

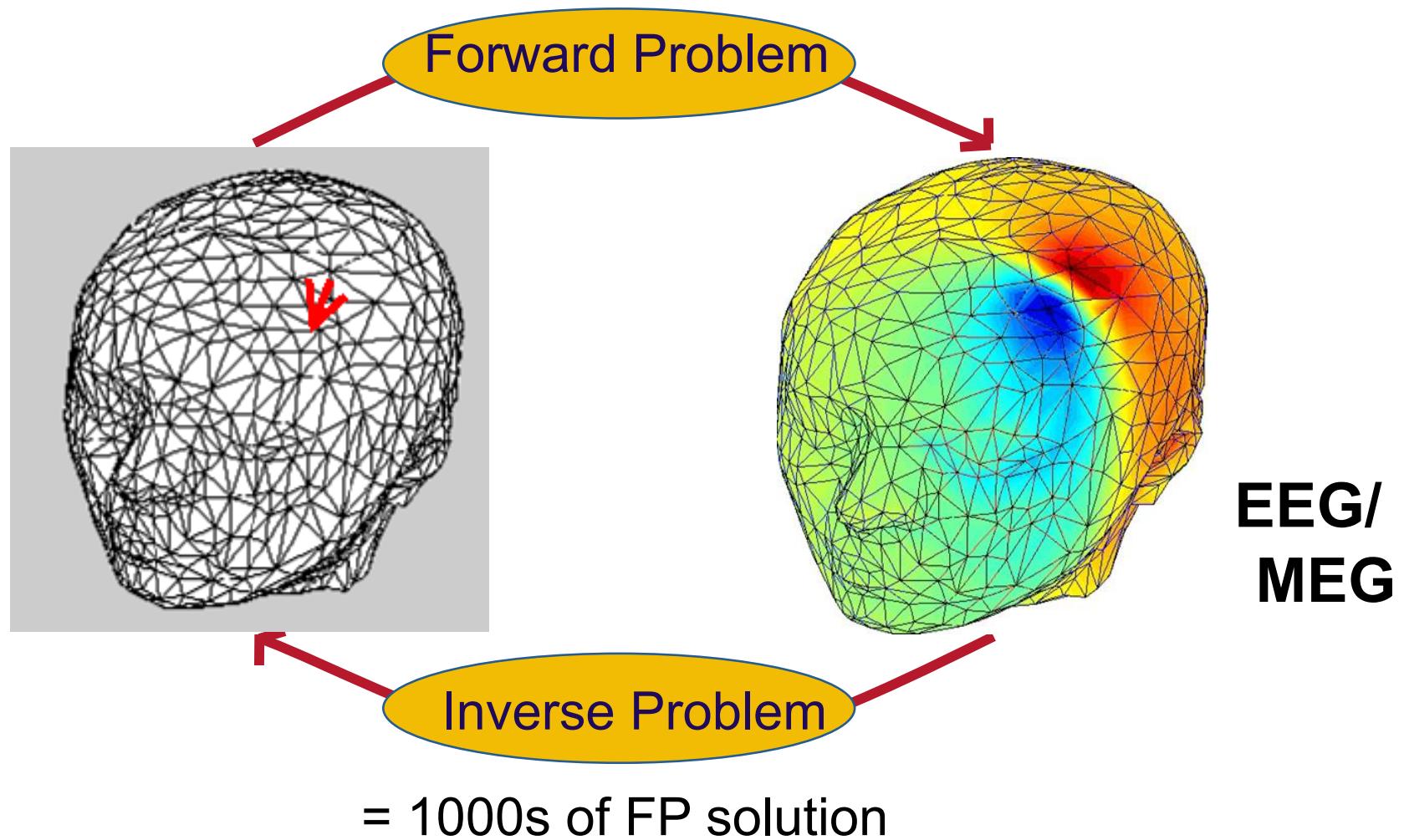
# Forward Problem of EEG

Zeynep AKALIN ACAR

14th EEGLAB Workshop, Mallorca

September, 2011

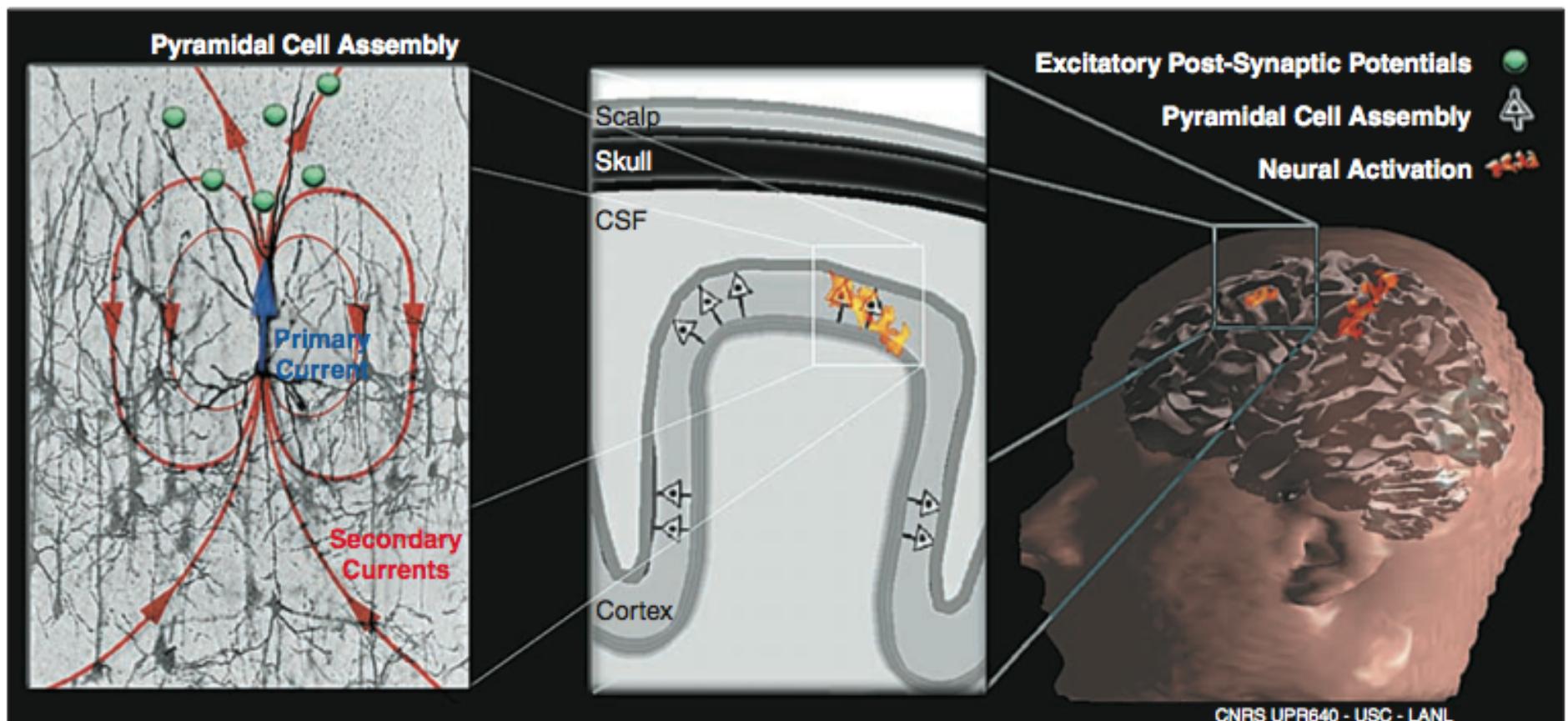
# Forward and inverse problem



# Components of source localization

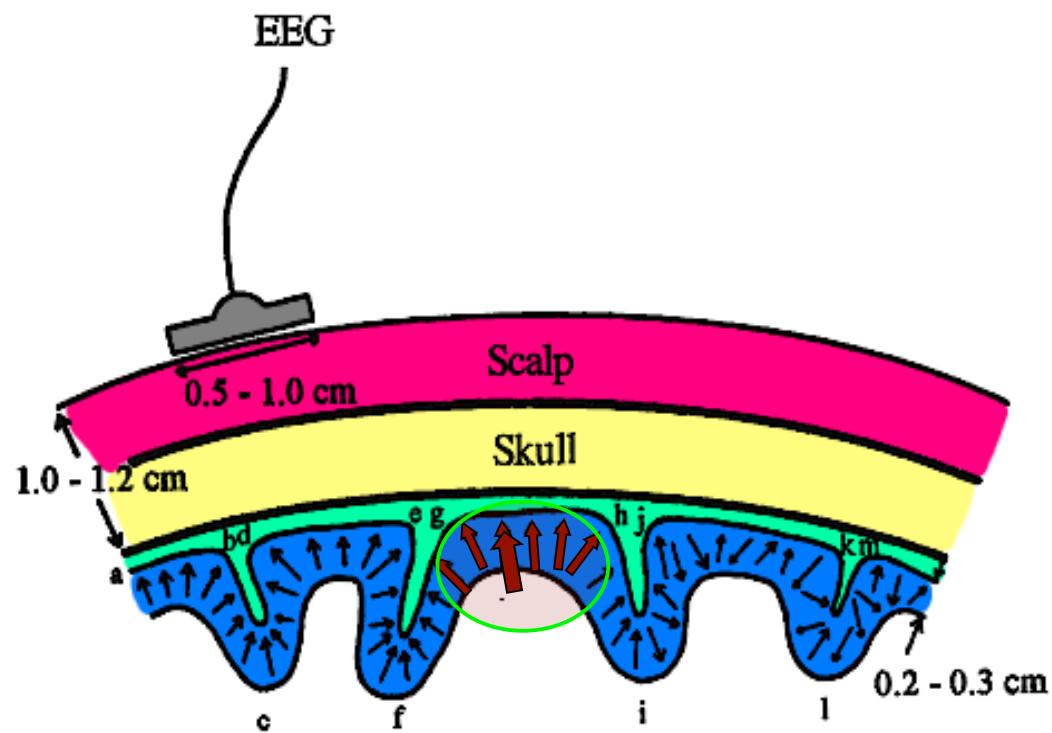
- ◆ Numerical head model
  - ◆ Choice of numerical method
  - ◆ Conductivity distribution in the head
  - ◆ Co-registration of EEG electrodes with MRI
  - ◆ Number/position of electrodes on the head surface
- 
- ◆ A priori information of source space
  - ◆ Processing of EEG signals
  - ◆ Choice of inverse model

# Generators of EEG



Baillet et al, 2001

# Equivalent cortical dipoles

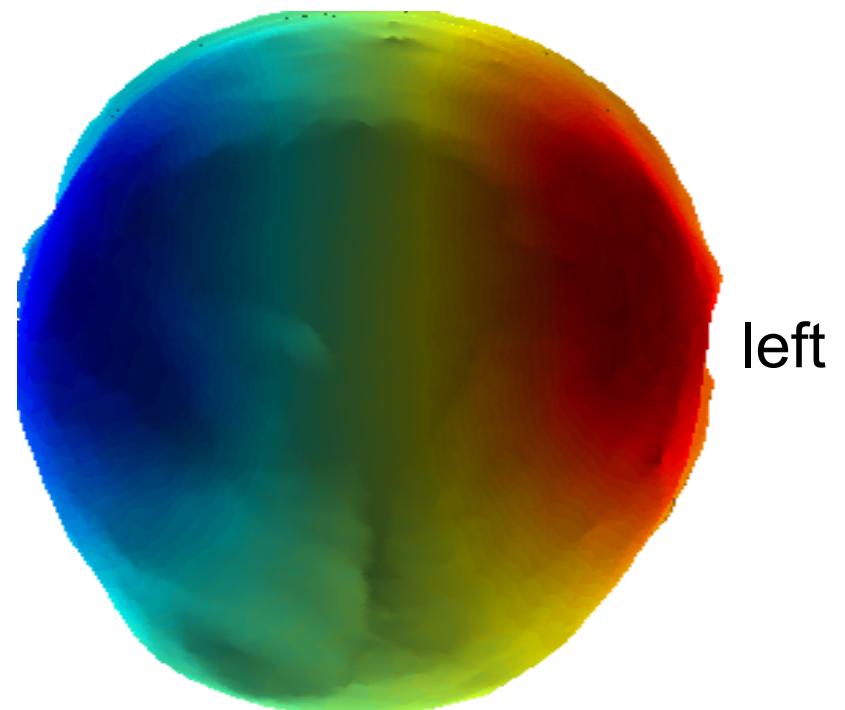
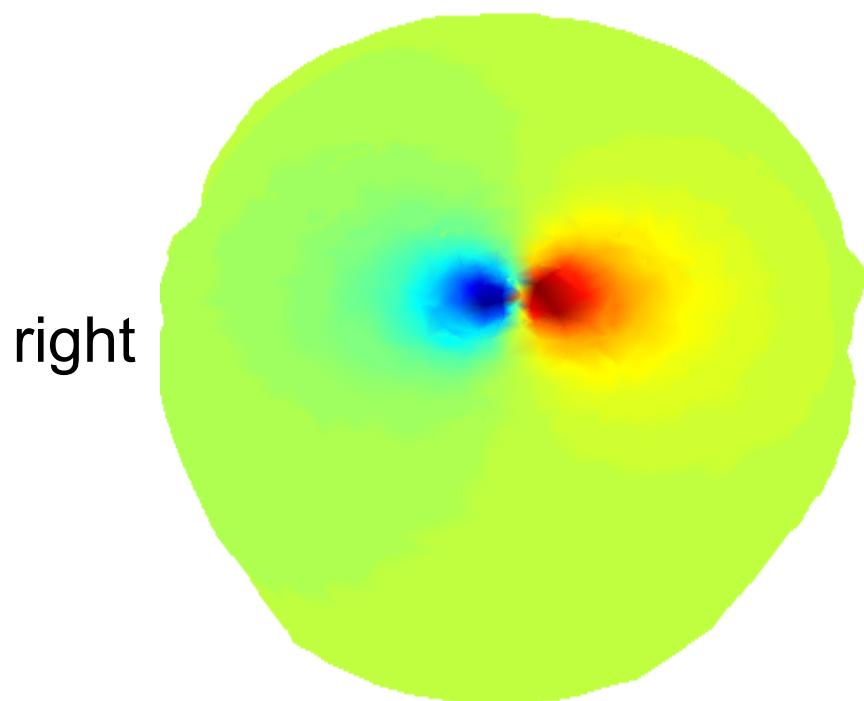
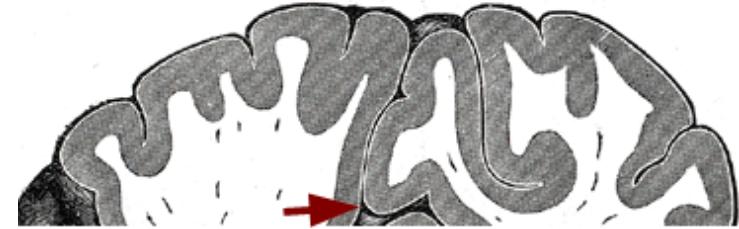


# Potential fields on the scalp

Shallow tangential source

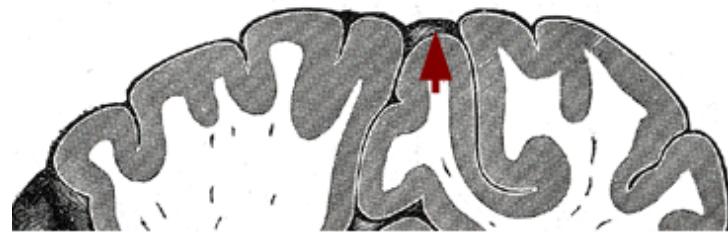


Deep tangential source



# Potential fields on the scalp

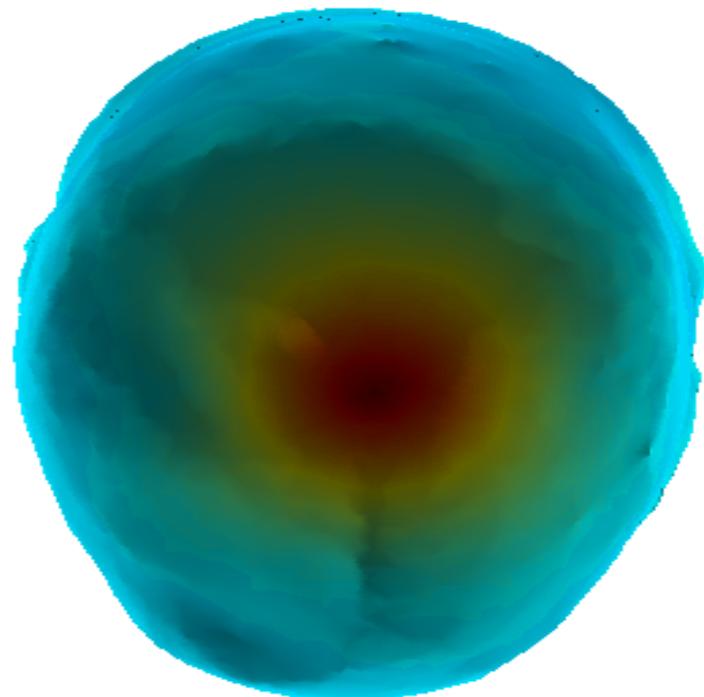
Shallow radial source



Deep radial source

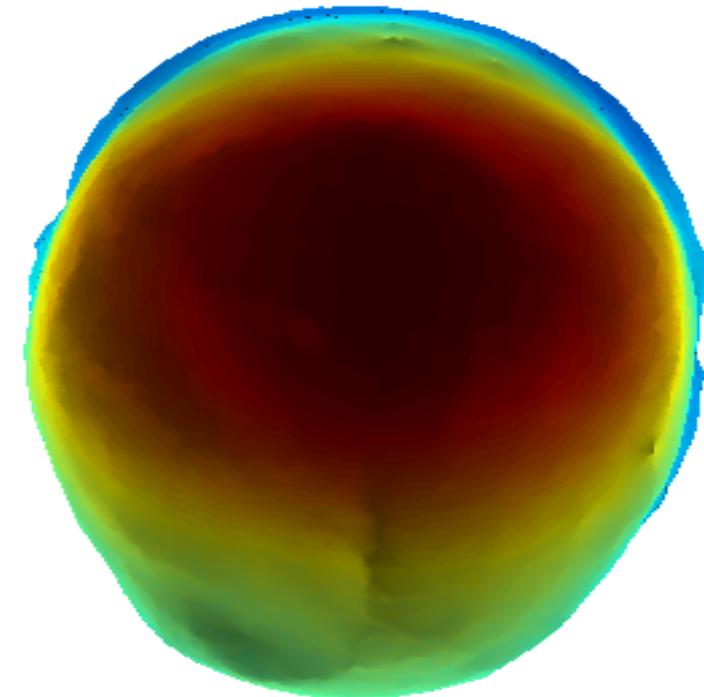


right



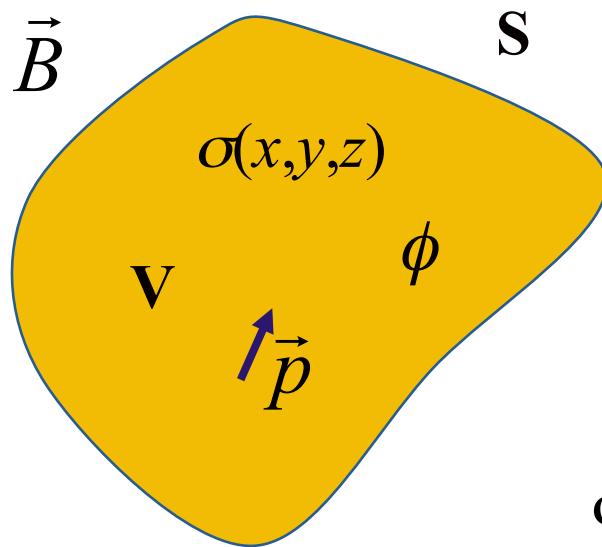
front

left



front

# Formulation of the FP



$$\nabla \cdot (\sigma \nabla \Phi) = -\nabla \cdot J^P \quad \text{inside } V$$

$$\sigma \frac{\partial \Phi}{\partial n} = 0 \quad \text{on } S$$

$\sigma(x,y,z)$  : conductivity distribution

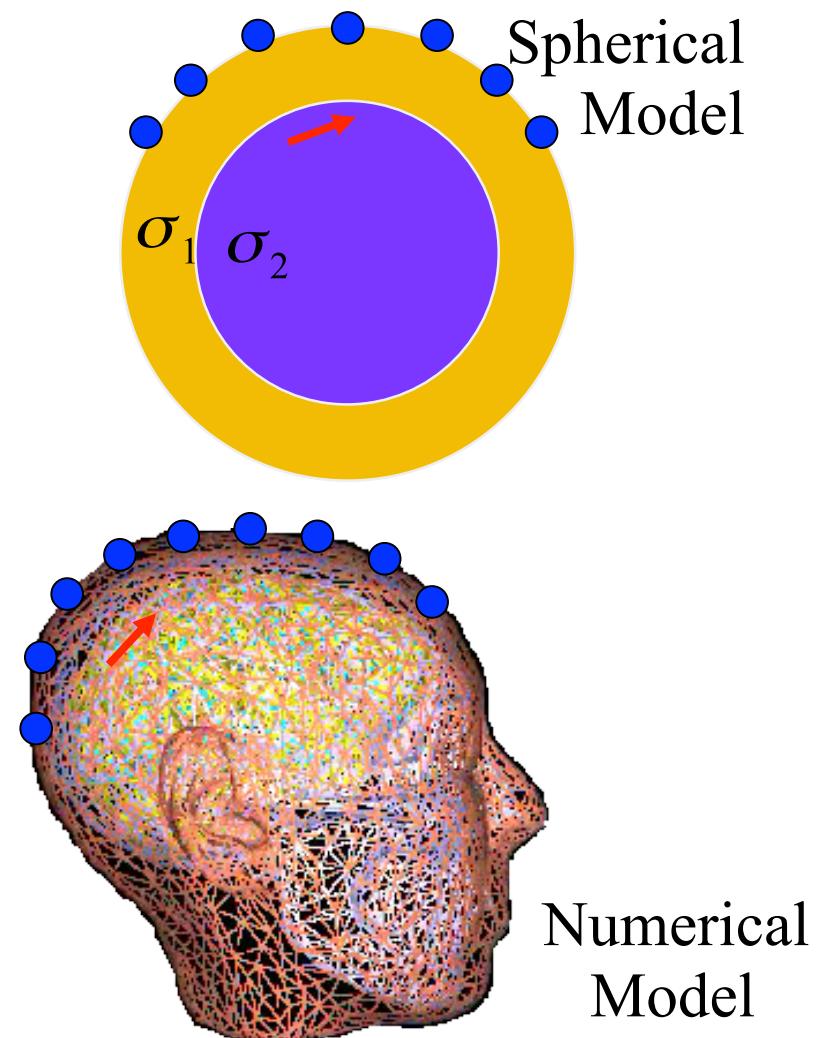
$\vec{p}$  : current source

Reference: Gulrajani, R., Bioelectricity and biomagnetism

# To Solve the Forward Problem

WE NEED

- ◆ Head Model
  - Conductivity values
  - Geometry
- ◆ Source distribution
  - Magnitude
  - Location
  - Direction
- ◆ Field Locations
- ◆ Solver



