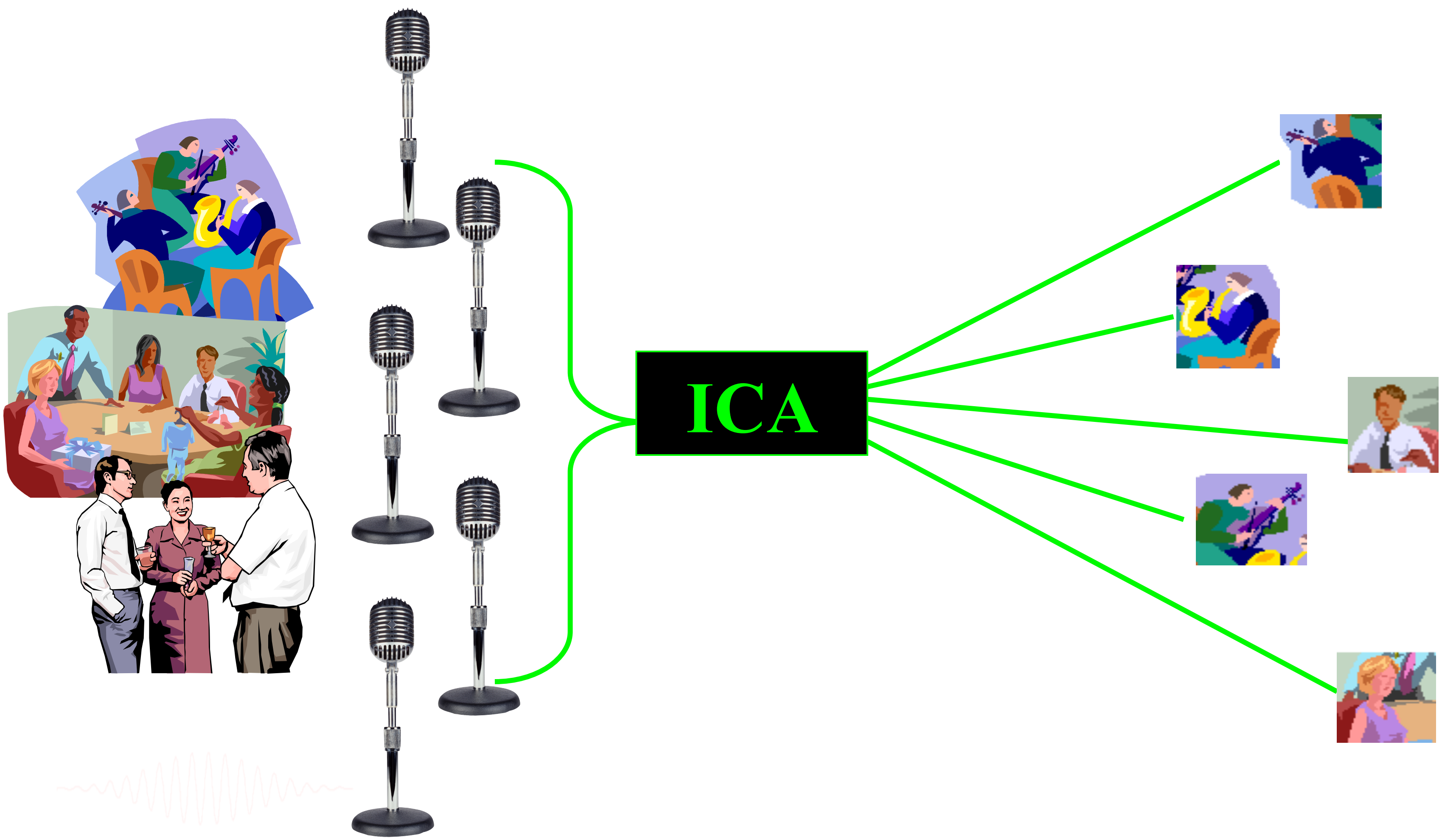
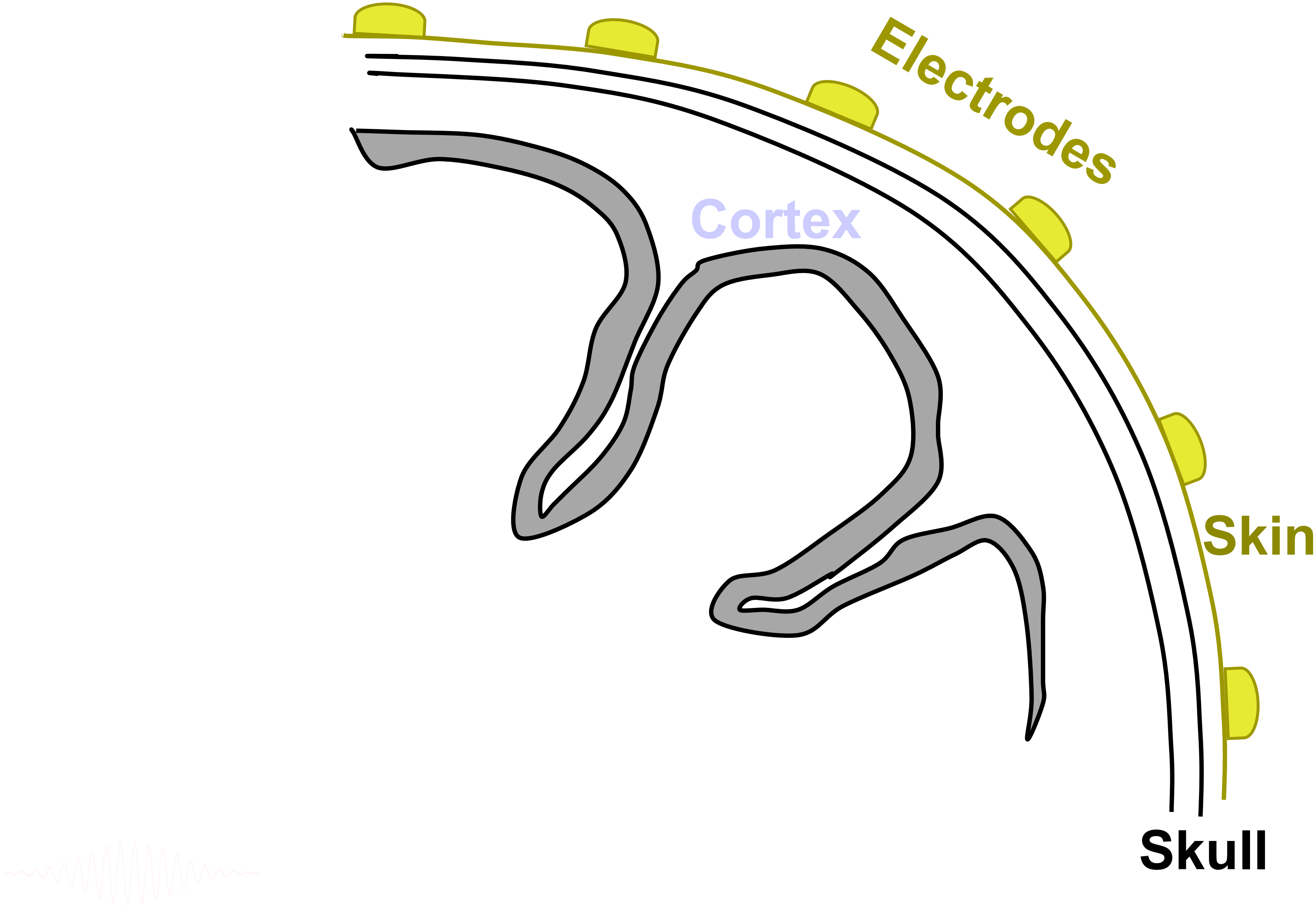
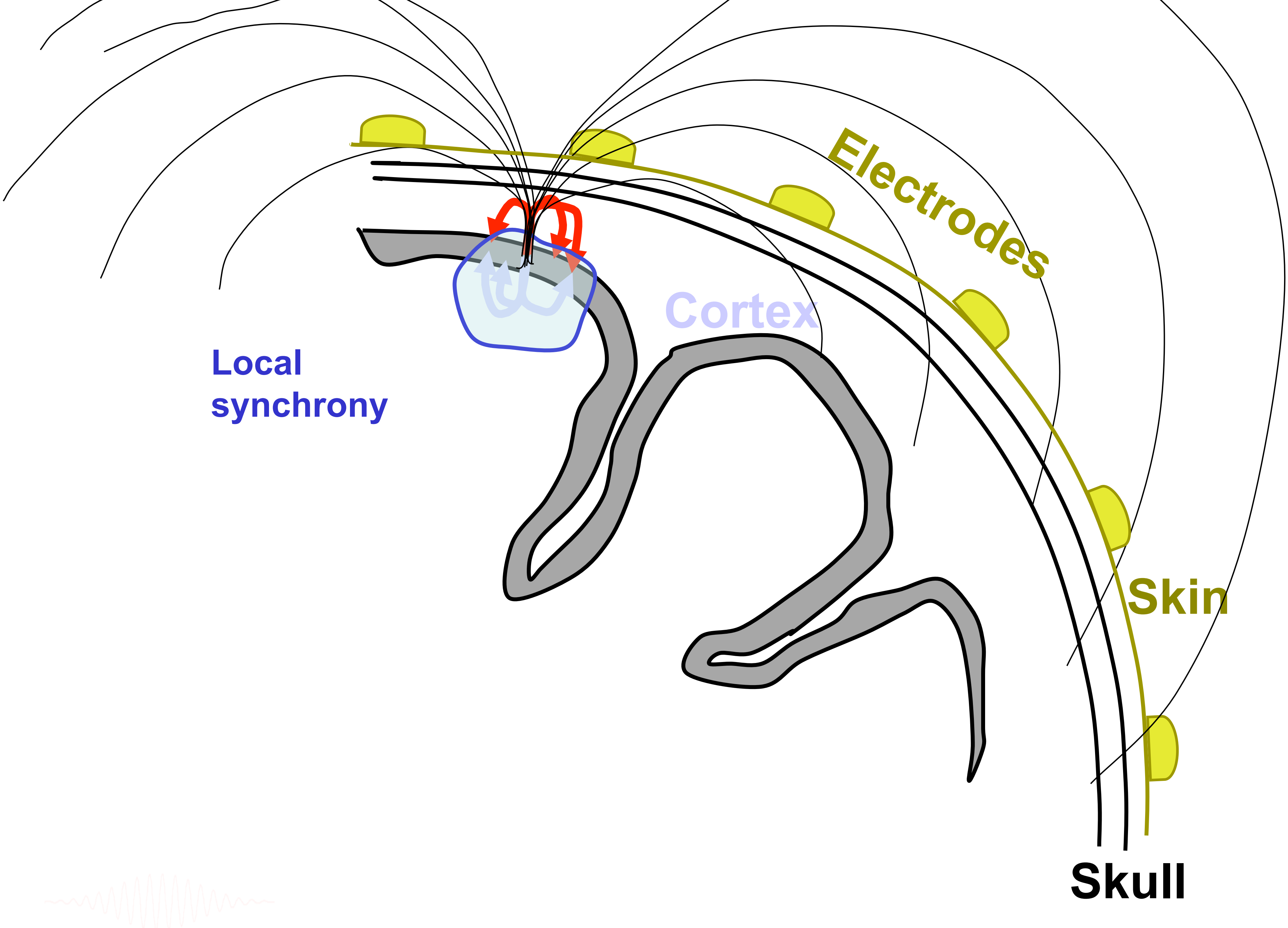
Independent component analysis applied to biophysical time series and EEG

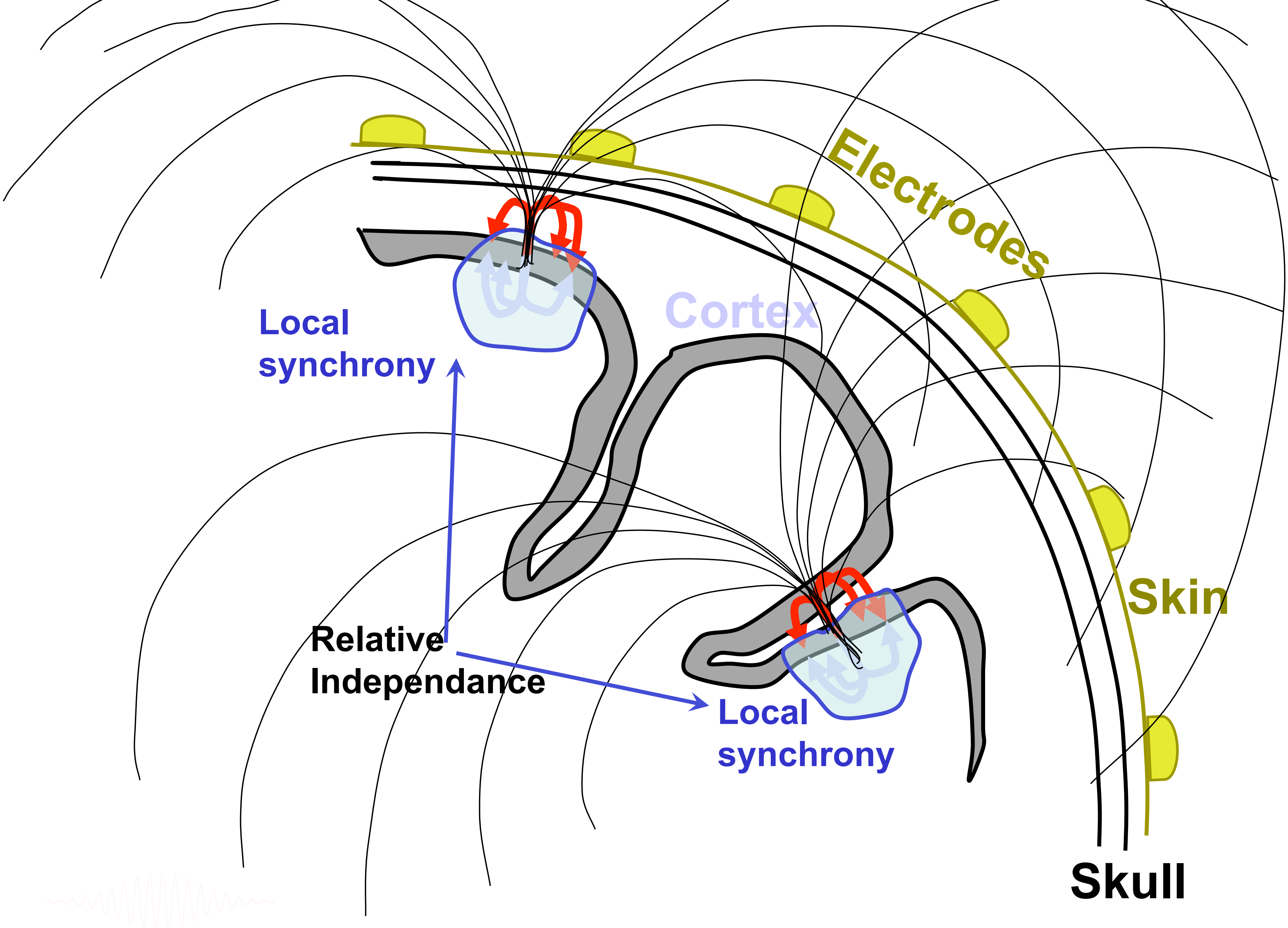
EEGLAB Workshop, Aspet, Arnaud Delorme

Example: Speech Separation

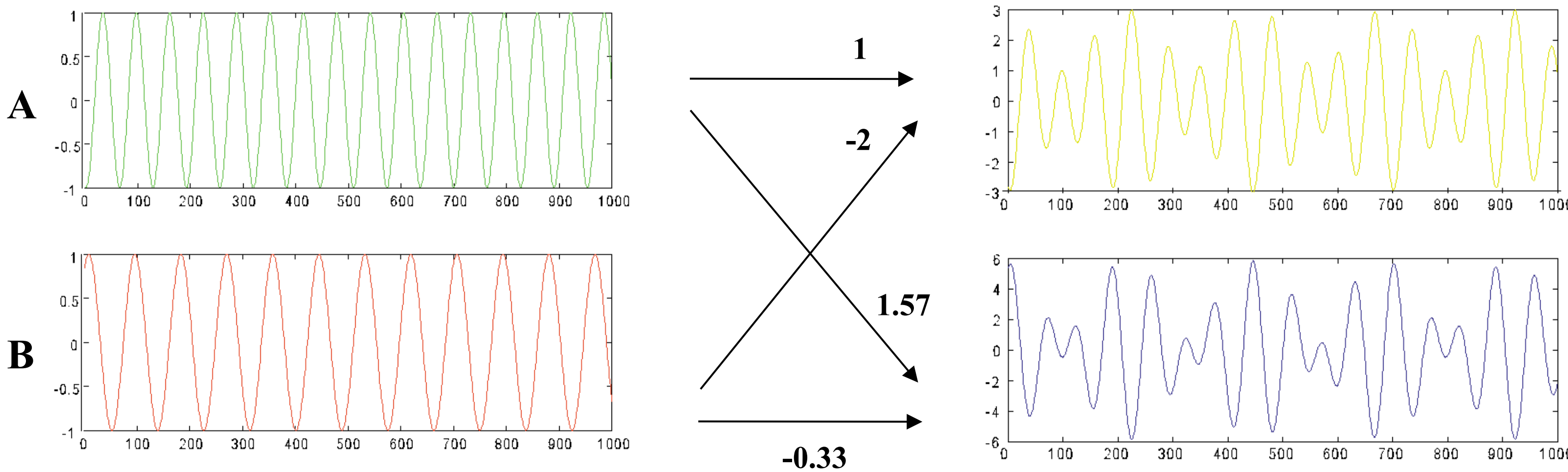



Electrodes
Cortex
Skin
Skull


Electrodes
Cortex
Local synchrony
Skin
Skull



Electrodes
Cortex
Local
synchrony
Skin
Relative
Independance
Local
synchrony
Skull



