

# Forward and inverse models

Localizing sources using DIPFIT

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# DIPFIT: localizing dipoles

- Motivation
- Ingredients
  - Source model
  - Volume conductor model
    - Analytical (spherical model)
    - Numerical (realistic model)
  - Comparison EEG and MEG
- Inverse modeling
  - Single and multiple dipole fitting
  - Distributed source models

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# Motivation

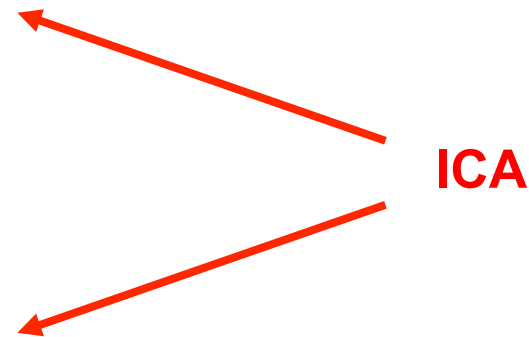
- Why fit dipoles?
- Why measure EEG?
- Why do ICA?
- Get extra information about brain processes
  - Time course of activity -----> **EEG**
  - Location of activity -----> **fMRI**

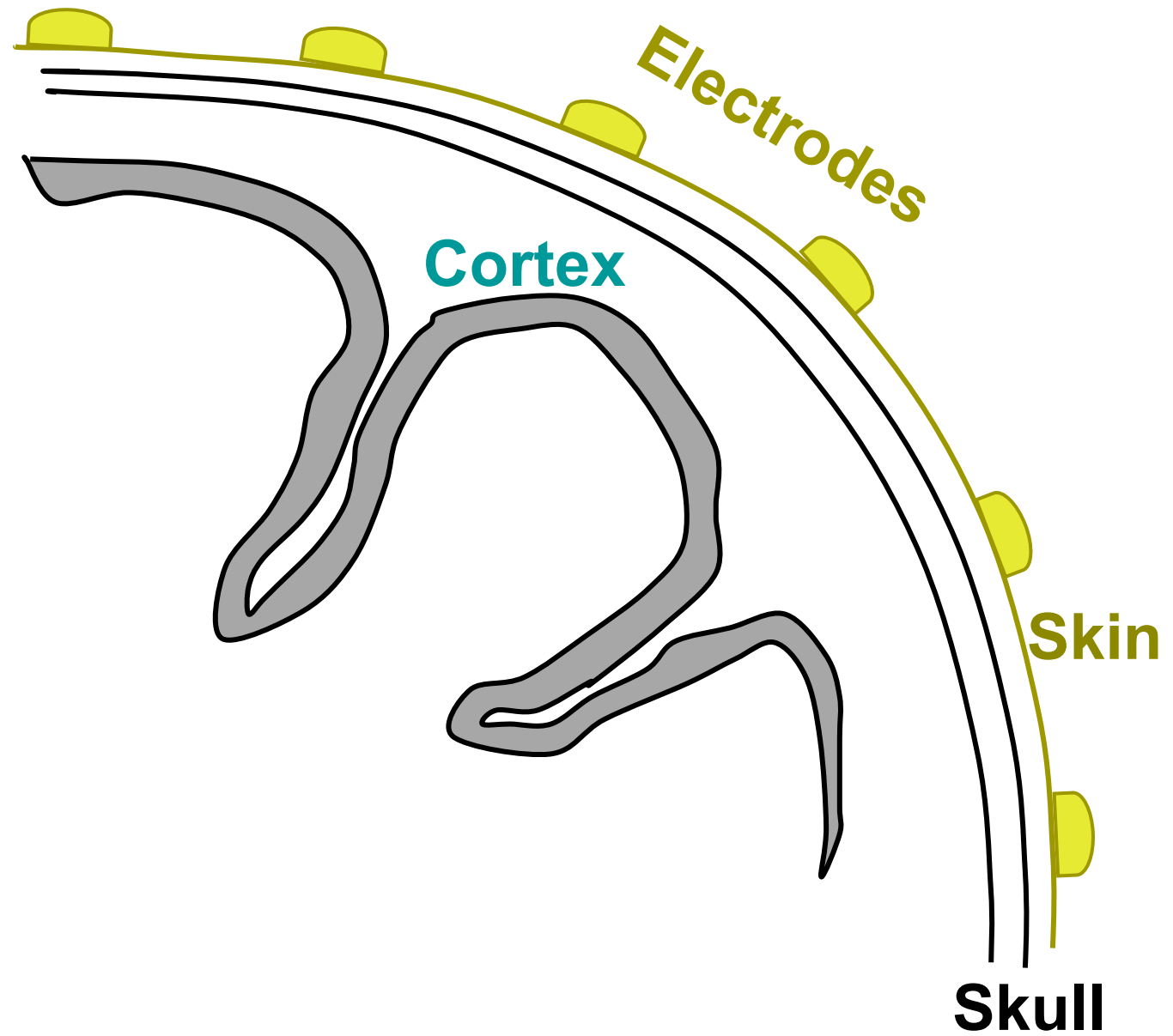
# Difference between EEG and fMRI

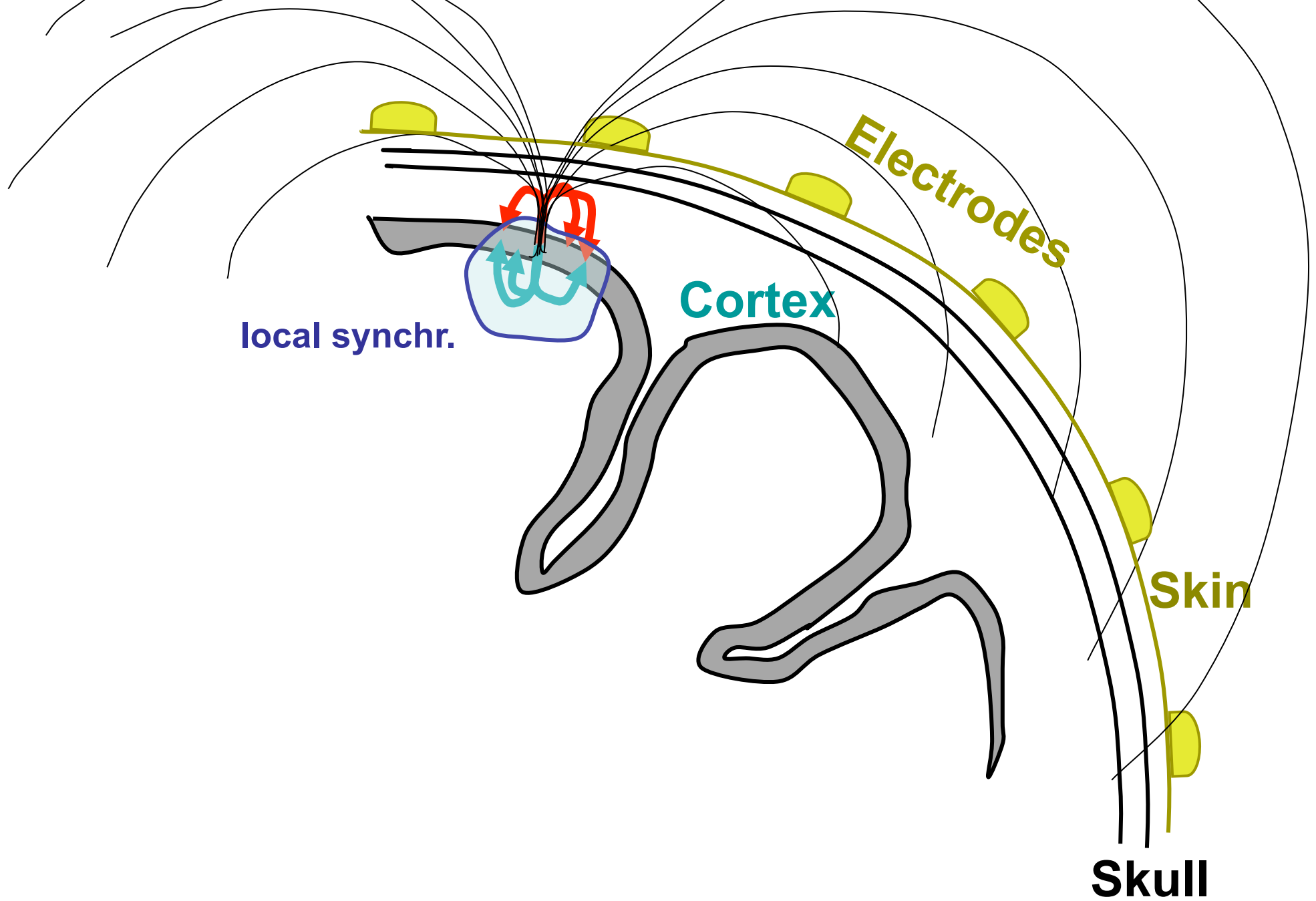
- EEG measures post-synaptic potentials
  - related to synchronized neuronal input
- fMRI measures BOLD
  - related to energy consumption
- Different characteristics in the time domain
- Different generators
- Timecourse *and* location

