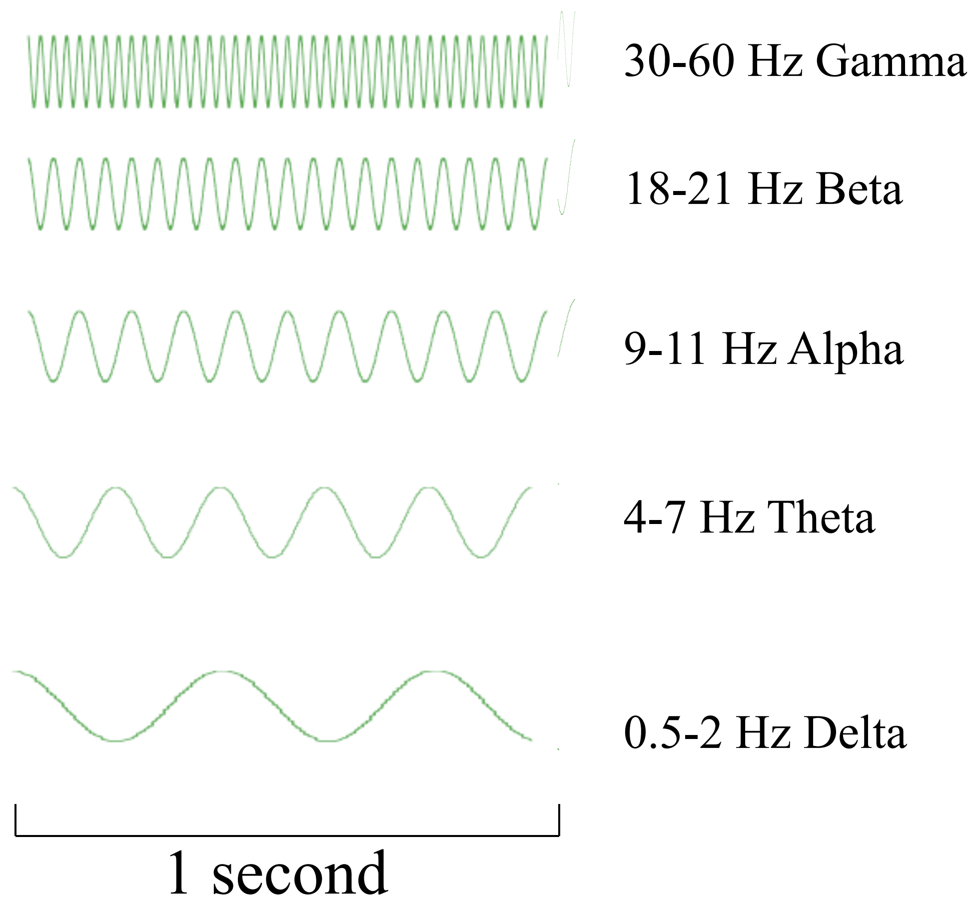
Time-Frequency Analysis of Biophysical Time series

June 20, 2011 Thirteenth EEGLAB Workshop, Aspet, France

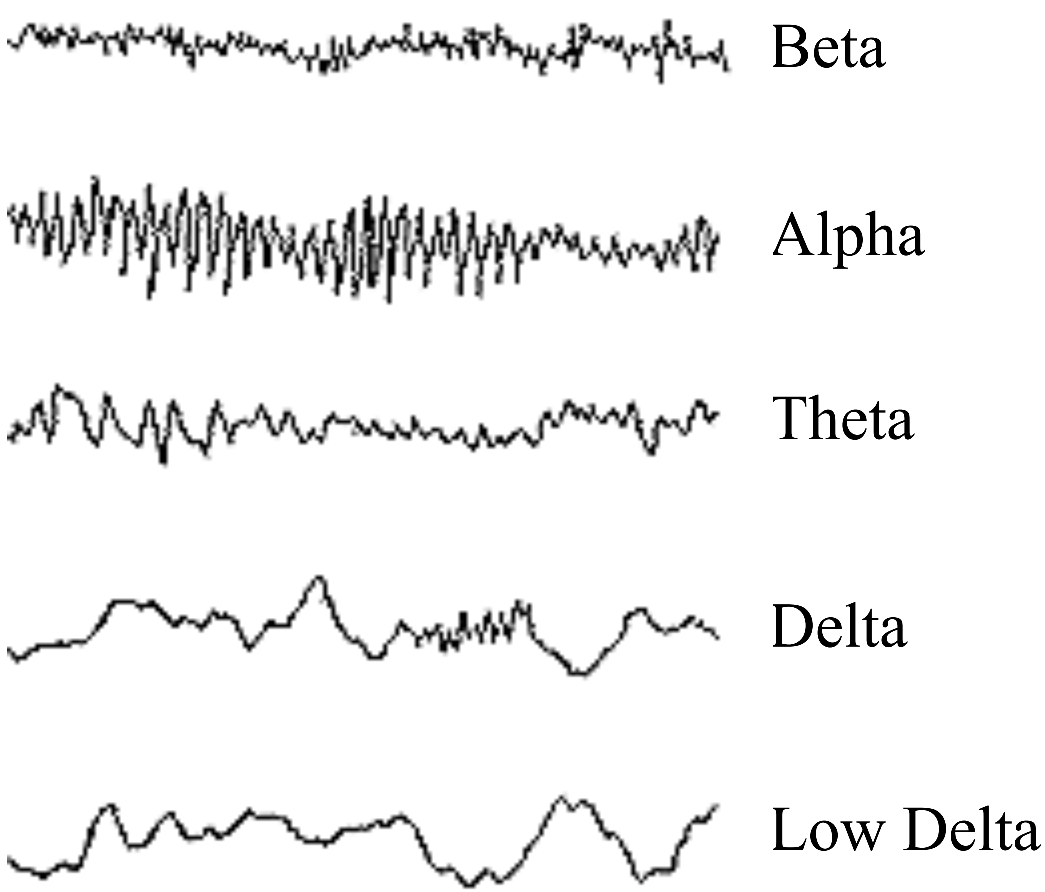
Tim Mullen
(with majority of slides modified from those of Arnaud Delorme)

Frequency analysis

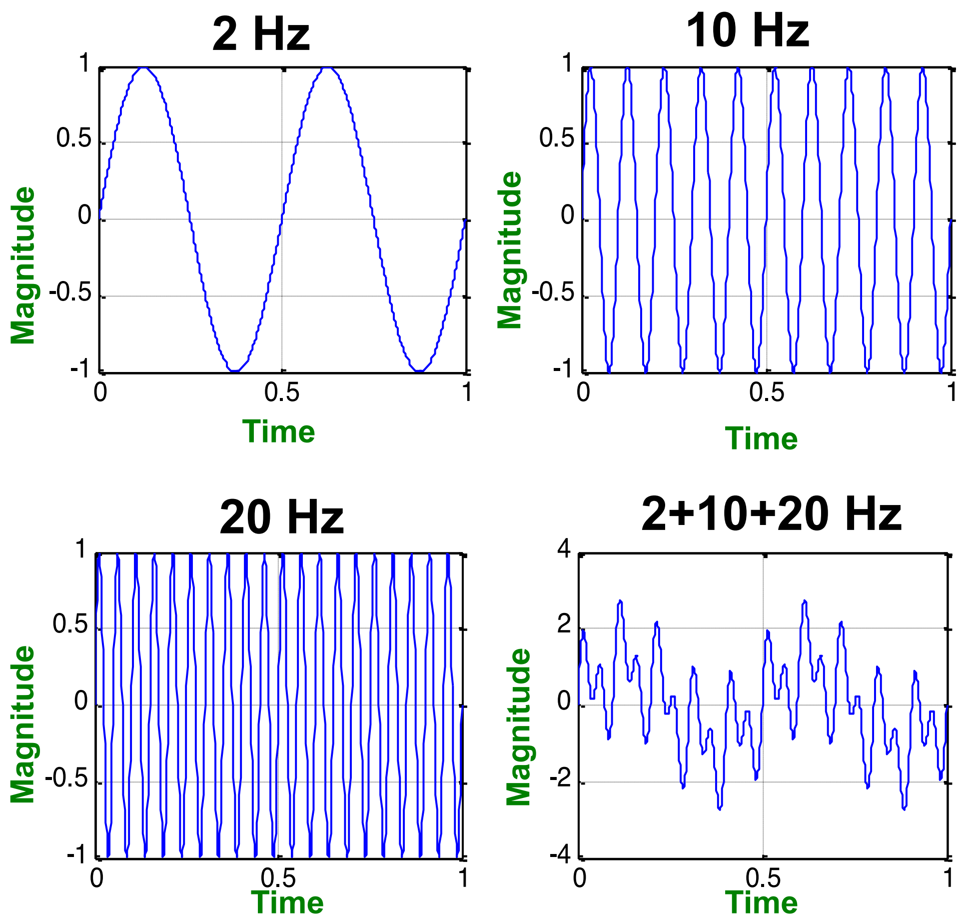
synchronicity of cell excitation determines amplitude and rhythm of the EEG signal



