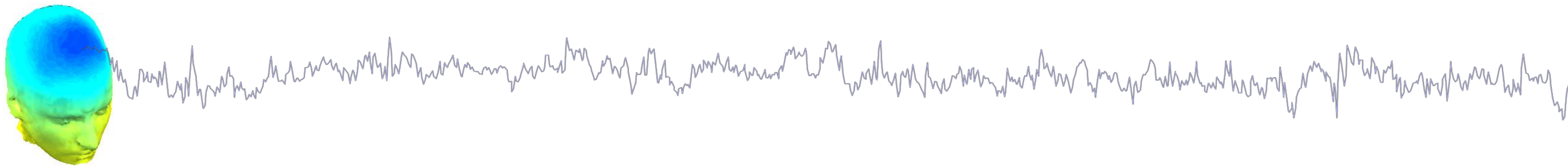


Time-frequency measures

Theory and Practice

EEGLAB Workshop 2018

UCSD

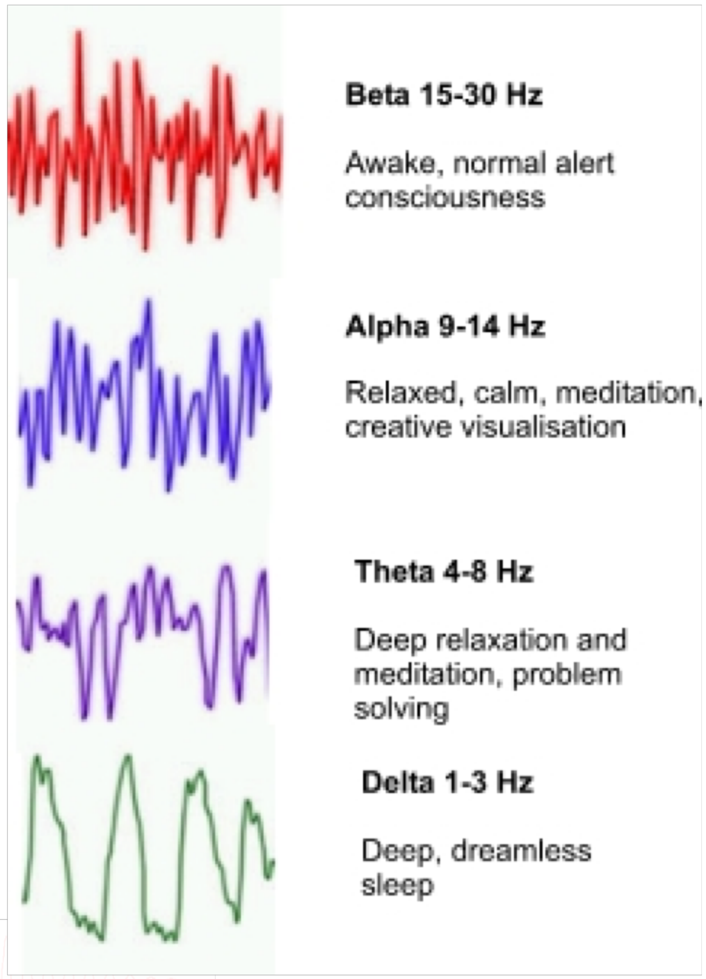
Day 2, 11:30

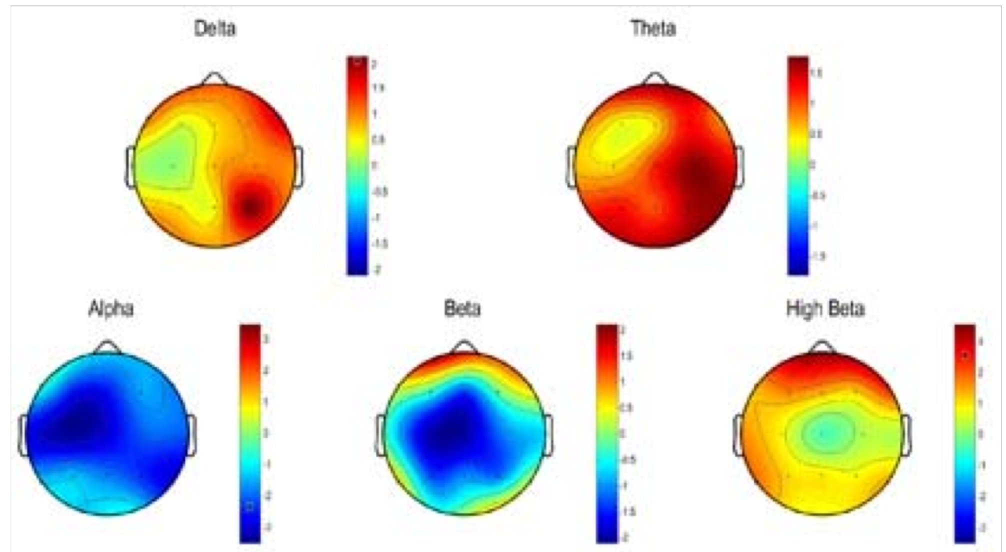
- Signals – M/EEG

- Goals
  - Describe dynamic characteristics of brain activity
  - Describe relation between different regions of brain

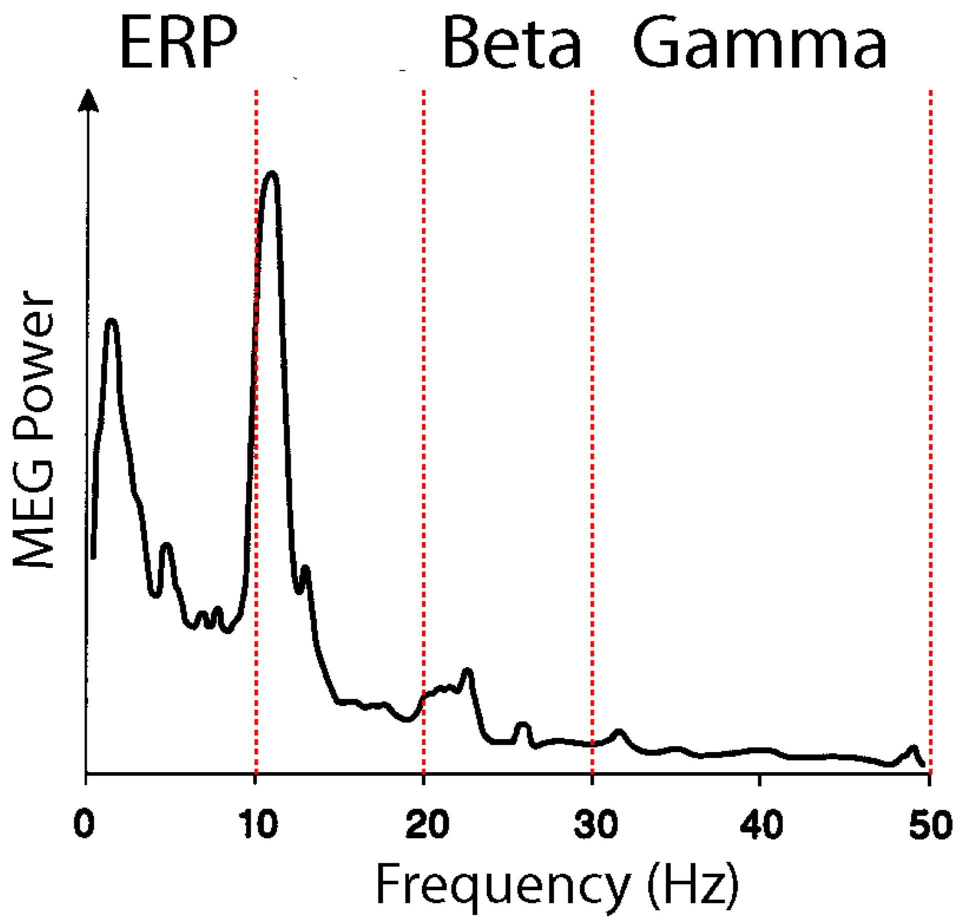
- Approaches
  - Time domain
  - Frequency domain
  - Time/Frequency

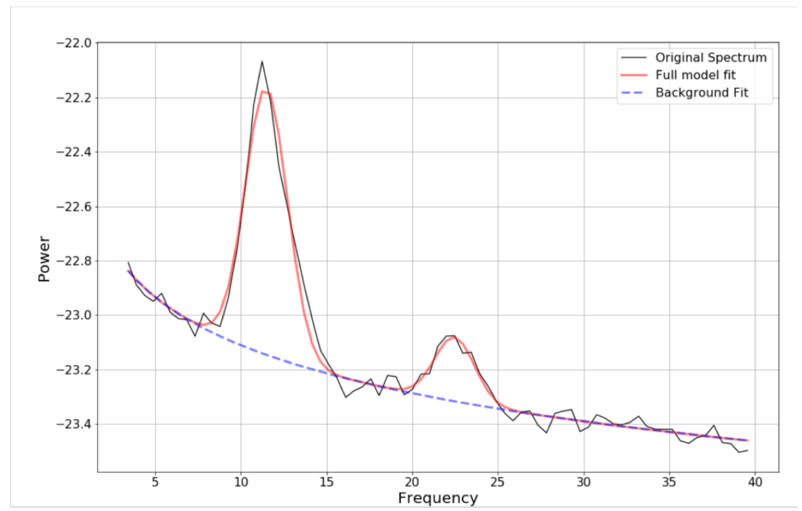
Different meanings traditionally given to different frequency bands





MEEG spectrum

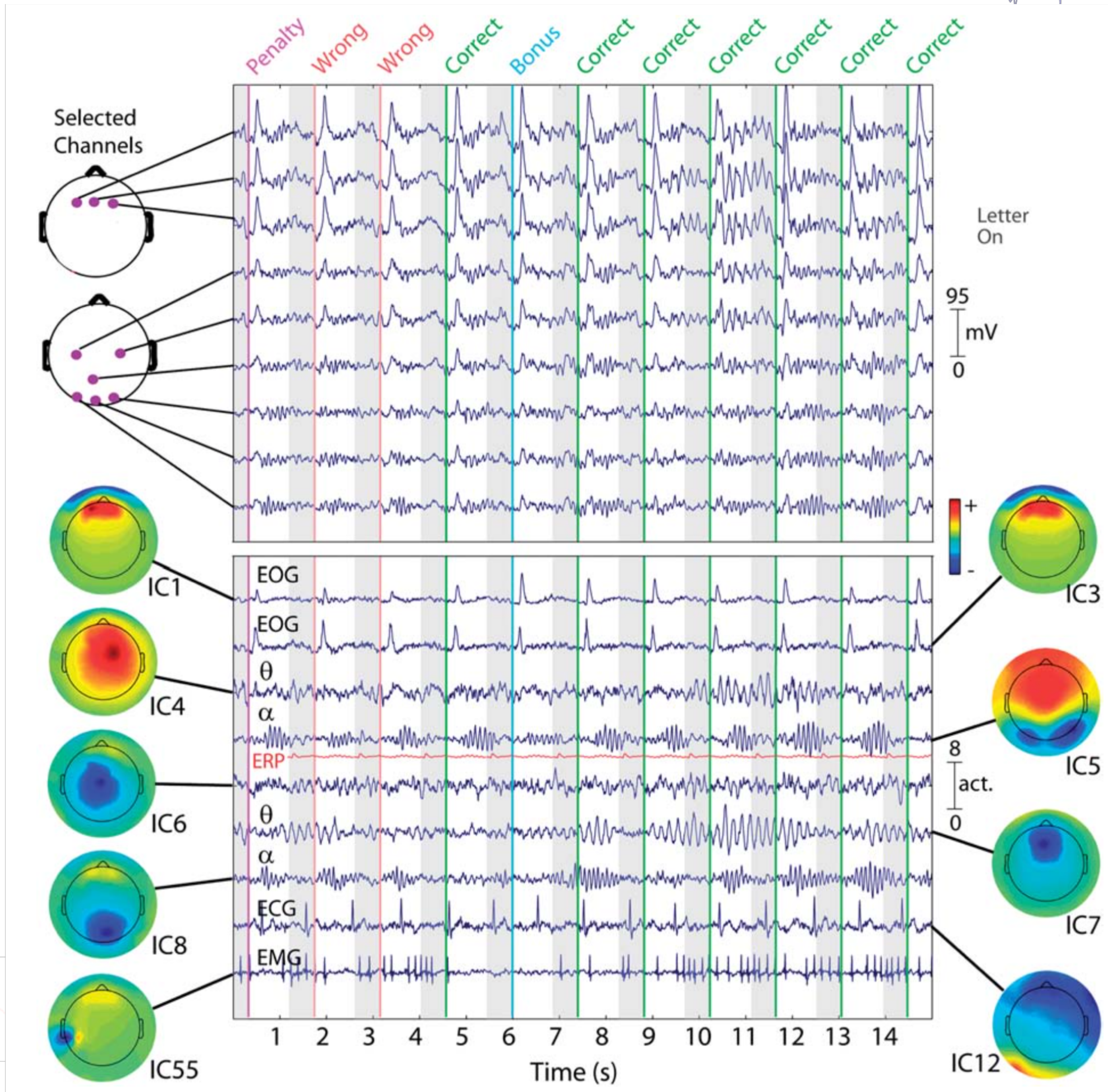




"FOOOF"

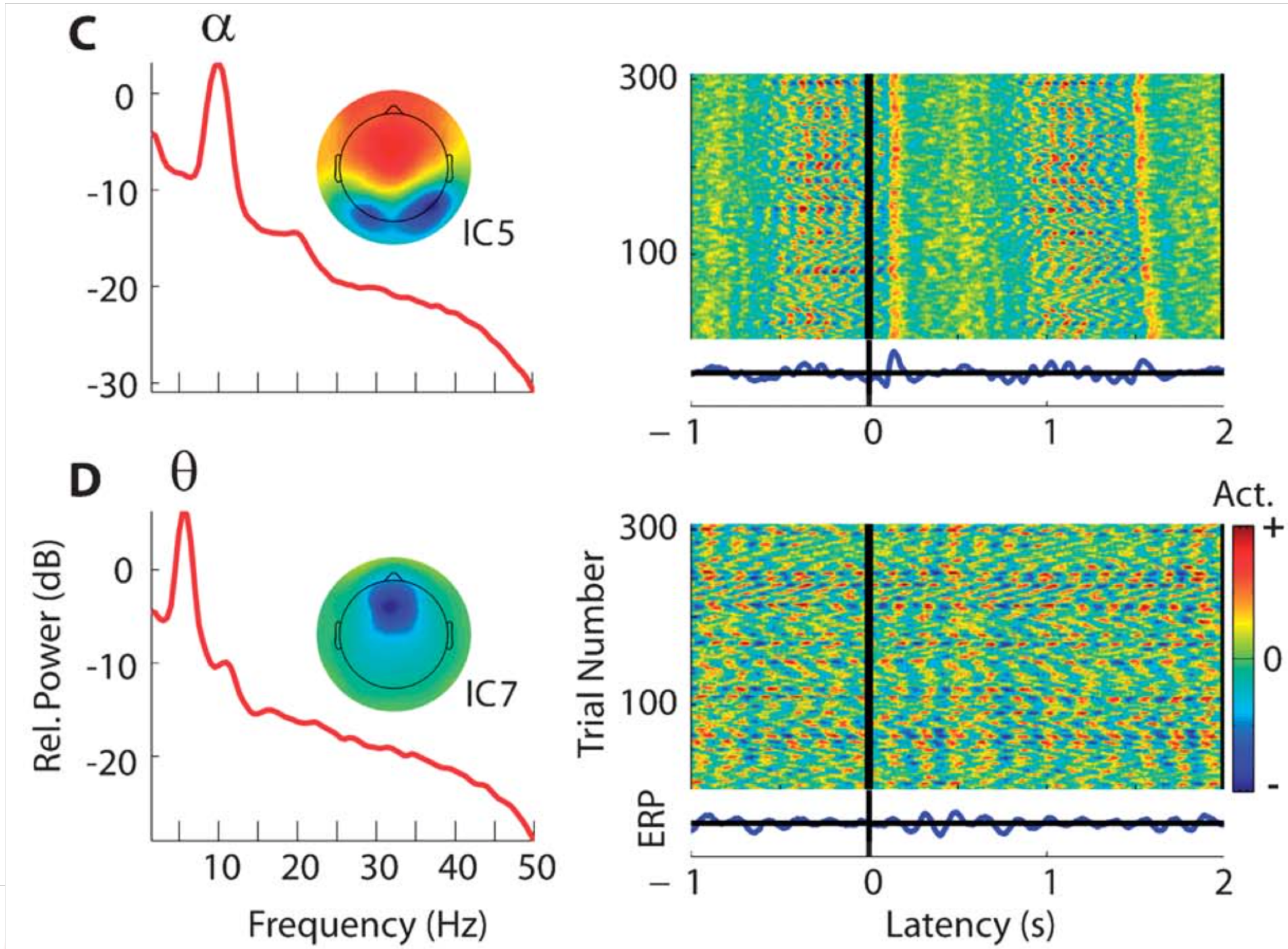
Haller M et al (2018) Parameterizing Neural Power Spectra. bioRxiv, 299859

