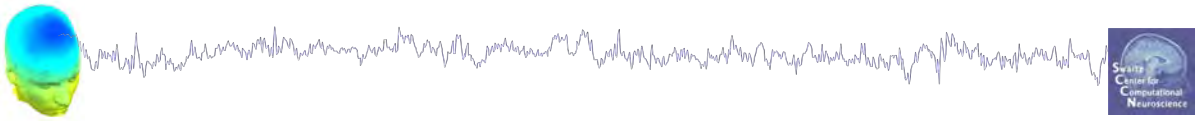
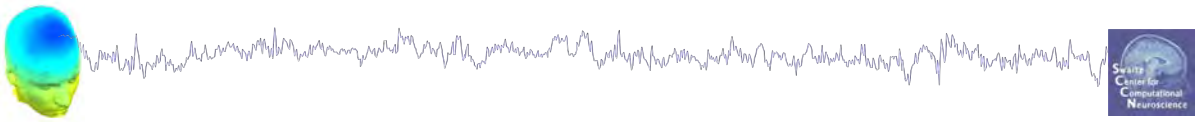


Time-frequency decomposition

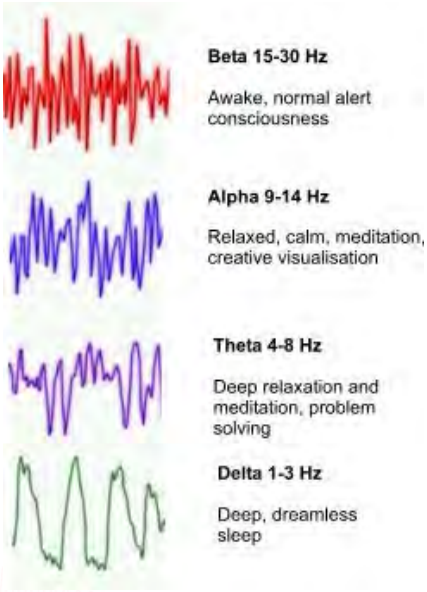
Theory and Practice

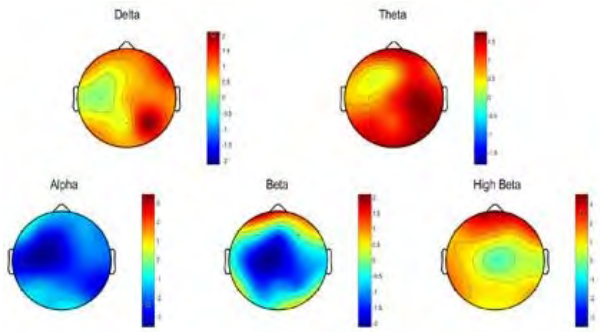
2021 Virtual EEGLAB Workshop



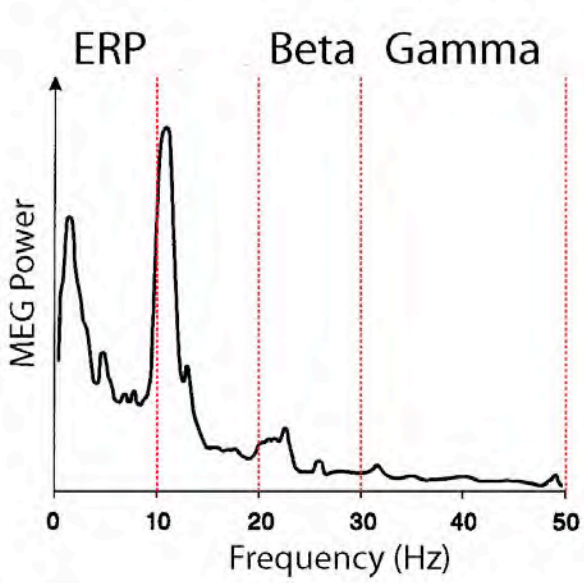
- Signals – EEG
- Goals
  - Describe dynamic characteristics of brain activity
  - Describe relation between different regions of brain
- Approaches
  - Time domain
  - Frequency domain
  - Time/Frequency

