A Two-Stage Based Approach for Extracting Periodic Signals

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Abstract. In many applications, such as biomedical engineering, it is often required to obtain specific periodic source signals. In this paper, we propose a two-stage based approach for extracting periodic signals. At the first stage, the autocorrelation property of the desired source signal is exploited to roughly extract the desired source signal. At the second stage, the extracted signal is further processed as cleanly as possible, based on the higher-order statistics. Simulations on artificially generated data and real-world ECG data have showed its better performance, compared with many existing extraction algorithms.

1 Introduction

Blind source extraction (BSE) [1] is a powerful technique that is closely related to blind source separation (BSS) [2]. The basic task of BSE is to estimate some components of source signals that are linearly combined in observations.

Compared with BSS, BSE has many advantages [1]. One advantage is that it can extract only the "interesting" signals from noisy mixed signals by exploiting their desired properties. That is to say, it requires certain additional *a priori* information of the desired source signal. Thus it generally is implemented in a semi-blind way. In many applications [3–6], such as the fetal ECG extraction [7, 8], the desired source signal is periodic or quasi-periodic. Therefore the period property can be used to extract the desired source signals.

Barros and Cichocki [3] first proposed an algorithm that can quickly extract the desired source signal with a specific period. But the algorithm's performance strongly depends on the precise estimation of an optimal time delay. In addition, the literature did not provide methods to find the optimal time delay, and only used the fundamental or multiple period of the desired source signal as the optimal time delay. In fact, the fundamental or multiple period is not necessarily the optimal time delay [7].

To overcome the drawbacks, several approaches recently have been proposed. Jafari *et al.* [6] proposed a fast algorithm that can instantaneously extract all the periodic source signals from the mixtures. It only needs to know the period

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of one of the source signals to extract, without knowing the optimal time delay. Another advantage is its tolerance of large estimate errors of the period. But its performance degrades if the period is not small enough, or if the periods of the source signals are close to each other.

On the other hand, we proposed an extraction algorithm [8], whose performance is not affected by the value of the period of the desired source signal. But the algorithm also needs to know the optimal time delay. In practice, it also takes the fundamental period of the desired source signal as the optimal time delay. However, compared with the one in [3], the algorithm can achieve better extraction performance due to exploitation of higher-order statistics information, and to some extent it is tolerant of estimate errors of the period. But the algorithm may fail in some complicated situations (see simulations in Section 4).

Recently, Lu *et al.* [9] proposed the so-called constrained ICA algorithm, which needs to elaborately design a reference signal that is closely related to the desired source signal. However, to design such reference signal, one should obtain lots of *a priori* information, which is not available in many cases.

In this paper, we propose an extraction algorithm, which only needs to estimate the period of the desired signal, and it is non-sensitive to the estimate errors of the period. Simulations on the artificially generated data and real-world data have showed its validity and good performance.

2 Problem Statement

Suppose one observes an n-dimentional stochastic signal vector $\mathbf{x}$ that is regarded as the linear transformation of an m-dimensional *mutually independent* zero-mean and unit-variance source vector $\mathbf{s}$, i.e., $\mathbf{x} = \mathbf{A}\mathbf{s}$, where $\mathbf{A}$ is an unknown mixing matrix. The goal of source extraction is to find a vector $\mathbf{w}$ such that $y = \mathbf{w}^T\mathbf{x} = \mathbf{w}^T\mathbf{A}\mathbf{s}$ is an estimated source signal up to a scalar. To cope with ill-conditioned cases and to make algorithms simpler and faster, whitening is often used to transform the observed signals $\mathbf{x}$ to $\tilde{\mathbf{x}} = \mathbf{V}\mathbf{x}$ such that $E\{\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T\} = \mathbf{I}$, where $\mathbf{V}$ is a whitening matrix. For convenience, in the following discussion we assume that $\mathbf{x}$ are the whitened observed signals and $n = m$.