Frequency analysis



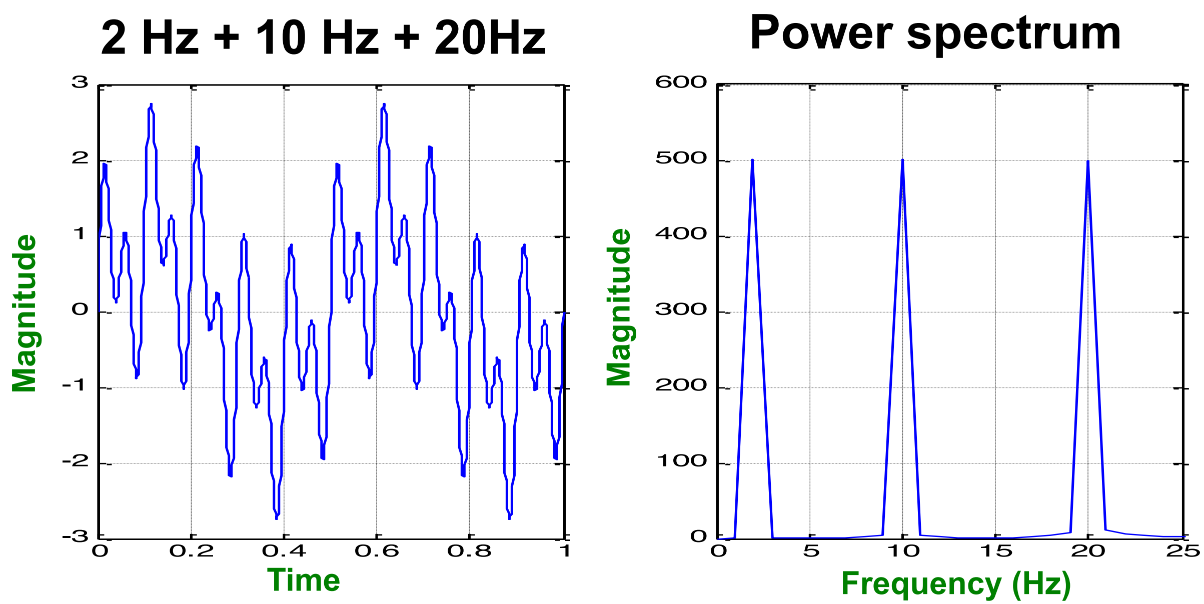
Stationary signals



Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Stationary signal

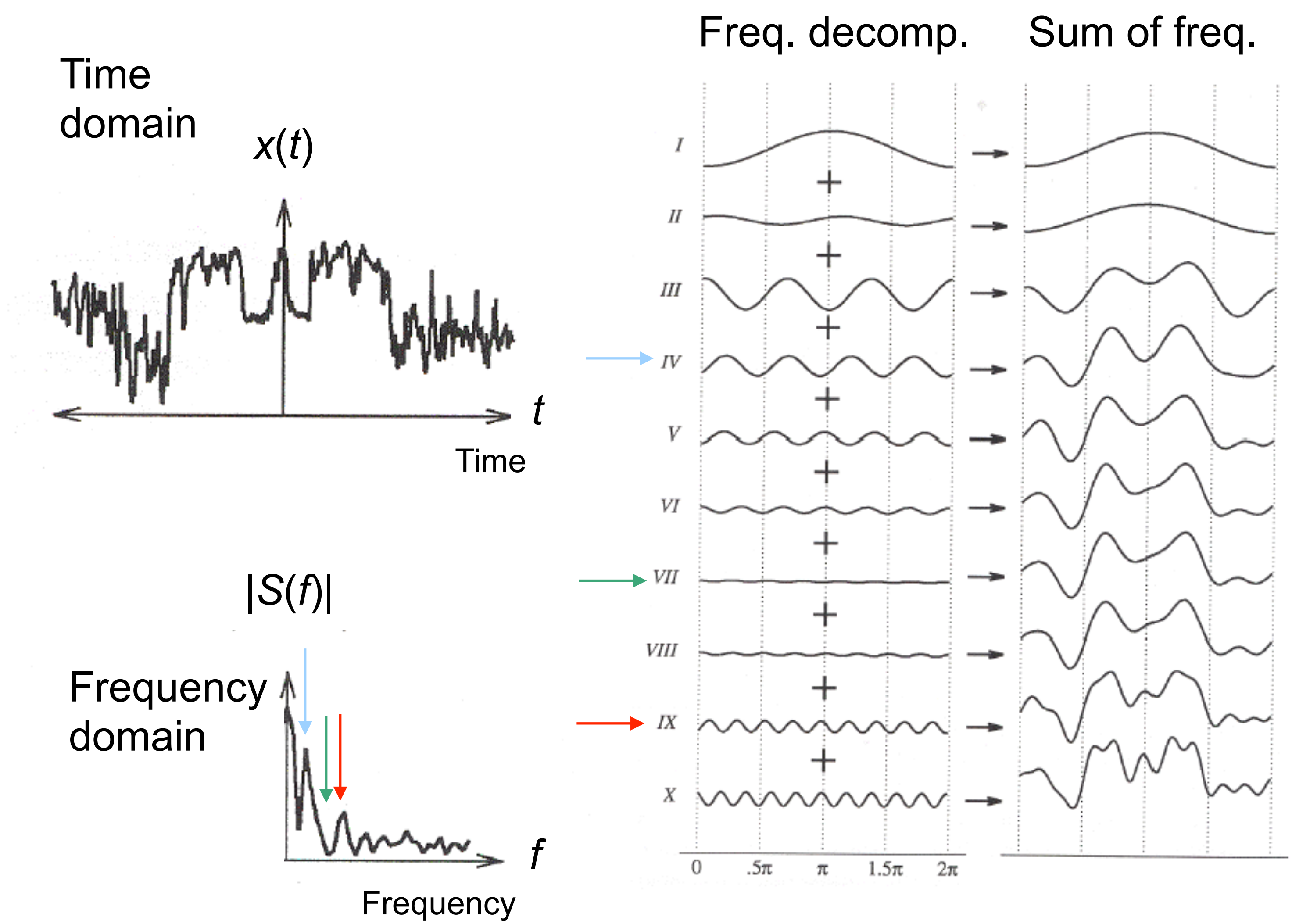
Stationary



By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2,10,20Hz respectively).