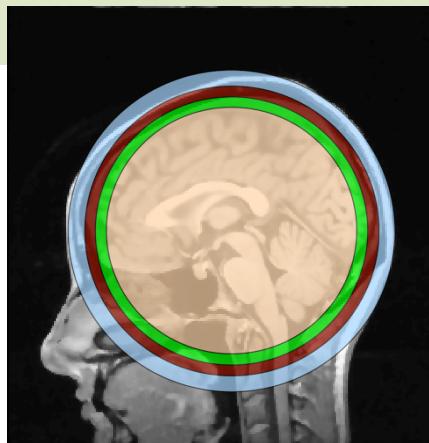
# Head Modeling Comparison

## Simple Head Models

- ◆ Single layer sphere, spheroid
- ◆ 3-4 layer sphere

### ANALYTICAL SOLVER

Simple, fast, but not accurate

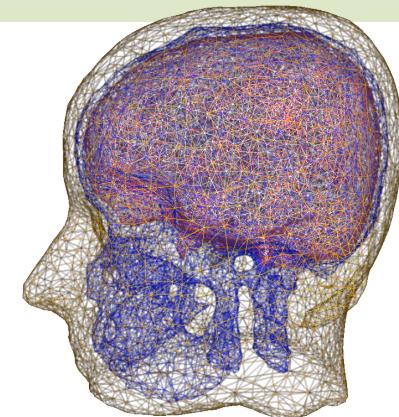


## Realistic Head Models

- ◆ Boundary Element
- ◆ Finite Element
- ◆ Finite Difference

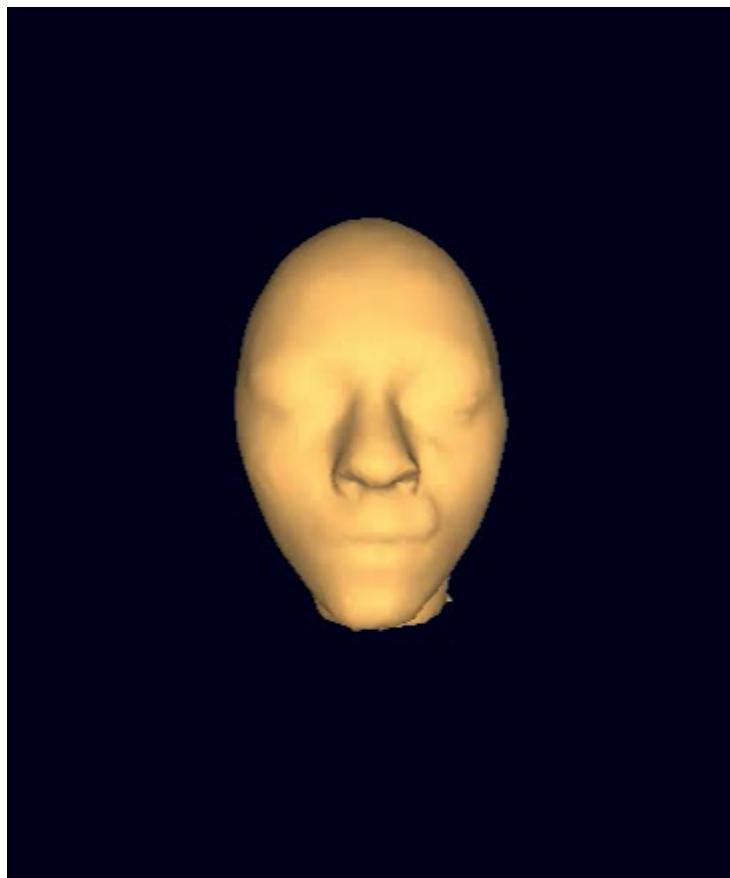
### NUMERICAL SOLVER

Represents head shape better, but computationally complex



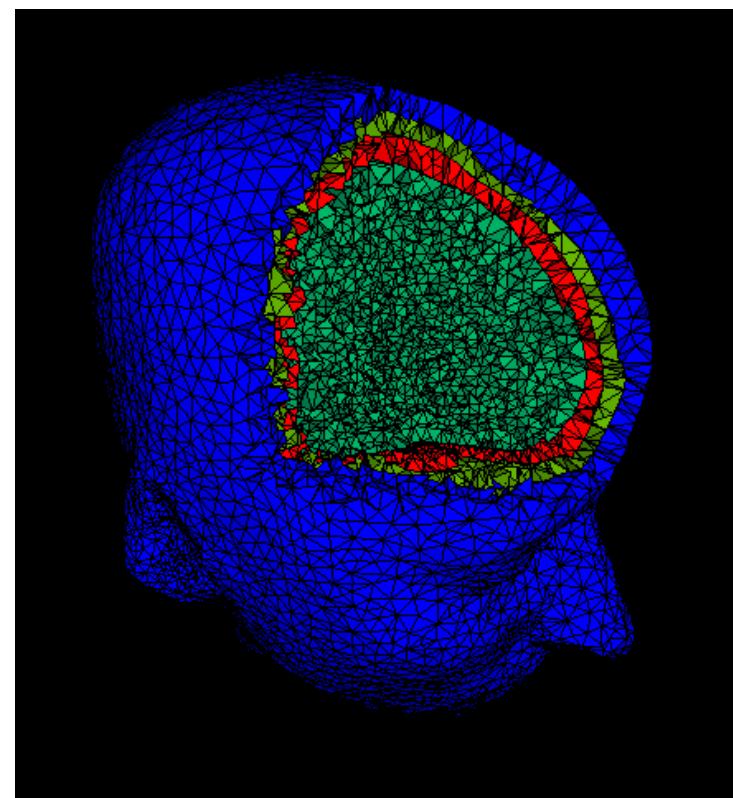
# Numerical Head Models

**BEM**



NFT BEM mesh

**FEM**

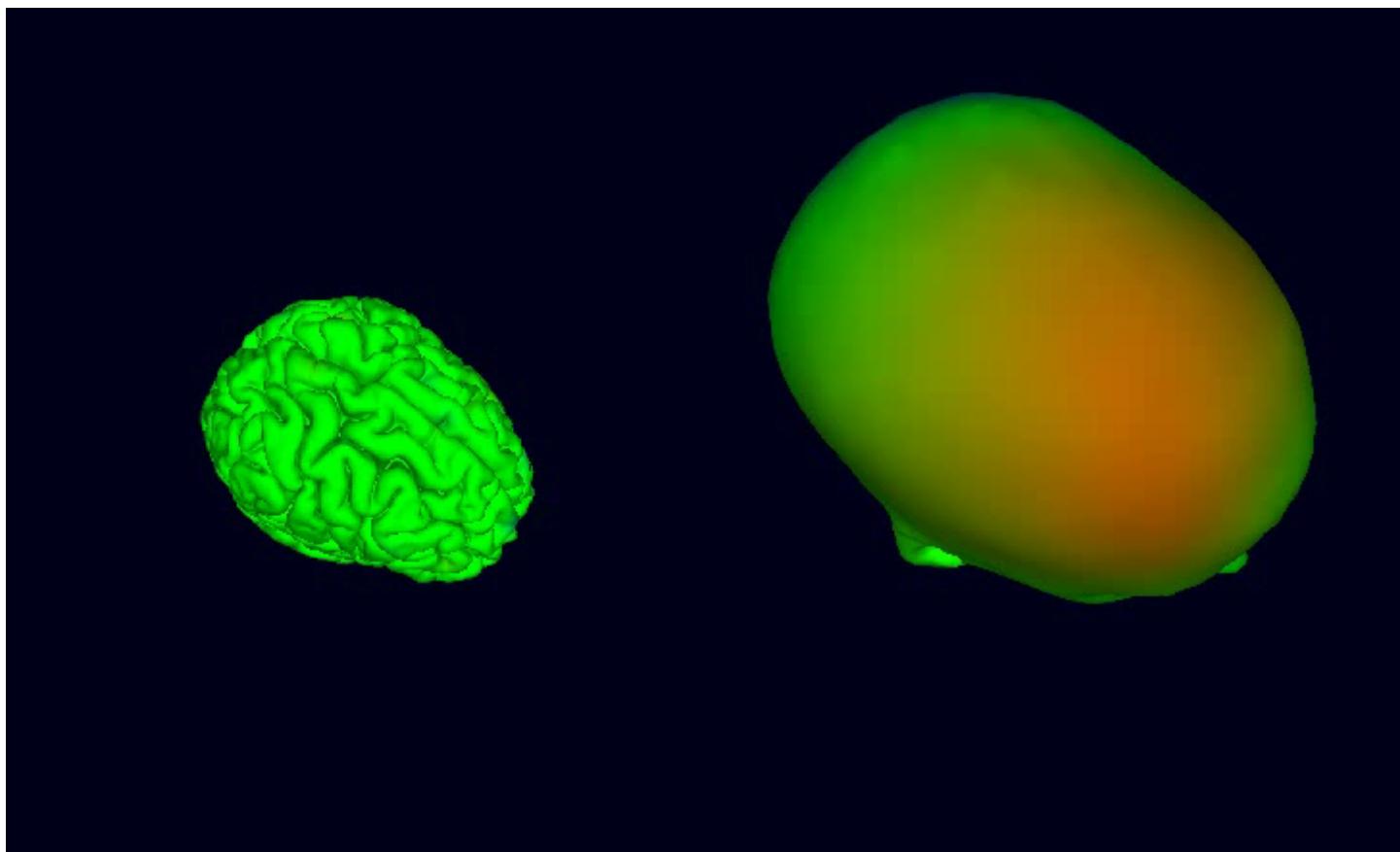


Generated using Tetgen  
from NFT BEM mesh

# FEM/BEM comparison

	BEM	FEM
Position of computational points	surface	volume
Free choice of computational points	yes	yes
System matrix	full	sparse
Solvers	direct	iterative
Number of compartments	small	large
Requires tessellation	yes	yes
Handles anisotropy	no	yes

# Potentials on the scalp



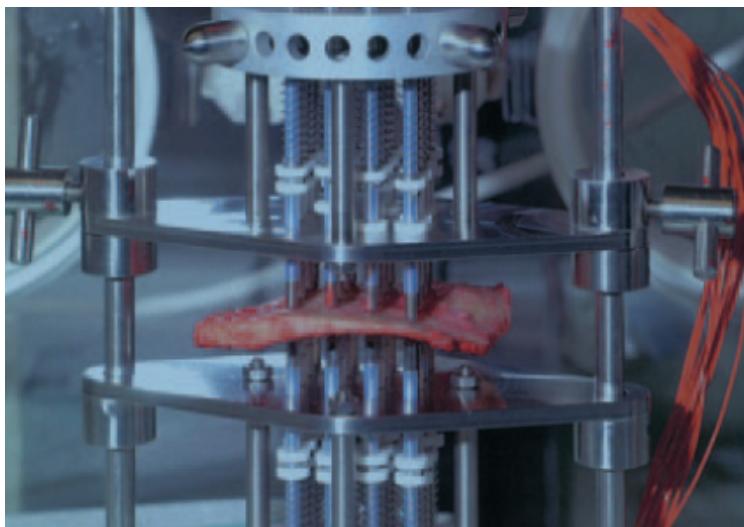
# Components of source localization

- ◆ Numerical head model
- ◆ Choice of numerical method
- ◆ **Conductivity distribution in the head**
- ◆ Co-registration of EEG electrodes with MRI
- ◆ Number/position of electrodes on the head surface
- ◆ A priori information of source space
- ◆ Processing of EEG signals
- ◆ Choice of inverse model

# Skull conductivity measurement

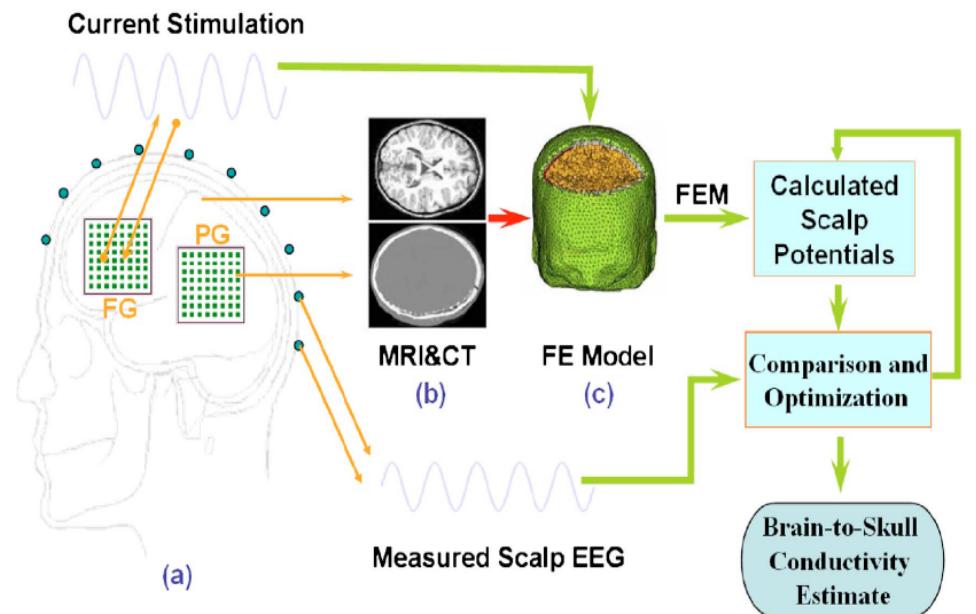
Measurement of skull conductivity

In vivo



Hoekama *et al*, 2003

In vitro



He *et al*, 2005

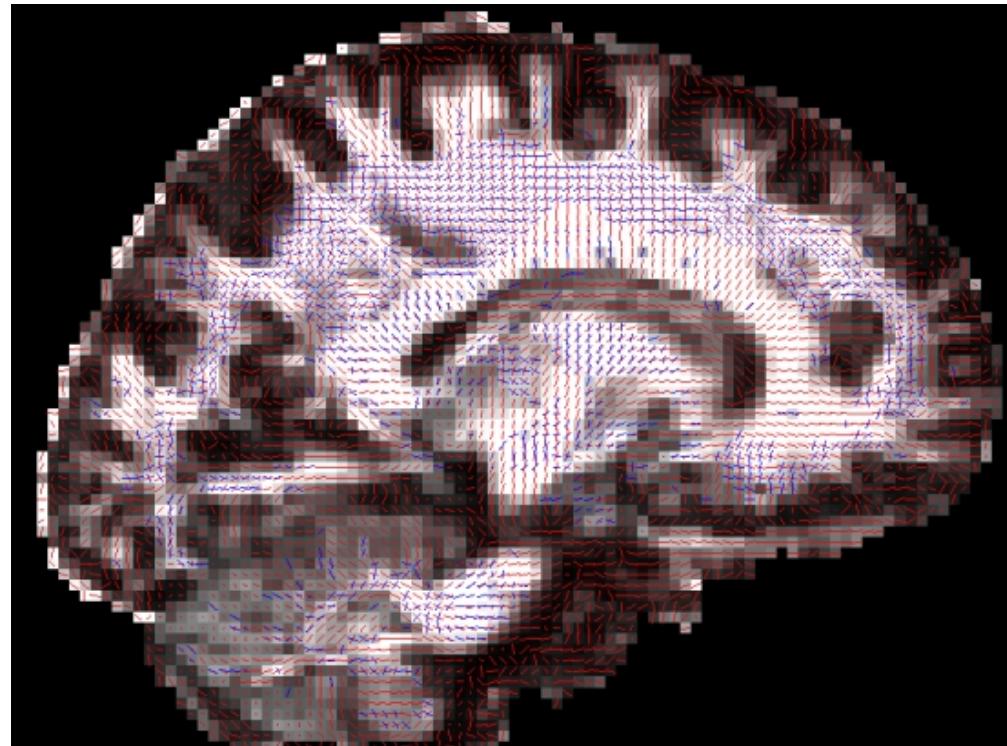
# Skull conductivity

Brain to skull ratio			
Rush and Driscoll	1968	80	
Cohen and Cuffin	1983	80	
Oostendorp et al	2000	15	
Lai et al	2005	25	

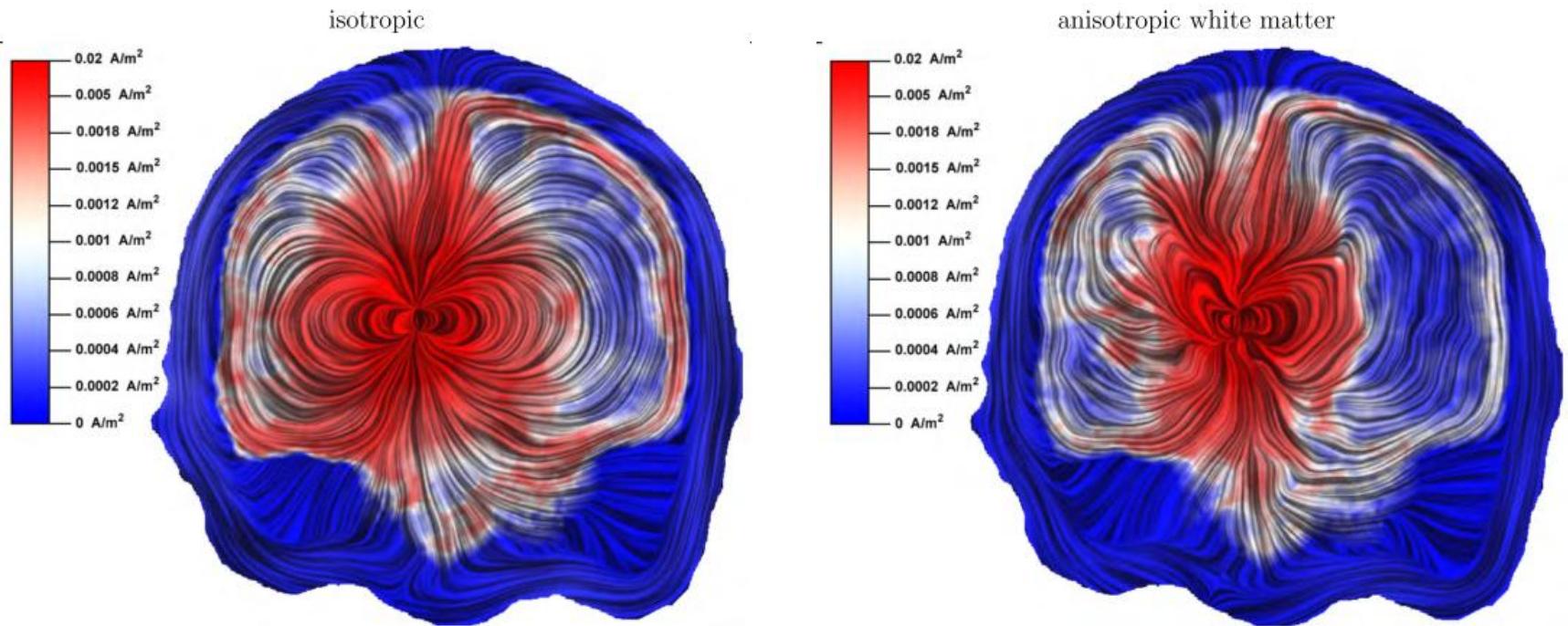
Measurement	Age	$\sigma$ (mS/m)	ratio
Agar-agar phantom	–	43.6	7.5
Patient 1	11	80.1	4
Patient 2	25	71.2	4.6
Patient 3	36	53.7	6.2
Patient 4	46	34.4	9.7
Patient 5	50	32.0	10.3
Post mortem skull	68	21.4	15.7

# Anisotropy

- ◆ Directional conductivity for skull and WM.
- ◆ WM anisotropy can be obtained from diffusion tensor imaging (DTI).
- ◆ WM  
anisotropy  
ratio = 9:1
- ◆ Skull  
ratio = 10:1



