The GLM family

- T-tests
- Simple regression
- ANOVA
- Multiple regression
- General linear model
  - Mixed effects/hierarchical
  - Timeseries models (e.g., autoregressive)
  - Robust regression
  - Penalized regression (LASSO, Ridge)
- Generalized linear models
  - Non-normal errors
  - Binary/categorical outcomes (logistic regression)

One-step solution
Iterative solutions (e.g., IWLS)

Slide from Tor Wager
A regression is a linear model

Varying factor: Contrast of image
Outcome: Reaction time

A regression is a linear model

- Given an experimental measure $x$ (e.g. contrast)
- We then do the experiment and collect data $RT$ (e.g. reaction time)
- Model: $RT = \beta_0 + x\beta_1 + \epsilon$
- Do some maths / run a software to find $\beta_1$ and $\beta_0$
- $RT = 23.6 + 2.7x$
A regression is a linear model

For each trial

\[ RT_1 = \beta_0 + 10*\beta_1 + \varepsilon_1 \]
\[ RT_2 = \beta_0 + 5*\beta_1 + \varepsilon_2 \]
\[ RT_3 = \beta_0 + 7*\beta_1 + \varepsilon_3 \]
...

To test for significance compare the original regression model \( RT_i = \beta_0 + c_i*\beta_1 + \varepsilon_i \) with the simplified model \( RT_i = \beta_0 + \varepsilon_i \)

Compare the fit

Test if 0 included in confidence interval
An ANOVA is a linear model

**Varying factor:** Type of image

**Outcome:** Reaction time (go/no-go)
\[ RT_{ij} = \beta_0 + \beta_i + \varepsilon_{ij} \]

that is to say the data (e.g. RT) = a constant term (grand mean \( \beta_0 \)) + the effect of a treatment (\( \beta_1 \) for fishes, \( \beta_2, \beta_3 \) for birds and reptiles) and the error term (\( \varepsilon_{ij} \))

For trial 4 (for example first trial of birds) we have

\[ RT_{2,1} = \beta_0 + 0^*\beta_1 + 1^*\beta_2 + 0^*\beta_3 + \varepsilon_{2,1} \]

For trial 13 (for example second trial of birds) we have

\[ RT_{2,2} = \beta_0 + 0^*\beta_1 + 1^*\beta_2 + 0^*\beta_3 + \varepsilon_{2,2} \]

Statistics: if there is an effect of treatment then error of the simplified model \( RT_{ij} = \beta_0 + \varepsilon_{ij} \) should be lower than the original model \( RT_{ij} = \beta_0 + \beta_i + \varepsilon_{ij} \)

Compare the fit

This is a GLM that is equivalent to an ANOVA
A GLM can do both a Regression and an ANOVA (ANCOVA)

**Varying factor:** Type of image AND contrast  
**Outcome:** Reaction time (go/no-go)

<table>
<thead>
<tr>
<th>Fishes</th>
<th>Birds</th>
<th>Reptiles</th>
<th>No-animal</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image 1" /></td>
<td><img src="image2.png" alt="Image 2" /></td>
<td><img src="image3.png" alt="Image 3" /></td>
<td><img src="image4.png" alt="Image 4" /></td>
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<tr>
<td><img src="image5.png" alt="Image 5" /></td>
<td><img src="image6.png" alt="Image 6" /></td>
<td><img src="image7.png" alt="Image 7" /></td>
<td><img src="image8.png" alt="Image 8" /></td>
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<td><img src="image9.png" alt="Image 9" /></td>
<td><img src="image10.png" alt="Image 10" /></td>
<td><img src="image11.png" alt="Image 11" /></td>
<td><img src="image12.png" alt="Image 12" /></td>
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<td><img src="image13.png" alt="Image 13" /></td>
<td><img src="image14.png" alt="Image 14" /></td>
<td><img src="image15.png" alt="Image 15" /></td>
<td><img src="image16.png" alt="Image 16" /></td>
</tr>
</tbody>
</table>

For example, for trial (first bird with contrast $c_{2,1}$) we have:

\[ RT_{2,1} = \beta_0 + 0^*\beta_1 + 1^*\beta_2 + 0^*\beta_3 + 0^*\beta_3 + c_{2,1}^*\beta_4 + \varepsilon_{2,1} \]

Categorical var. ANOVA  
Continous var. REGRESSION
The design matrix

\[
\begin{align*}
y(1..3) &= 1x\beta_1 + 0x\beta_2 + 0x\beta_3 + 0x\beta_4 + c + \text{error} \\
y(4..6) &= 0x\beta_1 + 1x\beta_2 + 0x\beta_3 + 0x\beta_4 + c + \text{error} \\
y(7..9) &= 0x\beta_1 + 0x\beta_2 + 1x\beta_3 + 0x\beta_4 + c + \text{error} \\
y(10..12) &= 0x\beta_1 + 0x\beta_2 + 0x\beta_3 + 1x\beta_4 + c + \text{error}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Y</th>
<th>Gp</th>
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<tbody>
<tr>
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<td>9</td>
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</tbody>
</table>

Design matrix
\[G_1, G_2, G_3, G_4, C\]
Central tendency and plots again

Categorical var.

Linear Modeling of EEG data: level 1

Electrode 1

Continuous var.

Electrode difference

Between conditions

Significance: bootstrap trials to get confidence interval of $\beta$s
Scalp topography of beta difference at a given latency

Scalp topography of potential difference (masked using beta signif.)

Limit of the regions masked for significance
1. Interaction design (EEGLAB default)

2. Factorial design

3. Full factorial design
Linear Modeling of EEG data: level 2

Participant 1
chan1  chan2  chan3  chan4

Participant 2

Participant 3

Level 2
Standard stats.
2nd level-GLM

GLM: ordinary least square (OLS) vs. weighted least square (WLS)
Linear Modeling of EEG data: level 2

Level 1

Participant 1

Participant 2

Participant 3

Level 2

2-way ANOVA:
- Main effect 1 (shape)
- Main effect 2 (color)
- Interaction
Linear Modeling of EEG data: level 2

Participant 1

Participant 2

Participant 3

Shape:
Group t-test on $\beta\circ - \beta\square$

Color
Group t-test on $\beta\bullet - \beta\bullet$

Interaction
- One sample t-test on $\beta\bullet, \beta\bullet, \beta\square, \beta\square$
Linear Modeling of EEG data: level 2

Mixed effect model
(still a GLM)

Hierarchical GLM

Level 1
- Participant 1
- Participant 2
- Participant 3

Level 2
2-way ANOVA:
- Main effect 1 (shape)
- Main effect 2 (color)
- Interaction

VS
The End