Since our goal is to extract the periodic source signal, we further assume the desired periodic signal s_i satisfies the following relations for a specific integer τ^*:

$$\begin{cases} E\left\{s_i(k)s_i(k-\tau^*)\right\} > 0 \\ E\left\{s_j(k)s_j(k-\tau^*)\right\} = 0 \quad \forall j \neq i \end{cases} \tag{1}$$

where s_j are other source signals, k is the time index, and τ^* is the so-called optimal time delay [3].

Ideally, under the constraint $\|\mathbf{w}\| = 1$, maximizing the objective function

$$J(\mathbf{w}) = E\{y(k)y(k-\tau^*)\} = \mathbf{w}^T E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\}\mathbf{w} \tag{2}$$

leads to perfect recovery of the desired periodic source signal s_i. The reason for this formulation is that for the desired signal s_i, this delayed autocorrelation has a large positive value, while for other source signals this value is zero.

Using the standard gradient approach and neglecting the small difference between $R_x(\tau^*)$ and $R_x(\tau^*)^T$, from the objective function (2) we can derive the Barros's algorithm [3]:

$$\begin{cases} \mathbf{w}^+ = R_x(\tau^*)\mathbf{w} \\ \mathbf{w} = \mathbf{w}^+/\|\mathbf{w}^+\| \end{cases} \tag{3}$$

where $R_x(\tau^*) = E\{\mathbf{x}(k)\mathbf{x}(k-\tau^*)^T\}$. Note here the Barros's algorithm is derived from a different aspect. The algorithm is simple and fast. However, some important practical issues should be considered.

One important issue is the optimal time delay τ^*. In most cases such optimal time delay does not exist. In other words, although the desired signal s_i is strongly autocorrelated at the time delay τ^*, several other source signals may be also autocorrelated at the delay τ^* (i.e., $E\{s_j(k)s_j(k-\tau^*)\} \neq 0, j \neq i$).

Another issue is the effect of finite samples [7, 10]. Even if the source signals are strictly mutually uncorrelated, in practice the calculated correlations of source signals using limited samples are generally non-zero, due to the fact that the expectation operator is replaced by the mathematical average. That is to say, even if $E\{s_i(k)s_j(k-\tau^*)\} = 0$ and $E\{s_j(k)s_j(k-\tau^*)\} = 0, j \neq i$, it is very possible that $\sum_{k=\tau^*}^{N-1} s_i(k)s_j(k-\tau^*)/(N-\tau^*) \neq 0$ and $\sum_{k=\tau^*}^{N-1} s_j(k)s_j(k-\tau^*)/(N-\tau^*) \neq 0$.

According to the results in [7], the performance of the Barros's algorithm (3) greatly degrades due to the joint effect of the above two issues.

The third issue is the exploitation of the higher-order statistics. In many applications the source signals are physically mutually independent. Therefore, suitable use of the higher-order statistics, rather than only using the second-order statistics, is expected to improve the extraction performance.

Considering the above issues, in the next section we will propose an efficient two-stage algorithm, which achieves better performance than many existing algorithms.

3 Proposed Algorithm

3.1 Framework of the Proposed Algorithm

First, we estimate the fundamental period τ of the desired source signal. For estimating τ, there are many methods, such as the autocorrelation method [3], and the instantaneous frequency estimation technique [4]. In addition, in some applications, e.g., biomedical signal processing, this type of information is often readily available [3, 5, 9]. Note that τ is not necessarily the optimal time delay.

The following procedure is roughly divided into two stages. The first stage is called the capture stage. In this stage, the algorithm coarsely extracts the desired source signal by using the estimated period. After the algorithm converges, we obtain the weight vector $\hat{\mathbf{w}}$. But due to some reasons (discussed below), $\hat{\mathbf{w}}$ is only close to the ideally optimal weight vector $\mathbf{w}_*$ (in the sense that the extracted desired source signal $y_* = \mathbf{w}_*^T\mathbf{x}$ is not mixed by any crosstalk noise). Therefore the captured source signal $\hat{y} = \hat{\mathbf{w}}^T\mathbf{x}$ is mixed by some noise and interference.

