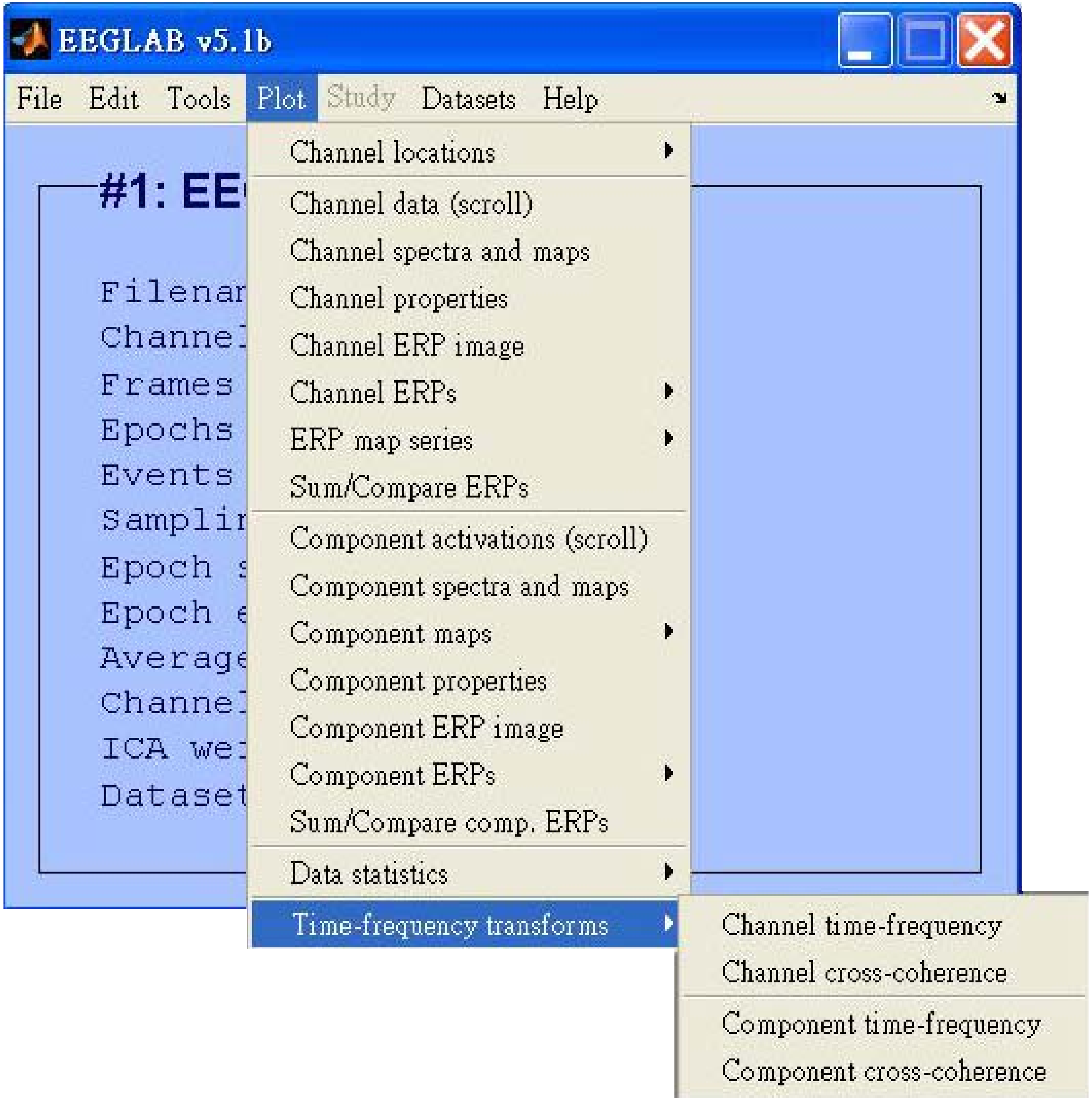
Time-Frequency analysis of biophysical time series

Courtesy of Arnaud Delorme


EEGLAB v5.1b
File Edit Tools Plot Study Datasets Help
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Channel
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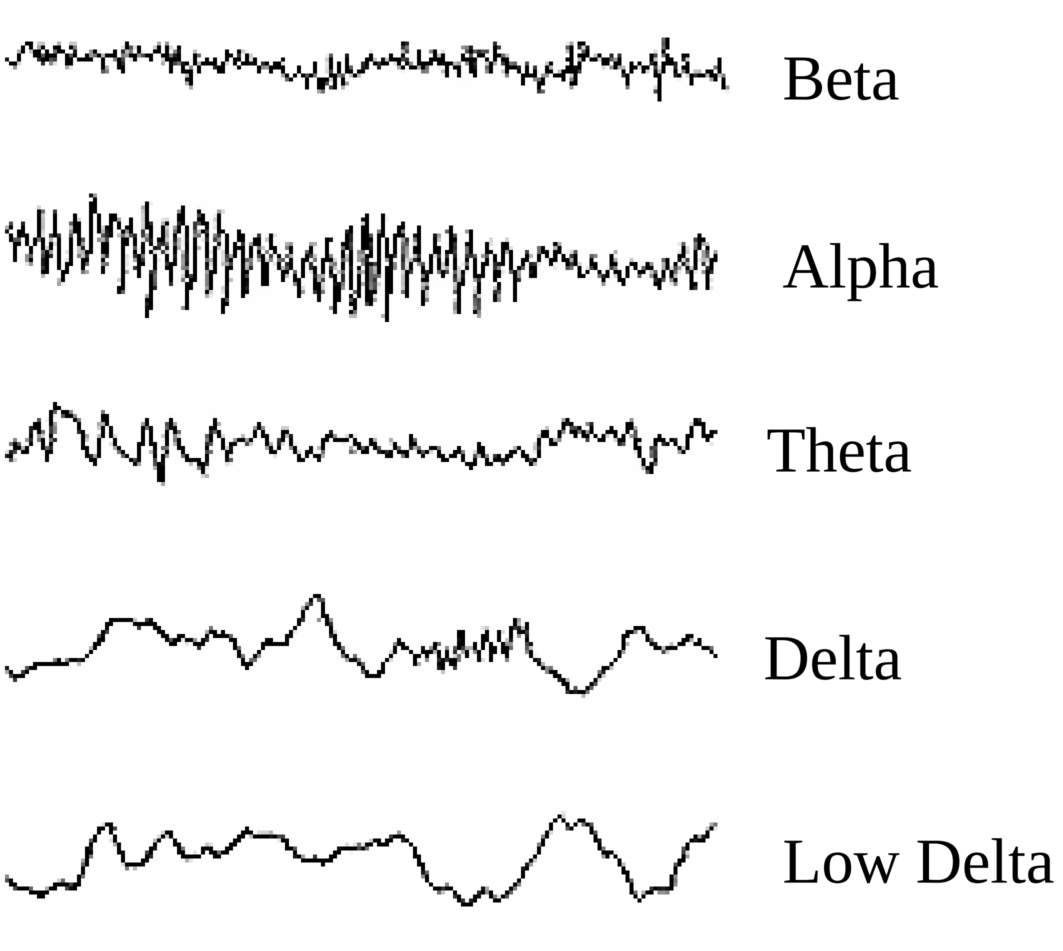
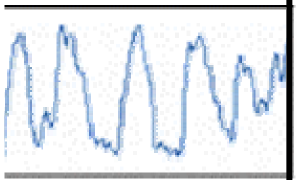
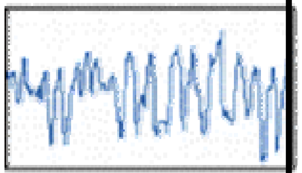
Dataset
Channel locations
Channel data (scroll)
Channel spectra and maps
Channel properties
Channel ERP image
Channel ERPs
ERP map series
Sum/Compare ERPs
Component activations (scroll)
Component spectra and maps
Component maps
Component properties
Component ERP image
Component ERPs
Sum/Compare comp. ERPs
Data statistics
Time-frequency transforms
Channel time-frequency
Channel cross-coherence
Component time-frequency
Component cross-coherence

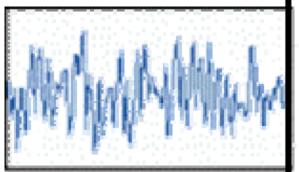
Why Frequency-domain Analysis

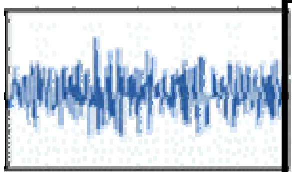
For many signals, the signal's frequency content is of great importance.







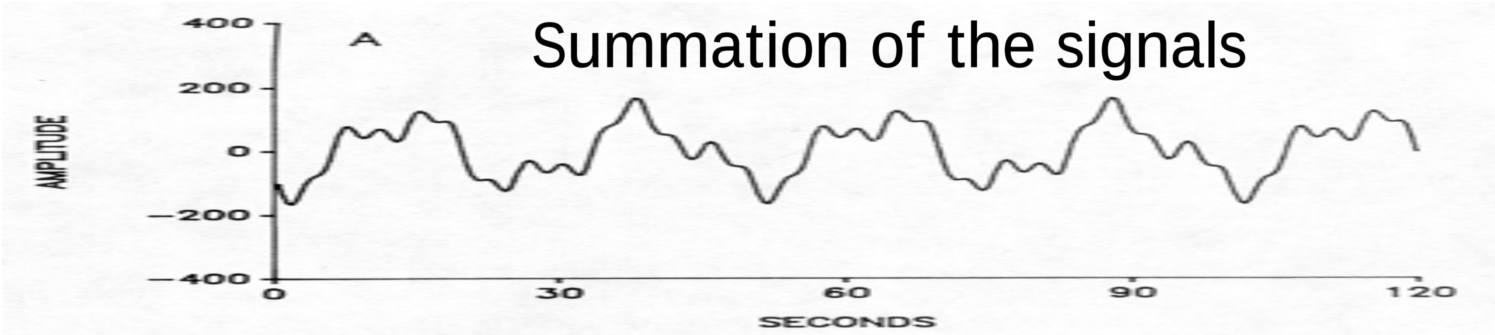




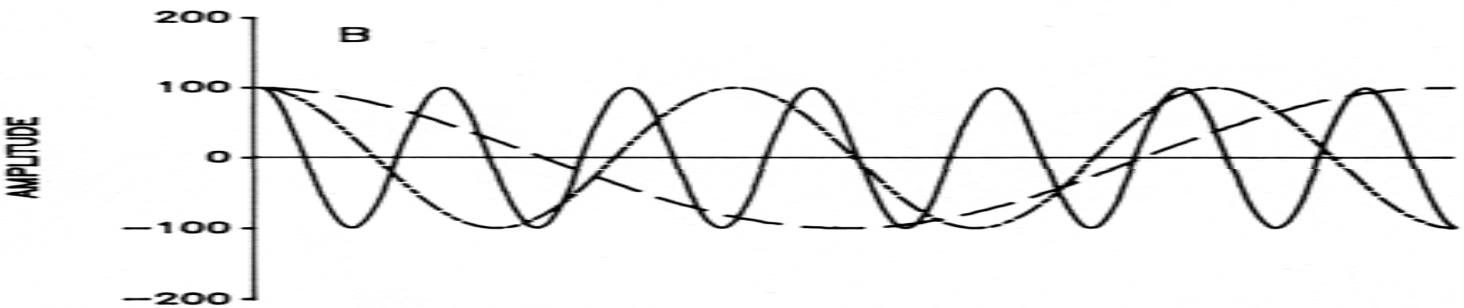
EEG Bands (Hz)	Distribution	Subjective feeling	Associated tasks & behaviors	Physiological correlates
Delta 0.1-3	Distribution: generally broad or diffused	deep, dreamless sleep, non-REM sleep, unconscious	lethargic, not moving, not attentive	not moving, low-level of arousal
Theta 4-8	usually regional, may involve many lobes	intuitive, creative, recall, fantasy, imagery, creative, dreamlike, drowsy	creative, intuitive; distracted, unfocused	healing, integration of mind/body
Alpha 8-12	regional, usually involves entire lobe	relaxed, not agitated, but not drowsy	meditation, no action	relaxed, healing
Beta 12-30	localized	alertness, agitation	mental activity, e.g. math	alert, active
Gamma >30	very localized	Focused arousal	high-level information processing, "binding"	information -rich task processing

Frequency-domain Analysis of the EEG

- Joseph Fourier (1768-1830)
- Any complex time series can be broken down into a series of superimposed sinusoids with different frequencies.



Summation of the signals



Fourier Analysis

Fourier-Transformation:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f t} dt \,; \quad h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i f t}$$

Discrete Fourier-Transformation傅利葉轉換 ($O(N^2)$):

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-ik\left(2\pi / N\right)n} \qquad k = 0,1,.., N-1$$

$$x[n] = \sum_{k=0}^{N-1} X(k) e^{ik(2\pi / N)n} \qquad n = 0,1,.., N-1$$

Fast Fourier Transform (FFT, $O(N\log_2 N)$), Cooley and Tukey (1965)

```
function [a,b] = dft (y)
%           DFT - The Discrete Fourier Transform
%           [a, b] = DFT (y)
%           a, b are the cosine and sine components

n = length (y);
t = 2*pi*(0:n-1)/n;
f = 2.0 / n;

for j = 0:n2
            cs = cos (j * t);
            ss = sin (j * t);
            a(j+1) = f * (cs * y);
            b(j+1) = f * (ss * y);
end

% boundaries
n2 = floor (n / 2);
a(1) = 0.5 * a(1);
a(n2+1) = 0.5 * a(n2+1);
b(1) = 0.0;
b(n2+1) = 0.0;
```