# Anisotropy

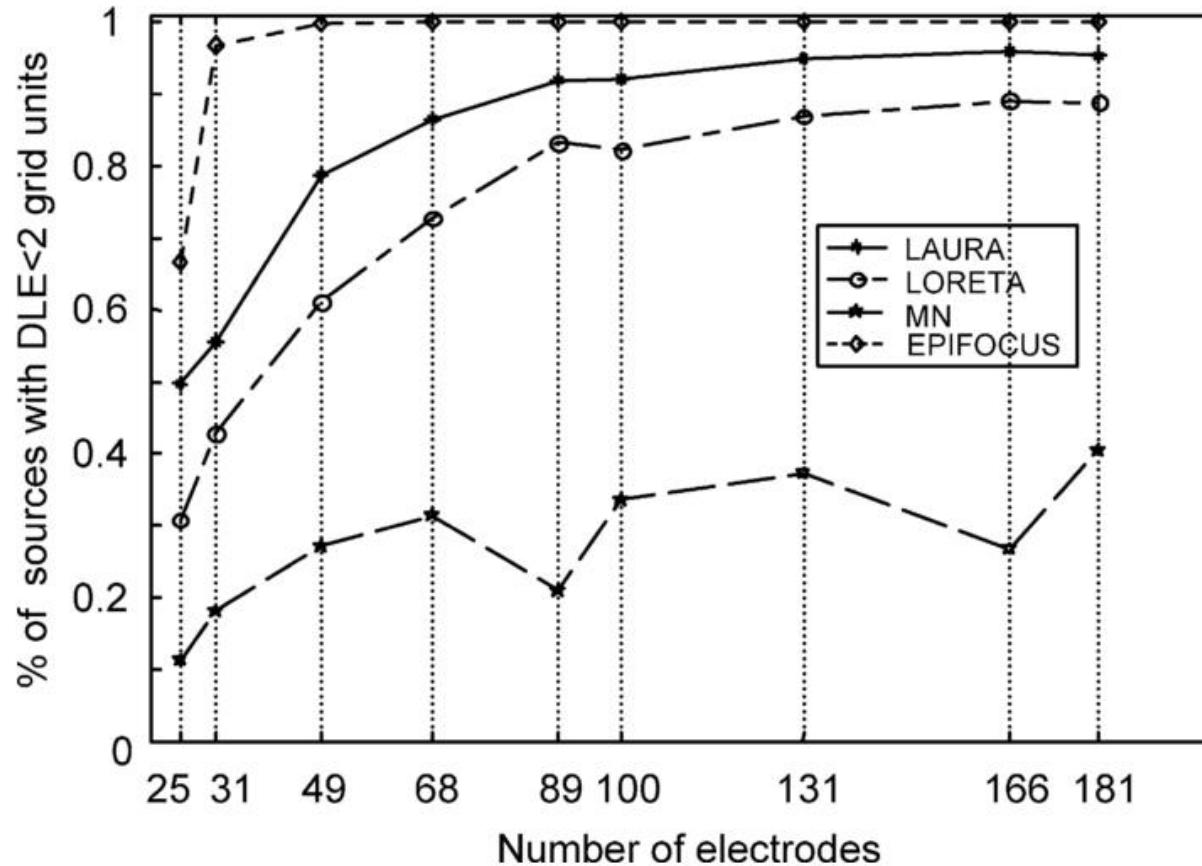


Return currents for a left thalamic source on a coronal cut  
Wolters et al, 2006

# Components of source localization

- ◆ Numerical head model
- ◆ Choice of numerical method
- ◆ Conductivity distribution in the head
- ◆ Co-registration of EEG electrodes with MRI
- ◆ Number/position of electrodes on the head surface
- ◆ A priori information of source space
- ◆ Processing of EEG signals
- ◆ Choice of inverse model

# Number of electrodes



Michel *et al,*  
2004

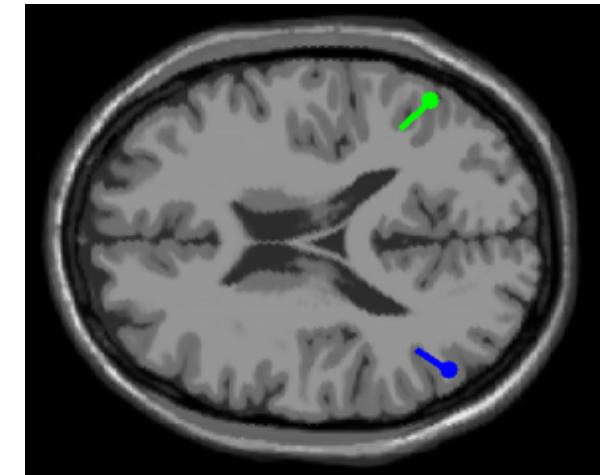
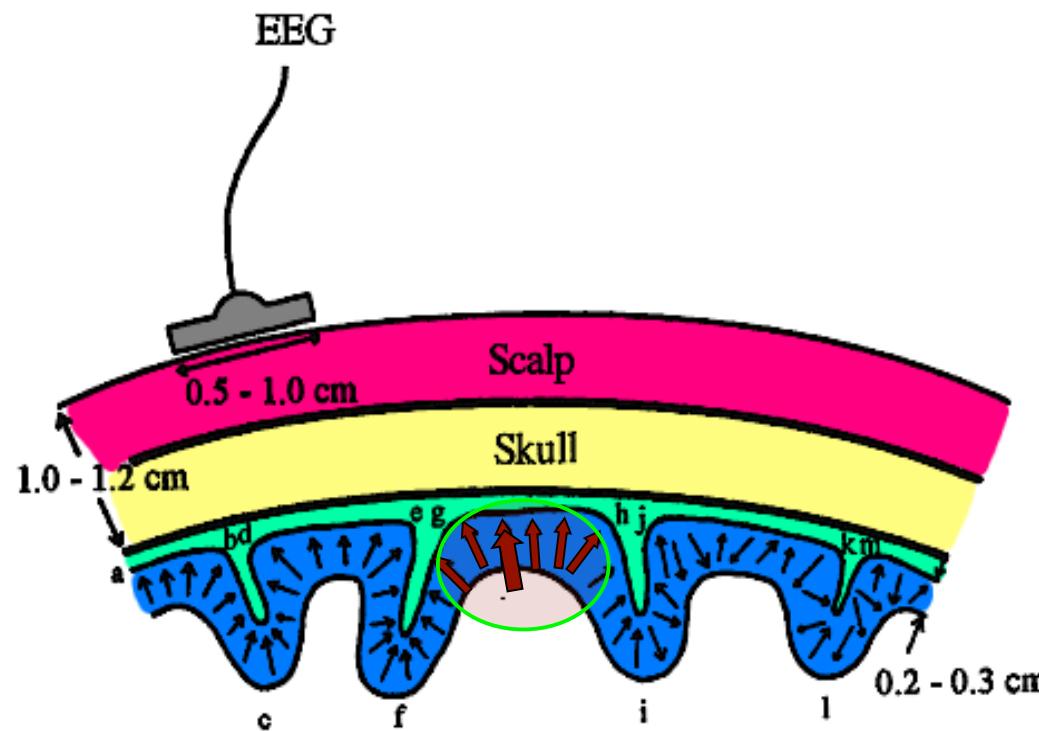
Simulation study, spherical model, 1152 dipoles

# Components of source localization

- ◆ Numerical head model
- ◆ Choice of numerical method
- ◆ Conductivity distribution in the head
- ◆ Co-registration of EEG electrodes with MRI
- ◆ Number/position of electrodes on the head surface
- ◆ A priori information of source space
- ◆ Processing of EEG signals
- ◆ Choice of inverse model

# Source models

Equivalent current dipole

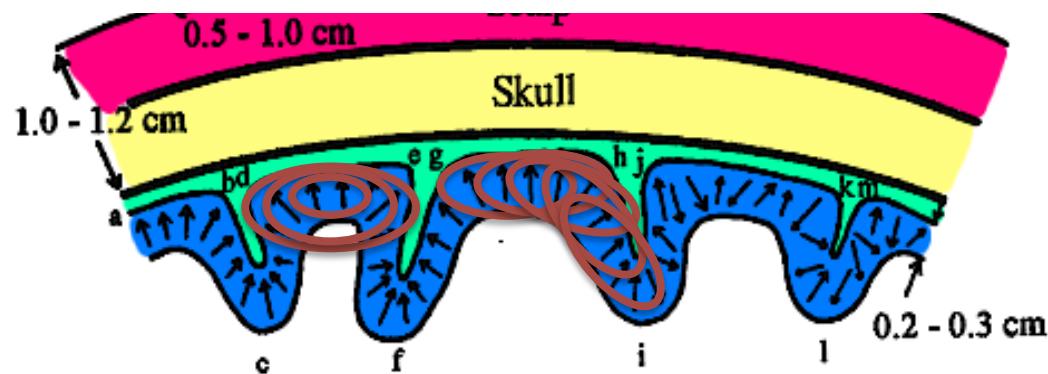


Overdetermined  
Nonlinear optimization

Source space:  
Brain volume

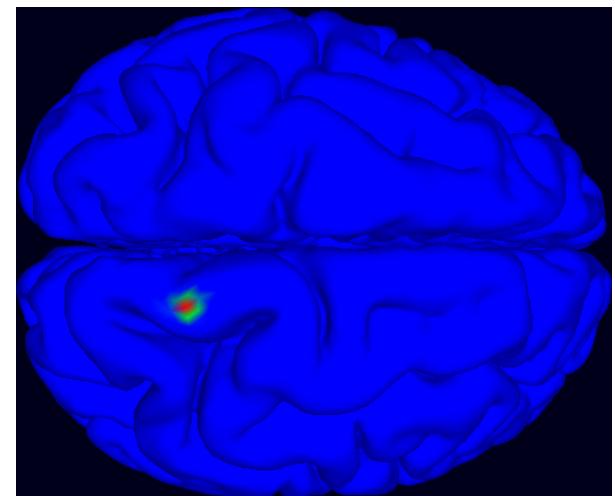
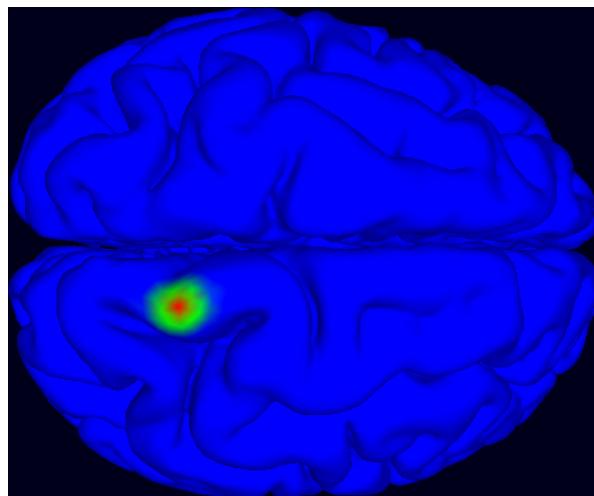
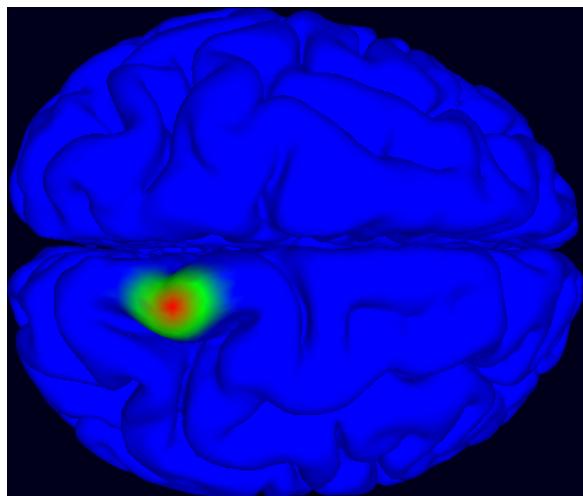
# Cortical patch sources

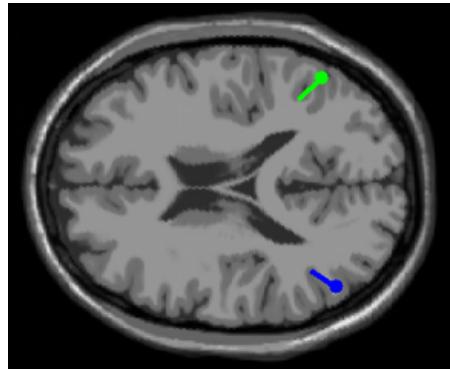
Distributed source models



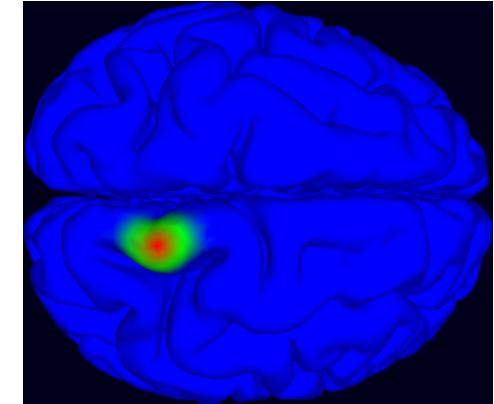
Source space:  
Cortical surface

Overlapping patches





# Inverse Problem



## Parametric Methods

- ◆ Overdetermined
- ◆ Searches for parameters of a number of dipoles
- ◆ Nonlinear optimization techniques
- ◆ May converge to local minima
- ◆ Non-linear least squares, beamforming, MUSIC, simulated annealing, genetic algorithms, etc.

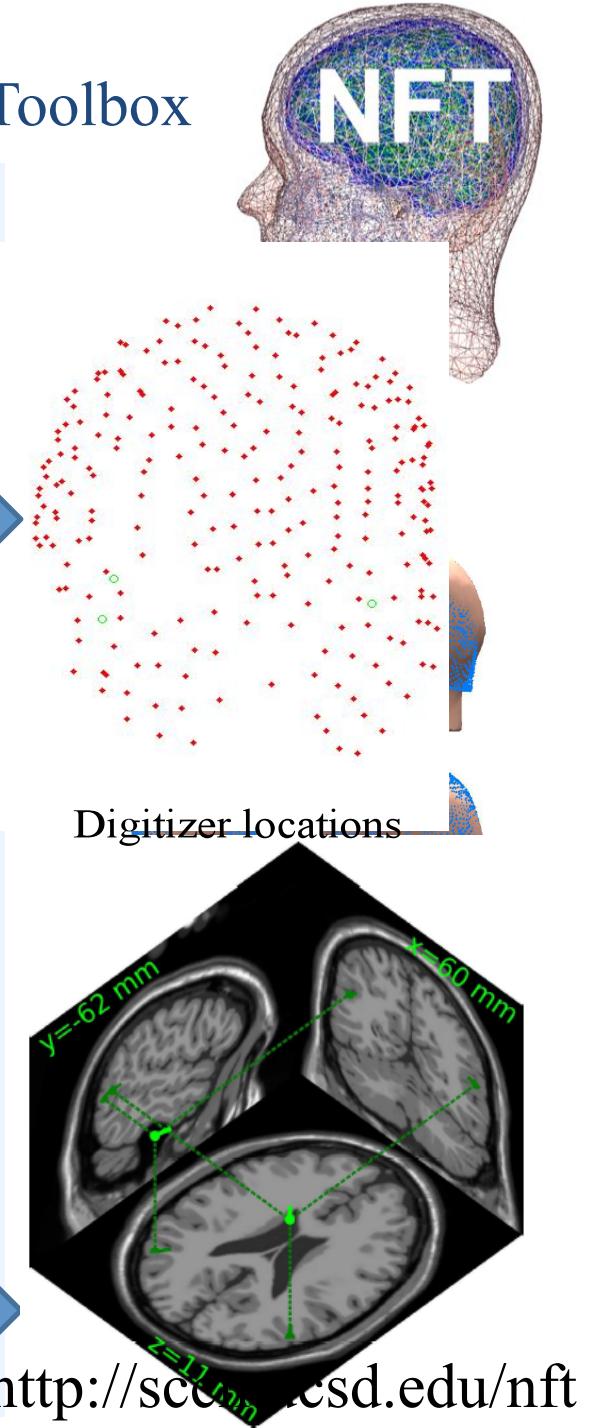
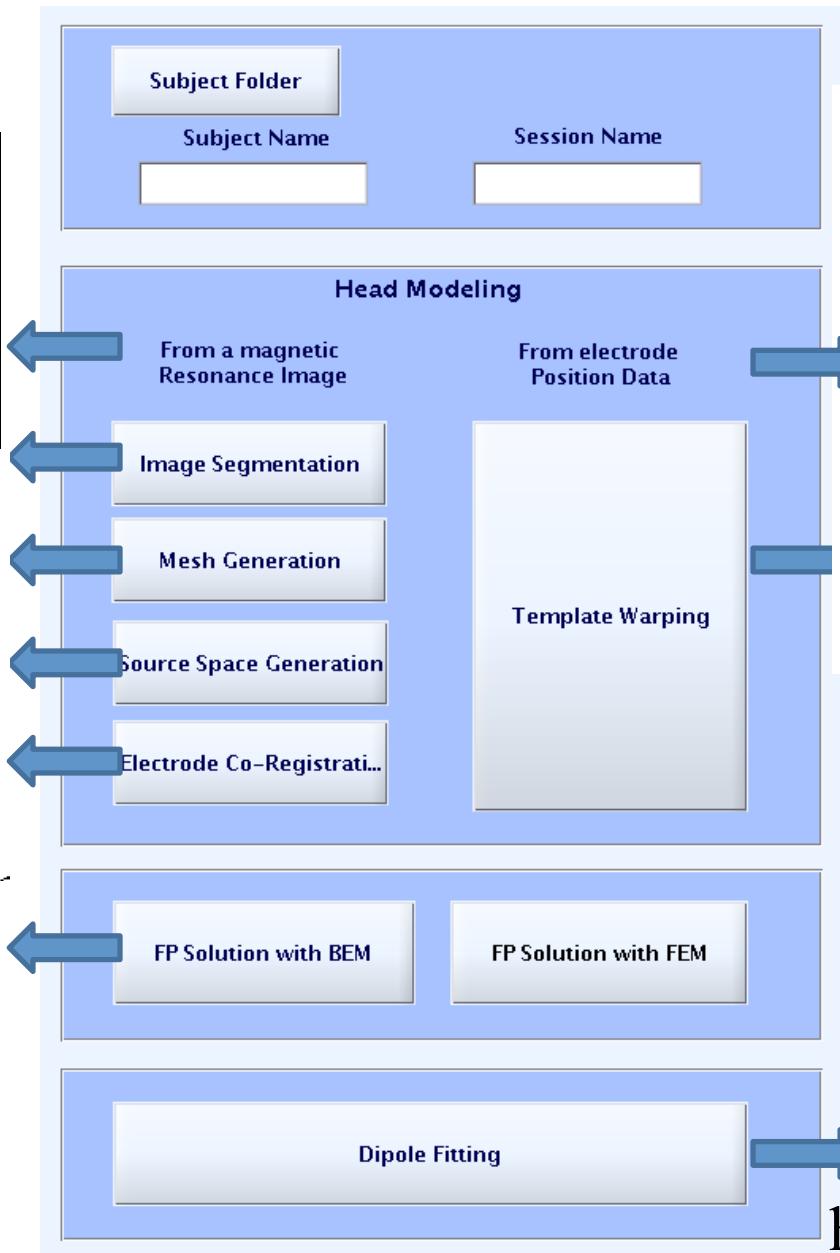
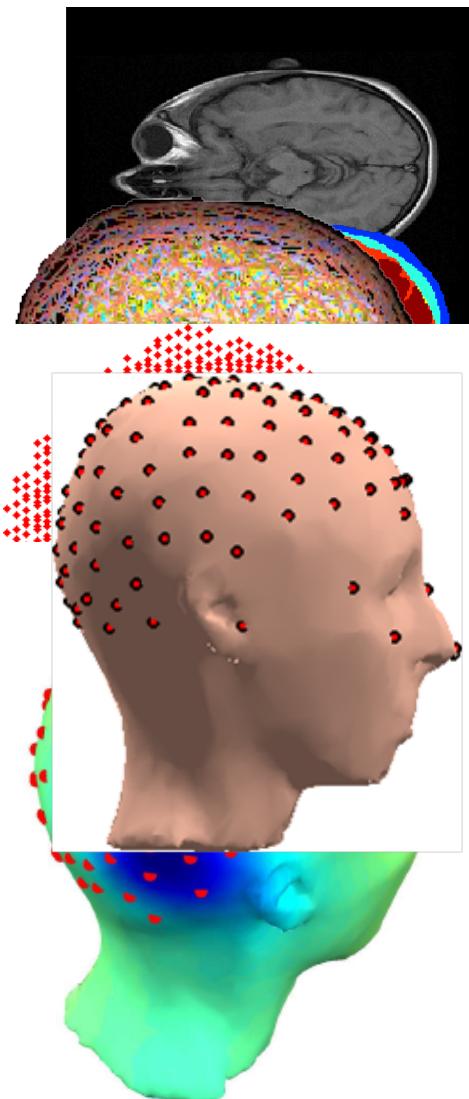
## Imaging Methods

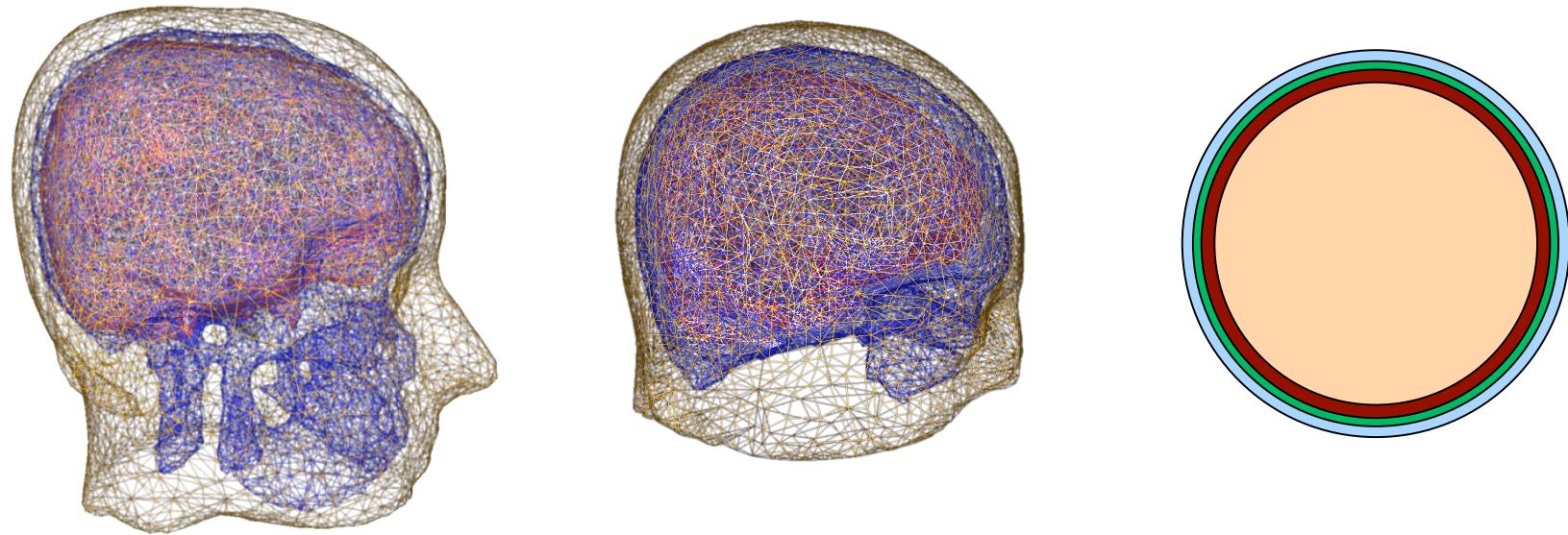
- ◆ Underdetermined
- ◆ Searches for activation in given locations.
- ◆ Linear optimization techniques
- ◆ Needs additional constraints
- ◆ Bayesian methods, MNE, LORETA, LAURA, etc.