Slide courtesy of Petros Xanthopoulos, Univ. of Florida

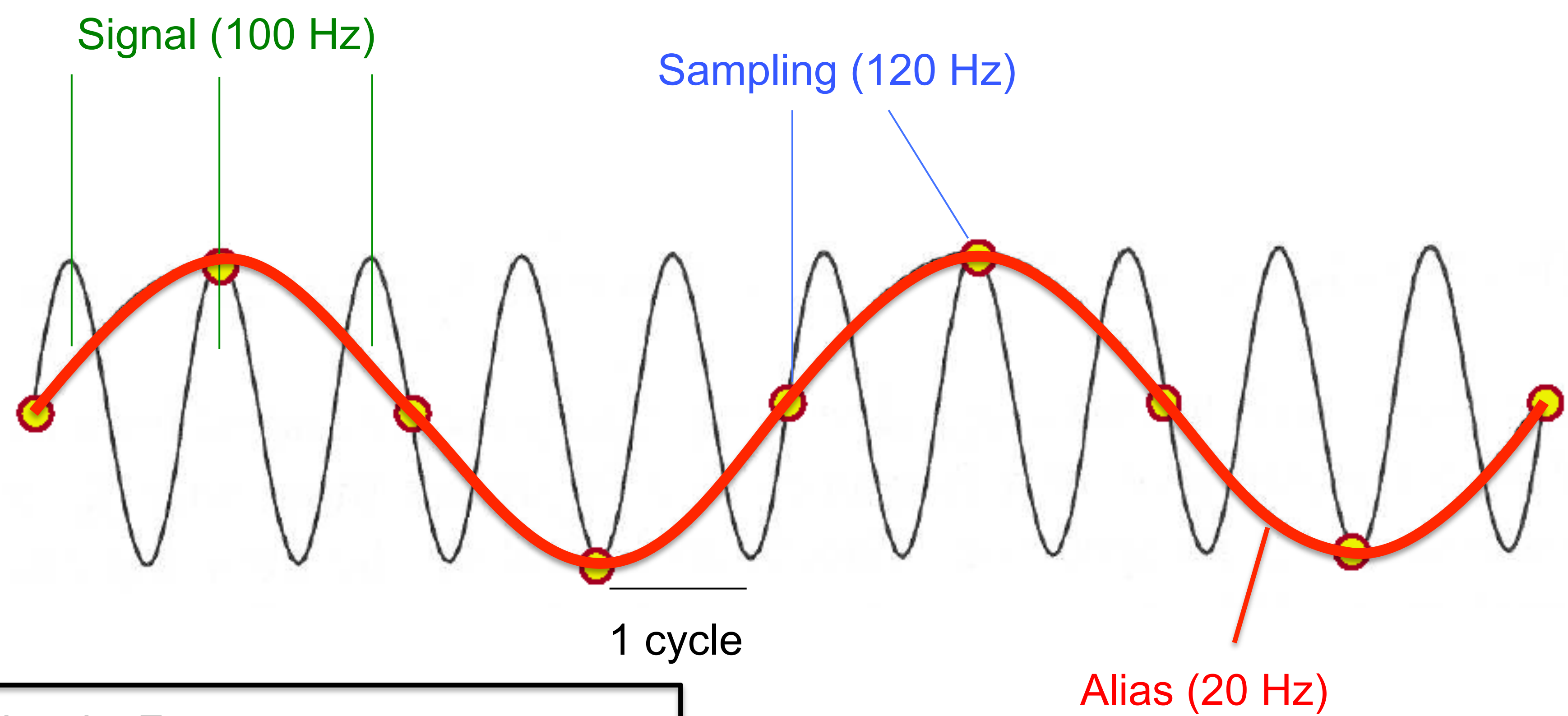
Fourier's Theorem



Figure, courtesy of Ravi Ramamoorthi & Wolberg

Nyquist frequency: Aliasing

e.g. 100 Hz signal sampled at 120 Hz



Nyquist Frequency:
Max frequency that can be uniquely recovered at sampling rate of f_s

$$f_N = f_s / 2$$

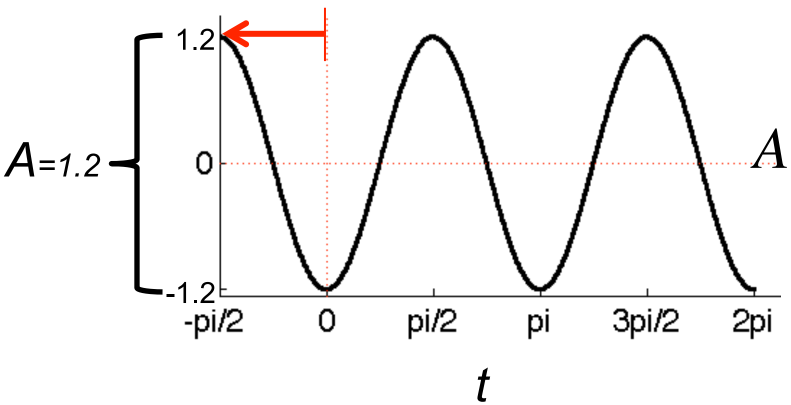
$$f_{alias}(N) = \left| f + N f_s \right|$$

$$f_s = \text{sampling rate}$$

Euler's Formula

phase shift
$\theta = \pi / 2$

angular frequency
$\omega = 2\pi f$



instantaneous complex power (amplitude and phase)

$$A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t + \theta)} + \frac{A}{2} e^{-i(\omega t + \theta)}$$

$$= \mathrm{Re}\{A e^{i(\omega t + \theta)}\} = \mathrm{Re}\{S(\omega, t)\}$$

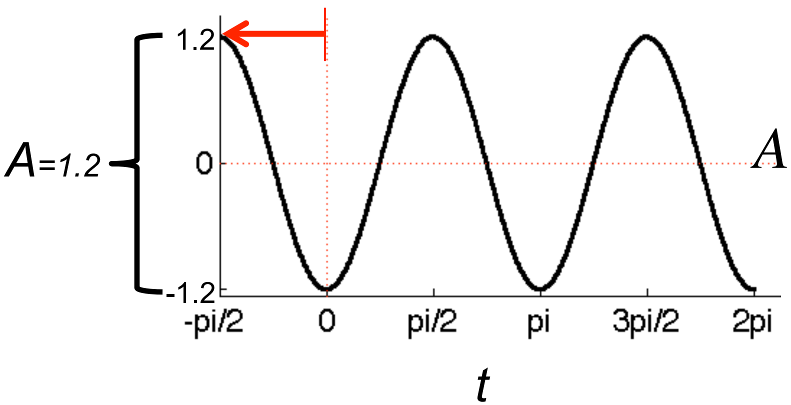
Euler's Formula

phase shift

$\theta = \pi / 2$

angular frequency

$\omega = 2\pi f$



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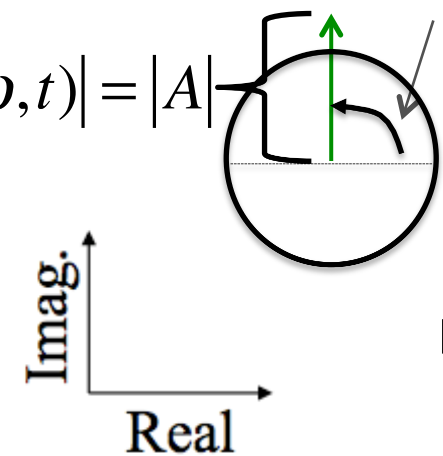
$$= \mathrm{Re}\{A e^{i(\omega t+\theta)}\} = \mathrm{Re}\{S(\omega,t)\}$$

$$\theta = \angle S(\omega,t)$$

$$= \pi / 2$$

$$|S(\omega,t)| = |A|$$



Phasor!

Euler's Formula

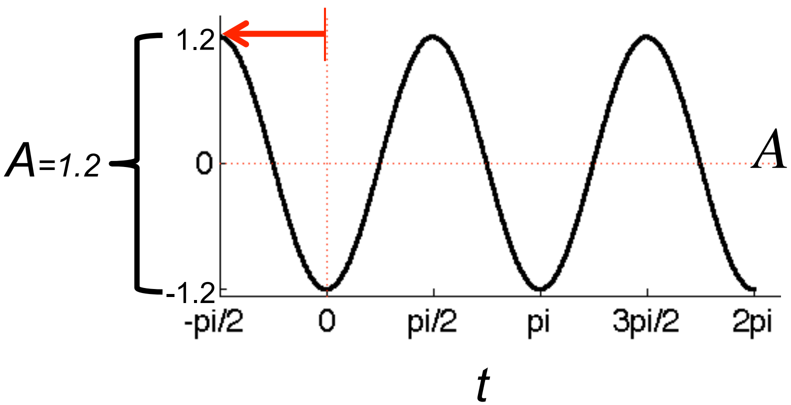
phase shift

$\theta = \pi / 2$

angular frequency

$\omega = 2\pi f$

instantaneous complex power (amplitude and phase)



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$$= \mathrm{Re}\{A e^{i(\omega t + \theta)}\} = \mathrm{Re}\{S(\omega, t)\}$$

Another version:

$$e^{i(\omega t + \theta)} = \cos(\omega t + \theta) + i \sin(\omega t + \theta)$$