Y=[A;B]

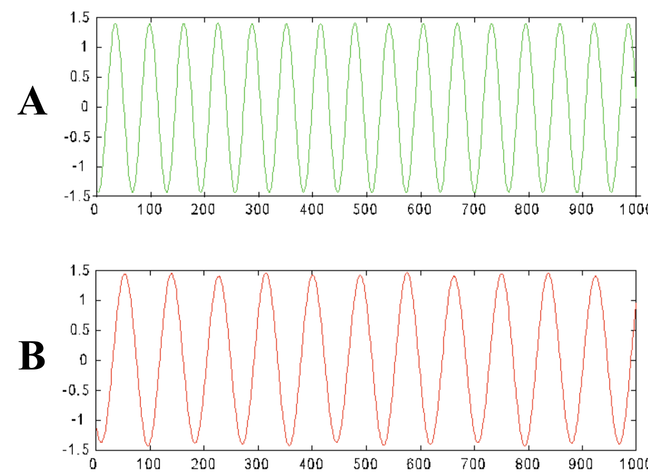
Linear Combination

X=YW

ICA

$\tilde{Y}=W^{-1}\tilde{X}$

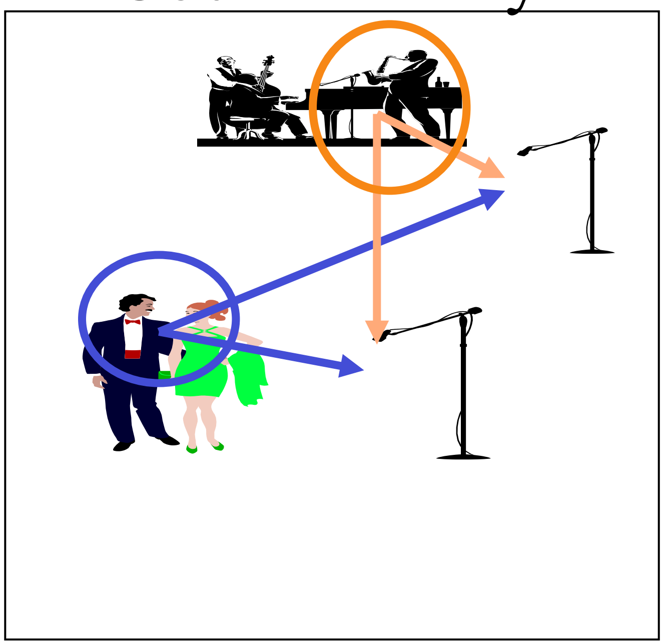
Infomax ICA

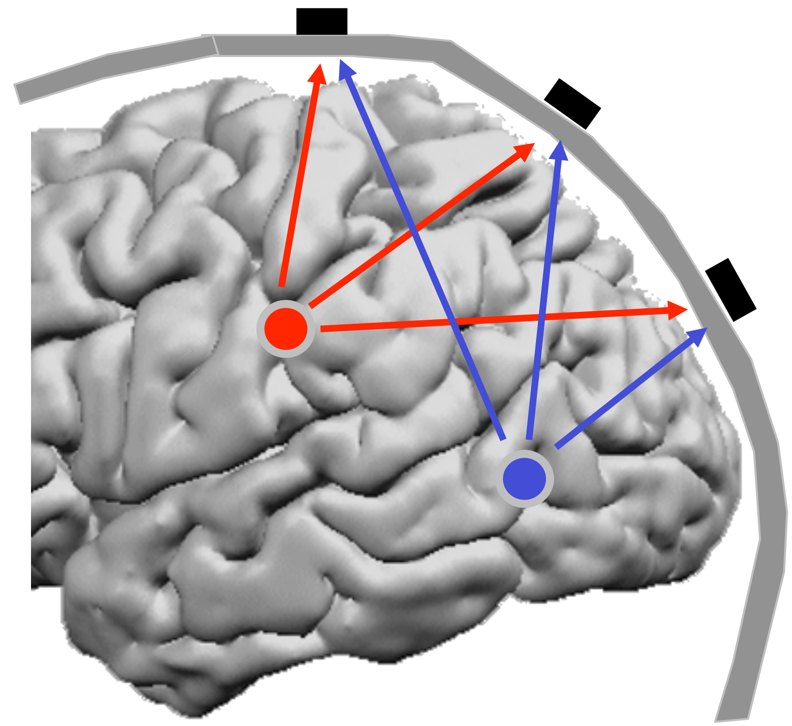


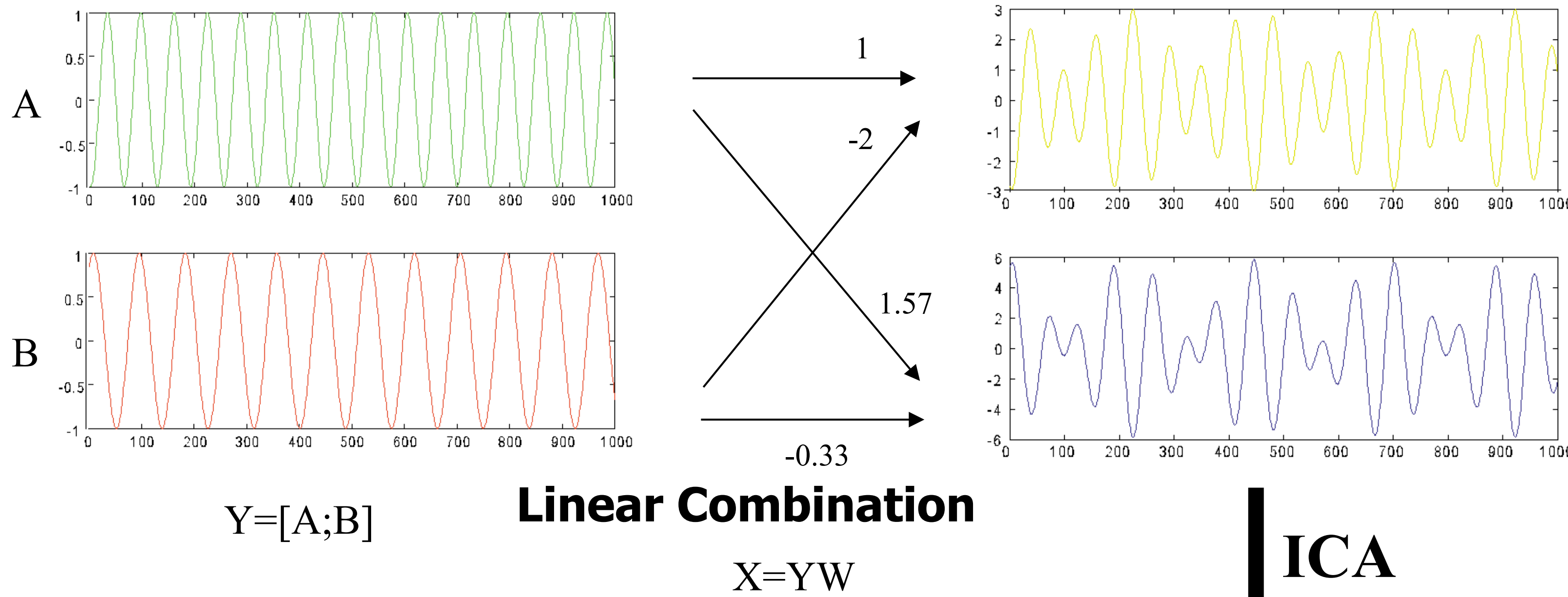
Independent component analysis

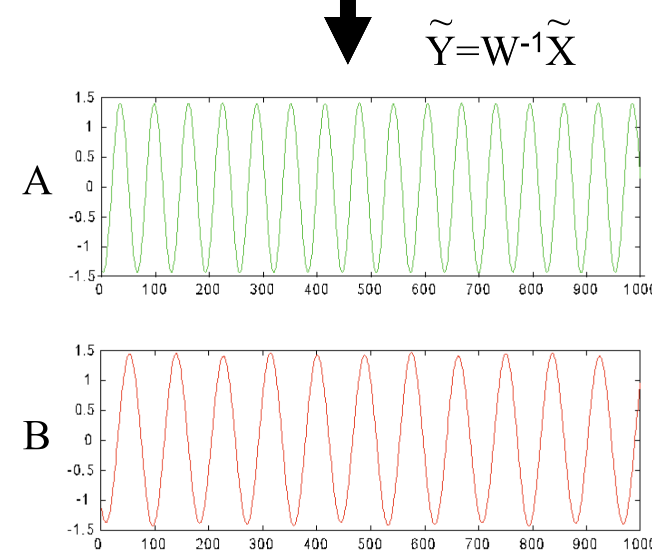
Mixture of Brain source activity

Cocktail Party









ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix

$$\mathbf{U} = \mathbf{W}\mathbf{X}$$

ICA activity

Data

Data X

$$\begin{array}{cl} \begin{bmatrix} 3 & 2 & 5 & 4 & 3 & 2 & \\ 0 & -2 & -5 & -1 & 1 & -1 & \ldots \\ -1 & 2 & 0 & 1 & 0 & -3 & \end{bmatrix} & \begin{array}{l} \leftarrow \text{Channel 1} \\ \leftarrow \text{Channel 2} \\ \leftarrow \text{Channel 3} \end{array} \end{array}$$

$$\begin{bmatrix} 5 & 3 & -2 \\ 1 & 2 & 4 \\ 0 & -1 & 3 \end{bmatrix} \longrightarrow \begin{array}{cl} \begin{bmatrix} 3*5+0*3-1*(-2) & 2*5+(-2)*3+2*(-2) & \\ 3*1+0*2-1*4 & 2*1+(-2)*2+2*4 & \ldots \\ 5*1-5*2+0*4 & 5*1-5*2+0*4 & \end{bmatrix} & \begin{array}{l} \leftarrow \text{Comp. 1} \\ \leftarrow \text{Comp. 2} \\ \leftarrow \text{Comp. 3} \end{array} \end{array}$$

Weight matrix W

ICA activity U

$$\mathbf{X}=\mathbf{W}^{-1}\mathbf{U}$$

Data

ICA activity U

$$\begin{array}{cl}\begin{bmatrix} 3 & 2 & 5 & 4 & 3 & 2 & \\ 0 & -2 & -5 & -1 & 1 & -1 & \dots \\ -1 & 2 & 0 & 1 & 0 & -3 & \end{bmatrix} & \begin{array}{l}\leftarrow \text{Comp. 1} \\ \leftarrow \text{Comp. 2} \\ \leftarrow \text{Comp. 3}\end{array}\end{array}$$

$$\begin{bmatrix} 5 & 3 & -2 \\ 1 & 2 & 4 \\ 0 & -1 & 3 \end{bmatrix} \longrightarrow \begin{array}{cl}\begin{bmatrix} 3*5+0*3-1*(-2) & 2*5+(-2)*3+2*(-2) & \\ 3*1+0*2-1*4 & 2*1+(-2)*2+2*4 & \dots \\ 5*1-5*2+0*4 & 5*1-5*2+0*4 & \end{bmatrix} & \begin{array}{l}\leftarrow \text{Chan 1} \\ \leftarrow \text{Chan 2} \\ \leftarrow \text{Chan 3}\end{array}\end{array}$$

Inverse weight matrix W^{-1}

Data X

Historical Remarks

- Herault & Jutten ("Space or time adaptive signal processing by neural network models“, *Neural Nets for Computing Meeting*, Snowbird, Utah, 1986): Seminal paper, neural network
- Bell & Sejnowski (1995): Information Maximization
- Amari et al. (1996): Natural Gradient Learning
- Cardoso (1996): JADE

- **Applications of ICA to biomedical signals**
  - EEG/ERP analysis (Makeig, Bell, Jung & Sejnowski, 1996).
  - fMRI analysis (McKeown et al. 1998)

ICA Theory – Cost Functions

Family of BSS algorithms

- Information theory (Infomax)
- Bayesian probability theory (Maximum likelihood estimation)
- Negentropy maximization
- Nonlinear PCA
- Statistical signal processing (cumulant maximization, JADE)

A unifying Information-theoretic framework for ICA

- Pearlmutter & Parra showed that InfoMax, ML estimation are equivalent.
- Lee et al. (1999) showed negentropy has the equivalent property to InfoMax.
- Girolami & Fyfe showed nonlinear PCA can be viewed from information-theoretic principle.

Independent Component Analysis

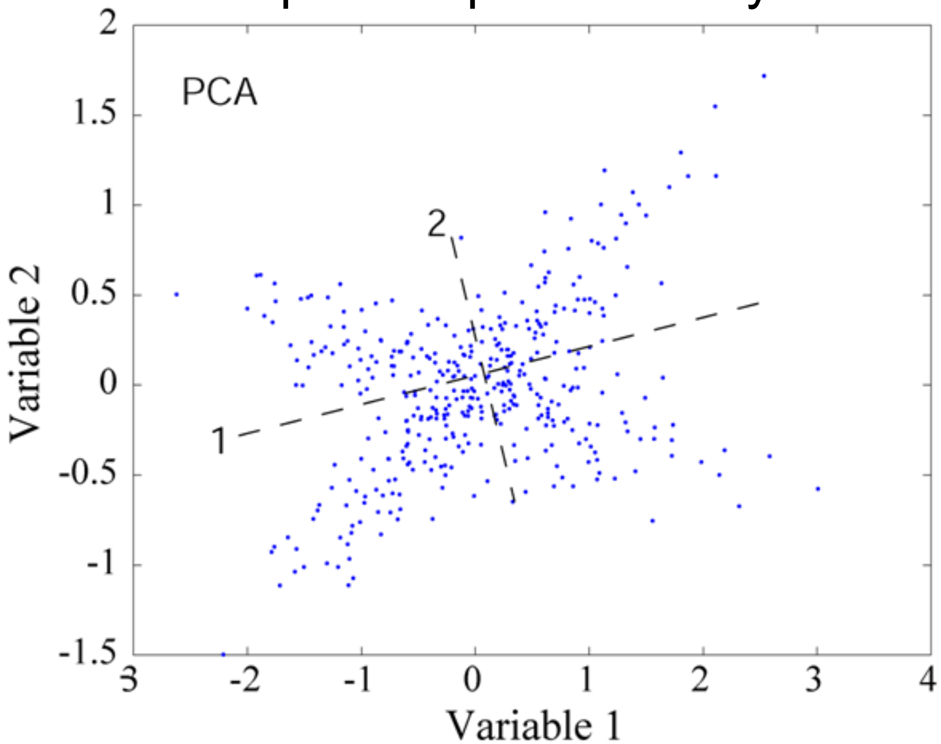
ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix,

$$\mathbf{U} = \mathbf{W}\mathbf{X},$$

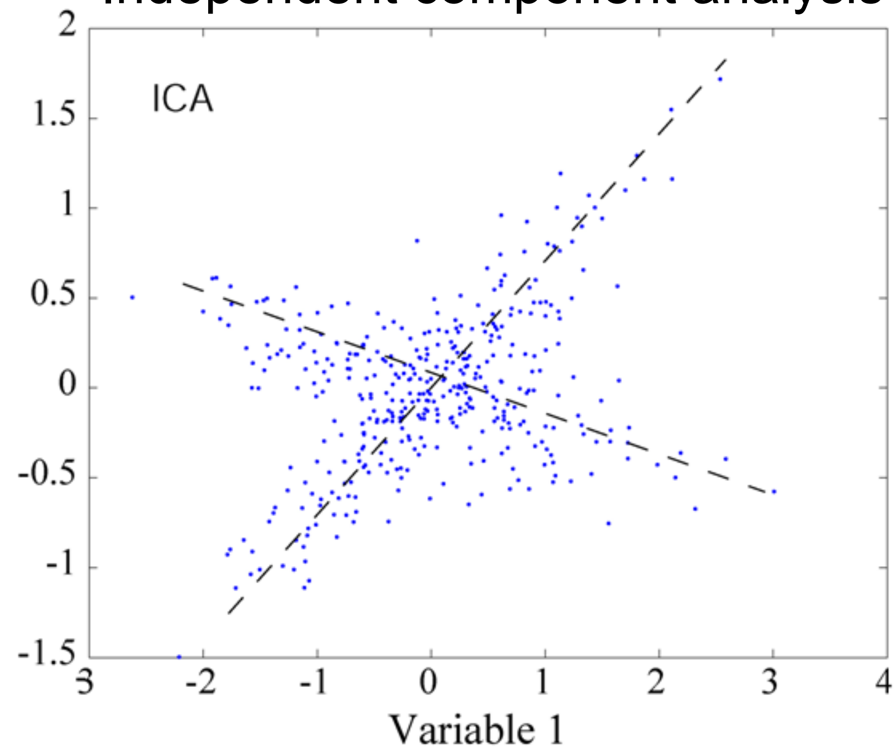
While PCA simply decorrelates the outputs (using an orthogonal matrix **W**), ICA attempts to make the outputs **statistically independent**, while placing no constraints on the matrix **W**.