# Why EEG: extra information

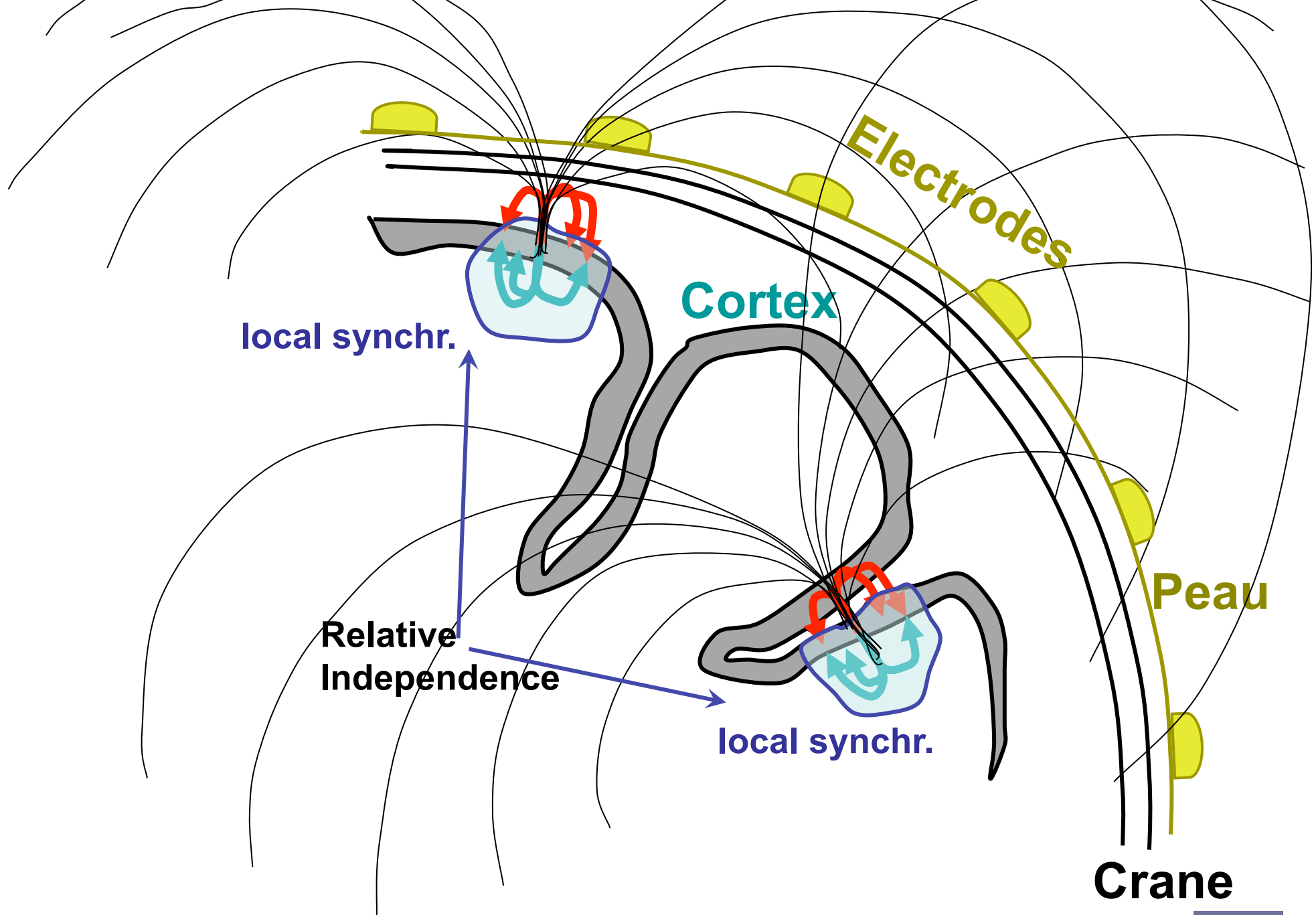
- Timecourse
  - ERSP
  - ERP
- Topography
  - Scalp distribution
  - Underlying generators





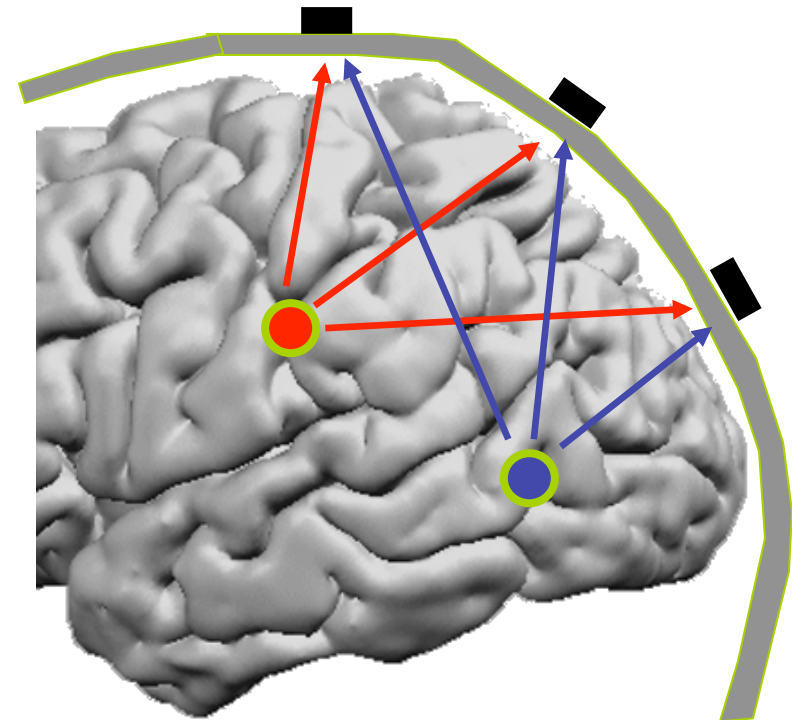
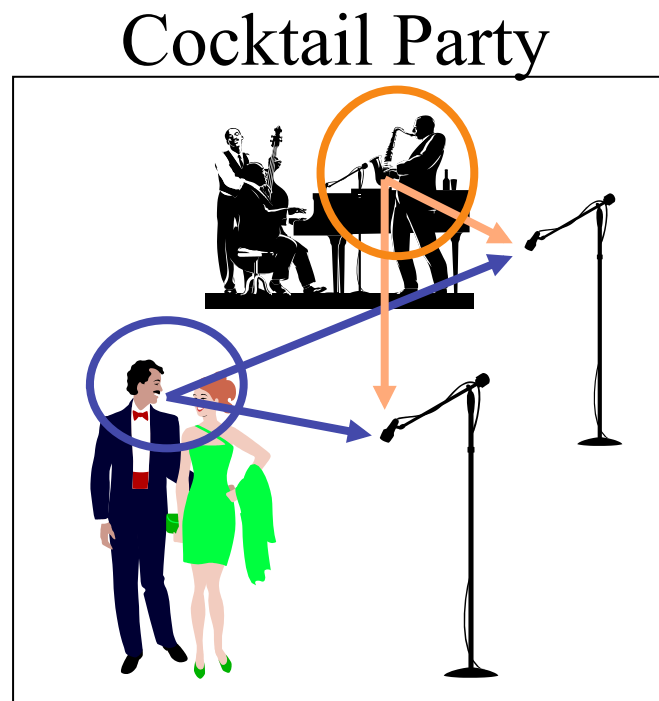


