Time-Frequency analysis of biophysical time series

Courtesy of Arnaud Delorme
Why Frequency-domain Analysis

For many signals, the signal's frequency content is of great importance.

- Beta
- Alpha
- Theta
- Delta
- Low Delta
<table>
<thead>
<tr>
<th>EEG Bands (Hz)</th>
<th>Distribution</th>
<th>Subjective feeling</th>
<th>Associated tasks &amp; behaviors</th>
<th>Physiological correlates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta 0.1-3</td>
<td>Distribution: generally broad or diffused</td>
<td>deep, dreamless sleep, non-REM sleep, unconscious</td>
<td>lethargic, not moving, not attentive</td>
<td>not moving, low-level of arousal</td>
</tr>
<tr>
<td>Theta 4-8</td>
<td>usually regional, may involve many lobes</td>
<td>intuitive, creative, recall, fantasy, imagery, creative, dreamlike, drowsy</td>
<td>creative, intuitive; distracted, unfocused</td>
<td>healing, integration of mind/body</td>
</tr>
<tr>
<td>Alpha 8-12</td>
<td>regional, usually involves entire lobe</td>
<td>relaxed, not agitated, but not drowsy</td>
<td>meditation, no action</td>
<td>relaxed, healing</td>
</tr>
<tr>
<td>Beta 12-30</td>
<td>localized</td>
<td>alertness, agitation</td>
<td>mental activity, e.g. math</td>
<td>alert, active</td>
</tr>
<tr>
<td>Gamma &gt;30</td>
<td>very localized</td>
<td>Focused arousal</td>
<td>high-level information processing, &quot;binding&quot;</td>
<td>information-rich task processing</td>
</tr>
</tbody>
</table>
Frequency-domain Analysis of the EEG

- Joseph Fourier (1768-1830)
- Any complex time series can be broken down into a series of superimposed sinusoids with different frequencies.

Summation of the signals
Fourier Analysis

Fourier-Transformation:

\[ H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i ft} dt ; \quad h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i ft} \]

Discrete Fourier-Transformation 傅利葉轉換 (O(N^2)):

\[ X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-ik(2\pi / N)n} \quad k = 0,1,..,N-1 \]
\[ x[n] = \sum_{k=0}^{N-1} X(k) e^{ik(2\pi / N)n} \quad n = 0,1,..,N-1 \]

Fast Fourier Transform (FFT, O(N\log_2 N), Cooley and Tukey (1965))
function [a,b] = dft (y)
% DFT - The Discrete Fourier Transform
% [a, b] = DFT (y)
% a, b are the cosine and sine components

n = length (y);
t = 2*pi*(0:n-1)/n;
f = 2.0 / n;

for j = 0:n/2
    cs = cos (j * t);
    ss = sin (j * t);
    a(j+1) = f * (cs * y);
    b(j+1) = f * (ss * y);
end

% boundaries
n2 = floor (n / 2);
a(1) = 0.5 * a(1);
a(n2+1) = 0.5 * a(n2+1);
b(1) = 0.0;
b(n2+1) = 0.0;
Spectral phase and amplitude

\[ F(k,t) \]

Real

Imaginary

\[ + \quad - \]

Imag.

Real

\[ \rightarrow \]
Spectral phase and amplitude

\[ F_k(f,t) \]
Pwelch method for computing spectrum
Spectral power

Power log(\(\mu V^2/Hz\))

Frequency (Hz)

Squared vector length

0 Hz 10 Hz 20 Hz 30 Hz 40 Hz 50 Hz
Overlap 50%

Squared amplitude

Average

Power [10^12μV²/Hz] vs. Frequency [Hz]
Zero padding
Plot data spectrum using EEGLAB

'winsize', 256 (change FFT window length)
'nfft', 256 (change FFT padding)
'overlap', 128 (change window overlap)
Disadvantage of Fourier Transform

In transforming to the frequency domain, time information is lost.
Frequency-domain Analysis of the EEG