# Review articles

- ◆ Michel *et al*, EEG source imaging, 2004
- ◆ Baillet *et al*, Electromagnetic brain mapping, 2001
- ◆ He *et al*, electrophysiological imaging of brain activity and connectivity – challenges and opportunities, 2011.
- ◆ Hallez *et al*, Review on solving the forward problem in EEG source analysis, J of Neuroeng and Rehab, 2007.
- ◆ Grech *et al*, Review on solving the inverse problem in EEG source analysis, J of Neuroeng and Rehab, 2008.

# Neuroelectromagnetic Forward Head Modeling Toolbox



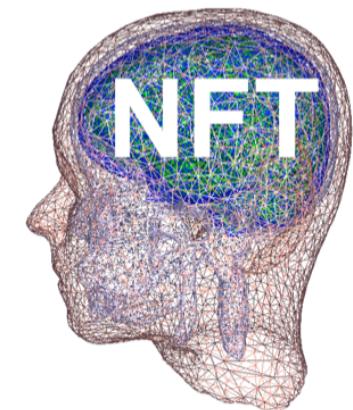


Effects of Forward Model Errors on EEG Source Localization

## MODELING ERRORS

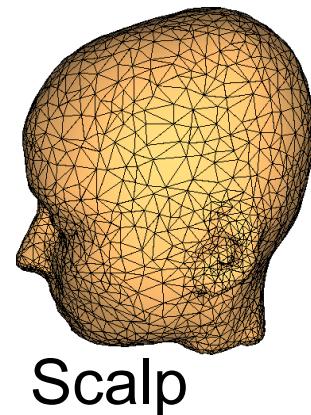
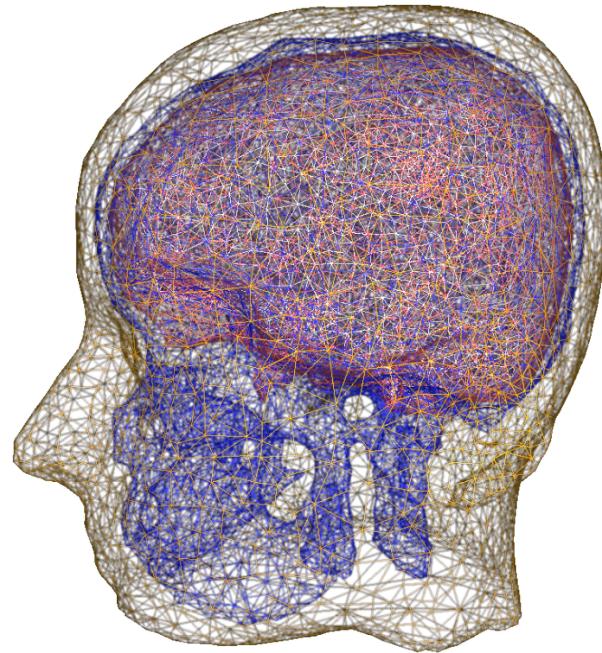
# Head Model Generation

- ◆ Reference Head Model
  - From whole head T1 weighted MR of subject
  - 4-layer realistic BEM model
- ◆ MNI Head model
  - From the MNI head
  - 3-layer and 4-layer template BEM model
- ◆ Warped MNI Head Model
  - Warp MNI template to EEG sensors
- ◆ Spherical Head model
  - 4-layer concentric spheres
  - Fitted to EEG sensor locations

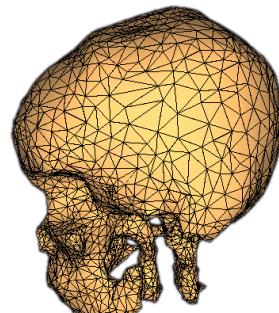


# The Reference Head Model

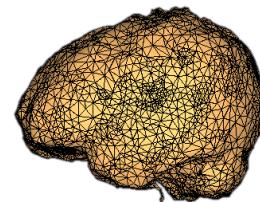
- ◆ 18541 nodes
- ◆ 37090 elements
  - 6928 Scalp
  - 6914 Skull
  - 11764 CSF
  - 11484 Brain



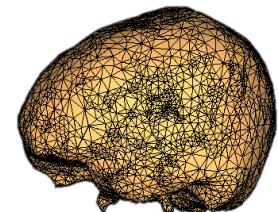
Scalp



Skull

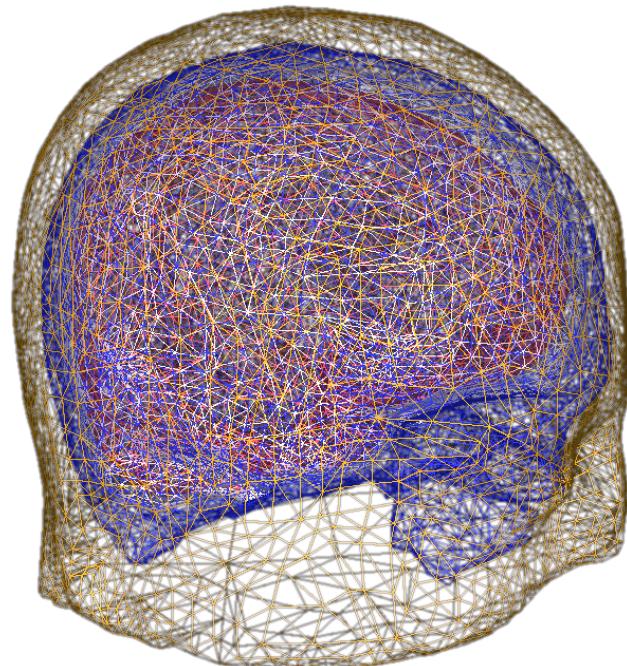


CSF



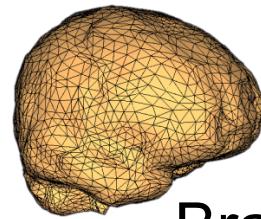
Brain

# The MNI Head Model

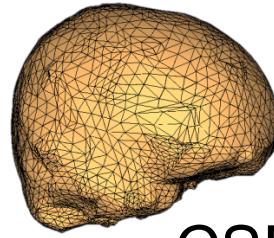


- ◆ 4-layer
  - 16856 nodes
  - 33696 elements

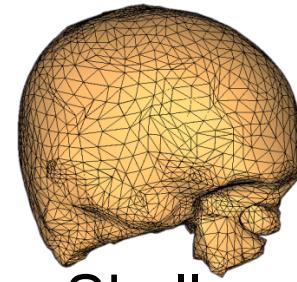
- ◆ 3-layer
  - 12730 nodes
  - 25448 elements



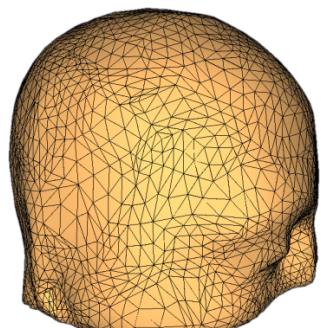
Brain



CSF

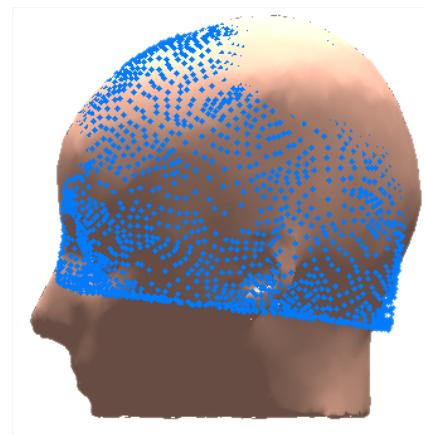
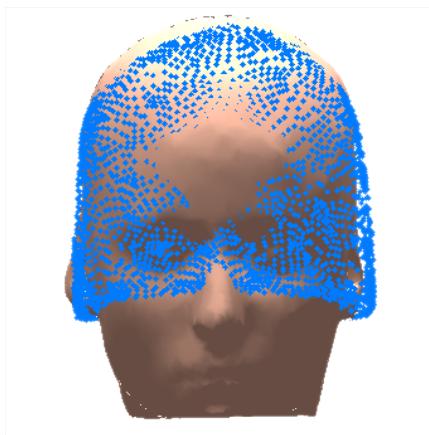


Skull

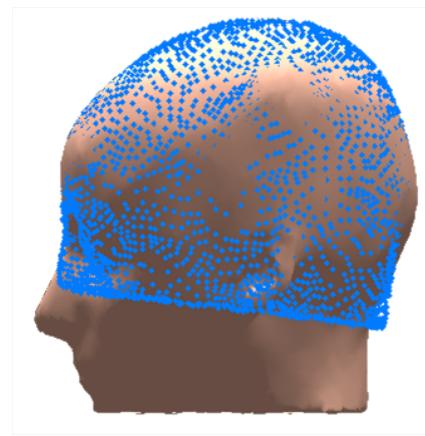
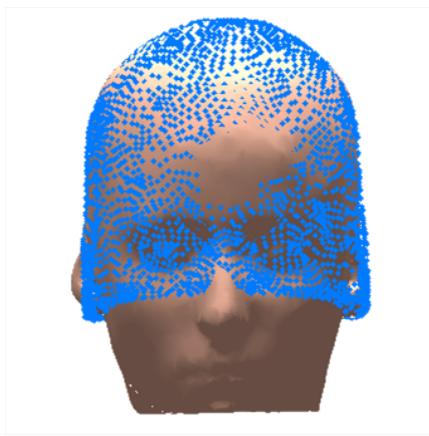


Scalp

# The Warped MNI Head Model

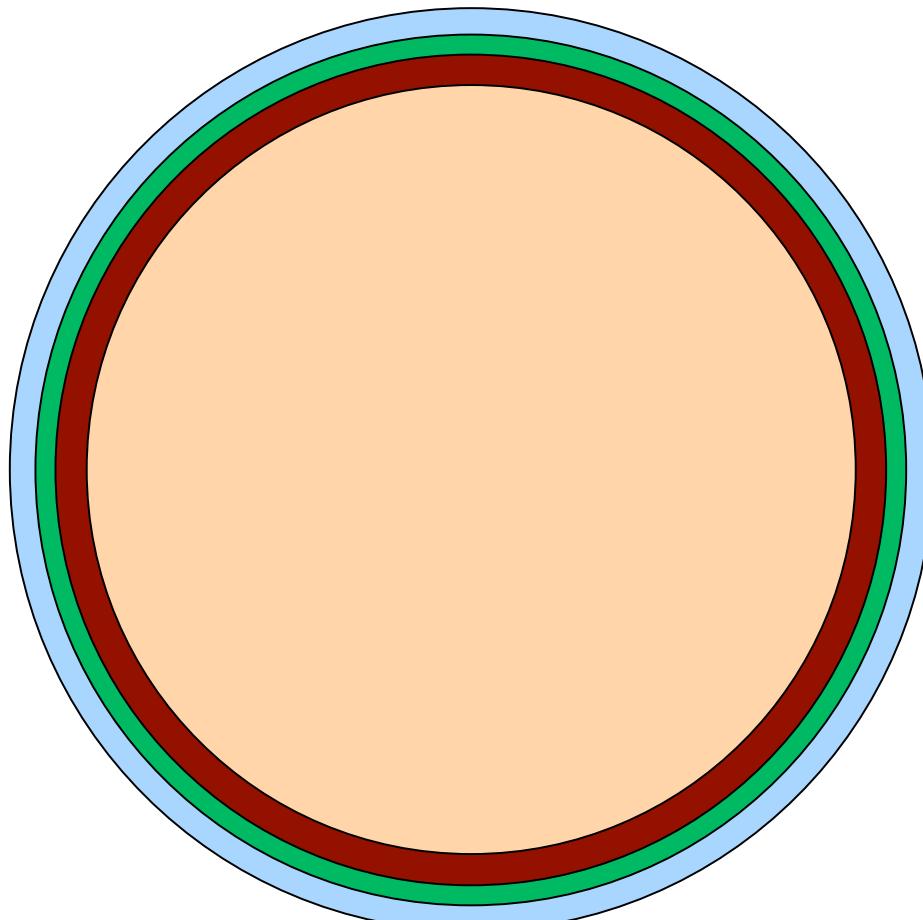


**Registered  
MNI template**



**Warped  
MNI mesh**

# The Spherical Head Model



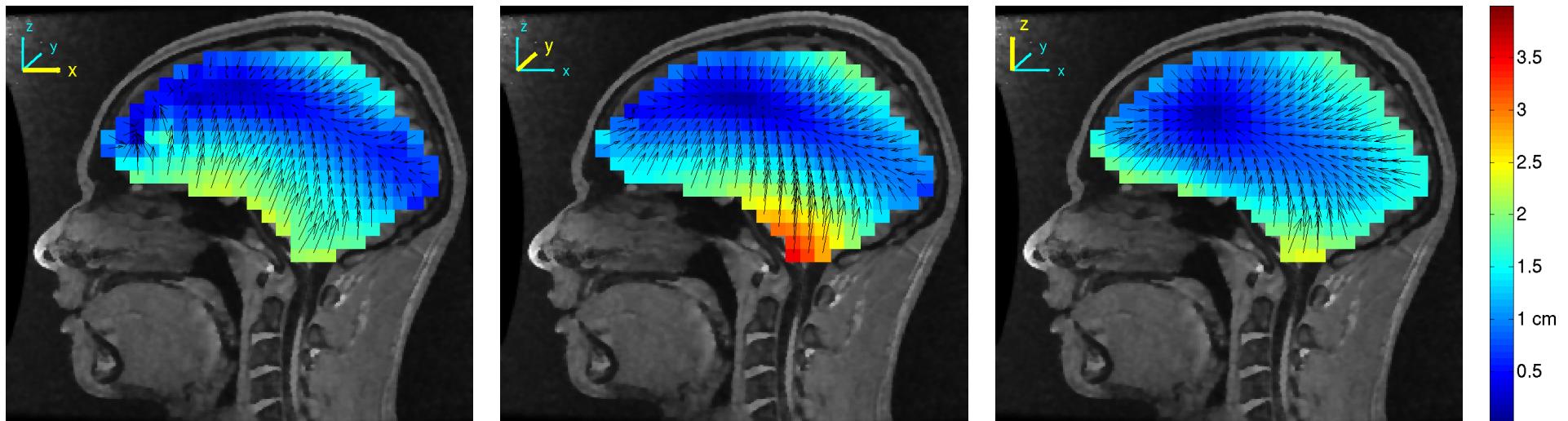
**4-Layer model**

**Outer layer is fitted to electrode positions**

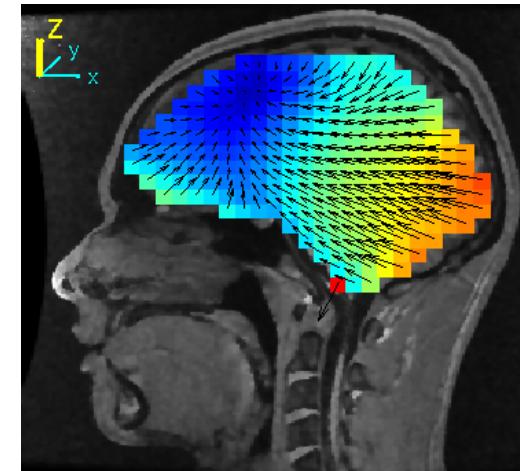
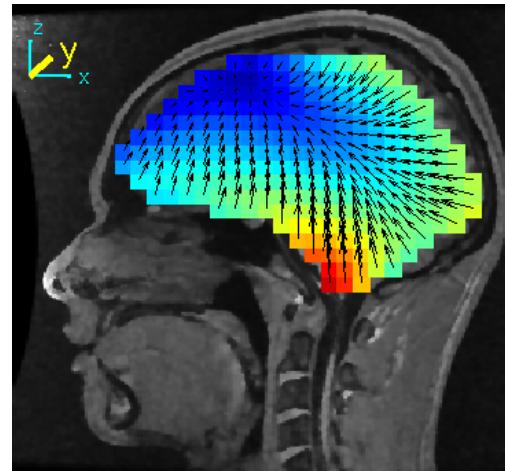
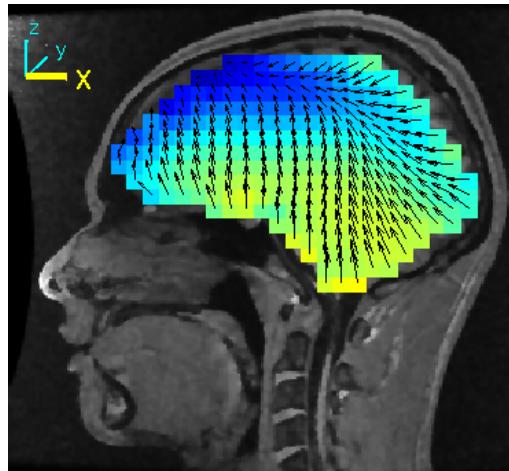
# Head Modeling Errors

- ◆ Solve FP with reference model
  - 3D grid inside the brain.
  - 3 Orthogonal dipoles at each point
  - ~7000 dipoles total
  - 4 subjects
- ◆ Localize using other head models
  - Single dipole search.
- ◆ Plot location and orientation errors

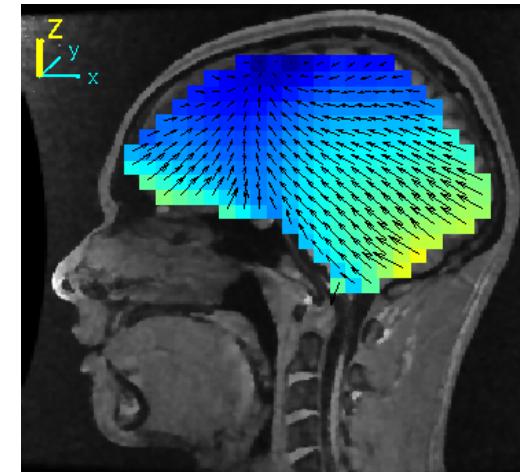
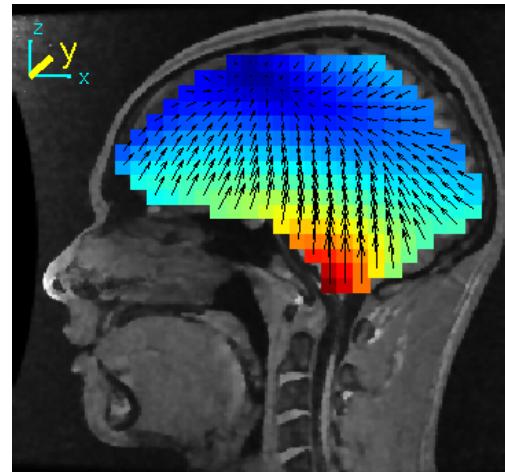
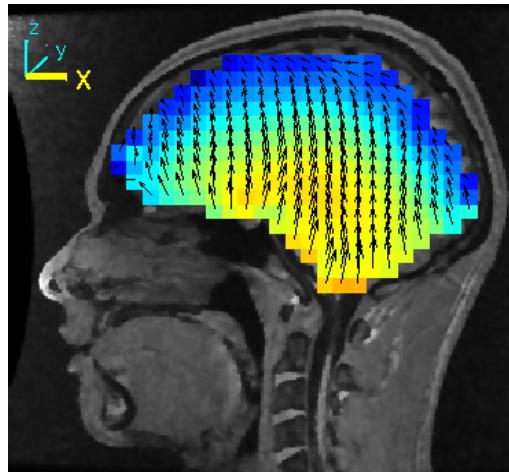
# Spherical Model Location Errors