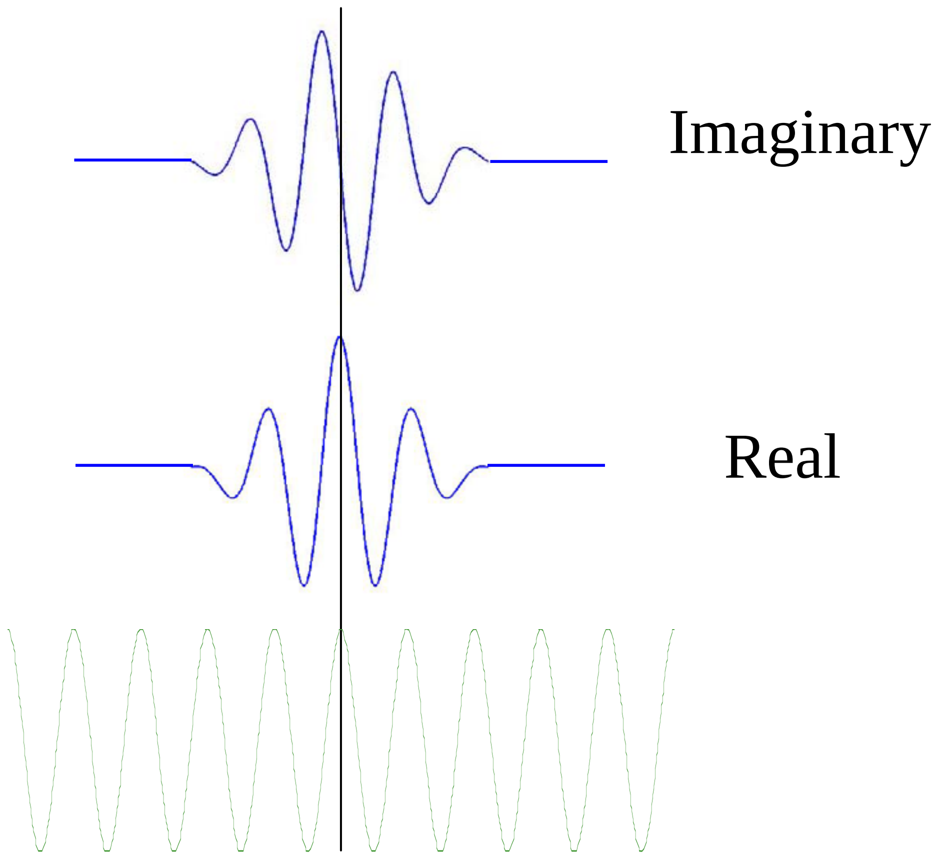
Loop on frequency

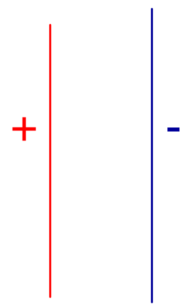
Cosine component

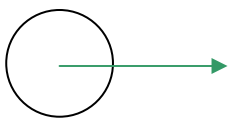
Sine component

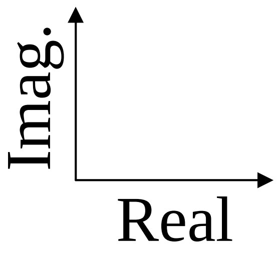
Multiply with signal

Spectral phase and amplitude

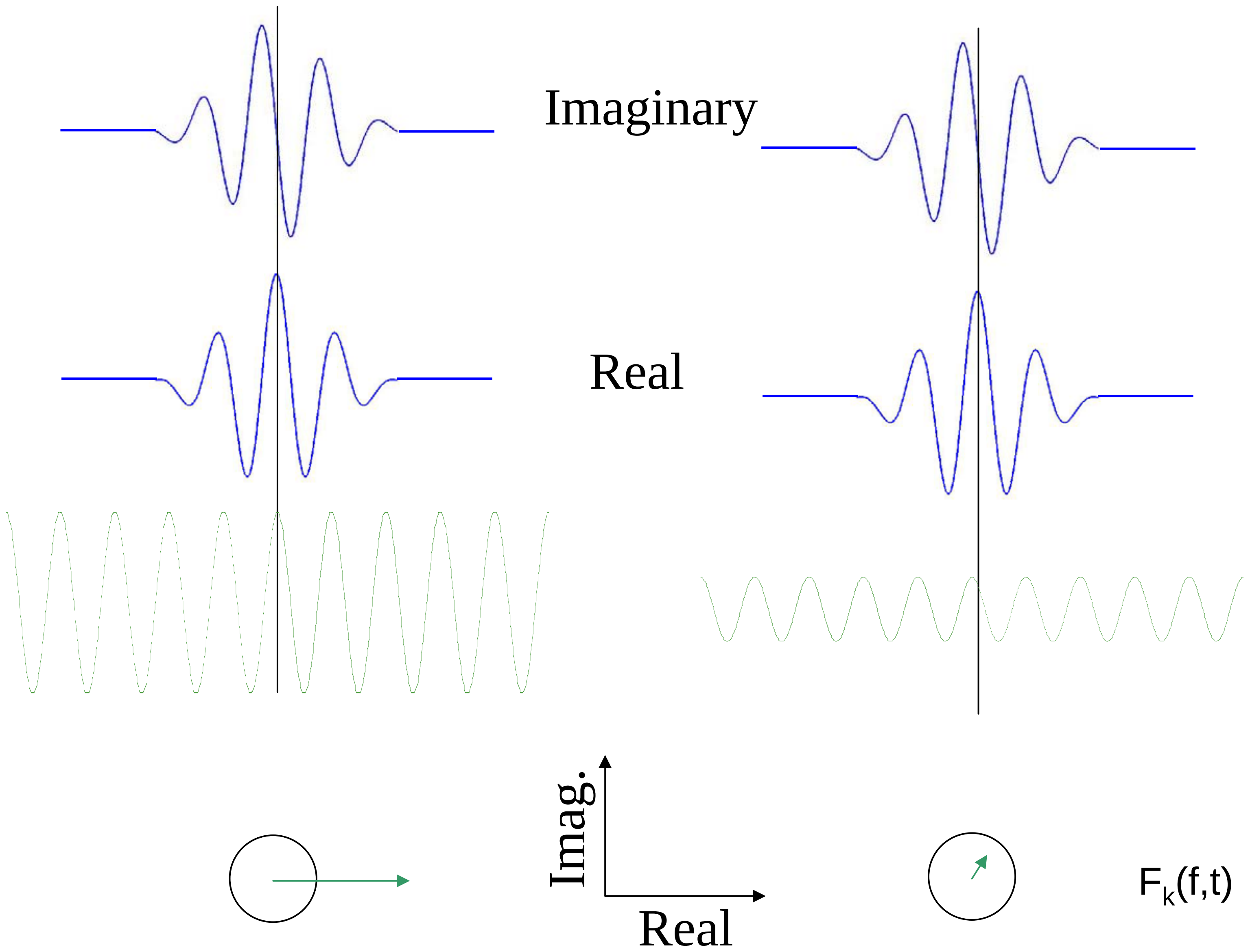




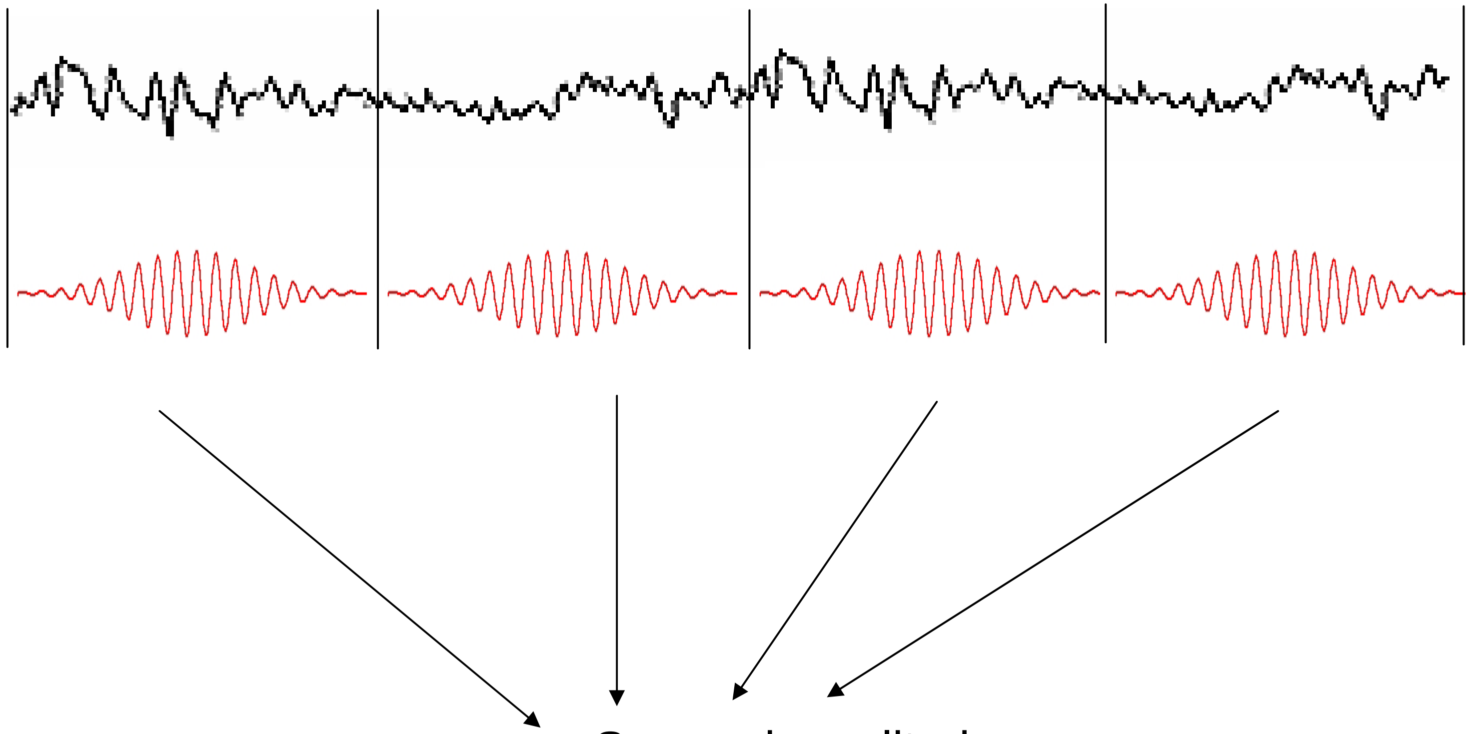




Spectral phase and amplitude

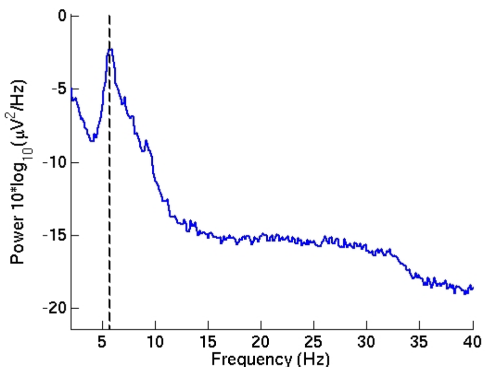


Pwelch method for computing spectrum

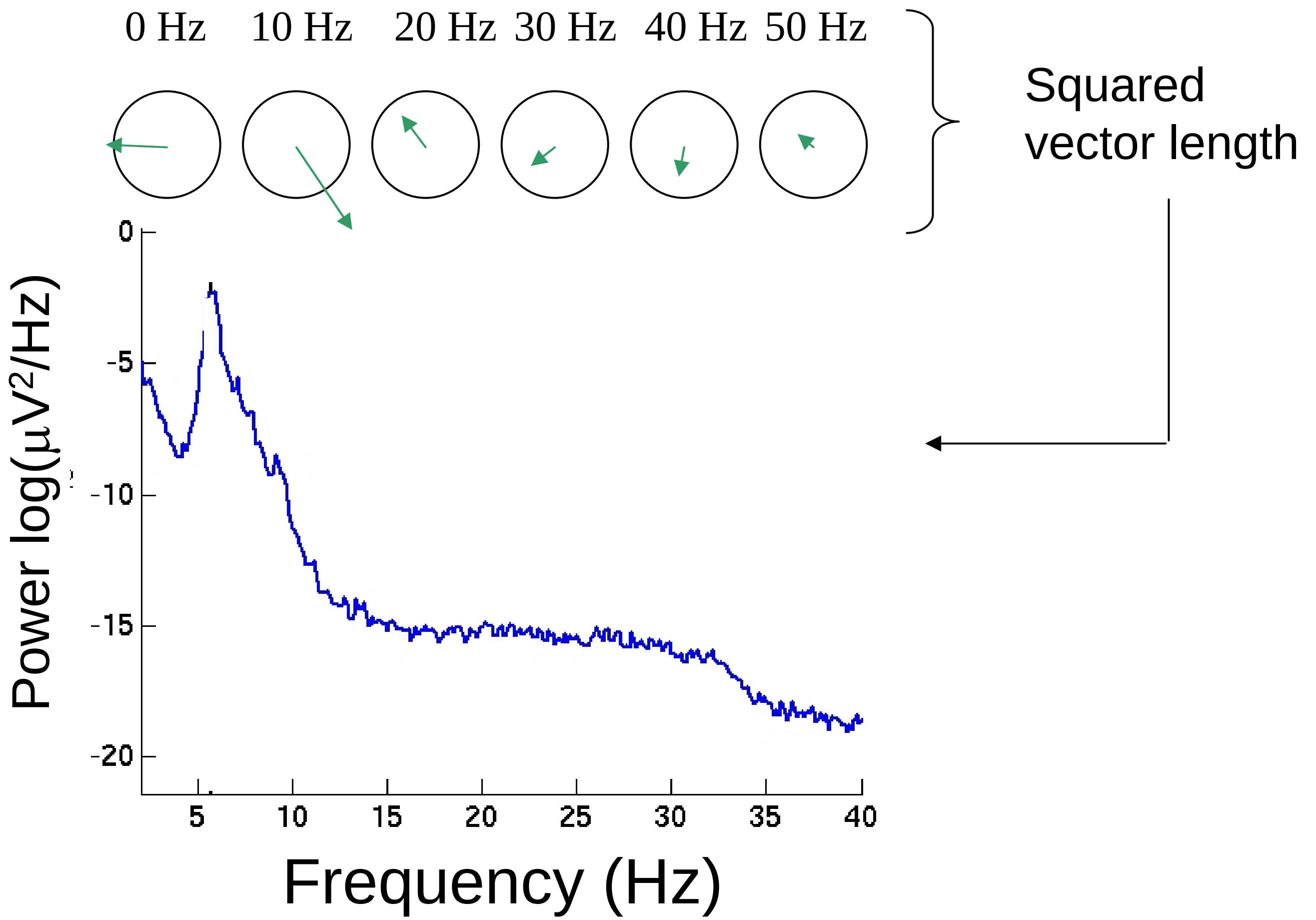


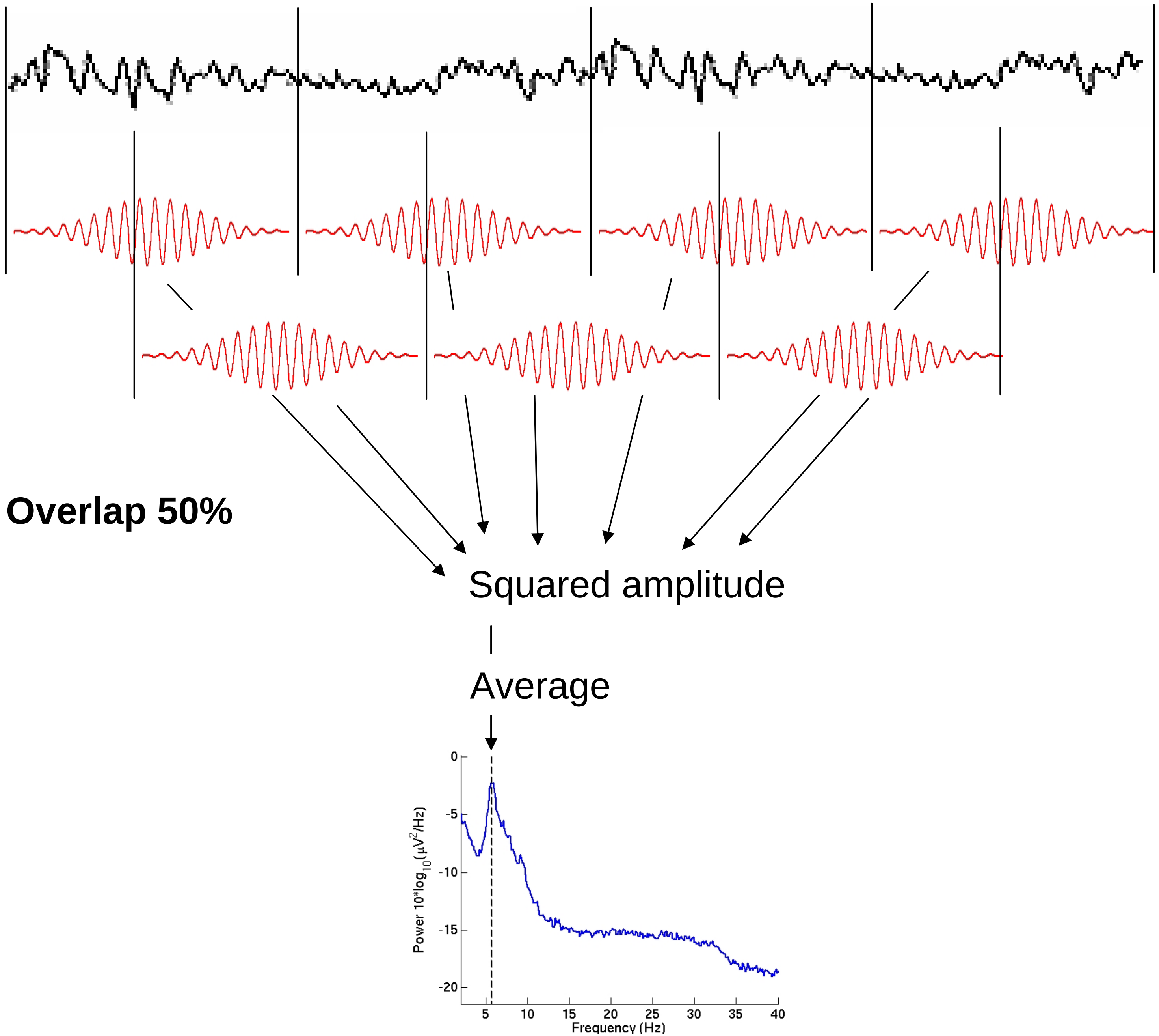
Squared amplitude

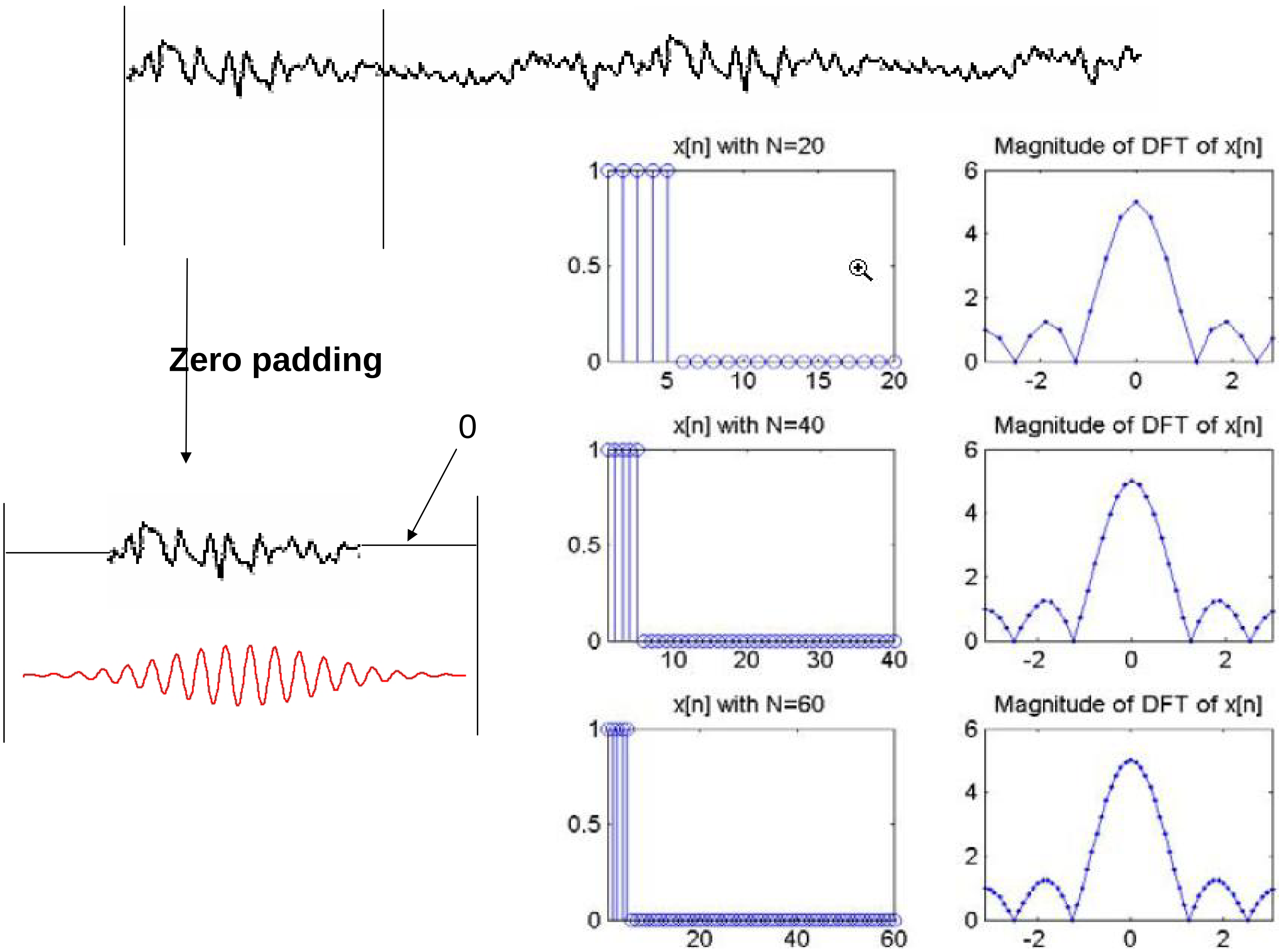
Average



Spectral power

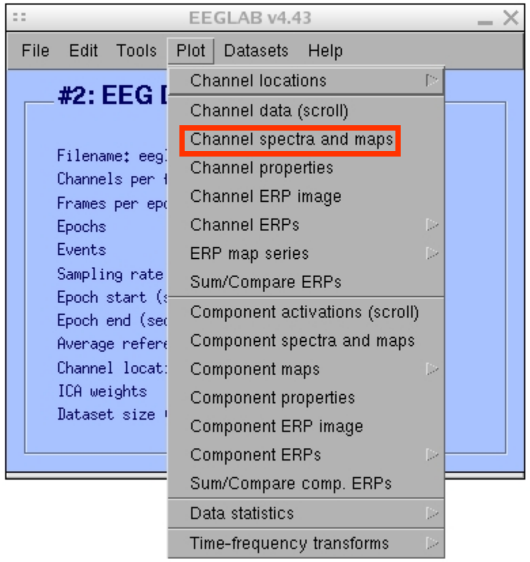


