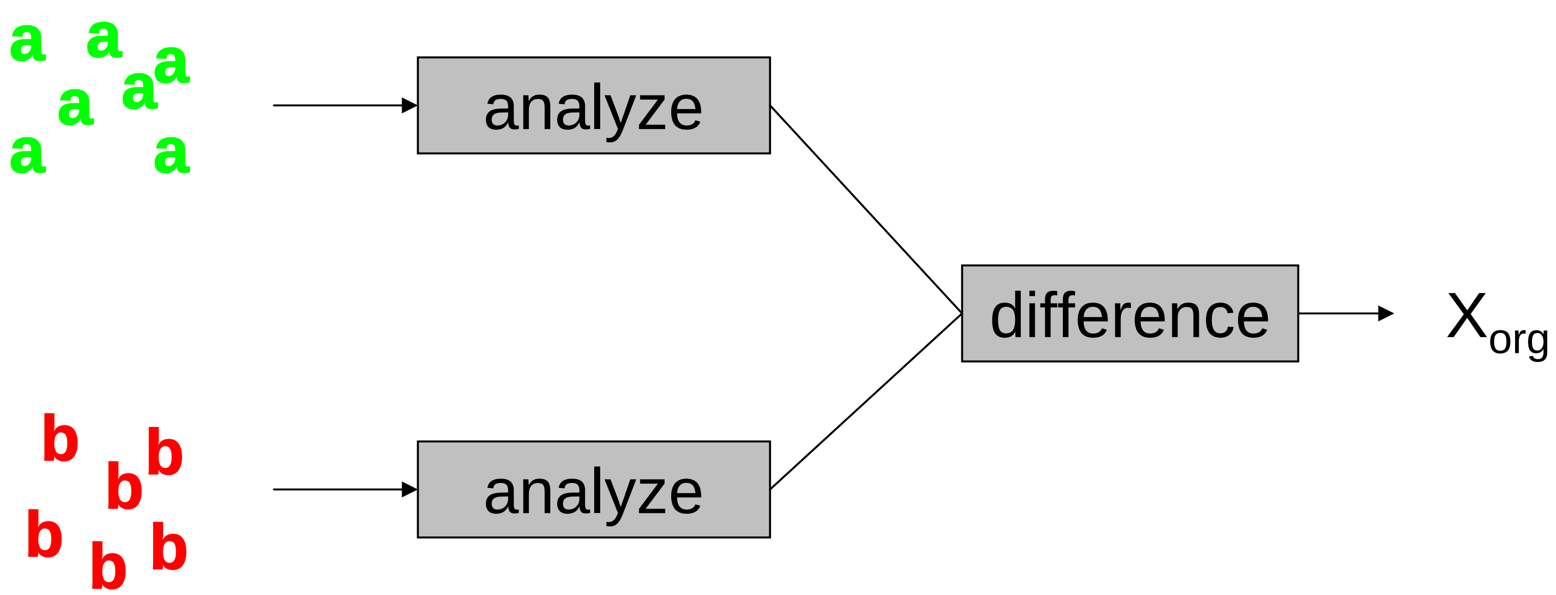
Randomization approach



Randomization approach



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