Time-varying frequency content

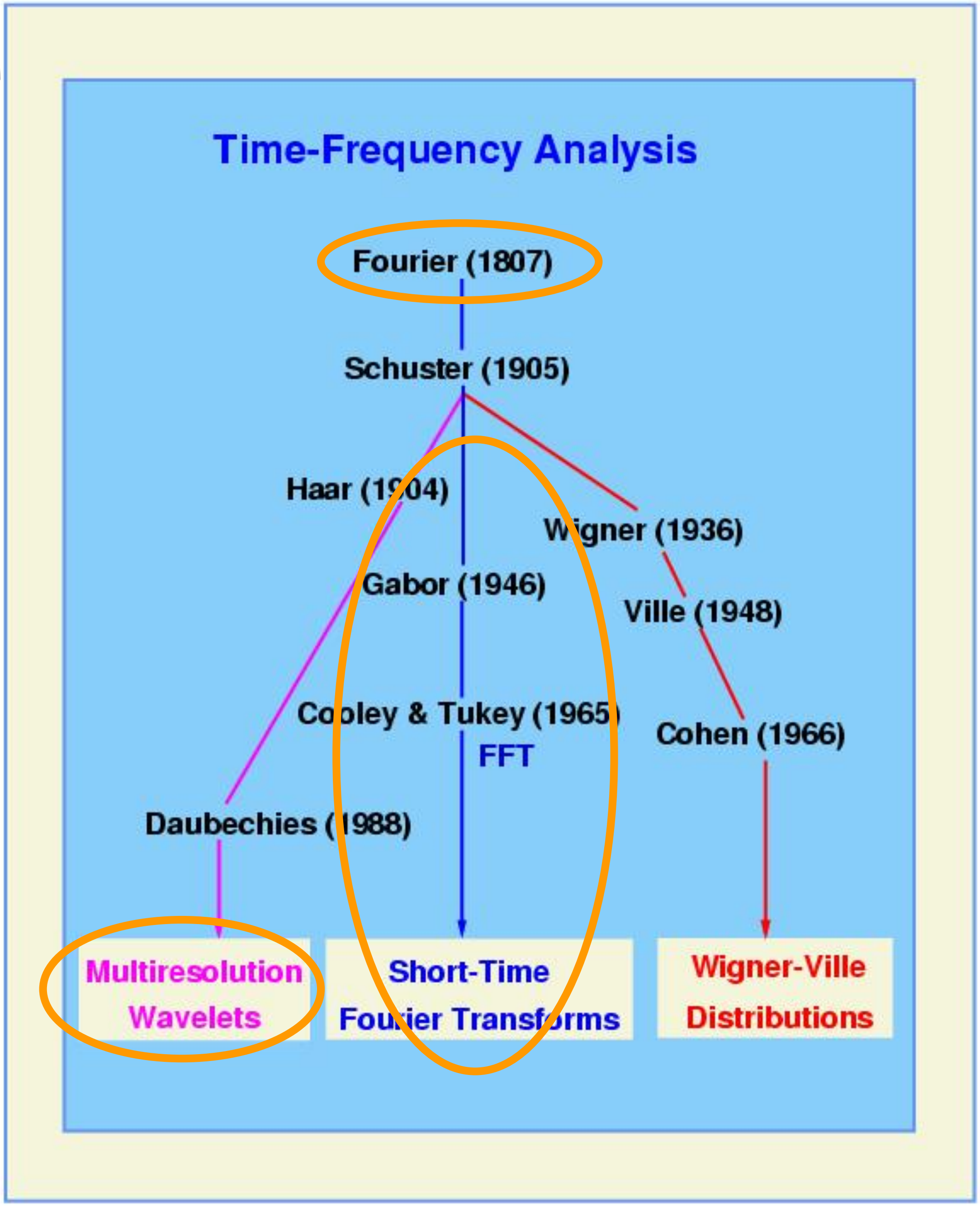


Onton & Makeig, 2006

Power Spectrum does not describe temporal variation

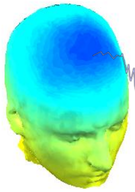


Onton & Makeig, 2006



S. Makeig, 2005

Plan

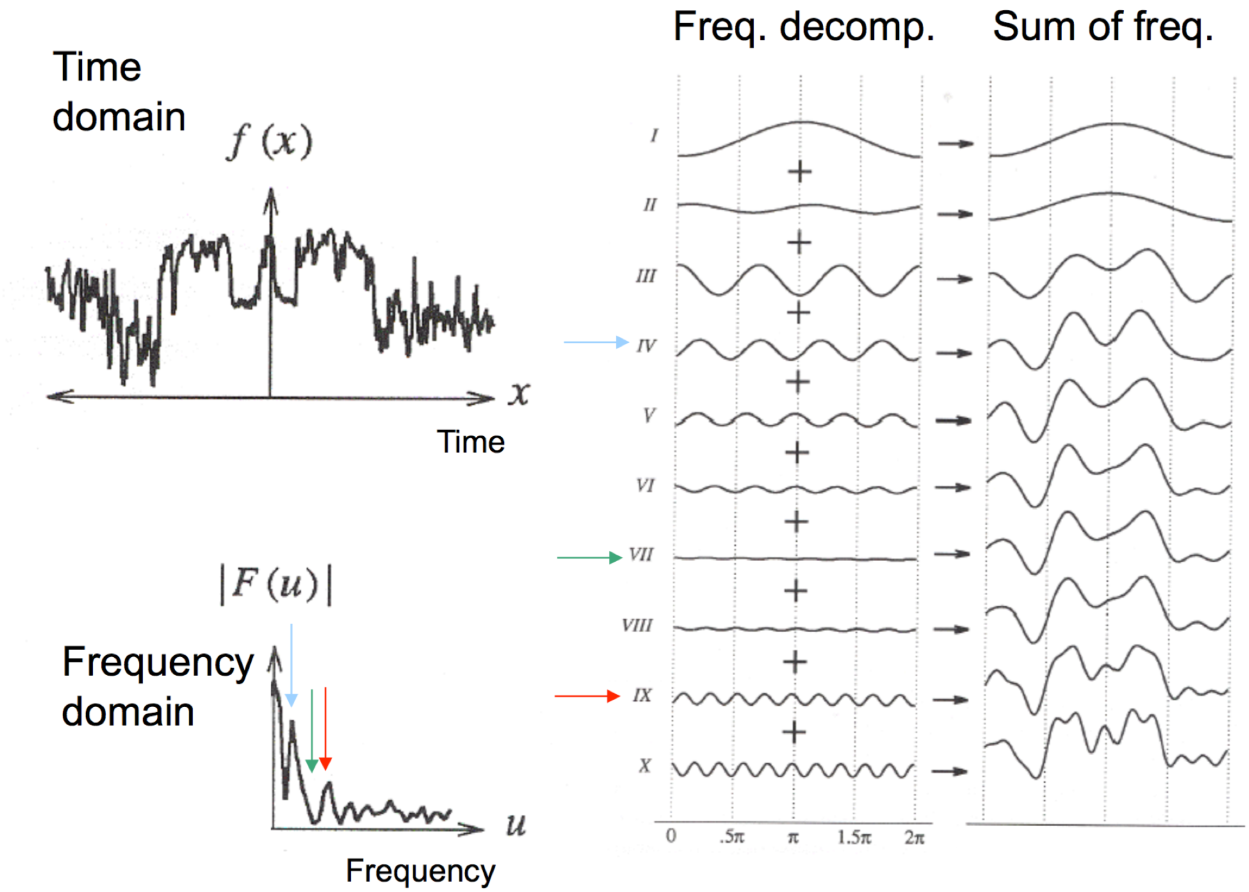


- **Part 1: Frequency Analysis**
  - Power Spectrum
    - Windowing
- **Part 2: Time-Frequency Analysis**
  - Short Time Fourier Transform
  - Wavelet Transform
  - ERSP
- **Part 3: Coherence Analysis**
  - Inter-Trial Coherence
  - Event-Related Coherence
- **Part 4: Other Applications**
  - TF Directional Causality (e.g. SIFT)
  - Cross-frequency analysis, e.g. Phase Amplitude Coupling (PAC)

Part 1: Frequency Analysis

- Goal: What frequencies are present in signal?
- What is power at each frequency?
- Considerations
  - Amplitude & phase
  - Windowing

Fourier Analysis



Forward transform $F(u) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi iux} dx$

Complex valued – $ae^{i\theta}$ amplitude & phase

Inverse transform $f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} du$

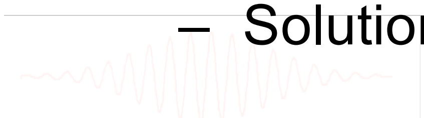
Figure, courtesy of Ravi Ramamoorthi & Wolberg

Power Spectrum. Approach 1: FFT

- Why not just take FFT of our entire signal of interest?
- Advantage – fine frequency resolution
  - ΔF = 1 / signal duration (s)
  - E.g. 100s signal has 0.01 Hz resolution
  - But, do we really need this?

- Disadvantage 1 – bias and variance
  - Solution: e.g. Welch's method

- Disadvantage 2 – no temporal resolution
  - Solution 1: Short-Time Fourier Transform

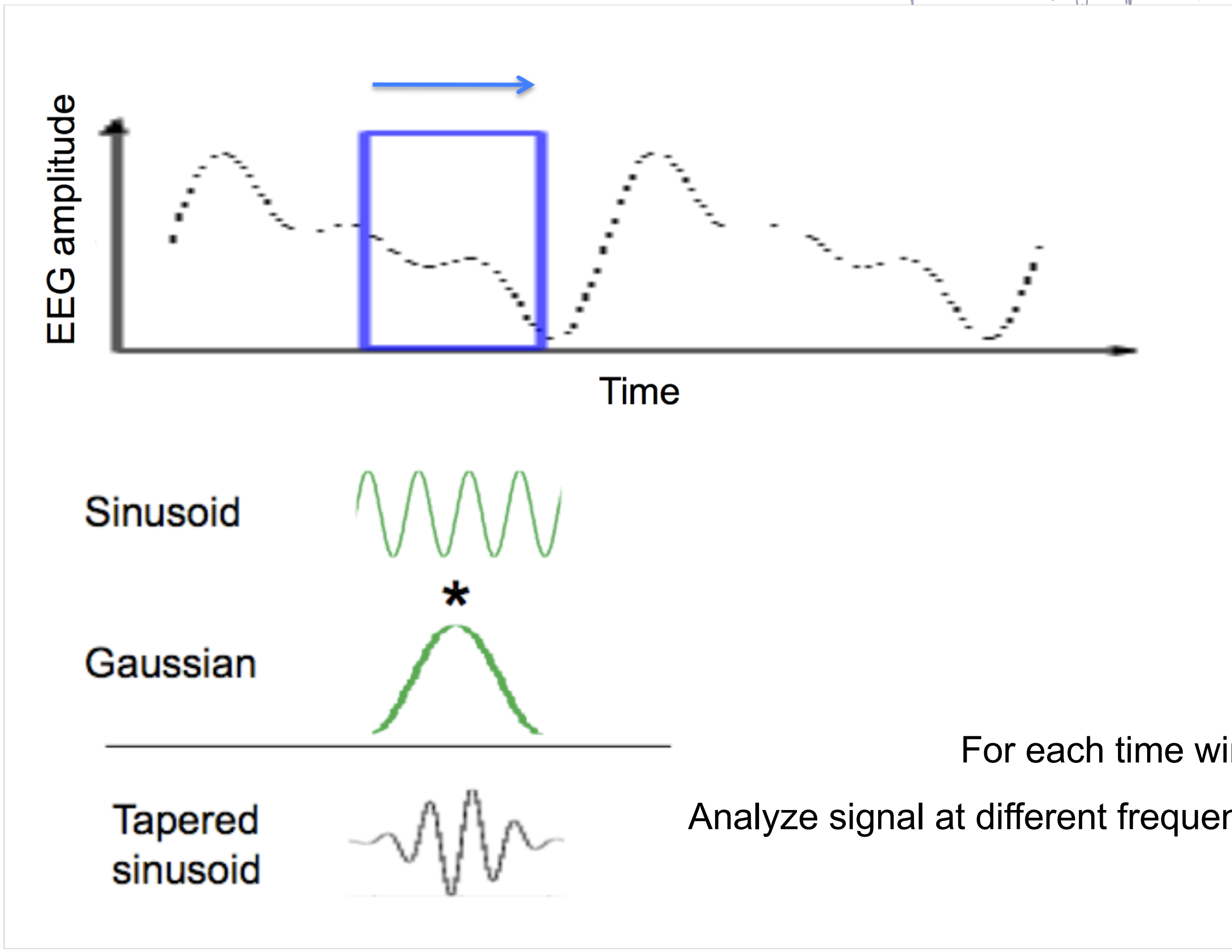


Part 2: Time-Frequency Analysis

- Short-Time Fourier Transform
  - Find power spectrum of short windows
  - Time-varying power spectrum ≡ "Spectrogram"

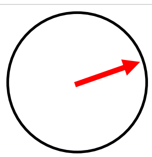
- Advantage: Analyze time-varying frequency content

- Disadvantage: Fixed temporal resolution is not optimal



For each time window

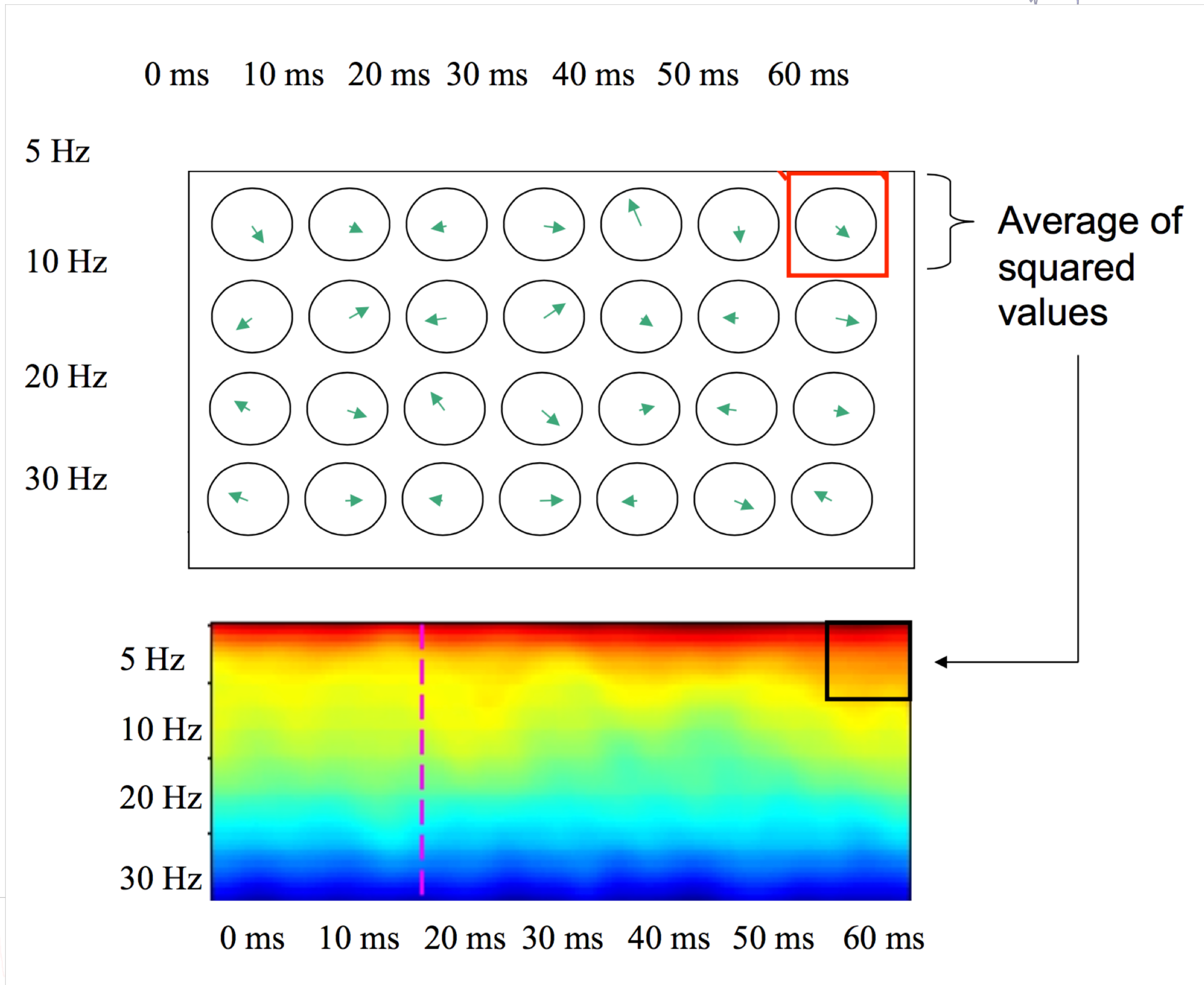
Analyze signal at different frequencies.