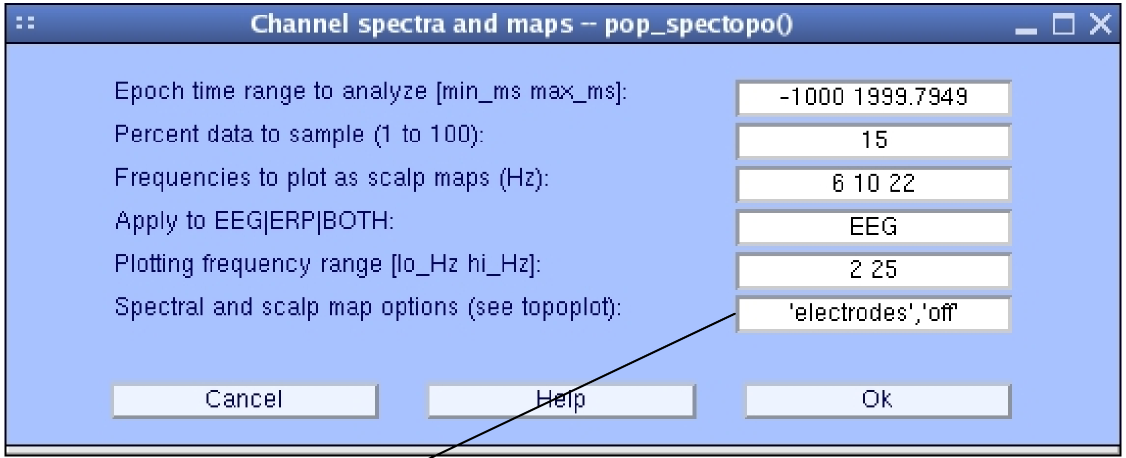

Overlap 50%
Squared amplitude
Average
Power 10*log10(μV²/Hz)
Frequency (Hz)
0
-5
-10
-15
-20
5
10
15
20
25
30
35
40

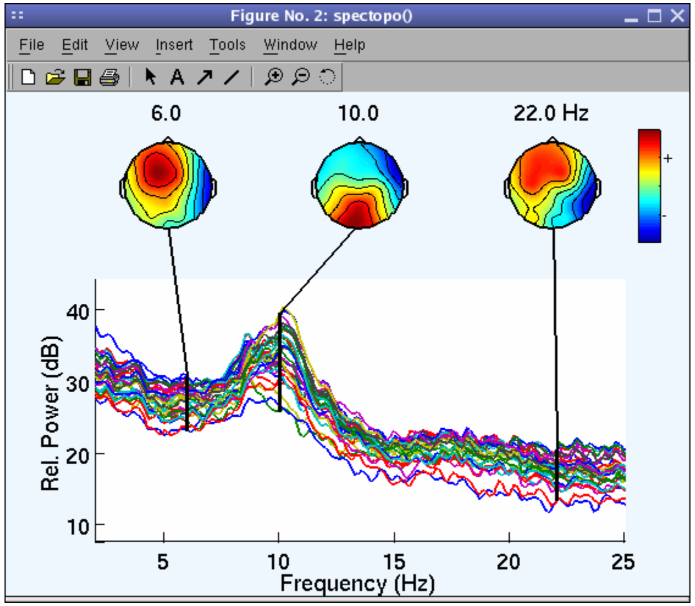


Zero padding
0
x[n] with N=20
Magnitude of DFT of x[n]
x[n] with N=40
Magnitude of DFT of x[n]
x[n] with N=60
Magnitude of DFT of x[n]

Plot data spectrum using EEGLAB

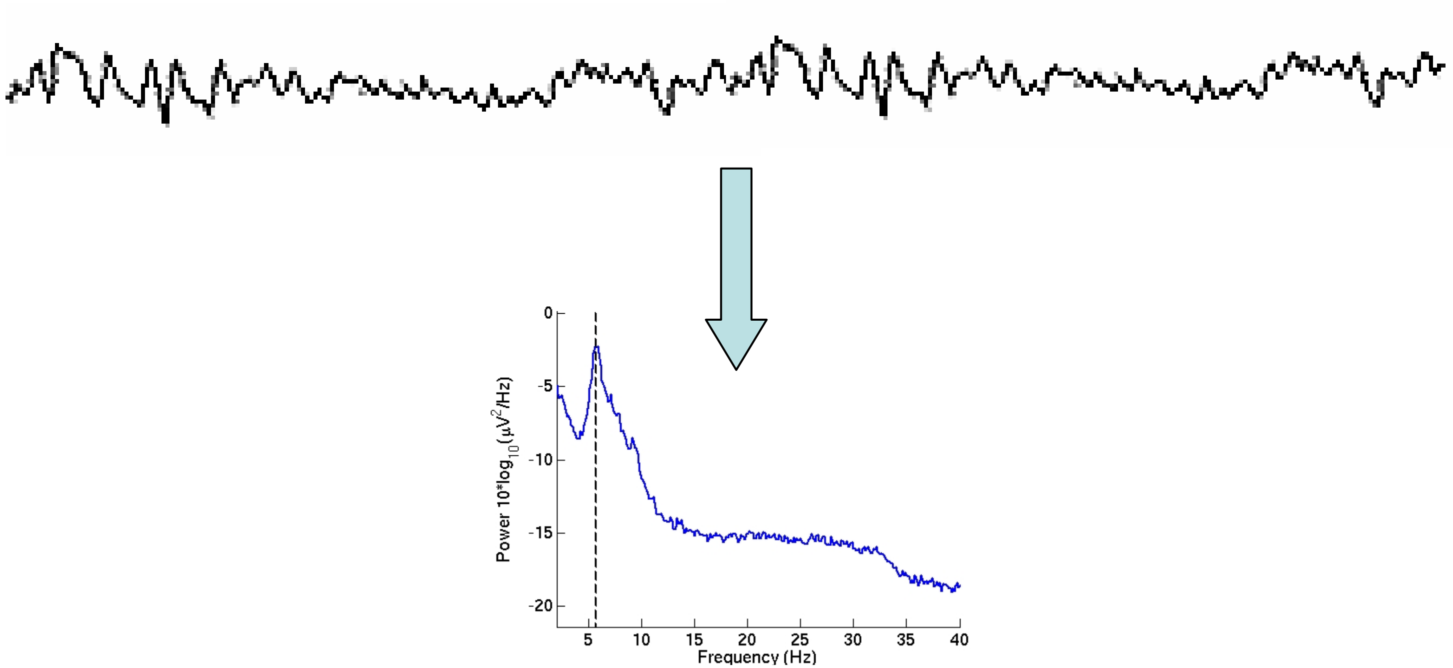






'winsize', 256 (change FFT window length)
'nfft', 256 (change FFT padding)
'overlap', 128 (change window overlap)

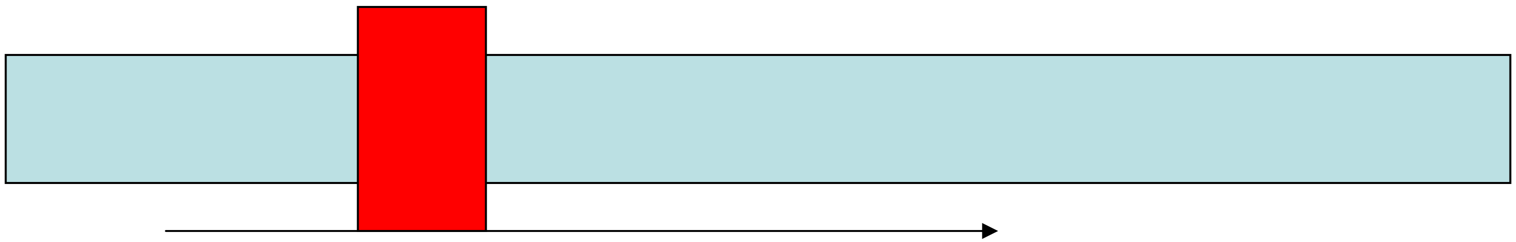
Disadvantage of Fourier Transform



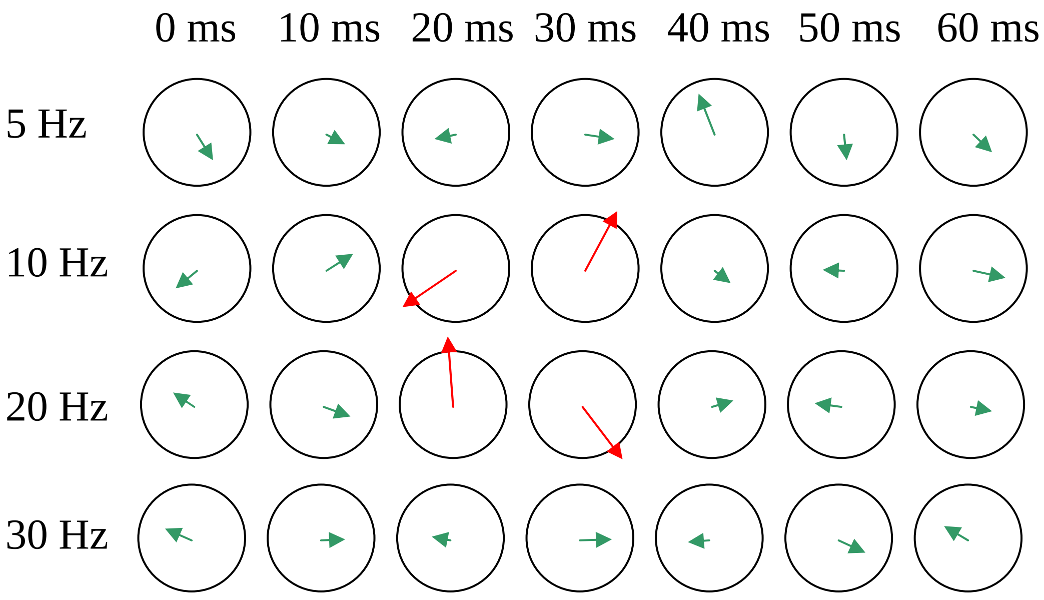
In transforming to the frequency domain, time information is lost.