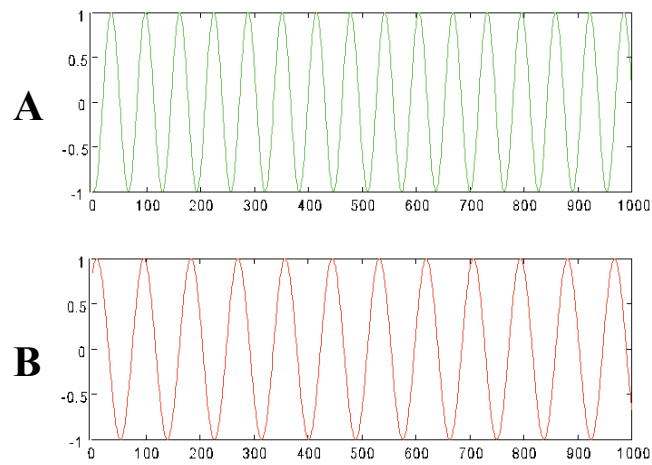




# Independent component analysis

Mixture of Brain source activity

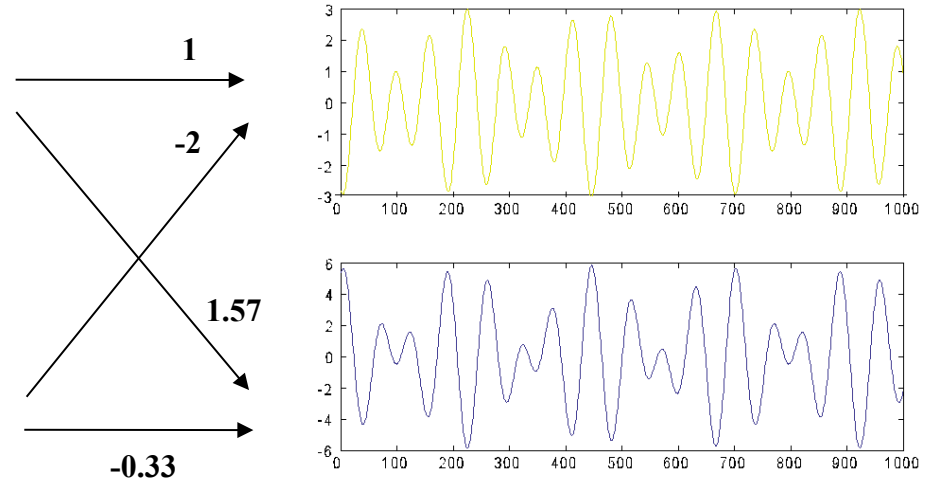




$$Y=[A;B]$$

**Linear Combination**

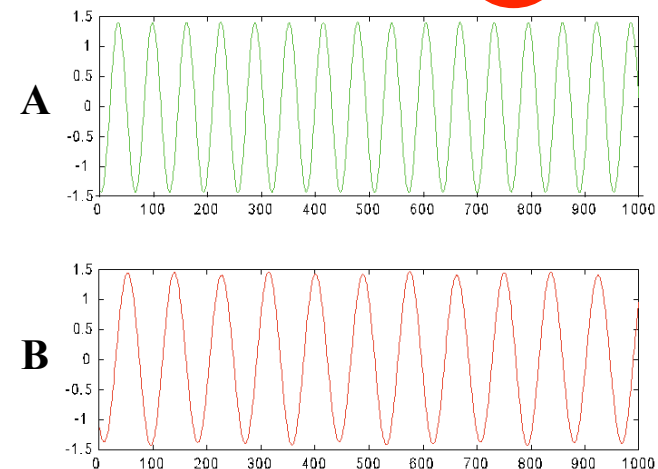
$$X=Y/W$$



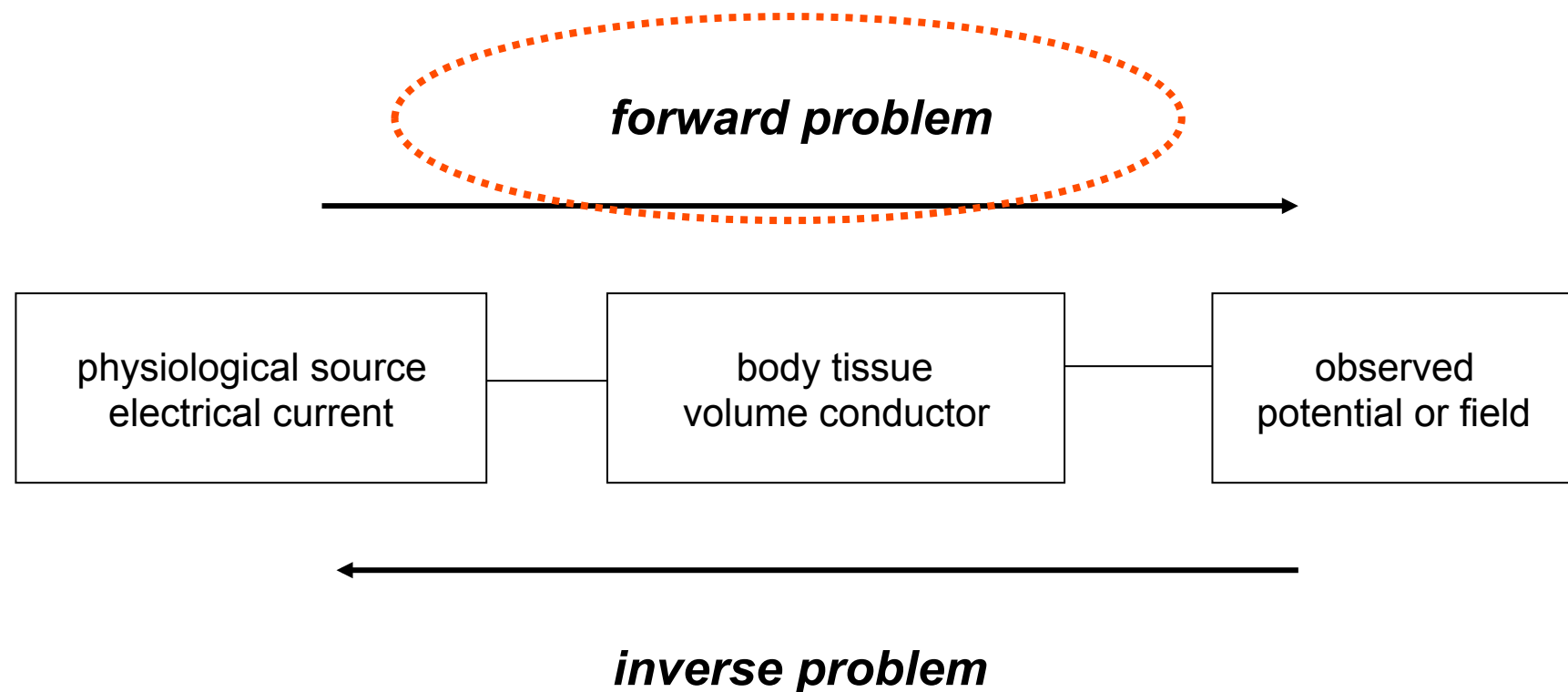
**Infomax ICA**

**ICA**

$$\tilde{Y}=W^{-1}X$$



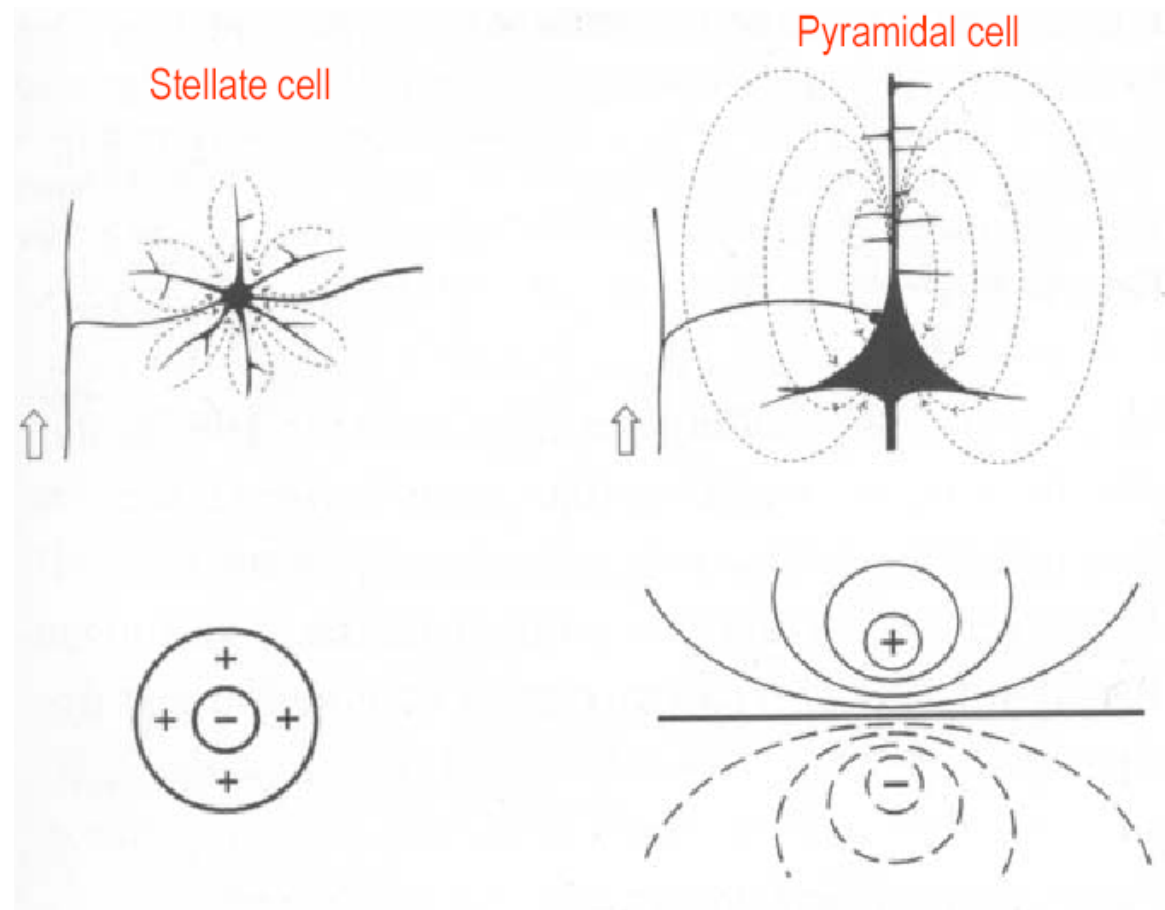
# Source modelling



# Overview

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# Neuronal currents



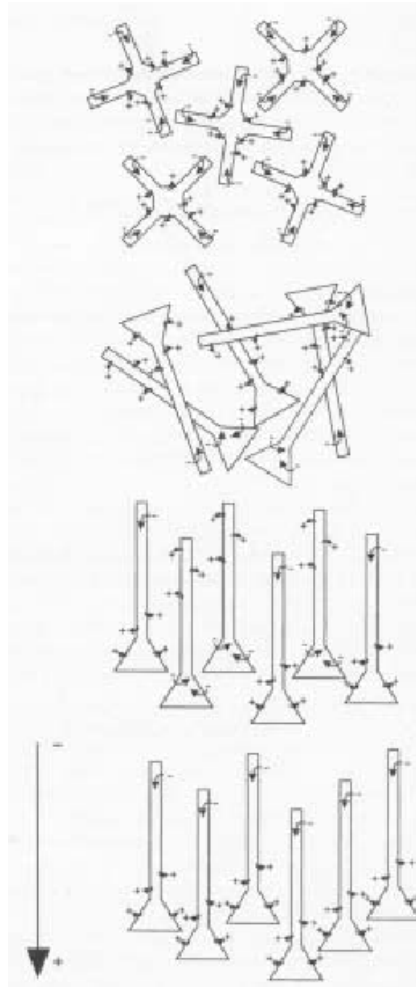
# Symmetry, orientation and activation

radial symmetric

random oriented

asynchronously activated

synchronously activated  
parallel oriented

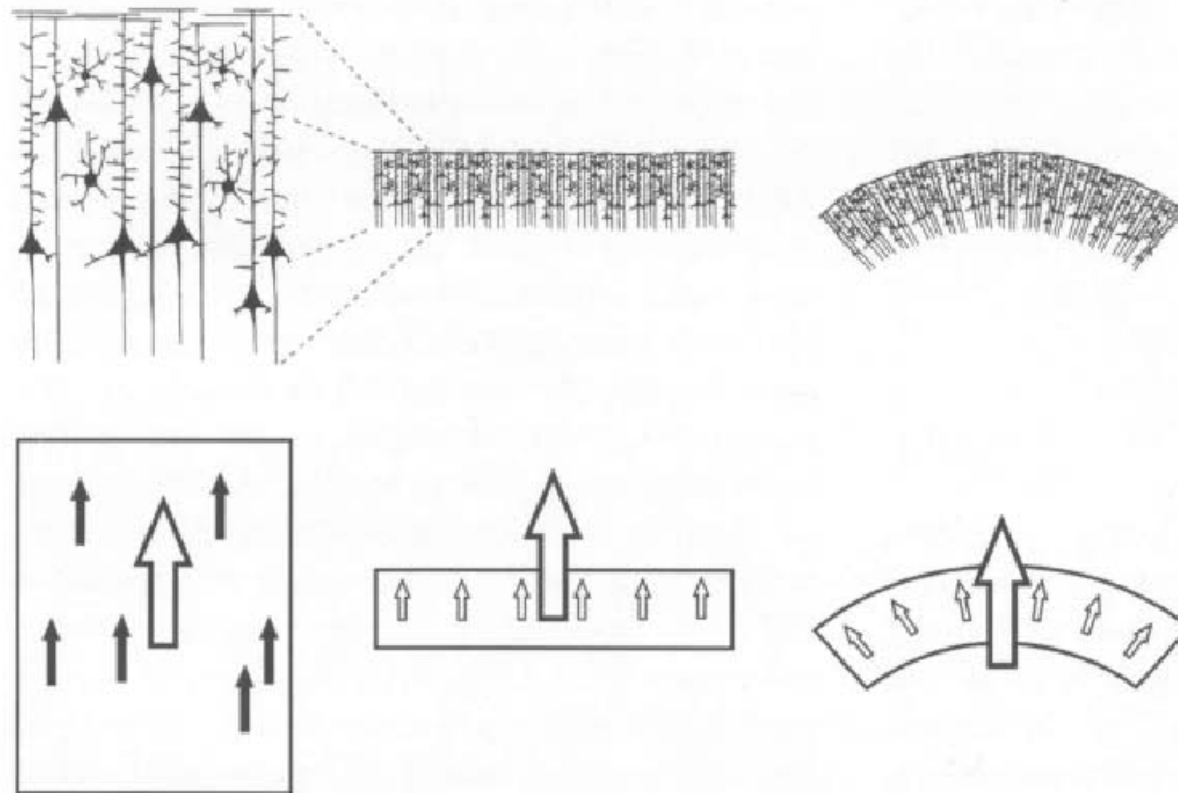


# Motivation for current dipoles

- Neurophysiological motivation



# Equivalent current dipoles



# Motivation for current dipoles

- Neurophysiological motivation
- Physical/mathematical motivation
  - Any current distribution can be written as a multipole expansion
