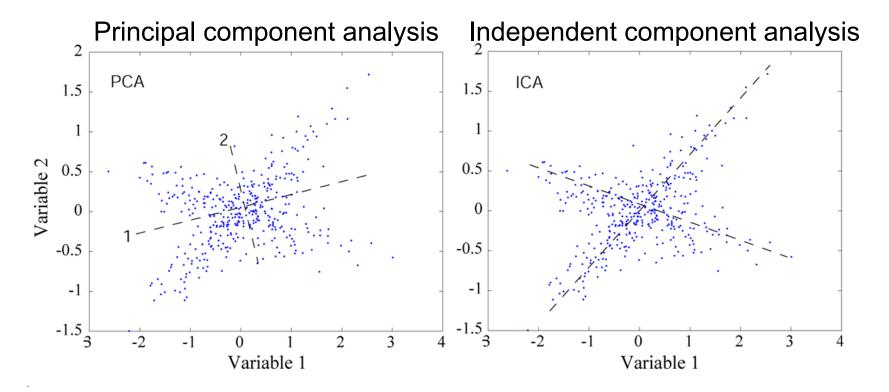
Clustering of ICA components

Arnaud Delorme

(with Julie Onton, Romain Grandchamp, Nima Bigdely Shamlo, Scott Makeig)

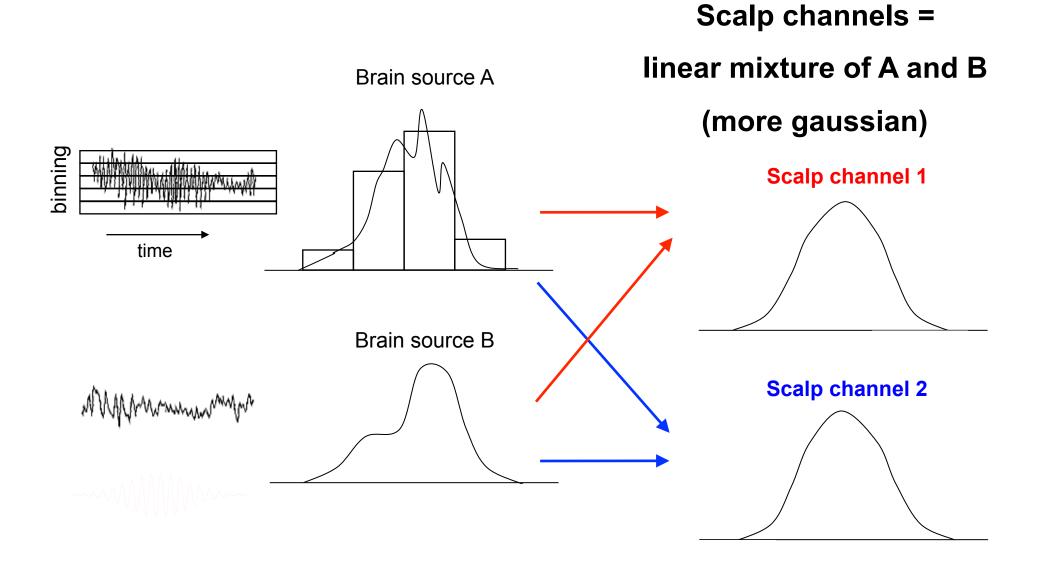
ICA and PCA

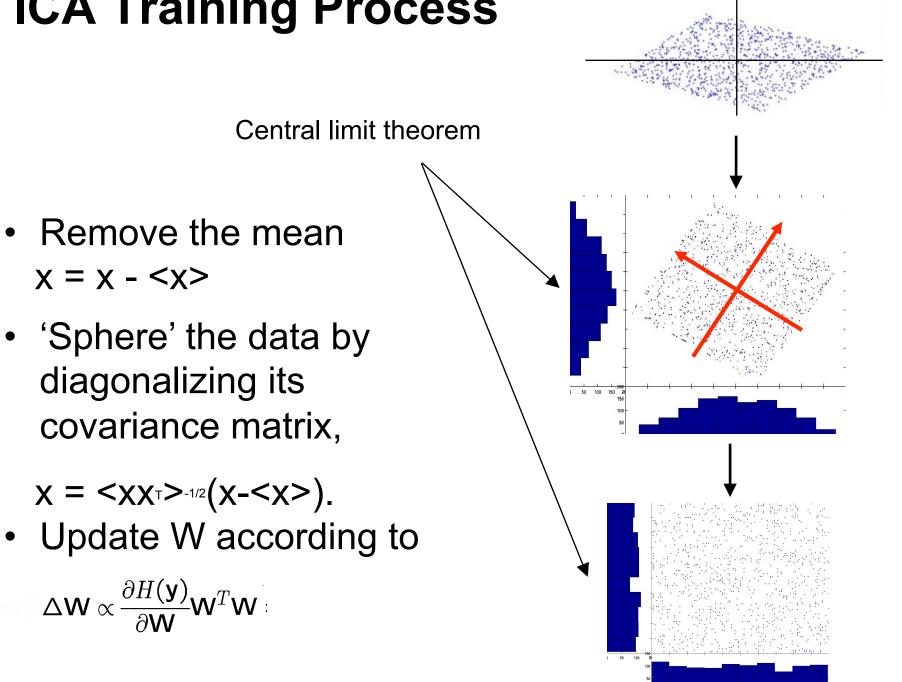
ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix,



While PCA simply decorrelates the outputs (using an orthogonal mixing matrix), ICA attempts to make the outputs statistically independent, while placing no constraints on the minxing matrix.