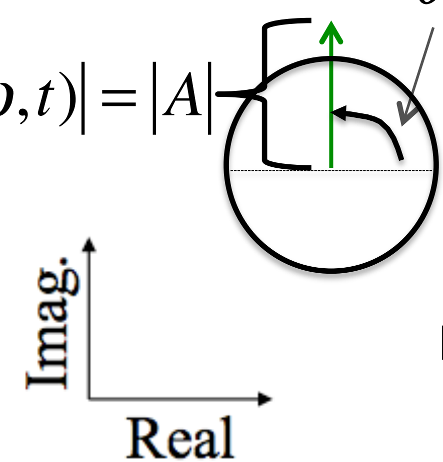
Real part
Cosine component

Imaginary part
Sine component

$\theta = \angle S(\omega, t)$

$= \pi / 2$

$|S(\omega, t)| = |A|$

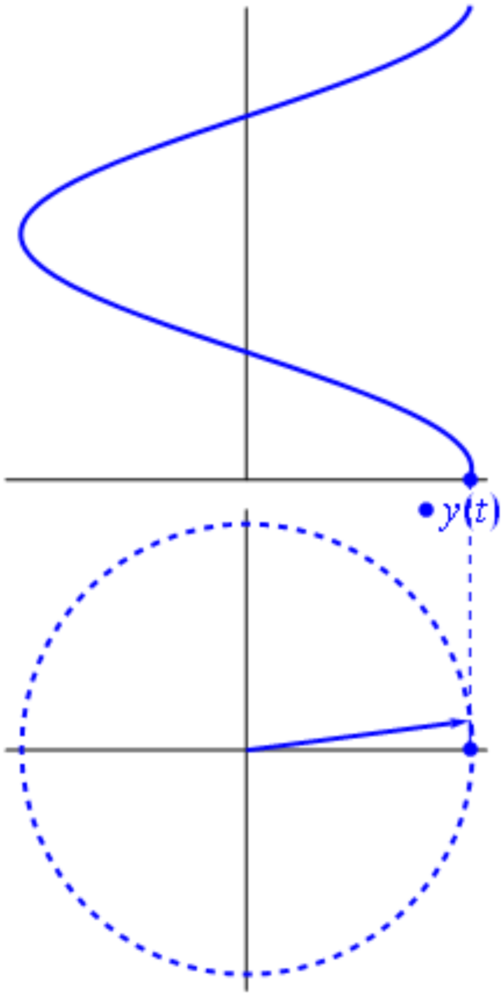


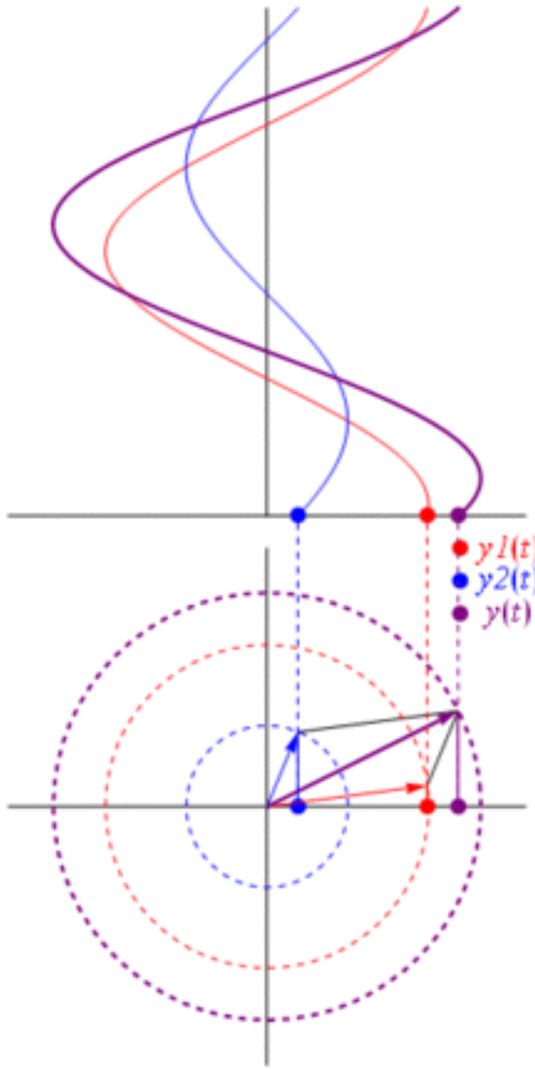
Phasor!

Phasers

Phasors

Rotation velocity (Rad/S; Hz)
= (angular) frequency (*w*; *f*)

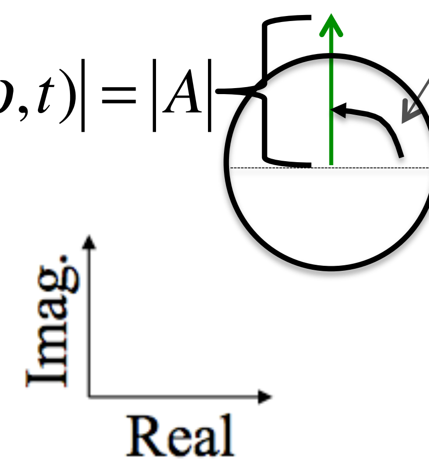




$$A \cdot \cos(\omega t + \theta) = \mathrm{Re}\{A e^{i(\omega t + \theta)}\}$$
$$= \mathrm{Re}\{S(\omega, t)\}$$

$$|S(\omega, t)| = |A|$$

$$\theta = \frac{\pi}{2}$$



Polar animations courtesy Wikipedia

Fourier Transform

Time → Frequency

Forward transform

$$S(f) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-2\pi i f t/N}$$

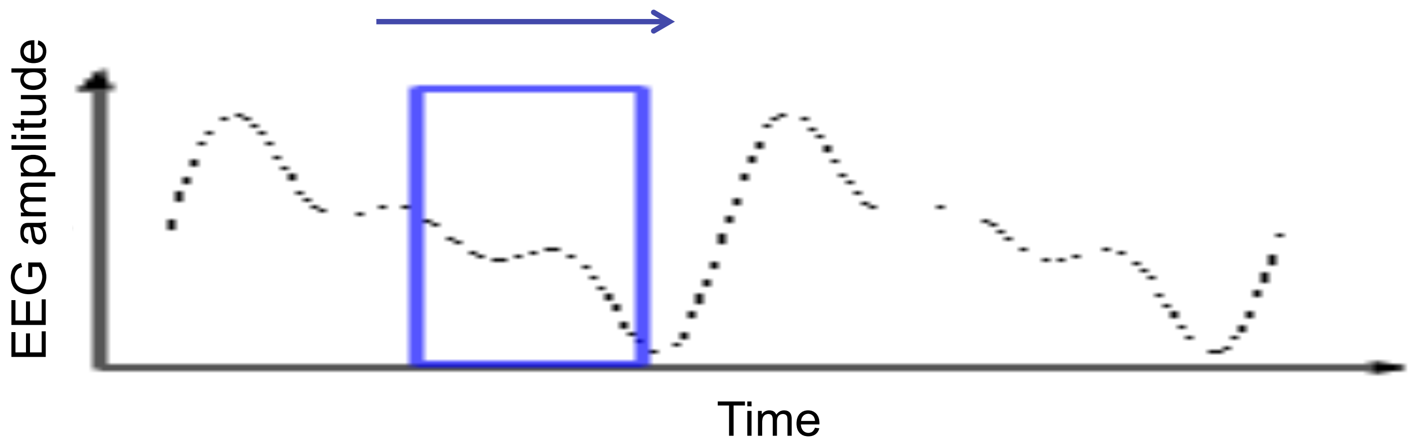
Frequency → Time

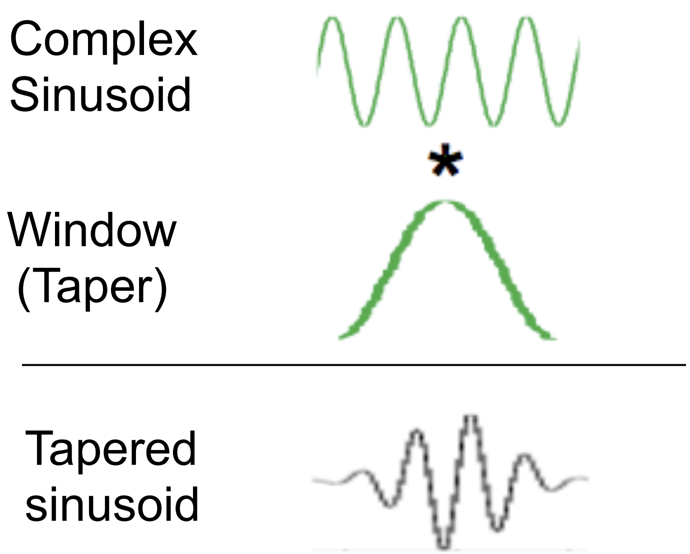
Inverse transform

$$x(t) = \frac{1}{N}\sum_{f=0}^{N} S(f)e^{2\pi i f t/N}$$

N = number of samples

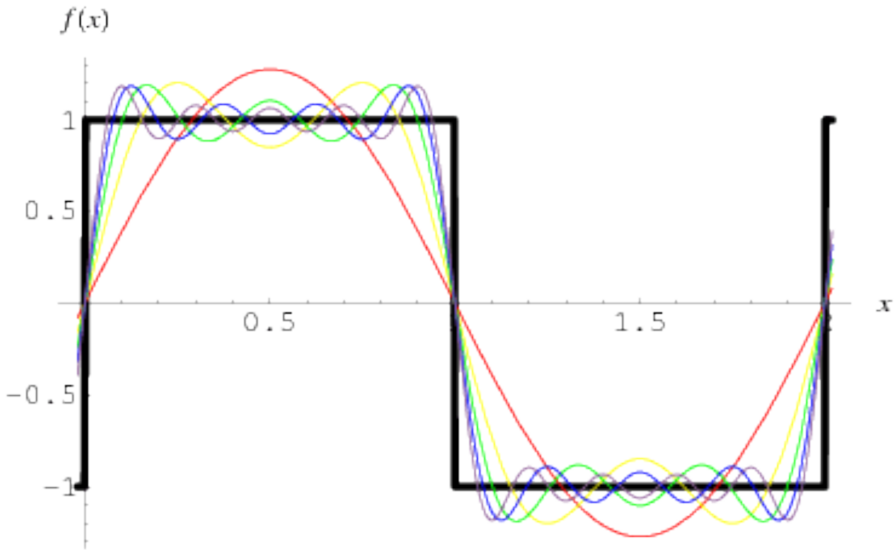
Fast Fourier Transform (FFT)