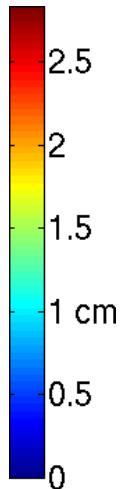
# 3-Layer MNI Location Errors



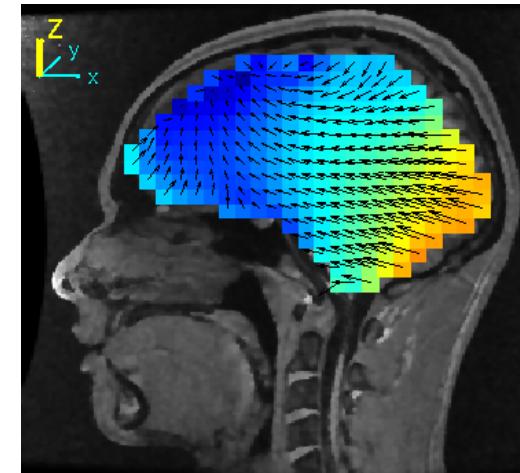
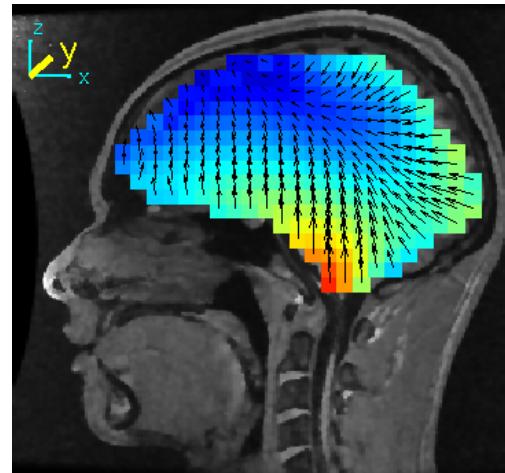
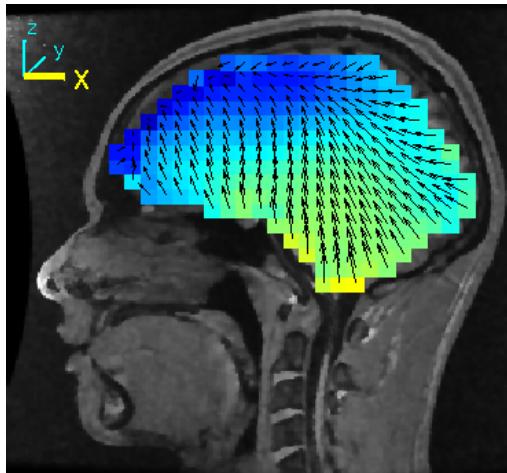
3-Layer MNI



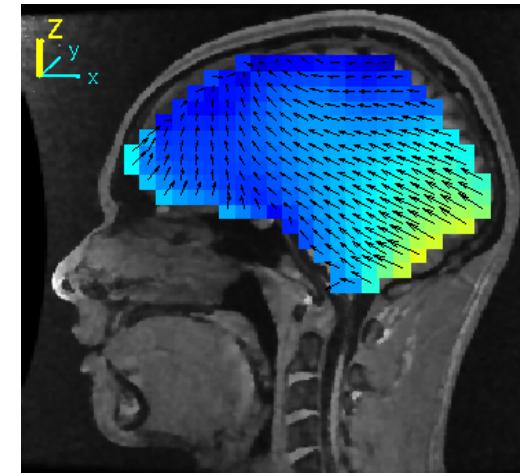
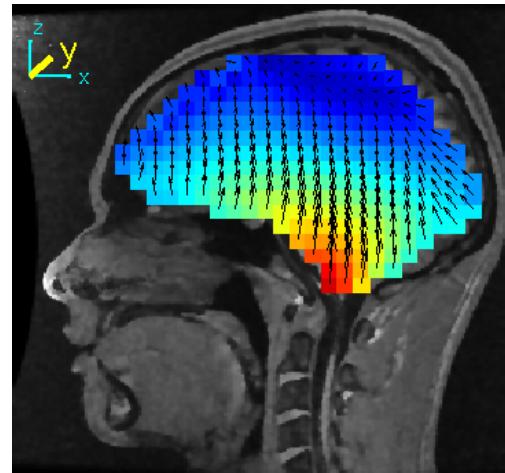
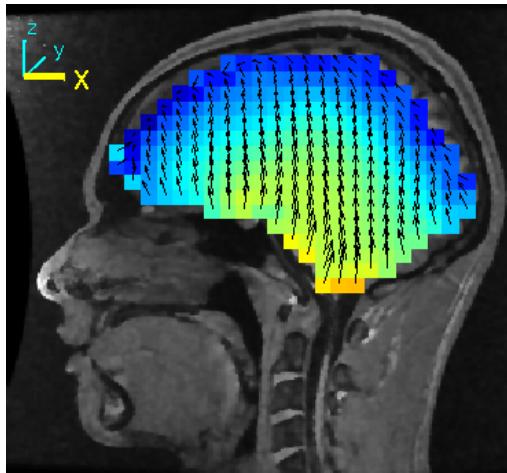
3-Layer Warped MNI



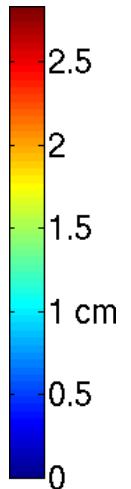
# 4-Layer MNI Location Errors



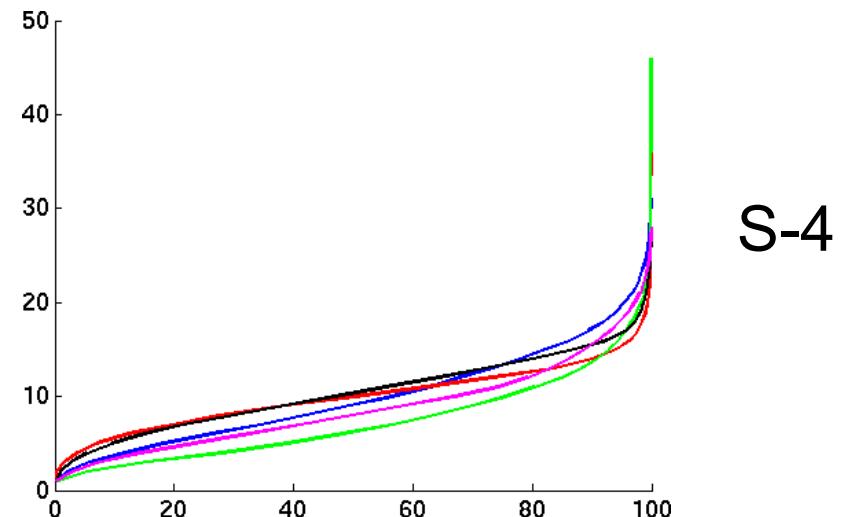
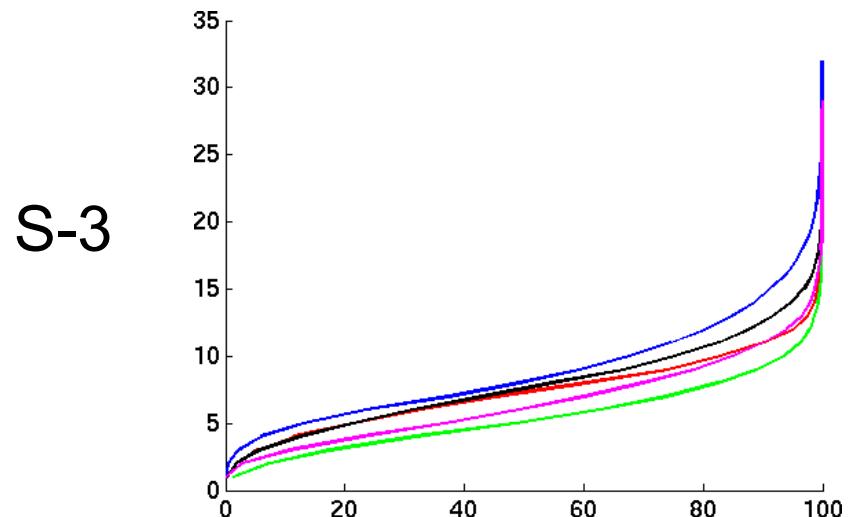
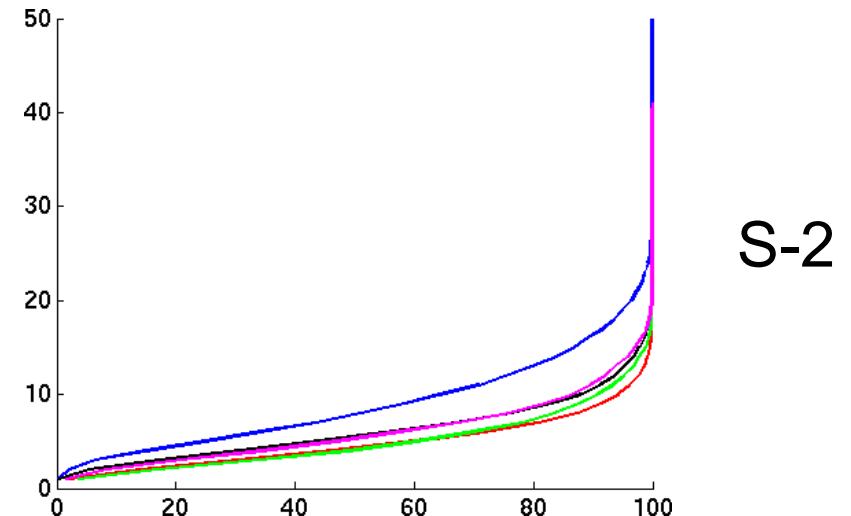
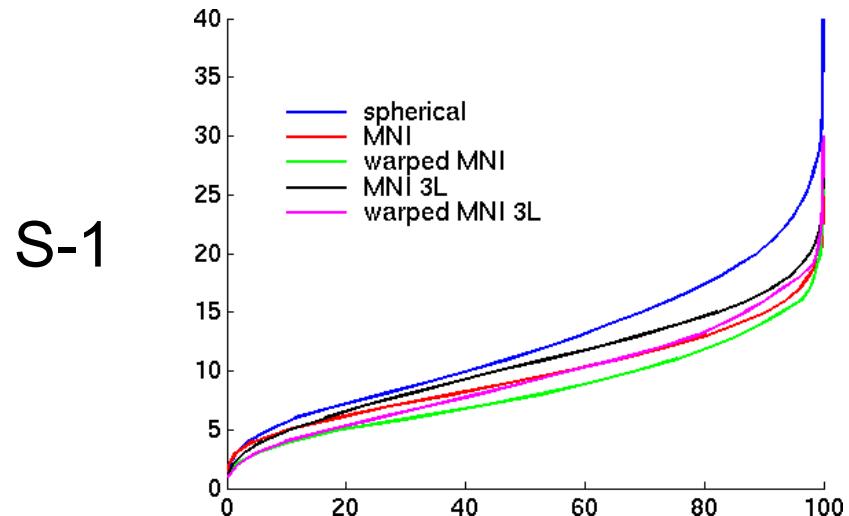
4-Layer MNI



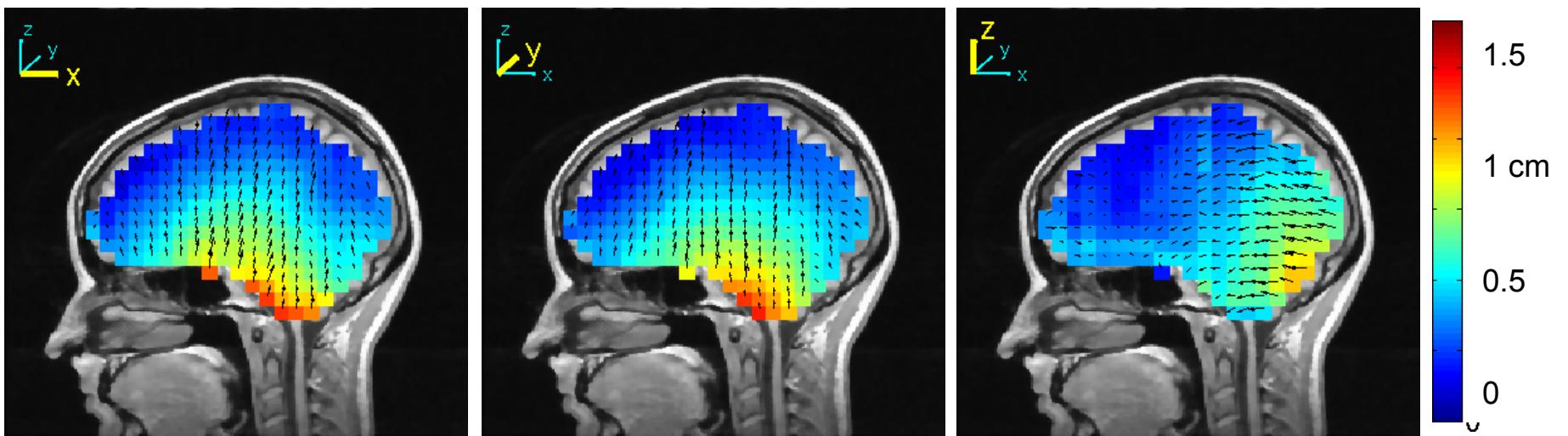
4-Layer Warped MNI



# Error Percentiles for Four Subjects



# Average Errors for Four Subjects



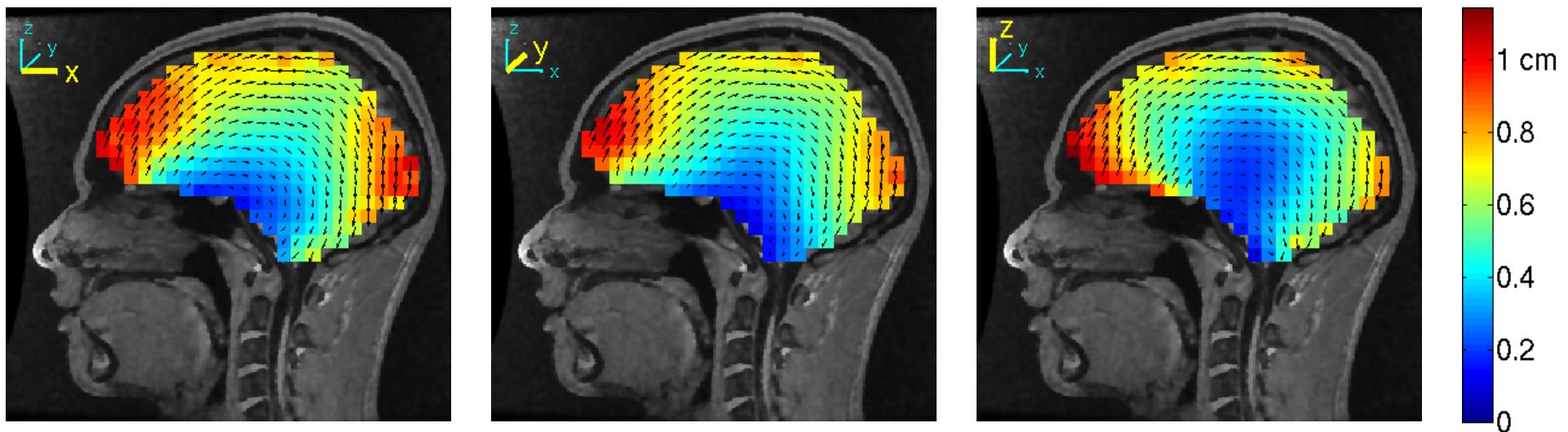
# Observations

- ◆ Spherical Model
  - Location errors up to 3.5 cm. Cortical areas up to 1.5 cm.
- ◆ 3-Layer MNI
  - Large errors where models do not agree.
  - Higher around chin and the neck regions.
- ◆ 4-Layer MNI
  - Similar to 3-Layer MNI.
  - Smaller in magnitude.

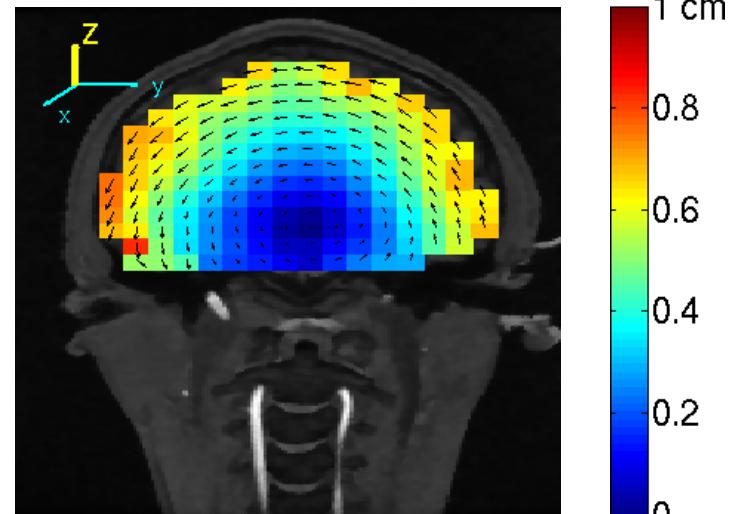
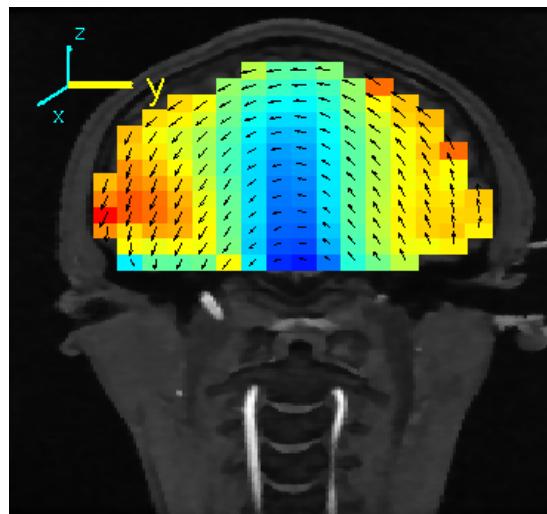
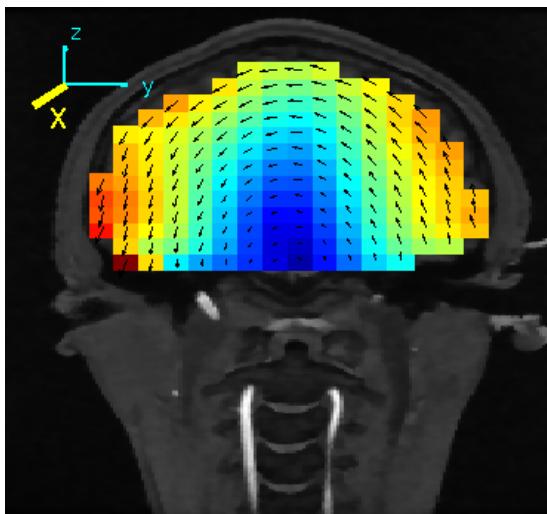
# Electrode co-registration errors

- ◆ Solve FP with reference model
- ◆ Shift all electrodes and re-register
  - 5° backwards
  - 5° left
- ◆ Localize using shifted electrodes
- ◆ Plot location and orientation errors

# 5° Backwards Location Errors



# 5° Left Location Errors



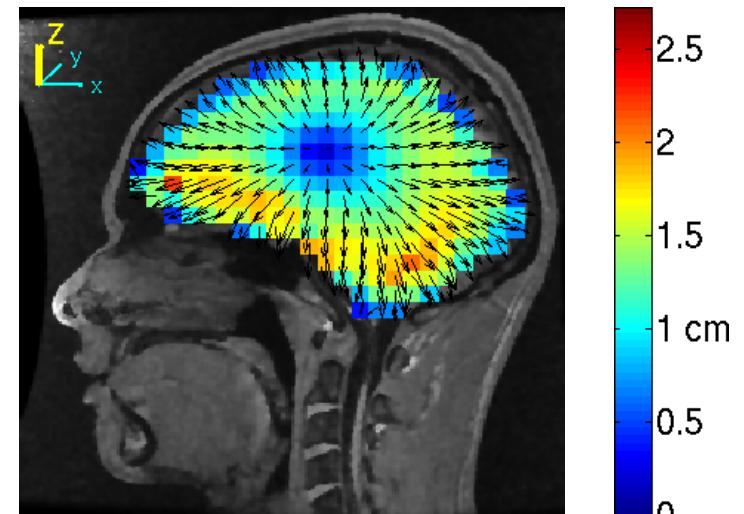
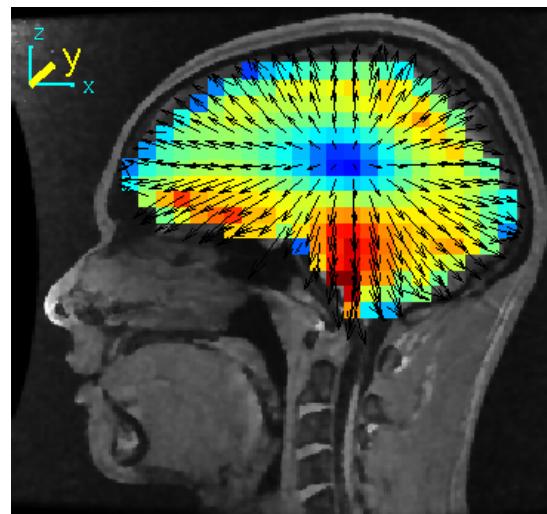
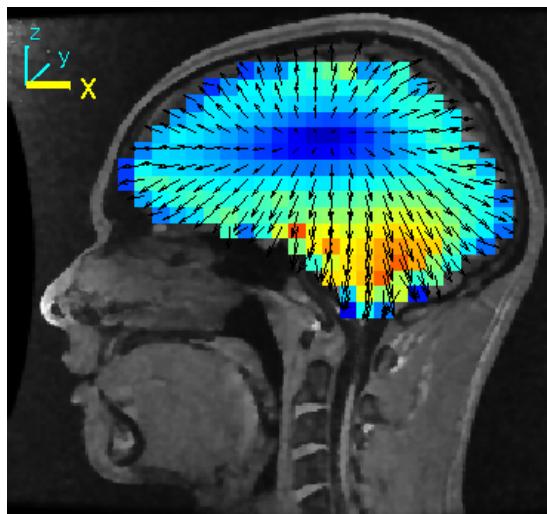
# Observations

- ◆ Errors increase close to the surface near electrode locations.
- ◆ Changing or incorrectly registering electrodes may cause 5-10 mm localization error.