Central limit theorem

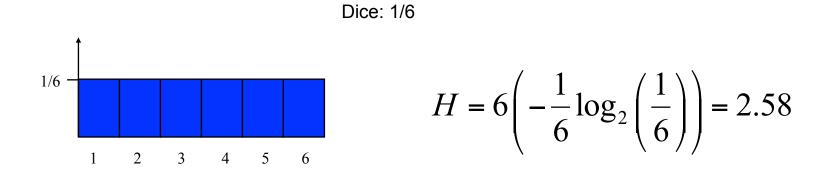




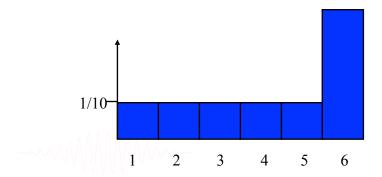
ICA Training Process

Entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x).$$



Fake dice (make a 6 half of the time): entropy 2.16 (base 2)



$$H = 5\left(-\frac{1}{10}\log_2\left(\frac{1}{10}\right)\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 2.16$$

Entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x).$$

Joint entropy

$$H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y)\log_b p(x,y),$$

Mutual Information

$$H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$$

Shannon in his landmark 1948 paper ``A Mathematical Theory of Communication." From http://planetmath.org/encyclopedia/ShannonsTheoremEntropy.html Contingency table for stress and emotionality

	STI	STRE					
	1	2	3	4	5	6	Total
EMOT=	1 19	4					23
	2 11	63	64	3	1		142
	3 2	16	18	20	2	2	60
	4 1	4	1	9	6	2	23
	5		1	2	4	3	10
	6			1	1	1	3
Tota	al 33	87	84	35	13	8	
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~							

From http://tecfa.unige.ch/~lemay/thesis/THX-Doctorat/node149.html

Contingency frequencies for stress and emotionality

	STRE					
	1	2	3	4	5	6
EMOT=1	0.07	0.02				
2	0.04	0.24	0.25	0.01		
3	0.01	0.06	0.07	0.08	0.01	0.01
4		0.02		0.03	0.02	0.01
5				0.01	0.02	0.01
6						

Joint entropy 3.46; exercise: compute mutual information  $H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y) \log_b p(x,y)$ 

### **ICA learning rule**

How to make the outputs statistical independent? Minimize their redundancy or mutual information. Consider the joint entropy of two components,

 $\Delta \mathbf{W} \propto \frac{\partial H(\mathbf{y})}{\partial \mathbf{W}} \mathbf{W}^T \mathbf{W}$ 

 $H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$ 

Maximizing  $H(y_1, y_2) \Longrightarrow$  minimizing  $I(y_1, y_2)$ .

The learning rule:

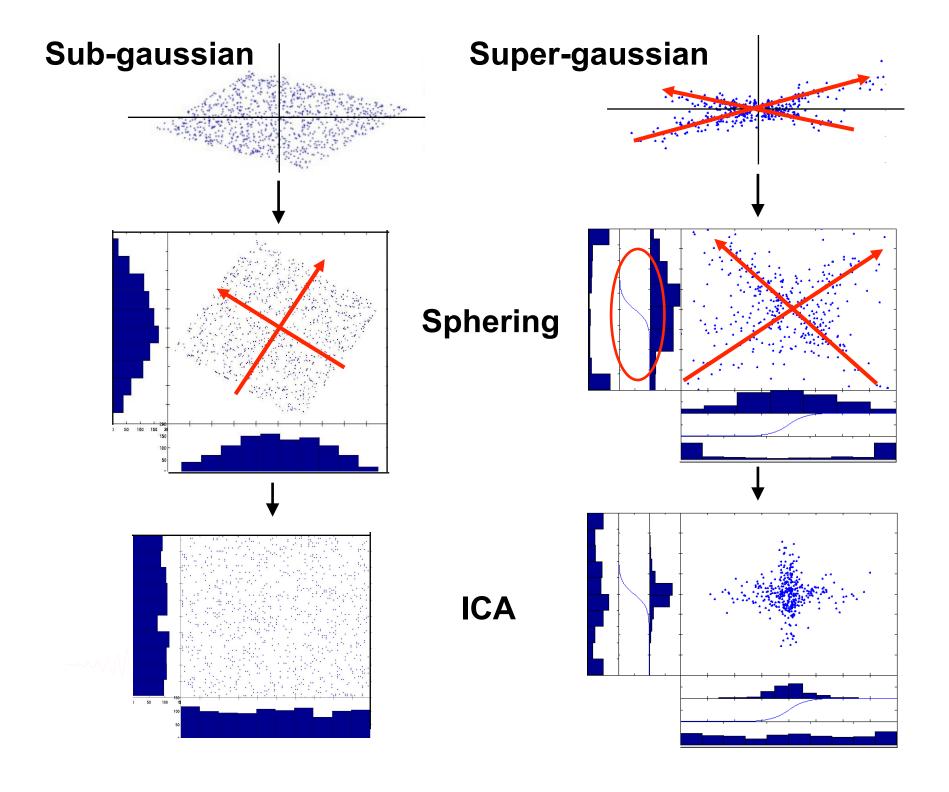
Entropy

extremum

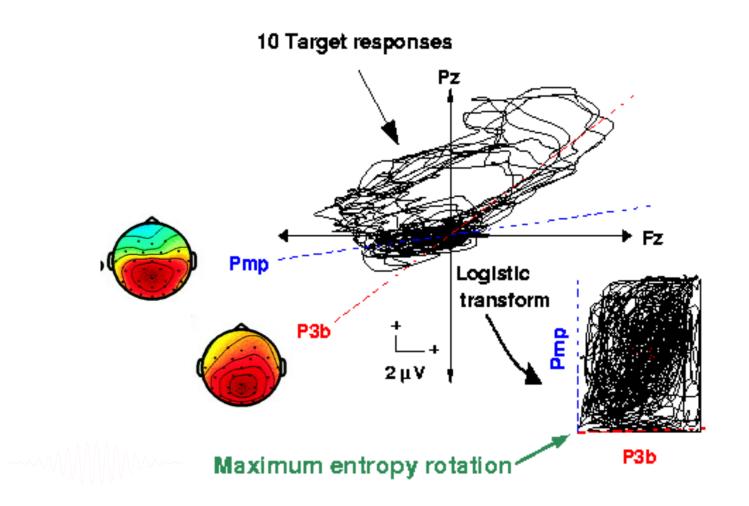
=0 if the two variables

are independent

Natural gradient (Amari)



#### Independent components of LEO/LIN



### **Steps of clustering**

- Select ICA components for clustering
- Precompute measures of interest
- Cluster measures
- Plot clusters and edit them if necessary