Different meanings traditionally given to different frequency bands

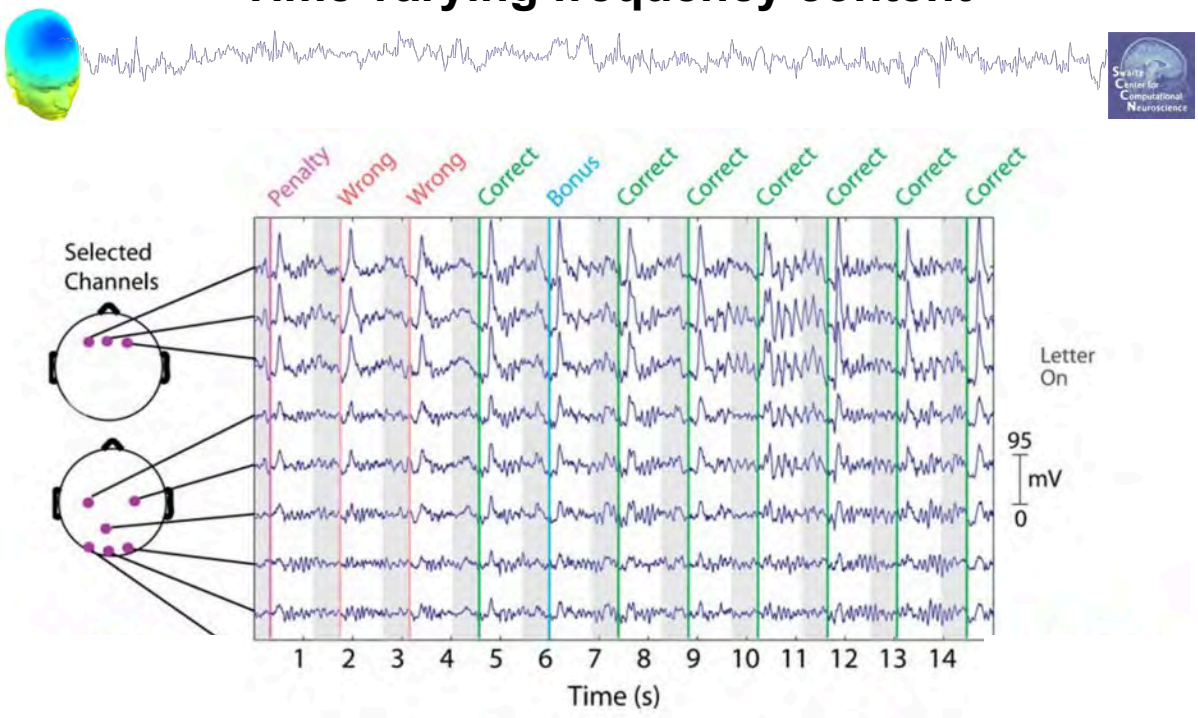




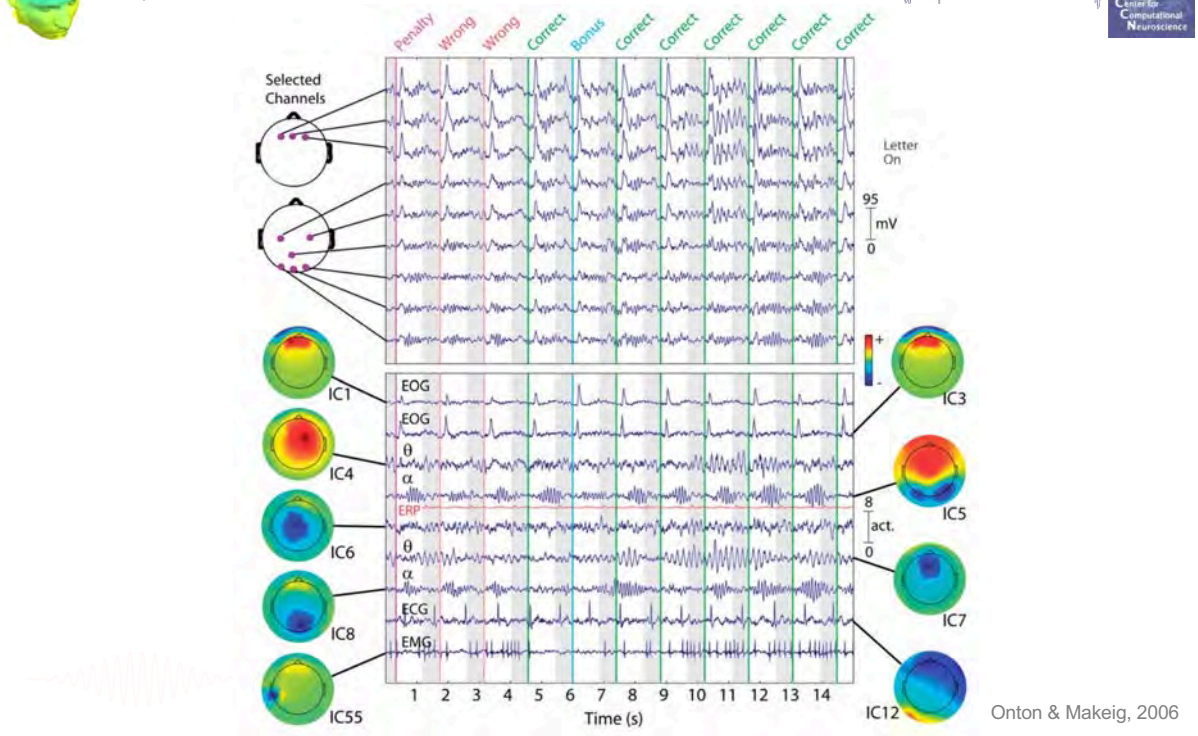
MEEG spectrum



Time varying frequency content

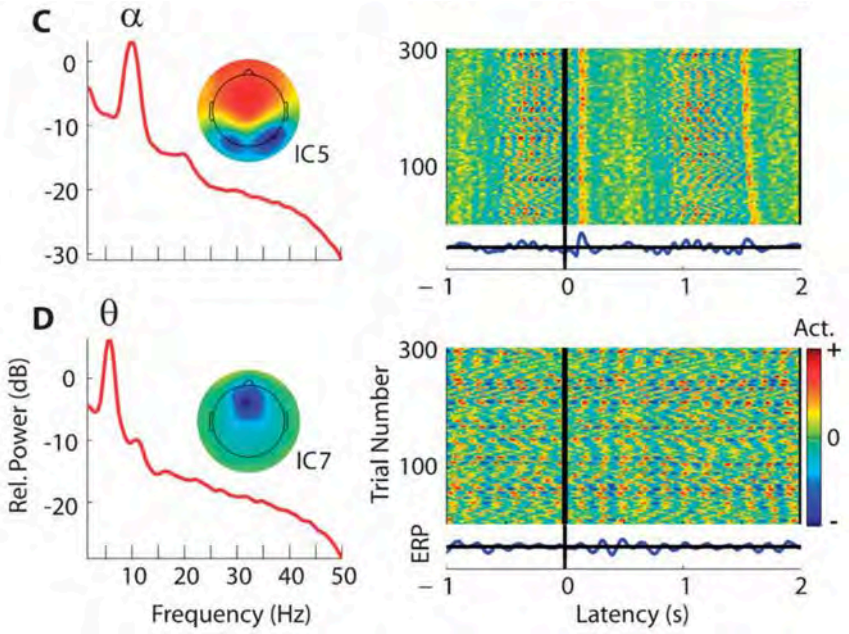


Time-varying frequency content

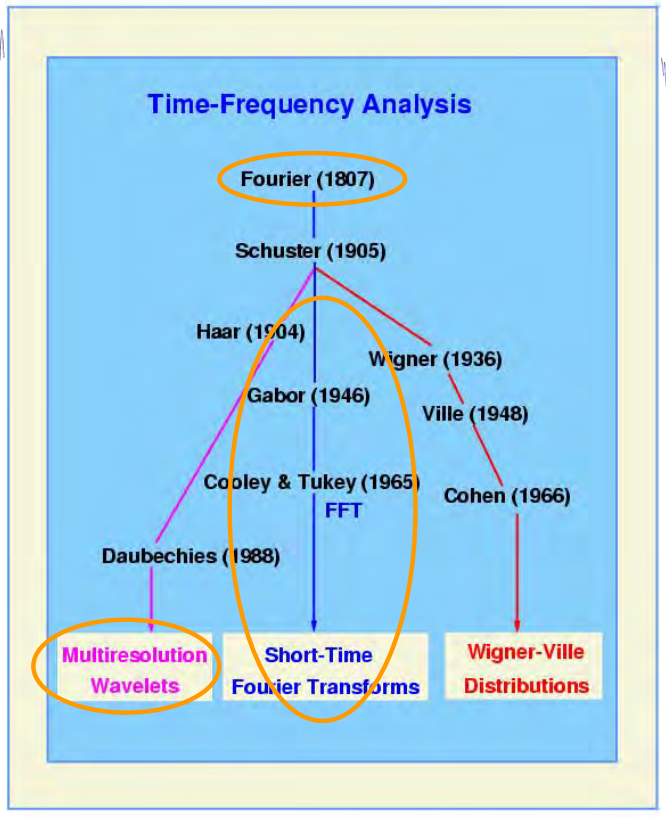


Onton & Makeig, 2006

Power Spectrum does not describe temporal variation

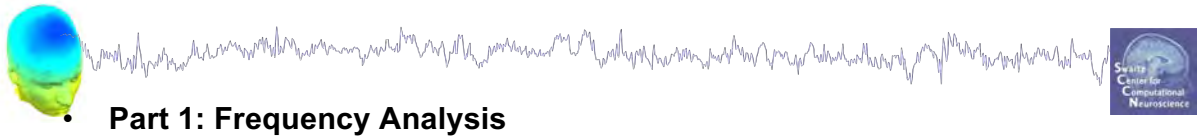


Onton & Makeig, 2006



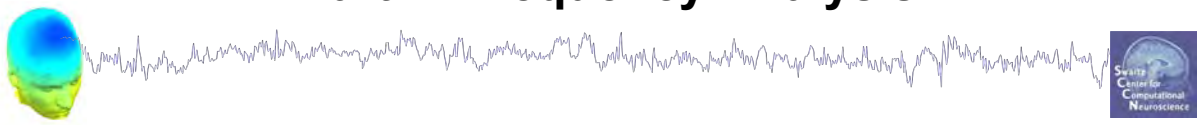
S. Makeig, 2005

Plan



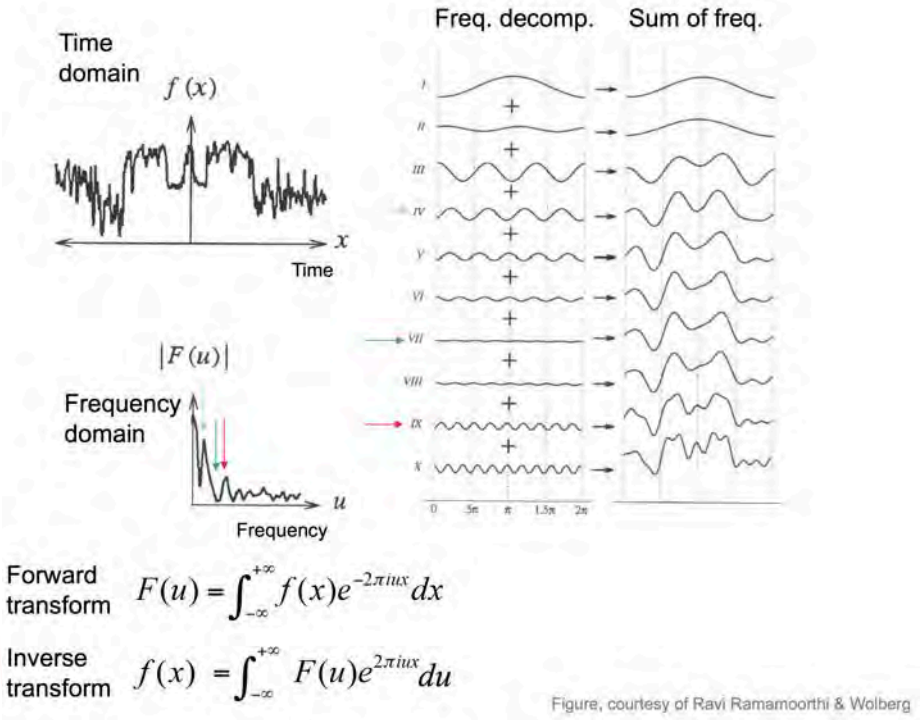
- **Part 1: Frequency Analysis**
  - Power Spectrum
    - Approaches
      - FFT
      - Welch's Method
    - Windowing
- **Part 2: Time-Frequency Analysis**
  - Short Time Fourier Transform
  - Wavelet Transform
  - ERSP
- **Part 3: Coherence Analysis**
  - Inter-Trial Coherence
  - Event-Related Coherence
- **Part 4: Other Applications**

Part 1: Frequency Analysis



- Goal: What frequencies are present in signal?
- What is power at each frequency?
- Principle: Fourier Analysis

Fourier Analysis



Figure, courtesy of Ravi Ramamoorthi & Wolberg

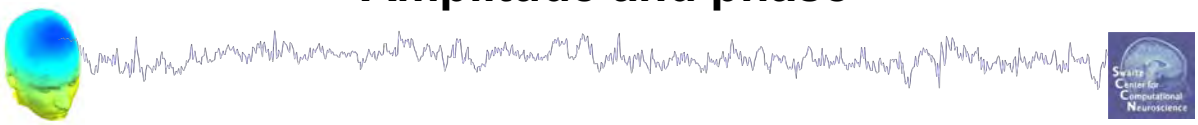
Power Spectrum. Approach 1: FFT

- Why not just take FFT of our signal of interest?
- Advantage – fine frequency resolution
  - ΔF = 1 / signal duration (s)
  - E.g. 100s signal has 0.01 Hz resolution
  - But, do we really need this?

- Disadvantage 1 – high variance
  - Solution: e.g. Welch's method

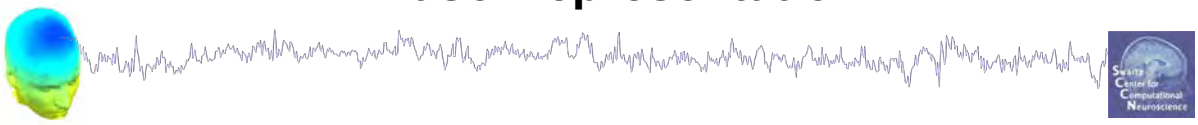
- Disadvantage 2 – no temporal resolution
  - Solution 1: Short-Time Fourier Transform

Amplitude and phase



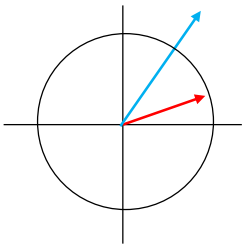
- Power spectra describe the *amount* of a given frequency present. Often expressed in **dB** [$10*\log_{10}$(Power)]
- Power is NOT a complete description of a signal: We also must know the *phase* at each frequency
- FFT/STFT/Wavelet return an amplitude and phase at each time and frequency (represented as complex #).
- To find power, we compute the magnitude, which discards phase.

Phasor representation

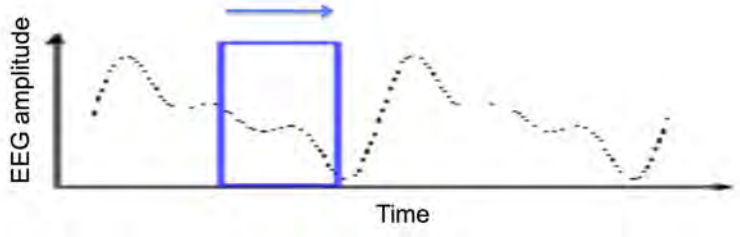


- A complex number $x + yi$ can be expressed in terms of amplitude and phase: $ae^{i\theta}$

```
                        amplitude*exp(1i*phase)
amplitude = sqrt(x^2 + y^2);
phase     = atan(y/x);
```



Approach 2: Welch's Method

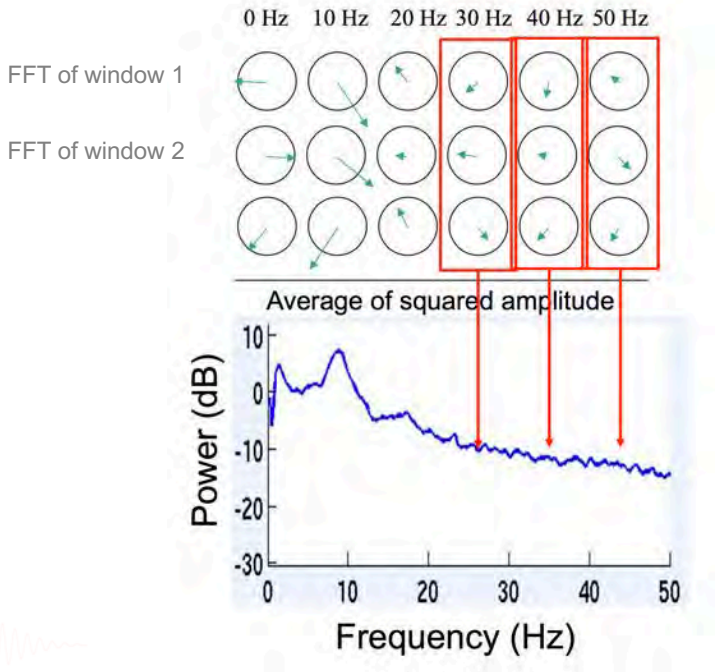


Calculate power spectrum of short signal windows, average.
Advantage: Smoother estimate of power spectrum

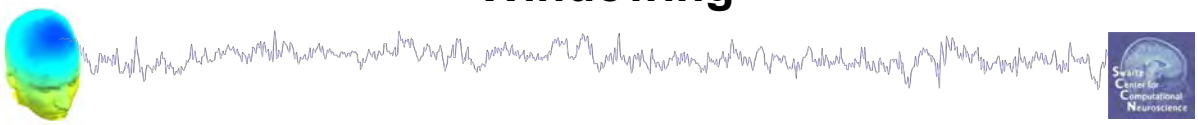
Frequency resolution now set by *window* length

e.g. 1s window -> 1 Hz resolution

In practice: *taper*, don't use rectangular window

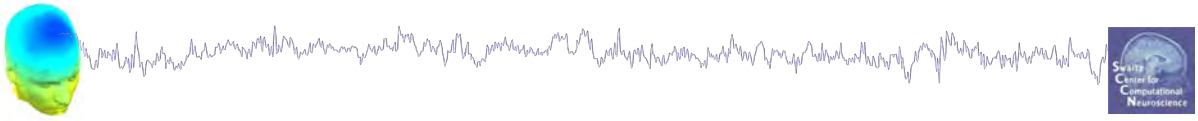


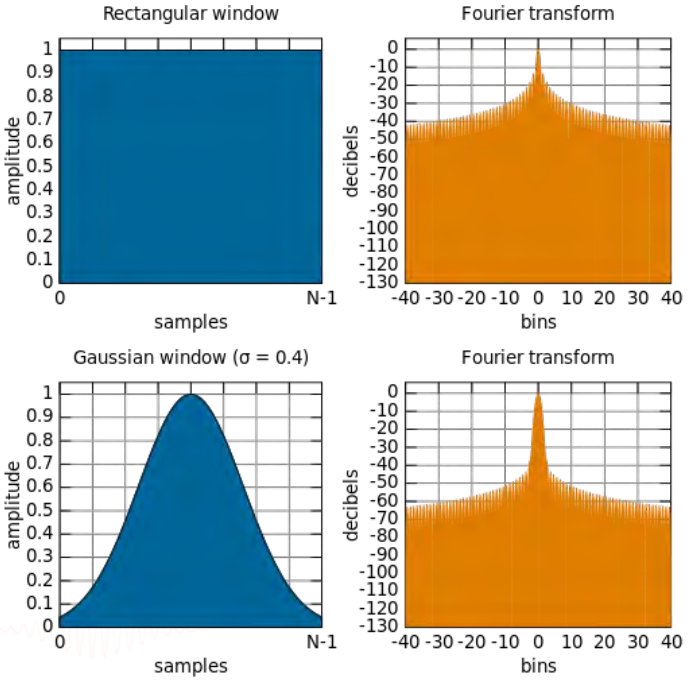
Windowing



- When we pick a short segment of signal, we typically window it with a smooth function (taper).
- Windowing in time = convolving (filtering) the spectrum with the Fourier transform of the window
- No window (=rectangular window) results in the most smearing of the spectrum
- There are many other windows optimized for different purposes: Hamming, Gaussian…