Complex Phasor representation:

`amplitude*exp(i*phase)`

Computing Spectrogram Power



Amplitude and phase

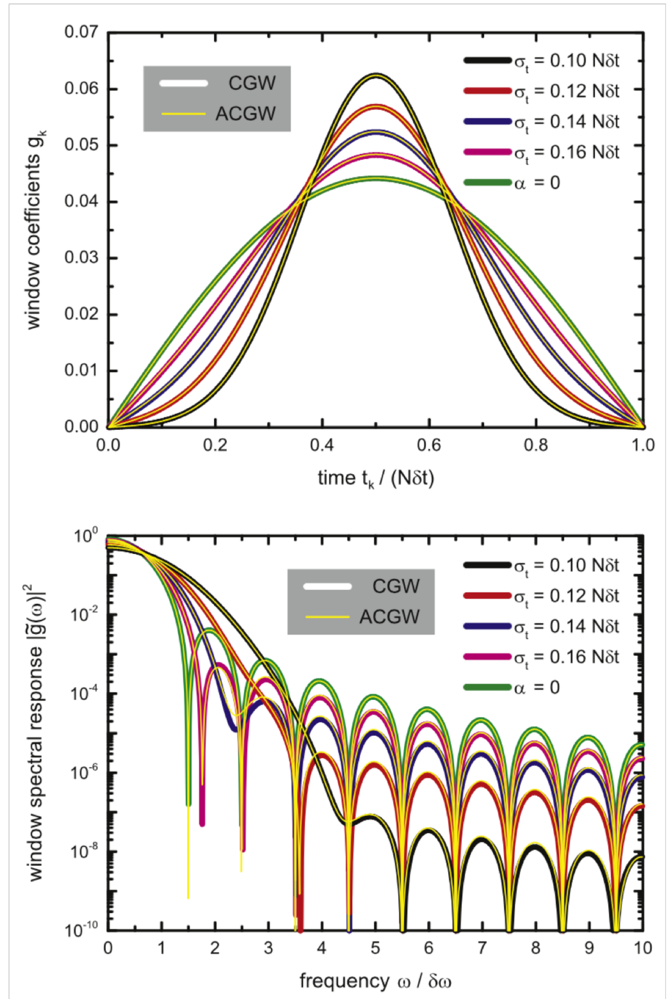
- Power spectra describe the *amount* of a given frequency present
- NOT a complete description of a signal: We also must know the *phase* at each frequency
- FFT/STFT/Wavelet return an amplitude and phase at each time and frequency (represented as complex #).
- To find power, we compute the magnitude, which discards phase.

Time-Frequency Uncertainty

- You cannot have both arbitrarily good temporal and frequency resolution!
  - $\sigma_t * \sigma_f \geq 1/2$

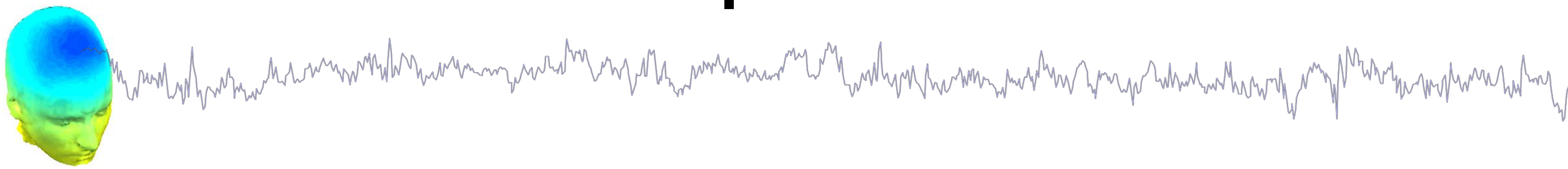
- If you want sharper temporal resolution, you will sacrifice frequency resolution, and vice versa.

- (Optimal: Confined Gaussian)



Starosielec S, Hägele D (2014) Discrete-time windows with minimal RMS bandwidth for given RMS temporal width. Signal Processing 102:240–6.

Consequence for STFT

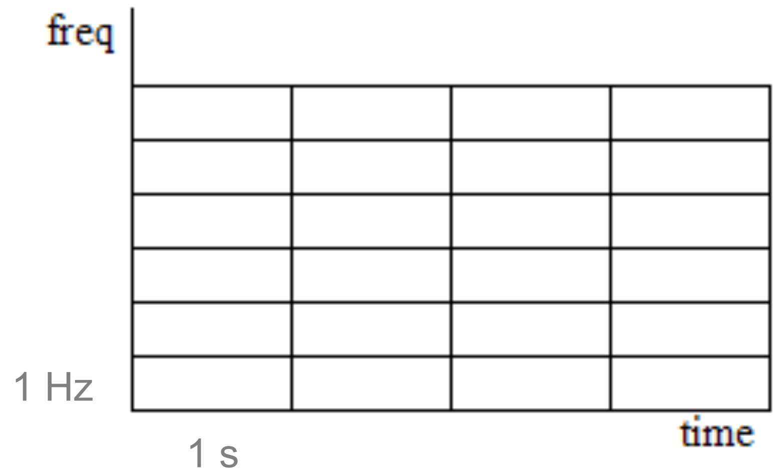


Shorter Windows

poorer frequency resolution

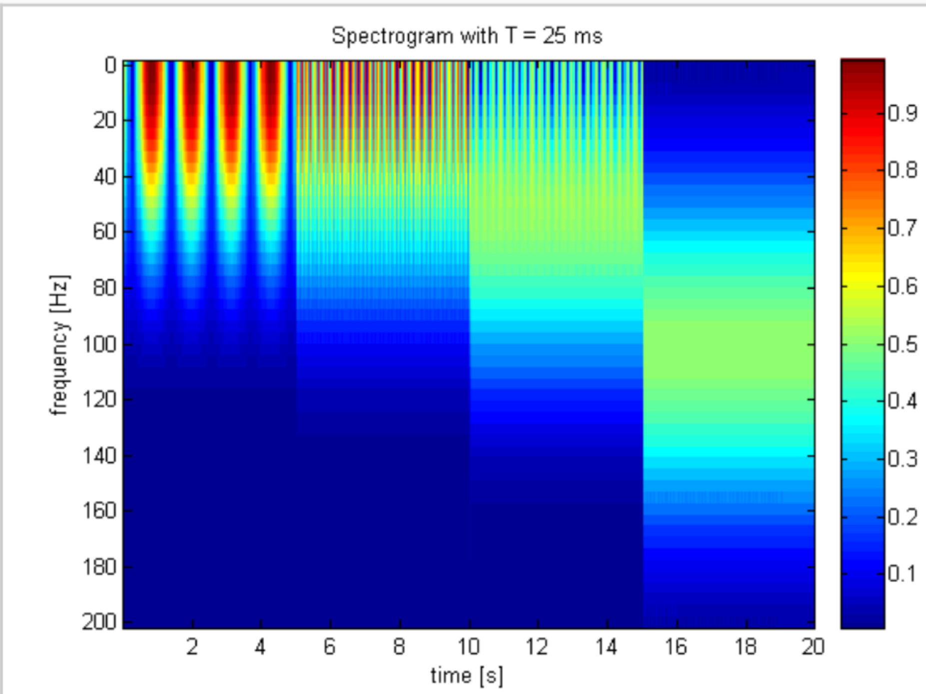
Longer Windows

finer frequency resolution

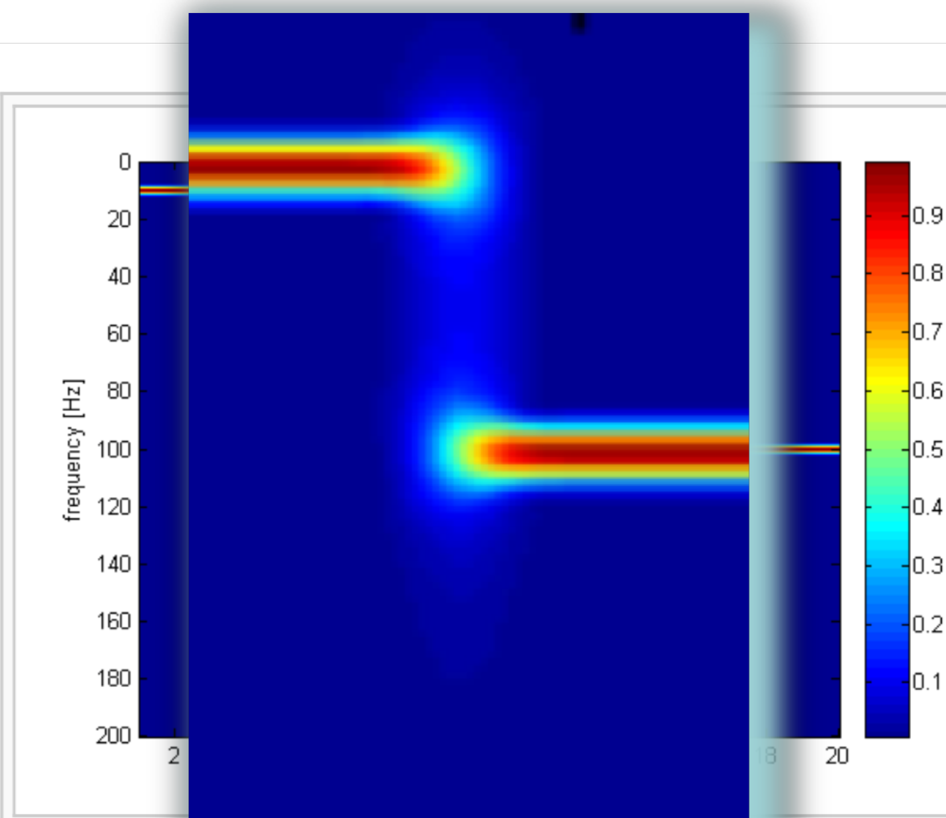


Time-Frequency Tradeoff

Signal: 10, 25, 50, 100 Hz

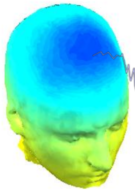


25 ms window



1000 ms window

One better way: Wavelet transform

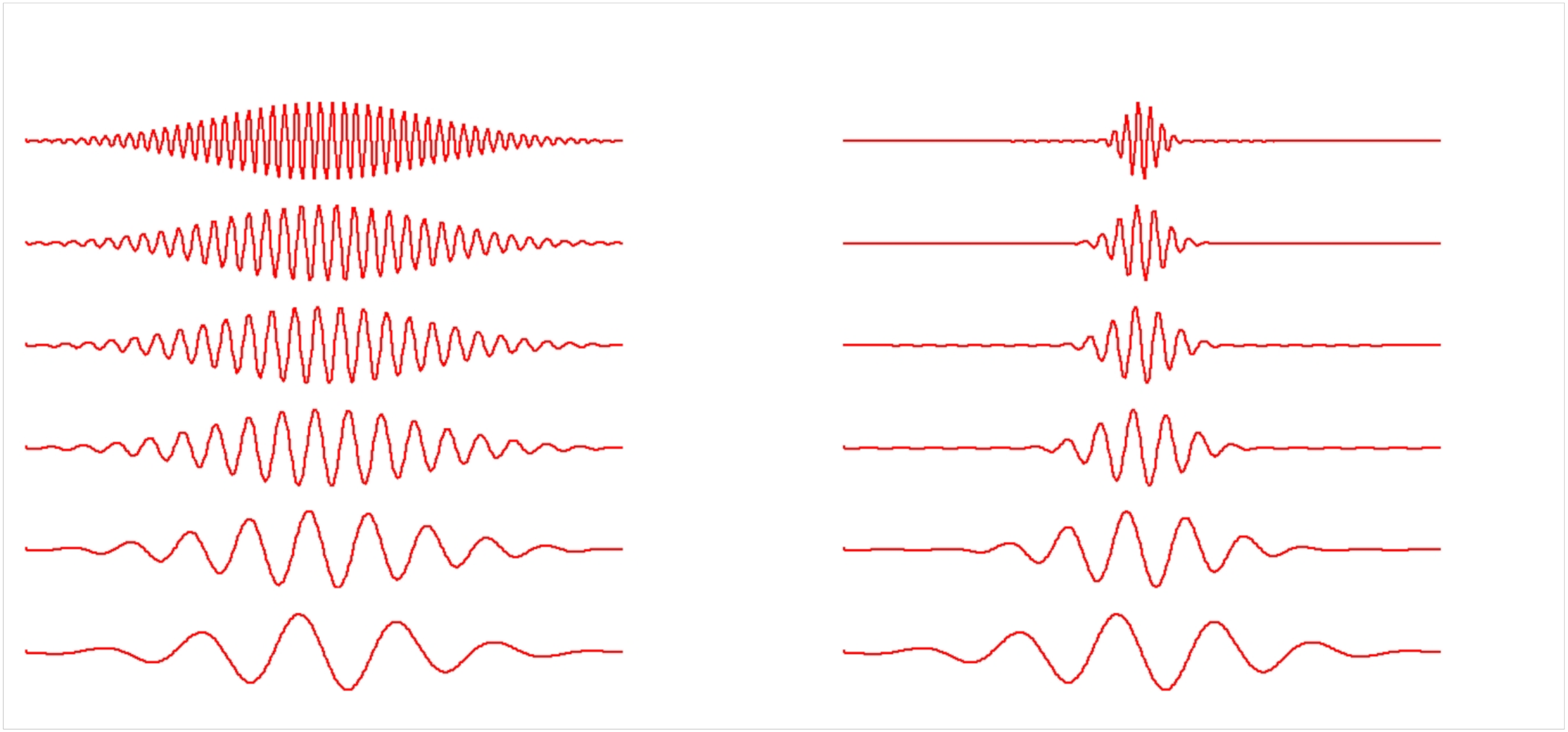


- Wavelet transform is a ‘multi-resolution’ time-frequency decomposition.