### Edit dataset info

				-		pop_study()	: Pre-select compone	ents		
					Enter maxir NOTE: This	num residual will delete ar	(topo map – dipole iy existing compon	eproj.) var. ent cluster:	(in %) s!	_
					Koon only	y in-brain dij	15			
					Reep off	y m-brain dij	Joies.			
Crea	ate a new STUDY set pop	_study()			Can	cel	Help		Ok	
-										
	STUDY set information - ren	nember to	save change	es	Channelson		1			
	STUDY set name:				Sternberg					
	STUDY set task name:				Sternberg					
	STUDY set notes:									
	dataset filename	browse		session		group	Select by r.v.			
1	C:\\Users\\julie\\Documents\W		S01		memorize		Comp.: 3 5	Clear		
2	C:\\Users\\julie\\Documents\\V		S01		ignore		Comp.: 3 5	Clear		
3	C:\\Users\\julie\\Documents\\V		S01		probe		Comp.: 3 5	Clear		
4	C:\\Users\\julie\\Documents\\V		S02		memorize		Comp.: 5 6	Clear		
5	C:\\Users\\julie\\Documents\\V	Vor	S02		ignore		Comp.: 5 6	Clear		
6	C:\\Users\\julie\\Documents\W	Vor	S02		probe		Comp.: 5 6	Clear		
7	C:\\Users\\julie\\Documents\W	Vor	S03		memorize		Comp.: 6 7	Clear		
8	C:\\Users\\julie\\Documents\\V	Vor	S03		ignore		Comp.: 6 7	Clear		
9	C:\\Users\\julie\\Documents\\V	Vor	S03		probe		Comp.: 6 7	Clear		
10	C:\\Users\\julie\\Documents\W	Vor	S04		memorize		Comp.: 1 2	Clear		
Impor	tant note: Removed datasets w	/ill not be sa	ved before be	ing delete	d from EEGLAB n	nemory				
			< P	age 1	>					
	Dataset info (condition, group,	) differs t	from study info	o. [set] = 0	verwrite dataset	info.				
_	Delete cluster information (to a						o.)			
	Help					Ca	ncel C	k		

### ICs to cluster

Edit	STUDY set inform	tion - remember to save changes						
	STUDY set task r	Sternberg						
	STUDY set notes	select components						
	dataset filename	ic 19 ic 20 sion condition group	Select by r.v.					
1	C:\\Users\\julie\\	ic 21 memorize	Comp.: 3 5	Clear				
2	C:\\Users\\julie\\	ic 22 ic 23	Comp.: 3 5	Clear				
3	C:\\Users\\julie\\	ic 24 probe	Comp.: 3 5	Clear				
4	C:\\Users\\julie\\	ic 25 ic 26 memorize	Comp.: 5 6	Clear				
5	C:\\Users\\julie\\	ic 27 ignore	Comp.: 5 6	Clear				
6	C:\\Users\\julie\\	ic 28 ic 29	Comp.: 5 6	Clear				
7	C:\\Users\\julie\\	ic 30 memorize	Comp.: 6 7	Clear				
8	C:\\Users\\julie\\	ic 31 ignore	Comp.: 6 7	Clear				
9	C:\\Users\\julie\\	Cancel Ok probe	Comp.: 6 7	Clear				
10	C:\\Users\\julie\\	Cancel Ok memorize	Comp.: 1 2	Clear				
Important note: Removed datasets will not be saved before being deleted from EEGLAB memory           Page 1       >          Dataset info (condition, group,) differs from study info. [set] = Overwrite dataset info.          Delete cluster information (to allow loading new datasets, set new components for clustering, etc.)								

### **Precompute data measures**

-	EEGLAB v6.0b										
File	Edit	Tools	Plot	Study	Datasets	Help	с.				
	-STUD	V se	4- A4	Edit	t study info						
	-3100	1 36	L AI		compute ch	annel measi	ures				
	Study f	ilenar	ne:	Plot	channel me	easures					
	Study task name			Precompute component measures							
		Nb of subjects Nb of conditions Nb of sessions			Build preclustering array						
					Cluster components						
	Nb of groups			Edit/plot clusters							
	Epoch consistency Channels per fram										
	Chann				25						
	Cluster	rs		1							
	Status Totol c	atus otal size (Mb)			eady to pr 0.4	ecluster					
	Totals	12e (M		5	0.4						

### **Precompute data measures**

#### TIP: Compute all measures so you can

#### test different combinations for clustering

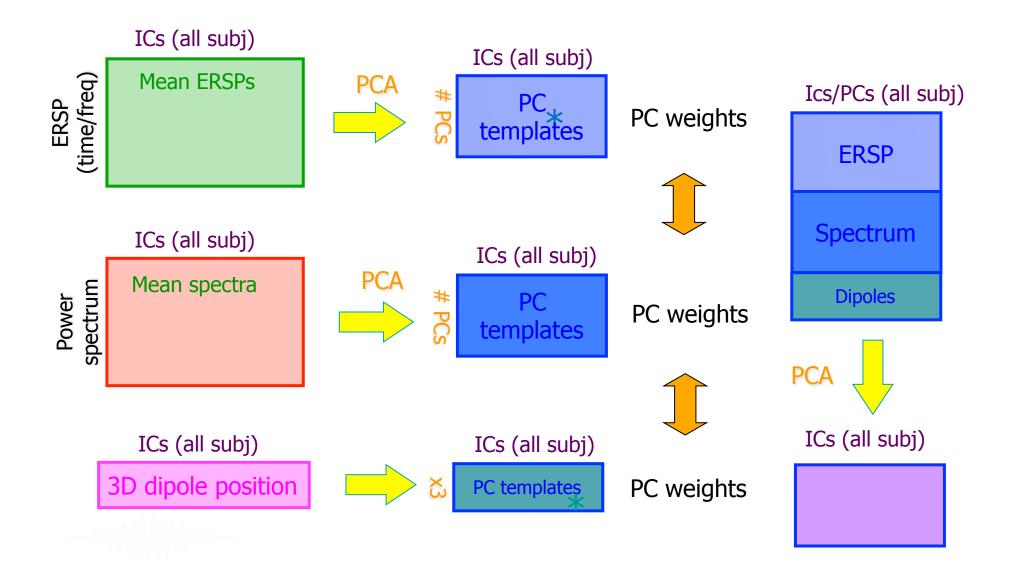
	Select and compute comp								
	Pre-compute component measures for STUDY 'Sternberg'								
l	Compute ERP/spectrum/ERSP only for components selected by RV (set) or for all components (unset)								
L	List of measures to prec	ompute							
L	🔽 ERPs	Baseline ([min max] in m	is)	[-200 0]					
L	Power spectrum	Spectopo parameters			Test				
	ERSPs     ITCs	Time/freq.parameters	'cycl	les', [3 0.5], 'nfreqs', 100 ᆽ	Test				
	Scalp maps       Time-frequency         Recompute even if present on disk       options								
	Help			Cancel	Ok				