Frequency-domain Analysis of the EEG

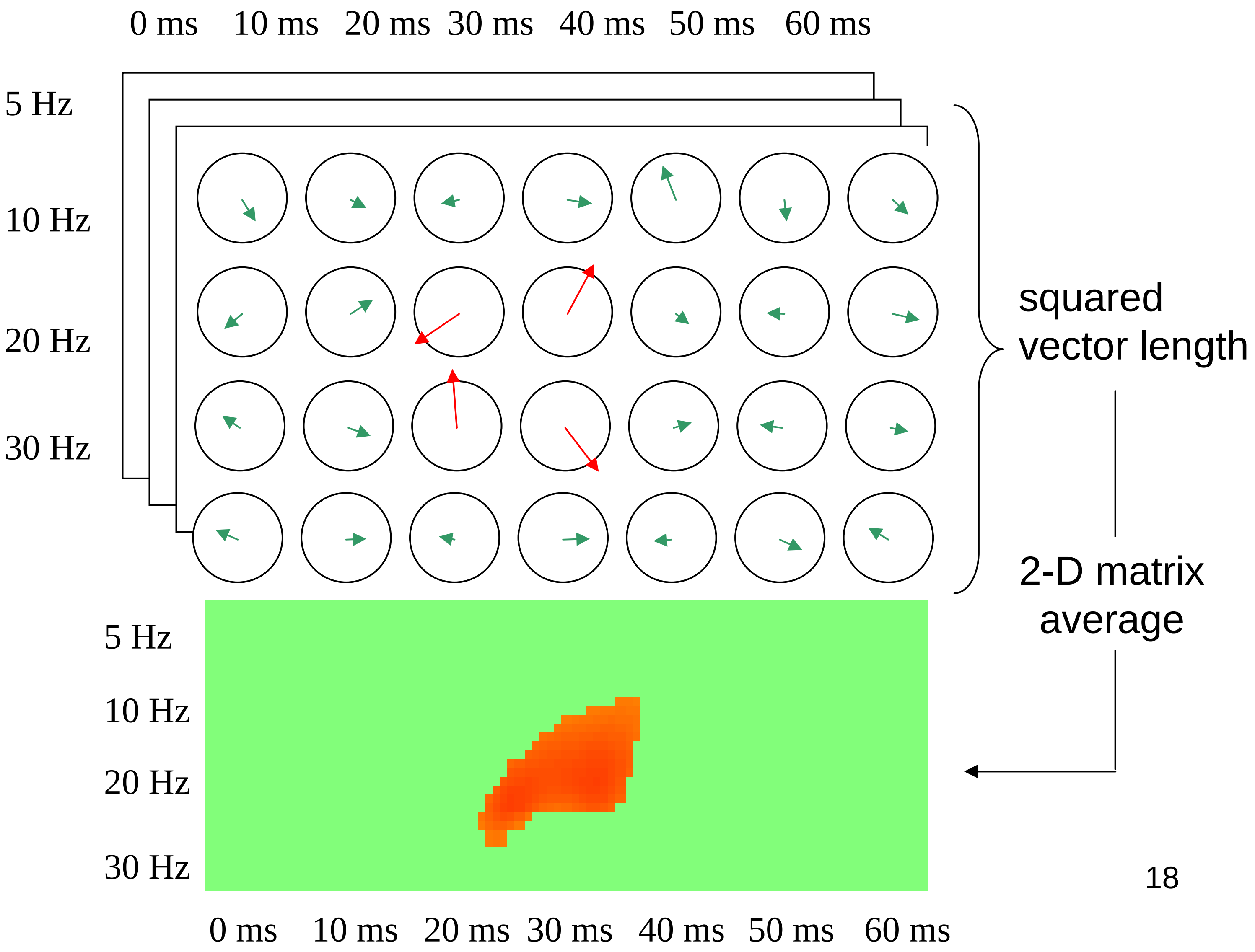
- We often apply a ‘window’ to the data.
- This simply means taking the amount we want from the data stream
- The window is moved along the data; we perform the FFT on this windowed data