- Intuition: Higher frequency signals have a shorter time scale

- So, vary window length with frequency!
  - longer window at lower frequencies
  - shorter window at higher frequencies

Comparison of FFT & Wavelet bases

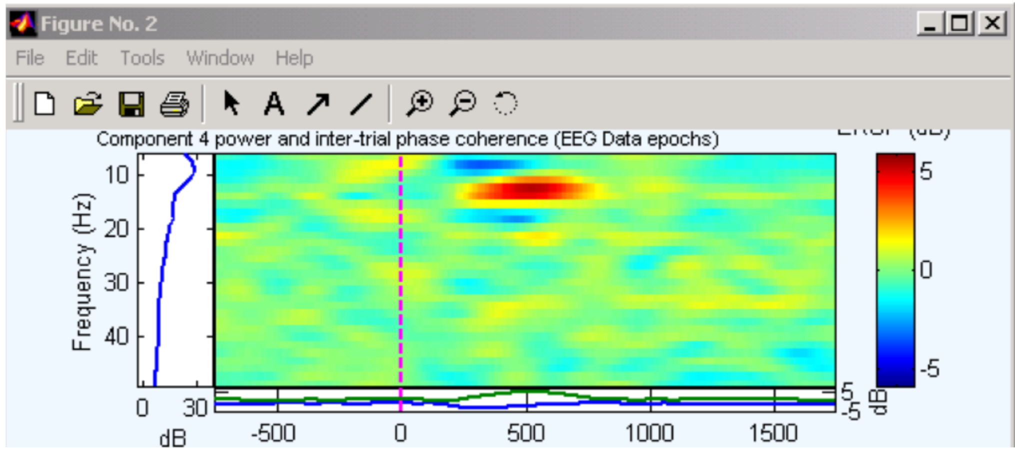


Scaled versions of one shape

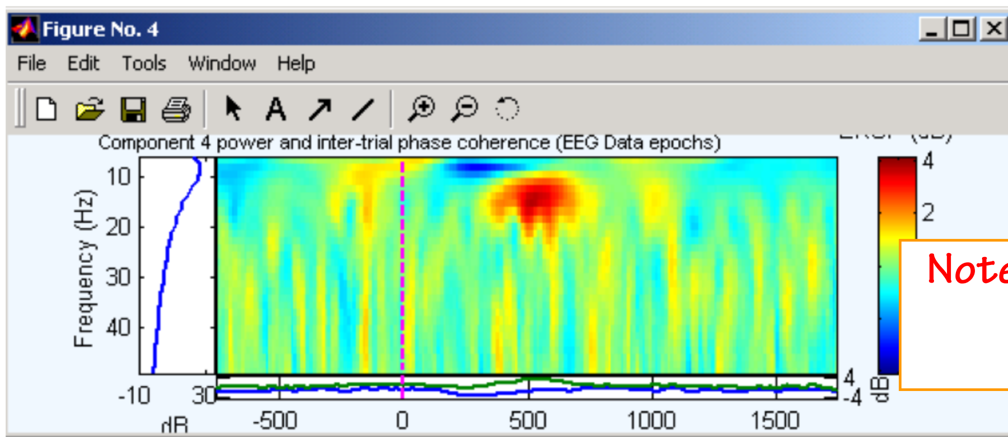
Constant* number of cycles

Comparison of FFT & Wavelet Spectrograms

FFT

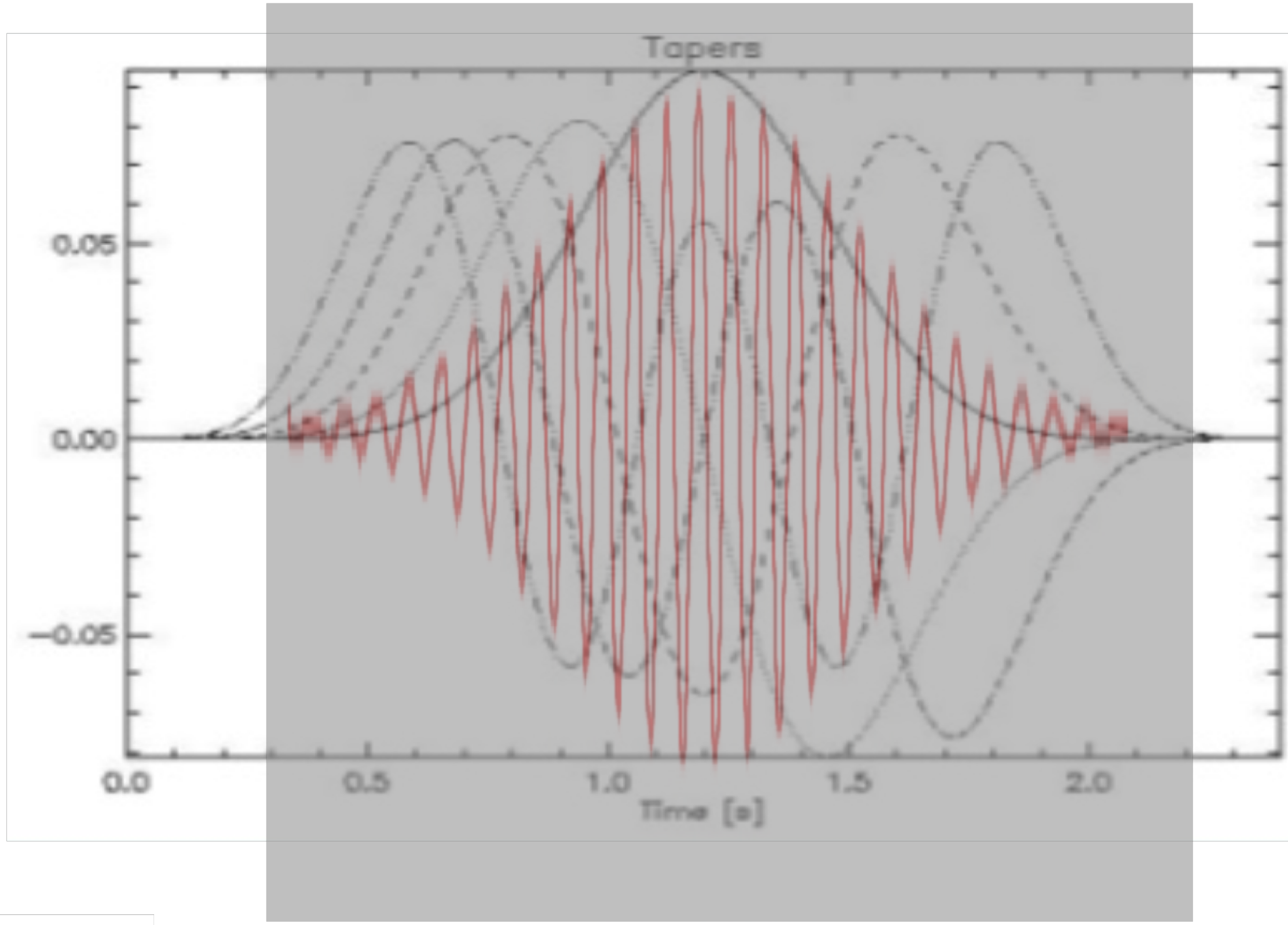


Wavelet



Note: Finer temporal resolution at higher frequencies

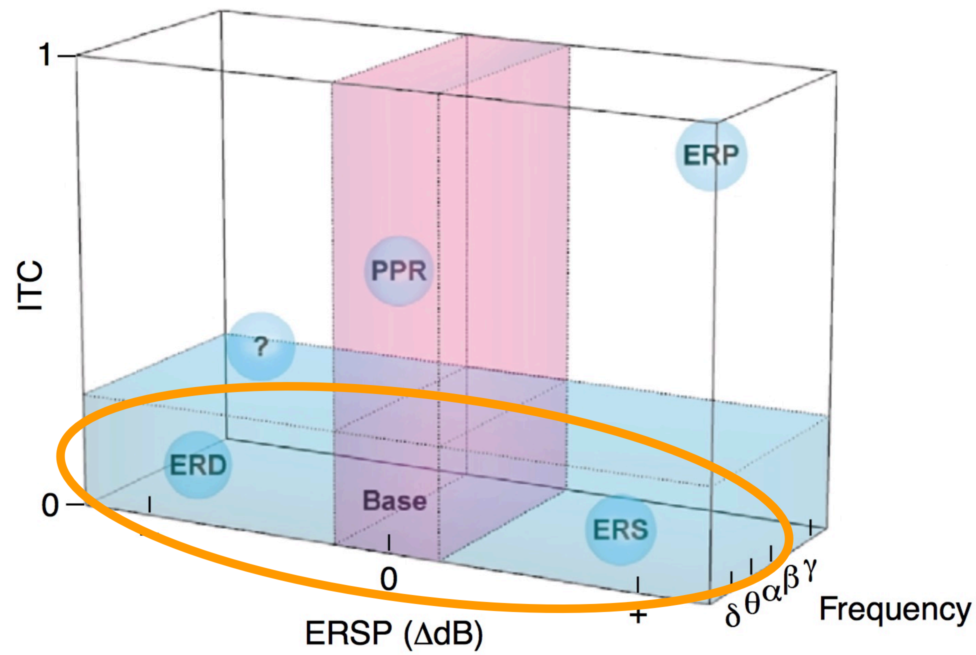
Multitaper Spectral Estimates



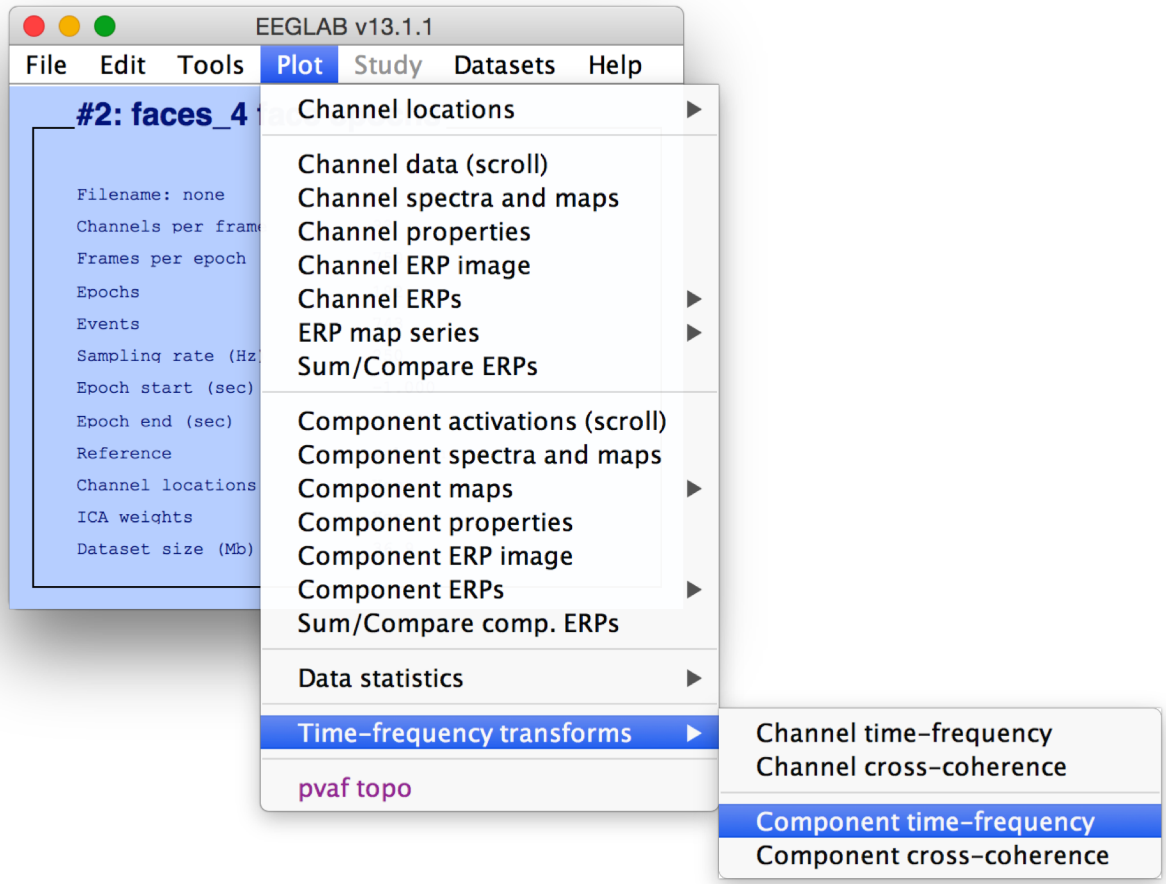
Definition: ERSP

- Event Related Spectral Perturbation

- Change in power in different frequency bands *relative to a baseline*. ERS , ERD

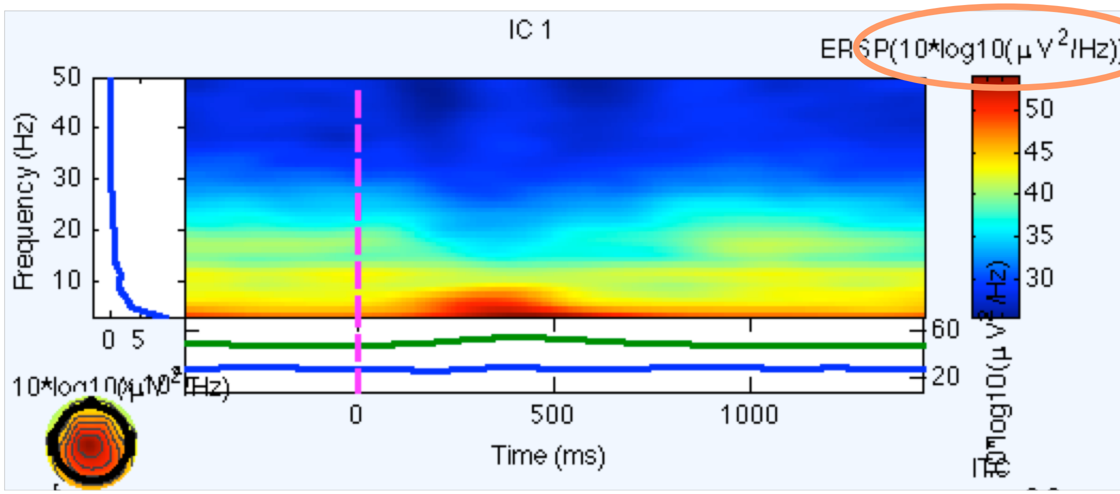


Try it out (faces_4.set)



ERS and ERS*P*

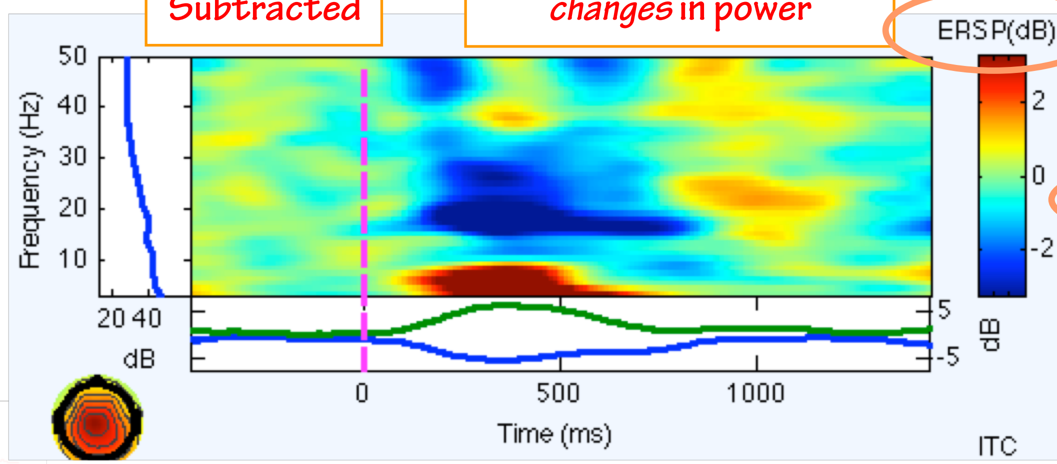
Event-related Spectrogram (ERS)



Baseline is Subtracted

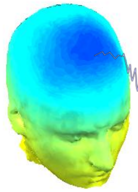
Revealing event-related *changes* in power

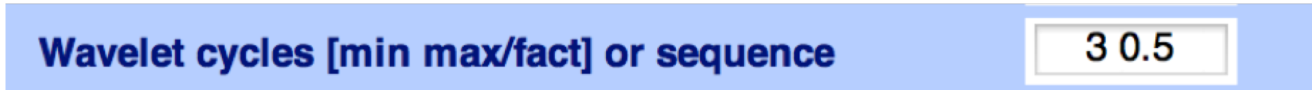
Event-Related Spectral *Perturbation* (ERSP)



$10*\log_{10}(SG(t,f) / baseline(f))$

Wavelet Specification





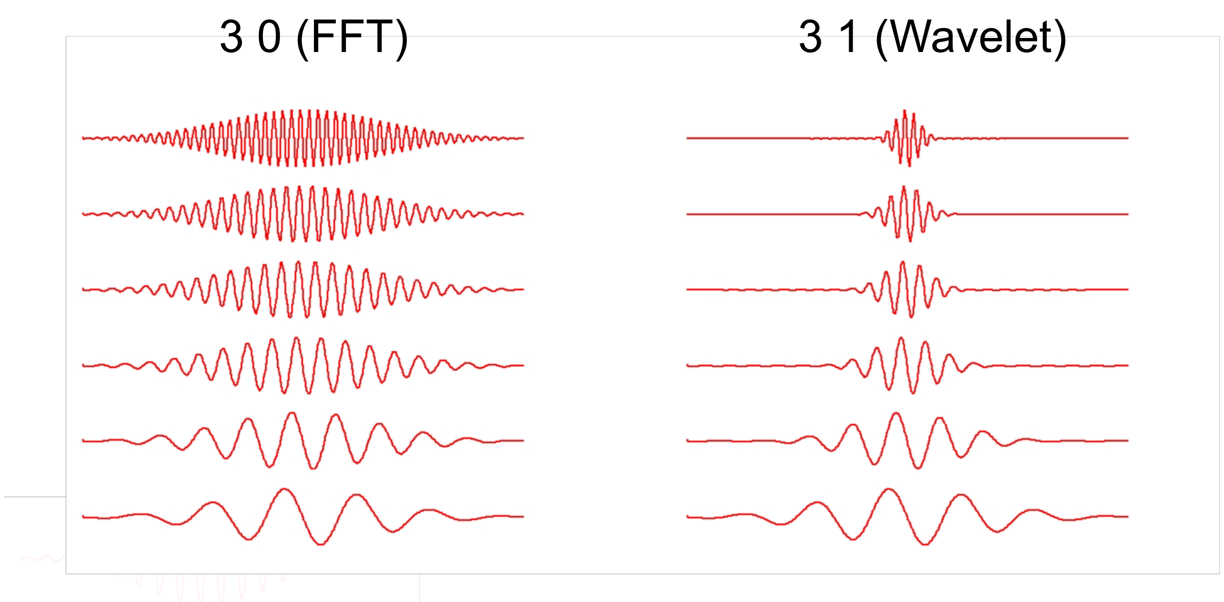
Answer: The first value (#cycles) controls the basic *duration of the wavelet in cycles*.