------

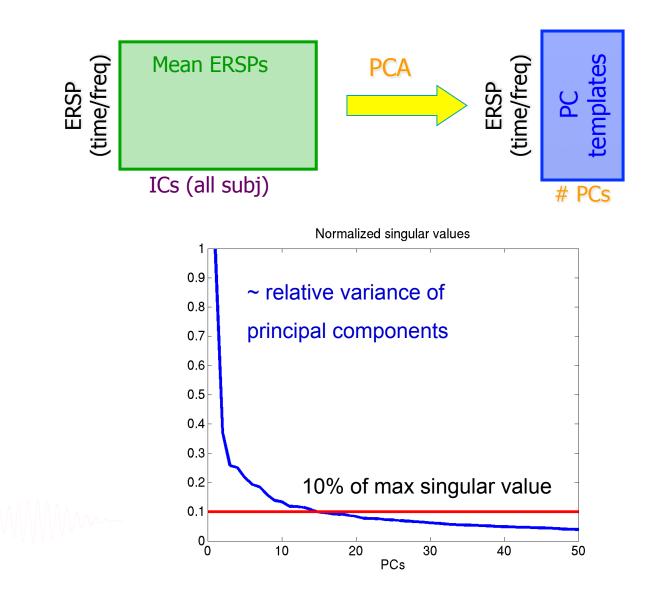
### **Cluster components**

-	EE	GLAB v6.0b								
File	Edit <b>Tools</b> Plot	Study Datasets Help 🛛 🛥								
	-STUDY set: AI	Edit study info								
	-STODT SEL AI	Precompute channel measures								
	Study filename:	Plot channel measures								
	Study task name	Precompute component measures								
	Nb of subjects Nb of conditions	Build preclustering array								
	Nb of sessions	Cluster components								
	Nb of groups	Edit/plot clusters	Select ar	nd compute com	ponent mea	asures for later clusteri	ng pop_p	reclust()		
	Epoch consistency Channels per fram									
	Channel locations	Du	ild pre-clustering ma ect the cluster to refine				archywill	he overwritten)		
	Clusters	1	ect the cluster to renne	-			<ul> <li>arcny win</li> </ul>	be overwritten)		
	Status Total size (Mb)	Pre-clustered 32.4								
	10(01)12((110)	32.1					-			
		(no	econly measures that l	have been prec	omputed r	mav be used)				
		Los		Dims. Norm						
		×		10	1	Freq.range [Hz]	3 25			
		<b>x</b>	ERPs dipoles	10 🗹	1 10	Time range [ms]	0 600			
			scalp maps	10	10	Use channel values	-	Absolute values		
		<b>X</b>	ERSPs	20 🖌	1	Time range [ms]	0 1500	Freq. range [Hz]	3 45	
		×	ITCs	10	1	Time range [ms]	0 600	Freq. range [Hz]	2 30	
			Final dimensions	10	Help					
			Save STUDY to file	/	home/juli	ie/WorkshopSD2007/	STUDY/ati	ention.study		
		/////	Cancel			Help		Ok		