Windows and their Fourier transforms

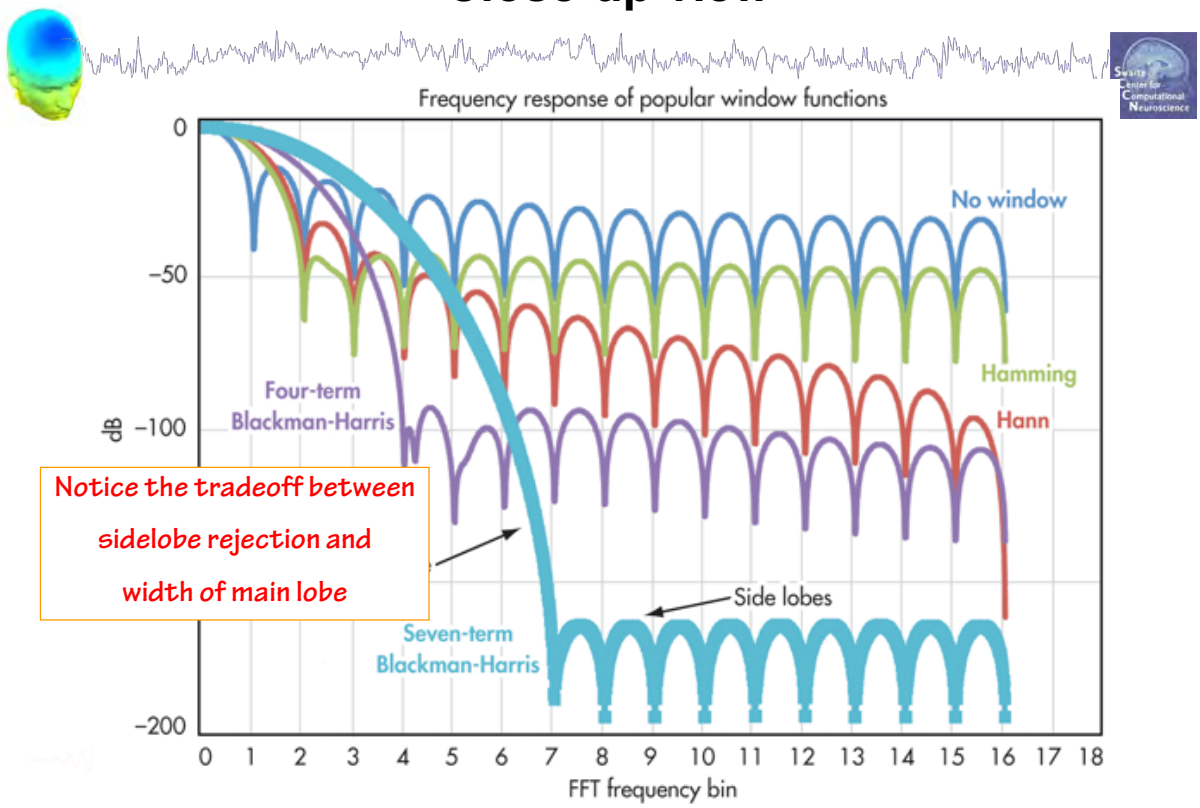




Narrowest main peak, but
Highest side-lobes
Most spectral 'smearing'

Wider main peak, but
much lower side-lobes

Close-up view

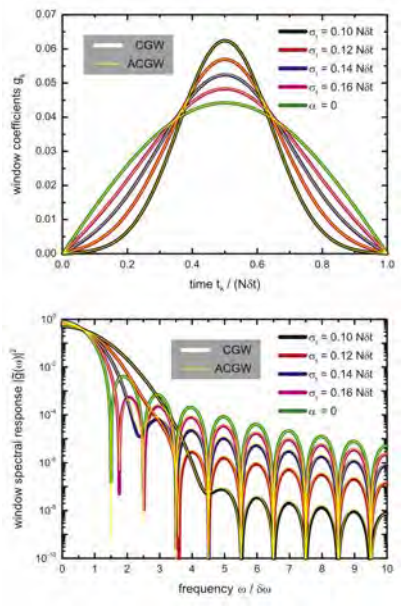


Part 2: Time-Frequency Analysis

- Short-Time Fourier Transform
  - Find power spectrum of short windows
  - “Spectrogram”
- Advantage: Can visualize time-varying frequency content
- Disadvantage: Fixed temporal resolution is not optimal

Time-Frequency Uncertainty

- You cannot have both arbitrarily good temporal and frequency resolution!
  - $\sigma_t * \sigma_f \geq 1/2$
- If you want sharper temporal resolution, you will sacrifice frequency resolution, and vice versa.
- (Optimal: Confined Gaussian)

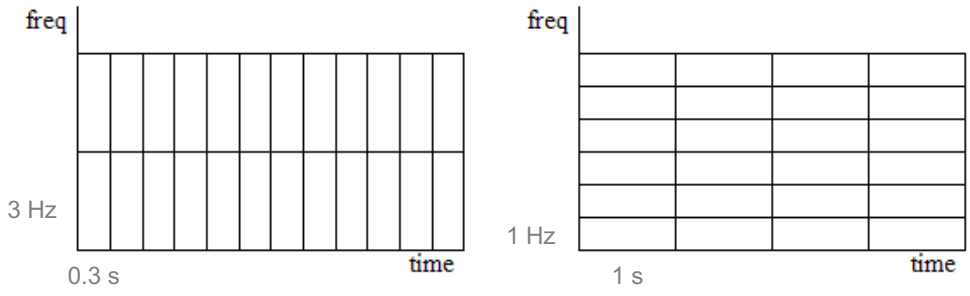


Starosielec S, Hägele D (2014) Discrete-time windows with minimal RMS bandwidth for given RMS temporal width. Signal Processing 102:240–6.

Consequence for STFT

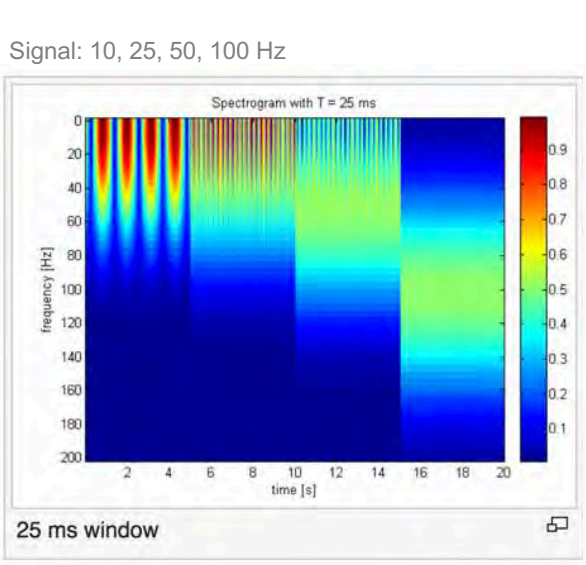
Shorter Windows
poorer frequency resolution

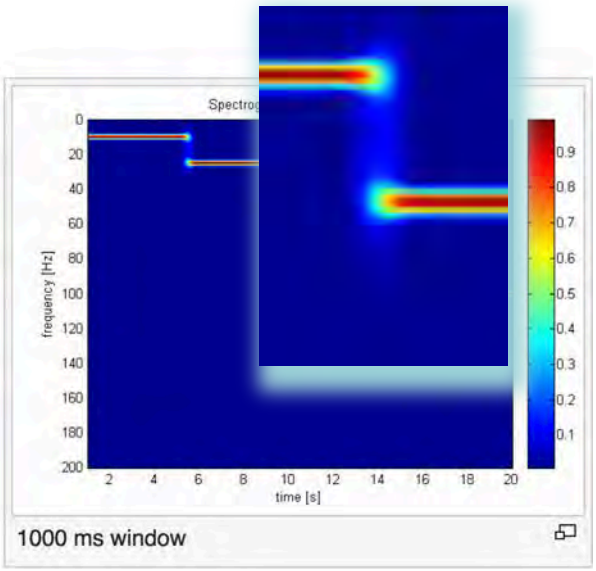
Longer Windows
finer frequency resolution



Time-Frequency Tradeoff

Signal: 10, 25, 50, 100 Hz

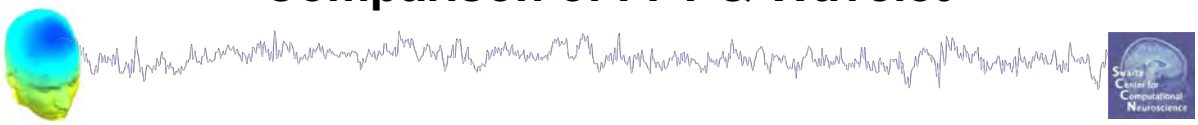




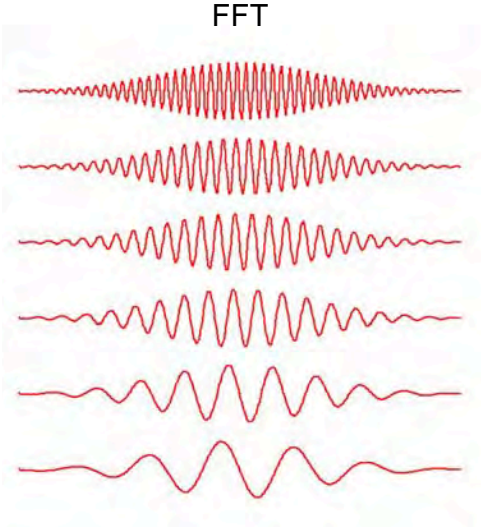
One better way: Wavelet transform

- Wavelet transform is a ‘multi-resolution’ time-frequency decomposition.
- Intuition: Higher frequency signals have a faster time scale
- So, vary window length with frequency!
  - longer window at lower frequencies
  - shorter window at higher frequencies