ICA and PCA

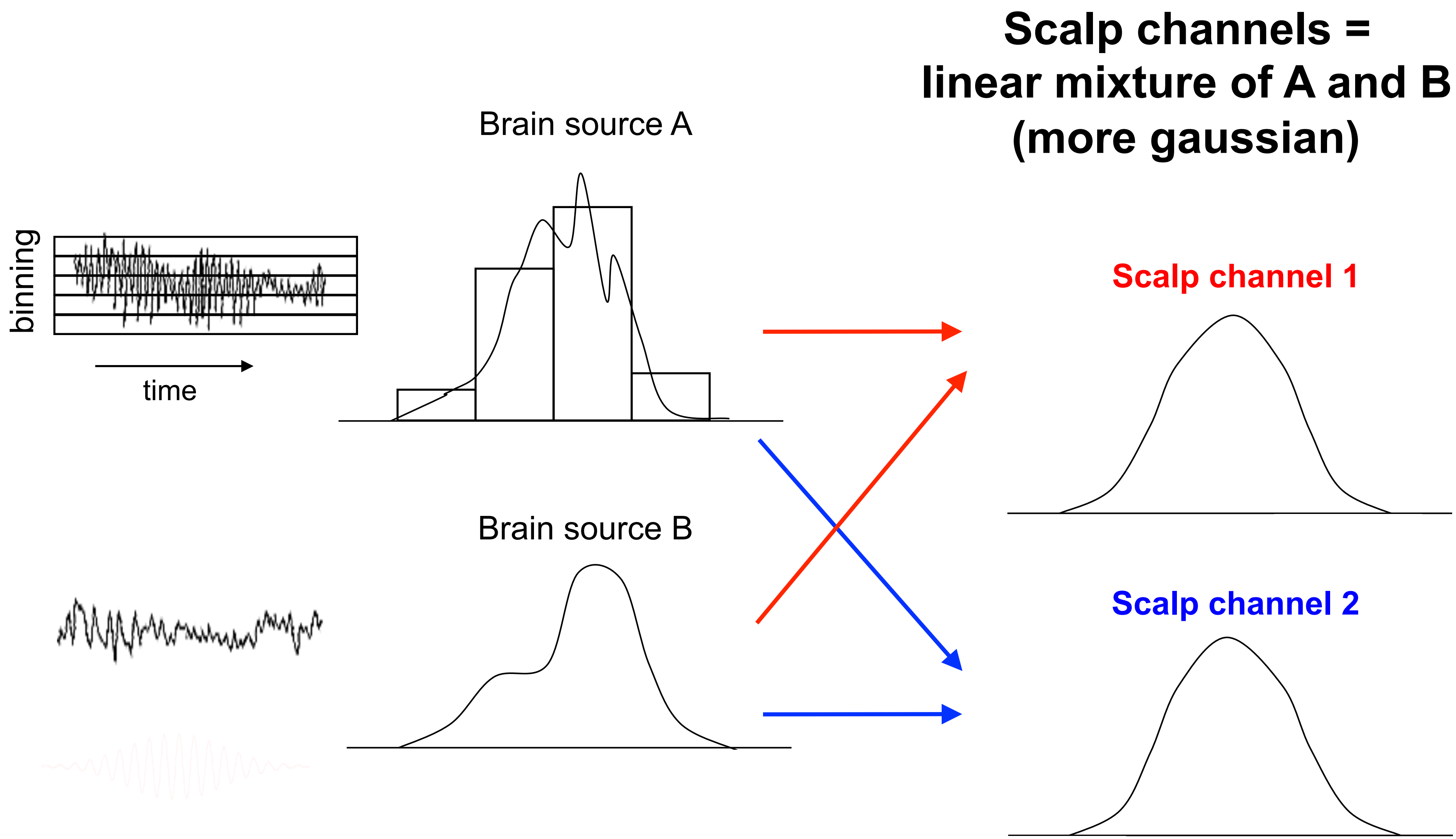
Principal component analysis



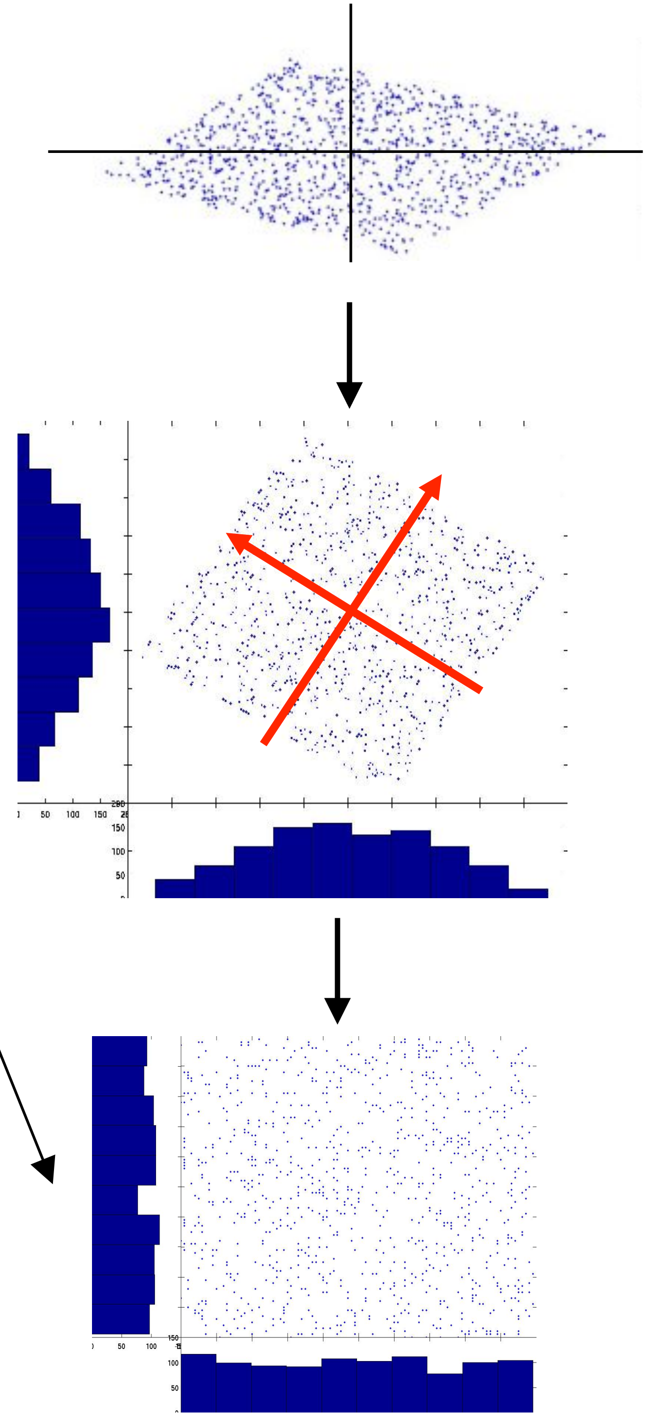
Independent component analysis



Central limit theorem



ICA Training Process



Central limit theorem

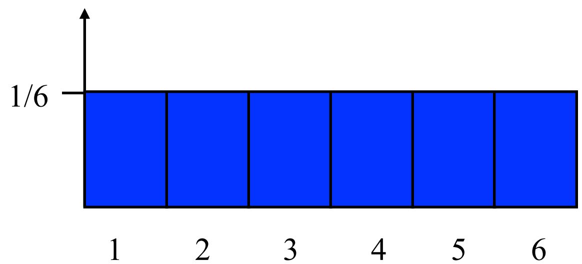
- Remove the mean
  $x = x - \langle x \rangle$
- 'Sphere' the data by diagonalizing its covariance matrix,
  $x = \langle xx^T \rangle^{-1/2}(x - \langle x \rangle)$.
- Update W according to

$$\Delta \mathbf{w} \propto \frac{\partial H(\mathbf{y})}{\partial \mathbf{w}} \mathbf{w}^T \mathbf{w}$$

Entropy

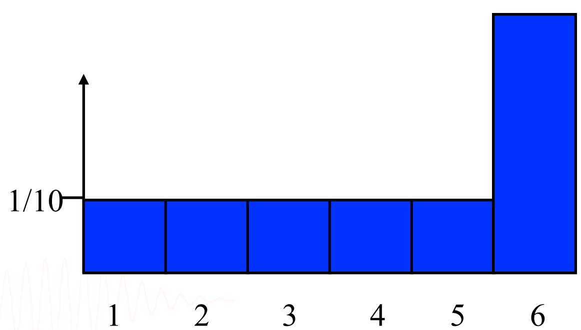
$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x).$$

Dice: 1/6

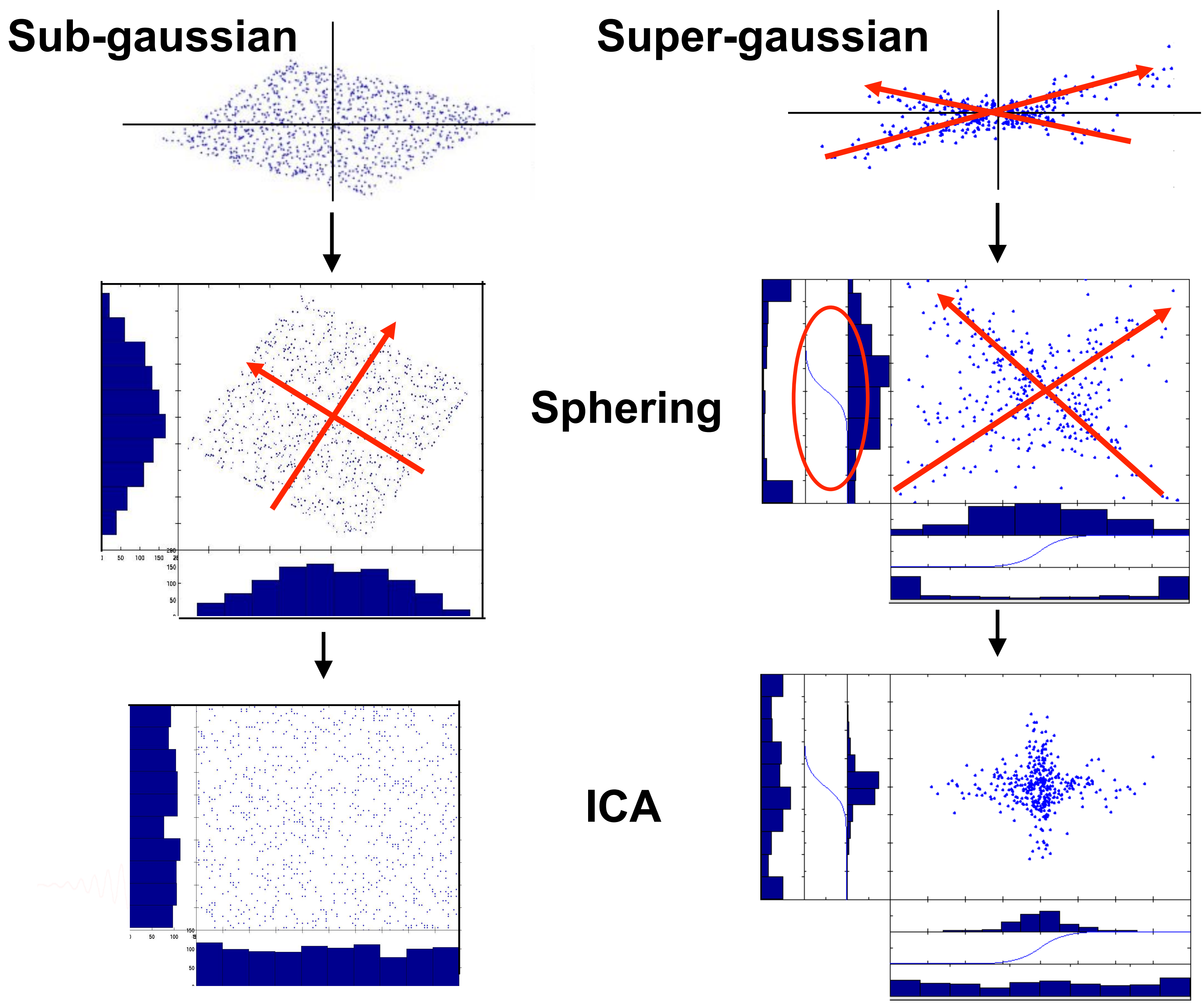


$$H = 6\left(-\frac{1}{6}\log_2\left(\frac{1}{6}\right)\right) = 2.58$$

Fake dice (make a 6 half of the time): entropy 2.16 (base 2)



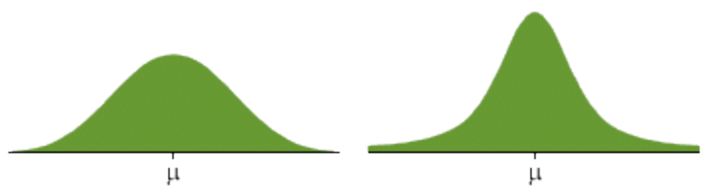
$$H = 5\left(-\frac{1}{10}\log_2\left(\frac{1}{10}\right)\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 2.16$$


Sub-gaussian
Super-gaussian
Sphering
ICA

Kurtosis, Super- and Sub-Gaussian

Kurtosis: a measure of how peaked or flat of a probability distribution is.

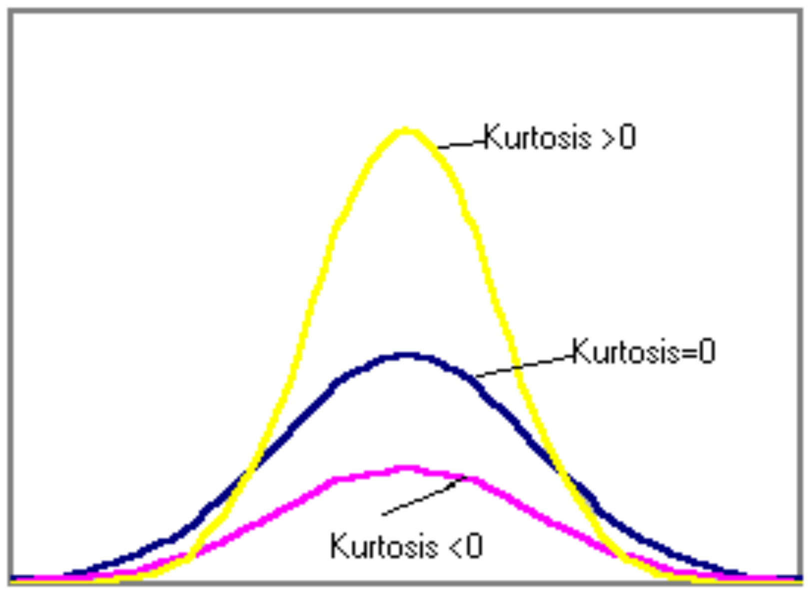
$$kurt(X) = \frac{E\left[(X-\mu)^4\right]}{\sigma^4}$$