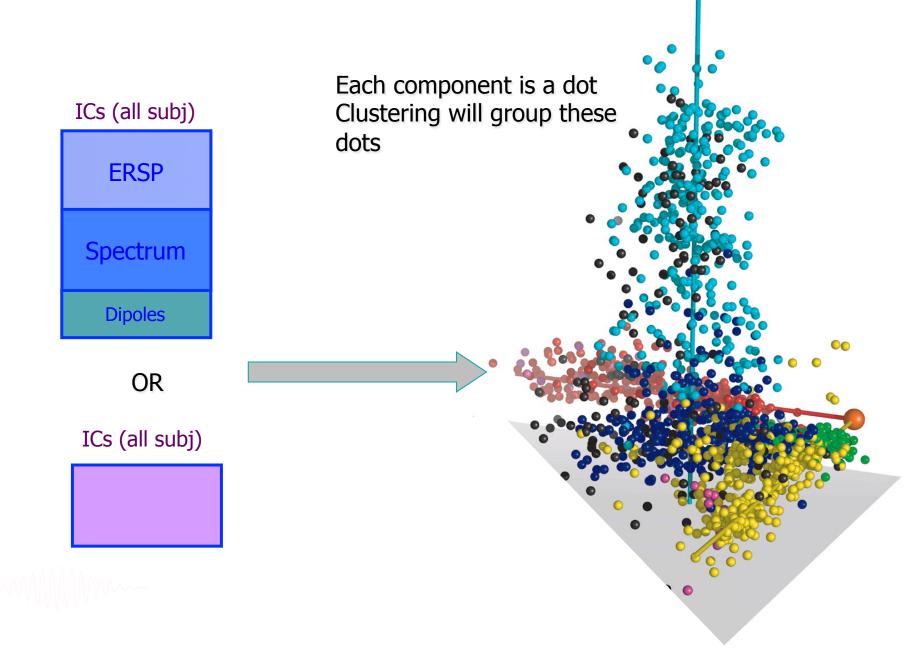
### **Precluster schematic**



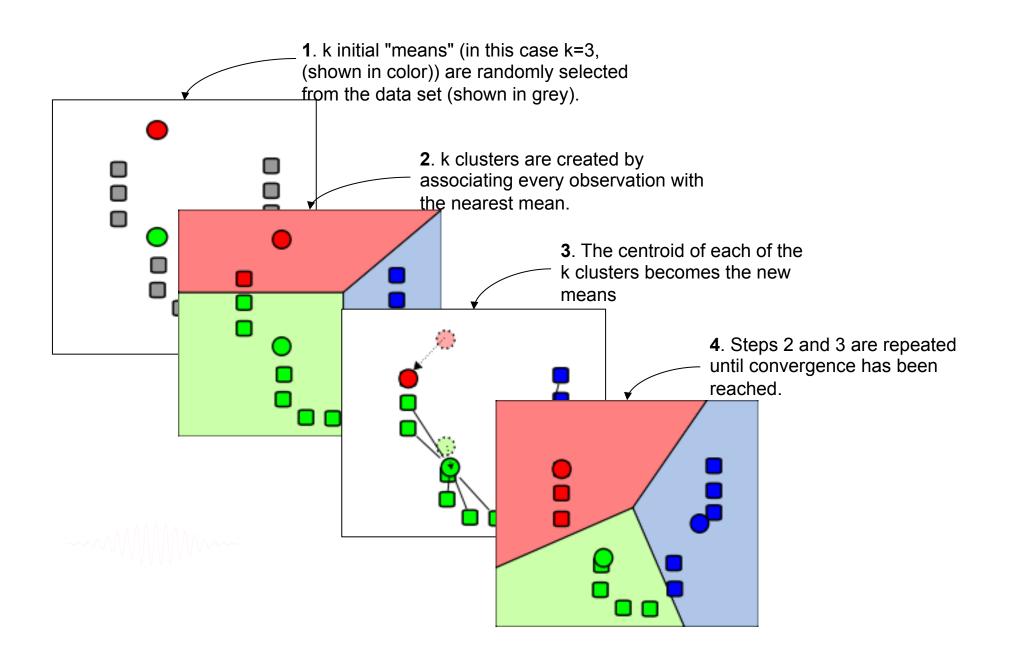
### **Precluster: Use singular values from PCA**



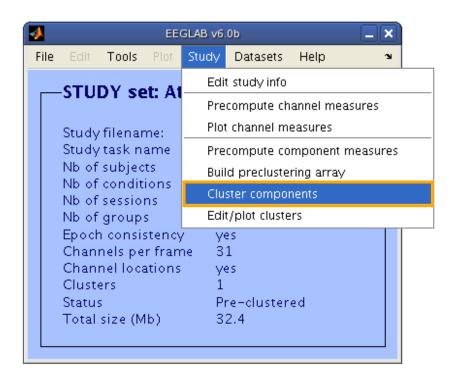
### **Precluster schematic**

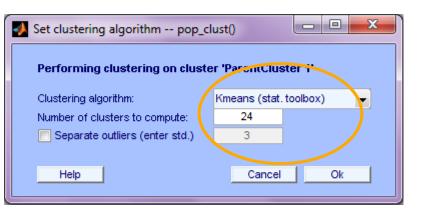


## **Classical KMean**



### **Cluster components**





### Choosing data measures

What measure(s) should you use?

It depends on your final cluster criteria...

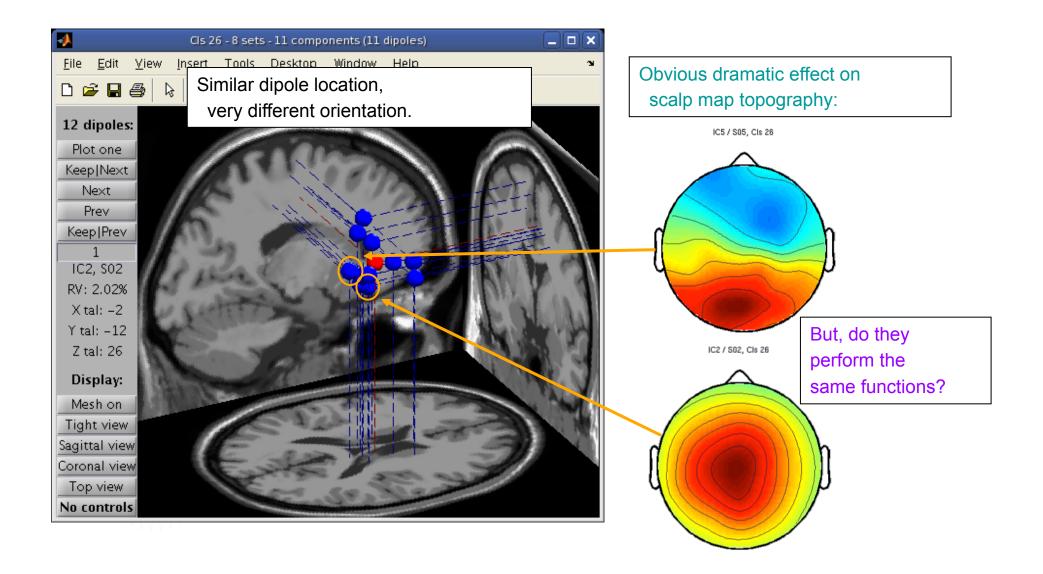
- If for example, your priority is dipole location, then cluster only based on dipole location...

But consider:

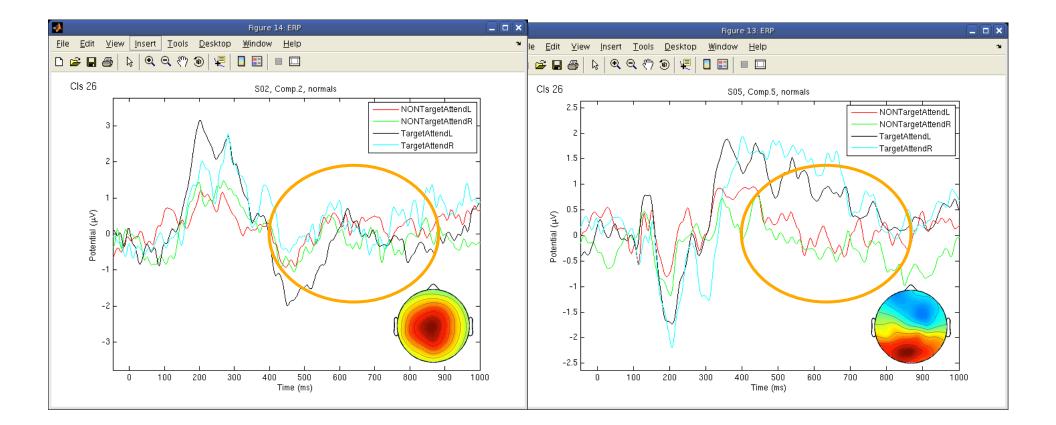
- What is the difference between these two components?