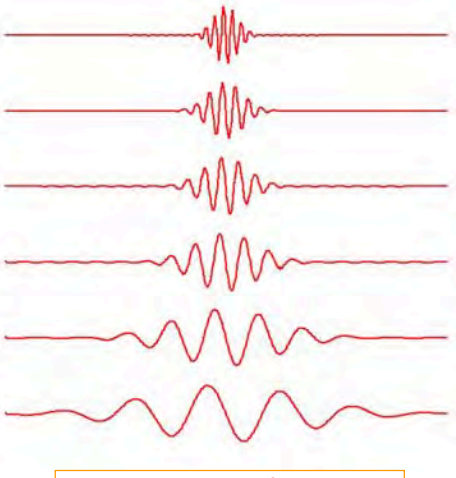
Comparison of FFT & Wavelet



FFT



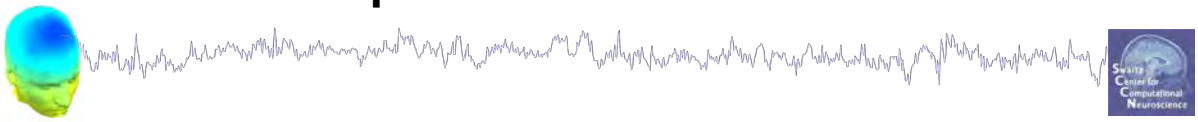
Wavelet



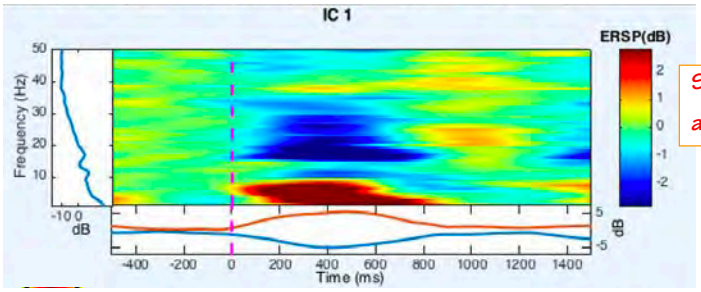
Scaled versions of one shape

Constant number of cycles

Comparison of FFT & Wavelet



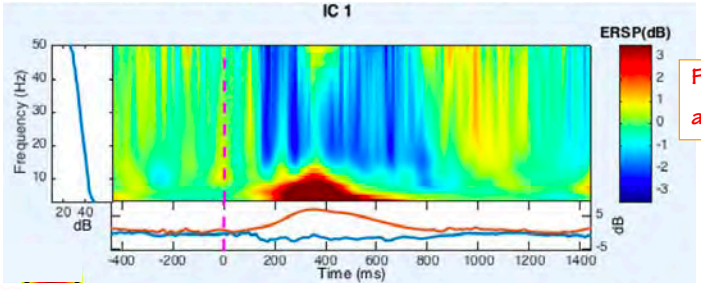
FFT



Similar time resolution

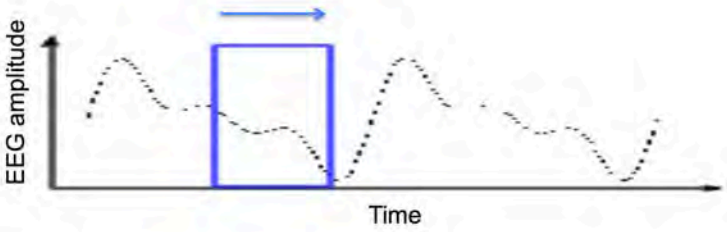
across frequencies

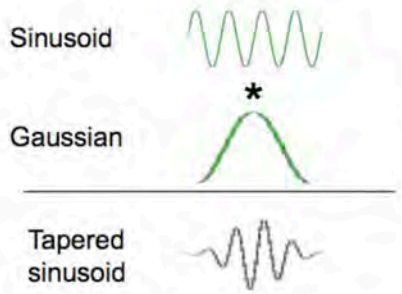
Wavelet



Finer time resolution

at high frequencies



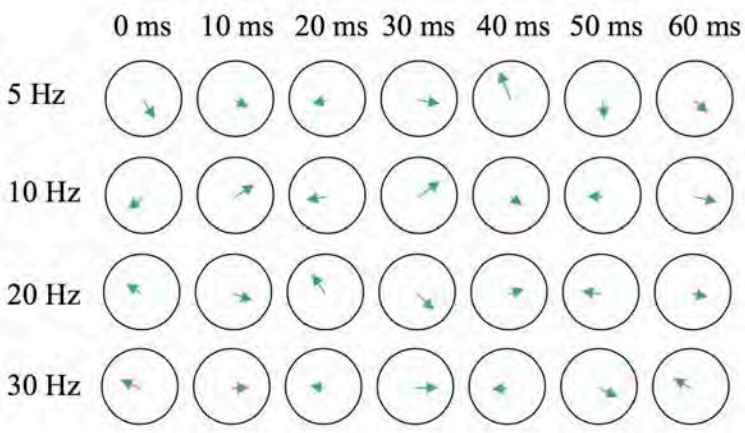


For each time point

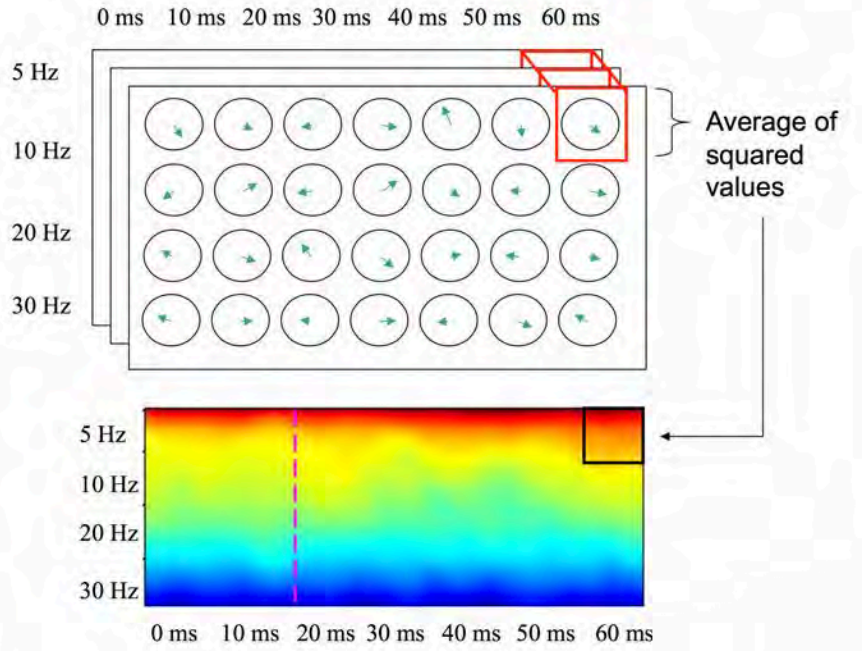
Analyze signal using the wavelets

for different frequencies.

Spectrogram of one epoch of data



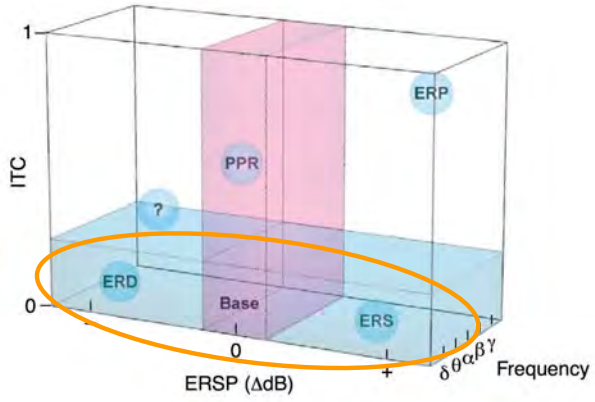
Computing Spectrogram Power



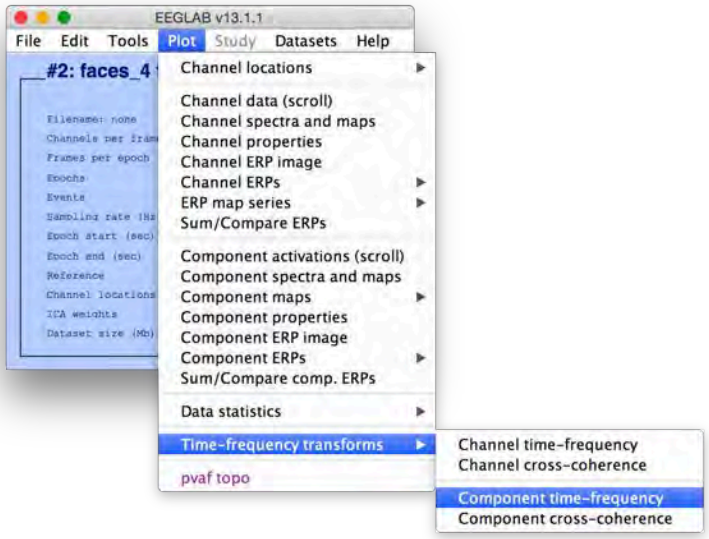
Definition: ERSP

- Event Related Spectral Perturbation

- Change in power in different frequency bands relative to a baseline. ERS (Event-Related *Synchronization*), ERD (Event-Related *Desynchronization*)



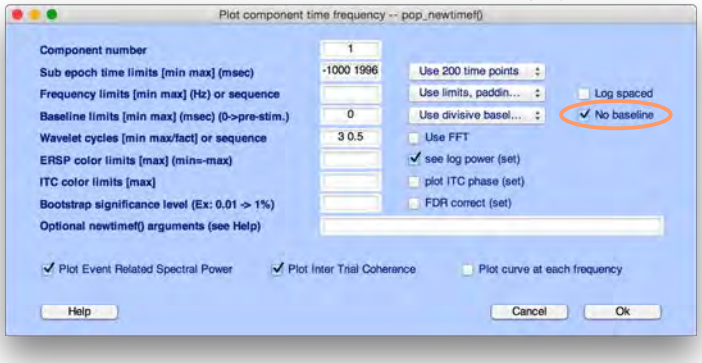
Try it out



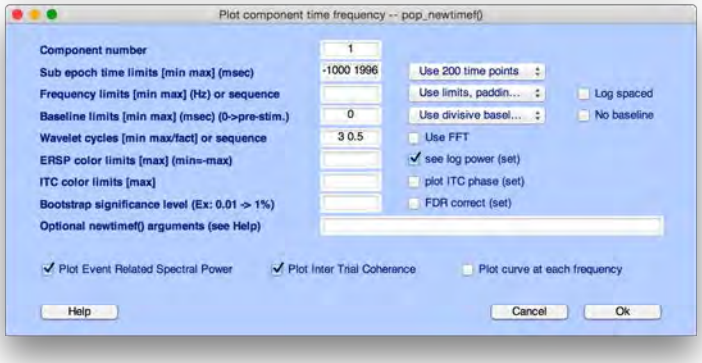
(Load faces_4.set
Epoch on 'face' event)

Display ERS vs. ERSP

Event-related Spectrogram



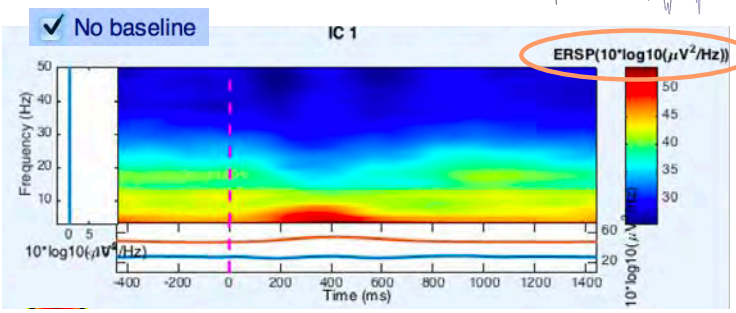
Event-Related Spectral Perturbation (ERSP)



Event-related

Spectrogram

SG(t,f)

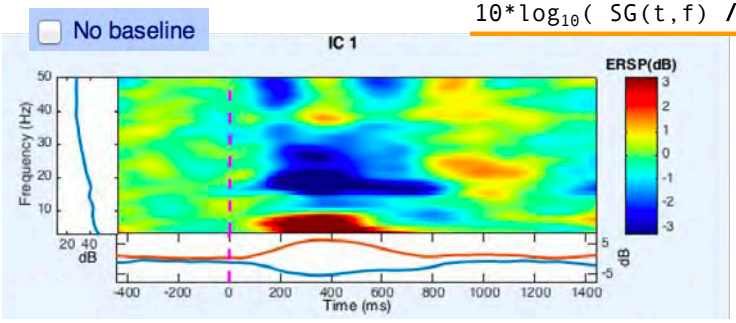


$10*\log_{10}(SG(t,f) / \mathbf{baseline(f)})$

Event-Related

Spectral **Perturbation**

(ERSP)



$10*\log_{10}(SG(t,f)) - 10*\log_{10}(baseline(f))$

Exercises

m

- Try different wavelet specifications

  Wavelet cycles [min max/fact] or sequence 3 0.5

  – Default: 3 0.5
    - 3 cycles. Try 2. How do the time limits of the plot change?
    - What is the 0.5? Try 0. Try 1…what do you observe?