Gaussian Dist. Kurtosis = 0

Super-Gaussian: kurtosis > 0

Sub-Gaussian: kurtosis < 0



Moments, Cumulants

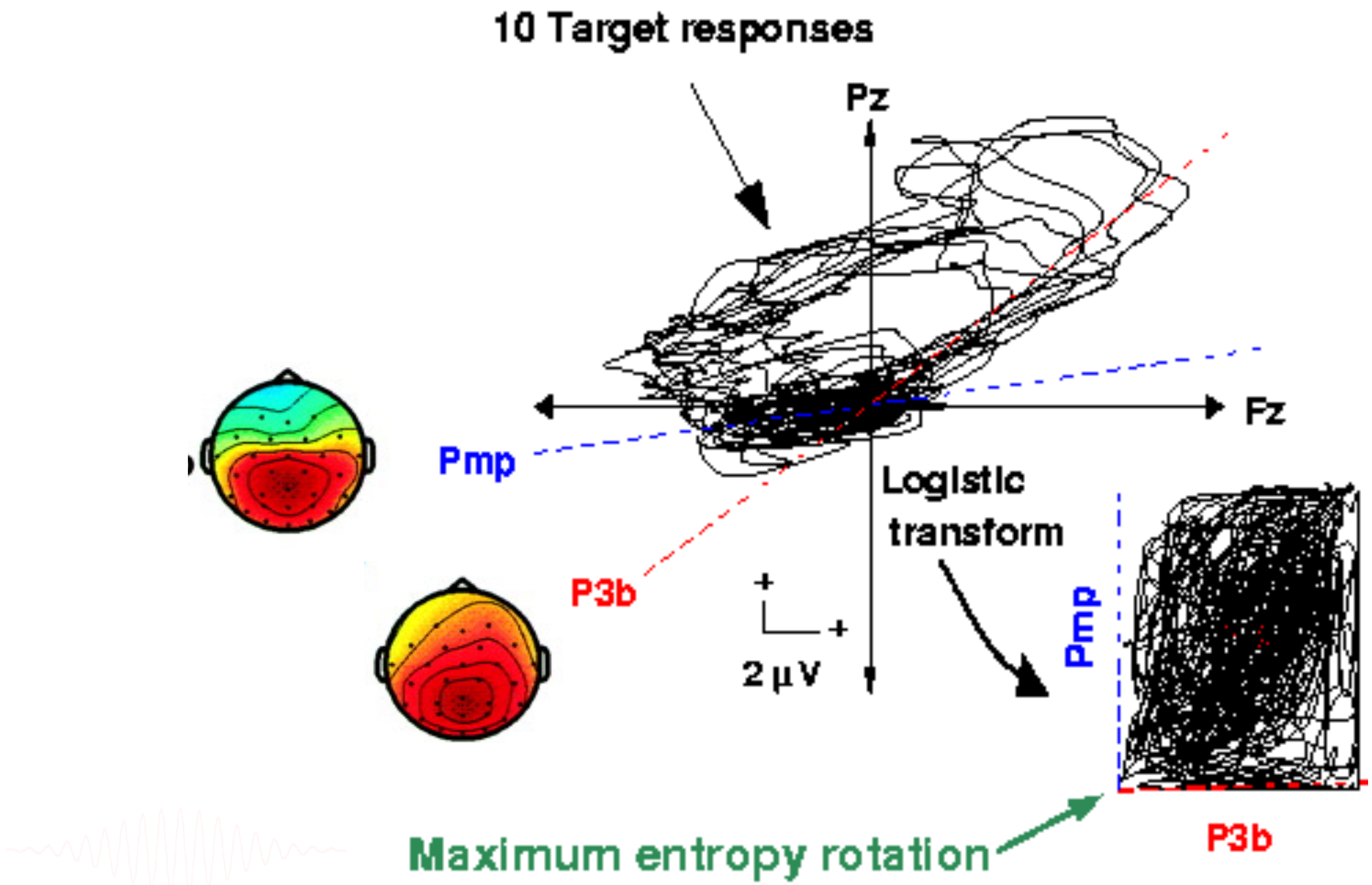
Moments $\mu_x(n) = E\{x^n\}$

Central moments $m_x(n) = E\{(x - m_x)^n\}$

Cumulants

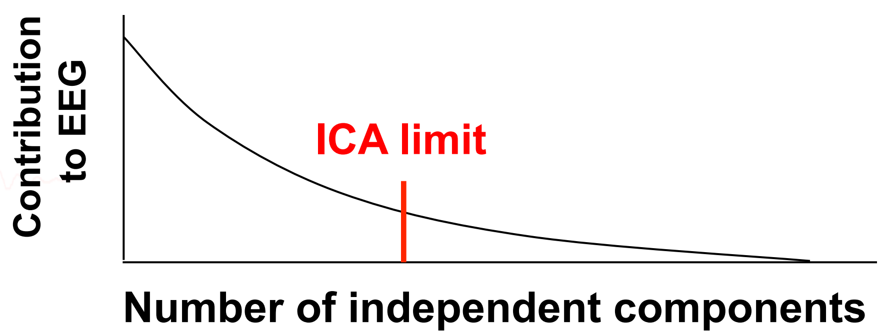
$$\begin{array}{rcll} c_1 & = & m_1 = \mu & \longleftarrow \text{mean} \\ c_2 & = & m_2 = \sigma^2 & \longleftarrow \text{variance} \\ c_3 & = & m_3 & \longleftarrow \text{skewness} \\ c_4 & = & m_4 - 3m_2^2 & \longleftarrow \text{kurtosis} \end{array}$$

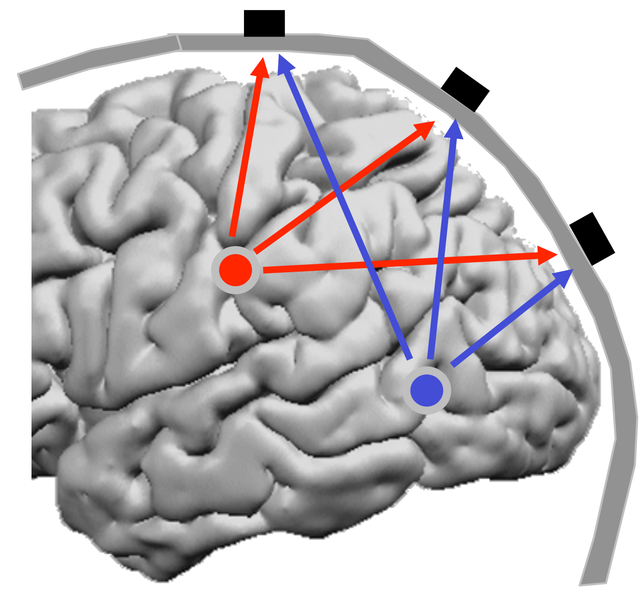
Independent components of EEG/ERP



ICA/EEG Assumptions

- Mixing is linear at electrodes **OK**
- Propagation delays are negligible **OK**
- Component time courses are independent **~**
- Number of components less than the number of channels.





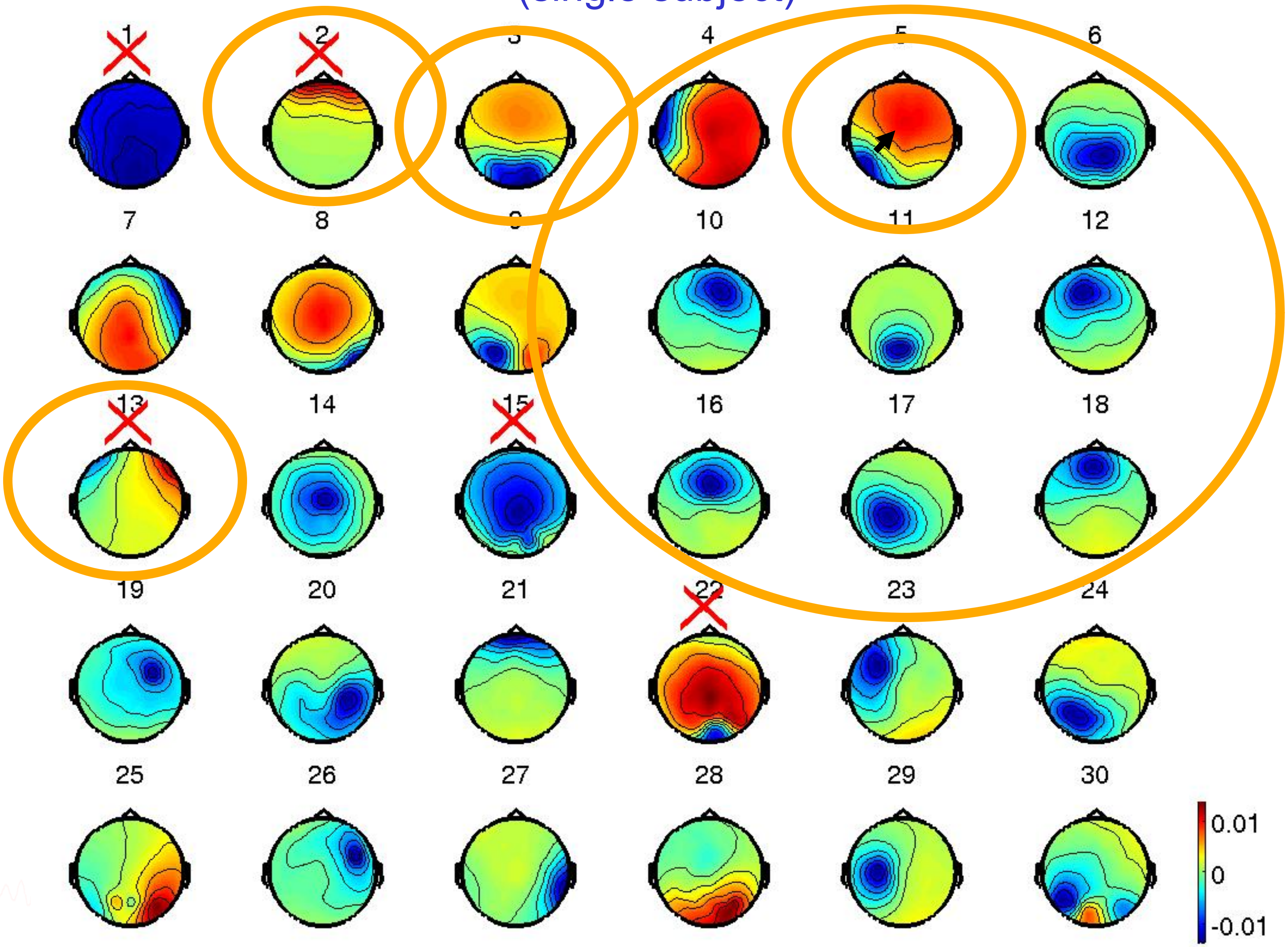
Independent Component Categories

- Artifacts
- Stimulus-locked activity
- Response-locked activity
- Non-phase locked activity
- Event-modulated oscillatory activity

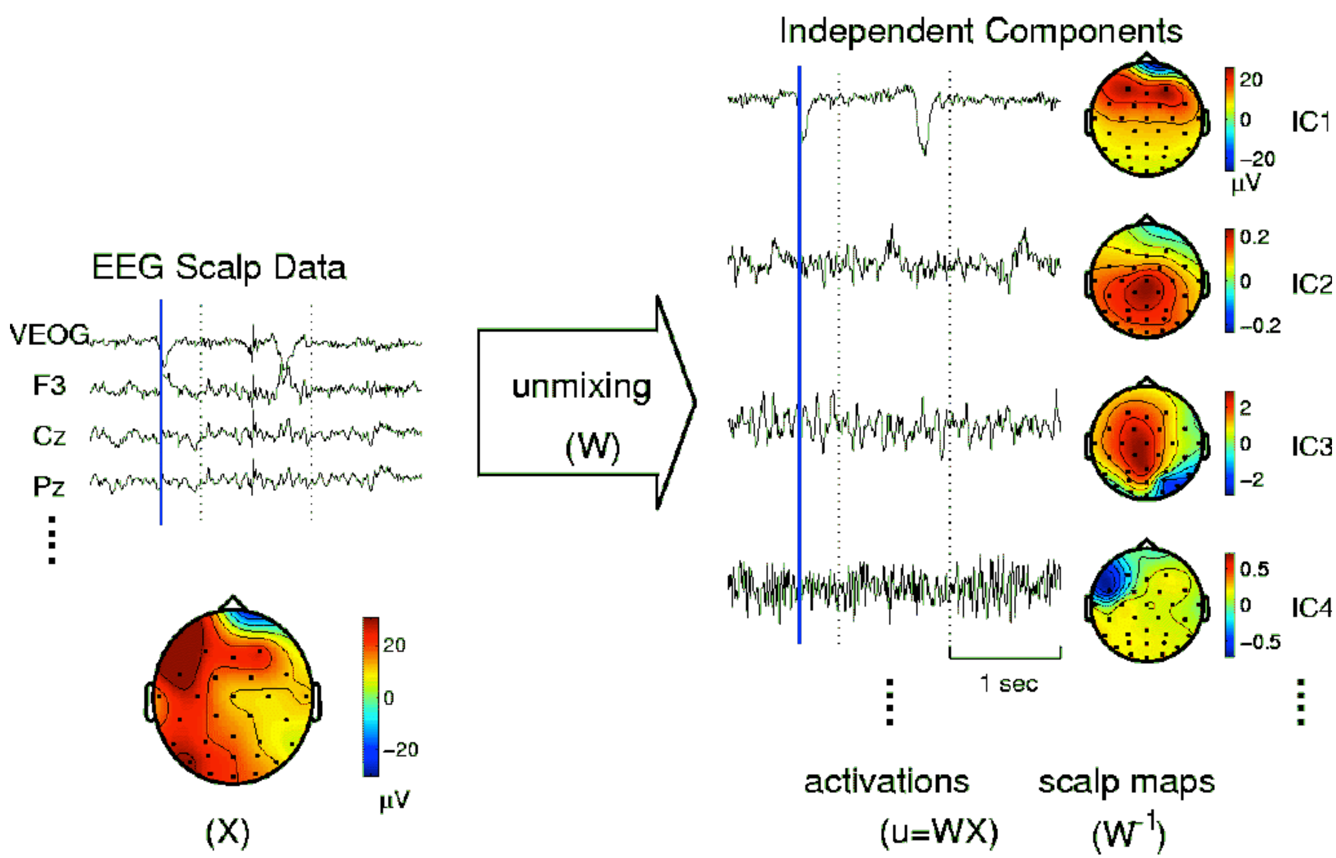
Characteristics of Independent Component of the EEG

- Concurrent Activity
- Maximally Temporally Independent
- Overlapping Maps and Spectra
- Dipolar Scalp Maps
- Functionally Independent
- Between-Subject Regularity

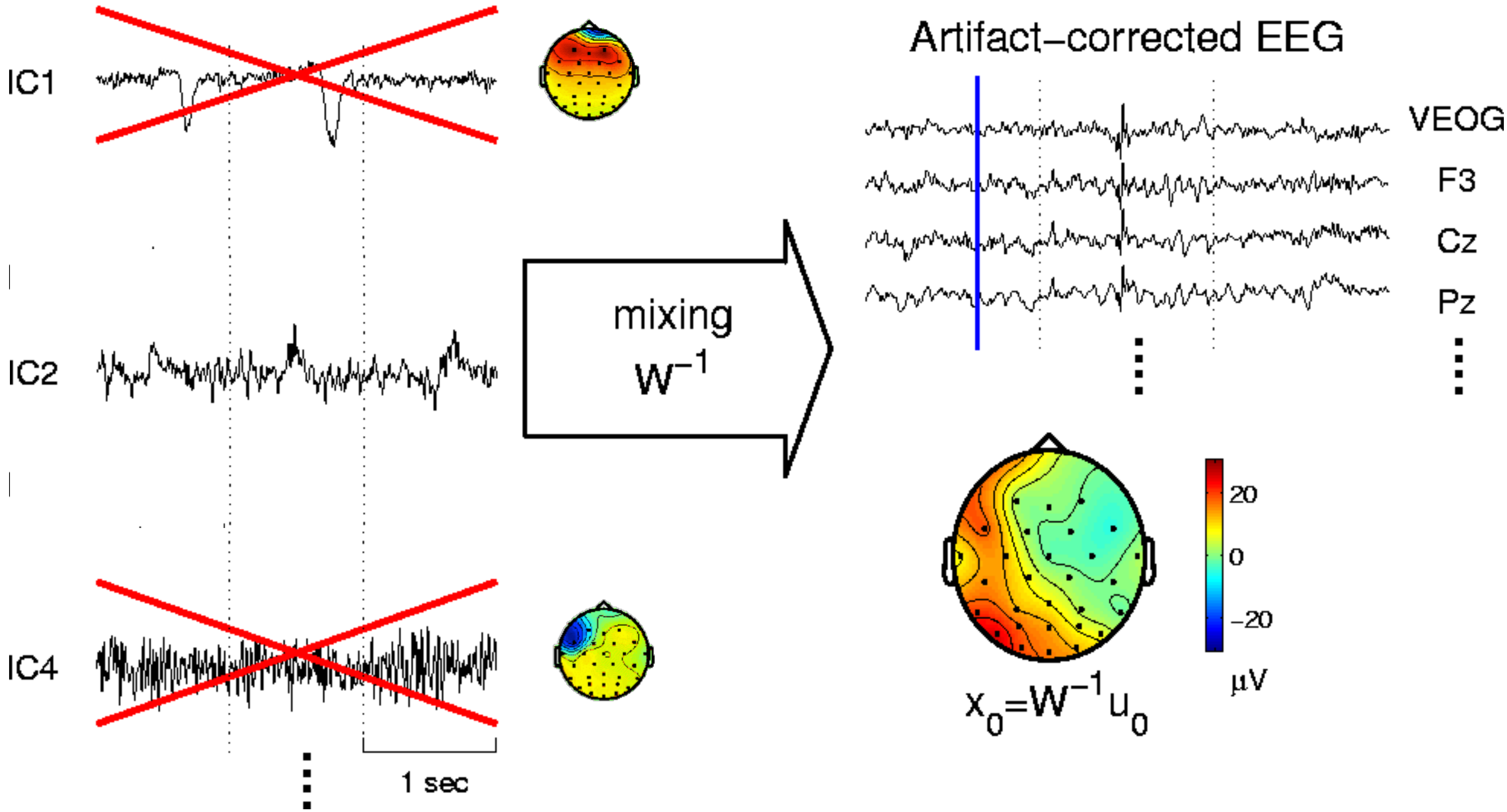
Largest 30 Independent Components
(single subject)