# Effect of Skull Conductivity

- ◆ Solve FP with reference model
  - Brain-to-Skull ratio: 80
- ◆ Generate test model
  - Same geometry
  - Brain-to-Skull ratio: 20
- ◆ Localize using test model
- ◆ Plot location and orientation errors

FP ratio: 80 IP ratio: 20



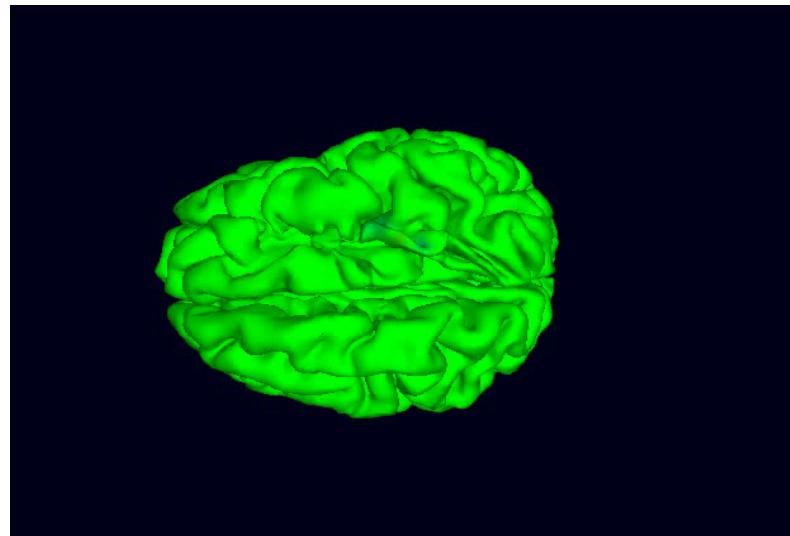
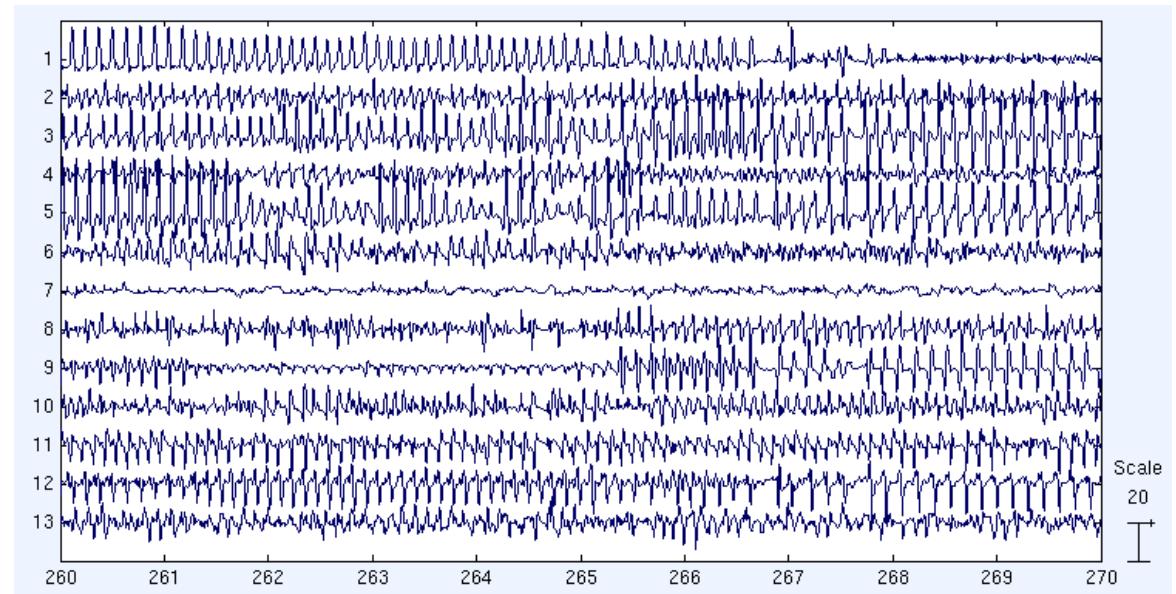
# Conclusion

- ◆ Head shape
  - Most impact on source localization accuracy.
- ◆ Incorrect electrode registration
  - Errors near the electrodes
  - Most studies investigate cortical activity close to the electrodes.
- ◆ Electrical properties
  - Number of layers
  - Relative conductivities (Brain-to-Skull ratio)

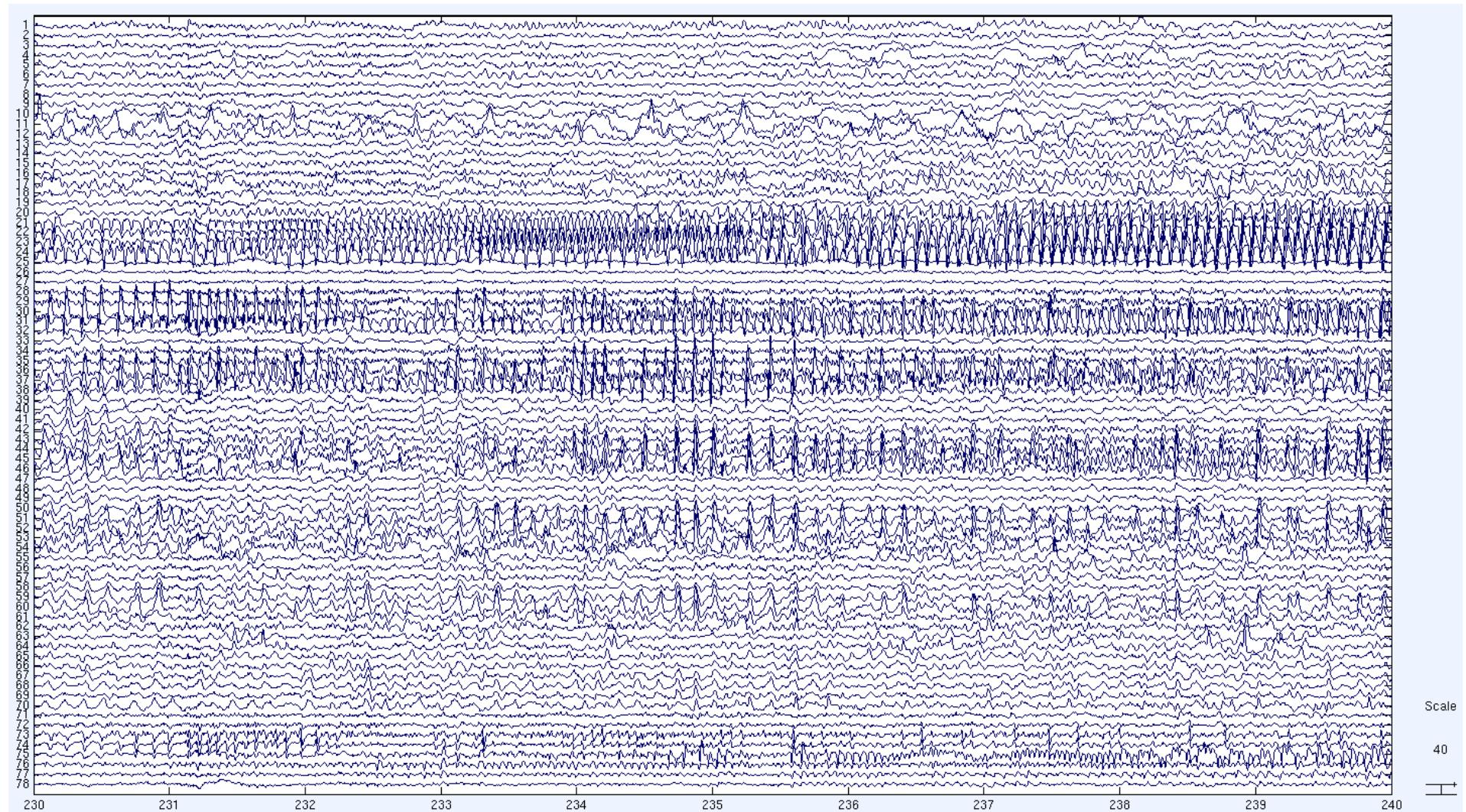
**ICON POSTER:** Forward model errors in EEG source localization  
Z Akalin Acar, S Makeig (Sunday afternoon)

Epilepsy Head Modeling

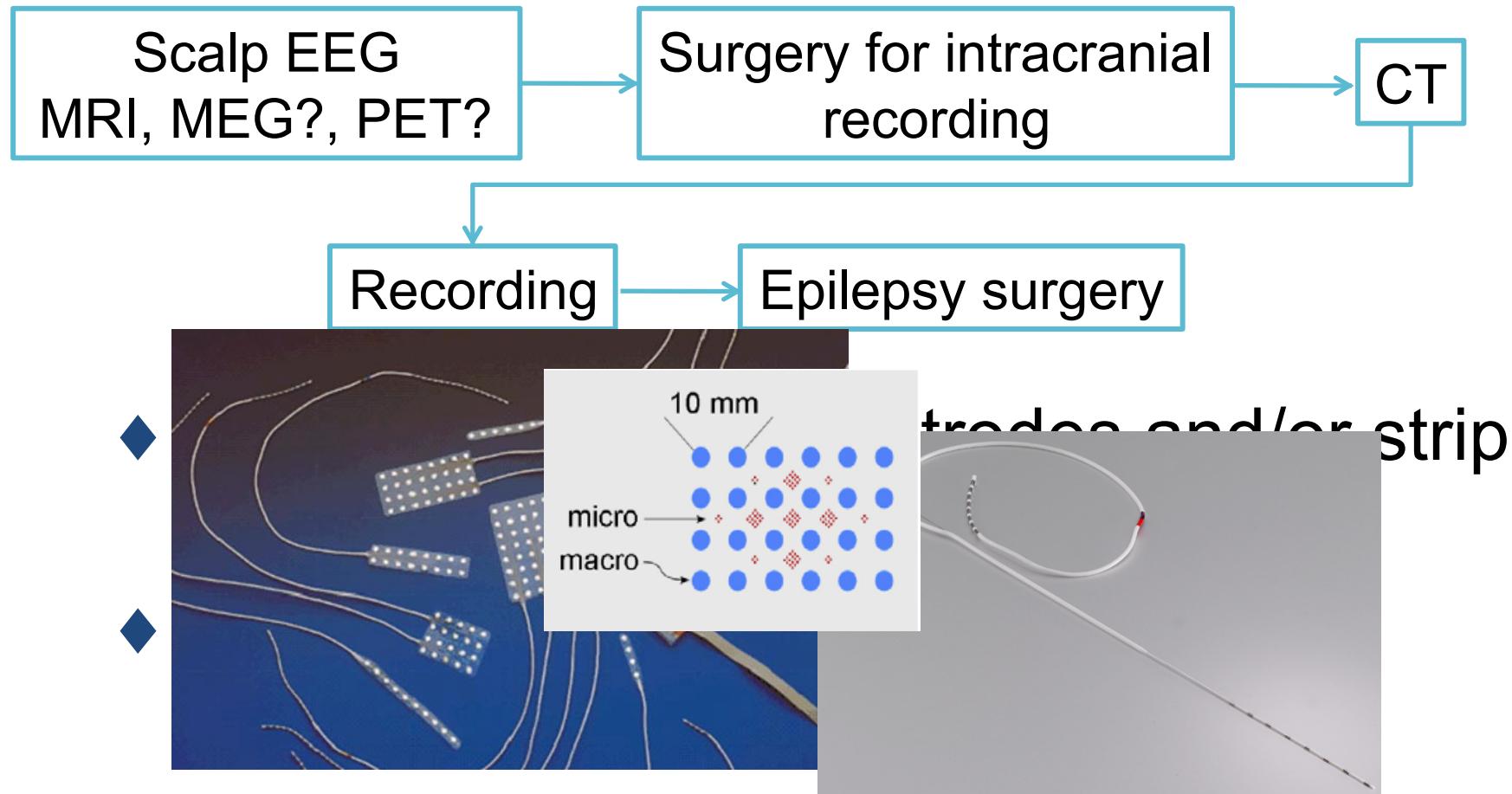
# CASE STUDY



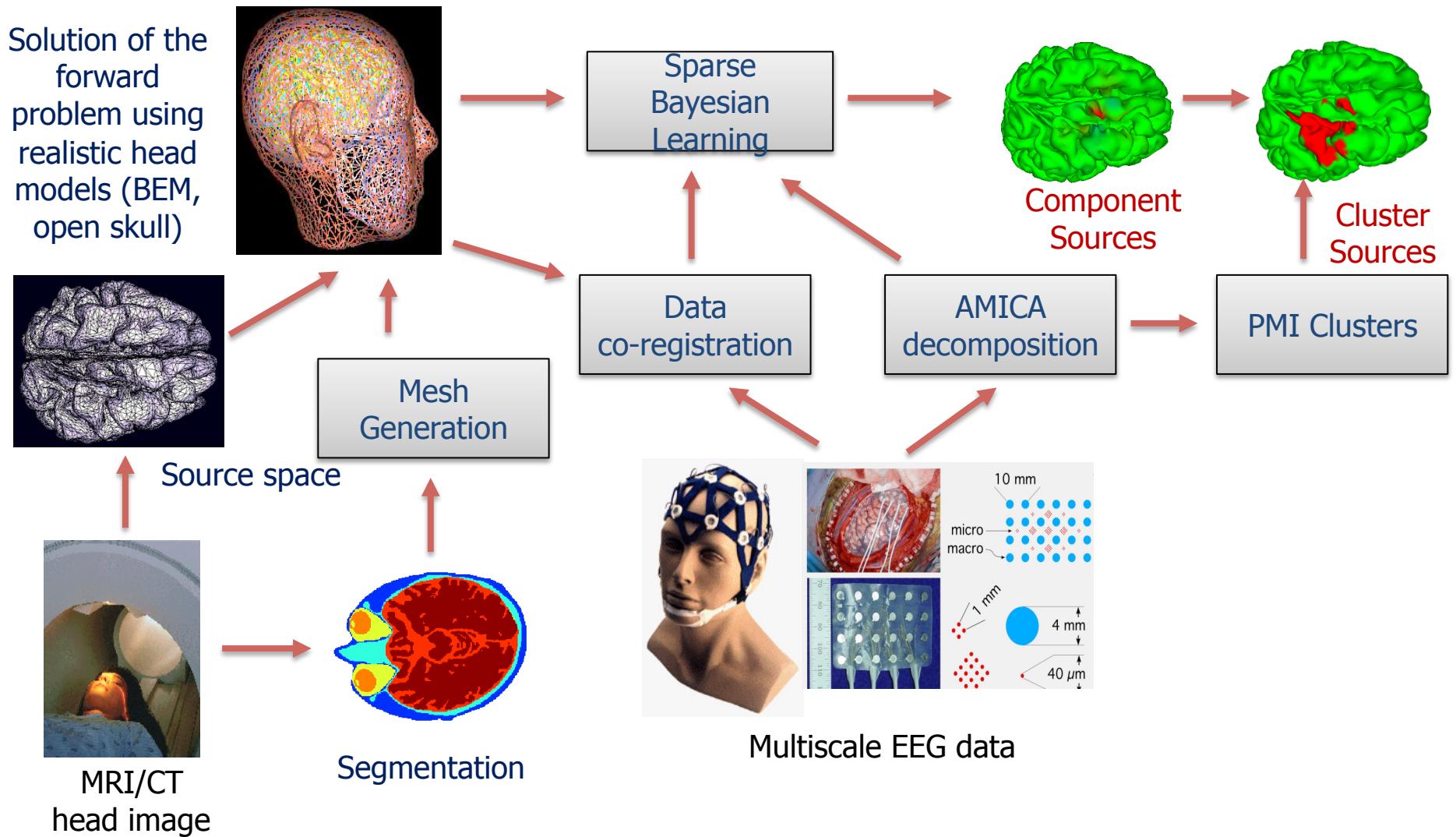
# Epilepsy



# Epilepsy surgery

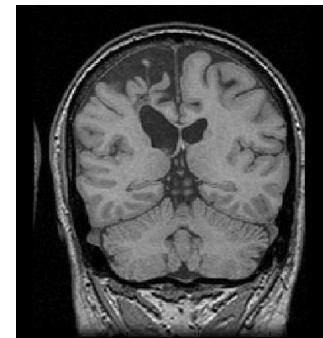


# Project Summary

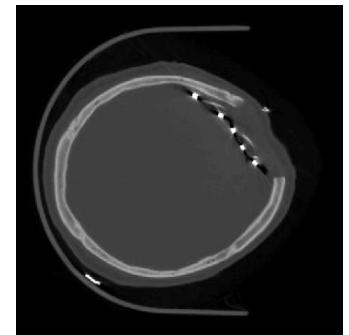


# Epilepsy Head Modeling

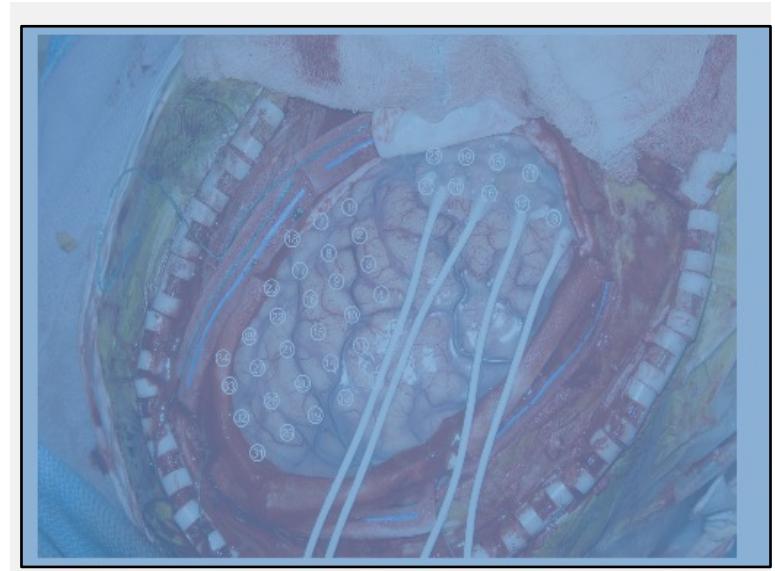
- ◆ Large hole in skull
- ◆ Plastic sheet
- ◆ A pre-surgery MR and post-surgery CT
- ◆ Differences in brain shape after surgery
- ◆ Co-registration of electrodes
  - Subdural – from CT segmentation
  - Scalp – no digitizer data



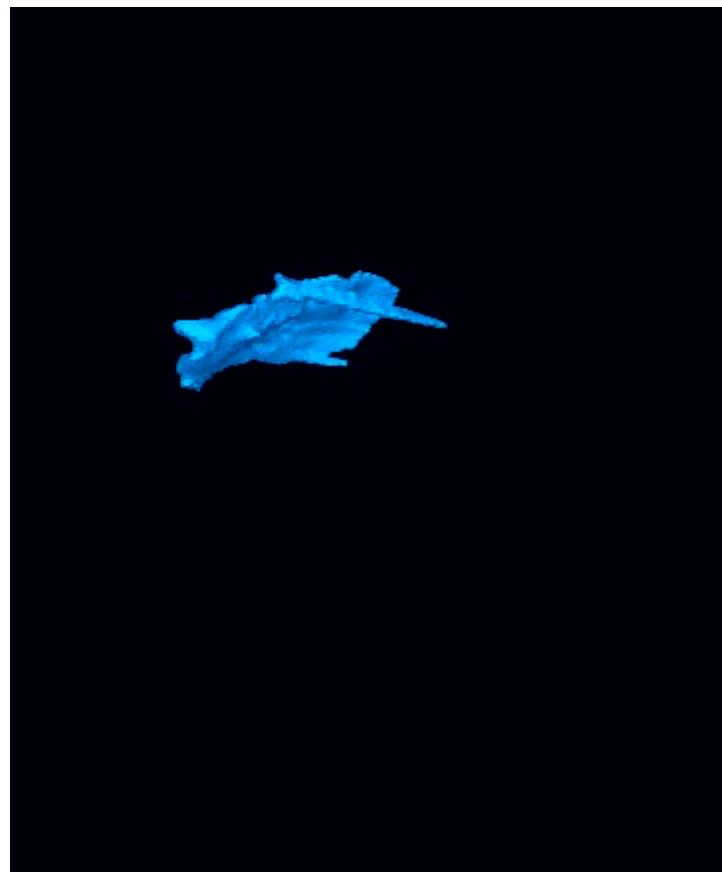
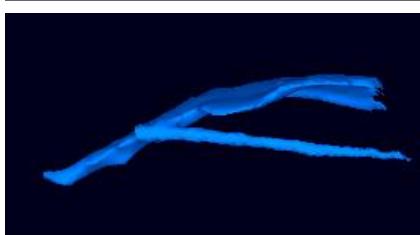
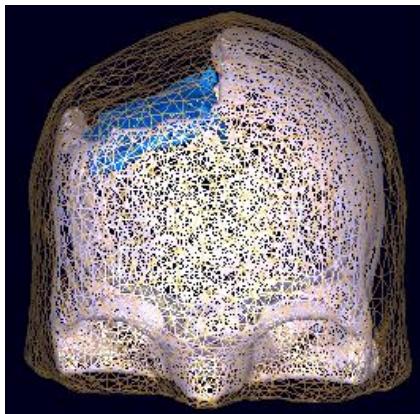
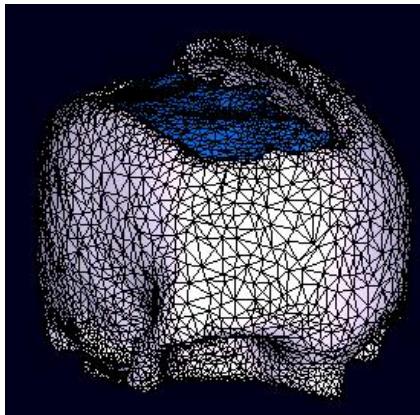
MR