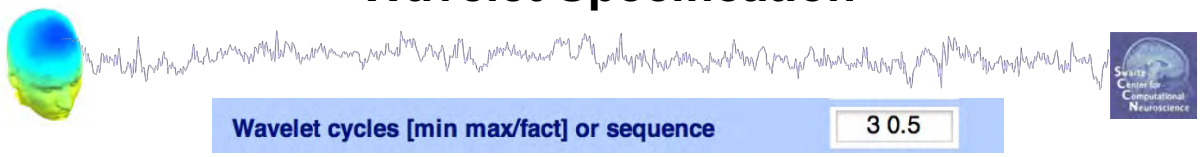
- Try different low-frequency limit

  Frequency limits [min max] (Hz) or sequence

  – what is the effect on the time limits of the ERSP?

- Try different baseline methods
  – divisive
  – standard deviation (express spectral perturbations in #sd relative to baseline sd)

Wavelet Specification

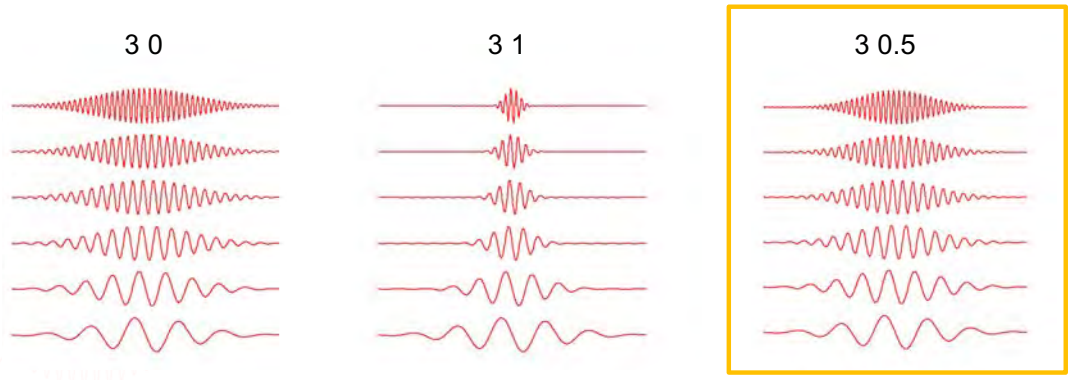


Answer: The first #cycles controls the basic duration of the wavelet in cycles.
The second factor controls the degree of shortening of time windows as frequency increases

0 = no shortening = FFT (duration remains constant with frequency)

1 = pure wavelet (#cycles remains constant with frequency)

0.5 = intermediate, a compromise that reduces HF time resolution to gain more frequency resolution.

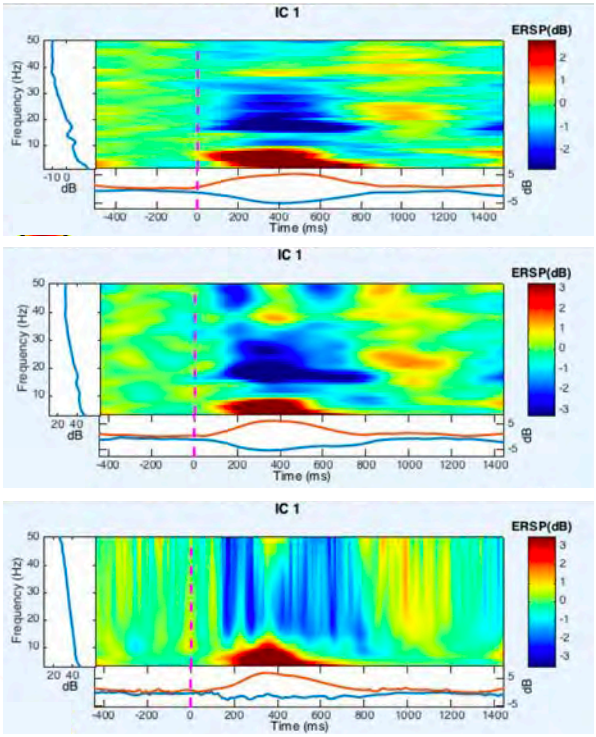


Comparison of FFT & Wavelet

[3 0] (FFT)

[3 0.5] Wavelet

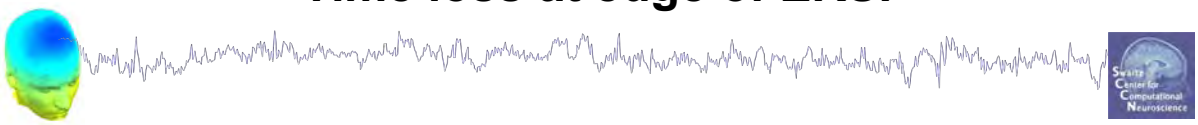
[3 1] Wavelet



A reasonable choice:

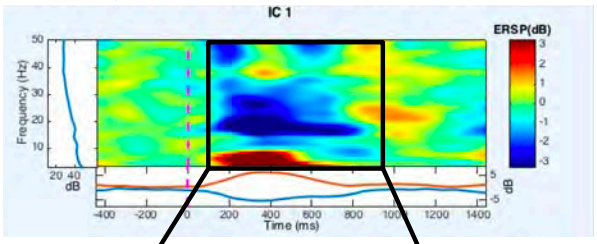
Notice: features have similar time and frequency resolution

Time loss at edge of ERSP

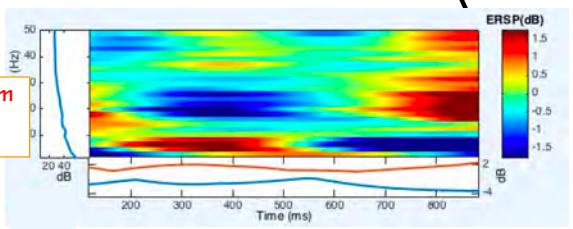


- Settings for 1) *wavelet cycles* and 2) *lowest frequency* impact the time limits of analysis

MIN FREQ: 3 Hz



MIN FREQ: 1 Hz



*more wavelet cycles, or a lower minimum frequency loses time at edges of epoch

Solution: If you need low frequencies in your ERSP, be sure to extract longer epochs to counteract this. If you can't re-epoch, then try reducing the number of wavelet cycles.

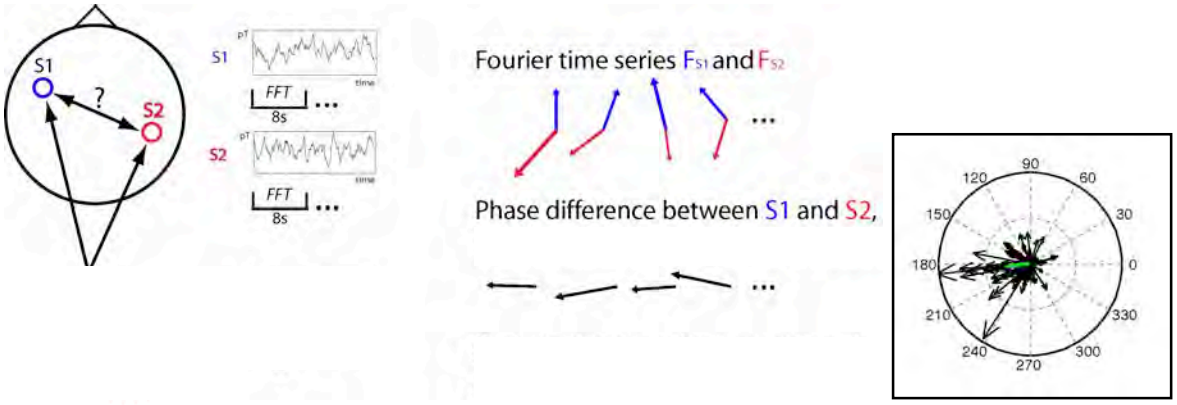
Part 3: Coherence Analysis

- Goal: How much do two signals resemble each other?
- Coherence = complex version of correlation: how similar are power and phase at each frequency?
- Variant: phase coherence (phase locking, etc.) considers only phase similarity, ignoring power
  - Regular coherence is simply a power-weighted phase coherence
  - Inter-trial coherence is useful!
- NOTE: For **understanding** connectivity between regions, *channel* coherence is a poor choice due to volume conduction. For IC connectivity, directional, 'causal' measures of connectivity have been developed (See SIFT lecture).

Coherence

$$C(f,t) \propto \sum_{k=trials} F1_k(f,t)\overline{F2_k(f,t)}$$

$$a_1 e^{i\theta_1} a_2 e^{-i\theta_2} \propto e^{i(\theta_1-\theta_2)}$$



Part 3a: Inter-Trial Coherence

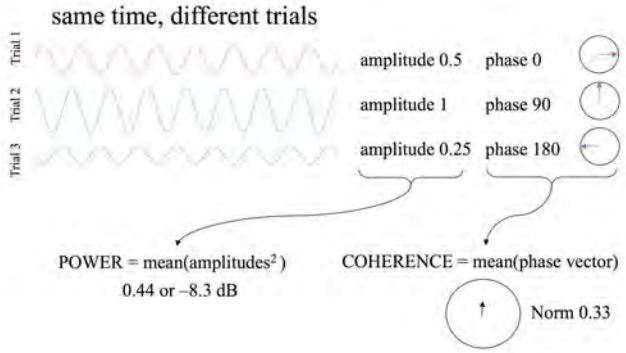
- Goal: How much do different trials resemble each other?
- Phase coherence not between two processes, but between multiple trials of the same process
- Defined over a (generally) narrow frequency range

EEGLAB's Inter-Trial Coherence is *phase* ITC

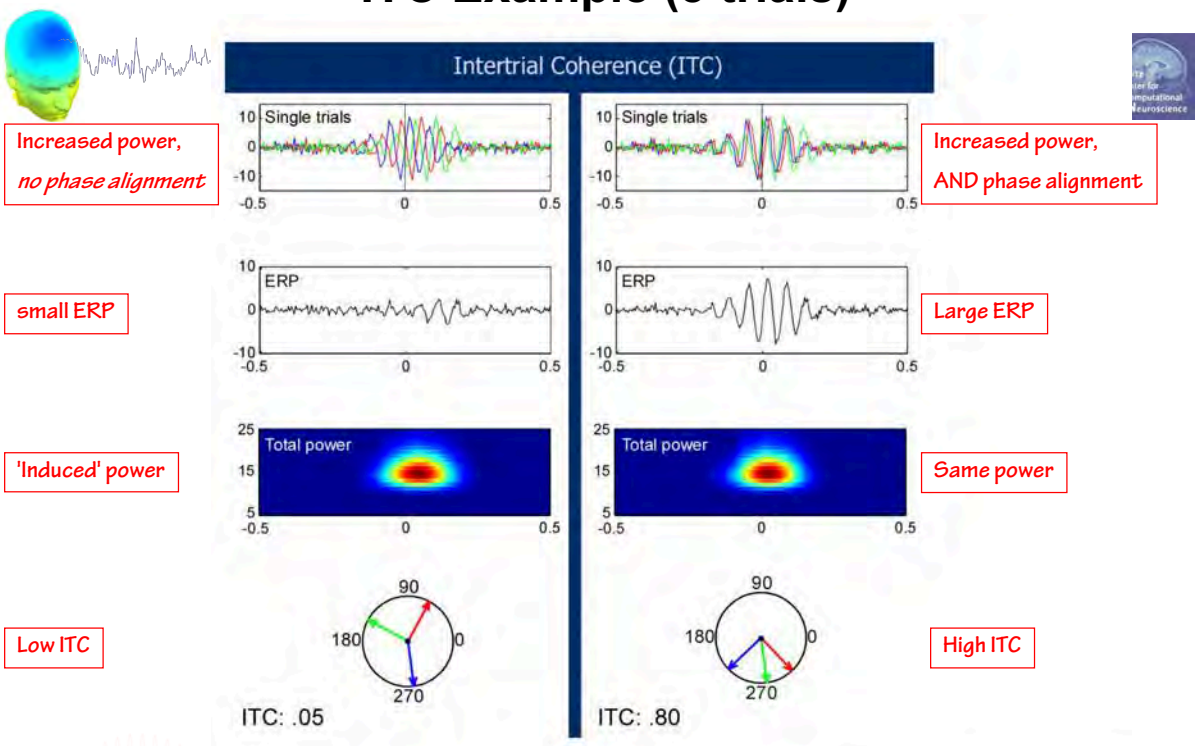
Phase ITC

$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k(f,t)}{\left|F_k(f,t)\right|}$$