  - First term: monopole (must be zero)
  - Second term: dipole
  - Higher order terms: quadrupole, octupole

# Motivation for current dipoles

- Neurophysiological motivation
- Physical/mathematical motivation
  - Any current distribution can be written as a multipole expansion
  - First term: monopole (must be zero)
  - Second term: dipole
  - Higher order terms: quadrupole, octupole
- Convenience
  - dipoles can be used as building block in distributed source models

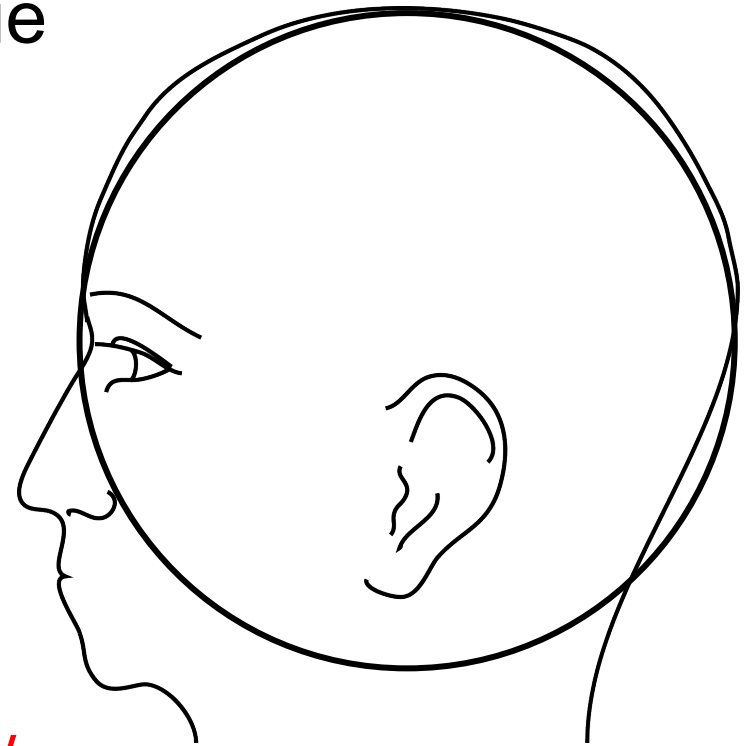
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# Volume conductor

- electrical properties of tissue
- geometrical description
- spherical model
- realistic shaped model

→ Describes how the currents flow,  
not where they originate from



# Volume conductor

- Advantages spherical model
  - mathematically accurate
  - reasonably accurate
  - computationally fast
  - easy to use
- Disadvantages spherical model
  - inaccurate, esp. in some regions