### **Choosing data measures**



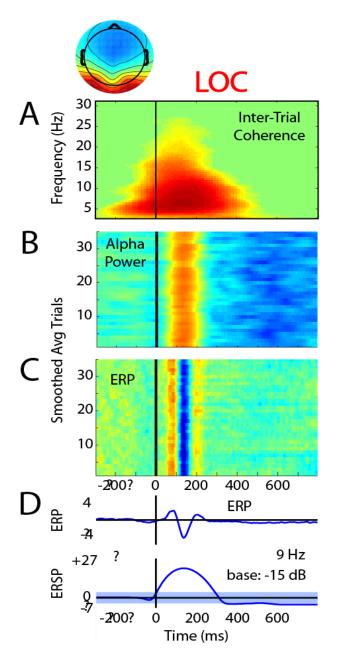
### **Choosing data measures**

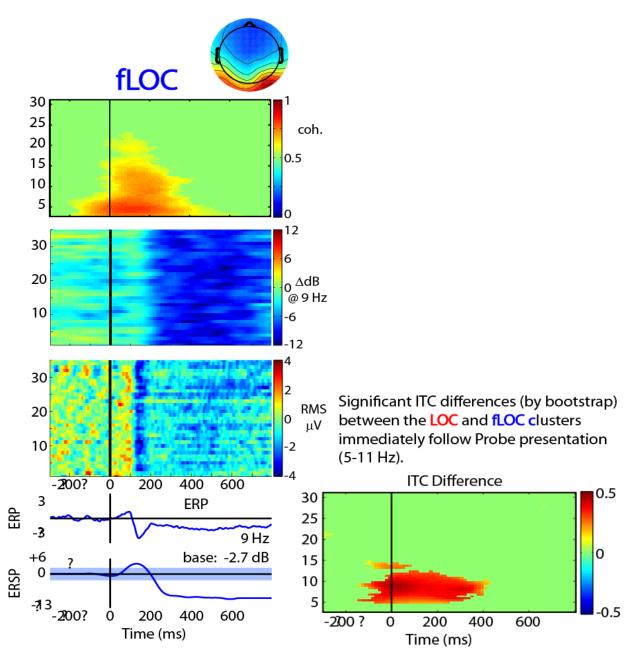


ERPs seem different...

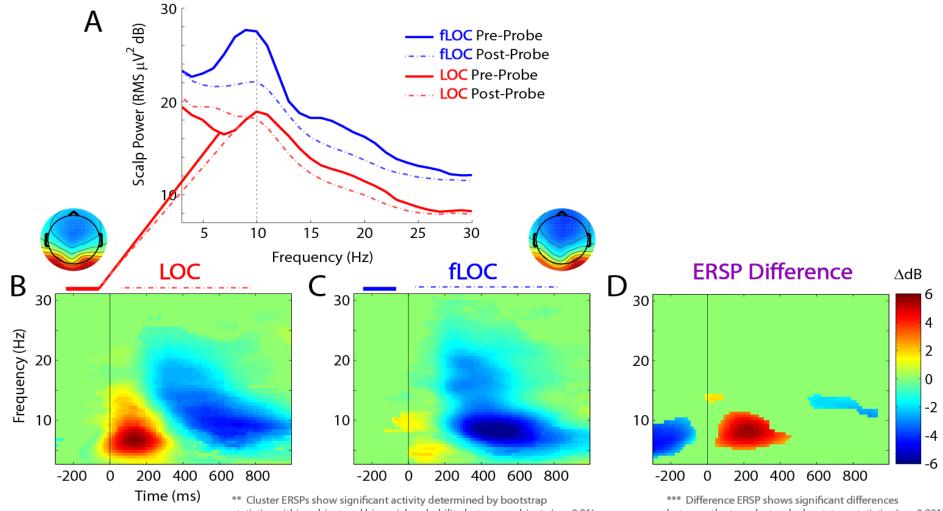


### **Subject differences?**





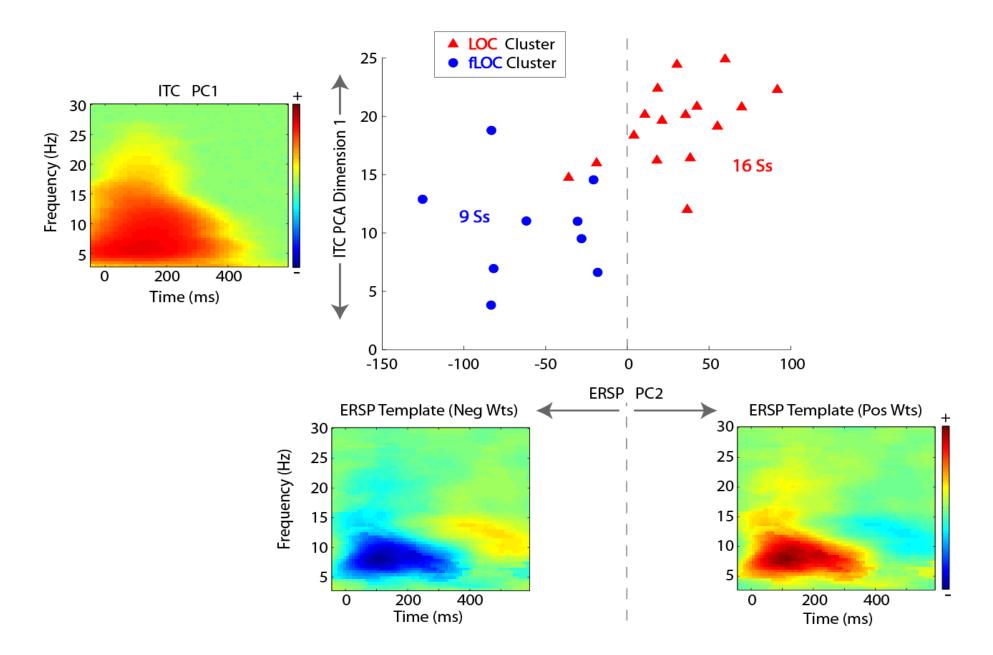
### **Subject differences?**



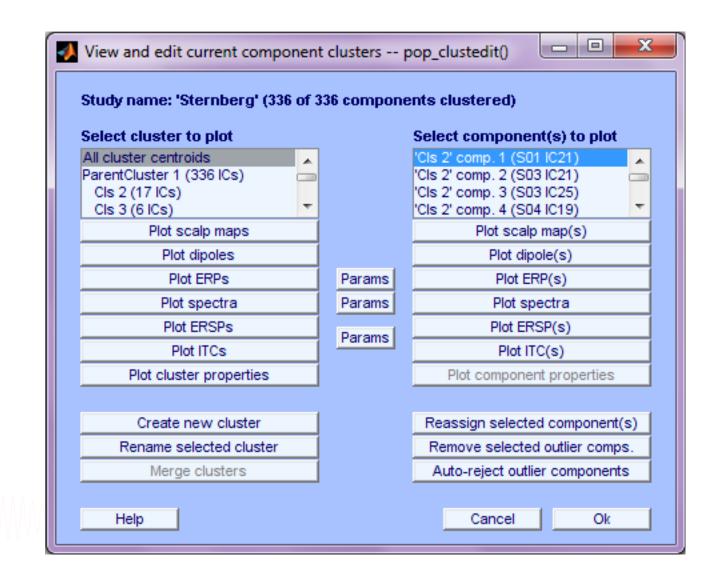
statistics within subject and binomial probability between subjects (p < 0.01)