Normalized
(no amplitude information)



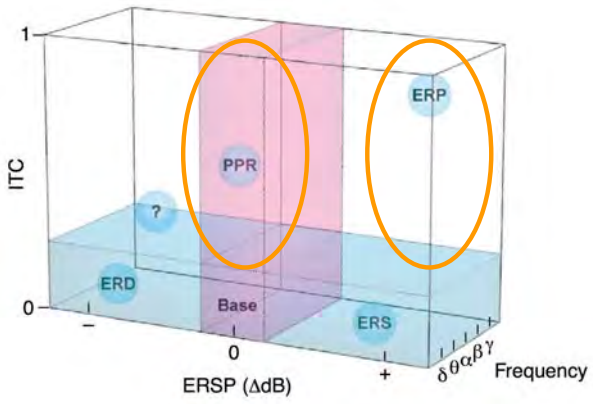
ITC Example (3 trials)

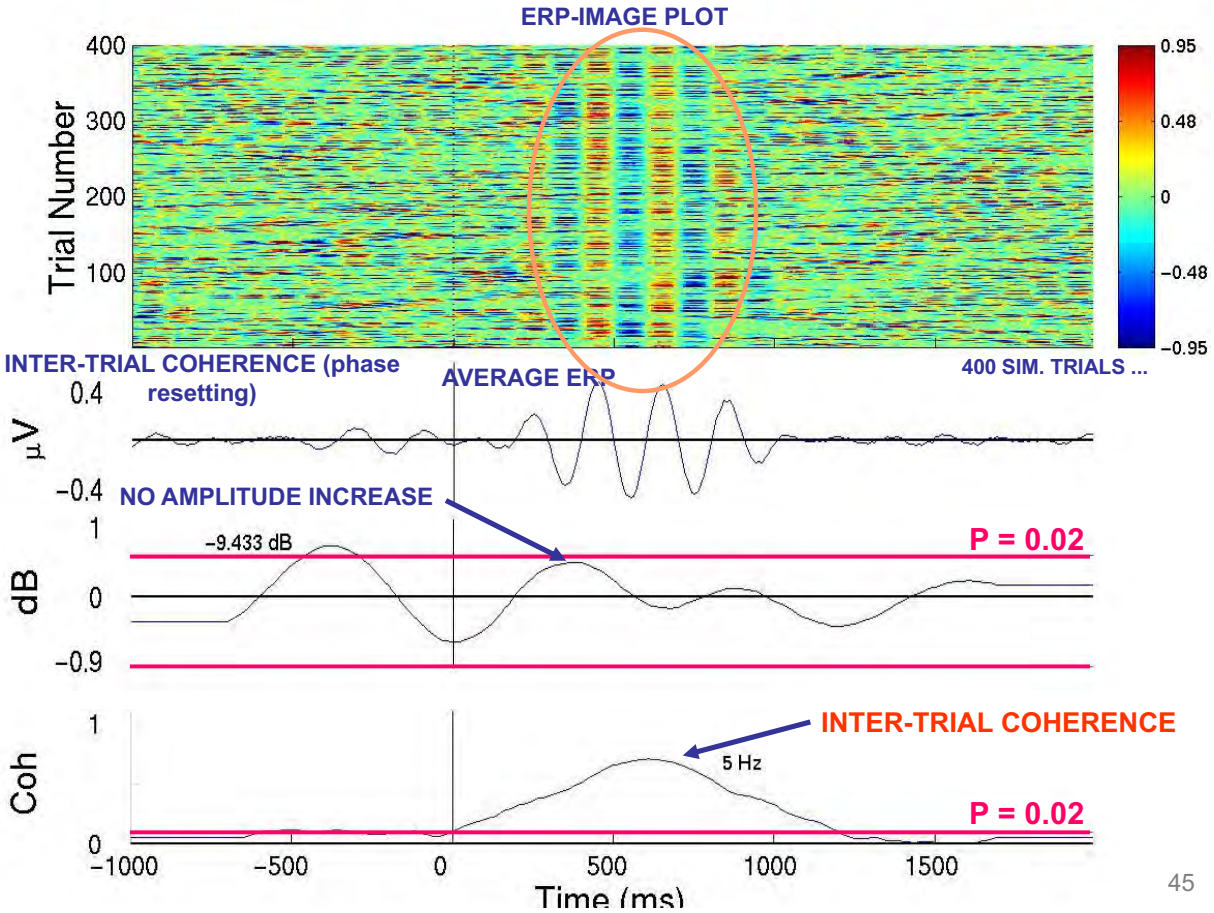


Slide courtesy of Stefan Debener

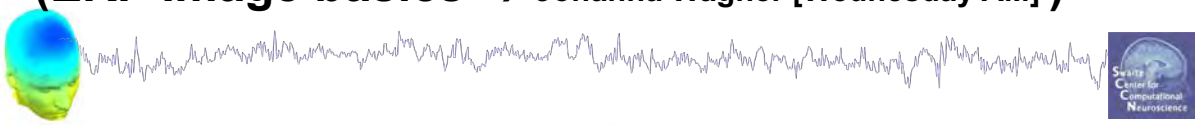
** Several possible origins of an ERP **

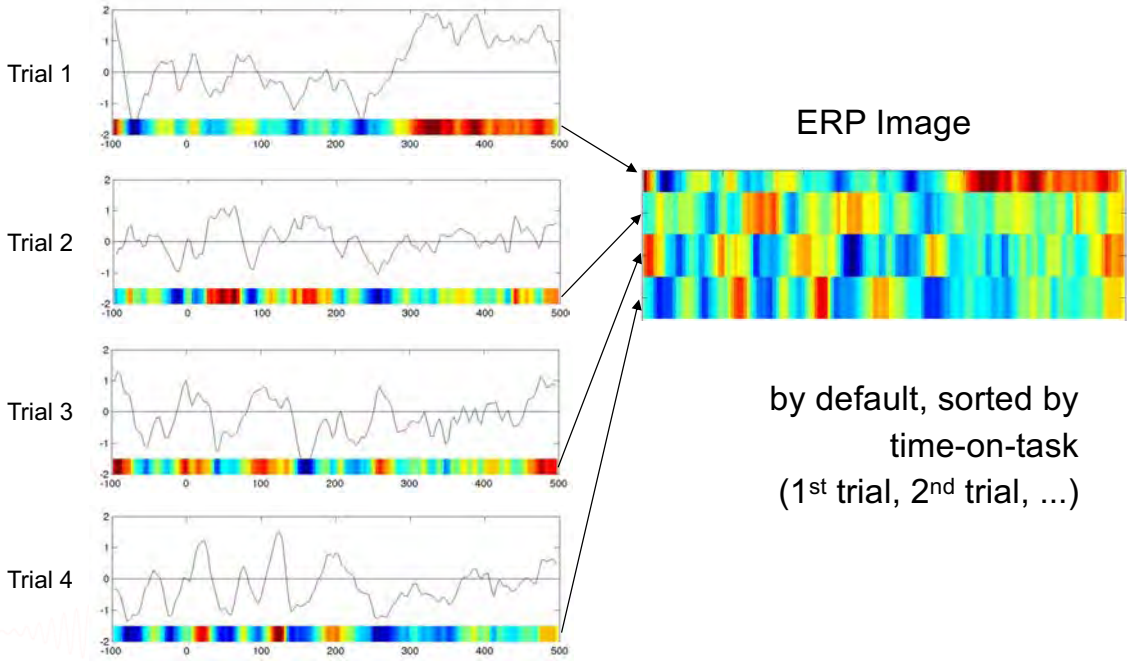
- Event Related Potential can result from
  - ITC increase (with no change in power)
  - ITC & Power change