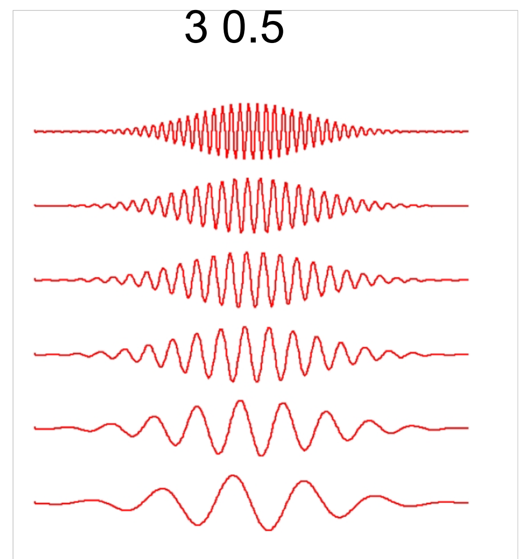
The second value controls the *degree of shortening of time windows as frequency increases*

0 = no shortening = FFT (duration remains constant with frequency)

1 = pure wavelet (#cycles remains constant with frequency)

0.5 = intermediate, a compromise that reduces HF time resolution to gain more frequency resolution





Part 3: Coherence Analysis

- Goal: How much do two signals resemble each other

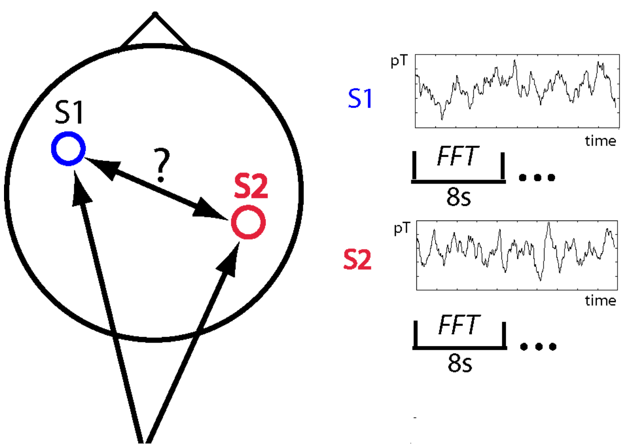
- Coherence = complex version of correlation: how similar are power and phase at each frequency?

- Variant: phase coherence (phase locking, etc.) considers only phase similarity, ignoring power
  - Regular coherence is simply a power-weighted phase coherence

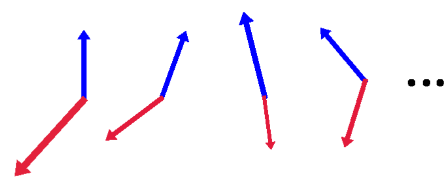
Coherence

$$C(f,t) \propto \sum_{k=trials} F1_k(f,t)\overline{F2_k(f,t)}$$

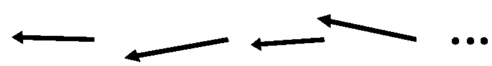
$$a_1 e^{i\theta_1} a_2 e^{-i\theta_2} \propto e^{i(\theta_1 - \theta_2)}$$

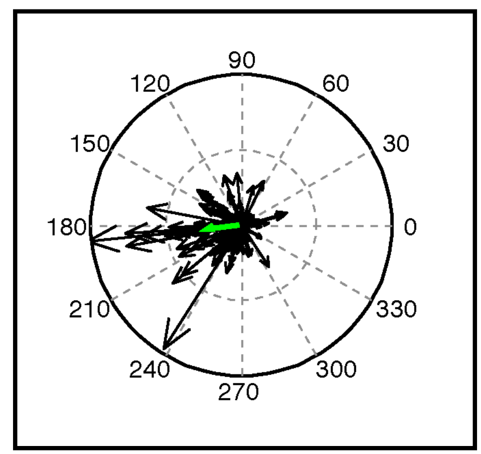


Fourier time series F_{S1} and F_{S2}



Phase difference between S1 and S2,





Part 3a: Inter-Trial Coherence

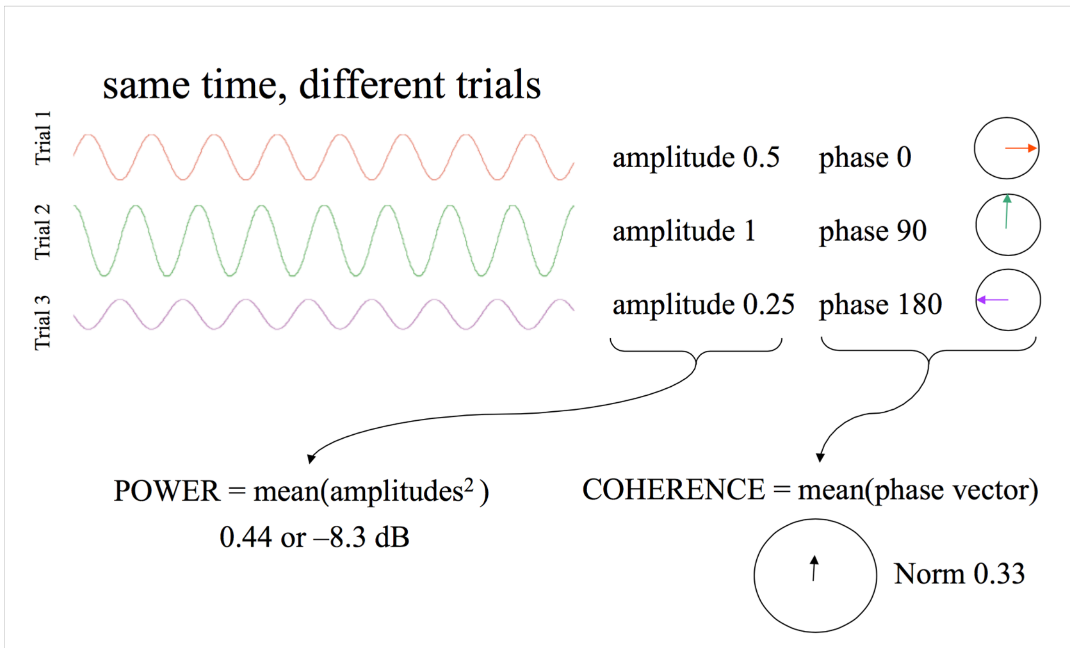
- Goal: How much do different trials resemble each other?
- Phase coherence not between two processes, but between multiple trials of the same process
- Defined over a (generally) narrow frequency range

EEGLAB's Inter-Trial Coherence is *phase* ITC

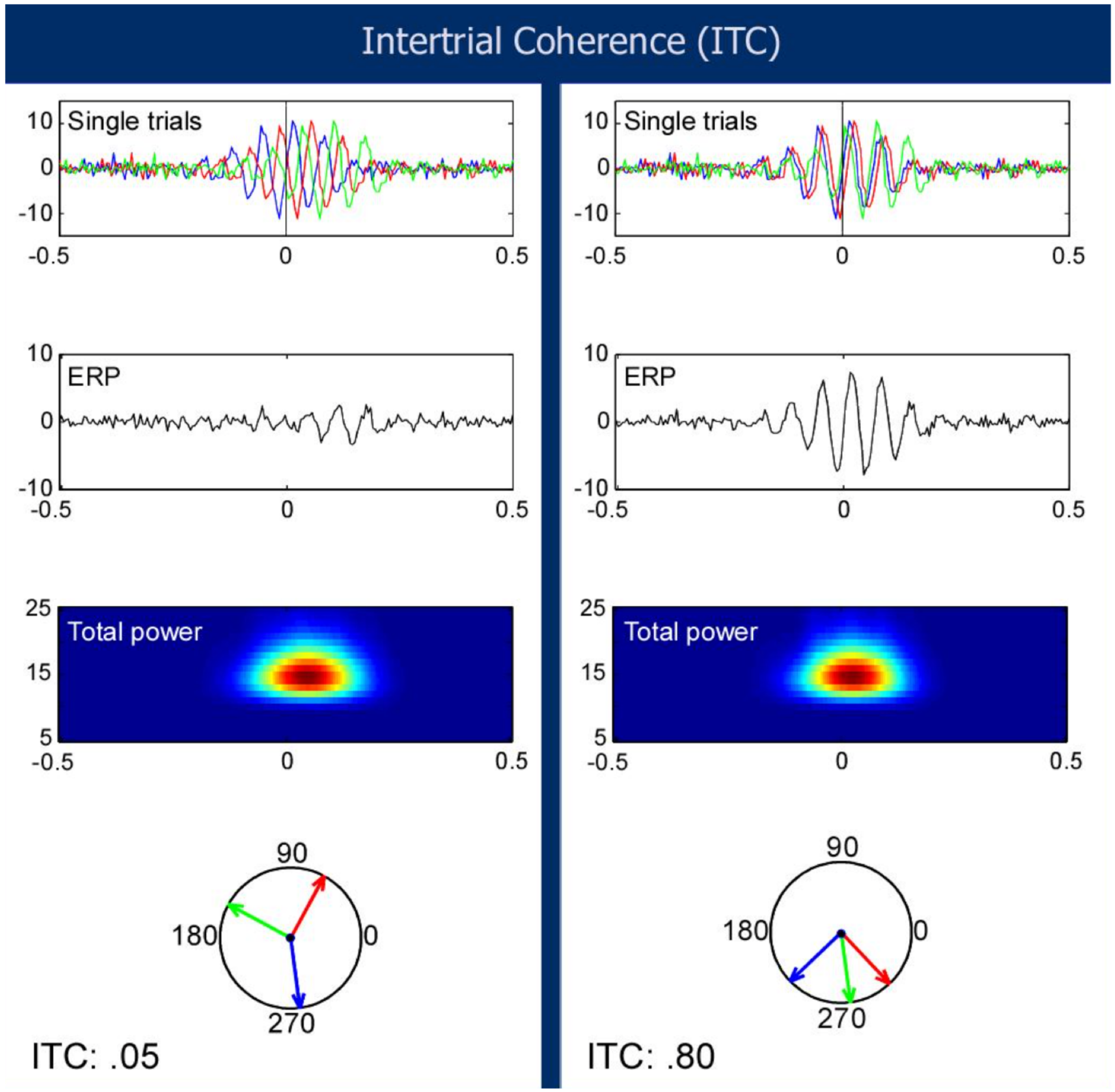
Phase ITC

$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k(f,t)}{\left|F_k(f,t)\right|}$$

Normalized
(no amplitude information)



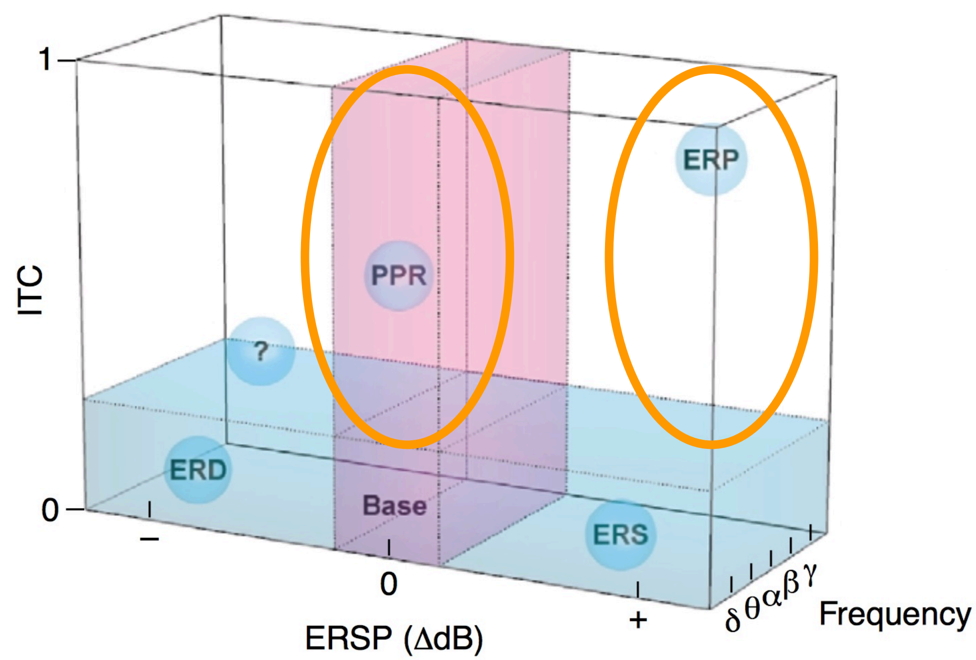
ITC Example (3 trials)