Spectrogram or ERSP



Spectrogram or ERSP

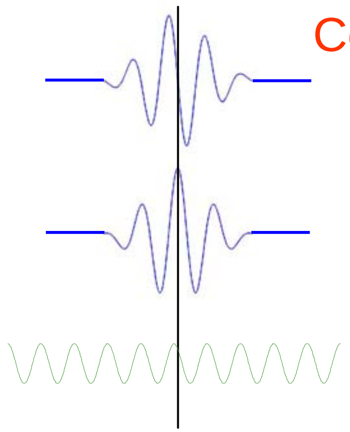


Power spectrum and event-related spectral perturbation

$$ERS(f,t) = \frac{1}{n}\sum_{k=1}^{n\ trials} \left|F_k(f,t)\right|^2$$

Complex number

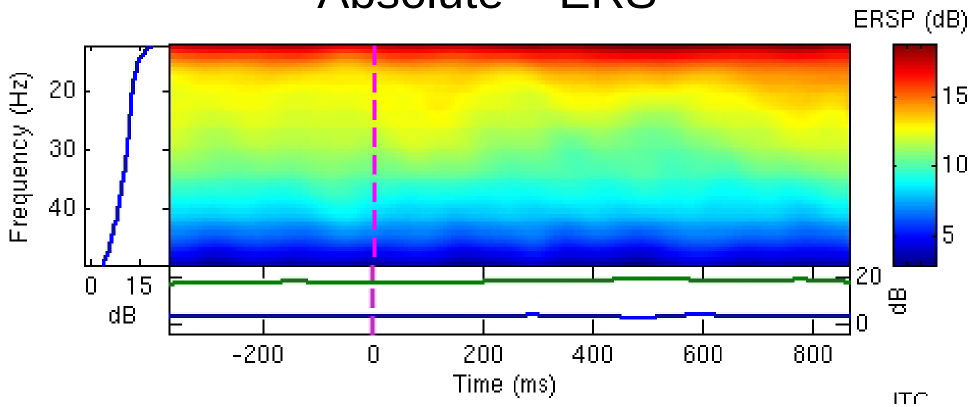
Scaled to dB $10Log_{10}(ERS)$



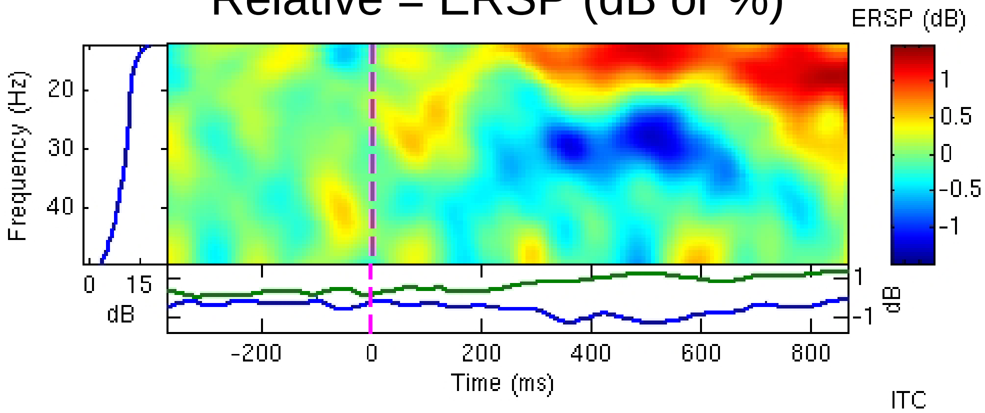
$F_k(f,t)$

Absolute versus relative power

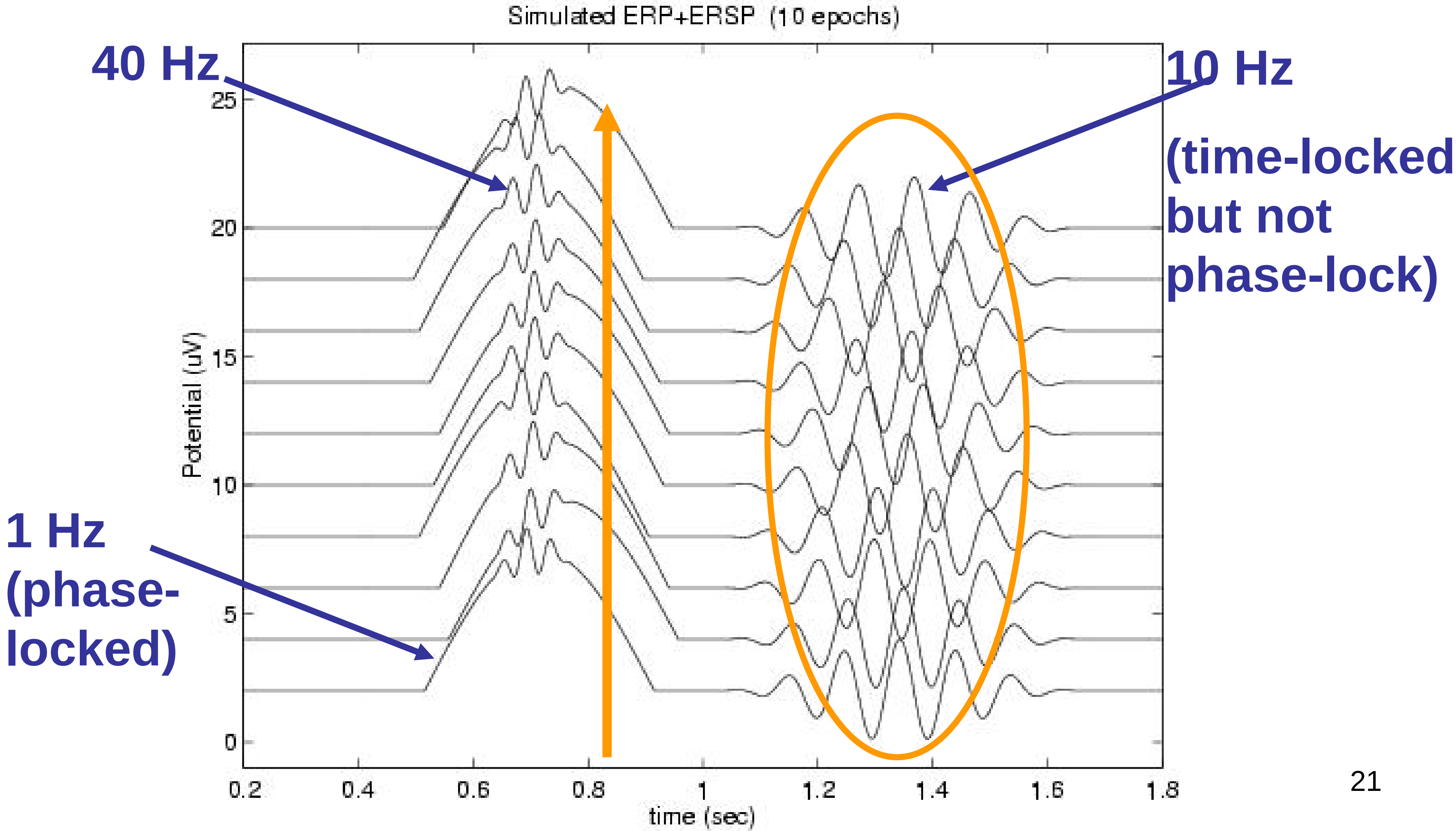
Absolute = ERS



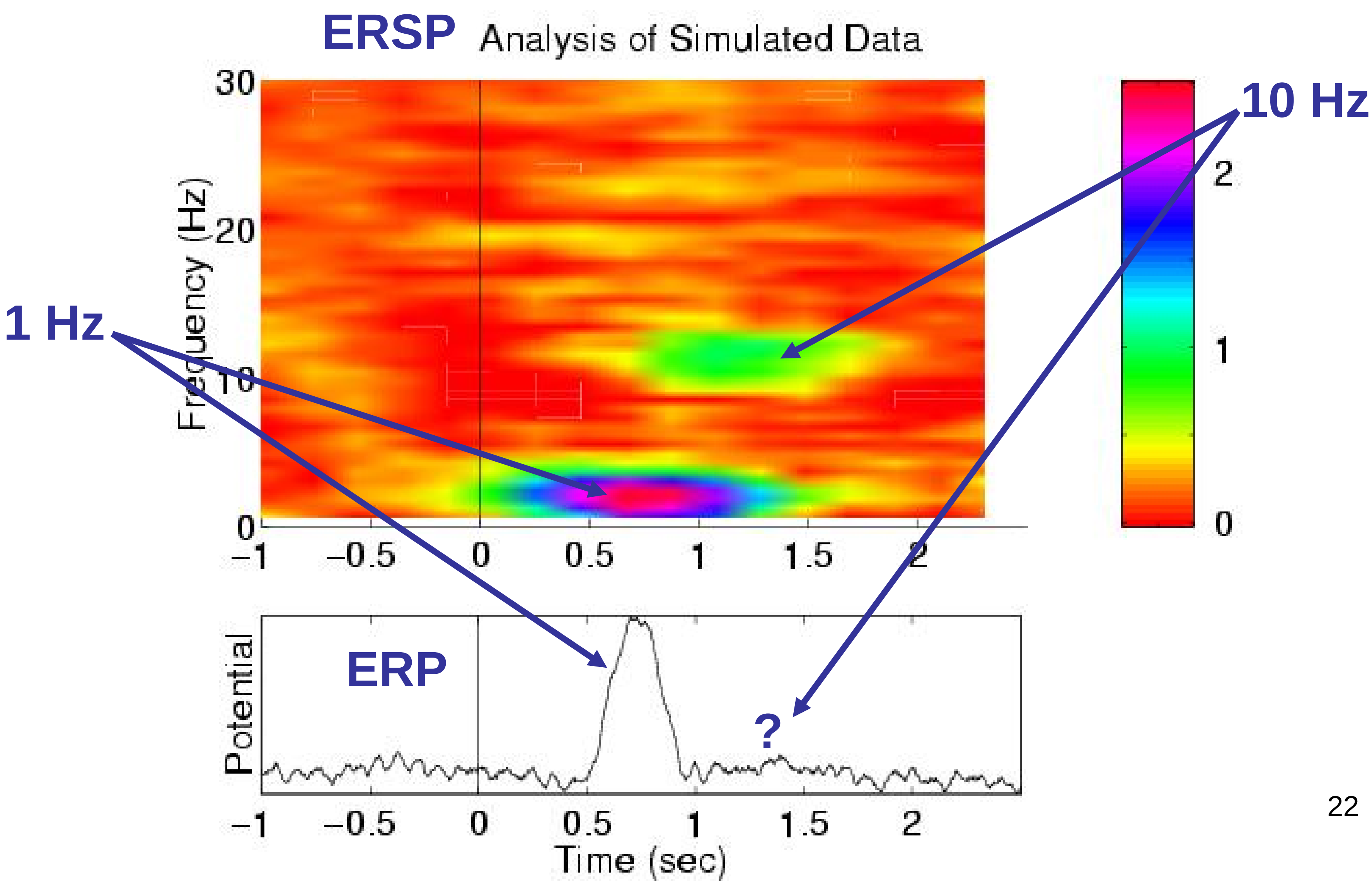
Relative = ERSP (dB or %)



Time-locked ERSP $\neq$ Time- & phase-locked ERP

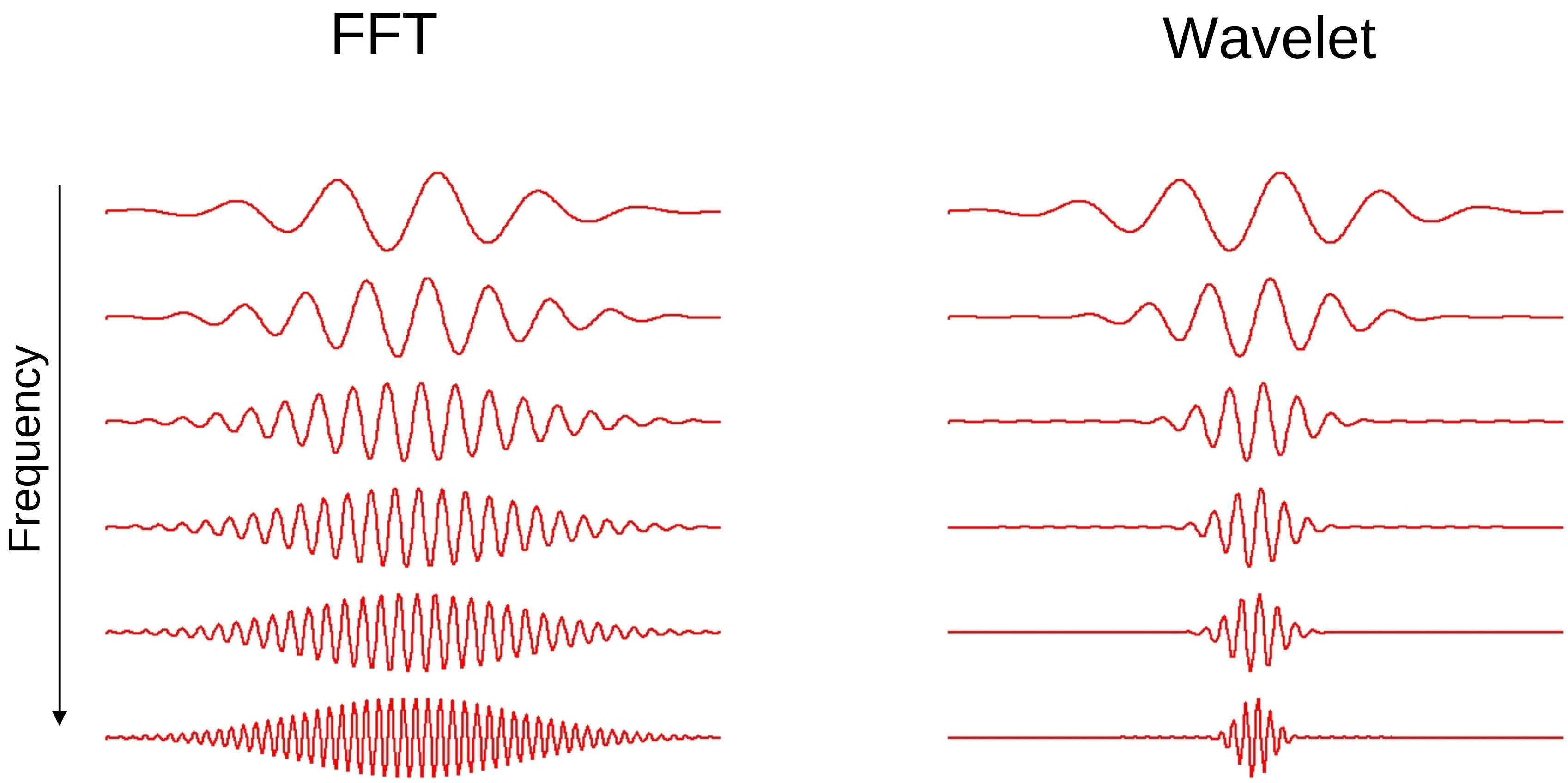


ERSP vs ERP

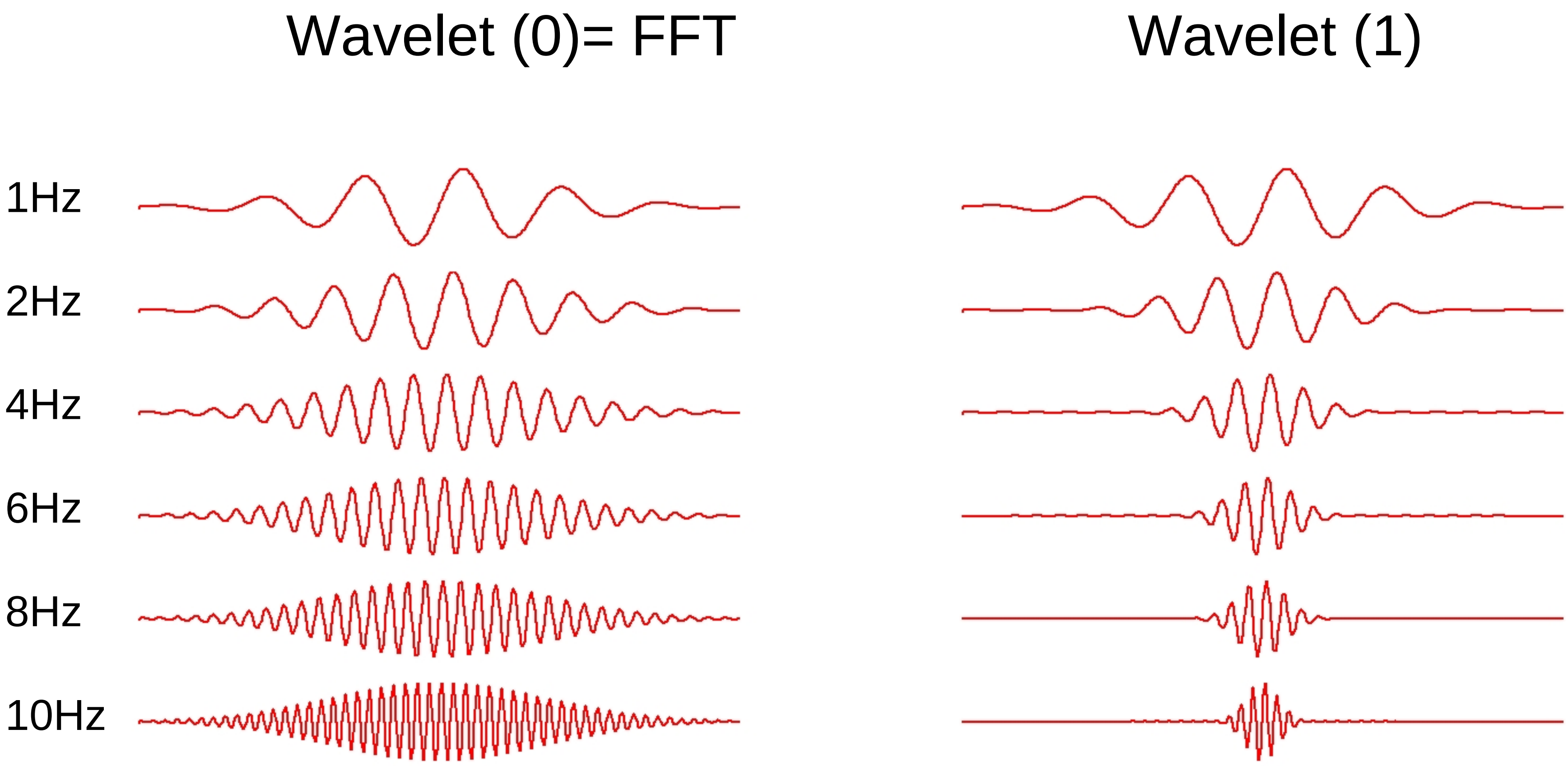


ERSP
Analysis of Simulated Data
Frequency (Hz)
30
20
10
0
−1
−0.5
0
0.5
1
1.5
2
1 Hz
10 Hz
2
1
0
ERP
?
Potential
Time (sec)

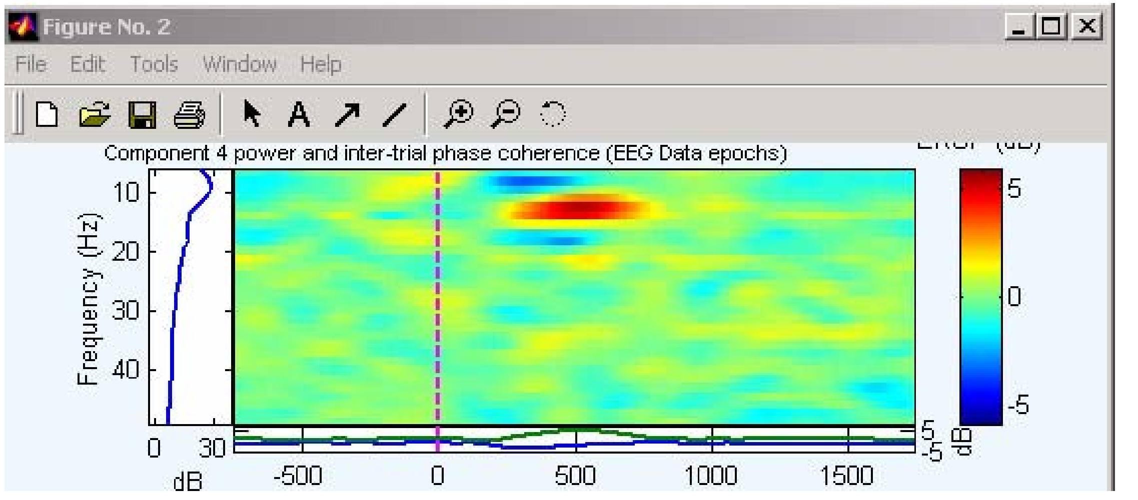
Difference between FFT and wavelets



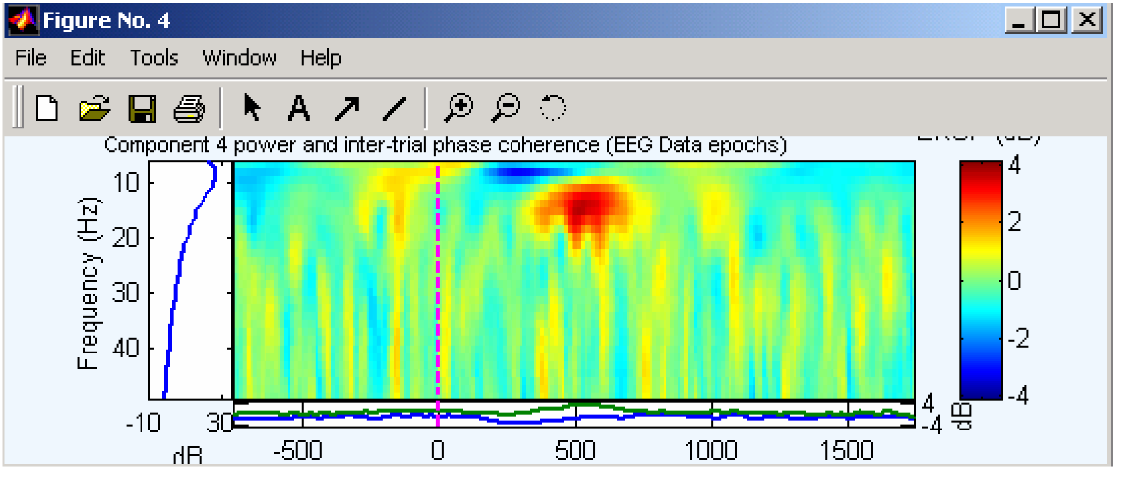
Wavelets factor



FFT

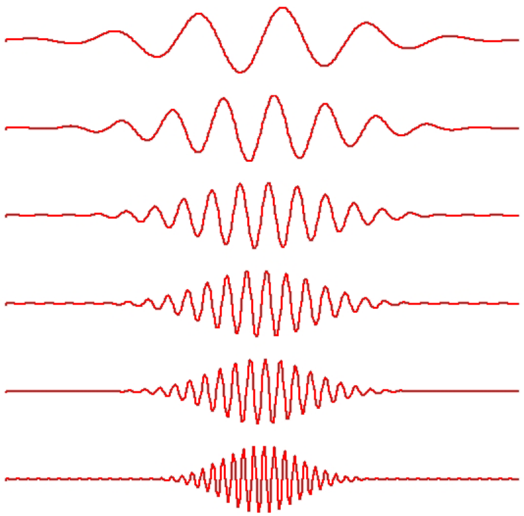


Pure wavelet

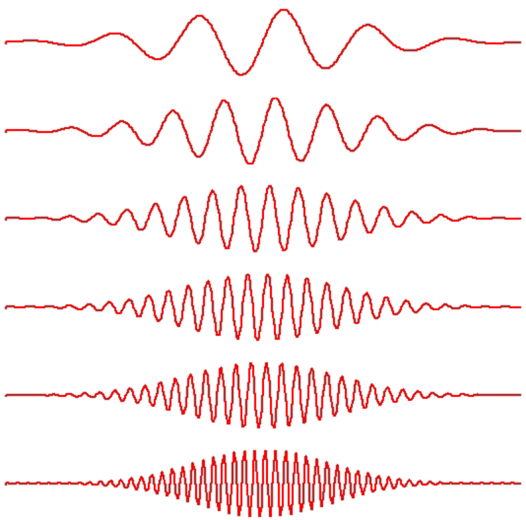


Modified wavelets

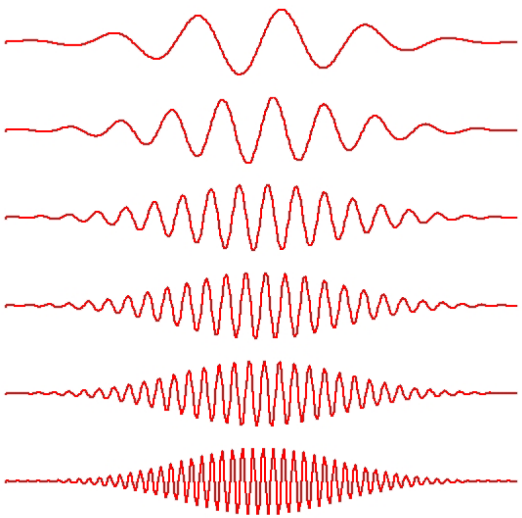
Wavelet (0.8)



