Supplementary Material to 'Recovery of Block Sparse Signals Using the Framework of Block Sparse Bayesian Learning'

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I. Download Links of the Compared Algorithms in the Experiments

- Block-OMP: downloaded from `http://igorcarron.googlepages.com/GroupSparseBox.zip`
- Block-CoSaMP: `http://dsp.rice.edu/software/model-based-compressive-sensing-toolbox`[1]
- CLuSS-MCMC: downloaded from `https://sites.google.com/site/link2yulei/cs`
- DGS: downloaded from `http://paul.rutgers.edu/~jzhuang/R_DGS.htm`
- Group Lasso: downloaded from `http://www.public.asu.edu/~jye02/Software/SLEP/index.htm`
- T-MSBL: downloaded from `http://dsp.ucsd.edu/~zhilin/TMSBL.html`
- Subspace Pursuit: downloaded from `http://igorcarron.googlepages.com/CSRec_SP.m`

II. Additional Experiment: Algorithms' Performance vs. SNR

Now we test algorithms' performance at different noise levels. Since Block-OMP and Block-CoSaMP are only suitable for noiseless cases, we excluded the two algorithms. Both the Cluster-SBL (Type I) and Cluster-SBL (Type 2) were compared here. In addition, we also performed Cluster-SBL (Type I) when it ignored the intra-block correlation. The matrix $\mathbf{\Phi}$ had the size of 80×162. The signal was partitioned into 27 blocks with identical sizes, only 4 of them were nonzero blocks. The intra-block correlation of each block was generated as in the paper [1] with $\beta = 0.95$. Gaussian white noise was added such that the SNR varied from 5 dB to 30 dB. The experiment results are shown in Fig.1. We can see our proposed algorithms had the best performance.

III. Application to Maternal-Fetal ECG Telemonitoring via Wireless Body-Area Networks

Compressed sensing has promising applications to ECG telemonitoring via wireless body-area networks since it can compress data for wireless transfer with low power-consuming. However, applying compressed sensing to maternal-fetal ECG telemonitoring is a big challenge. This is because to record fetal ECG the sensors should be put on the mother's abdomen, where strong noise (e.g. noise from muscle movements) is also recorded. This means in the compressed sensing model:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{v}, \tag{1}$$

although the measurement noise $\mathbf{v}$ is very small, the signal $\mathbf{x}$ itself contains large noise. Also, since $\mathbf{x}$ contains maternal ECG, fetal ECG and artifacts, it is much less sparse. So, recovering the fetal ECG (the main goal in this application) is a very difficult task for compressed sensing. The bottom picture in Fig.2 (a) shows a recorded signal with 500 samples from a pregnant woman's abdomen, where the downarrows indicate the fetal ECG.

Now we show our proposed algorithms have ability to recover the fetal ECG. The random Gaussian matrix Φ was of the size 200×500. Note that the block partition and block sizes of the signal were unclear (and unknown). For Cluster-SBL (Type II), we set $h = 15$ (the value was randomly chosen). Cluster-SBL (Type I) was also used here by assuming the signal could be approximated by a block sparse signal which was partitioned into 20 blocks with identical size 25 (the size was also randomly chosen). The experiment was repeated 100 trials.

The recovered signals by Cluster-SBL (Type I) and (Type II) in one trial are shown in Fig.2 and Fig.3, from which we can see the two proposed algorithms, if exploiting intra-block correlation, recovered the fetal ECG with high quality (indicated by the downarrows). Clearly, if not exploiting the intra-block correlation, the Cluster-SBL (Type I) could not recover the fetal ECG. The results in other 99 trials were similar to this (the Type I successed in all the 99 trials, but the Type II algorithm failed 13 times [2]).

Fig.4-Fig.5 show the recovered signals by Block-OMP (implemented for noiseless cases, since here the measurement noise $\mathbf{v}$ can be ignored.), Mixed ℓ_2/ℓ_1, CluSS-MCMC, and T-MSBL [3]. Clearly, all these algorithms failed to recover the fetal ECG.

We also chose other forms of $\mathbf{\Phi}$. For example, let $\mathbf{\Phi} = \mathbf{\Psi\Theta}$, where $\mathbf{\Psi}$ was a Gaussian random matrix, and $\mathbf{\Theta}$ was an orthogonal matrix constructed from wavelet transforms (To construct orthogonal wavelet transform matrices, the maternal-fetal ECG signal contained 512 samples in this case. But the measurement number, M, kept unchanged.). However, no matter what wavelet transform used, all

[1] When it was used in noiseless experiments, its build-in parameter 'tol = 1e-3' was changed to 'tol = 1e-10' for the best performance.

[2] Here we didn't use the MSE as performance index. This is because the signal itself contained noise, and thus a recovered signal with smaller MSE did not necessarily contain fetal ECG with higher quality.

[3] Note that although T-MSBL was derived for the multiple measurement vector model [2], it can be used for the basic compressed sensing model (1) as well.