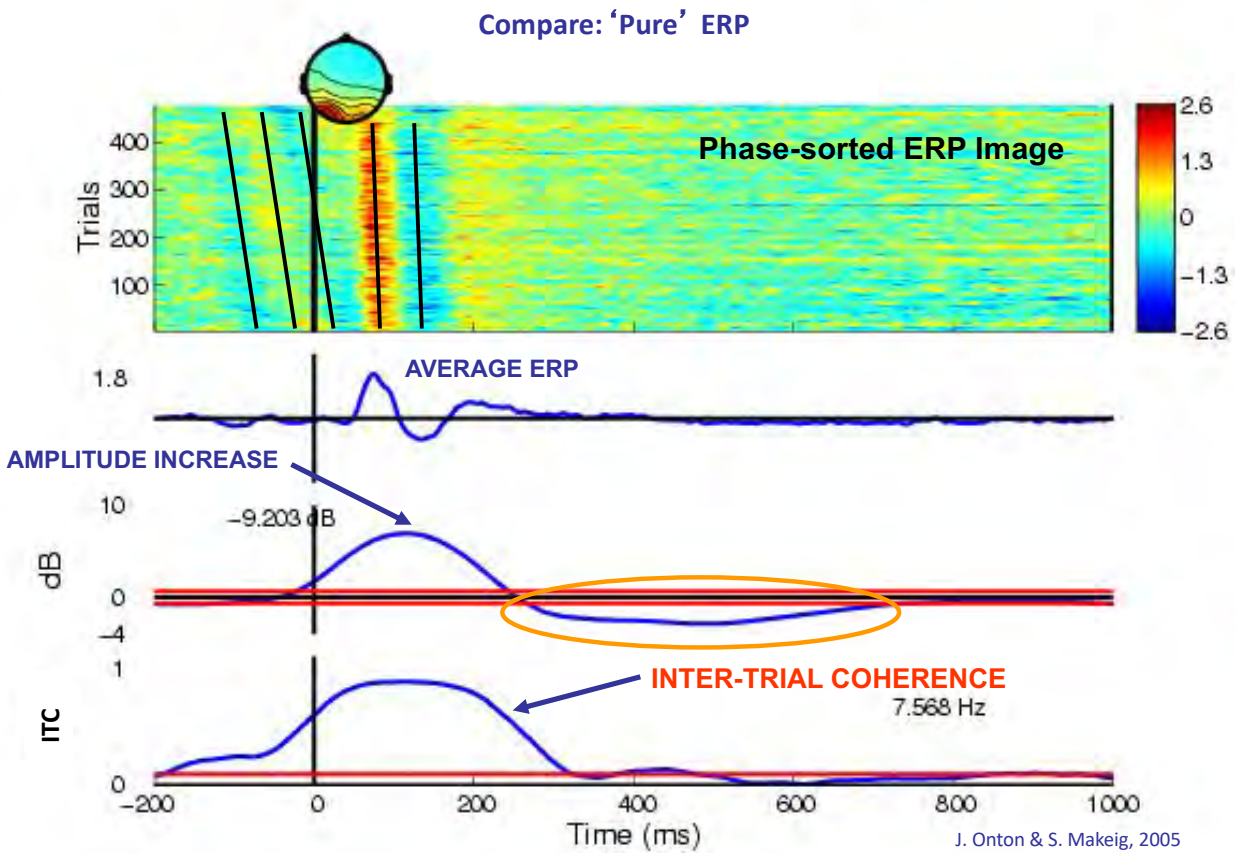




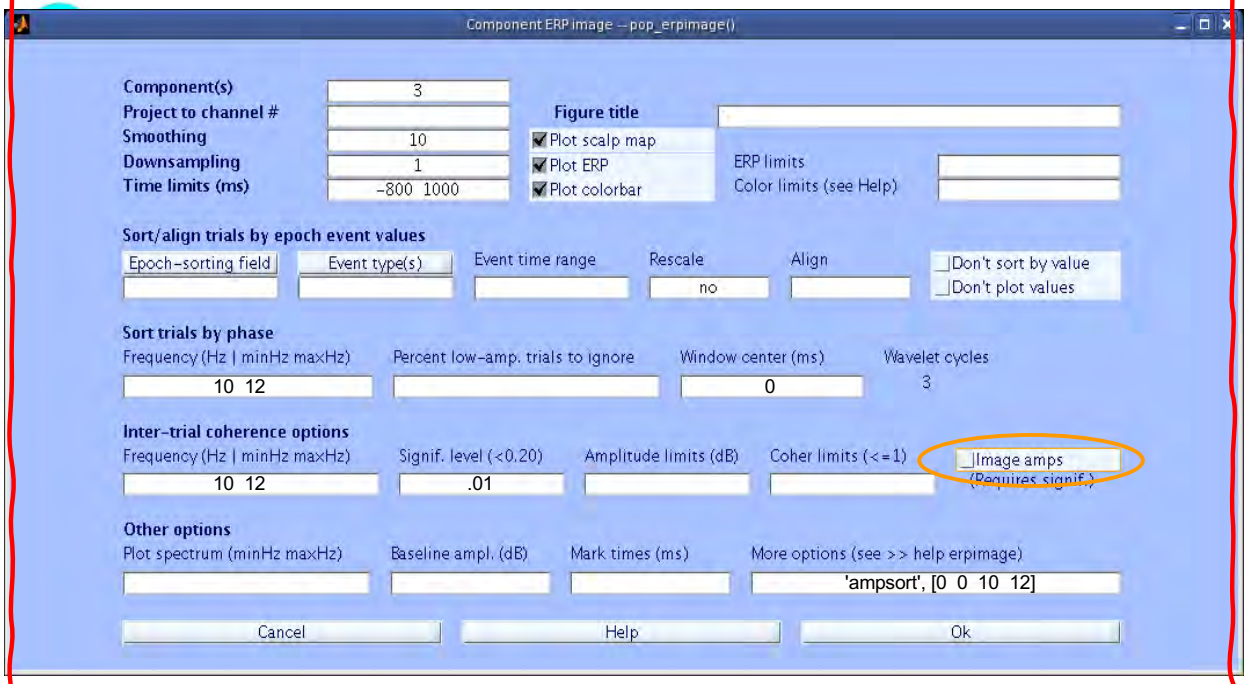
(ERP Image basics → Johanna Wagner [Wednesday AM])



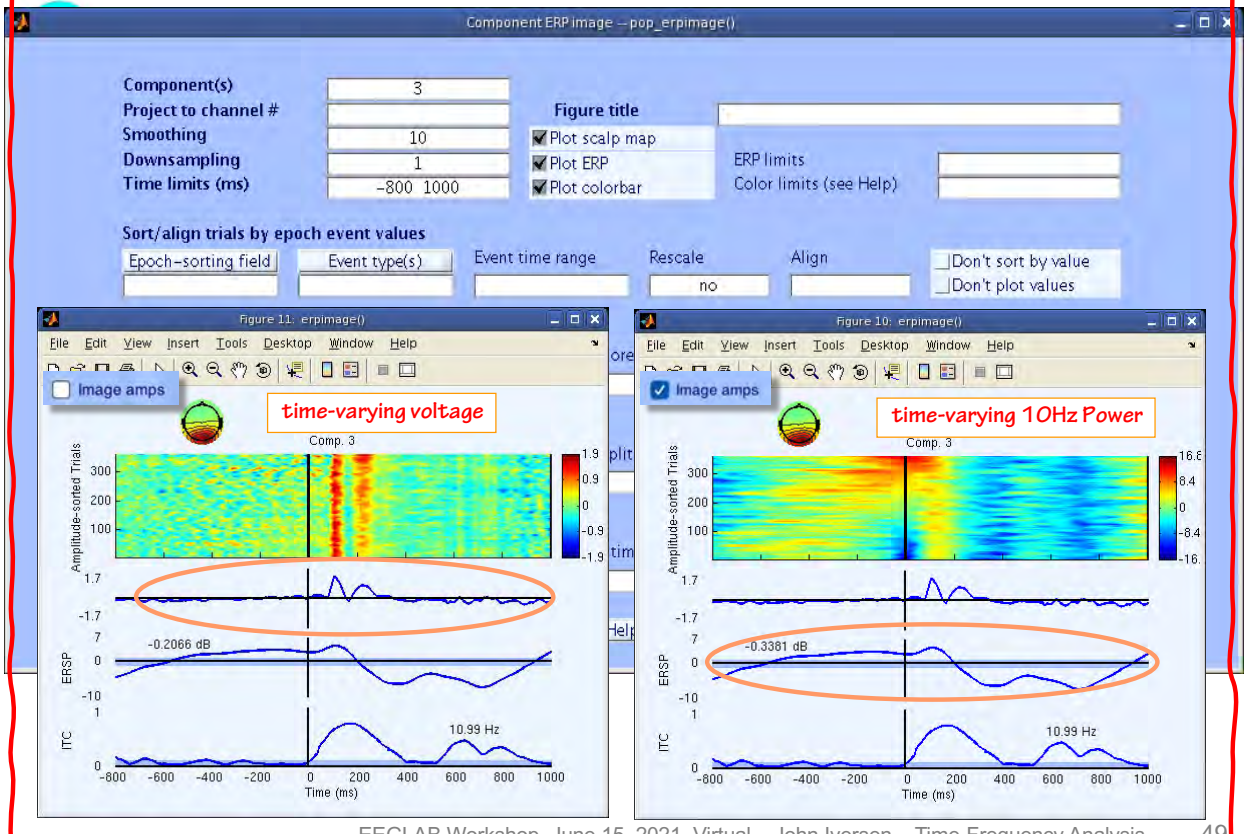




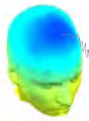
Component ERP Image: Activation vs. Amplitude

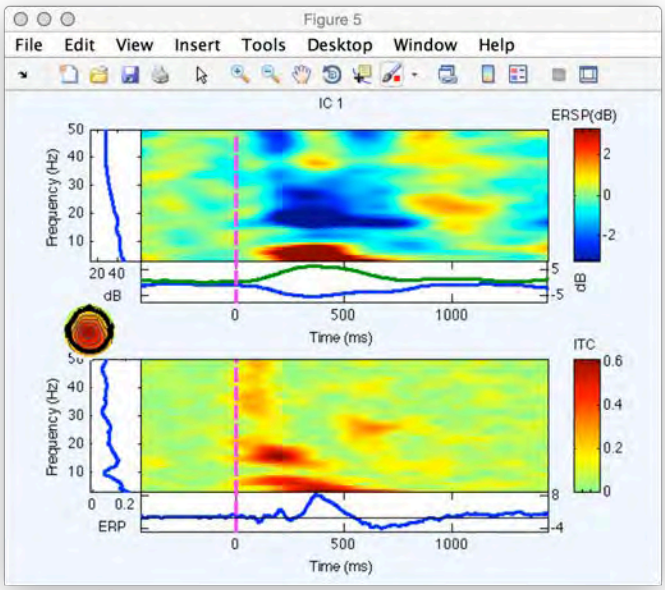


Component ERP Image: Activation vs. Amplitude



Putting it all together





Exercise

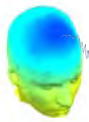
All: Compute ERSP/ITC for a component of your choice

Compute ERP Image (with ERSP and ITC displayed*)

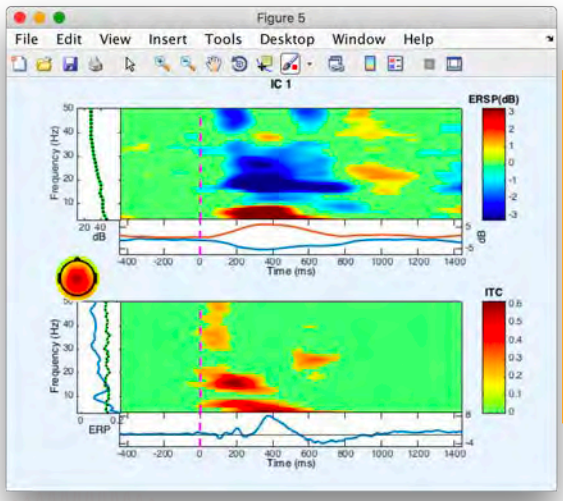
Use all of this information to explain the origin of the Evoked Response

Question: Which changes are significant? Use the options in ERP Image and ERSP dialogs to set significance threshold e.g. 0.01. Do the results survive?

Significance Testing



- Keep in mind: "is this significant?"



Method: Bootstrap

Green areas are not significant.

Scale of ERSP & ITC vales also give a clue:

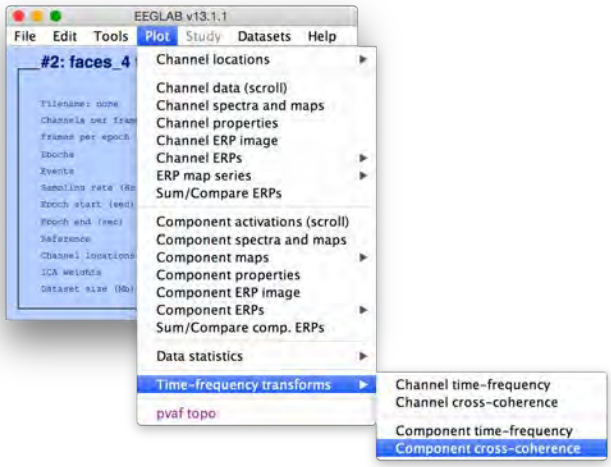
Large values are often encouraging of a significant effect

(Large ≈ > 1dB for ERSP; > 0.5 for ITC)

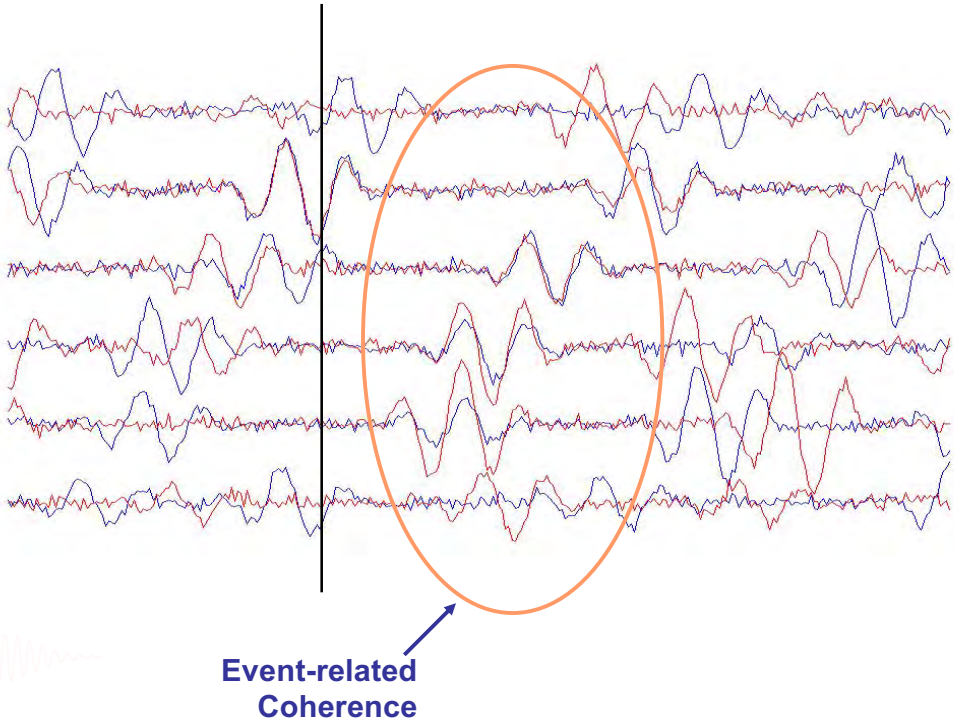
For exploratory purposes, can try 0.01 without FDR correction