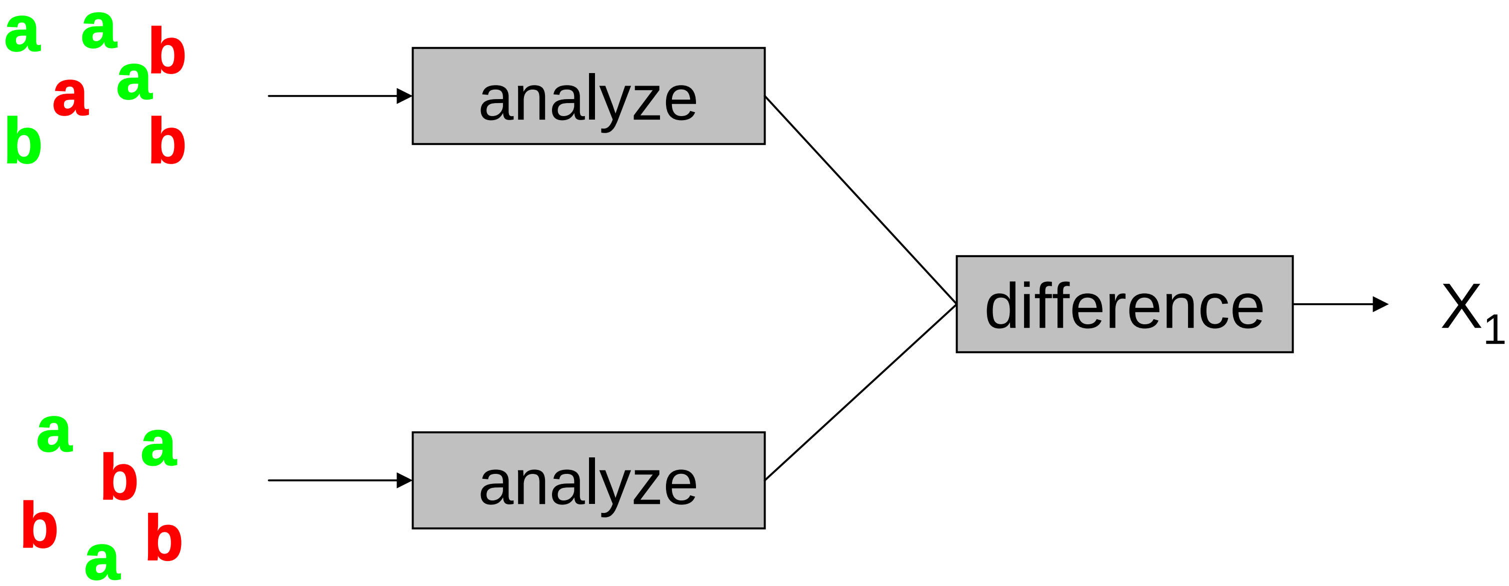
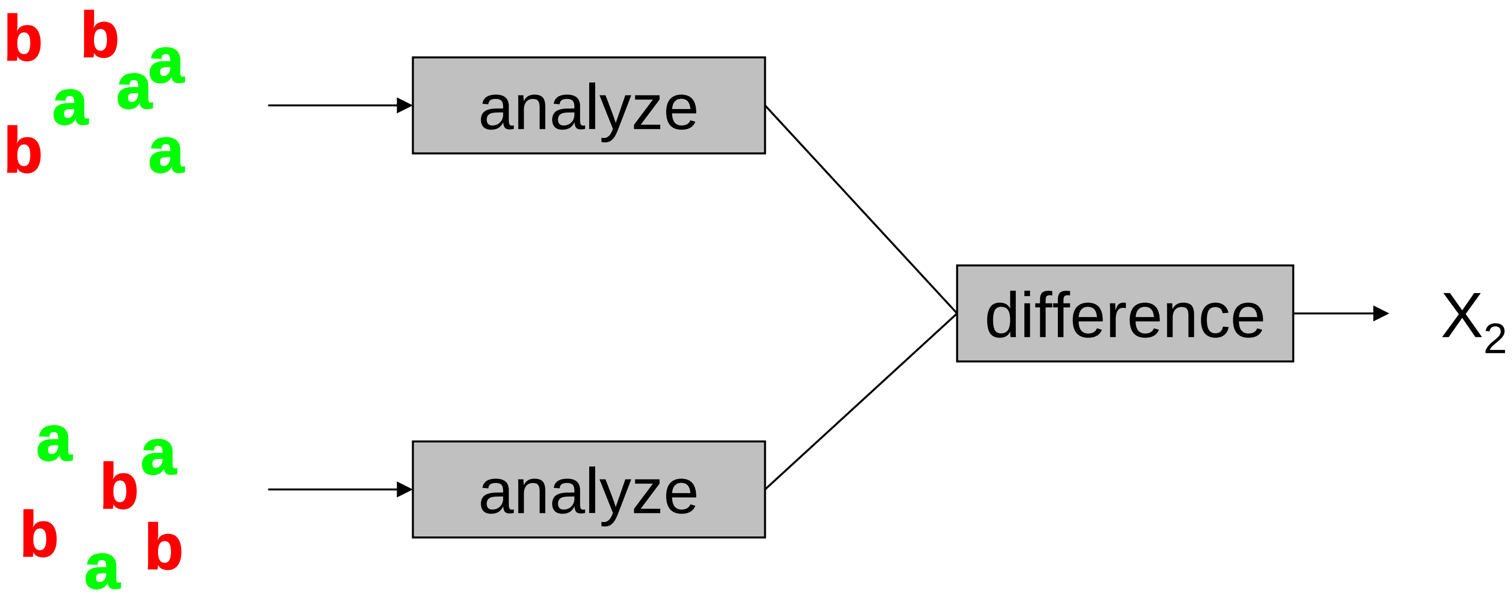
Randomization approach