between the two clusters by bootstrap statistics (p < 0.001)

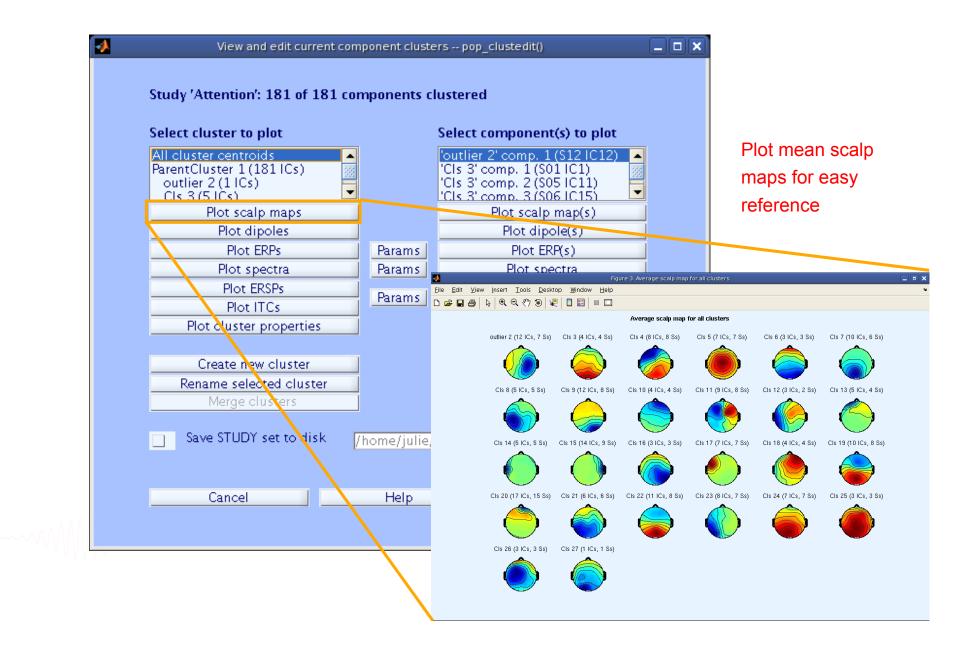
### **Subject differences?**



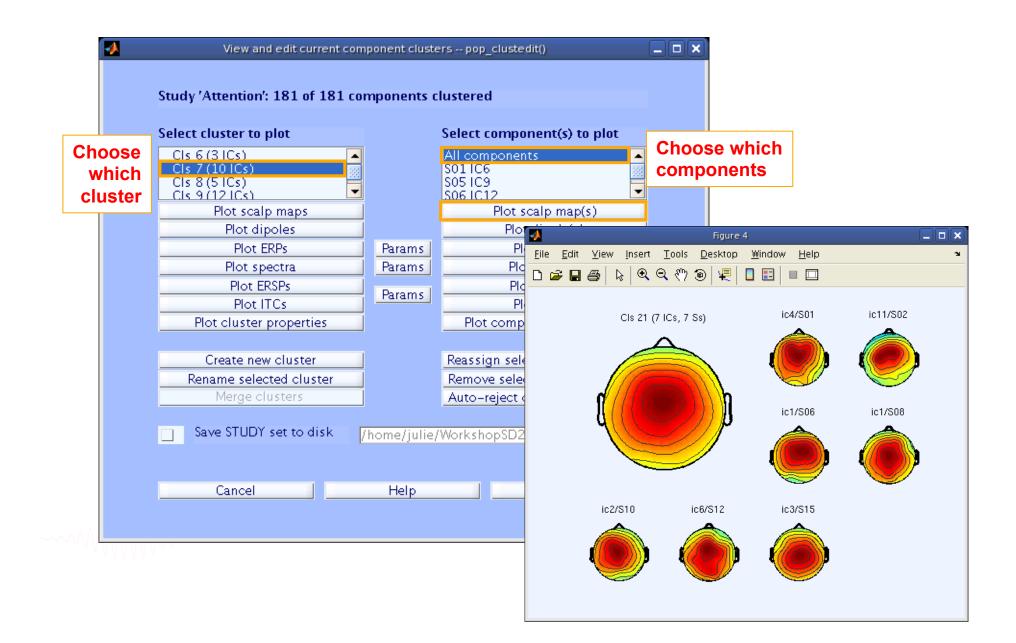
### **Plot/edit clusters**



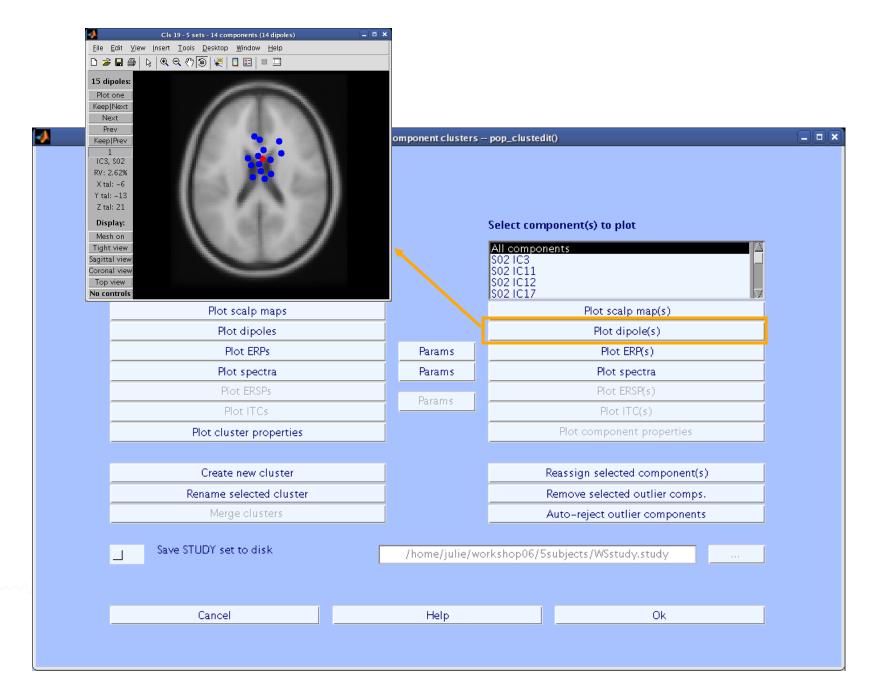
### **Plot cluster data**



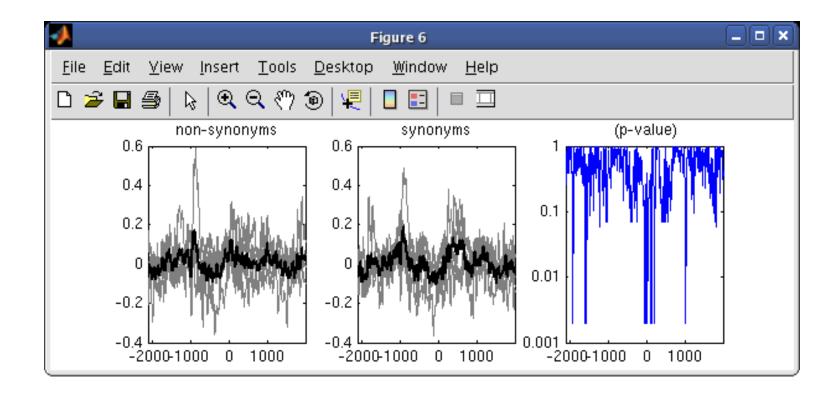
### **Plot cluster data**



### **Plot cluster data**



### **Plot cluster ERP**



### **Reassigning components**

View View	and edit current component clusters	pop_clustedit()	
Study ": 151 of 151 components clustere	d		
Select cluster to plot		Select component(s) to plot	
Cls 12 (6 ICs) Cls 13 (5 ICs) Cls 14 (11 ICs) Figure 5		All components S07 IC14 S07 IC33 S08 IC23 S10 IC60	
<u>File Edit View Insert Tools Desktop Window H</u> elp		7	
▷ ☞ 묘 鎶   ╘,   �. ♀. ᡧ ᢀ      □ ☷   ■ ⊐		Plot scalp map(s)	
CIs 17 average scalp map, 3Ss	ic14/S07	Plot dipole(s) Plot ERP(x)	
ic23/S08 ic23/S08 ic60/S10	ic33/S07	Cls 1: - 3 sets - 4 components (4 dipoles)         File       Edit       Yiew       Insert       Tools       Desktop       Window       Help         D       Image: Cls 1: - 3 sets - 4 components       Image: Cls 1: - 3 sets - 4 components       Image: Cls 1: - 3 sets - 4 components       Image: Cls 1: - 3 sets - 4 components       Image: Cls 1: - 3 sets - 4 components         Prev       Next       Prev       Image: Cls 1: - 25         Verkshop06/55       Y tal: - 25       Z tal: 46       Image: Cls 1: - 25       Image: Cls 1: - 25       Image: Cls 1: - 25         Mesh on       Tight view       Sagittal view       Image: Cls 1: - 25       Image: Cls 1: - 25       Image: Cls 1: - 25         Mesh on       Tight view       Sagittal view       Image: Cls 1: - 25       Image: Cls 1: - 25         Very Image: Cls 1: - 25       Z tal: 46       Image: Cls 1: - 25       Image: Cls 1: - 25         Very Image: Cls 1: - 25       X tal: - 46       Image: Cls 1: - 25       Image: Cls 1: - 25         Very Image: Cls 1: - 25       X tal: - 46       Image: Cls 1: - 25       Image: Cls 1: - 25         Very Image: Cls 1: - 25       X tal: - 46       Image: Cls 1: - 25       Image: Cls 1: - 25         Very Image: Cls 1: - 25       X tal: - 46	