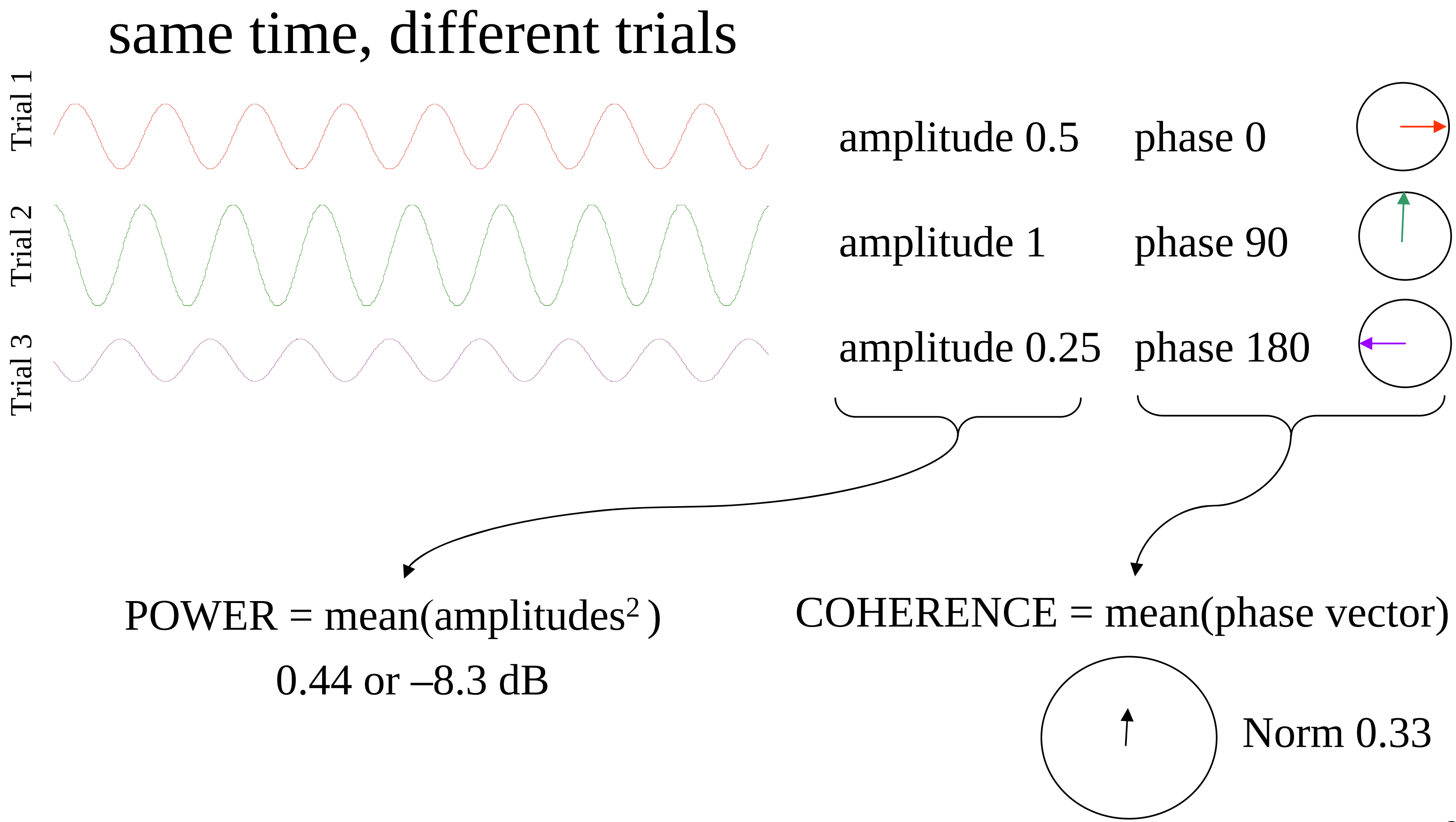
Wavelet (0.5)



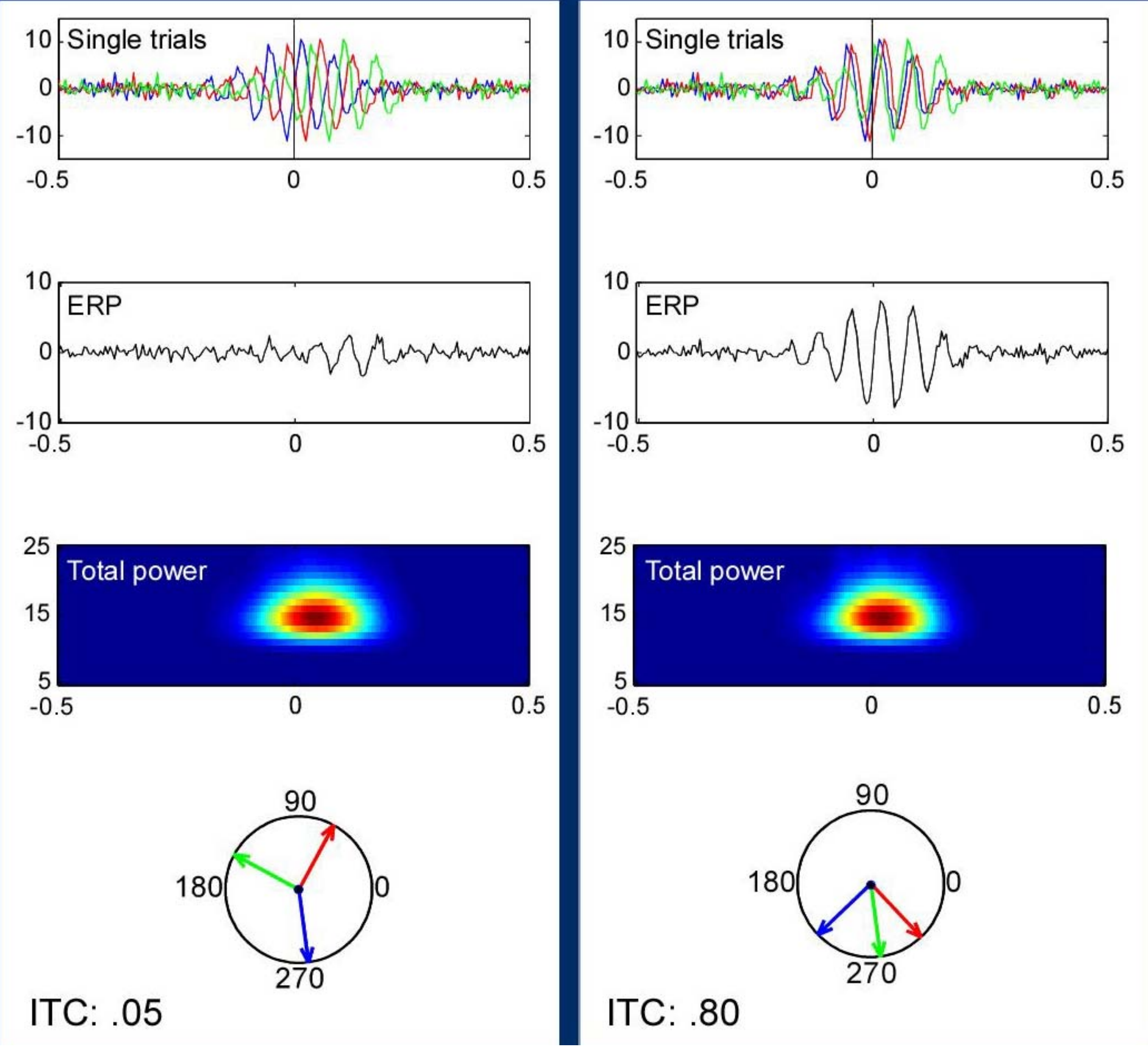
Wavelet (0.2)



Inter trial coherence



Intertrial Coherence (ITC)



Phase ITC

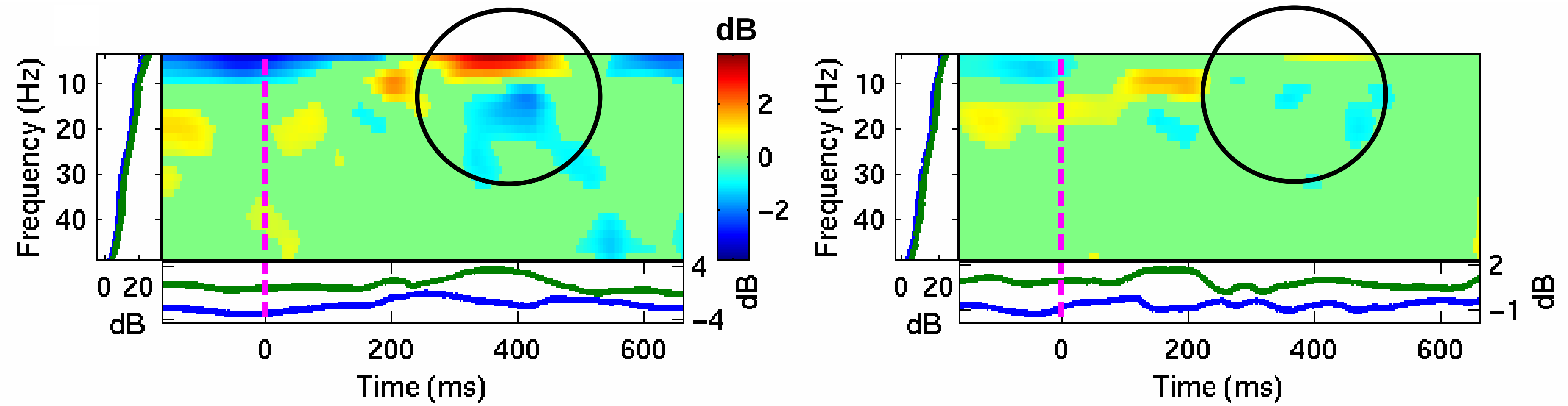
$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k(f,t)}{\left|F_k(f,t)\right|}$$

Normalized
(no amplitude information)