ICA Decomposition into Independent Components



Selective Projection onto Scalp Channels



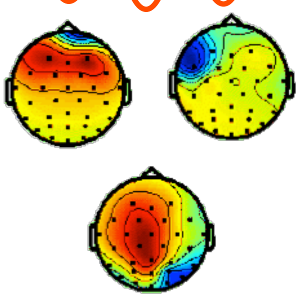
$$X = W^{-1}U$$

Data

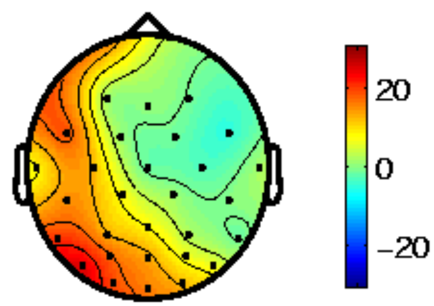
ICA activity U

$$
\begin{array}{c}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & -2 & -5 & -1 & 1 & -1 & \dots \\
0 & 0 & 0 & 0 & 0 & 0 & \dots \\
\dots & & & & & &
\end{bmatrix}
\end{array}
\begin{array}{l}
\leftarrow \text{Comp. 1} \\
\leftarrow \text{Comp. 2} \\
\leftarrow \text{Comp. 3} \\
\\
\end{array}
$$

$$
\begin{bmatrix}
5 & 3 & -2 & \dots \\
1 & 2 & 4 & \dots \\
0 & -1 & 3 & \dots \\
\dots & & &
\end{bmatrix}
\rightarrow
\begin{bmatrix}
3*5+0*3-1*(-2) & 2*5+(-2)*3+2*(-2) & \dots \\
3*1+0*2-1*4 & 2*1+(-2)*2+2*4 & \dots \\
5*1-5*2+0*4 & 5*1-5*2+0*4 & \dots \\
\dots & \dots &
\end{bmatrix}
\begin{array}{l}
\leftarrow \text{Chan 1} \\
\leftarrow \text{Chan 2} \\
\leftarrow \text{Chan 3} \\
\\
\end{array}
$$

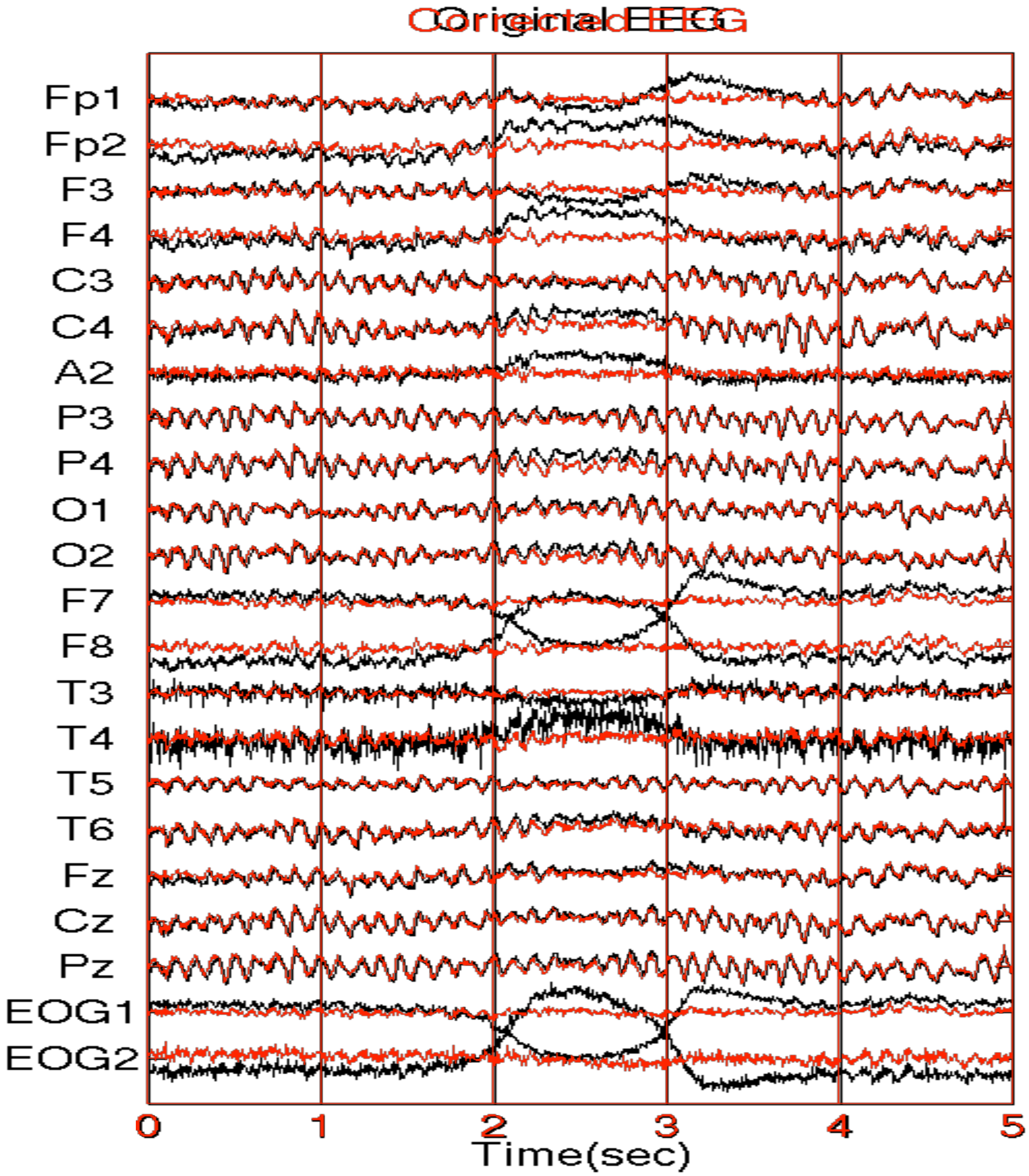


Data X

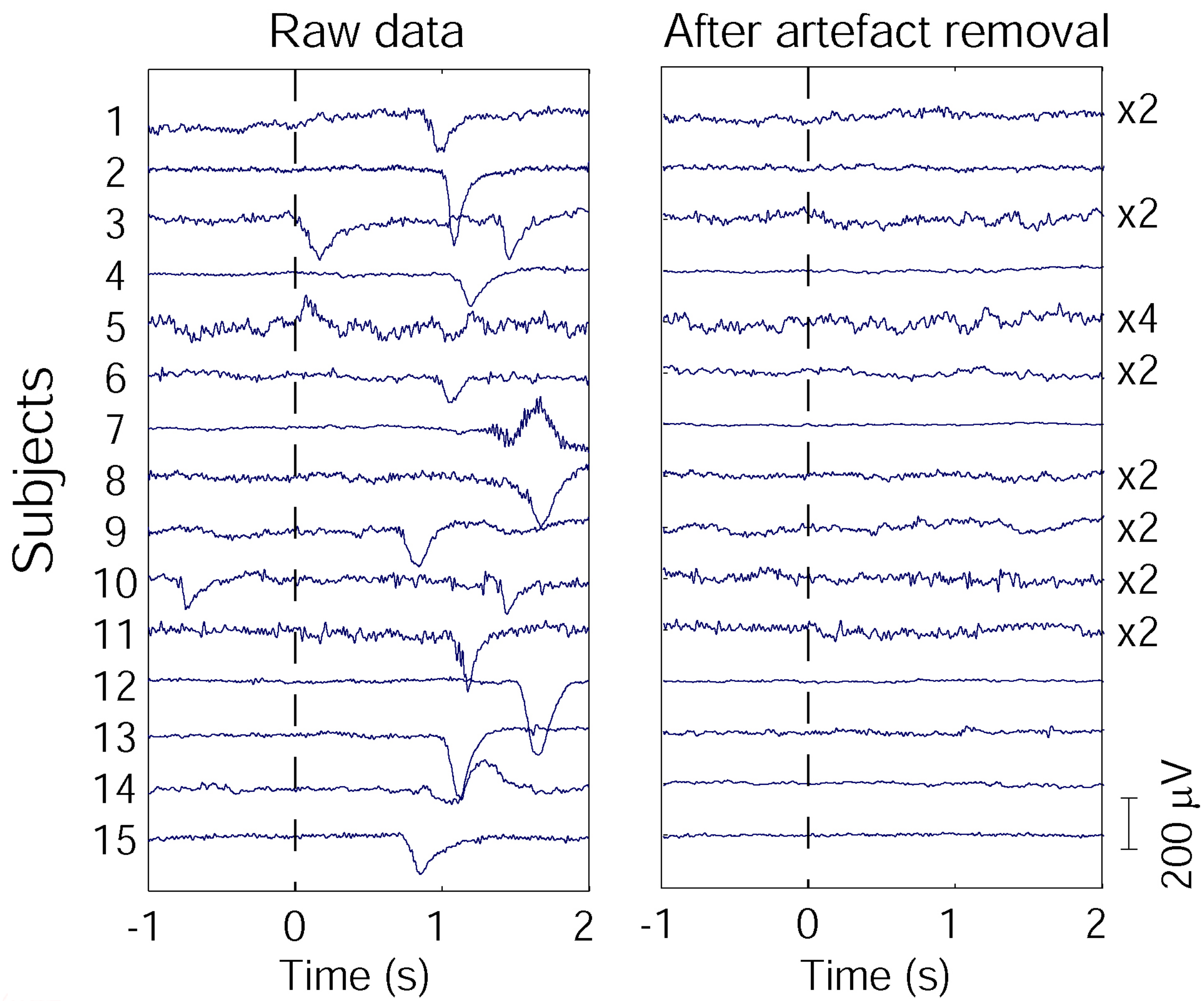


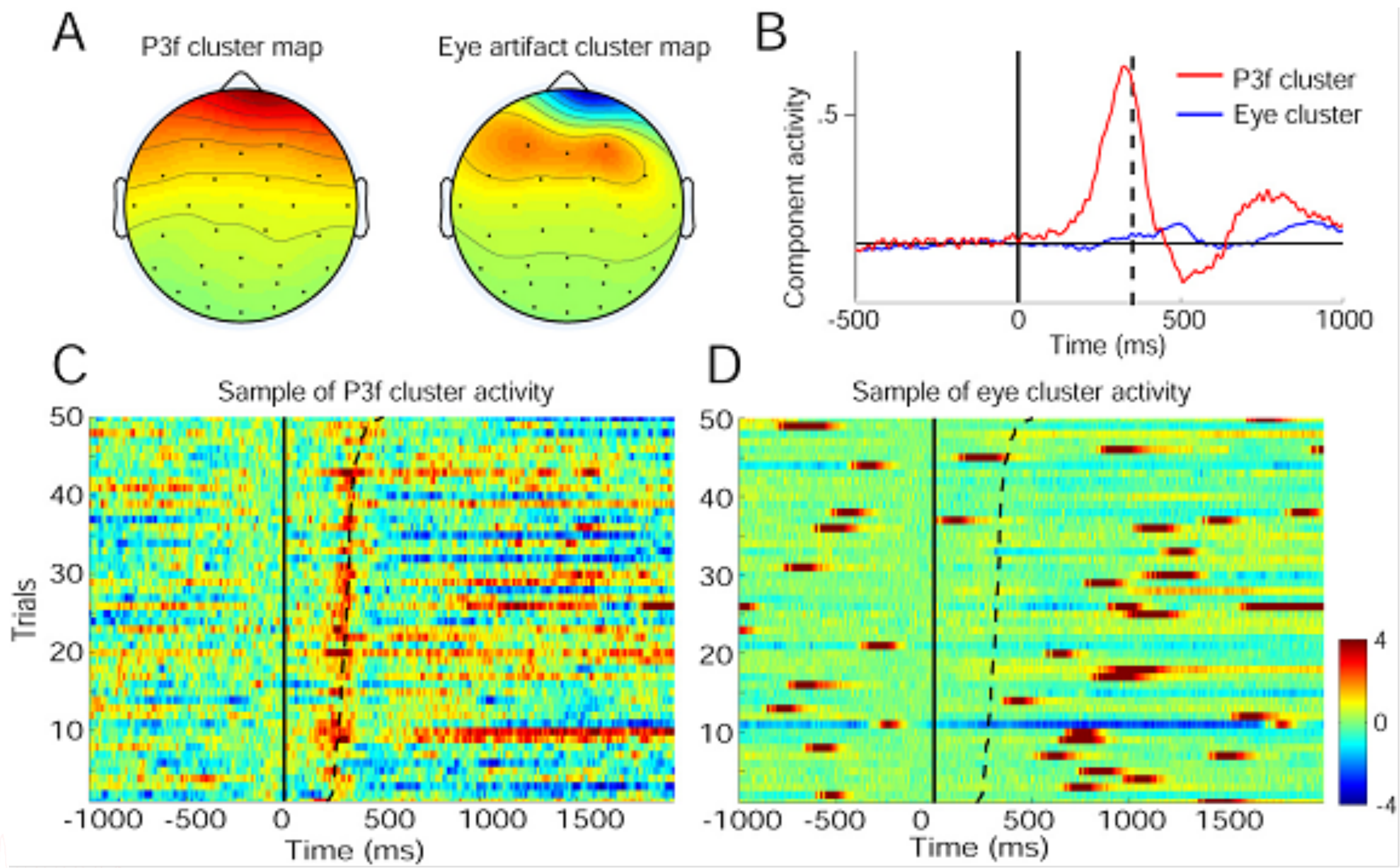
Inverse weight matrix W^{-1}

ICA-based Artifact Removal



Artifact removal using ICA

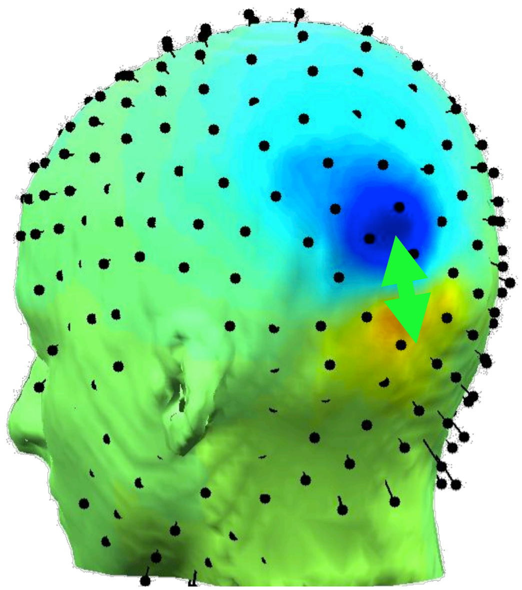




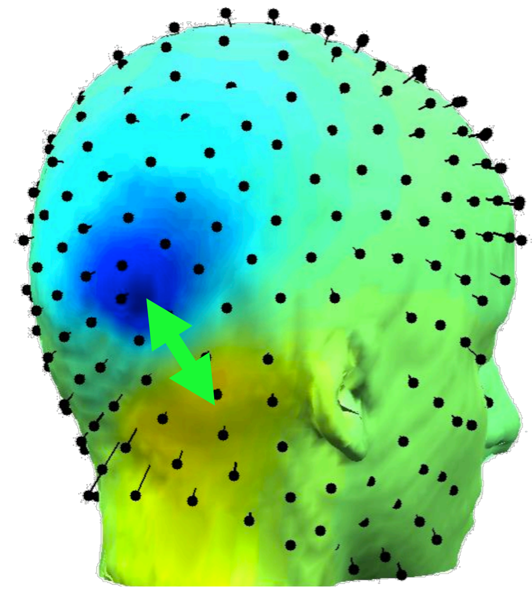
A
P3f cluster map
Eye artifact cluster map
B
Component activity
.5
P3f cluster
Eye cluster
-500
0
500
1000
Time (ms)
C
Sample of P3f cluster activity
Trials
50
40
30
20
10
-1000
-500
0
500
1000
1500
Time (ms)
D
Sample of eye cluster activity
50
40
30
20
10
-1000
-500
0
500
1000
1500
Time (ms)
4
0
-4