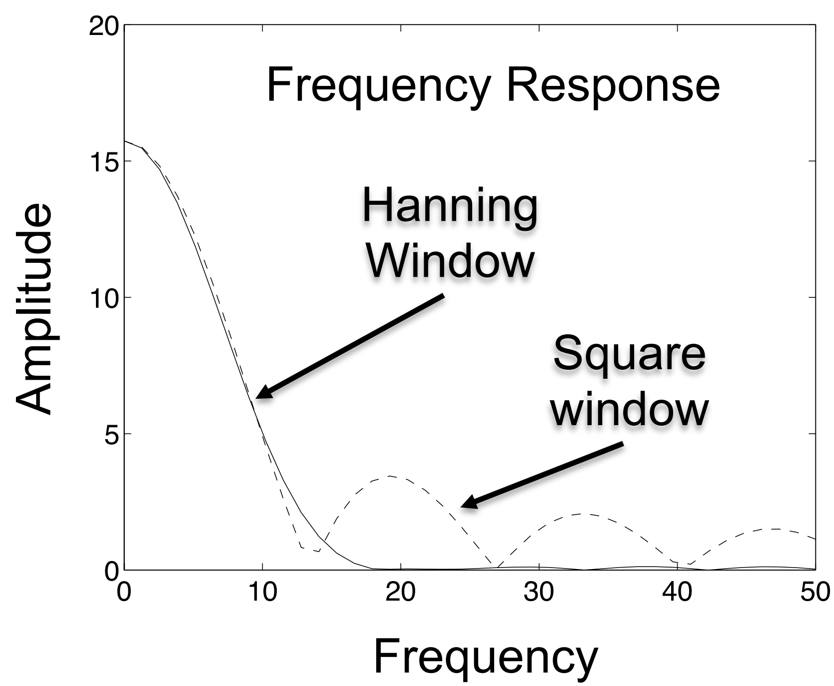


Performing Fourier transform by convolution with a (optionally tapered) complex sinusoid

Tapering

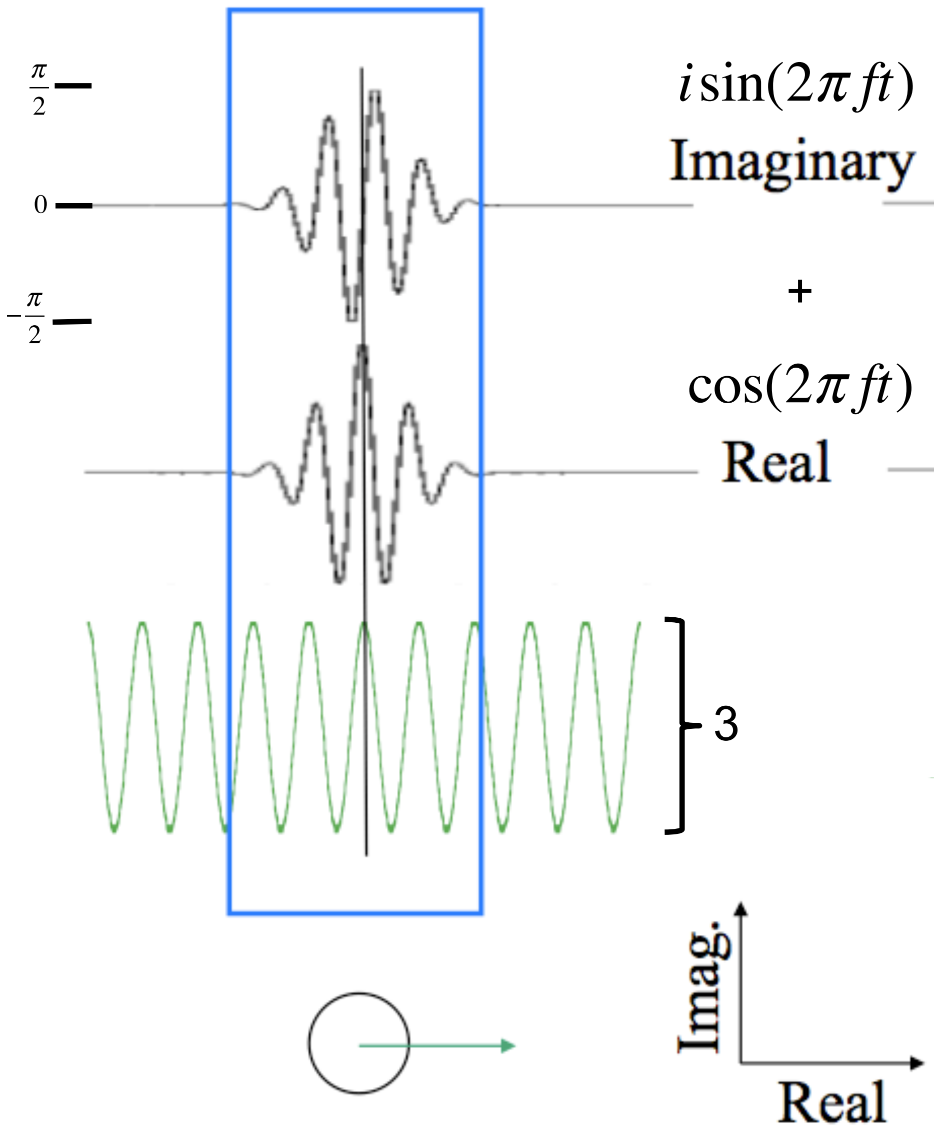


Gibbs Phenomenon
"Rippling" effect due to discontinuities in signal (e.g. edges of the truncated signal)