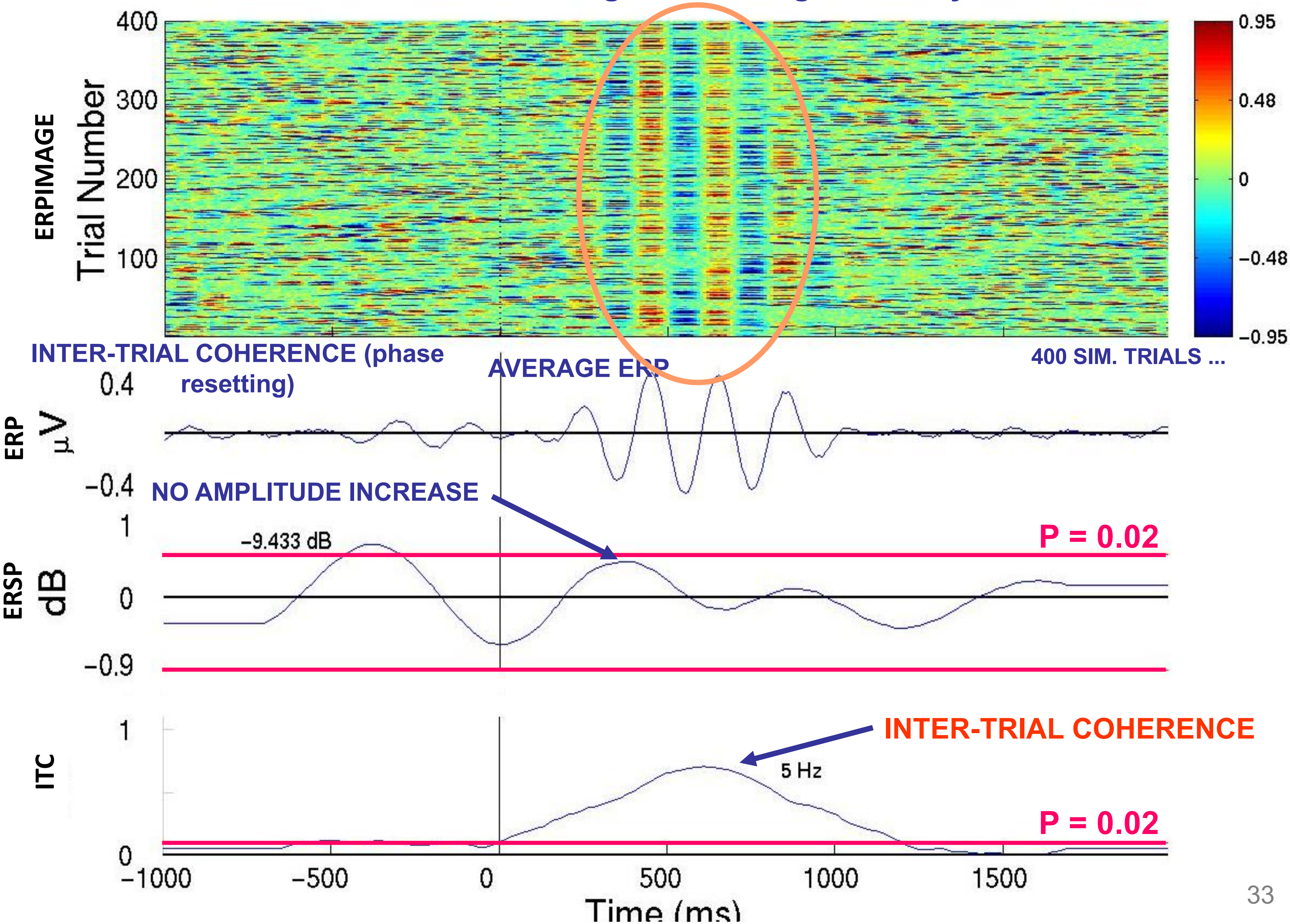
Slide courtesy of Stefan Debener

Several possible origins of an ERP

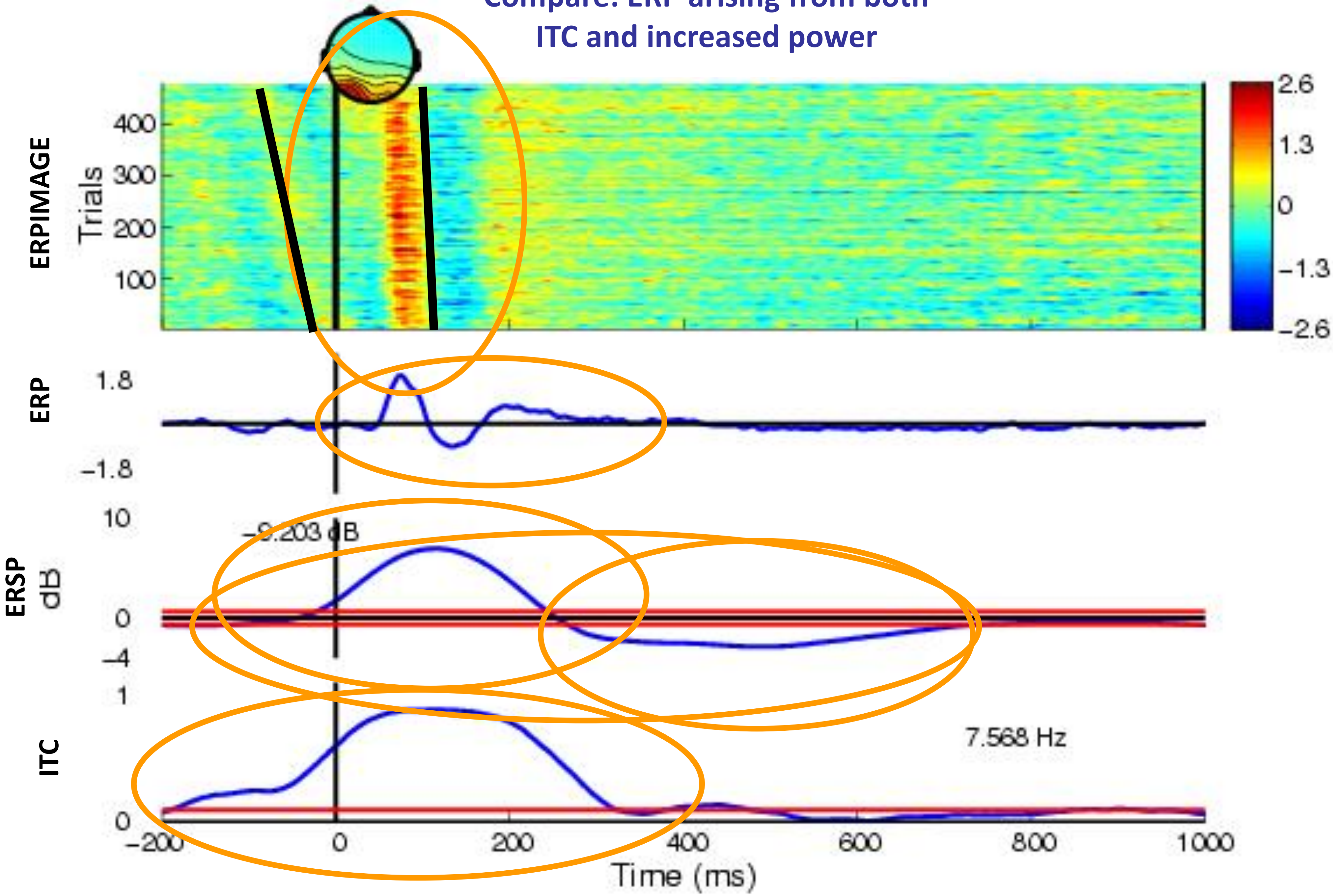
- Event Related Potential can result from
  - ITC increase (with no change in power)
  - ITC & Power change



ERPIMAGE showing ERP arising from only ITC



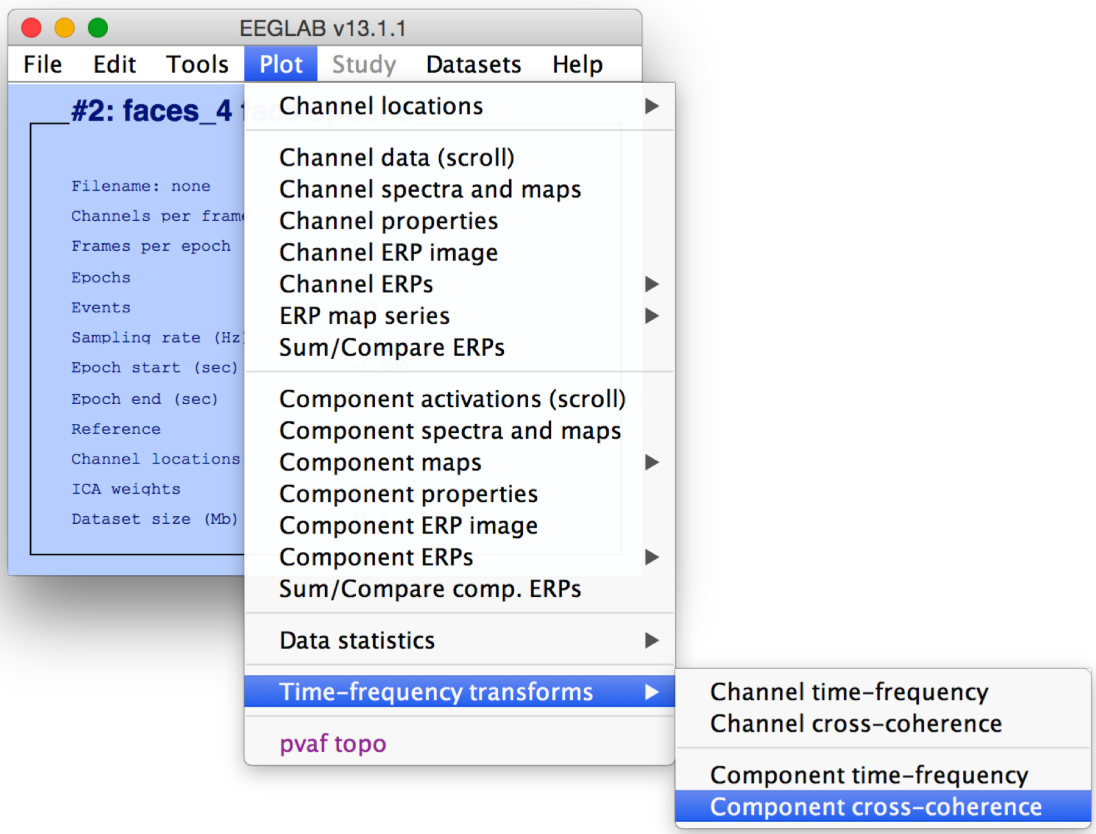
Compare: ERP arising from both ITC and increased power



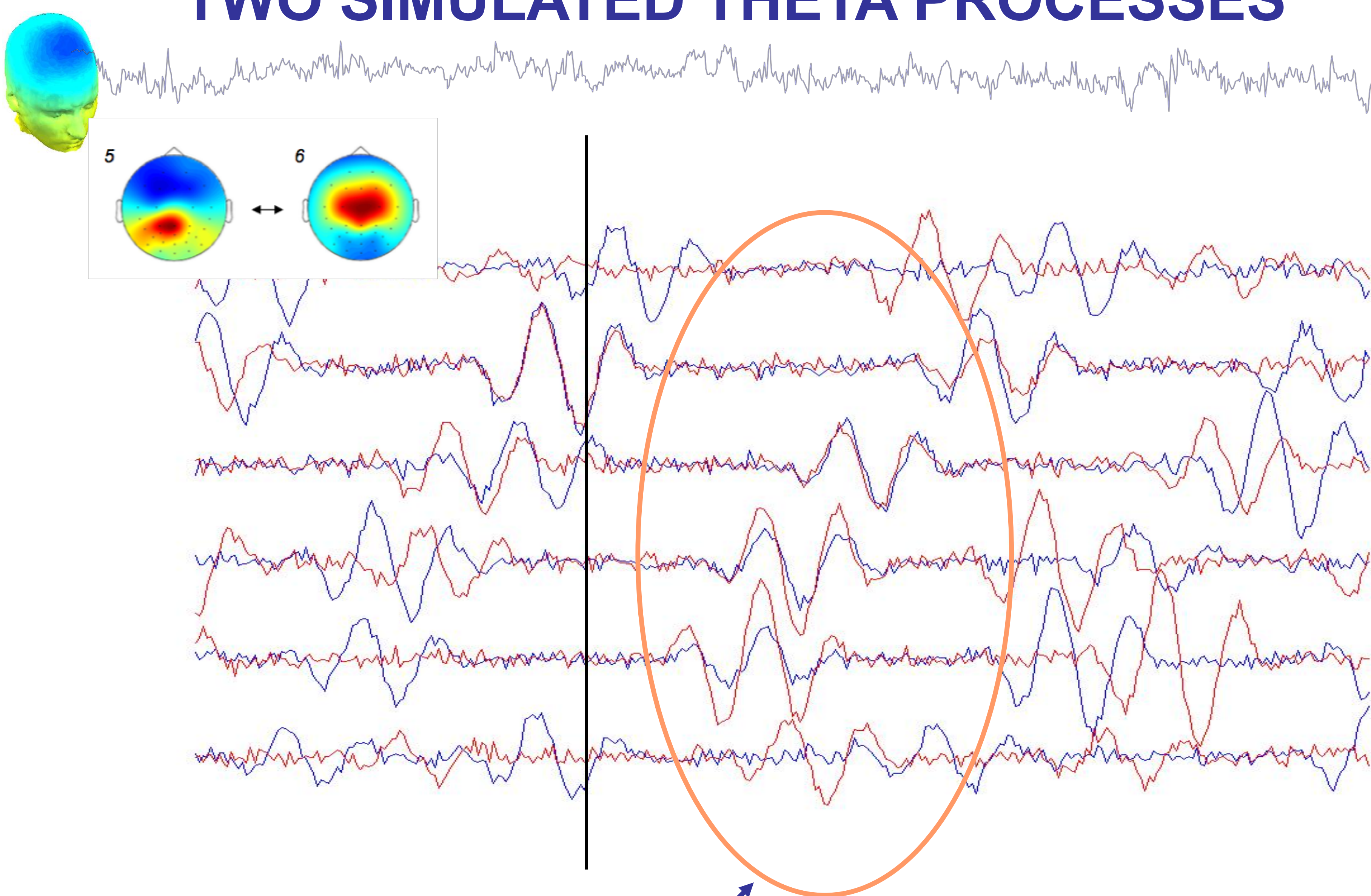
J. Onton & S. Makeig, 2005

Part 3b: Event Related Coherence

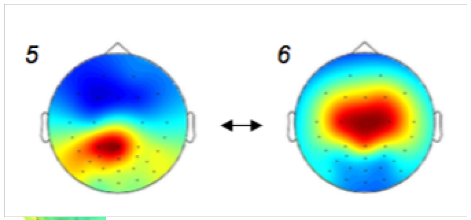
- Goal: How similar is the event-related response of two signals?
  - Traditionally between-channels (problematic due to volume conduction)
  - or *between-ICs*