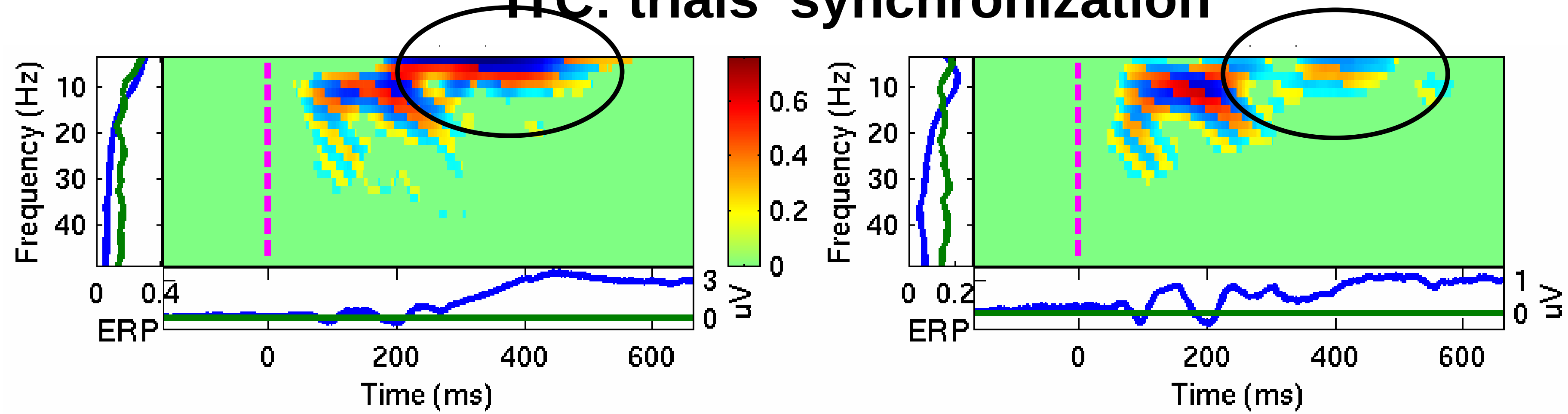
Power and inter-trial coherence

Time-frequency power

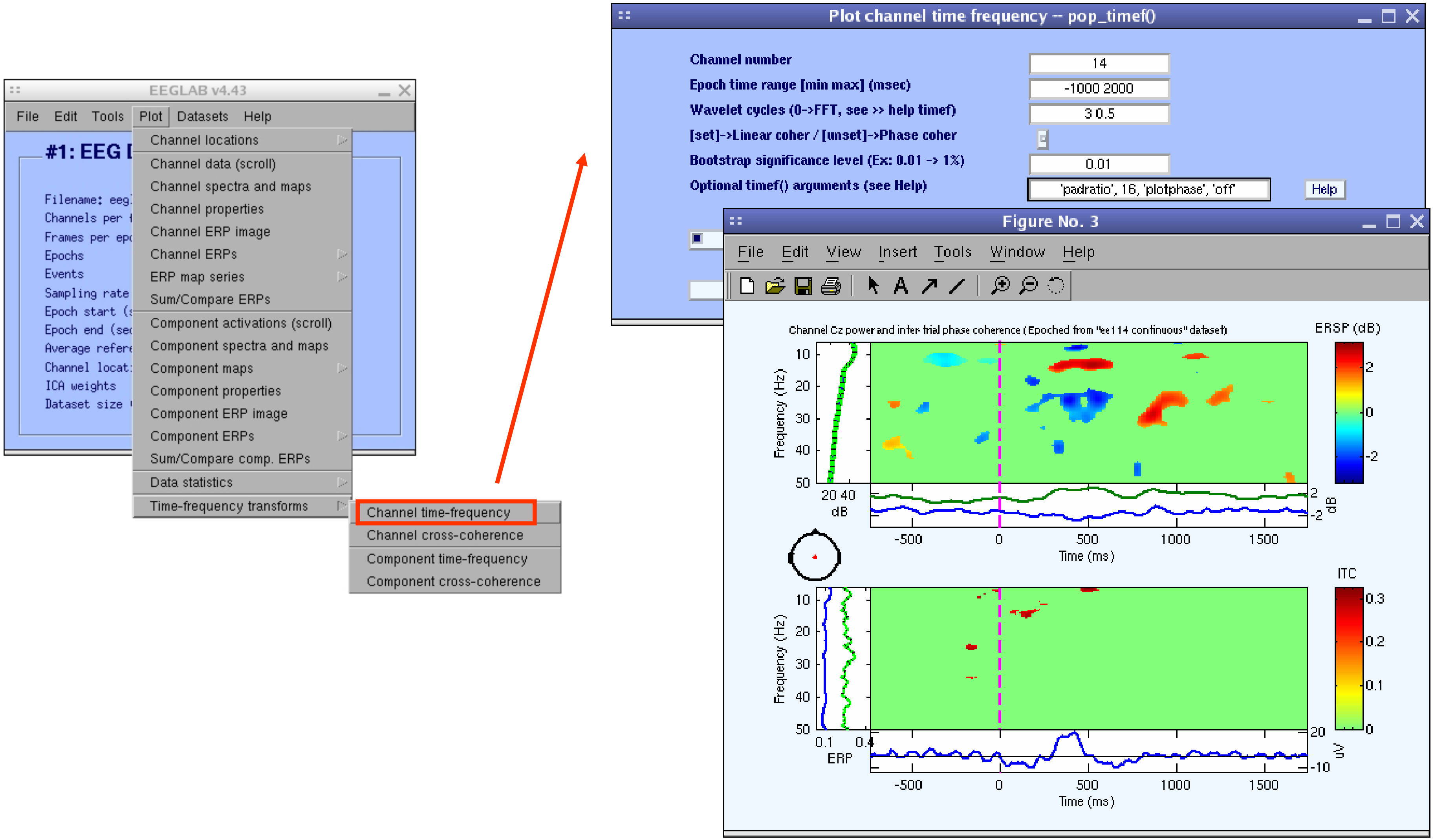
Condition 1 *Condition 2*



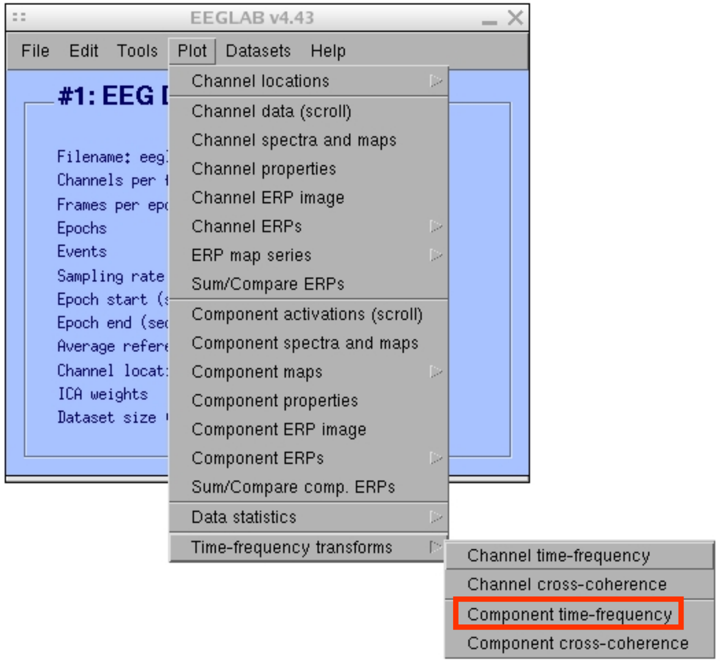
ITC: trials' synchronization

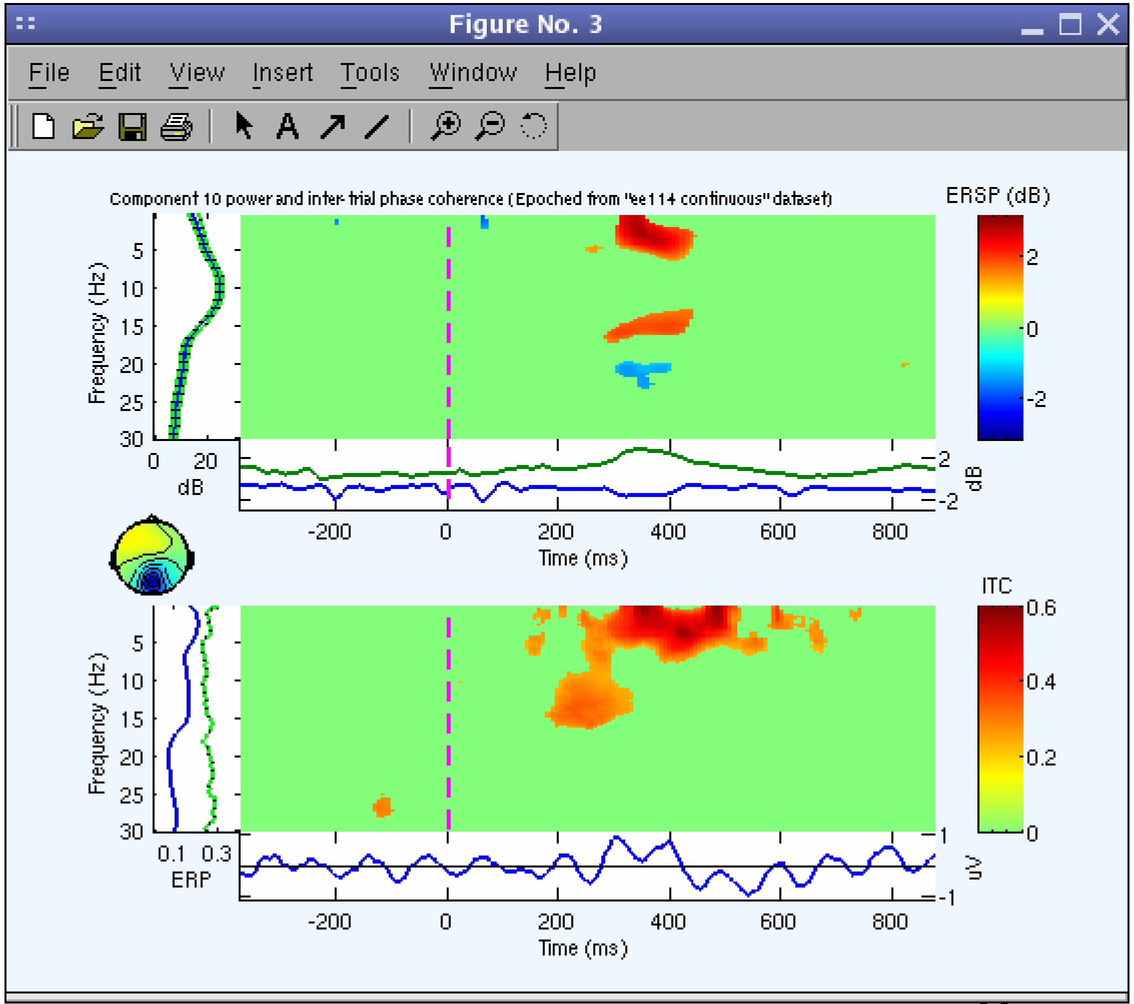


Channel time-frequency

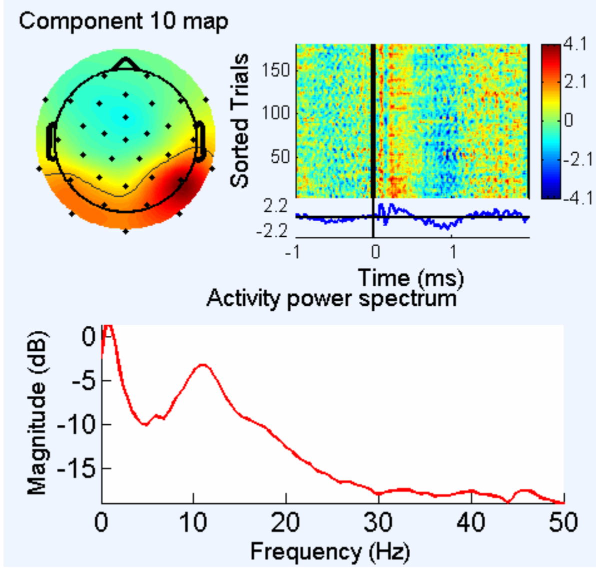


Component time-frequency

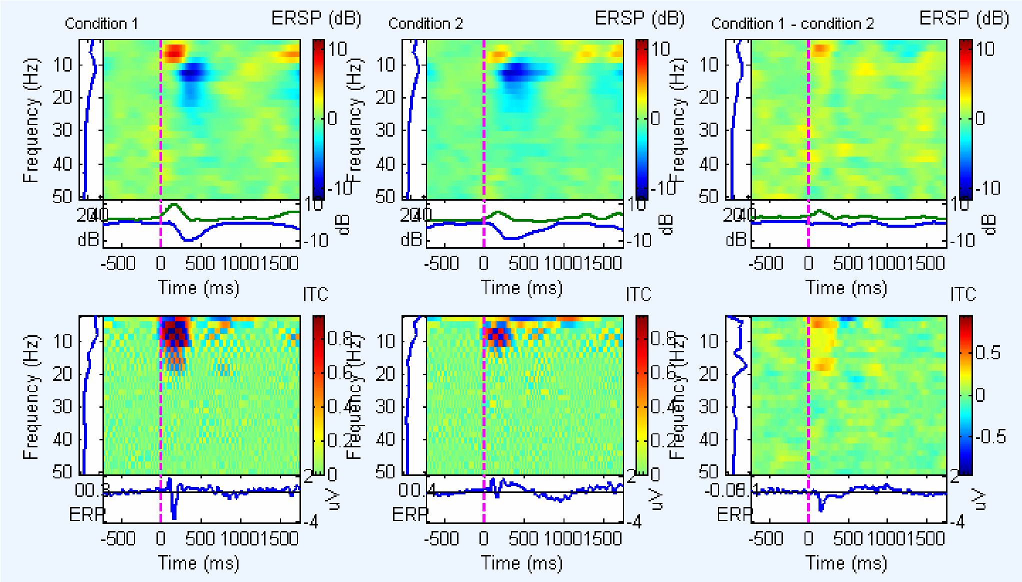




Compare between conditions



condition 1 condition 2 difference



ERSPs

ITCs

Component 10 for condition 1 (left) and condition 2 (right)

Number of data points

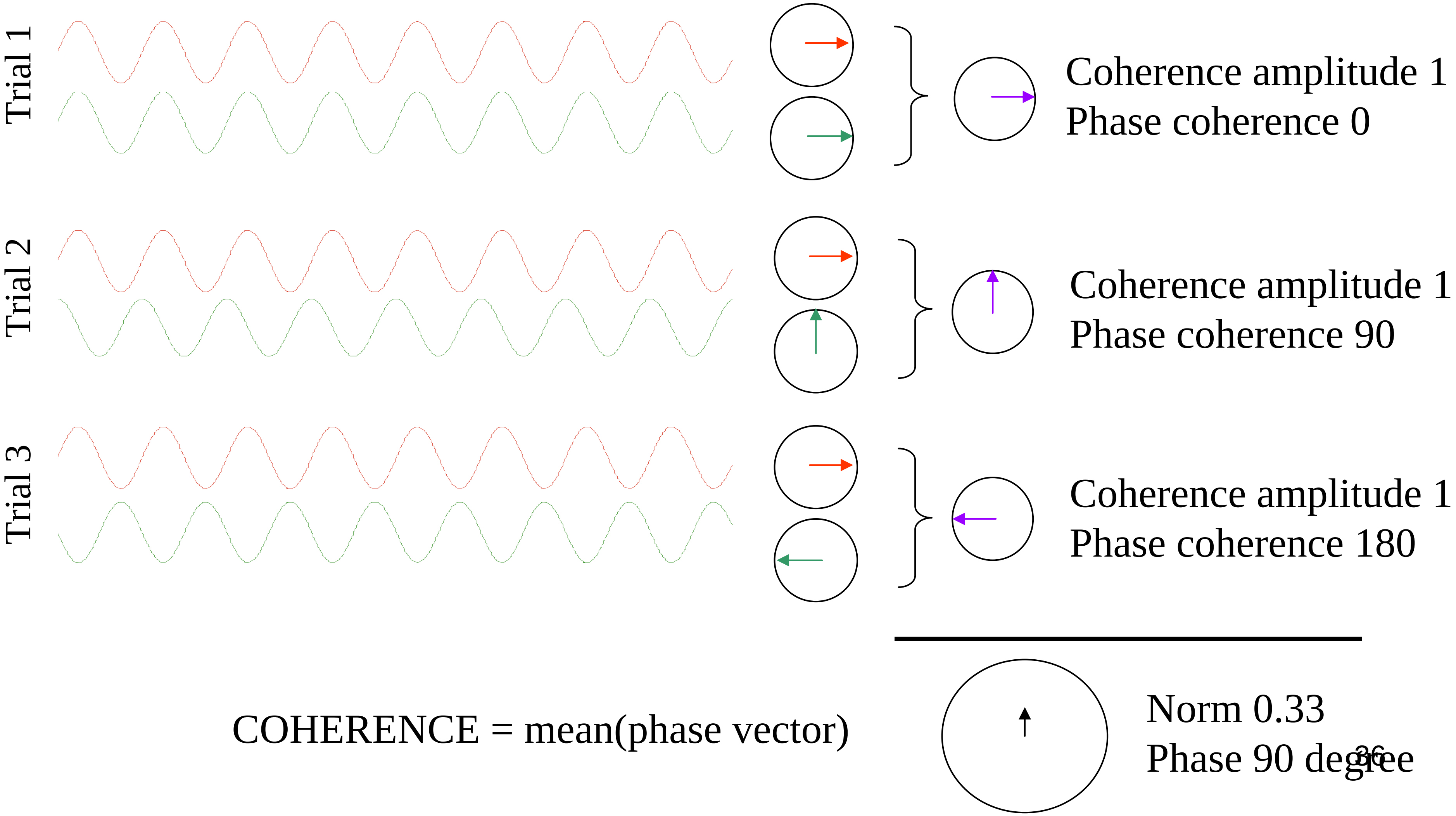
>> newtimef({ ALLEEG(2).icaact(10,:) EEG.icaact(10,:) }, EEG.pnts, …
[EEG.xmin EEG.xmax]*1000, EEG.srate, 0, 'padratio', 1);

Number of data points *Sampling rate* *Cycles (0=FFT)* *padding*

Do the activities of maximally independent EEG domains interact ?