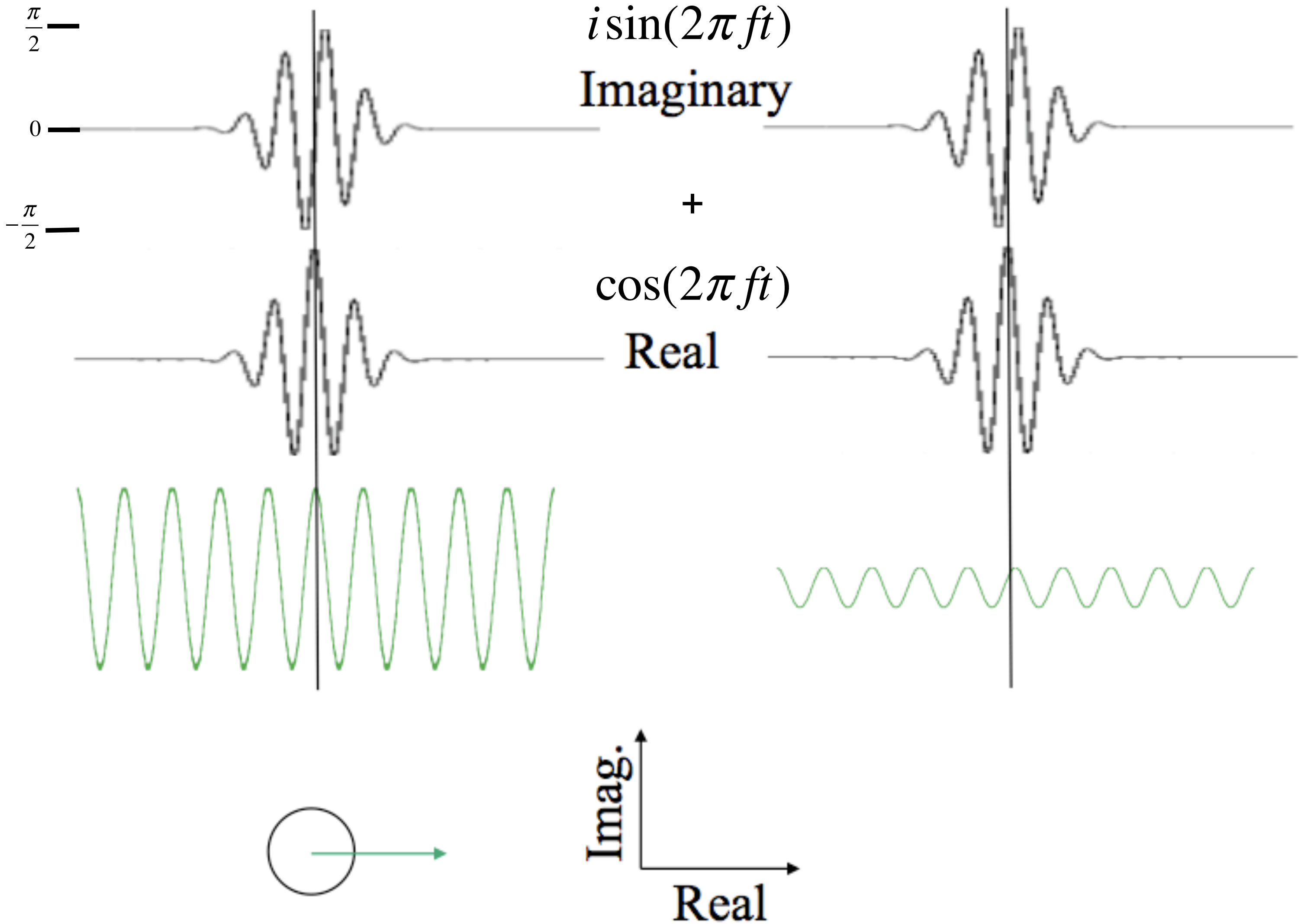


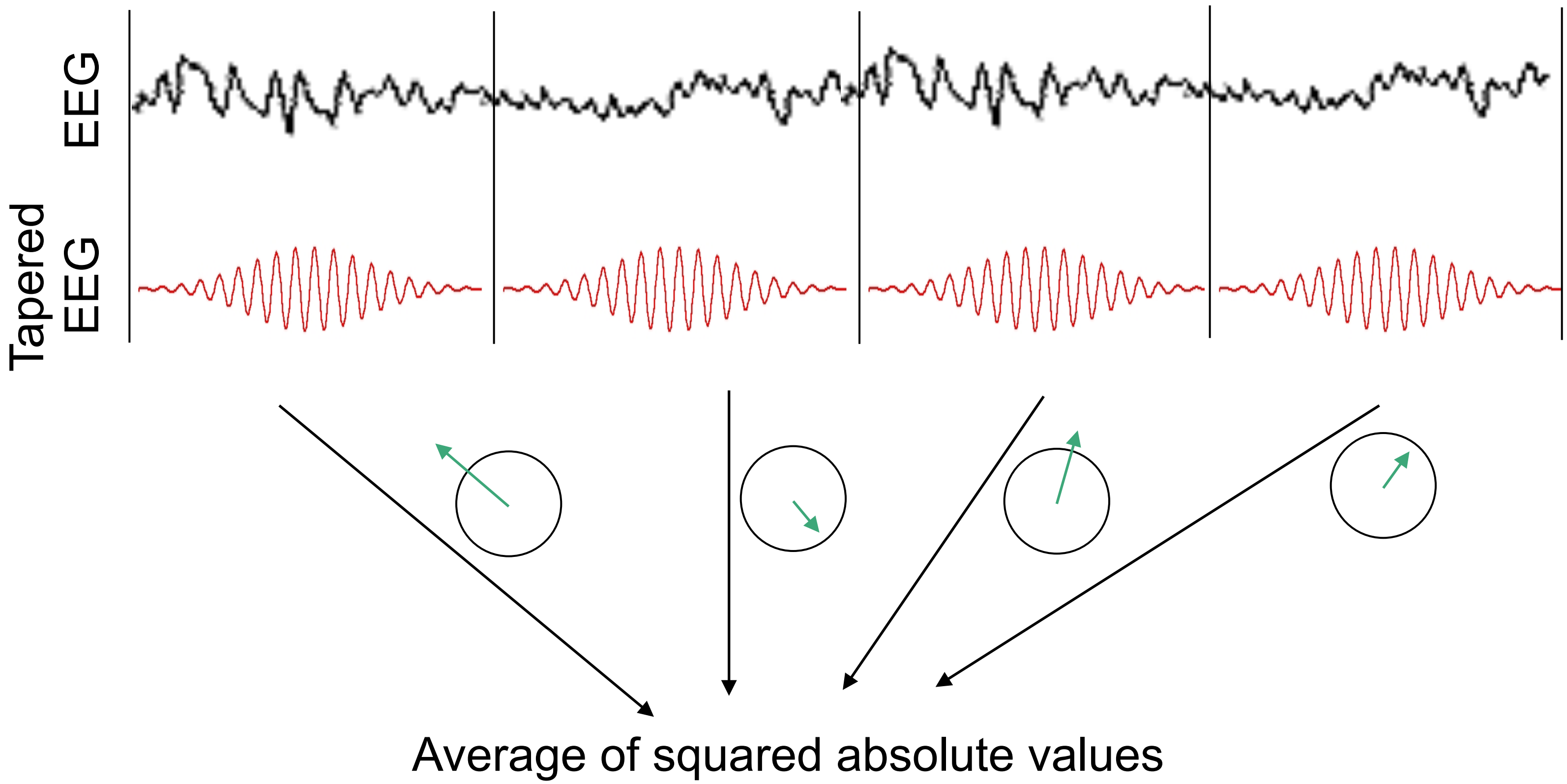
Tapering
Smoothly decay signal to zero at endpoints to avoid discontinuity