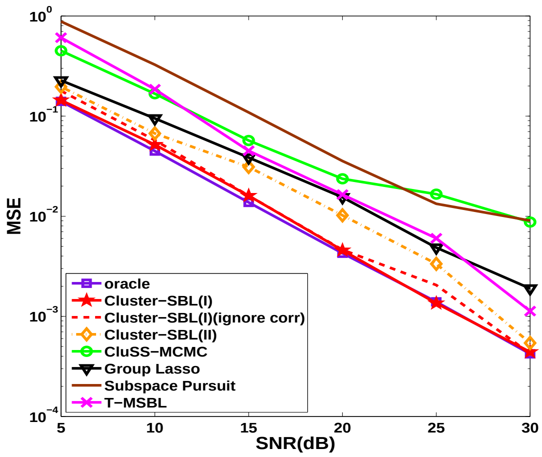

Fig. 1. Algorithm comparison at different noise levels.

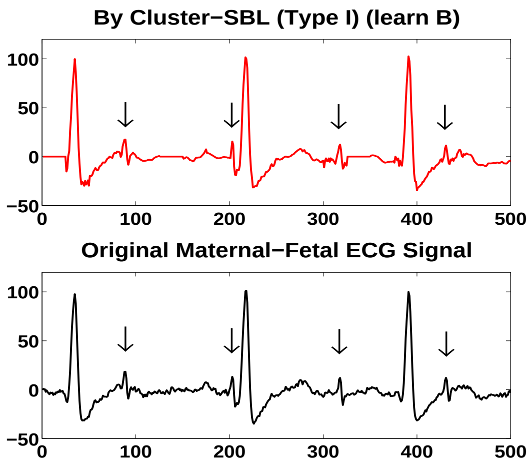

Fig. 2. (top) The ECG signal recovered by Cluster-SBL (Type I) exploiting intra-block correlation. (bottom) The original maternal-fetal ECG signal. Downarrows indicate peaks of the fetal ECG.

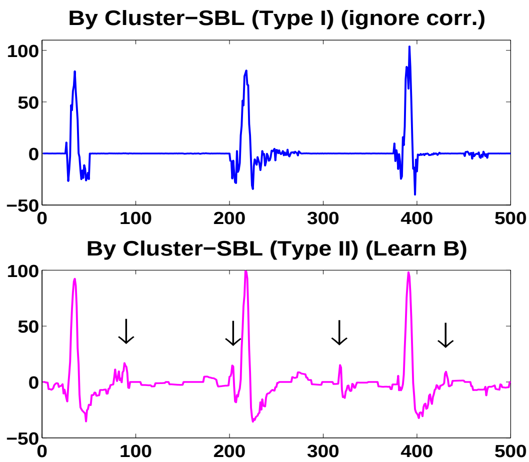

Fig. 3. (top) The signal recovered by Cluster-SBL (Type I) ignoring intra-block correlation. (bottom) The signal recovered by Cluster-SBL (Type II) exploiting the correlation. Downarrows indicate peaks of the fetal ECG.

the compared algorithms still could not recover the fetal ECG, since the vector $\mathbf{\Theta x}$ was still not sparse enough. In contrast, our proposed algorithms recovered the fetal ECG with higher quality.

In this experiment we clearly see that **only by exploiting intra-block correlation can we successfully recover the fetal ECG**.

References

[1] Z. Zhang and B. D. Rao, "Recovery of block sparse signals using the framework of block sparse Bayesian learning," in *ICASSP 2012*.

[2] ——, "Sparse signal recovery with temporally correlated source vectors using sparse Bayesian learning," *IEEE Journal of Selected Topics in Signal Processing*, vol. 5, no. 5, pp. 912–926, 2011.

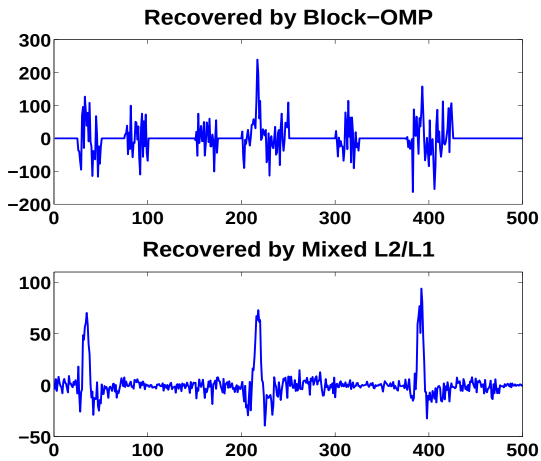

Fig. 4. The ECG signals recovered by Block-OMP (upper figure) and Mixed ℓ_2/ℓ_1 (bottom figure).

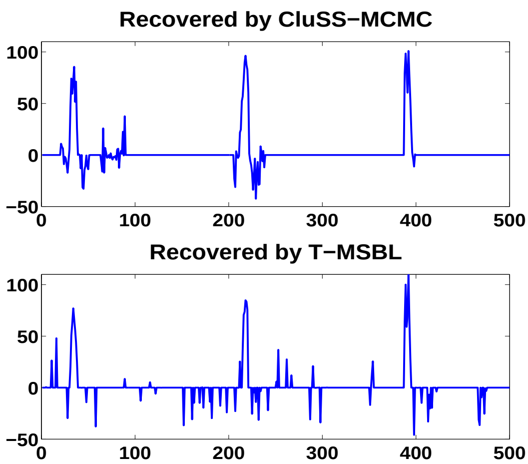

Fig. 5. The ECG signals recovered by CluSS-MCMC (upper figure) and T-MSBL (bottom figure).