Part 3b: Event Related Coherence

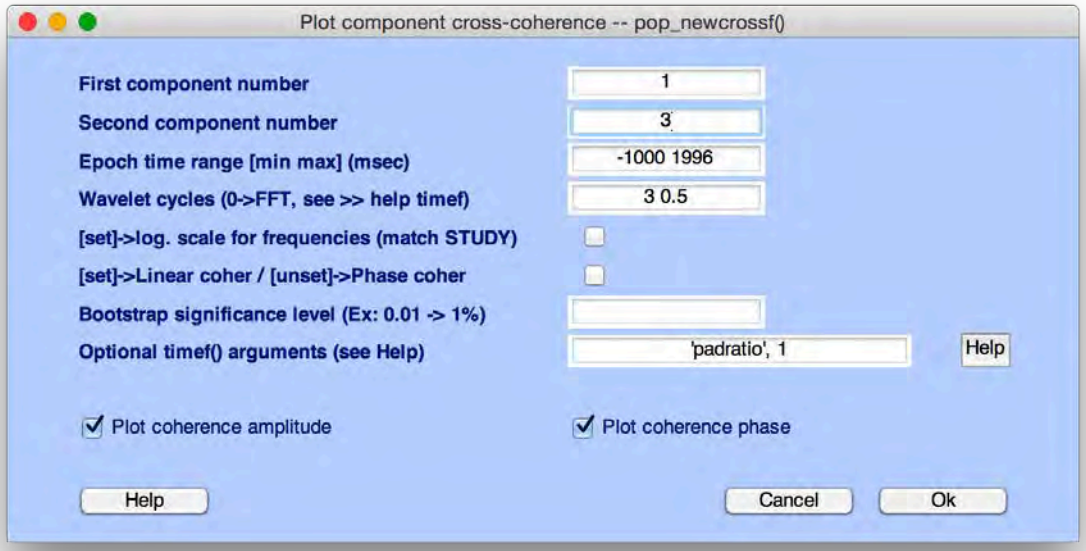
- Goal: How similar is the event-related response of two signals?
  - Between channels (problematic due to volume conduction)
  - Between ICs
  - Useful to quickly begin to understand relationships between components
  - SIFT provides more complete solution



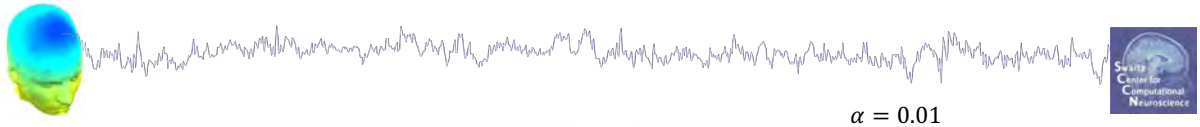
TWO SIMULATED THETA PROCESSES



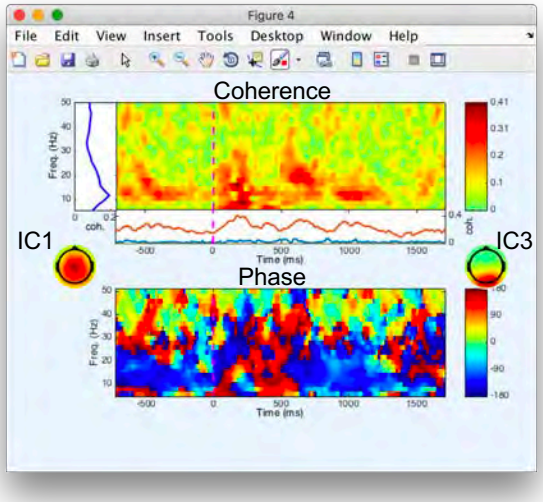
Try it!

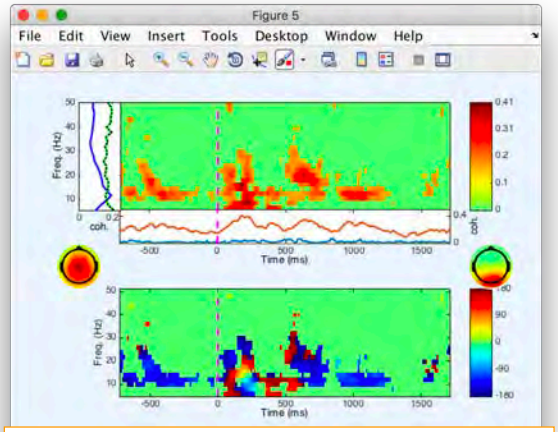


Cross coherence between IC 1 and IC 3



$\alpha = 0.01$



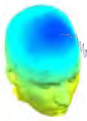


Significant event-related coherence (as well as tonic coherence) in alpha/beta bands

IC 1 tonically leads IC 3 (negative phase), but phase relationships are changed post-stimulus

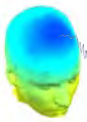
More advanced, directional, measures of effective connectivity *are present in the SIFT toolbox (a later lecture).*

Event-Related Coherence Exercise



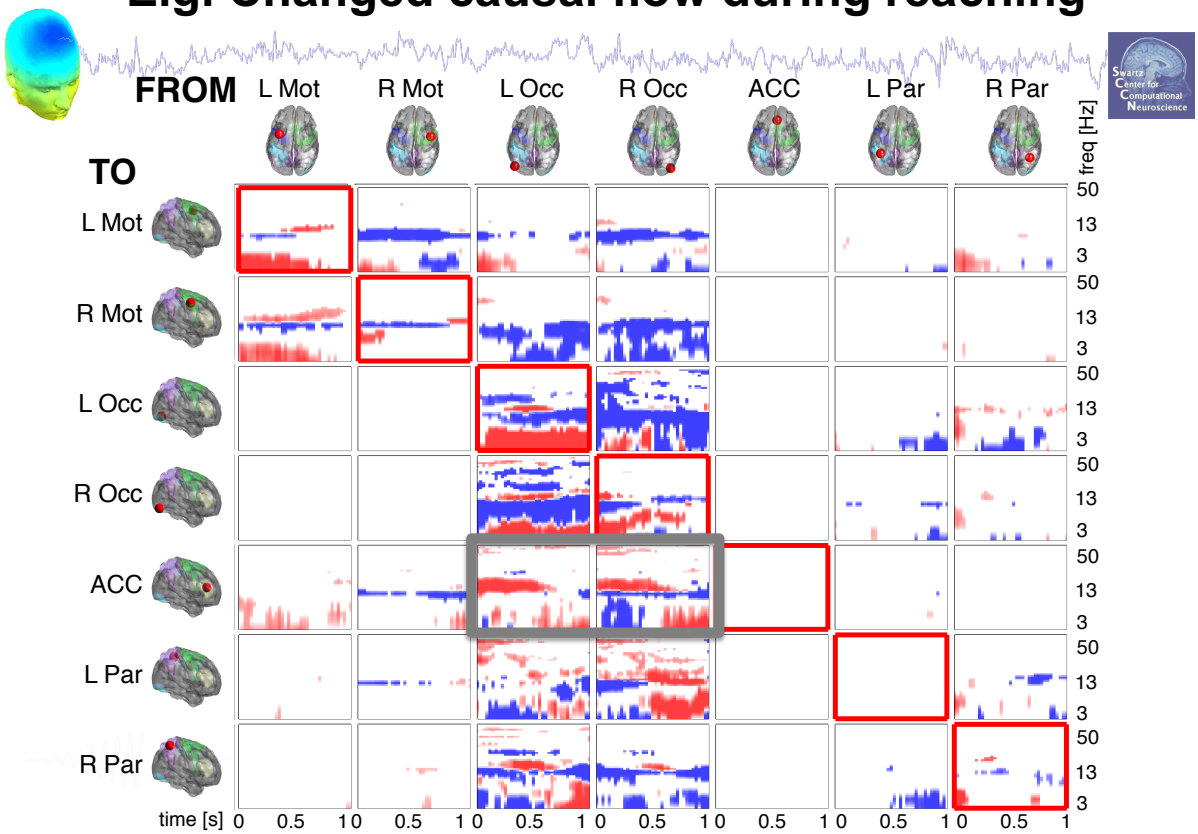
- Examine event-related coherence between two ICs
  - Which pair did you pick, and why? What do you predict?
  - What did you learn?
- Explore other options:
  - Significance threshold
  - Figure out how to subtract a baseline
  - Phase vs. Linear Coherence

Part 4: Other Applications

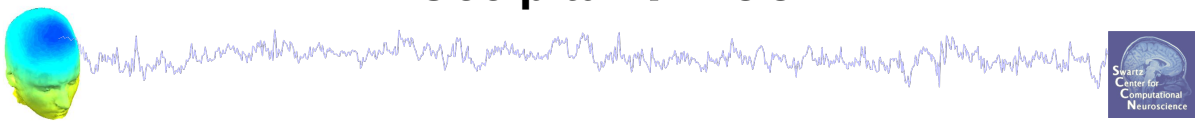


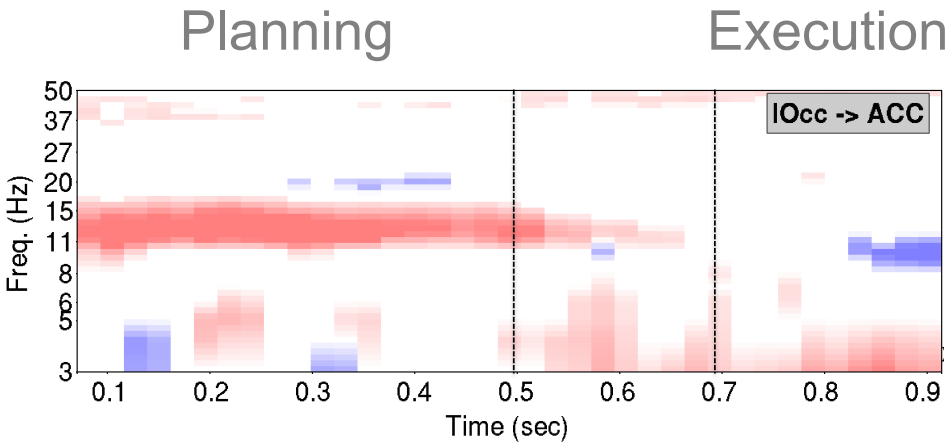
- Information Flow: Autoregressive modeling → time/frequency resolved directed information flow
  - **SIFT** – Tim Mullen [Tomorrow, Connectivity Analysis Track]
- Cross-frequency Analysis
  - **Phase/amplitude coupling (PAC)** - Ramón Martinez-Cancino [Right after this talk!]

E.g. Changed causal flow during reaching



Occipital → ACC





Iversen, et al, 2016; Courellis, et al, 2018

PRACTICUM

- Follow the red bordered slides, using the faces_4.set, epoched on the 'face' event.