Next, in the second stage, we run the fixed-point algorithm [1, 11] on the original mixtures, using $\hat{\mathbf{w}}$ as its initial weight vector. The initial weight vector $\hat{\mathbf{w}}$ can ensure the fixed-point algorithm converges to the sub-optimal solution $\bar{\mathbf{w}}$, which is much closer to $\mathbf{w}_*$ than $\hat{\mathbf{w}}$ is. Then we finally obtain the estimated desired source signal $\bar{y} = \bar{\mathbf{w}}^T\mathbf{x}$, which is almost not mixed by any interference.

3.2 The First Stage: Coarse Capture

In the first stage, the goal is to roughly extract the desired source signal, exploiting its autocorrelation structure. First, consider the objective function (2):

$$\begin{aligned} J(\mathbf{w}) &= \frac{1}{2}J(\mathbf{w}) + \frac{1}{2}J(\mathbf{w})^T \\ &= \frac{1}{2}\mathbf{w}^T E\{\mathbf{x}(k)\mathbf{x}(k-\tau)^T\}\mathbf{w} + \frac{1}{2}\mathbf{w}^T E\{\mathbf{x}(k-\tau)\mathbf{x}(k)^T\}\mathbf{w} \\ &= \frac{1}{2}\mathbf{w}^T\Big(\mathbf{R}_x(\tau) + \mathbf{R}_x(\tau)^T\Big)\mathbf{w} \\ &= \frac{1}{2}\mathbf{w}^T\overline{\mathbf{R}}\mathbf{w}, \end{aligned} \tag{4}$$

which implies that under the constraint $\|\mathbf{w}\| = 1$ maximizing (2) is equivalent to finding the eigenvector corresponding to the maximal eigenvalue of the real symmetric matrix $\overline{\mathbf{R}}$. Thus we directly obtain the following algorithm:

$$\mathbf{w} = EIG(\overline{\mathbf{R}}), \tag{5}$$

where $EIG(\overline{\mathbf{R}})$ is the operator that calculates the normalized eigenvector corresponding to the maximal eigenvalue of the real symmetric matrix $\overline{\mathbf{R}}$. After convergence, the algorithm gives the solution $\hat{\mathbf{w}}$.

If τ is the optimal time delay, then the solution $\hat{\mathbf{w}}$ is just the optimal solution $\mathbf{w}_*$, and the captured signal $\hat{y} = \hat{\mathbf{w}}^T\mathbf{x}$ is just the desired source signal, without any noise or distortion. However, the optimal time delay generally does not exist, then the optimal solution $\mathbf{w}_*$ is ideal. Therefore, in fact the solution $\hat{\mathbf{w}}$ is only near $\mathbf{w}_*$, and the captured desired source signal is noisy.

Note that if $\hat{\mathbf{w}}$ is not close enough to $\mathbf{w}_*$, the algorithms in the second stage may converge to other local maxima, and thus cannot obtain the desired source signal. Therefore we should make $\hat{\mathbf{w}}$ approximate $\mathbf{w}_*$ as closely as possible. In [7] we have showed that the larger the autocorrelations of undesired source signals at the delay τ are, and/or the larger the absolute value of the cross-correlation between any two source signals at the delay τ is, the farther $\hat{\mathbf{w}}$ deviates from $\mathbf{w}_*$. To ensure $\hat{\mathbf{w}}$ is close enough to $\mathbf{w}_*$, we modify the former objective function (2) as follows:

$$J(\mathbf{w}) = \mathbf{w}^T\Big\{\sum_{l=1}^{P}\big(\mathbf{R}_x(l\tau) + \mathbf{R}_x(l\tau)^T\big)\Big\}\mathbf{w}, \tag{6}$$

and its corresponding algorithm is given by

$$\mathbf{w} = EIG\Big(\sum_{l=1}^{P}\Big(\mathbf{R}_x(l\tau) + \mathbf{R}_x(l\tau)^T\Big)\Big), \tag{7}$$

where P is a positive integer, and τ is the fundamental period of the desired source signal. The algorithm is based on the averaged eigen-structure of correlation matrices of source signals over multi-delays. In [7] we have showed that both the averaged auto-correlations of undesired source signals and the averaged absolute value of cross-correlation between any two source signals at the delay τ tend to zero with P increasing. Thus the converged solution $\hat{\mathbf{w}}$ is closer to $\mathbf{w}_*$, ensuring the successful fine extraction of the second stage.