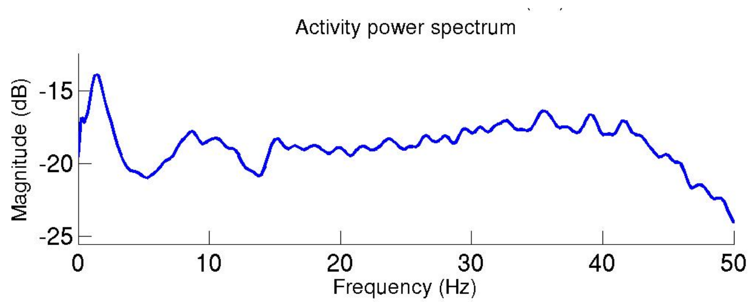
Two Neck Muscle Processes

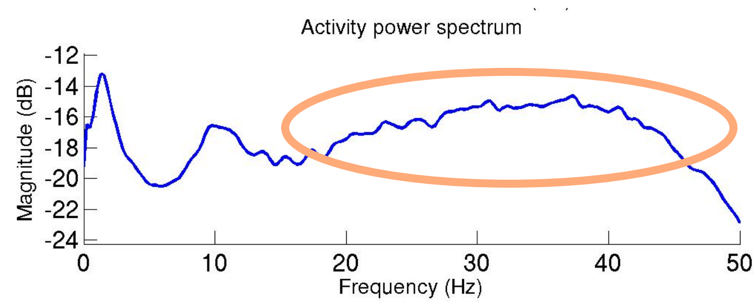
IC39



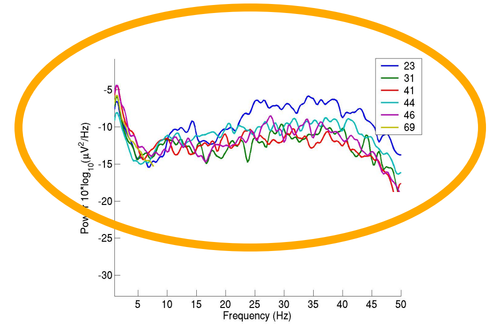
IC31

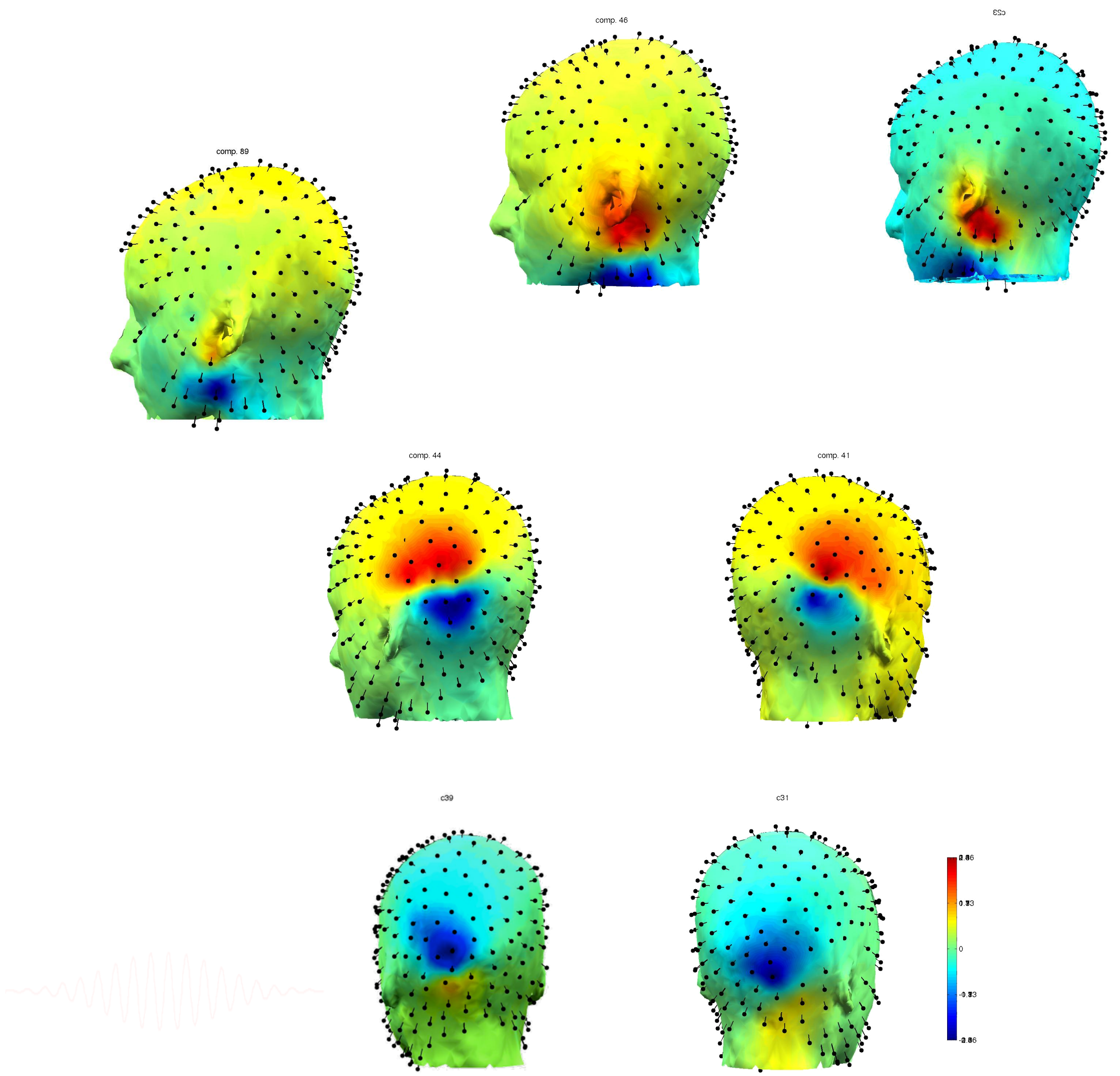


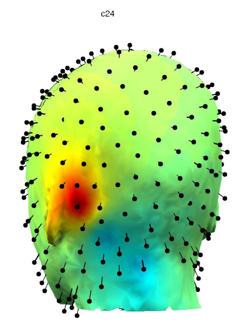


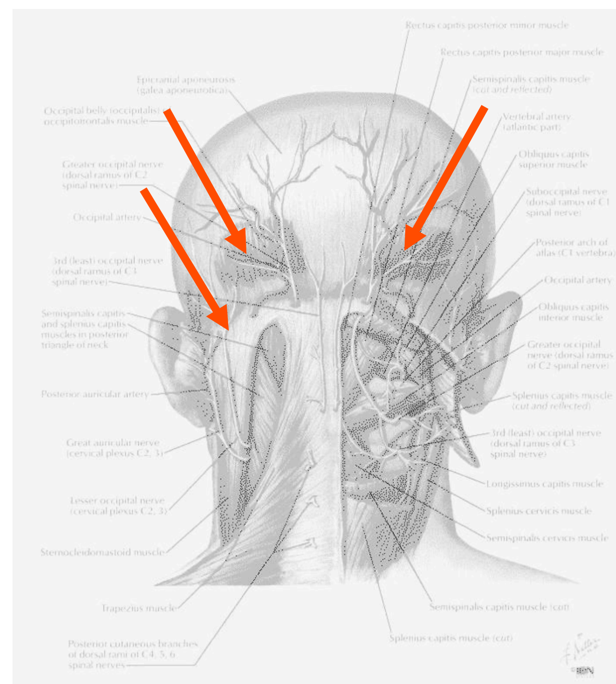


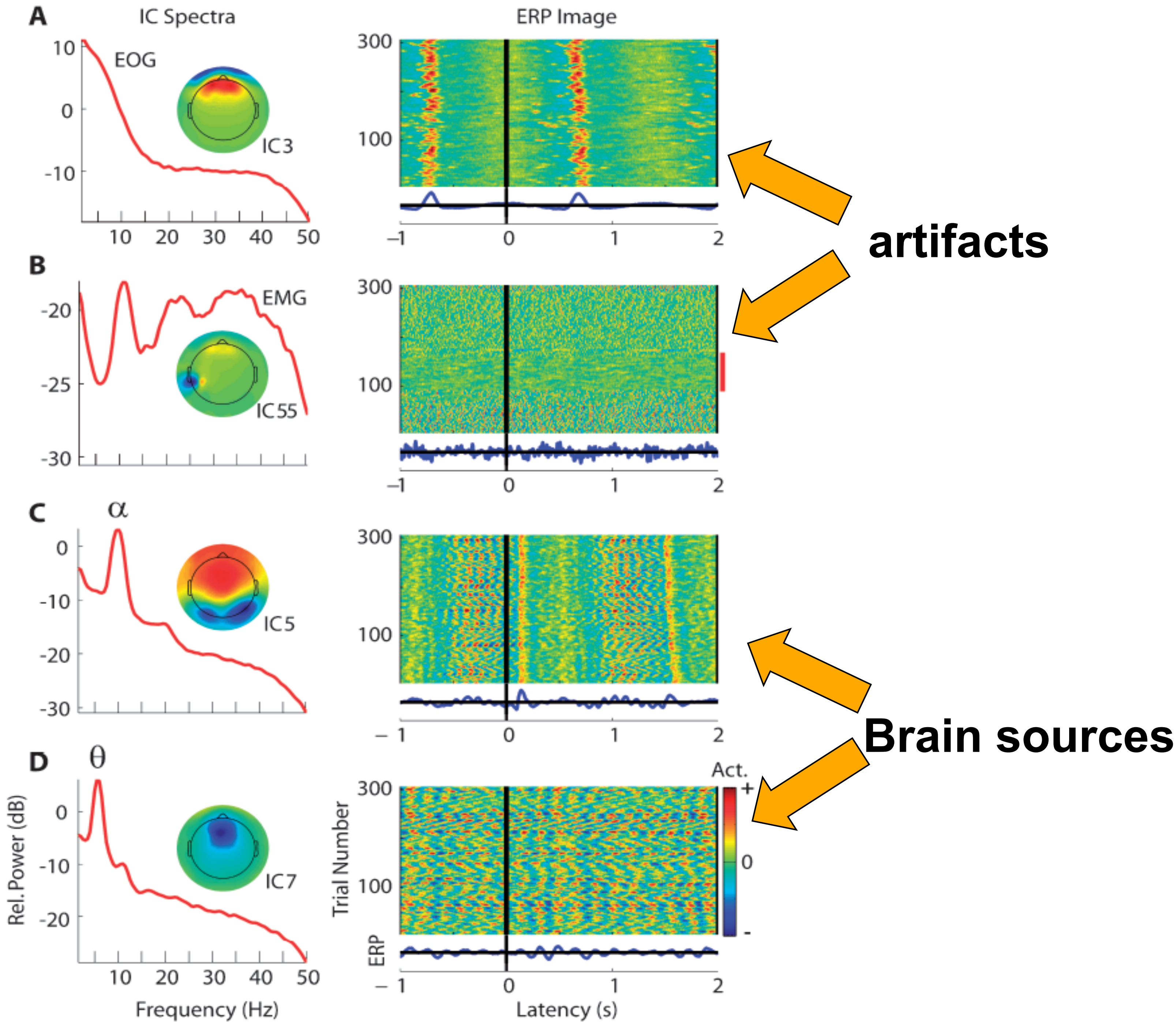
Some Independent EMG Components





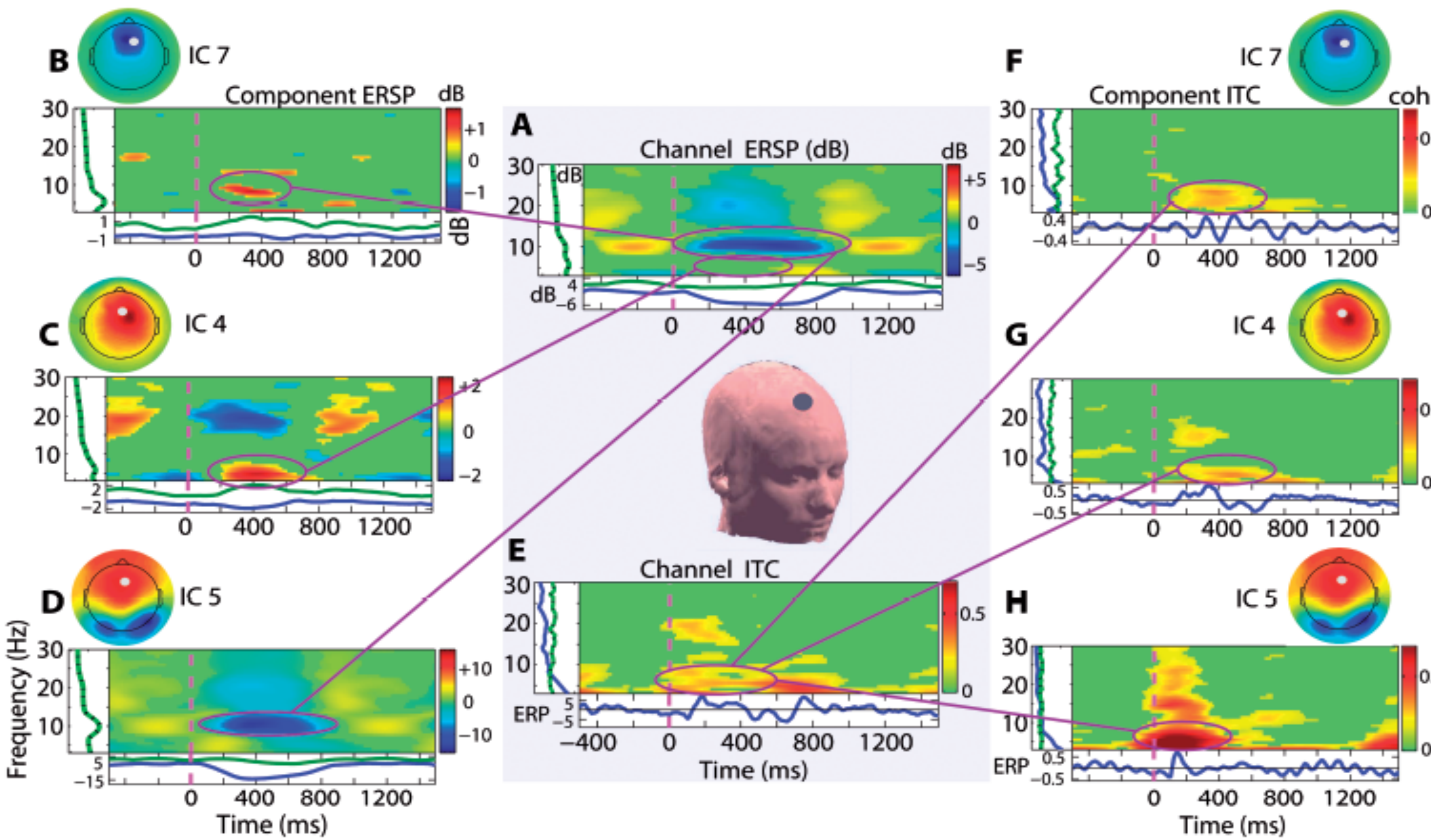






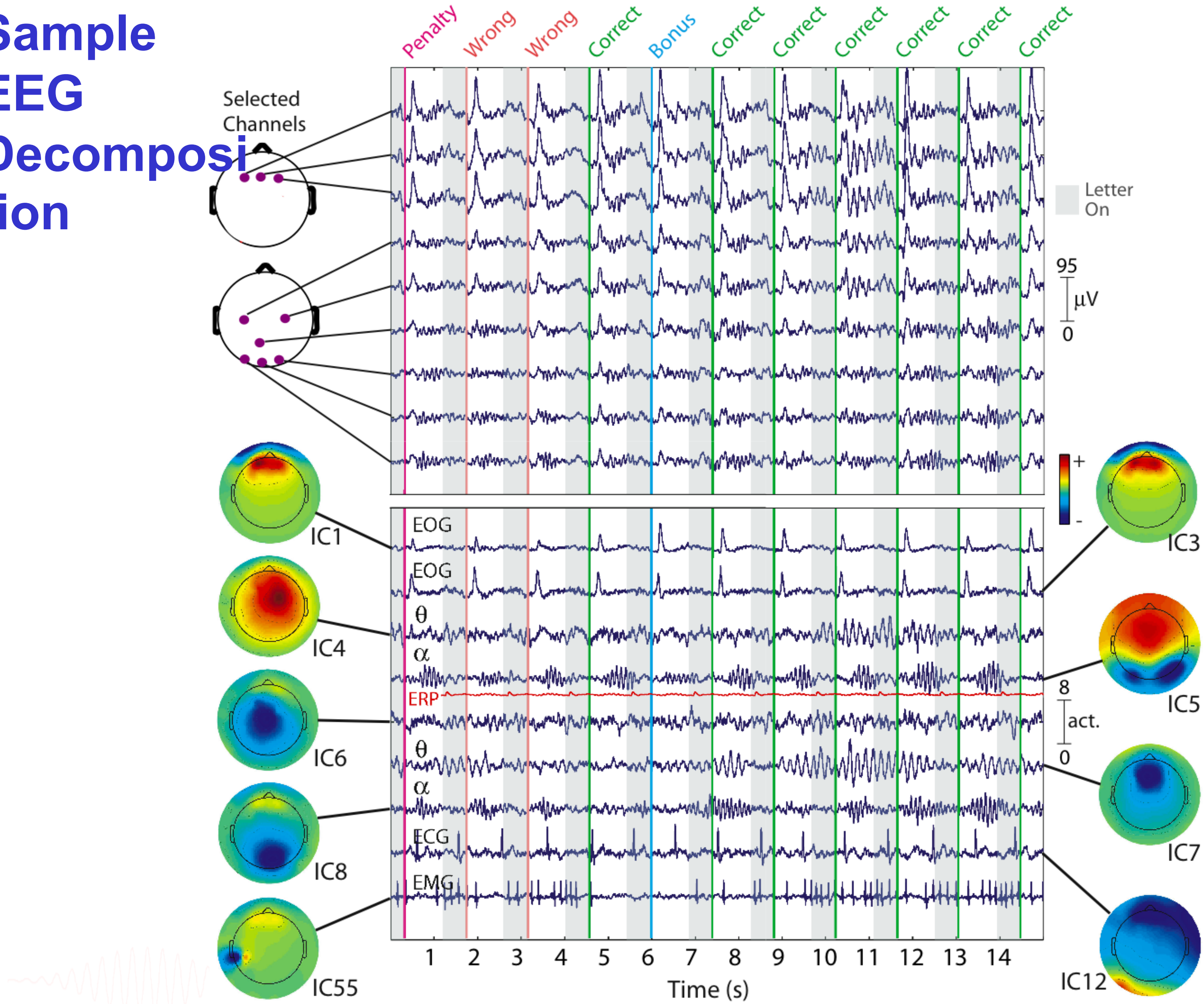
Credit: J. Onton

Why analyze source activity instead of channels?