Spectral phase and amplitude

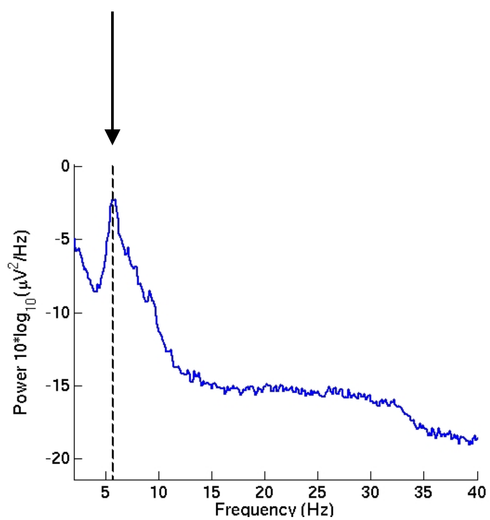


Spectral phase and amplitude

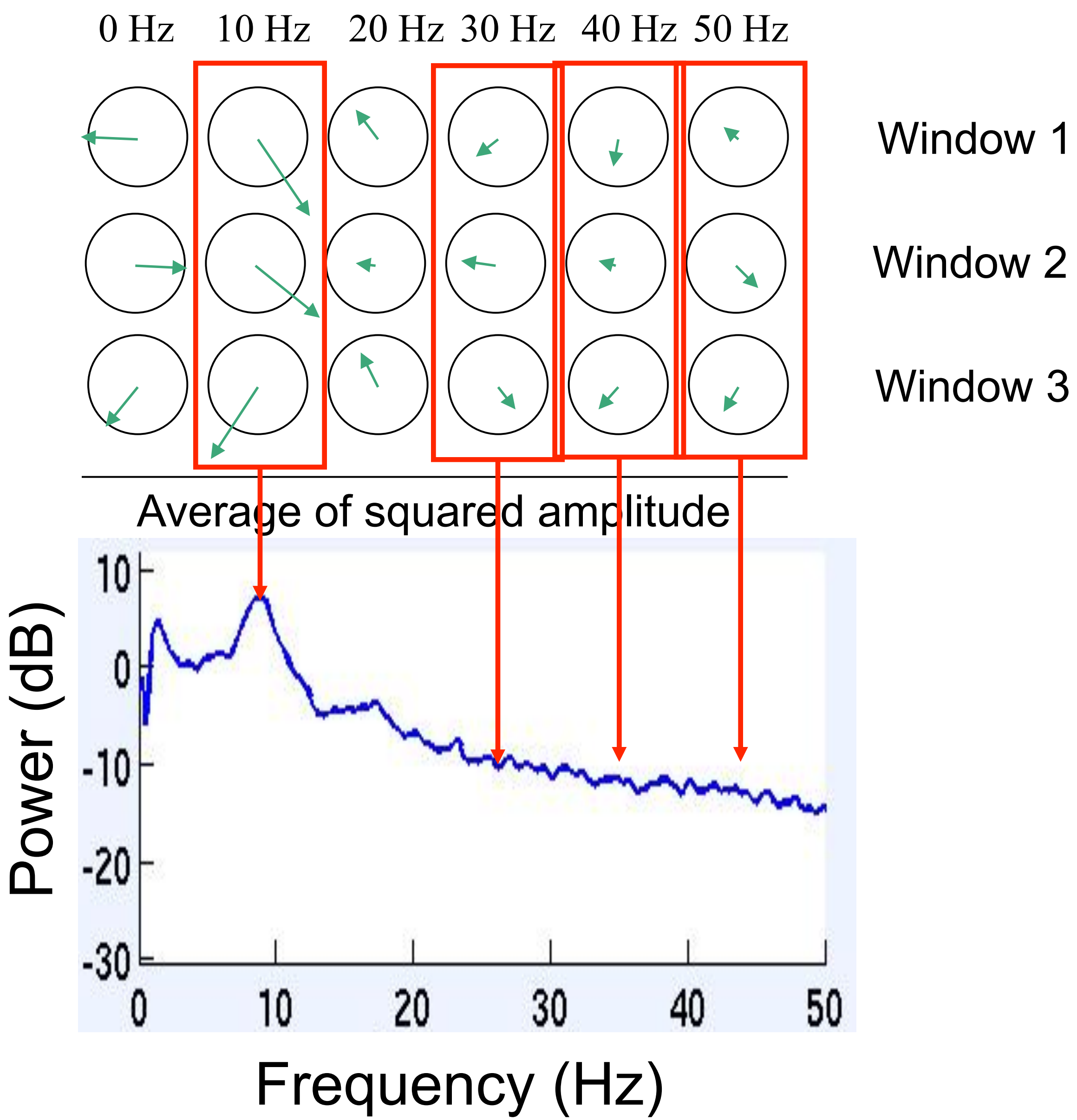




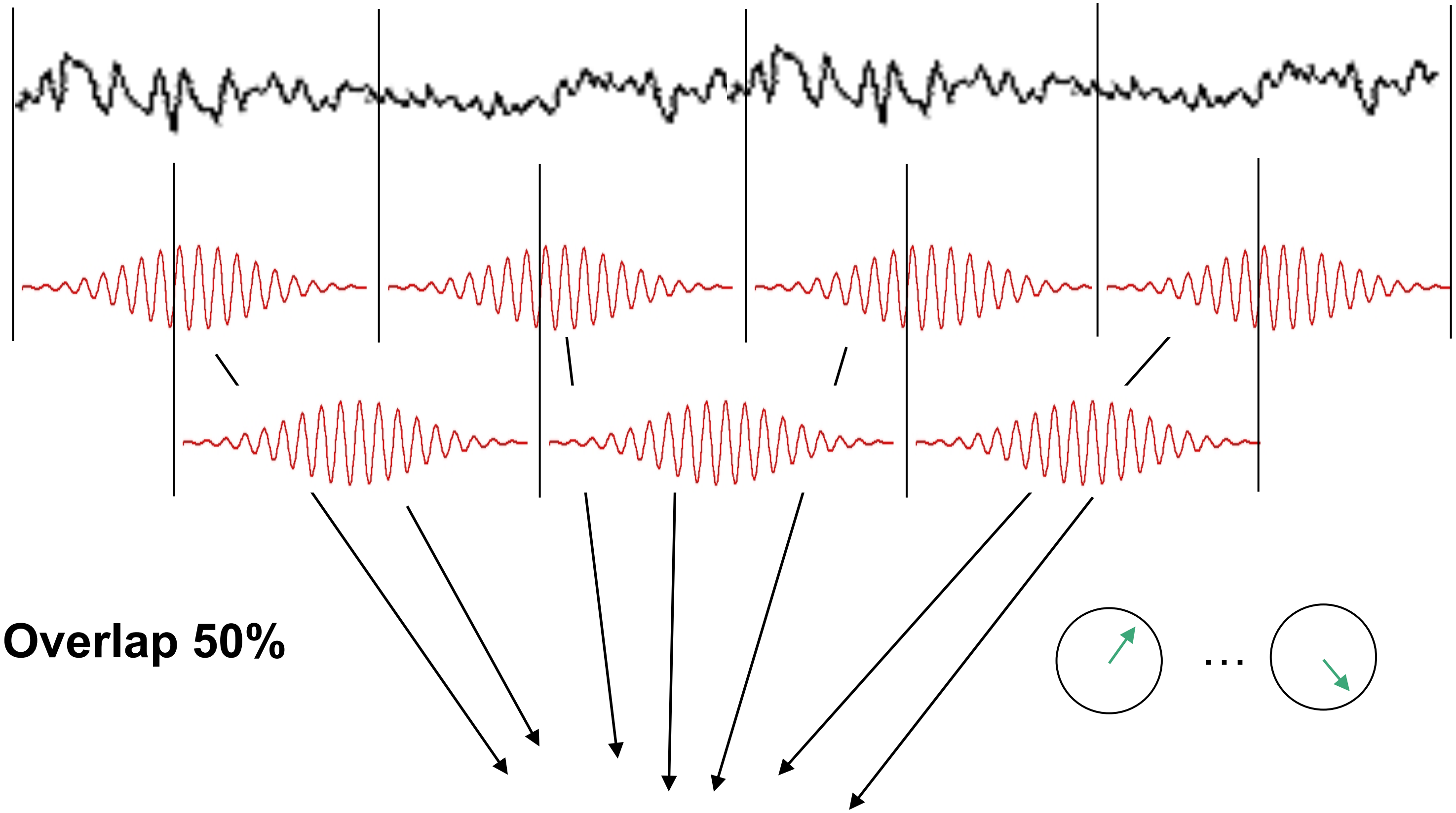
The Welch method



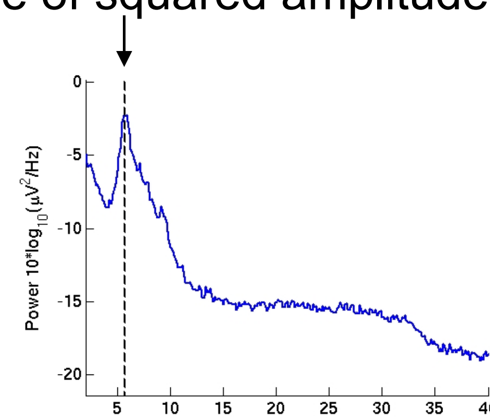
Spectral power

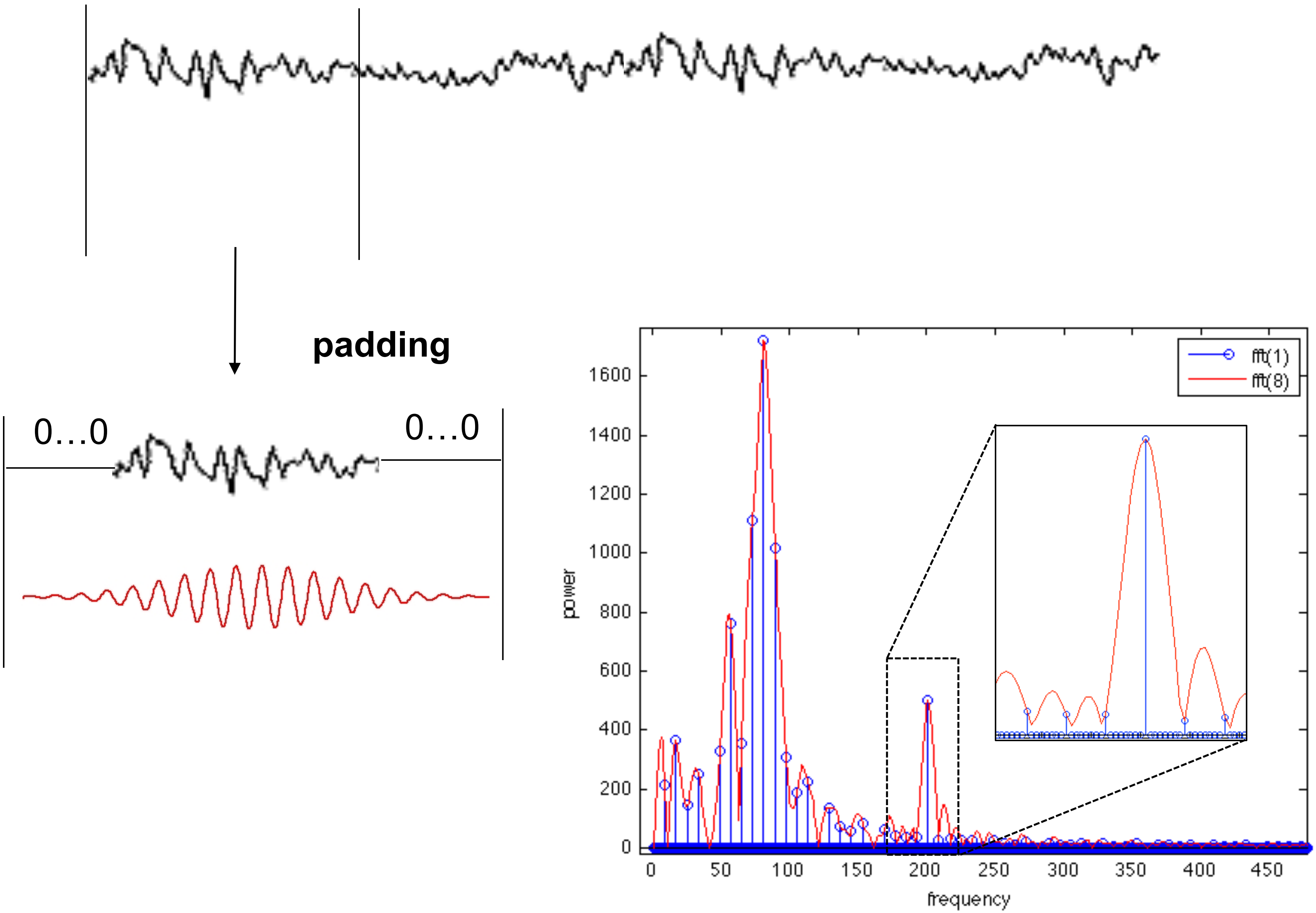