  - difficult alignment with anatomy

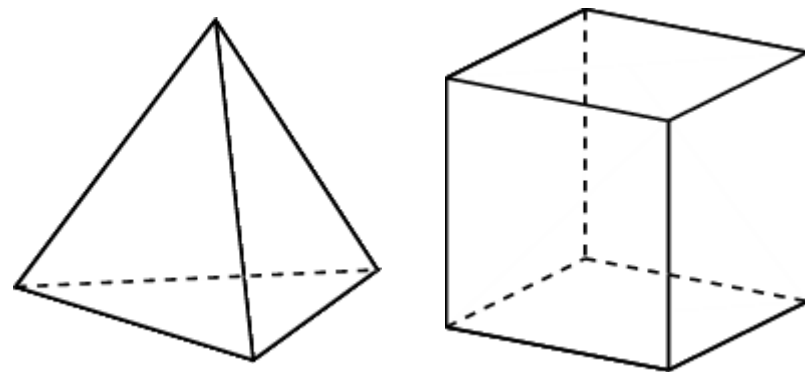
# Volume conductor

- Advantages realistic model
  - accurate solution for EEG
- Disadvantages realistic model
  - more work
  - individual anatomical MRI required
  - computationally slow(er)
  - numerically instable
  - difficult in interindividual comparison

→ The pragmatic solution is to use a standard realistic headmodel for EEG

# Realistic volume conductor

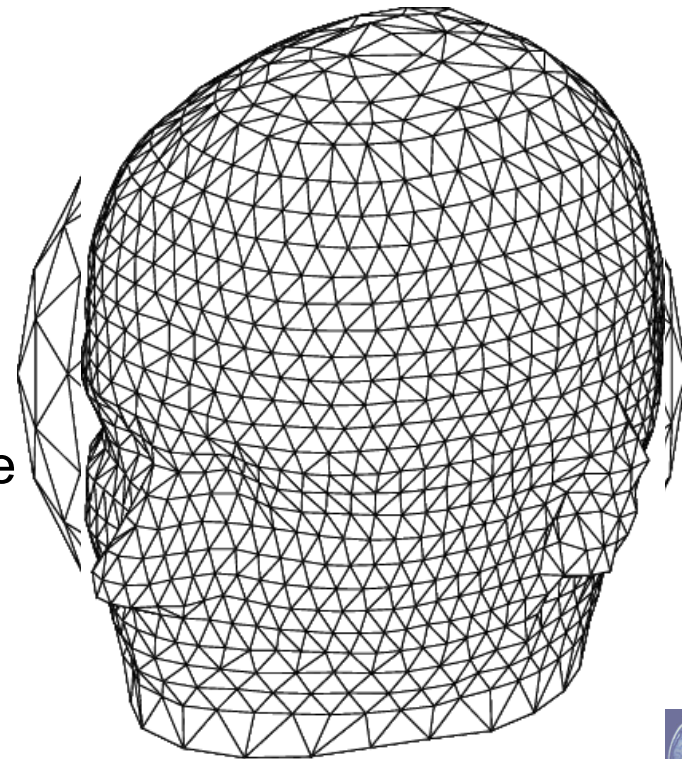
- Computational methods for volume conduction problem that allow realistic geometries
  - Boundary Element Method (BEM)
  - Finite Element Method (FEM)
- Geometrical description
  - triangles
  - tetraeders/voxels





# Volume conductor: BEM

- Boundary Element Method
  - description of geometry by compartments
  - each compartment is
    - homogenous
    - isotropic
  - important tissues
    - skin
    - skull
    - brain
    - (CSF)
  - triangulated surfaces as boundaries
  - surfaces should be closed