TWO SIMULATED THETA PROCESSES



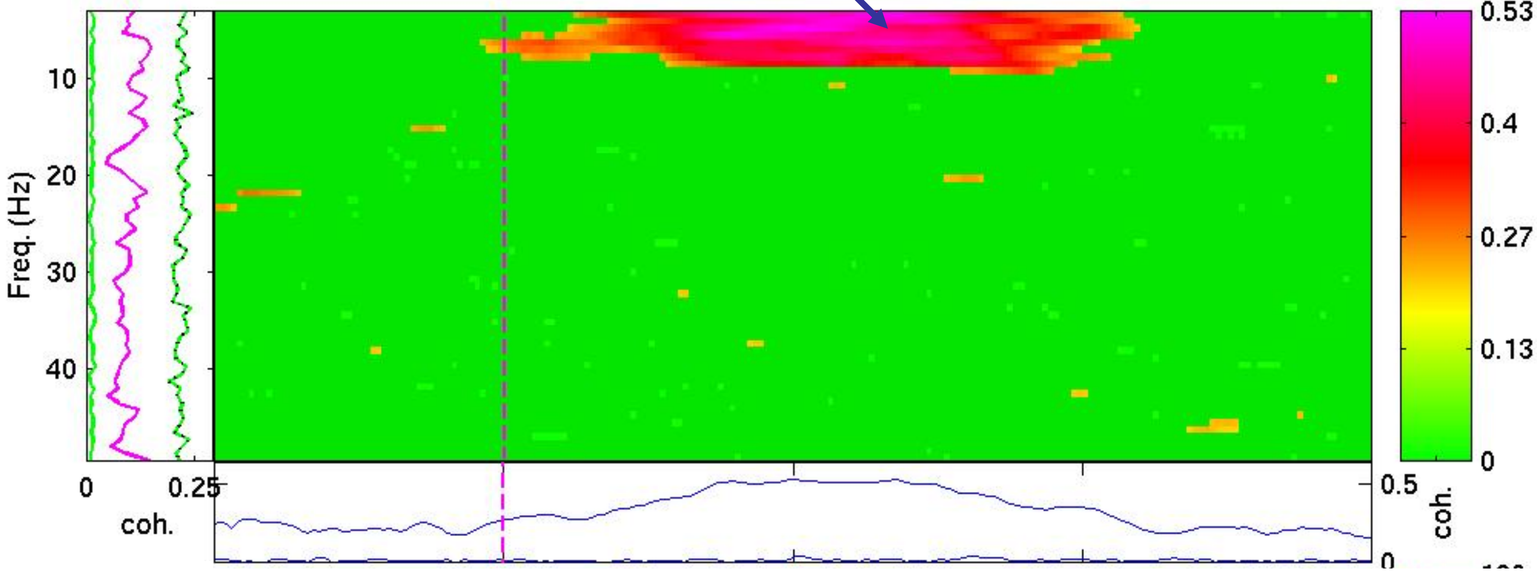
Event-related Coherence



EVENT-RELATED COHERENCE

ERCOH

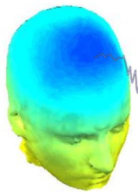
FREQUENCY



r

TIME

Part 4: Other Applications

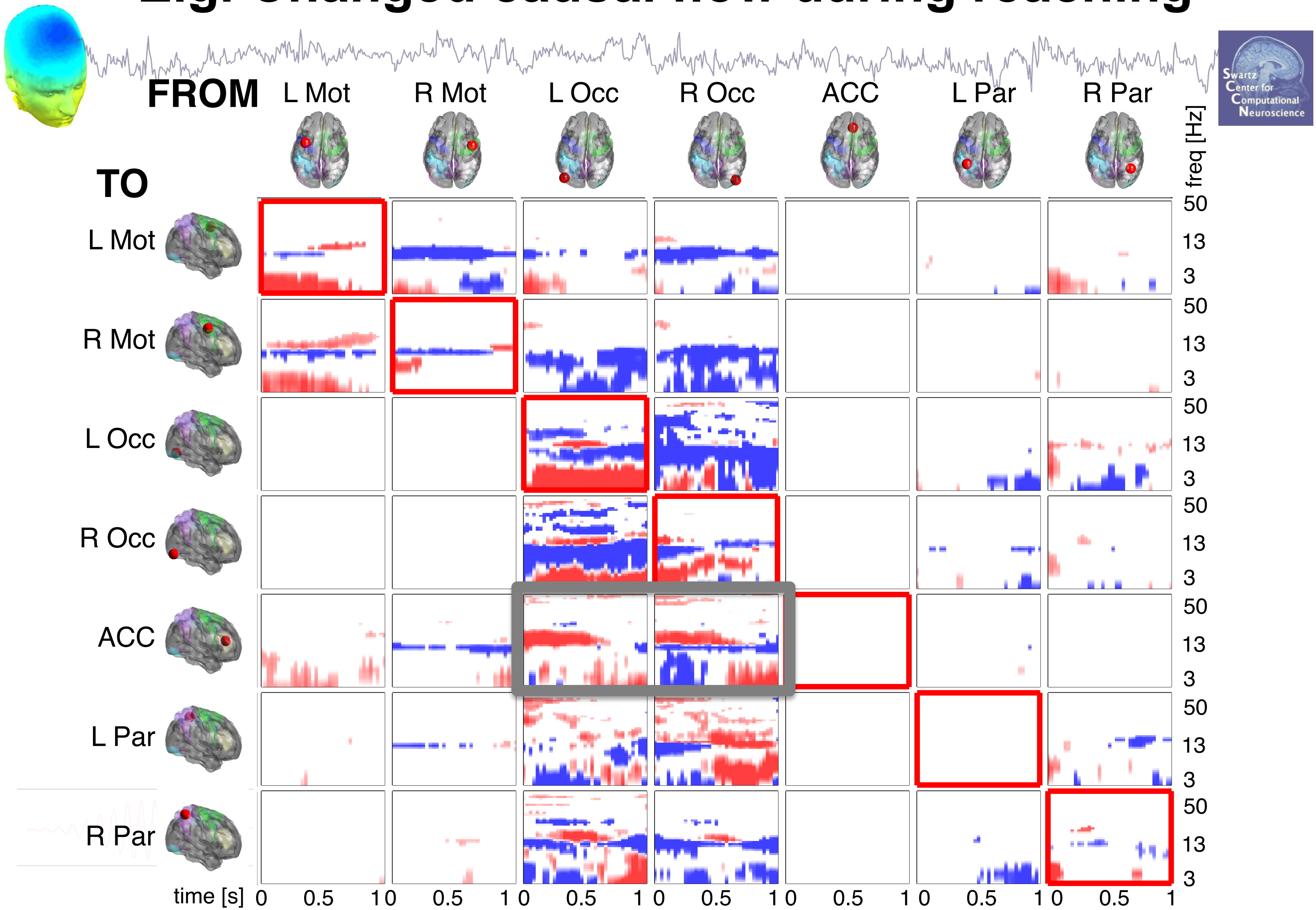


- Information Flow: Autoregressive modeling → time/frequency resolved directed information flow
  - SIFT: Day 3 & 4, Track C

- Cross-frequency Analysis

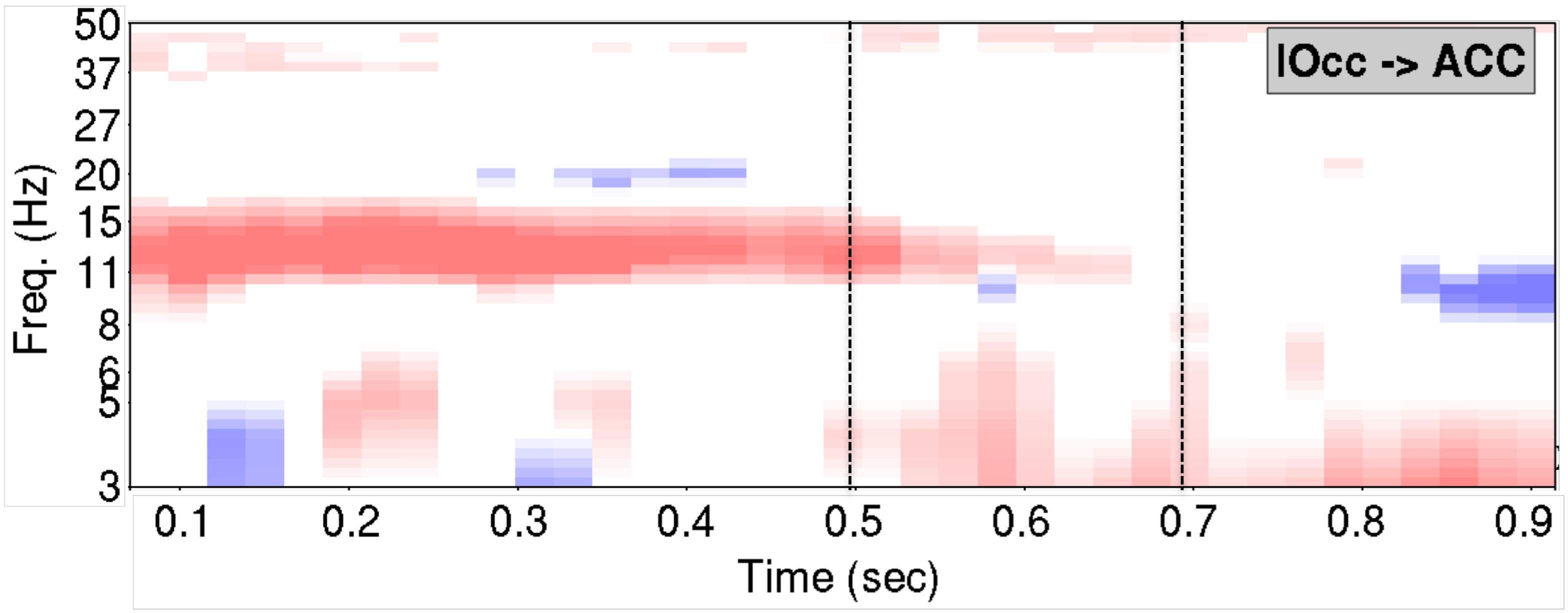
  - Day 2, Track D / Day 4, Track B
    - 3:50 **Phase/amplitude coupling (PAC)** - Ramón Martinez-Cancino

E.g. Changed causal flow during reaching



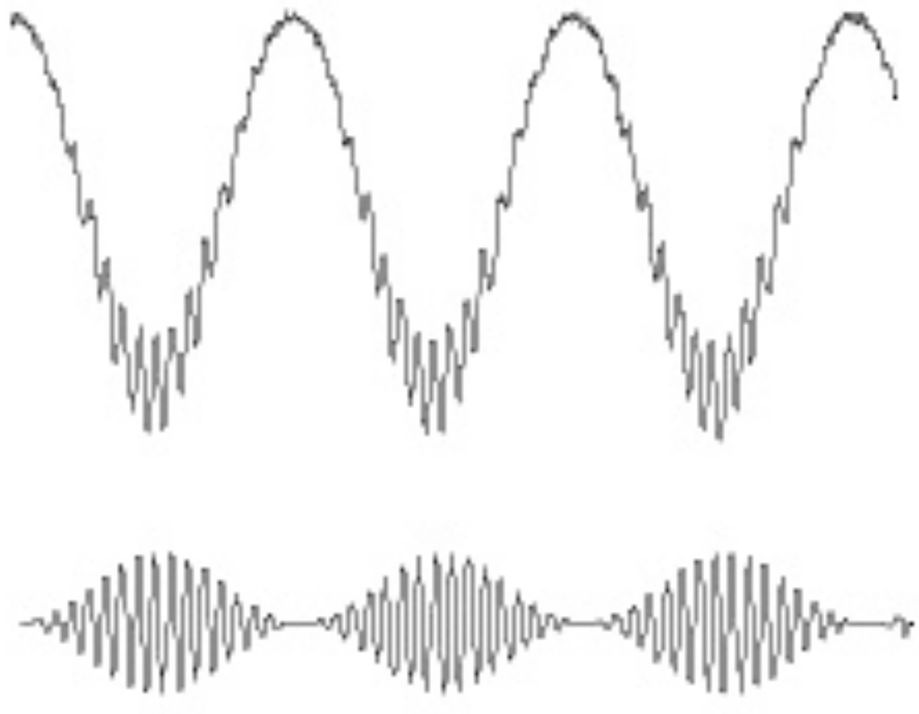
Occipital → ACC

Planning | Execution



Iversen, et al, 2016; Courellis, et al, 2018

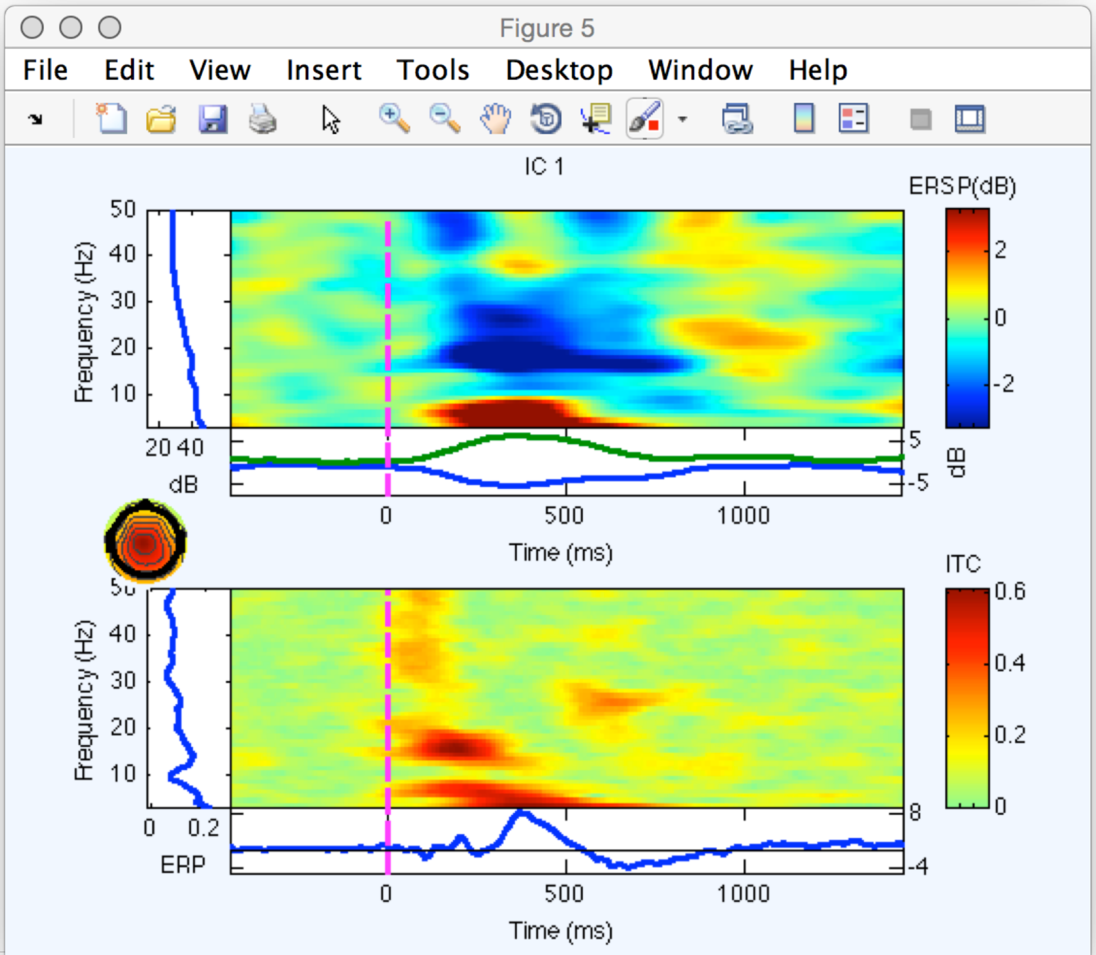
Phase-amplitude coupling



Day 2, Track D / Day 4, Track B

3:50 **Phase/amplitude coupling (PAC)** - Ramón Martinez-Cancino

Putting it all together



Exercise

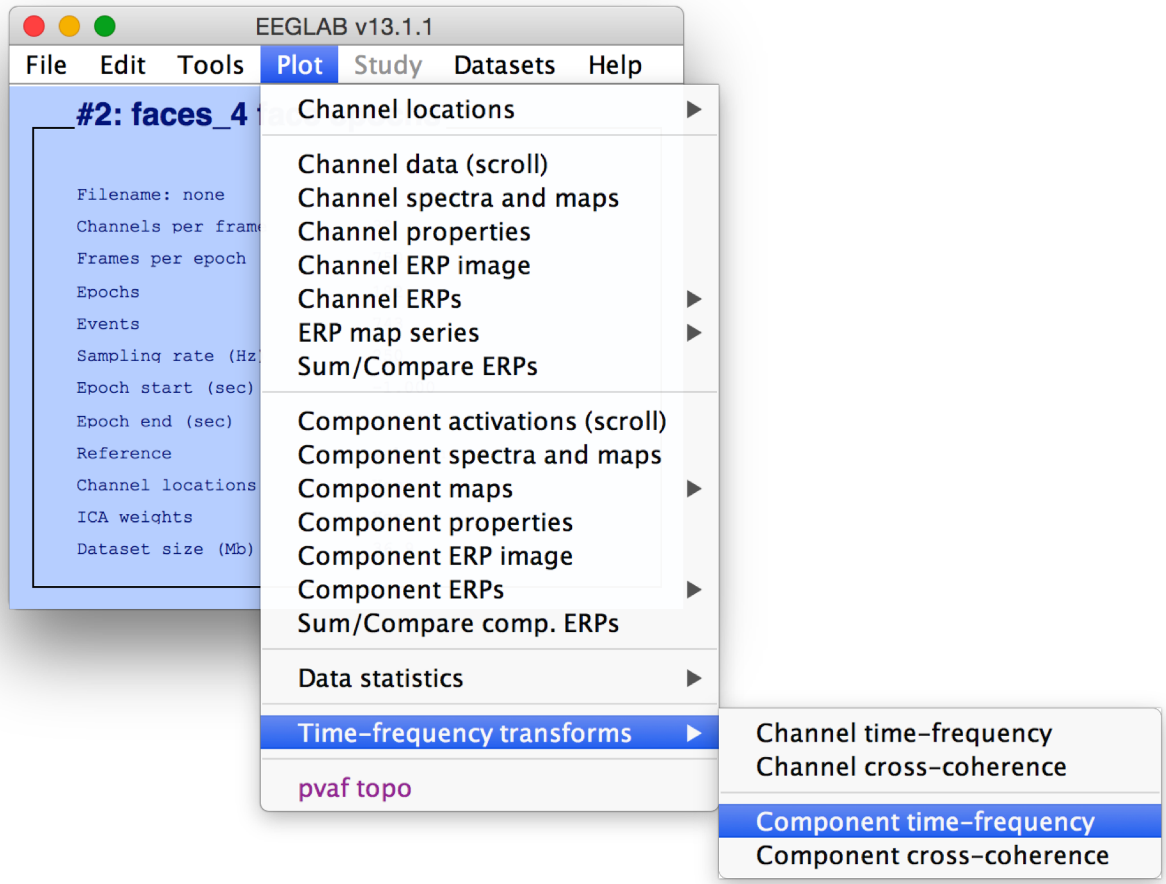
All: Compute ERSP/ITC for a component of your choice