3.3 The Second Stage: Fine Extraction

In this stage we use the one-unit fixed-point algorithm [11] to make the solution $\hat{\mathbf{w}}$ from the previous stage further close to the optimal solution $\mathbf{w}_*$, which implies the extracted source signal is cleaner, with less noise and interference. $\hat{\mathbf{w}}$ is taken as the initial weight value of the fixed-point algorithm:

$$\begin{cases} \mathbf{w}^+ = E\{\mathbf{x}(\mathbf{w}^T\mathbf{x})^3\} - 3\mathbf{w} \\ \mathbf{w} = \mathbf{w}^+/\|\mathbf{w}^+\|. \end{cases} \tag{8}$$

To improve the robustness to outliers and spiky noise, we can adopt the modified fixed-point algorithm [1]:

$$\begin{cases} \mathbf{w}^+ = \frac{E\{y^3\mathbf{x}\}}{E\{y^4\}} - \mathbf{w} \\ \mathbf{w} = \mathbf{w}^+/\|\mathbf{w}^+\|. \end{cases} \tag{9}$$

When the algorithm (8) or (9) has converged, we obtain the solution $\bar{\mathbf{w}}$, which is much closer to the optimal solution $\mathbf{w}_*$ than $\hat{\mathbf{w}}$ is. Therefore, we finally get the estimated desired source signal $\bar{y} = \bar{\mathbf{w}}^T\mathbf{x}$.

4 Simulations

In the first simulation, we generated five zero-mean and unit-variance source signals, shown in Fig.1(a). Each signal has 3000 samples. Three signals are periodic (or quasi-periodic), respectively given by $s_1(k) = \sin(2\pi f_1 k + 6\cos(2\pi 200k))$, $s_2(k) = \cos(2\pi f_2 k)$, and $s_3(k) = \cos(2\pi f_3 k+2)$, where $t_s = 1\times 10^{-4}$, $f_1 = 0.061$, $f_2 = 0.054$, and $f_3 = 0.028$. t_s is the sampling period, and $f_i(i = 1, 2, 3)$ are normalized frequencies. The other two signals are Gaussian noise. Note that s_1 is the desired source signal, and its fundamental period is assumed to be known.

The source signals were randomly mixed (Fig.1(b)). After whitening the mixed signals, we ran the Barros's algorithm [3], the algorithm in [8], the one in [7] and the proposed one in this paper. The results are shown in Fig.1(c), from which it is clear to see that the algorithms in [3] and the one in [8] extracted the wrong source signal, while the other two algorithms correctly extracted the desired source signals. To evaluate the extraction performance of the two algorithms that obtained the correct signal, we adopted the following measure:

$$PI = -10E\{lg(s(k) - \tilde{s}(k))^2\}, \quad (dB) \tag{10}$$

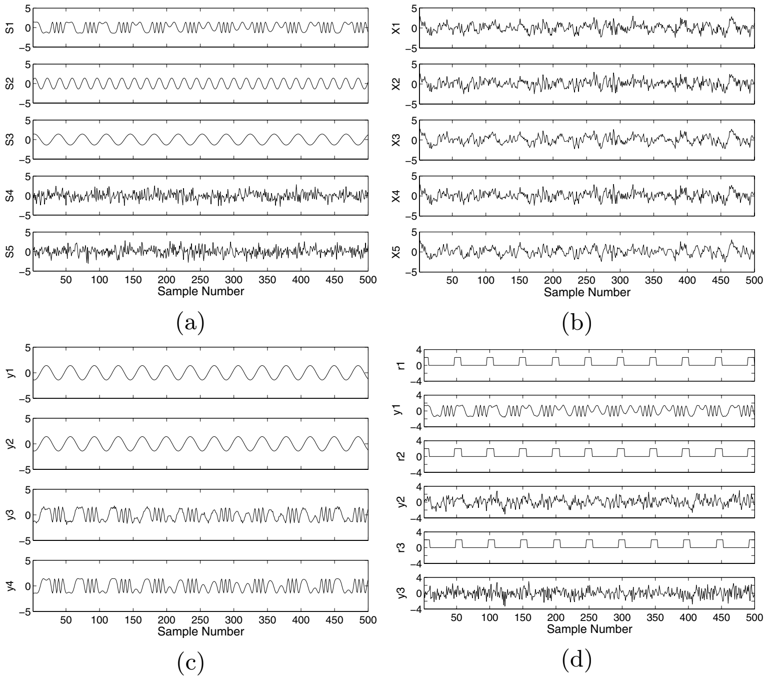

Fig. 1. Simulation on artificially generated data. (a) The five source signals. (b) The randomly mixed signals. (c) The extracted signals, respectively by the algorithm in [3] (y_1), the one in [8] (y_2), the one in [7] (y_3) and the proposed one in this paper (y_4). (d) The reference signals (r_1, r_2, r_3) and the corresponding extracted signals (y_1, y_2, y_3) by the constrained ICA [9].

where $s(k)$ was the desired source signal, and $\tilde{s}(k)$ was the extracted signal (both of them were normalized to be zero-mean and unit-variance). The higher PI was, the better the performance was. The averaged performance over 400 independent running of the algorithm in [7] and of the proposed algorithm was, respectively, 23.4 dB and 48.9 dB. The results showed the proposed algorithm has better extraction performance.

Next, we used the constrained ICA [9] to extract the desired source signal. Fig.1(d) shows the results, where r_1, r_2, r_3 are the very similar reference signals, having the same fundamental period. The rectangular pulse width of r_2 is larger than that of r_1 by only a sampling period, while keeping the same pulse occurrence time. r_3 has the same rectangular pulse width as r_1, but is delayed by a sampling period. y_1, y_2 and y_3 are the extracted signals, respectively by using the reference signal r_1, r_2 and r_3. Clearly, only y_1 was well recovered. This means that the algorithm's performance is greatly affected by the reference signal. To achieve good performance, the elaborately designed reference signal is necessary, which cannot be obtained in many cases. In addition, the selection of some parameters of the algorithm is crucial to the algorithm.

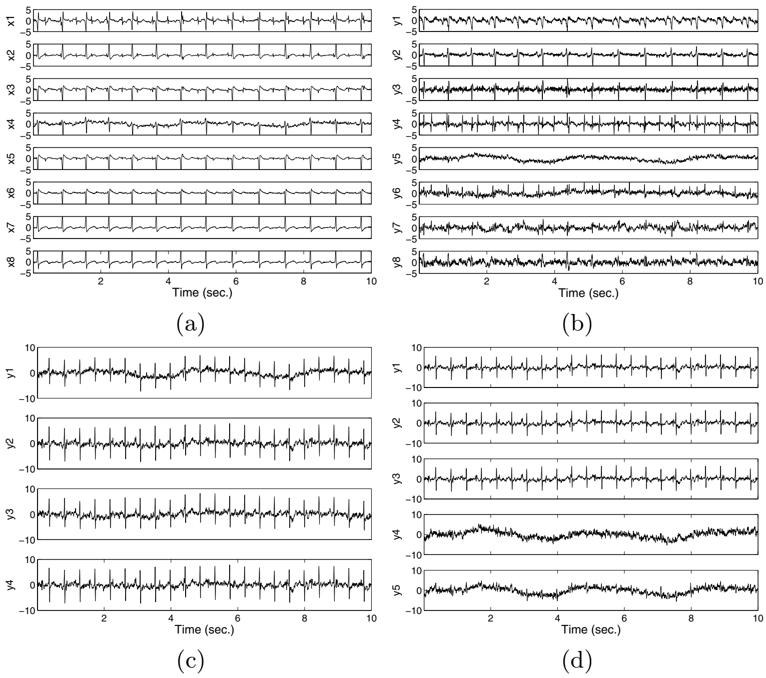

Fig. 2. Simulation on real-world ECG data. (a) The ECG data. (b) The separated signals by [6]. (c) The extracted signals, respectively by the algorithm in [3], the one in [8], the one in [7] and the proposed one by this paper, from the top down. (d)The extracted signals when the estimate errors were introduced. y_1-y_3 were extracted by the proposed algorithm, while y_4 and y_5 by the Barros's algorithm [3].