• We often apply a ‘window’ to the data.
• This simply means taking the amount we want from the data stream.
• The window is moved along the data; we perform the FFT on this windowed data.
Spectrogram or ERSP

<table>
<thead>
<tr>
<th>Frequency</th>
<th>0 ms</th>
<th>10 ms</th>
<th>20 ms</th>
<th>30 ms</th>
<th>40 ms</th>
<th>50 ms</th>
<th>60 ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 Hz</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Spectrogram or ERSP

0 ms 10 ms 20 ms 30 ms 40 ms 50 ms 60 ms

5 Hz

10 Hz

20 Hz

30 Hz

squared vector length

2-D matrix average
Power spectrum and event-related spectral perturbation

\[ ERS(f, t) = \frac{1}{n} \sum_{k=1}^{n \text{ trials}} |F_k(f, t)|^2 \]

Scaled to dB \( 10 \log_{10}(ERS) \)
Absolute versus relative power

Absolute = ERS

Relative = ERSP (dB or %)
Time-locked ERSP ≠ Time- & phase-locked ERP

Simulated ERP+ERSP (10 epochs)

- 40 Hz (phase-locked)
- 10 Hz (time-locked but not phase-lock)
- 1 Hz (phase-locked)
ERSP vs ERP

ERSP Analysis of Simulated Data

1 Hz

10 Hz

ERP
Difference between FFT and wavelets

FFT

Wavelet

Frequency
Wavelets factor

Wavelet (0) = FFT

Wavelet (1)

1Hz
2Hz
4Hz
6Hz
8Hz
10Hz
FFT

Pure wavelet
Modified wavelets

Wavelet (0.8)  Wavelet (0.5)  Wavelet (0.2)
Inter trial coherence

same time, different trials

Trial 1
amplitude 0.5 phase 0

Trial 2
amplitude 1 phase 90

Trial 3
amplitude 0.25 phase 180

POWER = mean(amplitudes^2)
0.44 or −8.3 dB

COHERENCE = mean(phase vector)
Norm 0.33
Intertrial Coherence (ITC)

Single trials

ERP

Total power

ITC: .05

ITC: .80
Phase ITC

\[ ITPC(f, t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k(f, t)}{|F_k(f, t)|} \]

Normalized
(no amplitude information)
Power and inter-trial coherence

Time-frequency power

**Condition 1**

**Condition 2**

**ITC: trials’ synchronization**
Channel time-frequency
Component time-frequency
Compare between conditions

Component 10 for condition 1 (left) and condition 2 (right)

$$\text{Component 10 for condition 1 (left) and condition 2 (right)}$$

$$\text{ERSPs}$$

$$\text{ITCs}$$

$$\text{Number of data points}$$

$$\text{Sampling rate}$$

$$\text{Cycles (0=FFT)}$$

$$\text{padding}$$

$$\text{Number of data points}$$

$$\text{>> newtimef(\{ ALLEEG(2).icaact(10,:); EEG.icaact(10,:); \}, EEG.pnts, \ldots \ [EEG.xmin EEG.xmax]*1000, EEG.srate, 0, 'padratio', 1);}$$

$$\text{Number of data points}$$

$$\text{Sampling rate}$$

$$\text{Cycles (0=FFT)}$$

$$\text{padding}$$

$$\text{34}$$
Do the activities of maximally independent EEG domains interact?
Cross-coherence amplitude and phase

2 components, comparison on the same trials

Trial 1
Coherence amplitude 1
Phase coherence 0

Trial 2
Coherence amplitude 1
Phase coherence 90

Trial 3
Coherence amplitude 1
Phase coherence 180

$\text{COHERENCE} = \text{mean(phase vector)}$

Norm 0.33
Phase 90 degree
Phase coherence (default)

\[ ERPCOH^{a,b}(f,t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k^a(f,t)F_k^b(f,t)^*}{|F_k^a(f,t)| |F_k^b(f,t)|} \]
Cross-coherence amplitude and phase

Condition 1

Condition 2
Component phase coherence
Summary