CT



# Scalp, skull and sheet models



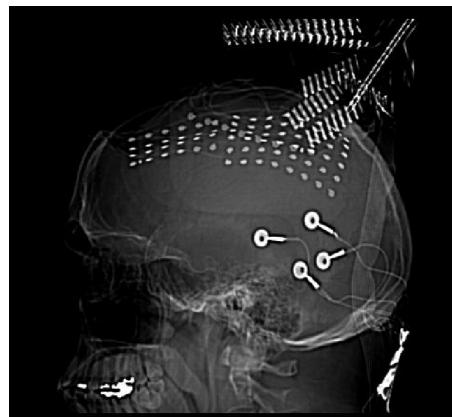
**Number of elements**

Scalp: 10000

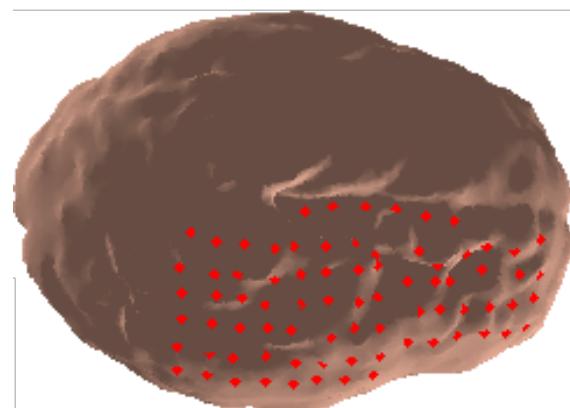
Skull: 30000

Plastic sheet :  
7000

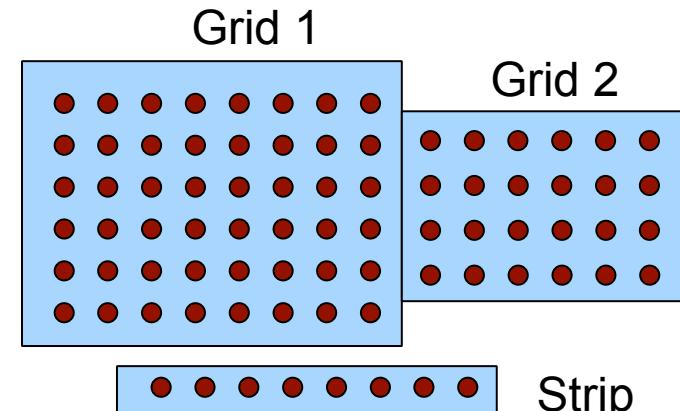
# Analyzing Epilepsy Recordings



CT image of the implanted grid electrodes



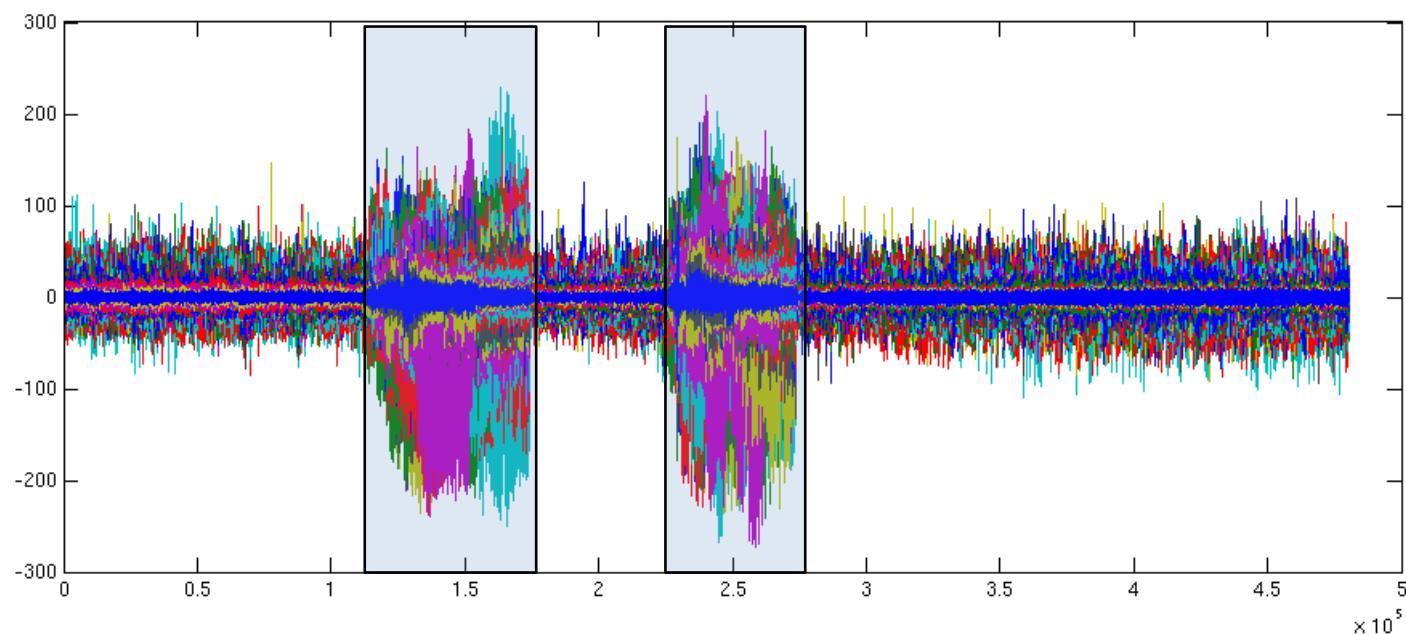
- ◆ Pre-Surgical Evaluation
- ◆ Rest Data
- ◆ Simultaneous recordings
  - 78 iEEG electrodes
  - 29 scalp electrodes
- ◆ Provided by Dr. Greg Worrell, Mayo Clinic



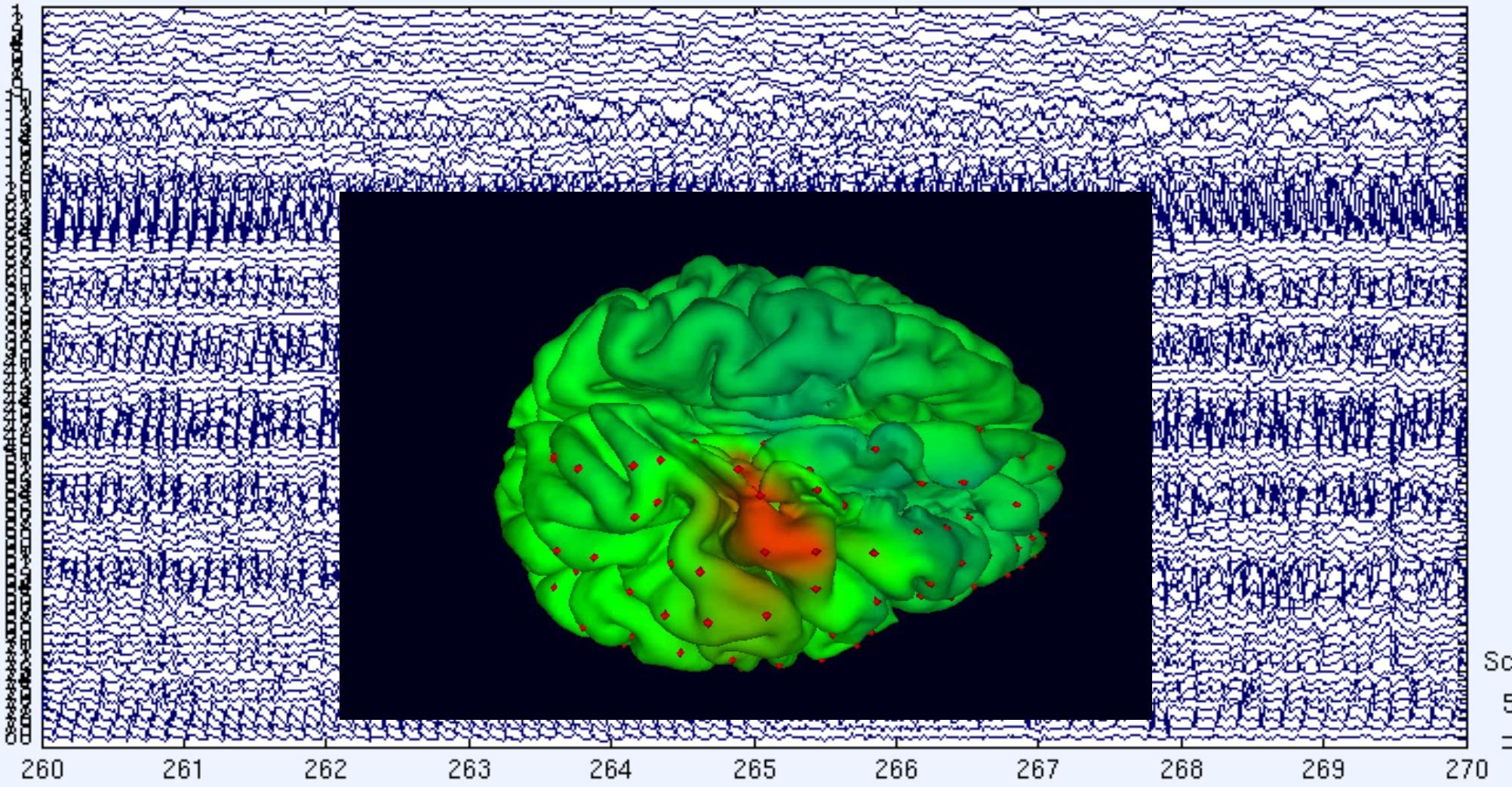
Akalin Acar et al, 2008

# Data

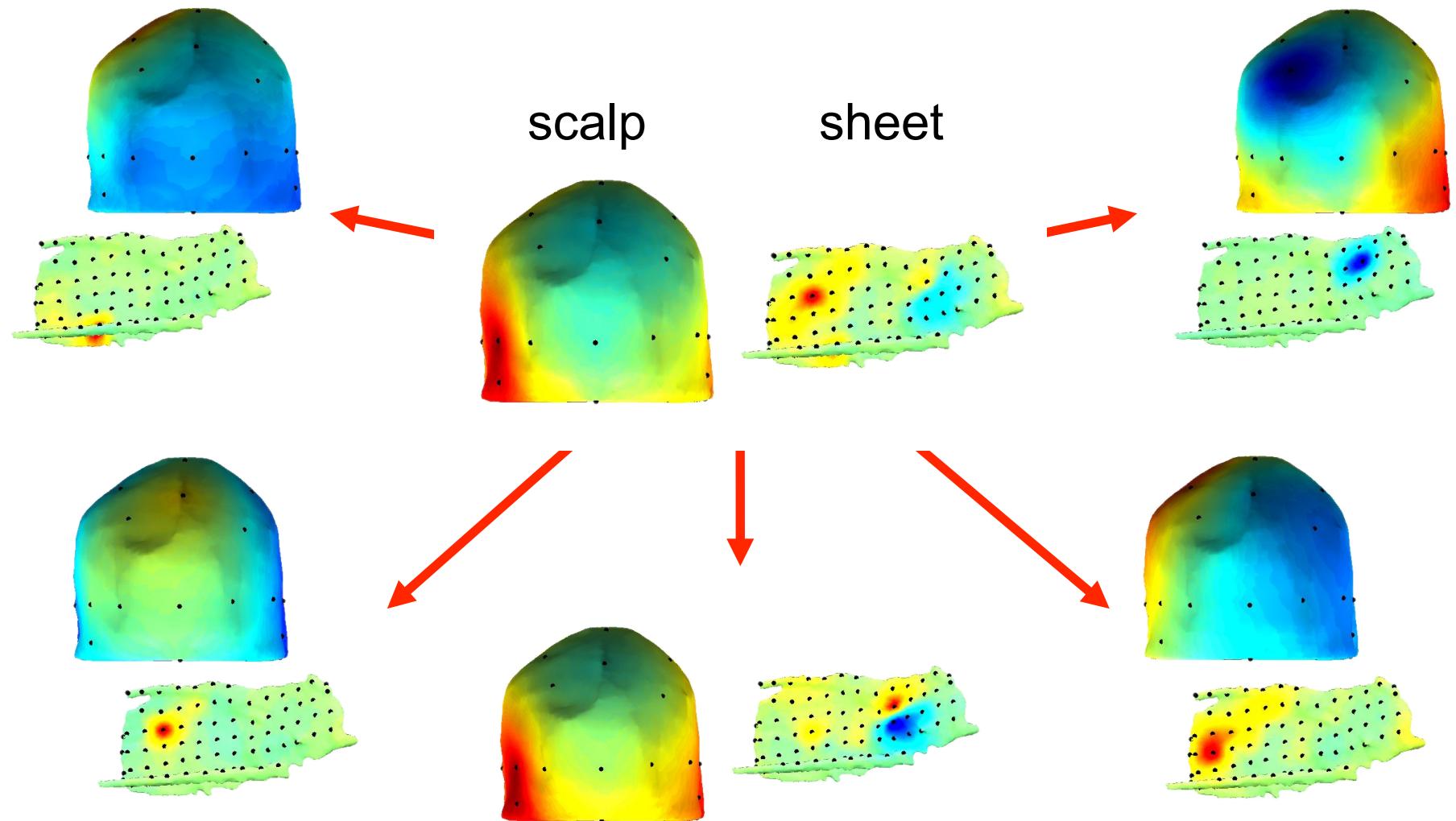
- ◆ 16 minutes ECoG + EEG data
- ◆ 2 seizures (1.9 min + 1.5 min)
- ◆ ECoG = 78 channels, EEG = 29 channels



# iEEG data

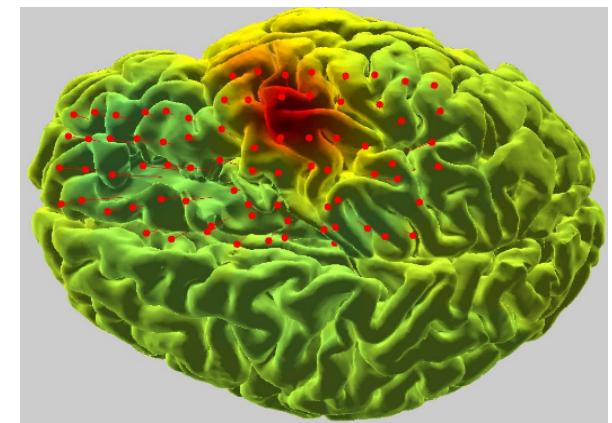
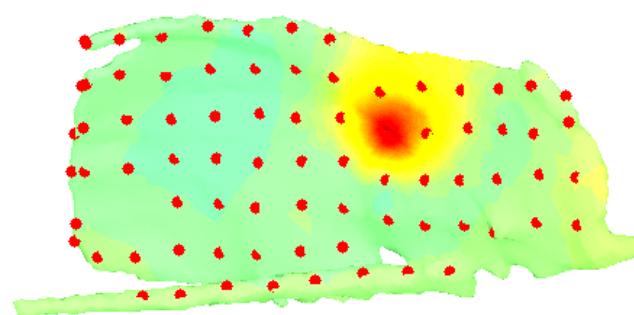
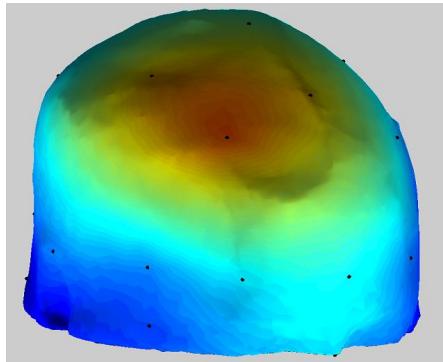


# Independent Component Analysis



# Independent Components

IC 1

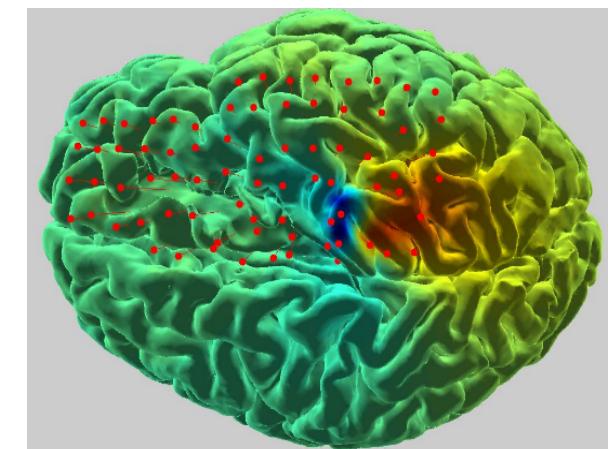
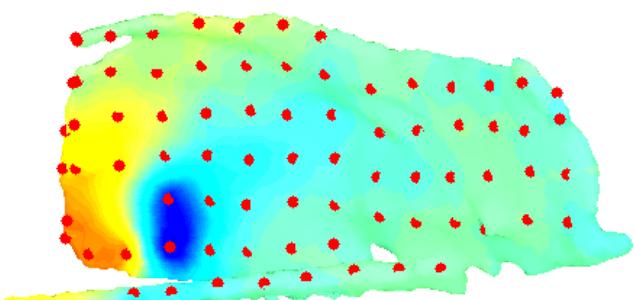
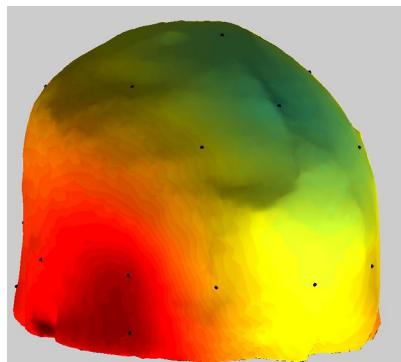