Next we used the real-world ECG data [12] (Fig.2(a)). Our goal was to extract the fetal ECG, which was very weak and almost only visible in x_1. Using the method in [8] we estimated that the period of the fetal ECG was 112 sampling periods. After whitening the sensor signals, we respectively ran the algorithm in [6], the one in [3], the one in [8], the one in [7] and the proposed one. The results are shown in Fig.2(b) and (c). We can see that the algorithm in [6] had poor performance. The reason is that the period of the fetal ECG was not small, violating the basic assumption of the algorithm. In addition, the extracted fetal ECG by the Barros's algorithm [3] also showed the poor performance.

In practice, the estimate errors of the period of the desired source signal are inevitable. Suppose the estimated period of the Fetal ECG deviates from its true value. The extraction results are shown in Fig.2(d), where y_1, y_2, y_3 were the corresponding extracted signals by the proposed algorithm when the estimated period was 108, 114, and 118 sampling periods, respectively. y_4, y_5 were the extracted signals by the Barros's algorithm [3] when the estimated period was 110 and 113 sampling periods, respectively. Clearly, the proposed algorithm is not sensitive to the estimate errors.

5 Conclusions

In this paper we present a two-stage algorithm for extracting periodic signals. It converges quickly, due to the use of efficient eigenvalue decomposition methods at the first stage and the fast fixed-point algorithm in the second stage. And it has good extraction performance. Furthermore, the algorithm is tolerant of estimate errors of the period of the desired source signal. In addition, the algorithm does not need to design the so-called reference signals.

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References

1. Cichocki, A., Amari, S.: Adaptive Blind Signal and Image Processing: Learning Algorithms and Applications. John Wiley & Sons, New York (2002)
2. Cruces-Alvarez, S.A., Cichocki, A., Amari, S.: From Blind Signal Extraction to Blind Instantaneous Signal Separation: Criteria, Algorithm, and Stability. IEEE Trans. Neural Networks **15 (4)** (2004) 859-873
3. Barros, A.K., Cichocki, A.: Extraction of Specific Signals with Temporal Structure. Neural Computation **13 (9)** (2001) 1995-2003
4. Barros, A.K., Ohnishi, N.: Heart Instantaneous Frequency (HIF): An Alternative Approach to Extract Heart Rate Variability. IEEE Trans. Biomedical Engineering **48 (8)** (2001) 850-855
5. Hansen, L.K., Nielsen, F.Å., Larsen, J.: Exploring FMRI Data for Periodic Signal Components. Artificial Intelligence in Medicine **25 (1)** (2002) 35-44
6. Jafari, M.G., *et al.*: Sequential Blind Source Separation Based Exclusively on Second Order Statistics Developed for a Class of Periodic Signals. IEEE Trans. Signal Processing (in press)
7. Zhang, Z.-L., Yi, Z.: Robust Extraction of Specific Signals with Temporal Structure. Neurocomputing (in press)
8. Zhang, Z.-L., Yi, Z.: Extraction of Temporally Correlated Sources with Its Application to Non-invasive Fetal Electrocardiogram Extraction. Neurocomputing (in press)
9. Lu, W., Rajapakse, J.C.: Approach and Applications of Constrained ICA. IEEE Trans. Neural Networks **16 (1)** (2005) 203-212
10. Bermejo, S.: Finite Sample Effects in Higher Order Statistics Contrast Functions for Sequential Blind Source Separation. IEEE Signal Processing Letters **12 (6)** (2005) 481-484
11. Hyvärinen, A., Oja, E.: A Fast Fixed-Point Algorithm for Independent Component Analysis. Neural Computation **9 (7)** (1997) 1483-1492
12. De Moor, D.(eds.): Daisy: Database for the Identification of Systems. Available online at: http://www.esat.kuleuven.ac.be/sista/daisy