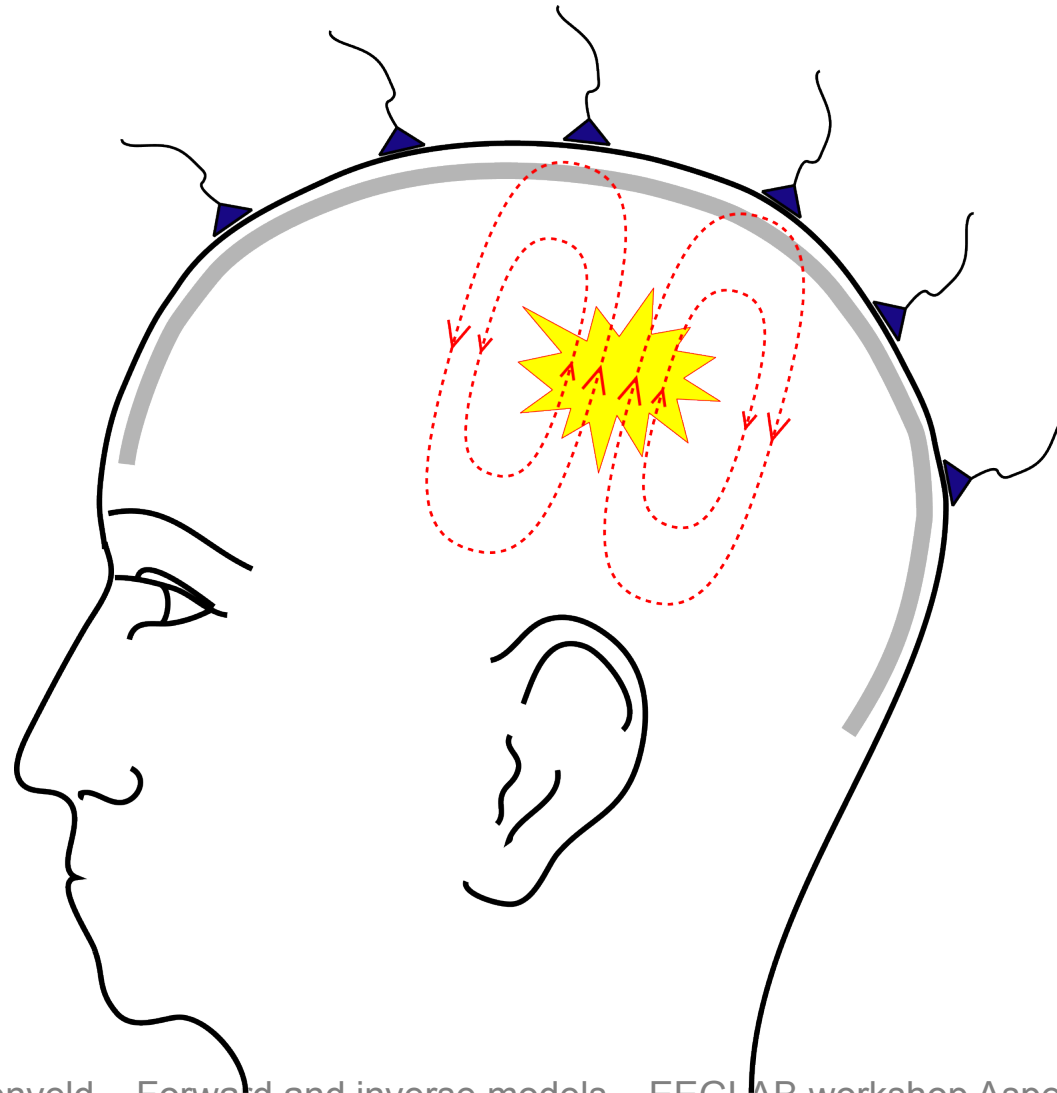
# Volume conductor: FEM

- Tessellation of 3D volume in tetraeders
- Large number of elements
- Each tetraeder can have its own conductivity
- FEM is most accurate numerical method
- Computationally expensive
- Accurate conductivities are not (well) known

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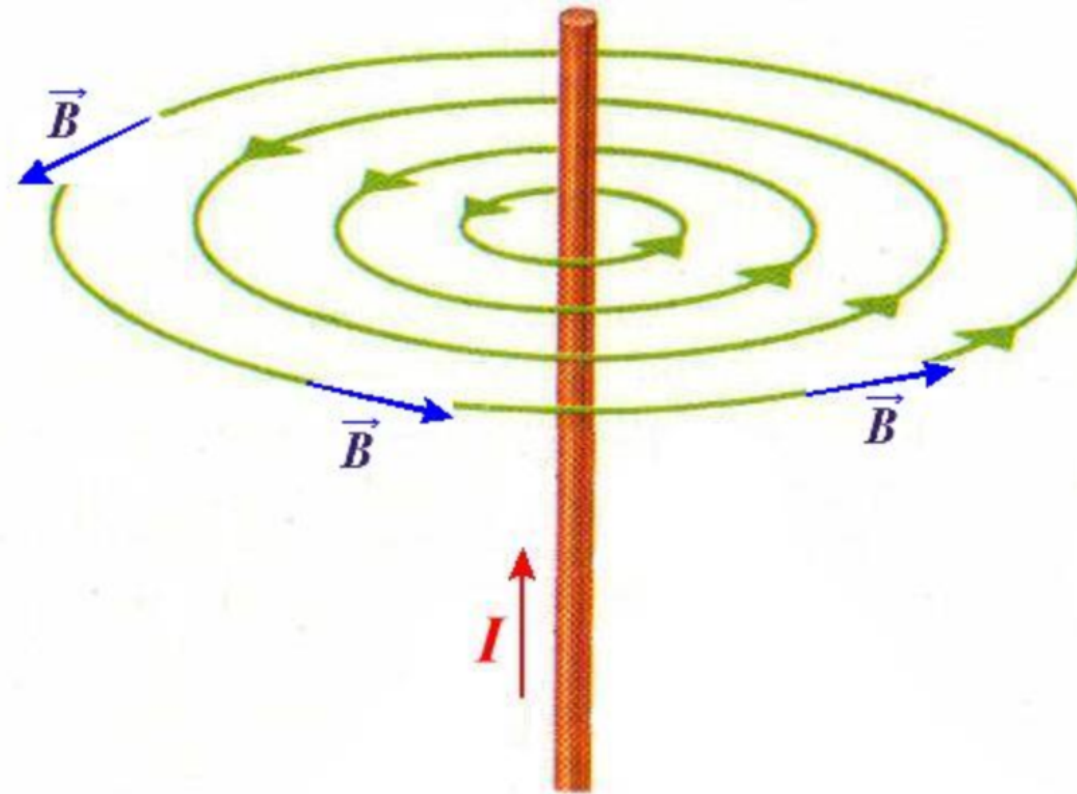
# EEG volume conduction