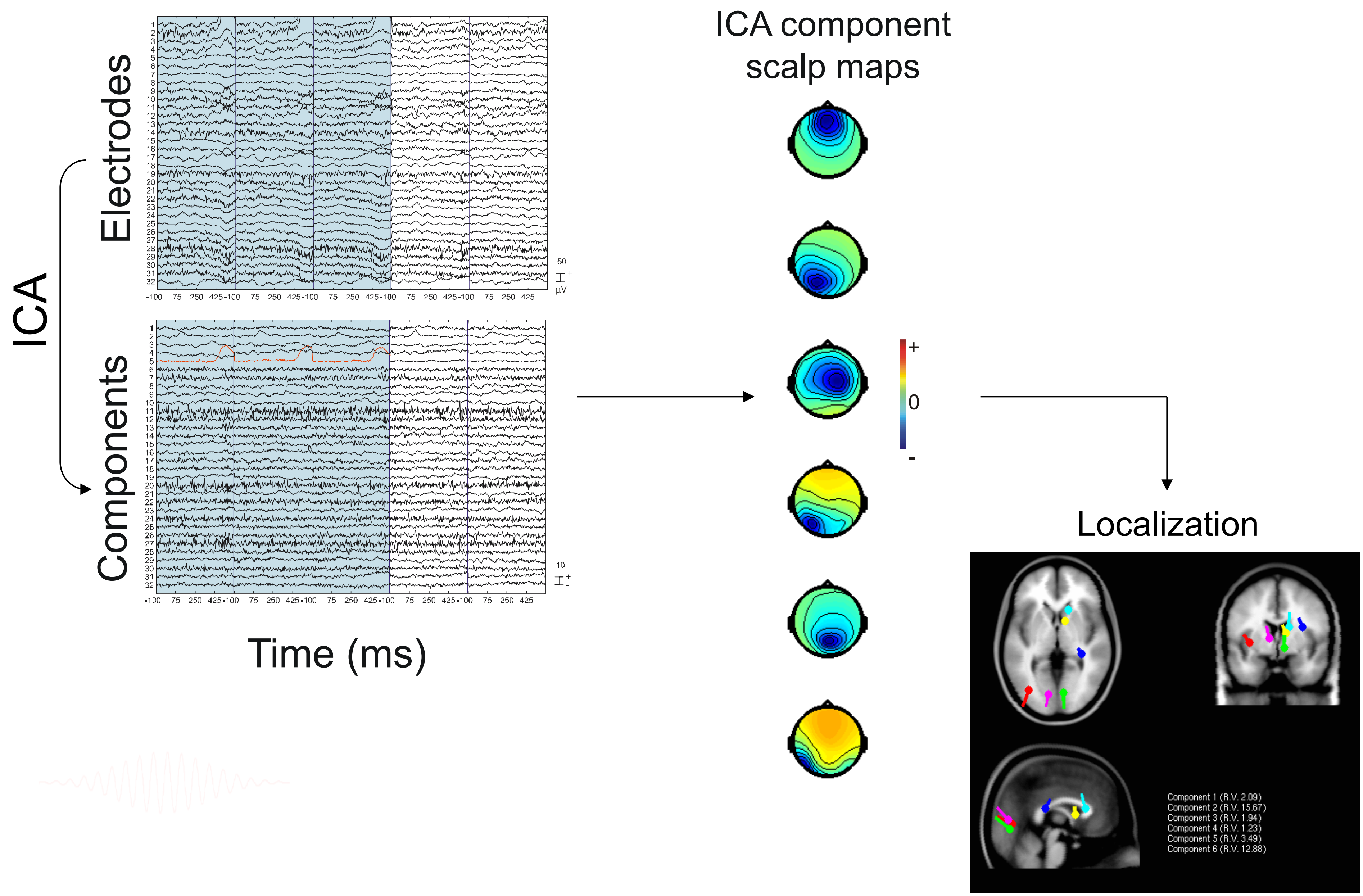
Credit: J. Onton

Sample EEG Decomposition



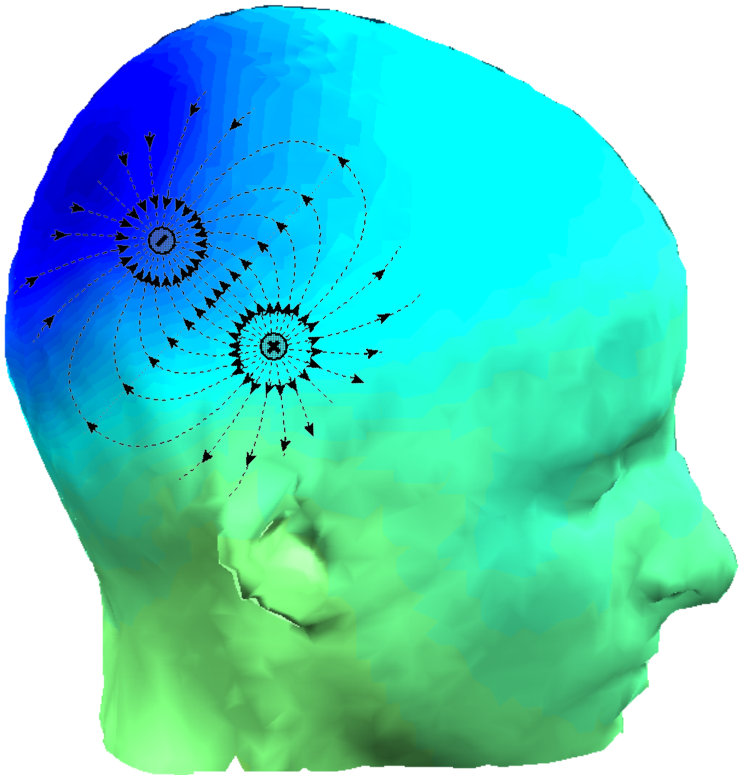
Credit: J. Onton

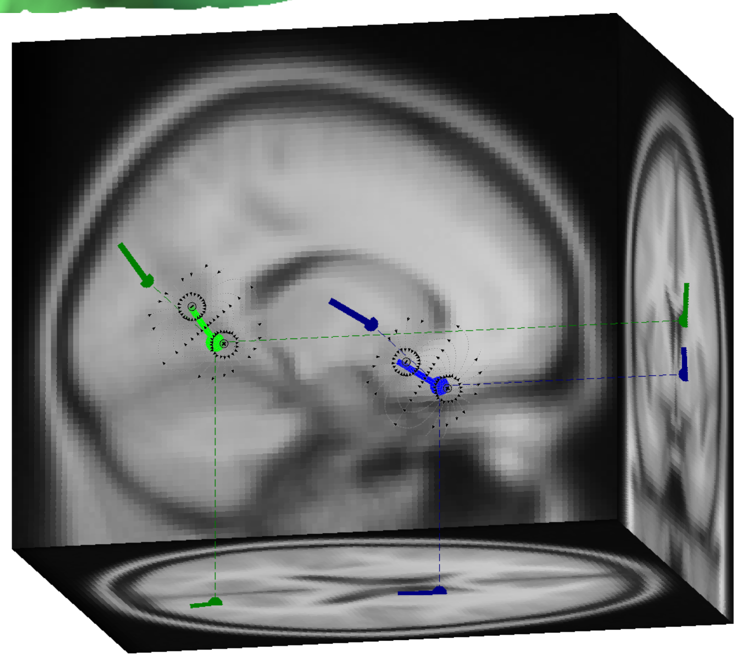
Localization

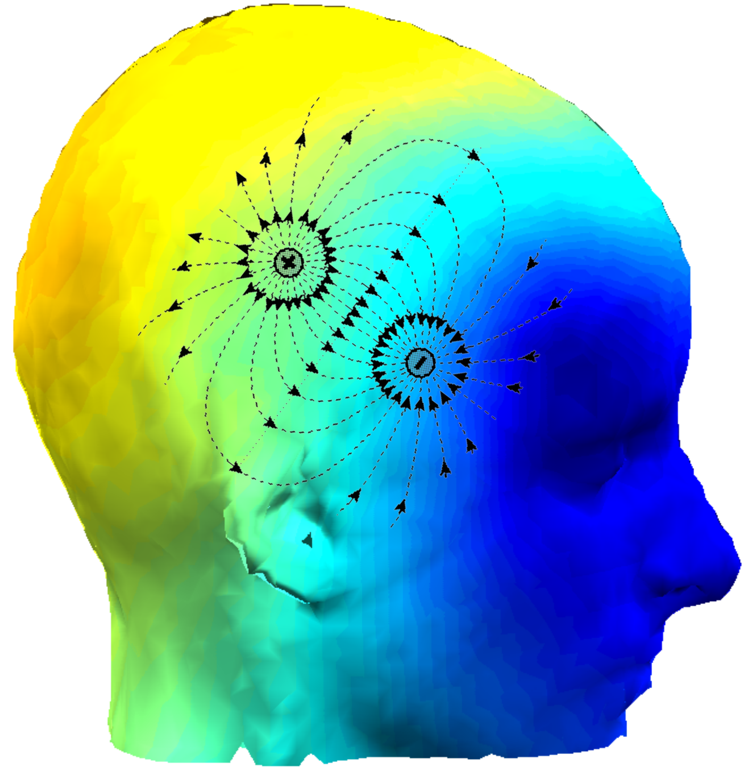


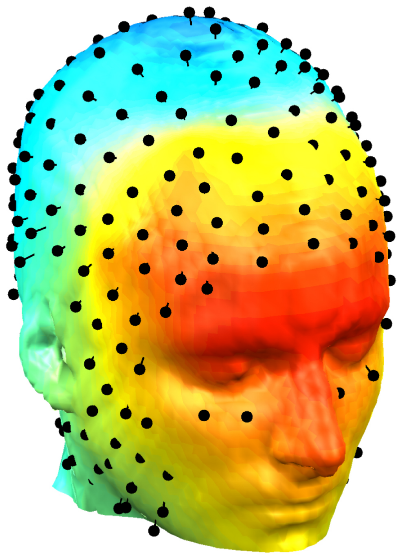
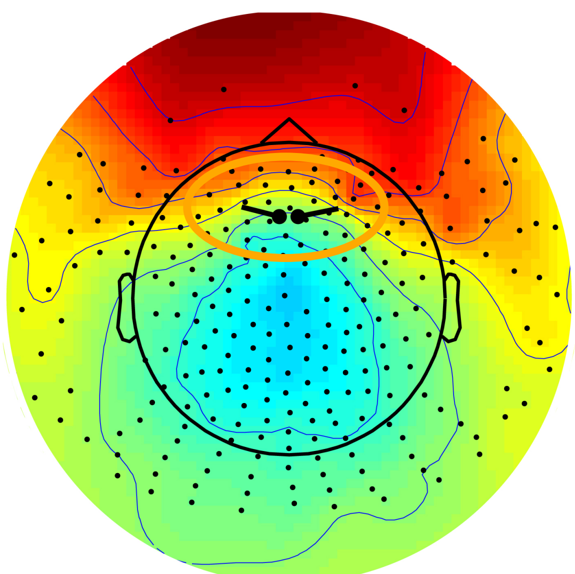
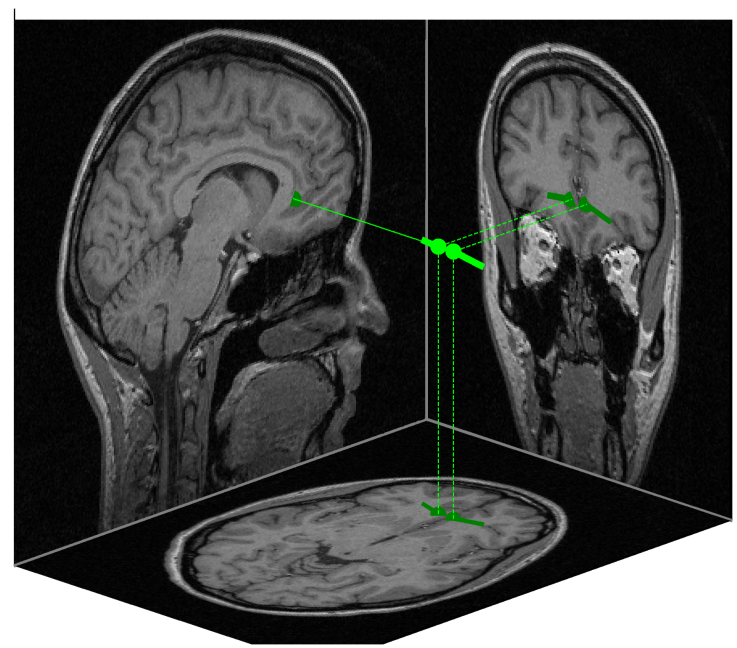
Dipolar Scalp Projections