Randomization approach



Non-parametric statistics

- Randomization of independent variable
- Hypothesis is about data, not about the specific parameter
- Randomization distribution of the statistic of interest “x” is approximated using Monte-Carlo approach
- H0 is tested by comparing observed statistic v.s. randomization distribution

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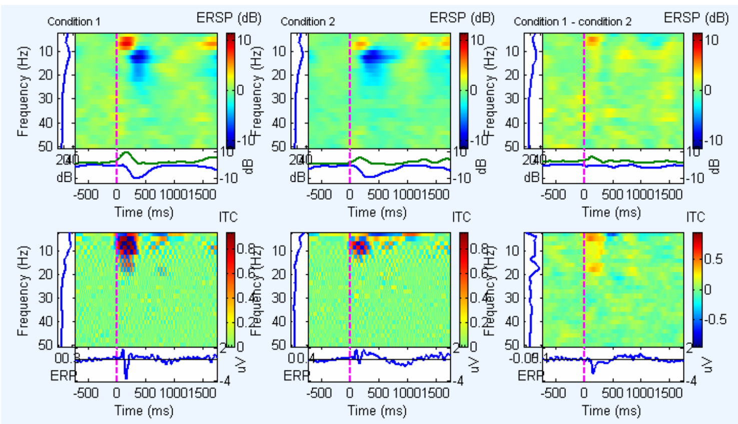
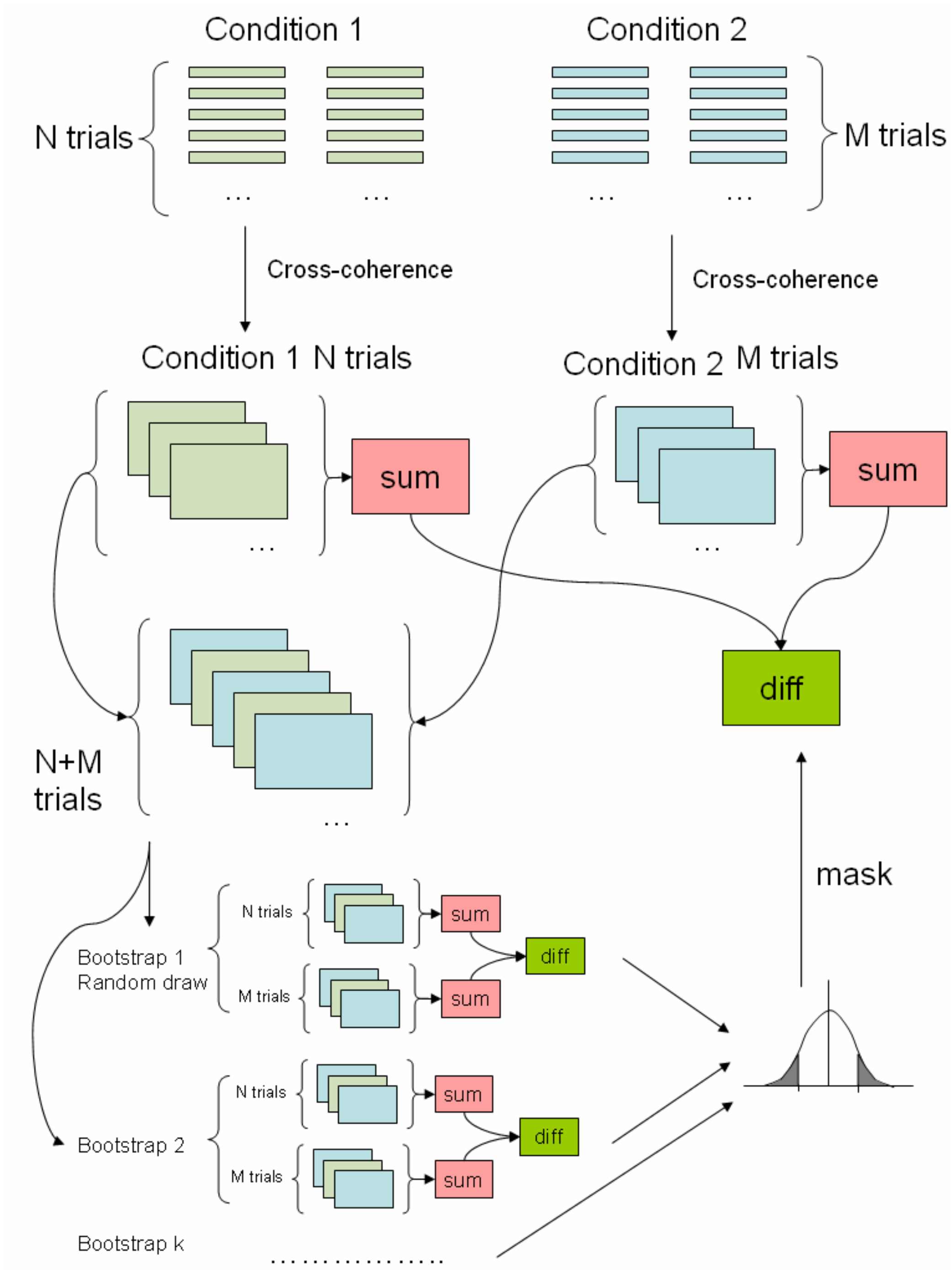
Permutation for comparing between conditions