# EEG volume conduction

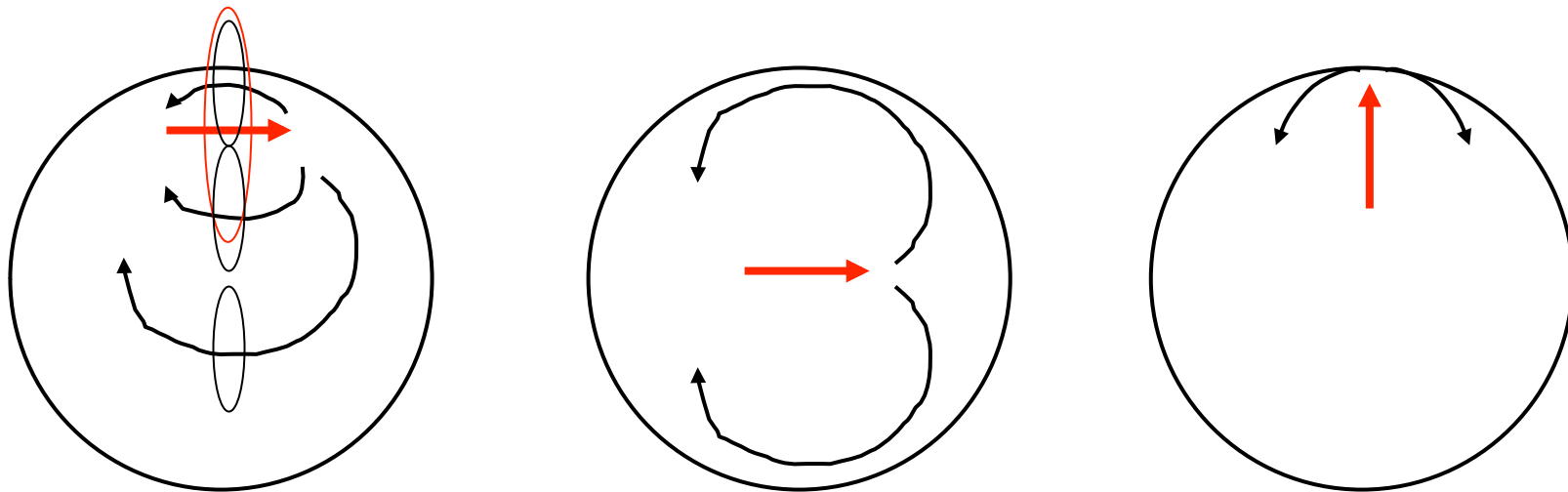
- Potential difference between electrodes corresponds to current flowing through skin
- Only tiny fraction of current passes through skull
- Therefore the model should describe skull and skin as accurately as possible
- Problems with skull
  - Not visible in anatomical MRI
  - Thickness varies
  - Conductivity is not homogeneous
  - Complex geometry at base of skull

# Electric current $\rightarrow$ magnetic field



# MEG volume conduction

- Measures sum of fields associated with
  - Primary currents
  - Secondary currents !!!



# MEG volume conduction

- Only tiny fraction of current passes through the poorly conductive skull
- Therefore skull and skin **can be neglected** in the MEG model
- Local conductivity around dipole important
  - geometry
  - conductivity



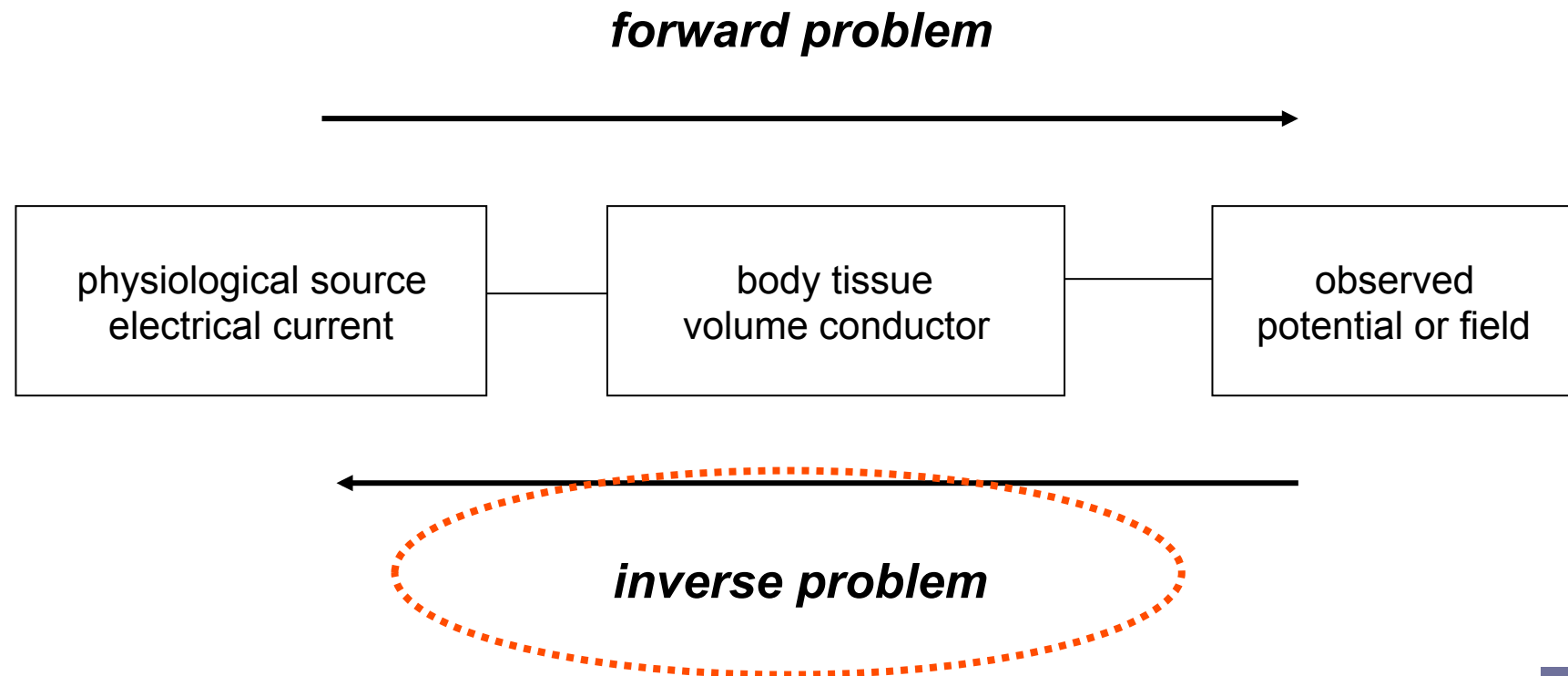
# Differences between EEG and MEG

- more blurred
- deep sources
- electrode noise
- reference electrode
- fixed to head
- skull+skin important for modelling
- no radial or deep sources
- environmental noise
- independent sensors
- head can move
- skull+skin not important

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# Source modelling



# Inverse methods

- Single and multiple dipole models
  - Minimize error between model and measured potential/field
- Distributed dipole models
  - Perfect fit of model to the measured potential/field
  - Minimize additional constraint on sources
  - LORETA (smoothness)
  - Minimum Norm (L2)
  - Minimum Current (L1)
- Spatial filtering
  - Scan whole brain with single dipole and compute the filter output at every location
  - MUSIC
  - Beamforming (e.g. LCMV, SAM, DICS)