### Issue with standard clustering

Large parameter space problem: many different clustering solutions can be produced by changing parameters and measure subsets. Which one should we choose?

Select and compute component measures for later clustering -- pop_preclust()  $-\Box \times$ Pre-compute measures on which to cluster components from study 'N400STUDY' **EEGLAB** clustering Select the cluster to refine during sub-clustering (any existing sub-hierarchy will be overwritten) has ~12 parameters ParentCluster 1 (151 ICs) Pre-compute or Load Norm. Rel. Wt. Dims. Frequency range [Hz] 3 25 ×. spectra 10 ×. 1 ERPs. Latency range in ms [lo hi] × 10 × 1 -2100 1995 × dipoles. 3 10 ×. × scalp maps ×. 10 1 Use channel values Absolute values **ERSPs** Time/freq. parameters e', [3 25], 'cycles', [3 0.5], 'pa ×. 10 1 × ITCs. 10 × 1 Time/Ded_bareneters t, [3-25], "cyclest, [3-0-5], "pa Final dimensions 10 Help Save STUDY to file /data/common4/amo/Ssubjects/N400preclust.study Cancel Help Ok.

### **Measure projection**

# (EEGLAB extension by Nima Bigdely Shamlo) only has one pre-clustering parameter.

🛃 Measure Product clustering -	- pop_mpcluster() 💷 🛛
Number of clusters to compute: Relative dipole weight (between 0 ar Select measuretures to be used in th	
♥ Dipole ♥ ERP ERSP ITC ♥ Spectra Scalp map	
Separate outliers (enter std.)	3
Help	<u>Cancel</u> Ok

(Affinity clustering by Pernet, Martinez, Delorme)

## Exercise

- Load the STUDY
- Precluster and cluster components using spectrum and dipoles location
- Look at your cluster. Identify frontal midline theta cluster and occipital alpha cluster
- Plot significant difference for one component cluster spectrum between the two conditions in the default design