Potentials on scalp

Potentials on plastic sheet

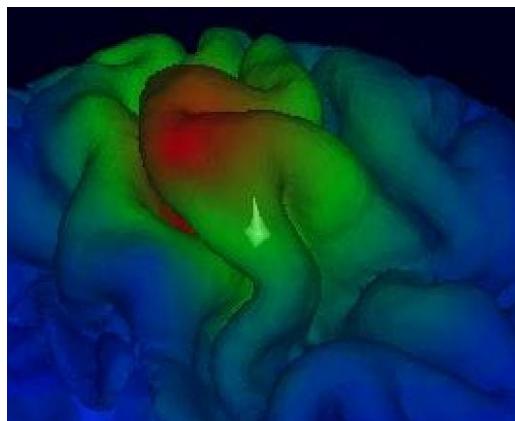
On the brain surface

IC 52



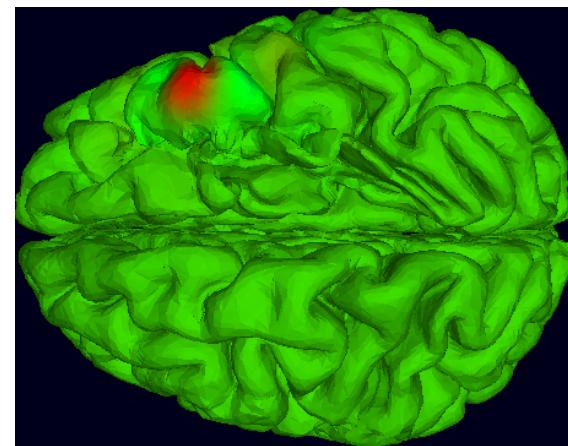
# Source Localization Results

Dipole source localization

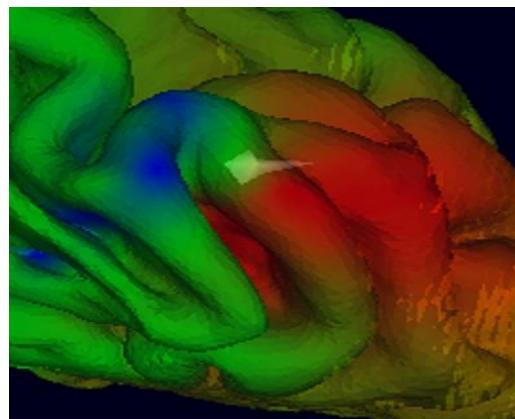


IC 1

Distributed source localization - SBL

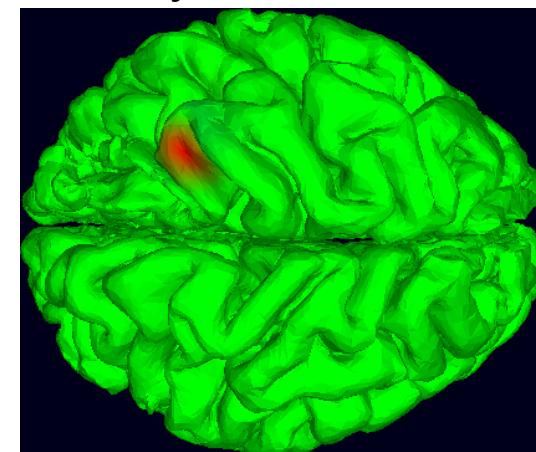


Radial source



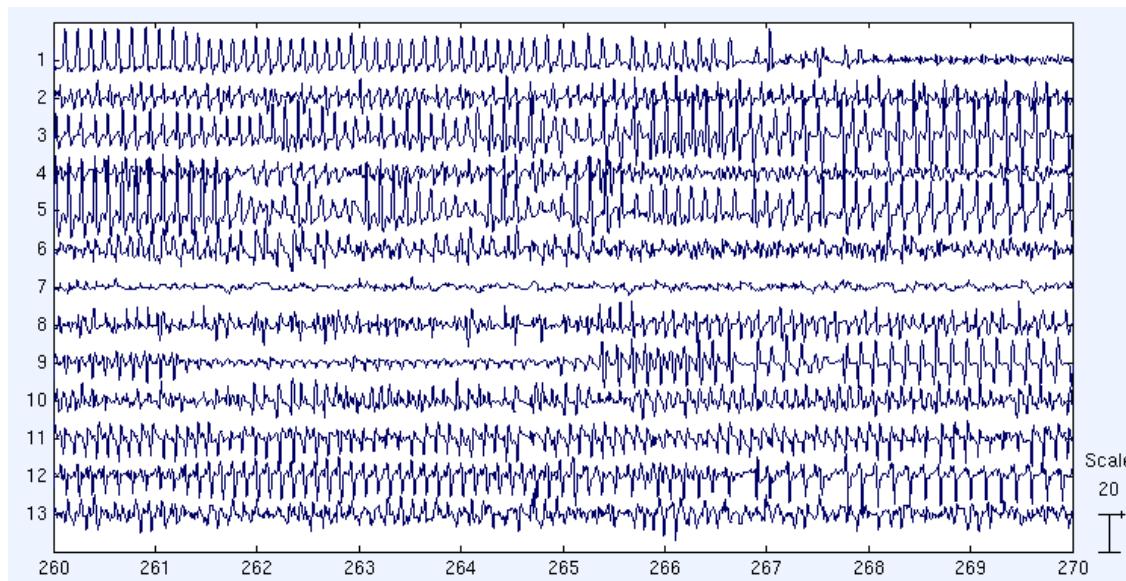
IC 52

Tangential source



Sulcal source

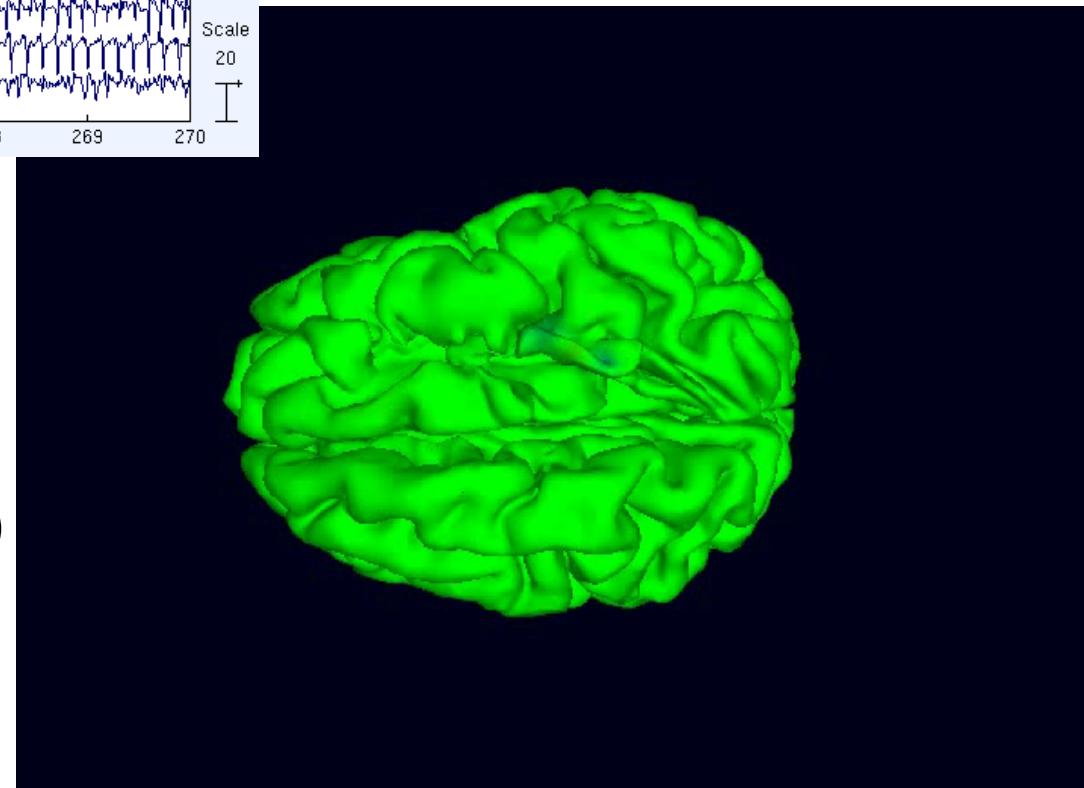
# Cortical activity of seizure components



Activations of 13  
seizure components

Cortical activity of  
Seizure components

$$Movie(t) = \sum_{i=1}^{13} S_i \times Act_i(t)$$



# Conclusion

- ◆ ICA can detect and identify seizure components in the EEG data.
- ◆ Correct source localization requires correct forward problem solution.

**ICON POSTER:** Independent Component Analysis and source localization of ECoG data

S Makeig, J Palmer, G Worrell, Z Akalin Acar (Sunday afternoon)

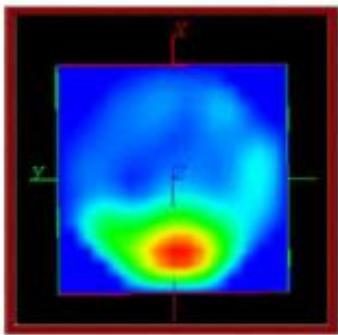
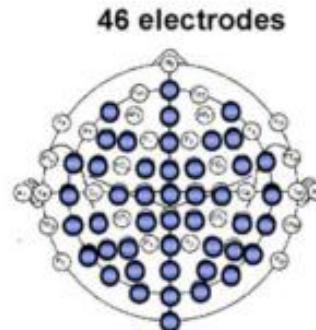
# References

1. Z. Akalin Acar, S. Makeig, "Neuroelectromagnetic Forward Head modeling Toolbox", *J. of Neuroscience Methods*, vol. 190 (2), 258-270, 2010.
2. Z. Akalin Acar, N. Gencer, "An advanced boundary element method (BEM) implementation for the forward problem of electromagnetic source imaging", vol. 49, 5011-5028, 2004.
3. Z. Akalin Acar, G. Worrell, S. Makeig, "Patch-based cortical source localization in epilepsy", Proc. of IEEE EMBC 2009, Minneapolis.
4. Z. Akalin Acar, S. Makeig, "Effect of head models in EEG source localization", Sfn 2010, San Diego.
5. Z. Akalin Acar, S. Makeig, G.Worrell, "Head modeling and cortical source localization in epilepsy", Proc. of IEEE EMBC 2008, Vancouver.
6. Z. Akalin Acar, J. Palmer, G. Worrell, S. Makeig, "Electrocortical source imaging of intracranial EEG data in epilepsy", Proc. of IEEE EMBC 2011, Boston.

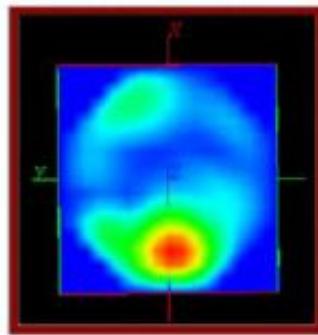
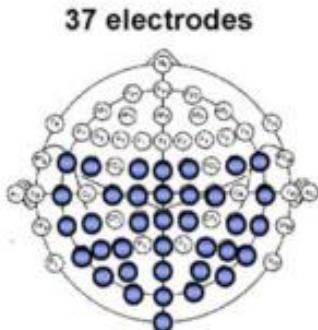
**THANK YOU**

# Distribution of electrodes

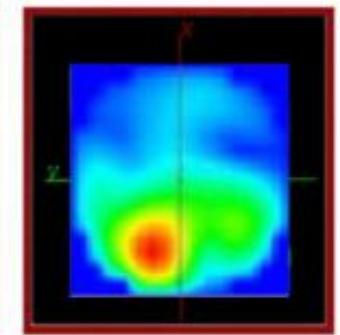
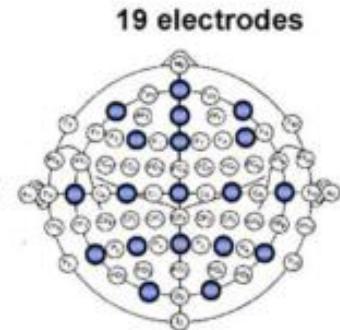
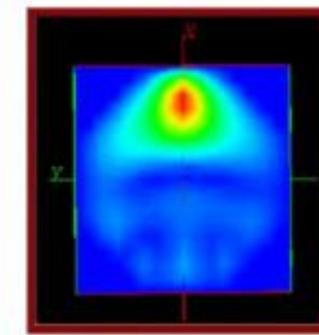
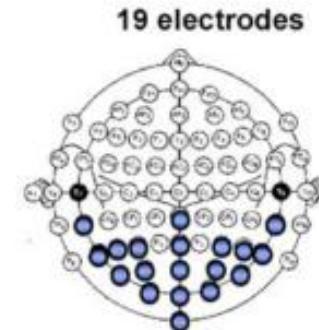
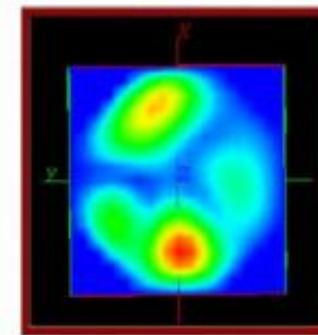
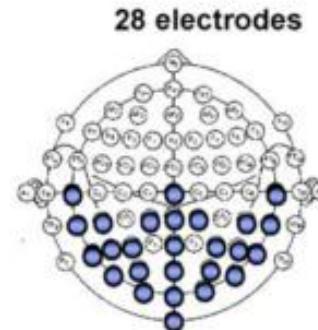
Original montage



Down-sampled montages restricted  
to the posterior scalp

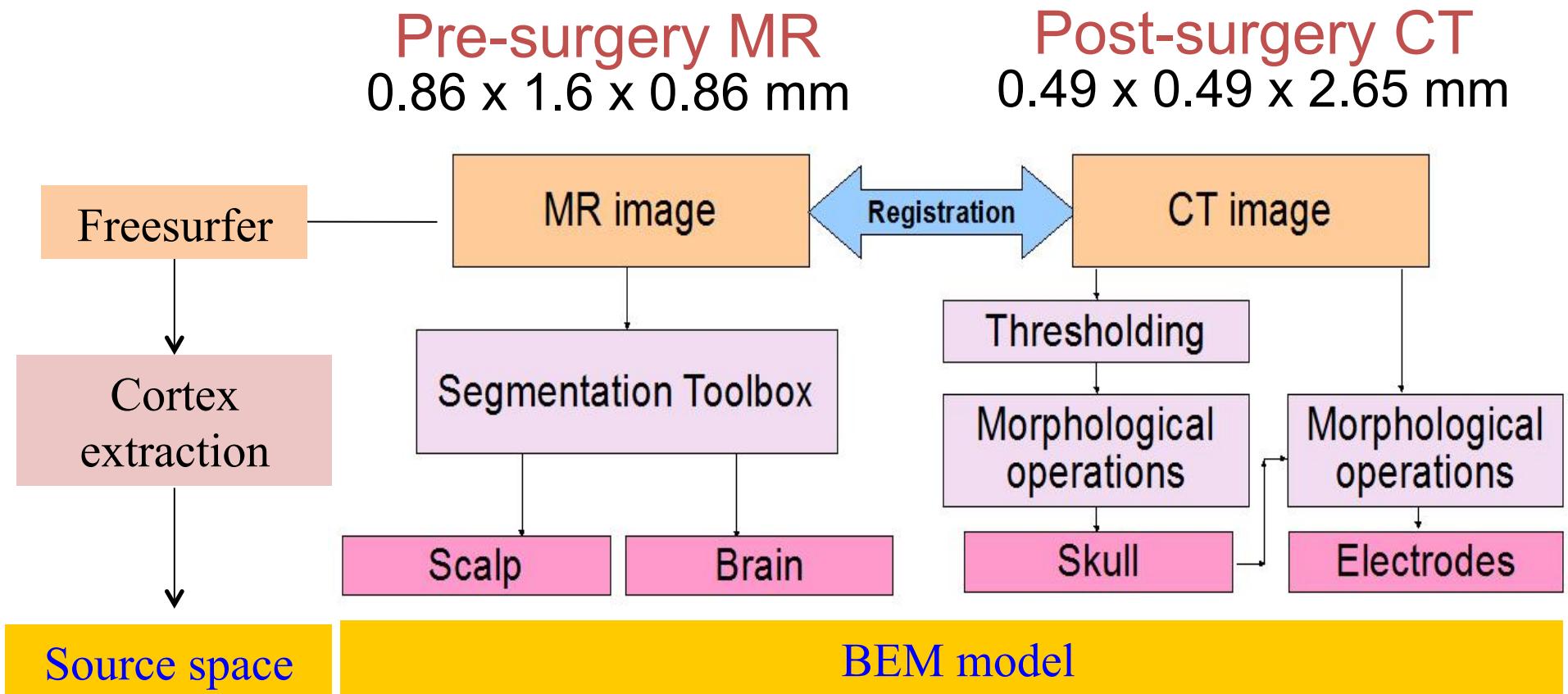


Down-sampled montage  
evenly distributed



Michel et al, 2004

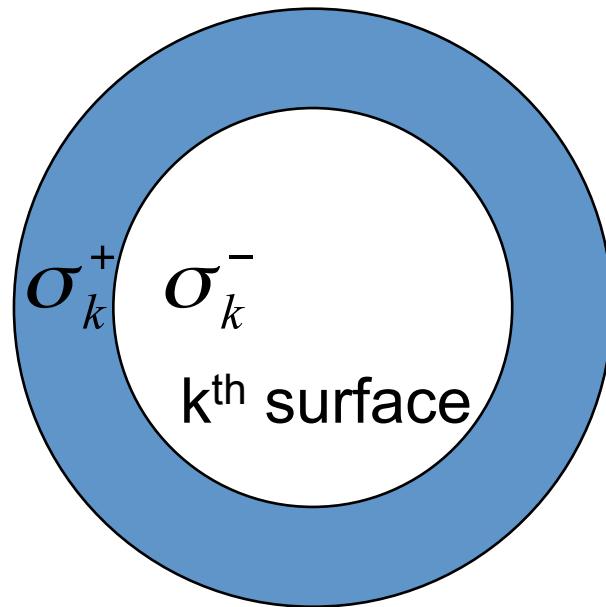
# Head modeling in epilepsy



# BEM Formulation

Integral equation for Potential Field:

$$\phi(\vec{r}) = 2g(\vec{r}) + \frac{1}{2\pi} \sum_{k=1}^n \left( \frac{\sigma_k^- - \sigma_k^+}{\sigma_i^- + \sigma_i^+} \right) \int_{S_k} \phi(\vec{r}') \frac{\vec{R}}{R^3} \cdot d\vec{S}_k(\vec{r}')$$



# BEM Formulation

Integrating the previous integral equation over all elements a set of equations are obtained.

In matrix notation **for the potential field** we obtain

$$\Phi_{M \times 1} = C_{M \times M} \Phi + g_{M \times 1} \quad \Phi = [I - C]^{-1} g \quad \Phi = A^{-1} g$$

$M$ : number of nodes

The expression **for the magnetic field**:

$$B_{n \times 1} = B_0 + H_{n \times M} \Phi$$

$n$ : number of magnetic sensors

# Algebraic formulation of the FP

Scalp potentials for N electrodes and p dipoles:

$$V(r) = \sum_i^p g(r, r_{dip}, d_i) = \sum_i^p g(r, r_{dip}, e_{d_i}) d_i$$

$$V = \begin{bmatrix} V(r_1) \\ \vdots \\ V(r_N) \end{bmatrix} = \begin{bmatrix} g(r_1, r_{dip}, e_{d1}) & \cdots & g(r_1, r_{dip}, e_{dp}) \\ \vdots & \ddots & \vdots \\ g(r_N, r_{dip}, e_{d1}) & \cdots & g(r_N, r_{dip}, e_{dp}) \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix} = G(\{r_j, r_{dip_i}, e_{d_i}\}) \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}$$

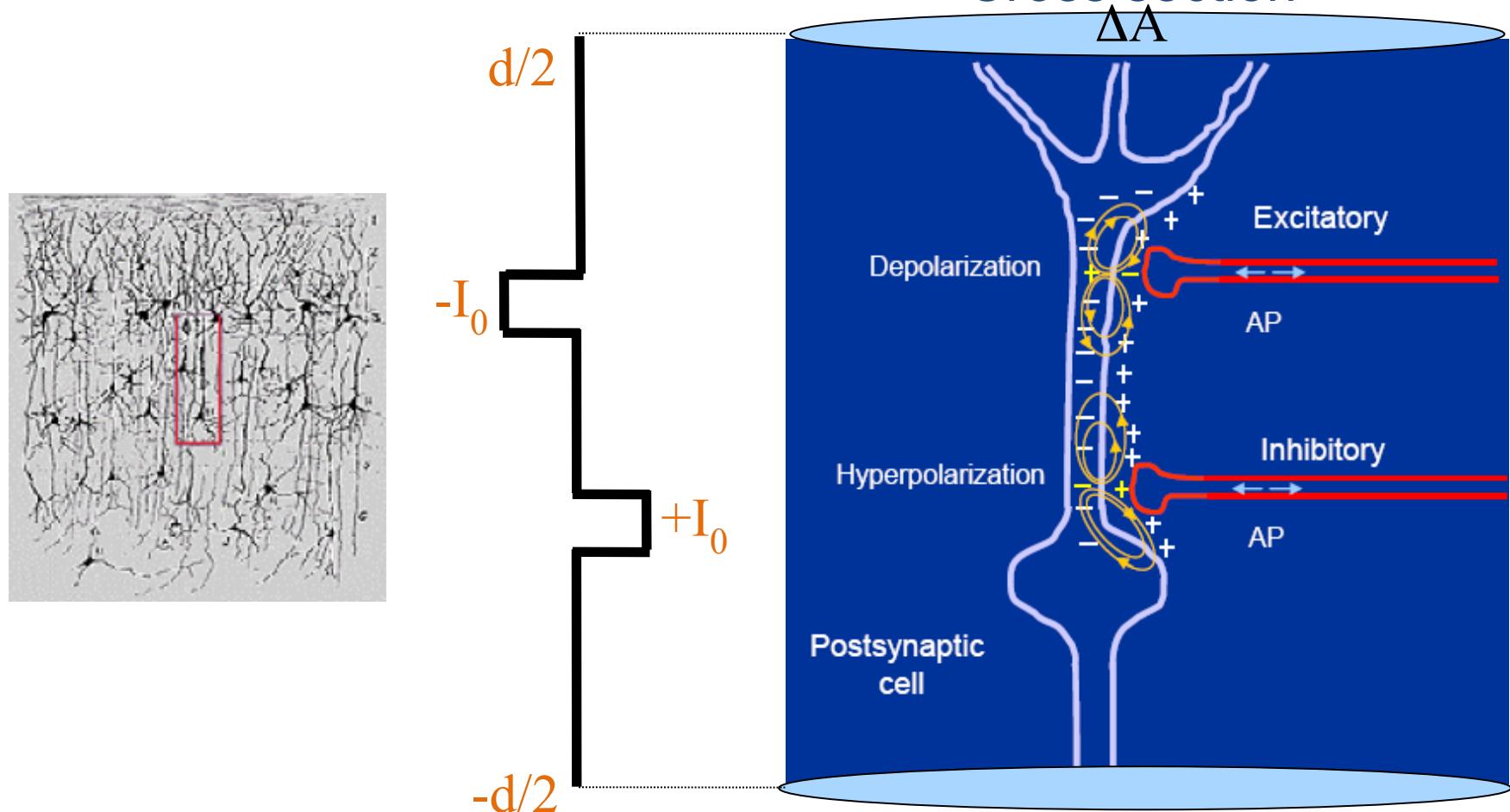
For N electrodes and p dipoles and T discrete time samples:

$$V = \begin{bmatrix} V(r_1, 1) & \cdots & V(r_1, T) \\ \vdots & \ddots & \vdots \\ V(r_N, 1) & \cdots & V(r_N, T) \end{bmatrix} = G(\{r_j, r_{dip_i}, e_{d_i}\}) \begin{bmatrix} d_{1,1} & \cdots & d_{1,T} \\ \vdots & \ddots & \vdots \\ d_{p,1} & \cdots & d_{p,T} \end{bmatrix}$$

$$V = GD + n$$

# Generators of EEG

## Cross section



diameter: 3mm  $\sim 10^5\text{-}10^6$  pyramidal cells  
A large pyramidal cell would have  $\sim 10^4\text{-}10^5$  synapses