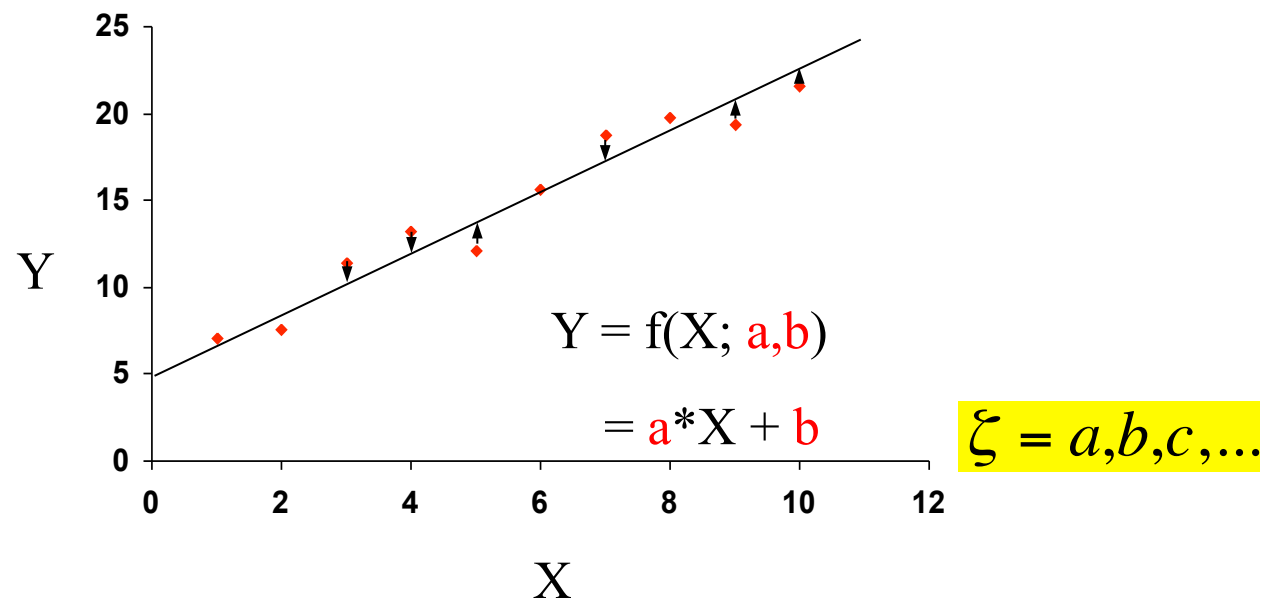
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# Single or multiple dipole models

- Manipulate source parameters to minimize error between measured and model data
  - Location of each source
  - Orientation of each source
  - Strength of each source
- Orientation and strength together correspond to the “dipole moment” and can be estimated linearly
- Position is estimated non-linearly
- Source **parameter estimation**

# Parameter estimation



# Parameter estimation: model

measured potential

$$V_i = V(\vec{r}_i) + \text{noise}$$

forward model for the data

$$Y_i = Y(r_i; \xi) + \text{noise}$$

select “optimal” model

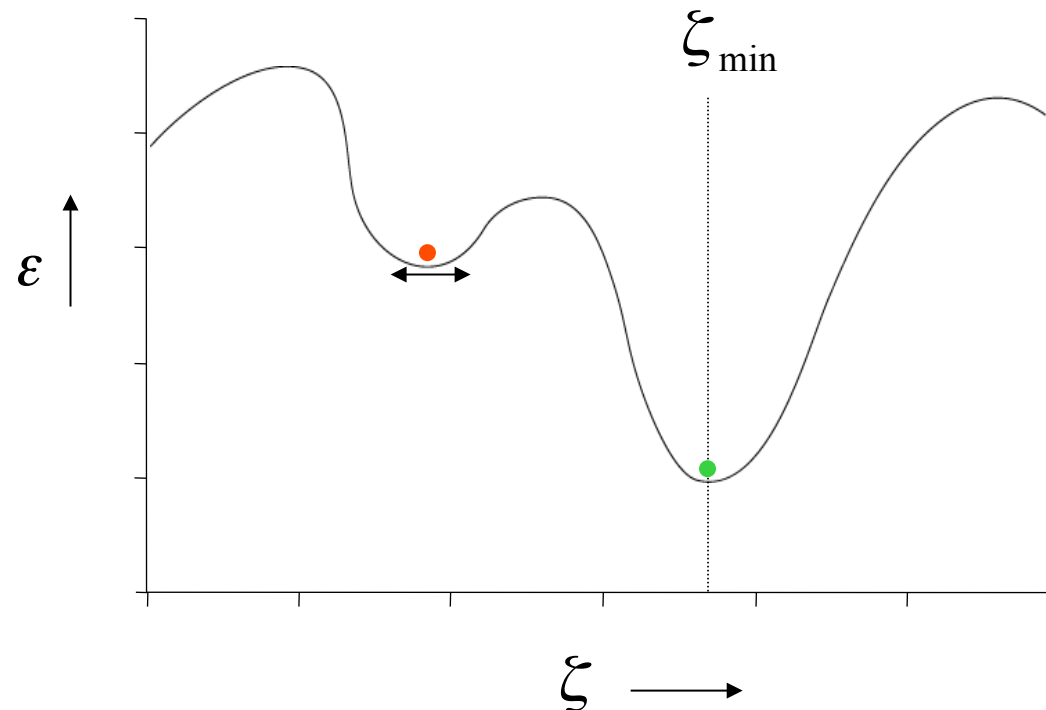
$$\min_{\xi} \left\{ \sum_{i=1}^N (Y(r_i; \xi) - V(r_i))^2 \right\}$$



# Select optimal model

$$error(\xi) = \sum_{i=1}^N (Y_i(\xi) - V_i)^2 \Rightarrow \min_{\xi} (error(\xi))$$

$$\xi = a, b, c, \dots$$



# Dipole *scanning*: grid search

- define grid with allowed dipole locations
  - compute optimal dipole moment for each location
  - compute value of goal-function
  - plot value of goal-function on grid
- 
- number of evaluations:
    - single dipole, 1 cm grid: ~4 000
    - single dipole,  $\frac{1}{2}$  cm grid: ~32 000
    - two dipoles, 1cm grid: ~16 000 000

# Dipole *fitting*: nonlinear search

- start with an initial guess
  - evaluate the local derivative of goal-function
  - “walk down hill” to the most optimal solution
- 
- number of evaluations: ~100

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# Distributed source model

- Position of the source is **not estimated** as such
  - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
  - In principle easy to solve, however...
  - More “unknowns” (parameters) than “knowns” (measurements)
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem

# Distributed source model

- Linear estimation

$$\vec{\Psi} = q_1 \vec{\Psi}_1 + q_2 \vec{\Psi}_2 + \dots = \begin{bmatrix} \Psi_{1,1} & \Psi_{2,1} & \dots \\ \Psi_{1,2} & \Psi_{2,2} & \dots \\ \vdots & \vdots & \ddots \\ \Psi_{1,N} & \Psi_{2,N} & \dots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = L \cdot \vec{q}$$

$$\vec{q} = L^{-1} \cdot \vec{\Psi}$$

# Distributed source model

$$V = L \cdot q + \text{Noise}$$

$$\min_q \{ \|V - L \cdot q\|^2 \} = 0 \quad !!$$

- Regularized linear estimation:

$$\min_q \{ \|V - L \cdot q\|^2 + \lambda^2 \cdot \|D \cdot q\|^2 \}$$

- Constrained linear estimation:

$$\min_q \{ q^T \cdot W \cdot q \} \quad \text{while} \quad \|V - L \cdot q\|^2 = 0$$

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# Spatial filtering

- position of the source as such is not estimated
- scanning the whole brain
  - single dipole as source
  - estimate activity at each grid location
    - that explains a part of the data
    - that suppresses other activity
- various methods
  - multiple signal classification (MUSIC)
  - beamforming
  - LCMV, SAM, DICS, ...
- not a distributed source model, but a distributed representation of the single dipole estimate
- unmixing of data into “signal source” and “noise sources” using assumptions on temporal relation between sources

# Summary 1

- Forward modelling
  - Required for the interpretation of scalp topographies
  - Interpretation of scalp topography *is* “source estimation”
  - Mathematical techniques are available that aid in interpreting scalp topographies -> inverse modeling

# Summary 2

- Inverse modeling
  - Model assumption for volume conductor
  - Model assumption for source (I.e. dipole)
  - Additional assumptions on source
    - Single point-like source
    - Multiple point-like sources
    - Distributed source
  - Different mathematical solutions
    - Dipole fitting (linear and nonlinear)
    - Linear estimation (regularized)