ICA creates a spatial filter for each temporally independent source

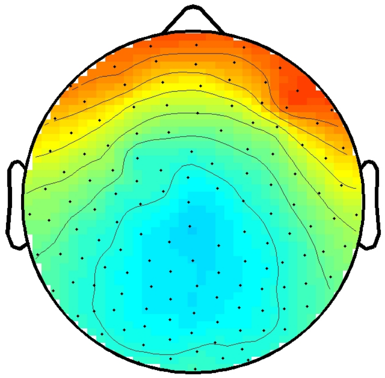


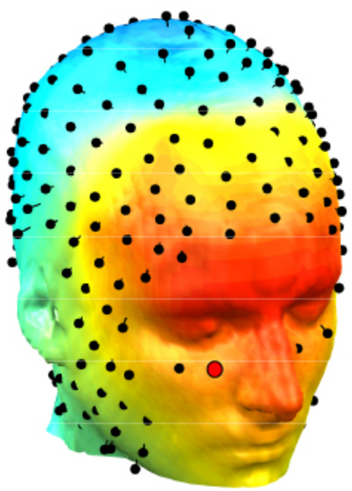




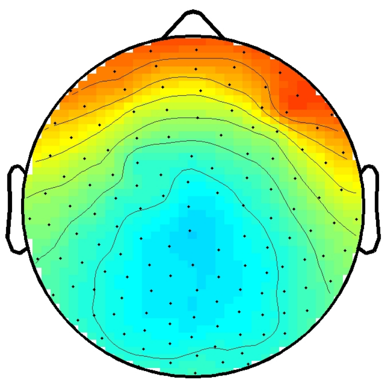




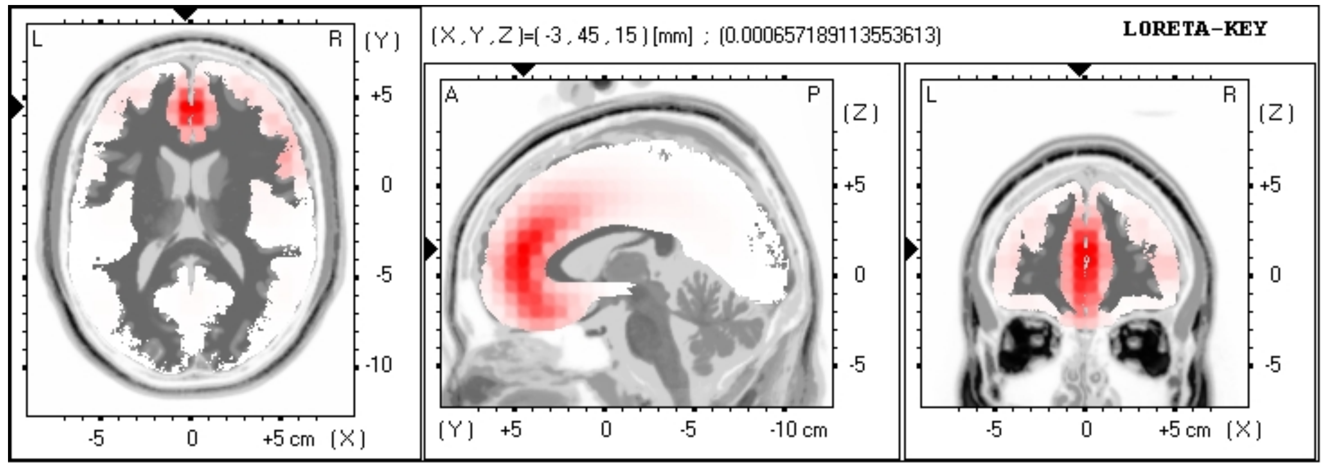
Localization of activity

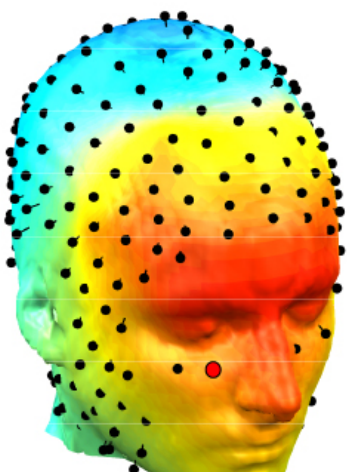


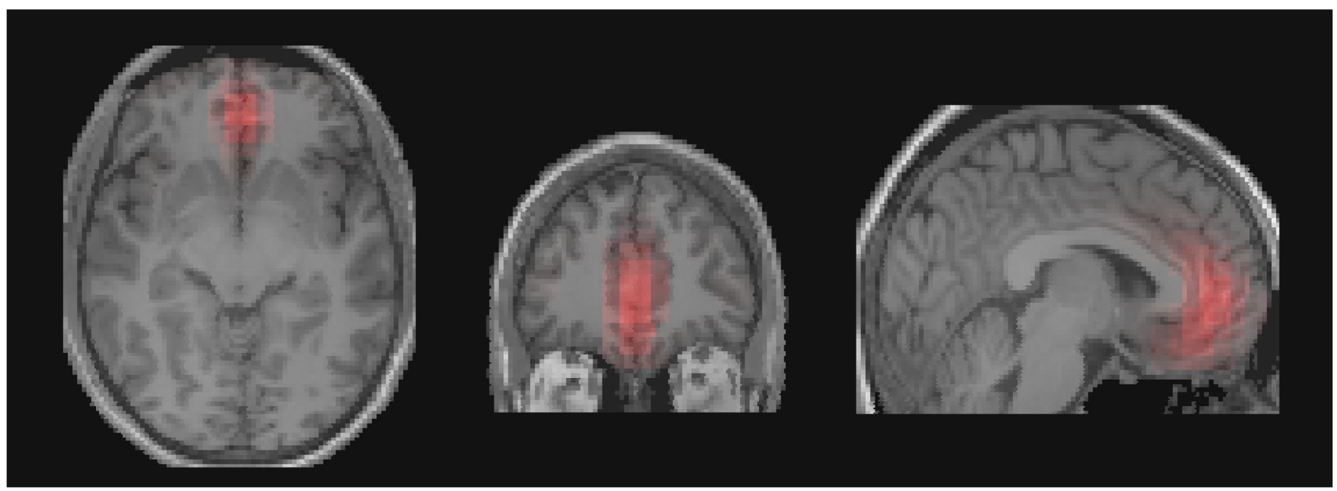


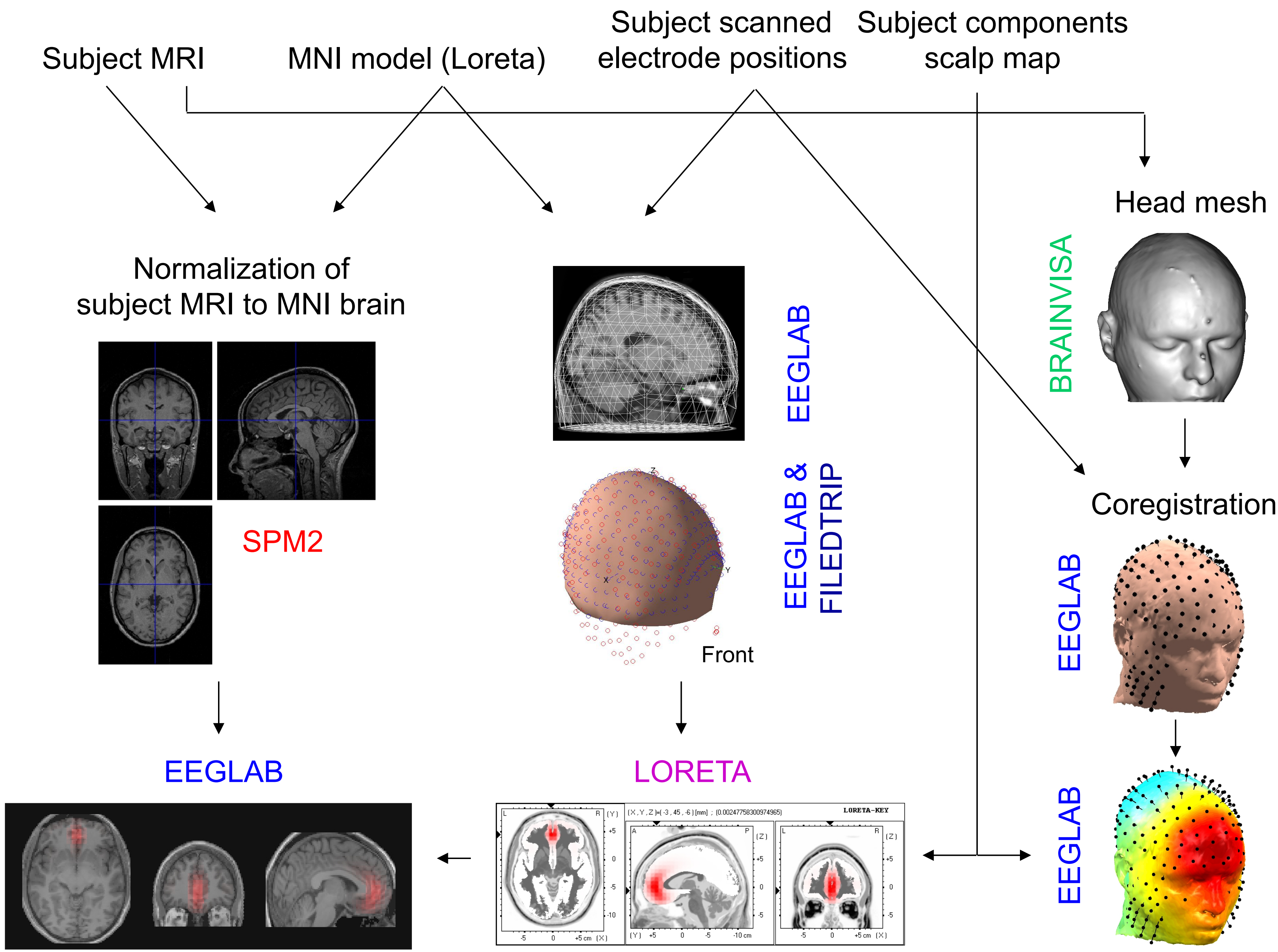
Localization of activity



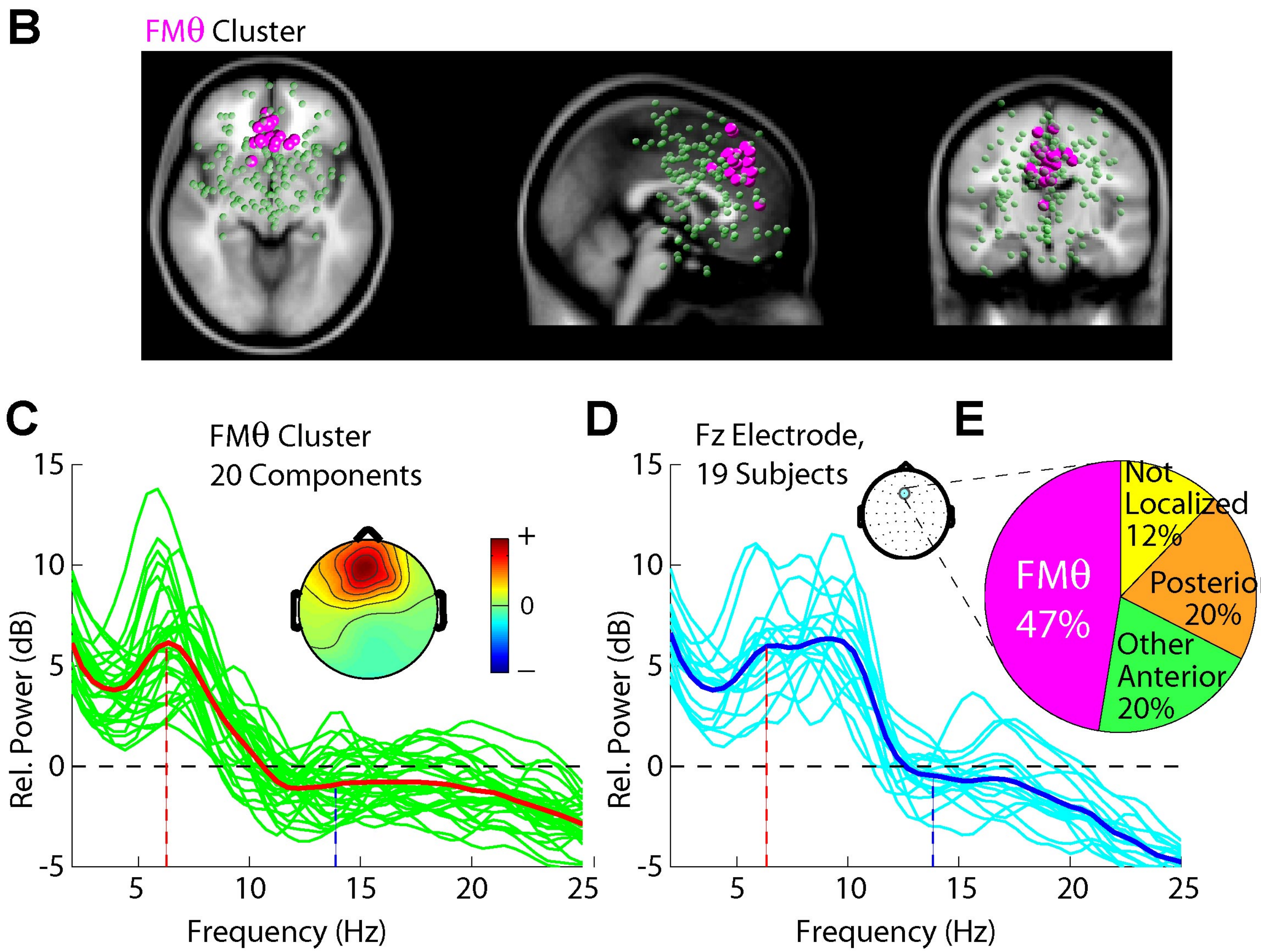






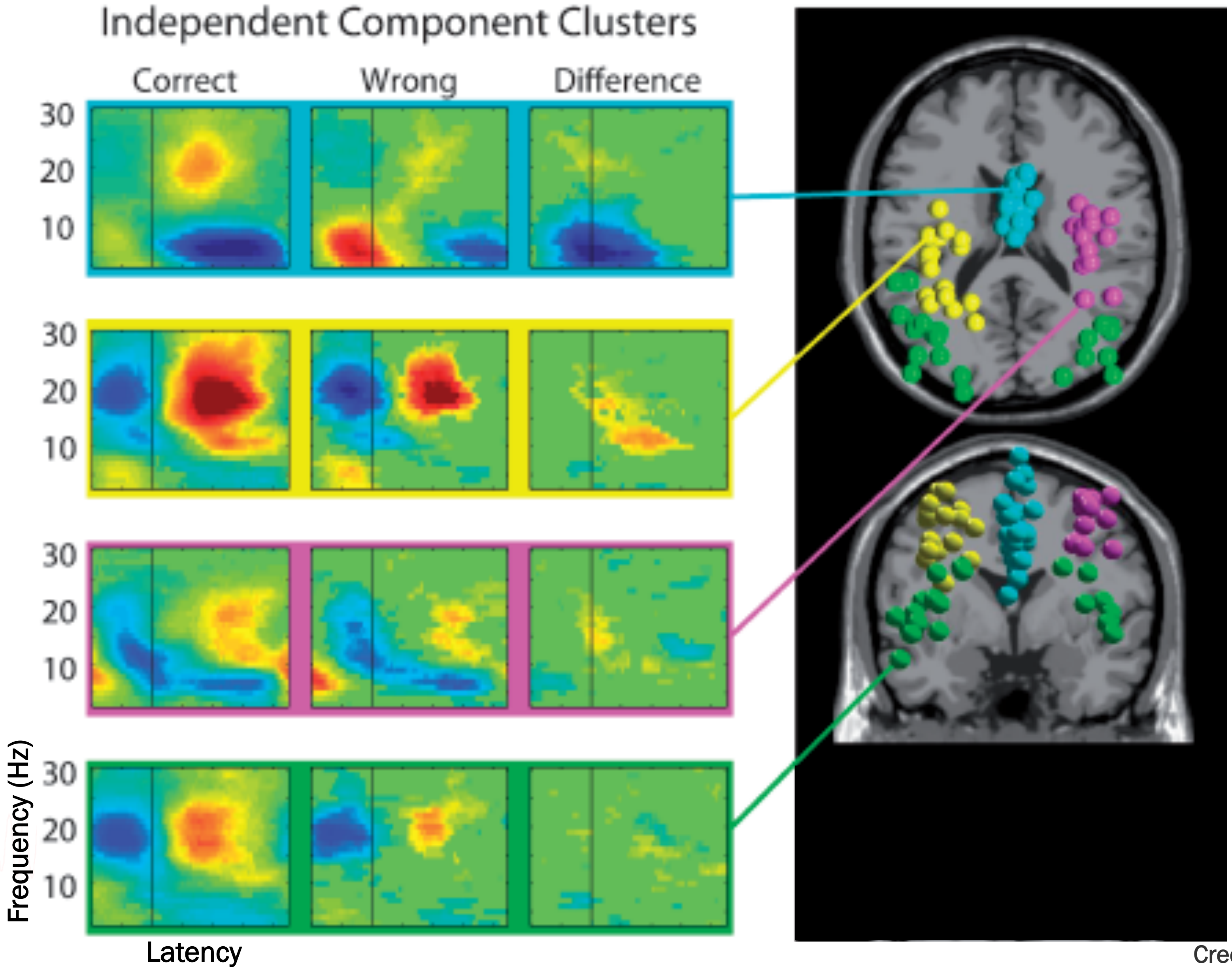

Subject MRI
MNI model (Loreta)
Subject scanned electrode positions
Subject components scalp map
Head mesh
BRAINVISA
Normalization of subject MRI to MNI brain
EEGLAB
EEGLAB & FILEDTRIP
Coregistration
SPM2
Front
EEGLAB
EEGLAB
LORETA
EEGLAB
LORETA-KEY

For example: frontal midline theta cluster



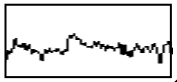
Onton, Delorme and Makeig, NeuroImage 27 (2005) 341 - 356

Goal: to cluster matching ICs across subjects



Credit J. Onton

Temporal ICA



ICA activity U

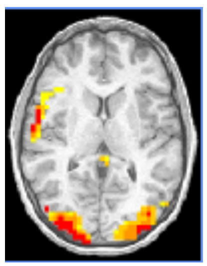
$$\begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} & u_{1,4} & \\ u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & \cdots \\ u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & \end{bmatrix} \begin{array}{l} \leftarrow \text{Comp. 1} \\ \leftarrow \text{Comp. 2} \\ \leftarrow \text{Comp. 3} \end{array}$$

$$\mathbf{X}=\mathbf{W}^{-1}\mathbf{U}$$

$$\begin{bmatrix} w^{-1}_{1,1} & w^{-1}_{1,2} & w^{-1}_{1,3} \\ w^{-1}_{2,1} & w^{-1}_{2,2} & w^{-1}_{2,3} \\ w^{-1}_{3,1} & w^{-1}_{3,2} & w^{-1}_{3,3} \end{bmatrix} \longrightarrow \begin{bmatrix} u_{1,1}w^{-1}_{1,1}+u_{2,1}w^{-1}_{1,2}+u_{3,1}w^{-1}_{1,3} & u_{1,2}w^{-1}_{1,1}+u_{2,2}w^{-1}_{1,2}+u_{3,2}w^{-1}_{1,3} & \\ u_{1,1}w^{-1}_{2,1}+u_{2,1}w^{-1}_{2,2}+u_{3,1}w^{-1}_{2,3} & u_{1,2}w^{-1}_{2,1}+u_{2,2}w^{-1}_{2,2}+u_{3,2}w^{-1}_{2,3} & \cdots \\ u_{1,1}w^{-1}_{3,1}+u_{2,1}w^{-1}_{3,2}+u_{3,1}w^{-1}_{3,3} & u_{1,2}w^{-1}_{3,1}+u_{2,2}w^{-1}_{3,2}+u_{3,2}w^{-1}_{3,3} & \end{bmatrix} \begin{array}{l} \leftarrow \text{Chan 1} \\ \leftarrow \text{Chan 2} \\ \leftarrow \text{Chan 3} \end{array}$$

Inverse weight matrix W^{-1}

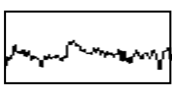
Data X (EEG/MEG time series)



Spatial ICA

ICA activity U

$$\begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} & u_{1,4} & \\ u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & \cdots \\ u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & \end{bmatrix} \begin{array}{l} \leftarrow \text{Comp. 1} \\ \leftarrow \text{Comp. 2} \\ \leftarrow \text{Comp. 3} \end{array}$$



$$\mathbf{X}=\mathbf{W}^{-1}\mathbf{U}$$

$$\begin{bmatrix} w^{-1}_{1,1} & w^{-1}_{1,2} & w^{-1}_{1,3} \\ w^{-1}_{2,1} & w^{-1}_{2,2} & w^{-1}_{2,3} \\ w^{-1}_{3,1} & w^{-1}_{3,2} & w^{-1}_{3,3} \end{bmatrix} \longrightarrow \begin{bmatrix} u_{1,1}w^{-1}_{1,1}+u_{2,1}w^{-1}_{1,2}+u_{3,1}w^{-1}_{1,3} & u_{1,2}w^{-1}_{1,1}+u_{2,2}w^{-1}_{1,2}+u_{3,2}w^{-1}_{1,3} & \\ u_{1,1}w^{-1}_{2,1}+u_{2,1}w^{-1}_{2,2}+u_{3,1}w^{-1}_{2,3} & u_{1,2}w^{-1}_{2,1}+u_{2,2}w^{-1}_{2,2}+u_{3,2}w^{-1}_{2,3} & \cdots \\ u_{1,1}w^{-1}_{3,1}+u_{2,1}w^{-1}_{3,2}+u_{3,1}w^{-1}_{3,3} & u_{1,2}w^{-1}_{3,1}+u_{2,2}w^{-1}_{3,2}+u_{3,2}w^{-1}_{3,3} & \end{bmatrix} \begin{array}{l} \leftarrow \text{Time 1} \\ \leftarrow \text{Time 2} \\ \leftarrow \text{Time 3} \end{array}$$

Inverse weight matrix W^{-1}

Data X (EEG/MEG voxel activities)