Cross-coherence amplitude and phase

2 components, comparison on the same trials



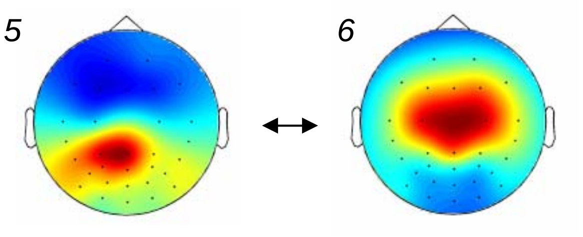
Phase coherence (default)

$$ERPCOH^{a,b}(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k^a(f,t)F_k^b(f,t)^*}{\left|F_k^a(f,t)\right|\left|F_k^b(f,t)\right|}$$

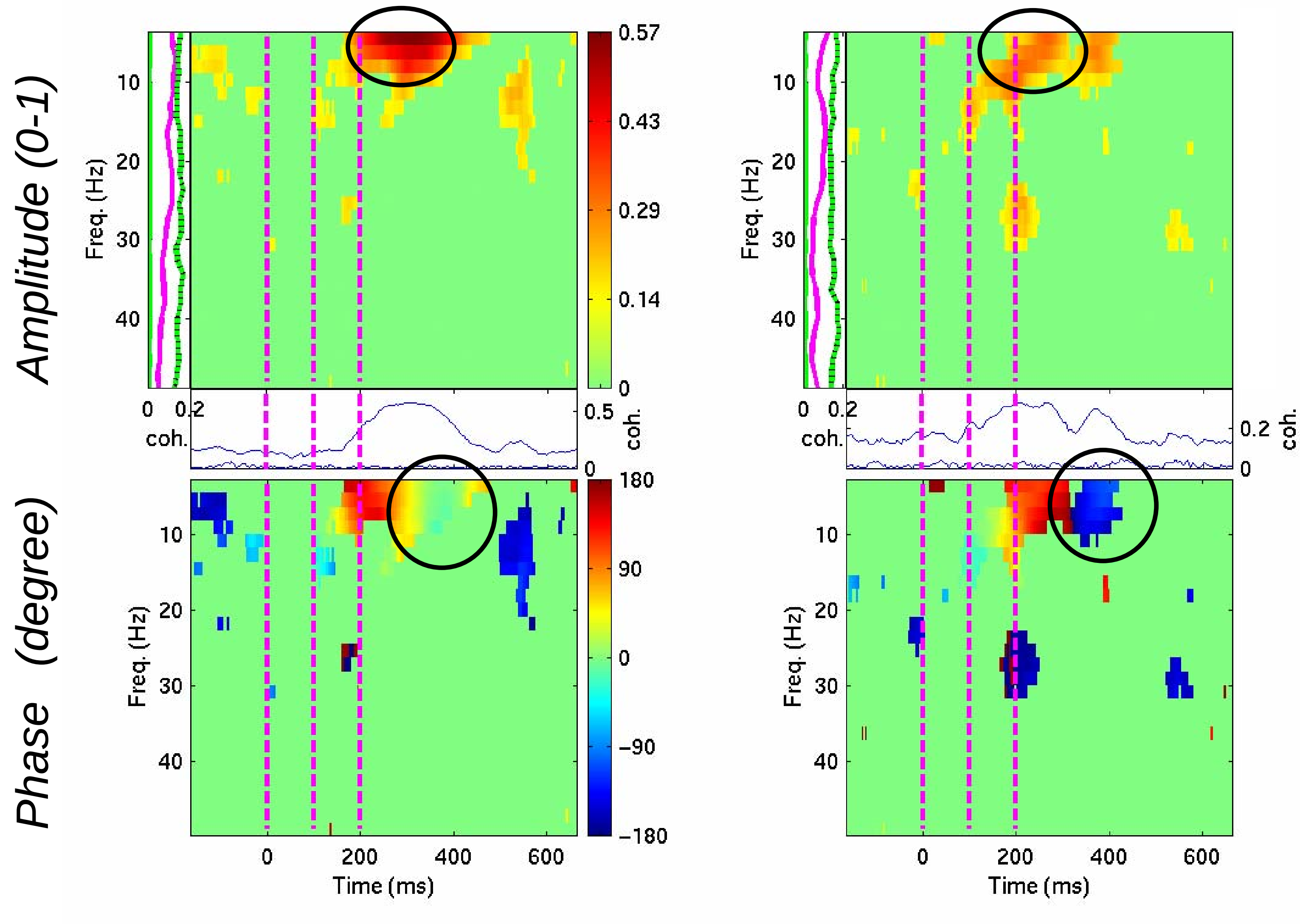
Only phase information component b

Only phase information component a

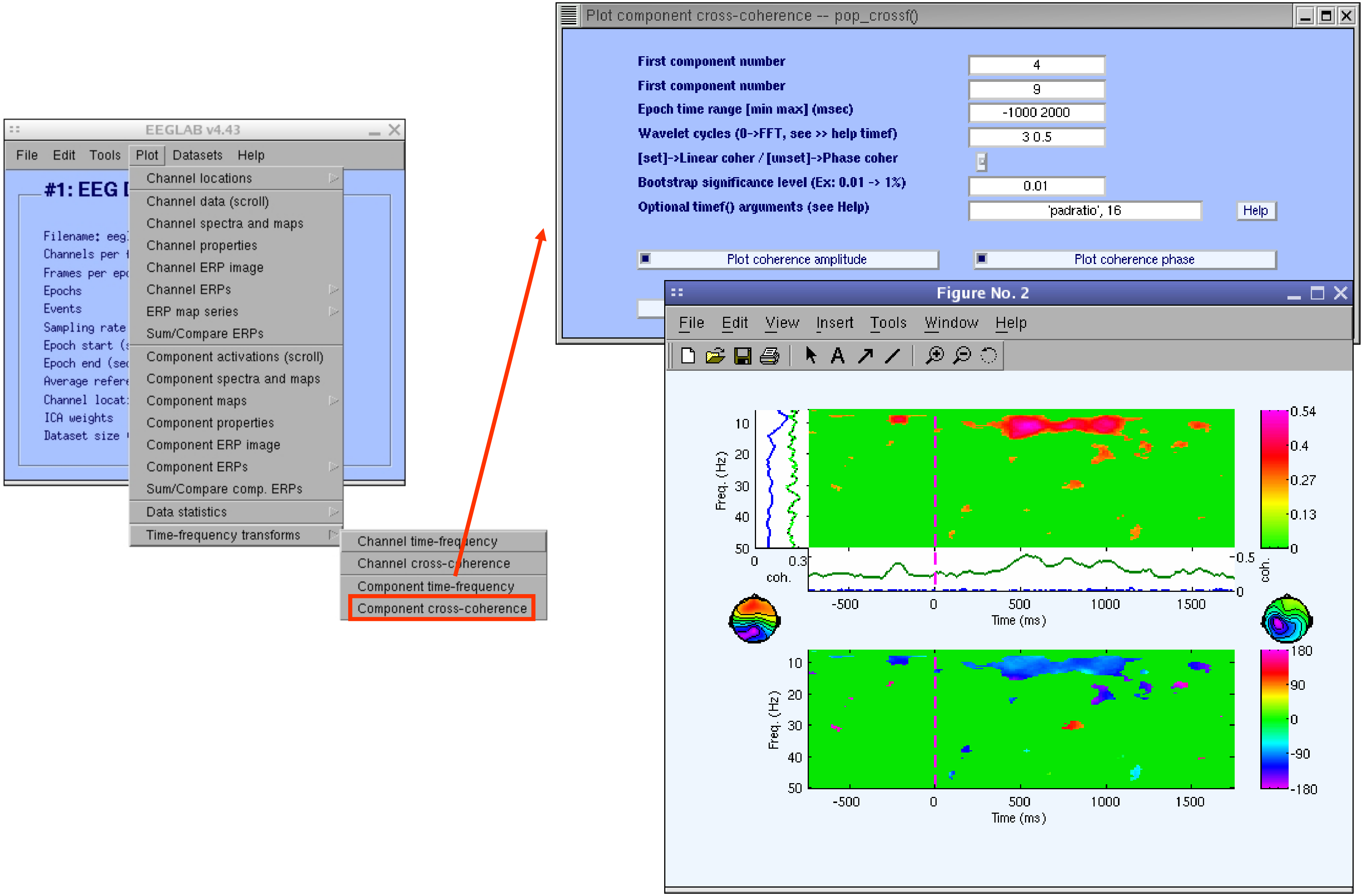
Cross-coherence amplitude and phase



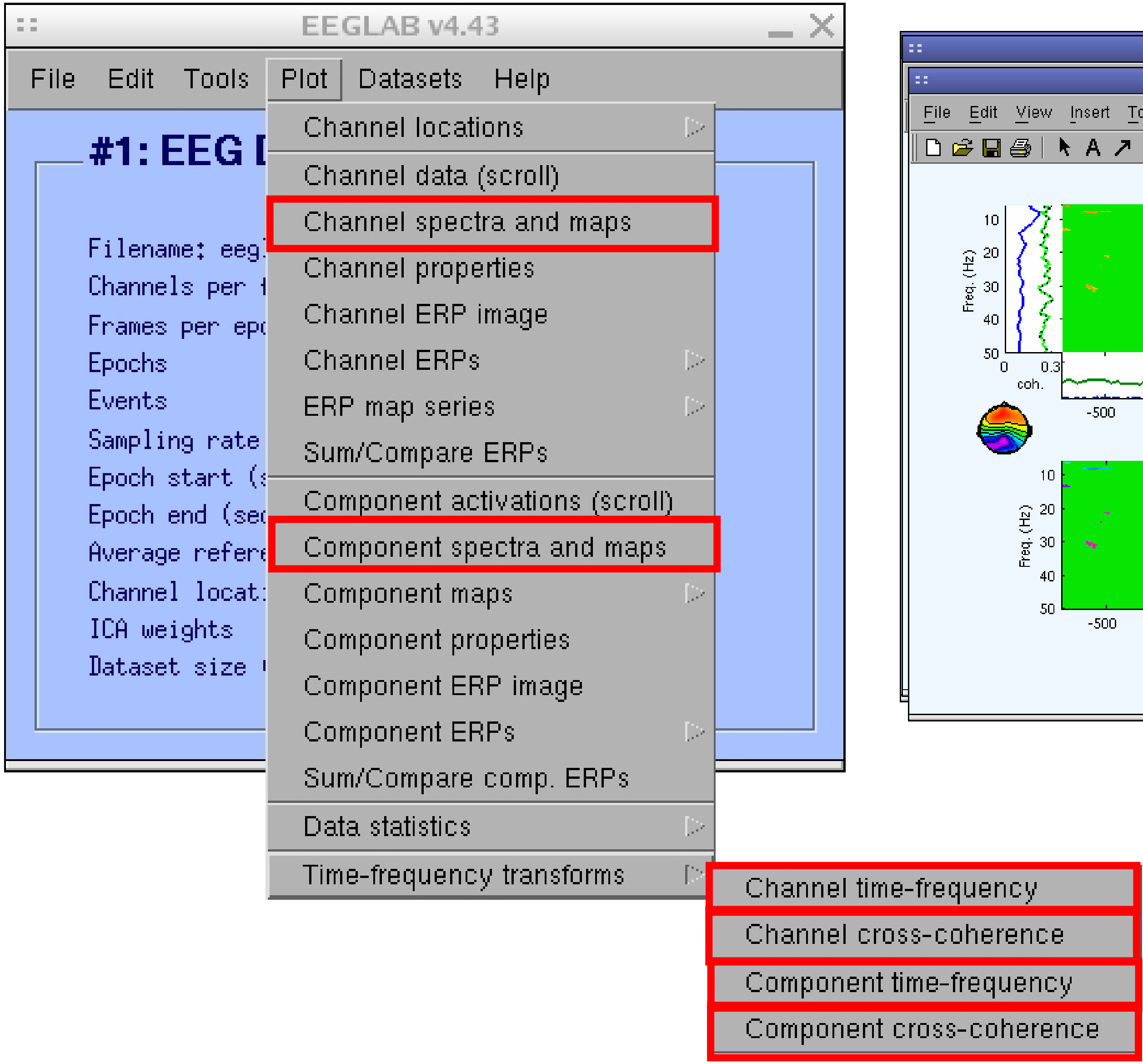
Condition 1 *Condition 2*



Component phase coherence



Summary


EEGLAB v4.43
File Edit Tools Plot Datasets Help
#1: EEG D
Filename: eeg
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Frames per epo
Epochs
Events
Sampling rate
Epoch start (s
Epoch end (sec
Average refere
Channel locat
ICA weights
Dataset size
Channel locations
Channel data (scroll)
Channel spectra and maps
Channel properties
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Component activations (scroll)
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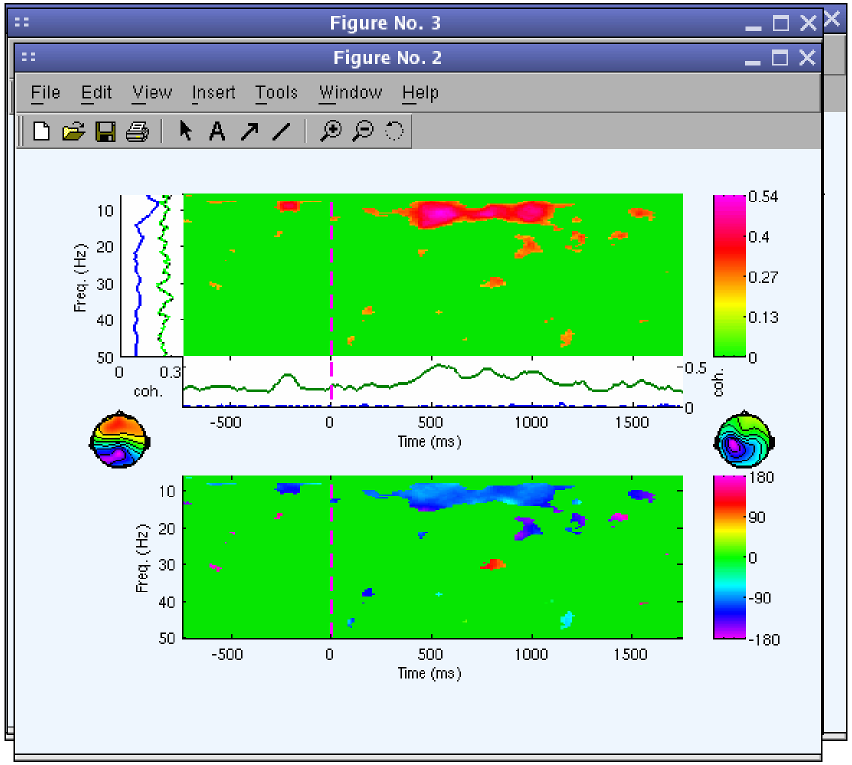
Figure No. 3
Figure No. 2
File Edit View Insert Tools Window Help
Freq. (Hz)
coh.
Time (ms)