Help mask cross-coherence difference

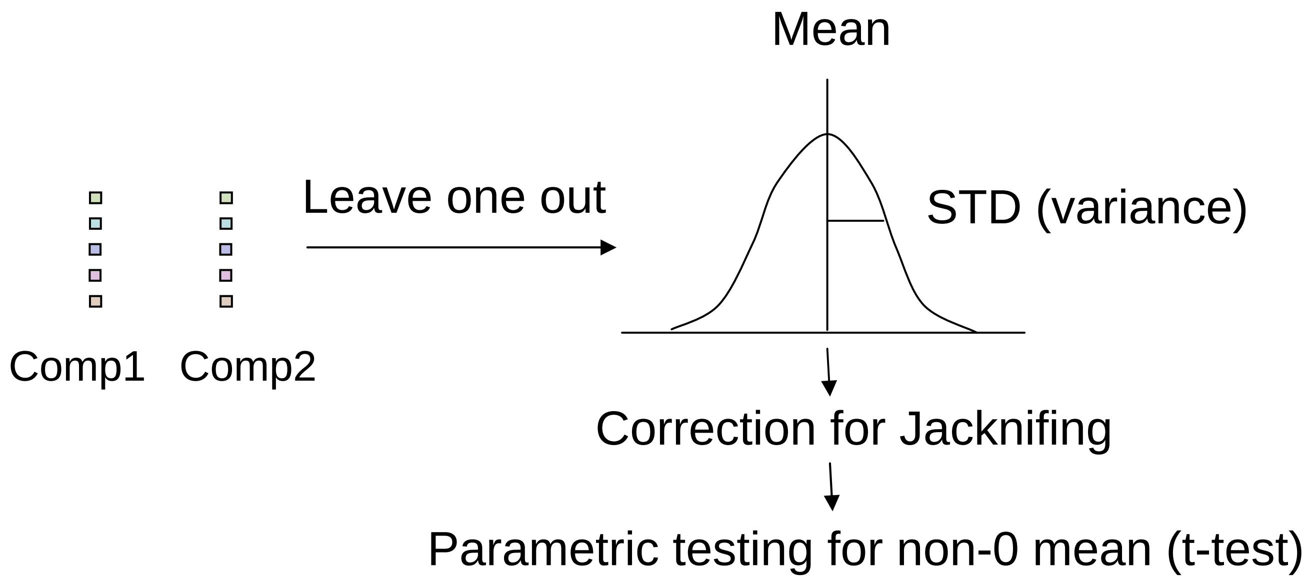




Bootstraping & Jacknife (parametric testing)

- Estimating variance for non-linear measures
  - Jacknife: leaving one out
  - Bootstrap: taking sub-samples with replacement

E.g. testing for coherence



Correcting for multiple comparisons

- Bonferoni correction: divide by the number of comparisons (Bonferroni CE. Sulle medie multiple di potenze. Bollettino dell'Unione Matematica Italiana, 5 third series, 1950; 267-70.)
- Holms correction: sort all p values. Test the first one against α/N, the second one against α/(N-1)
- For time-domain: find minumum resolution where effect is visible, then divide significance threshold by the total number of points
- For time-frequency decomposition:
  - minimal resolution in frequency is real wavelets
  - Minimal resolution in time is the time at which neighboring time points are significantly correlated (Approximatelly 200 in most cases).
  - For 5% significance between 2-100Hz, the new p value should be $0.05 * 1/200 = 2.5 * 10^{-4}$ (Bonferoni)