Compute ERP Image (with ERSP and ITC displayed*)

Use all of this information to explain the origin of the Evoked Response

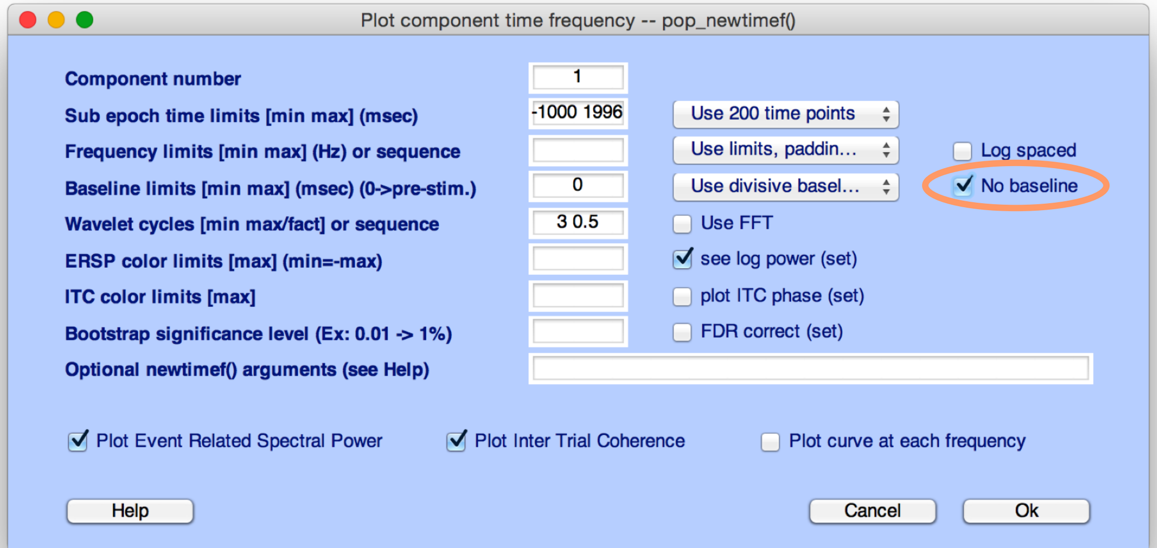
Question: Which changes are significant? Use the options in ERP Image and ERSP dialogs to set significance threshold e.g. 0.01. Do the results survive?

Try it out (faces_4.set)

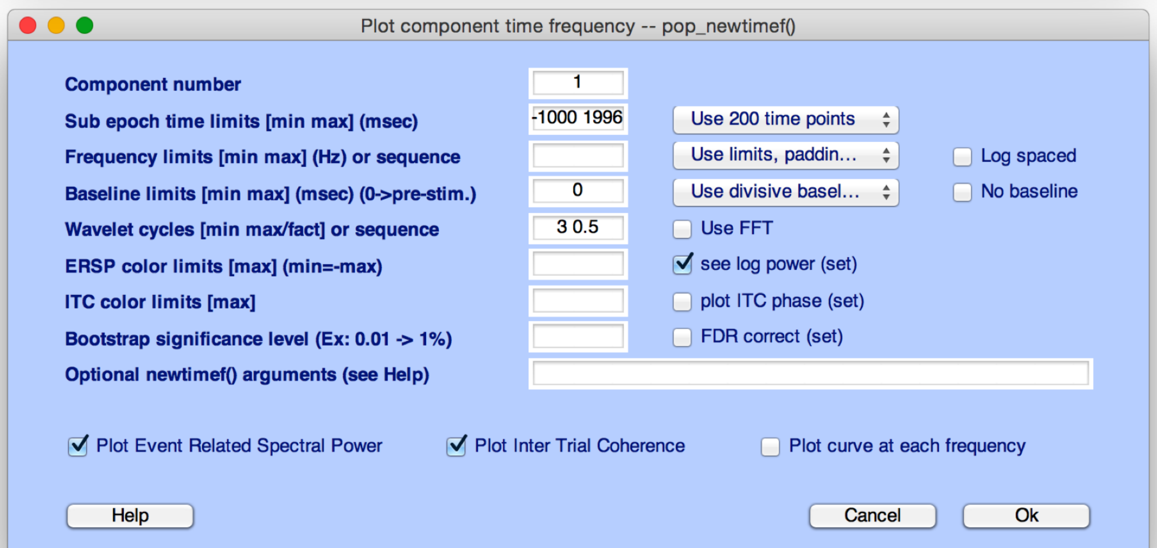


Display ERS vs. ERSP (faces_4.set)

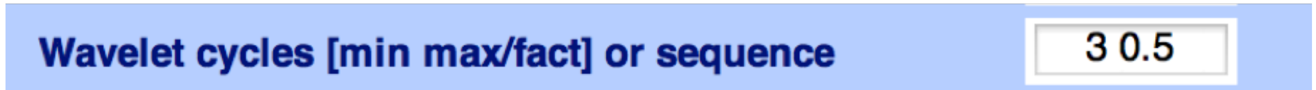
Event-related Spectrogram (ERS)



Event-Related Spectral Perturbation (ERSP)



Try different wavelet parameters



Answer: The first value (#cycles) controls the basic *duration of the wavelet in cycles*.

The second value controls the *degree of shortening of time windows as frequency increases*

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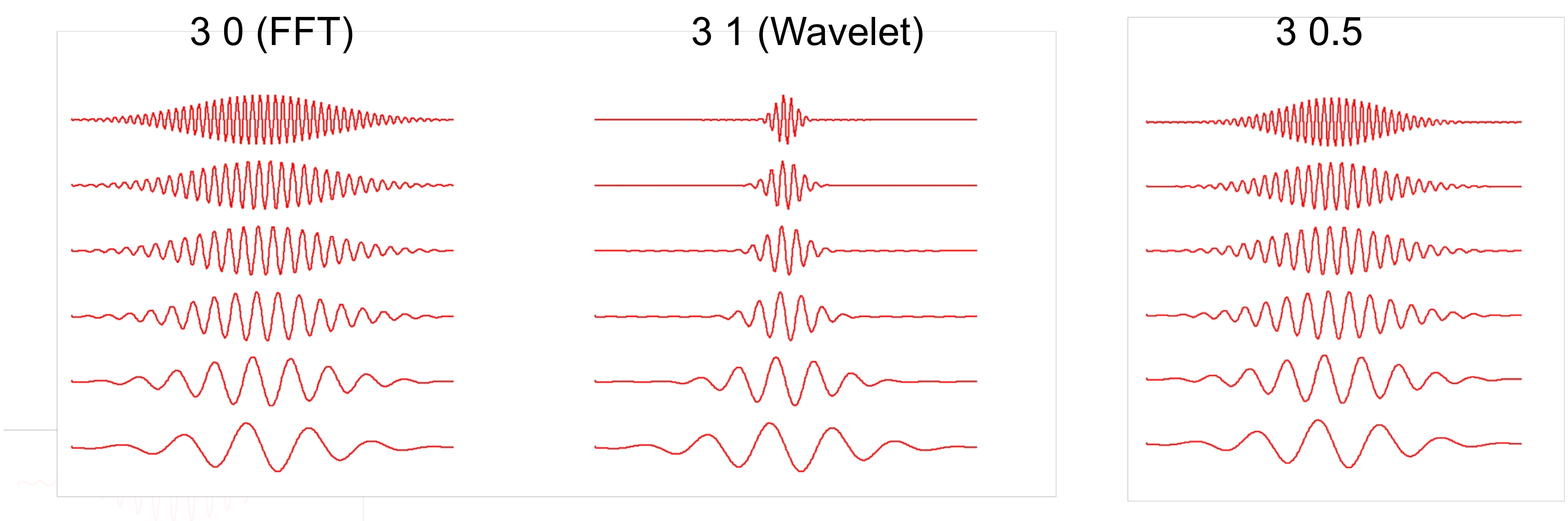
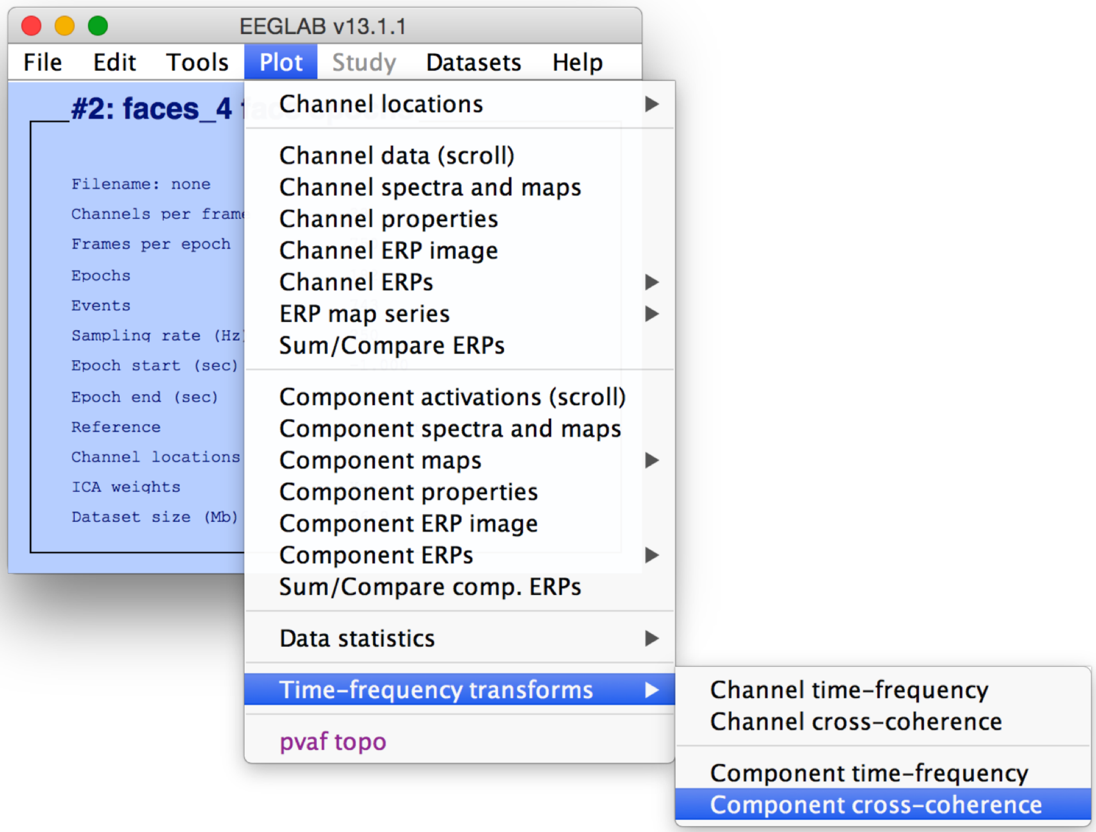
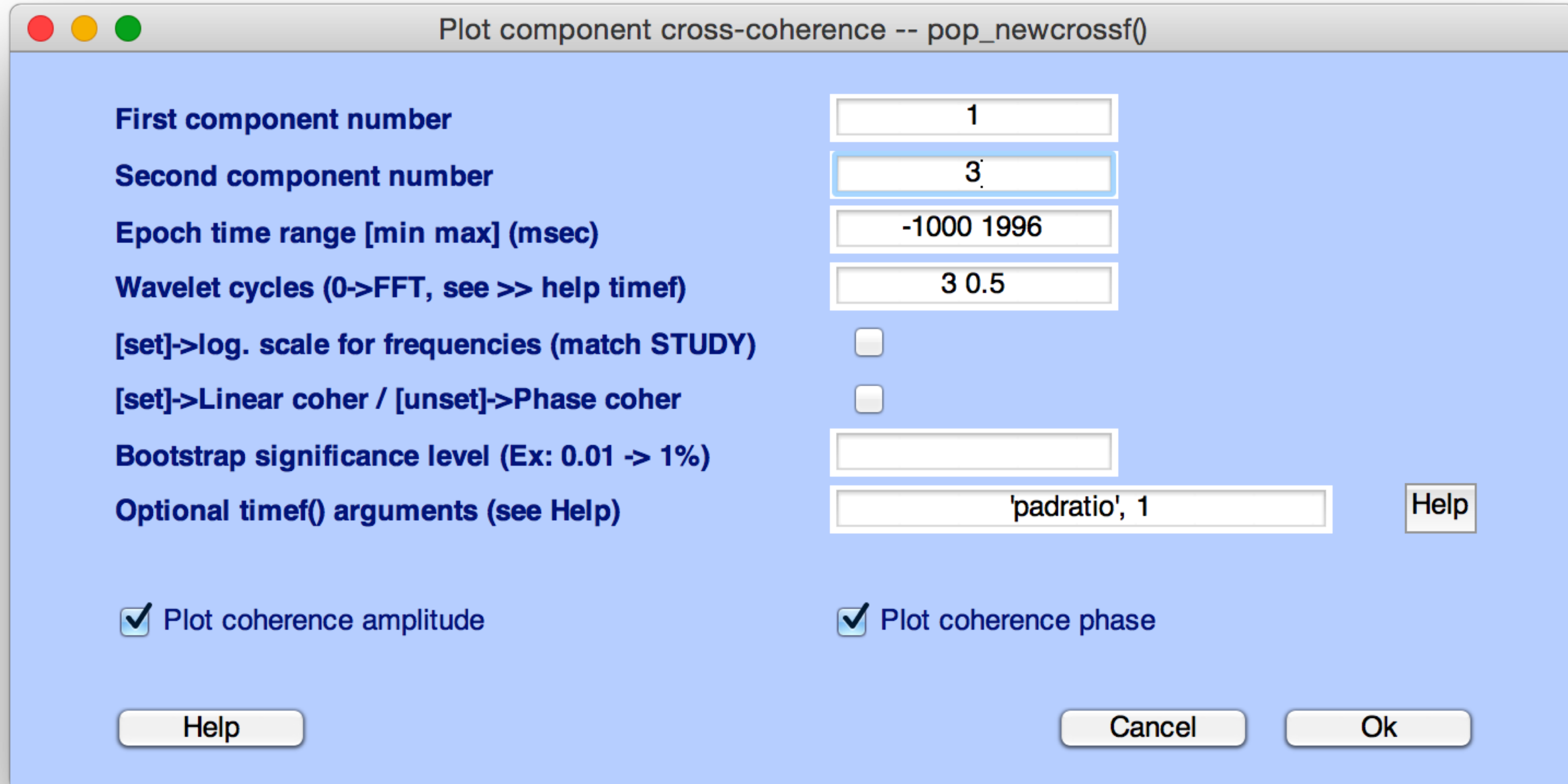


Part 3b: Event Related Coherence

- Goal: How similar is the event-related response of two signals
  - Typically between channels (problematic due to volume conduction)
  - or between ICs



Try it!


Plot component cross-coherence -- pop_newcrossf()
First component number
1
Second component number
3
Epoch time range [min max] (msec)
-1000 1996
Wavelet cycles (0->FFT, see >> help timef)
3 0.5
[set]->log. scale for frequencies (match STUDY)
[set]->Linear coher / [unset]->Phase coher
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional timef() arguments (see Help)
'padratio', 1
Help
Plot coherence amplitude
Plot coherence phase
Help
Cancel
Ok

Event-Related Coherence Exercise

- Examine event-related coherence between two ICs
  - Which pair did you pick, and why? What do you predict?
  - What did you learn?

- Explore other options:
  - Significance threshold
  - Figure out how to subtract a baseline
  - Phase vs. Linear Coherence