The Welch method



Average of squared amplitudes

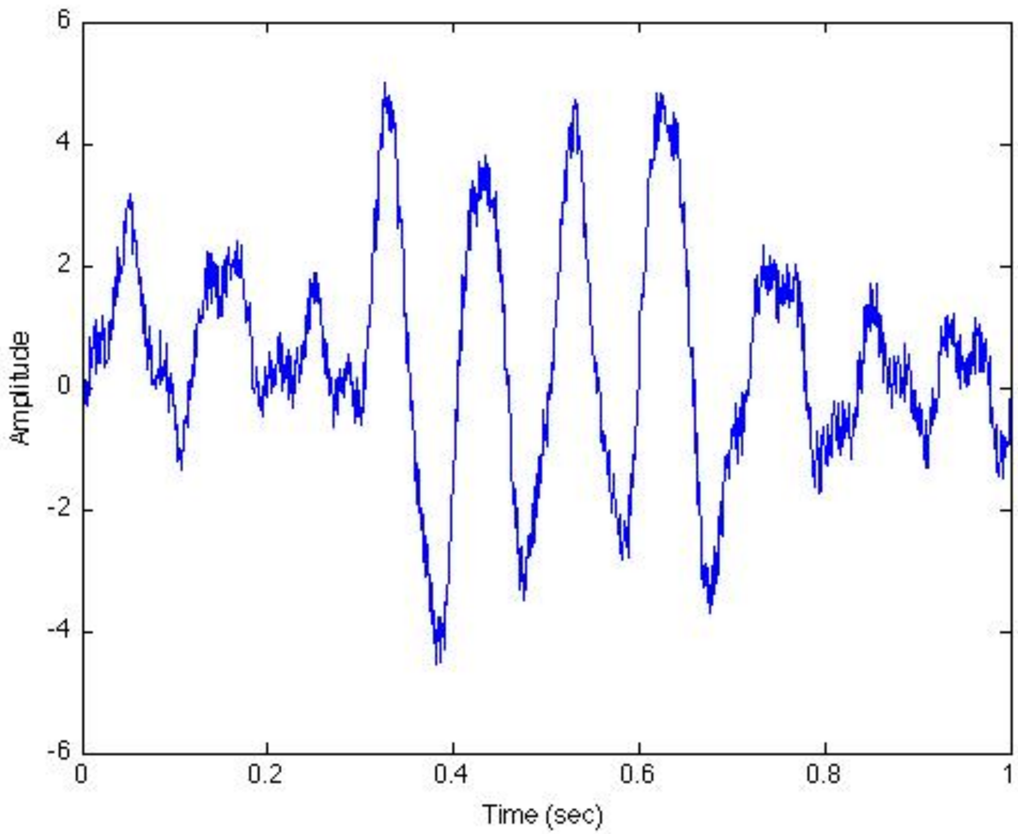




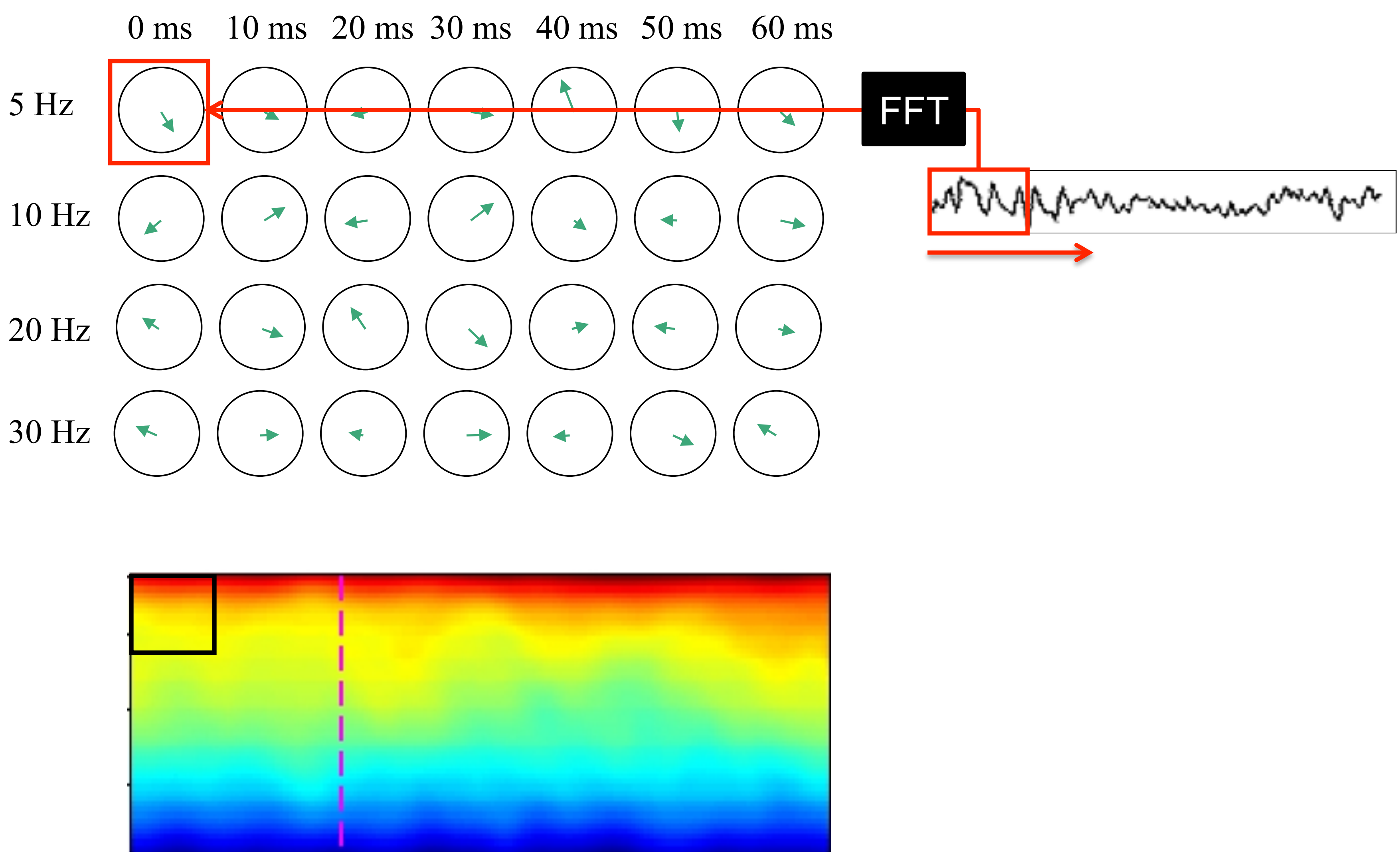
padding
0…0
0…0
power
frequency
fft(1)
fft(8)
0
200
400
600
800
1000
1200
1400
1600
0
50
100
150
200
250
300
350
400
450

Non-Stationary Signals

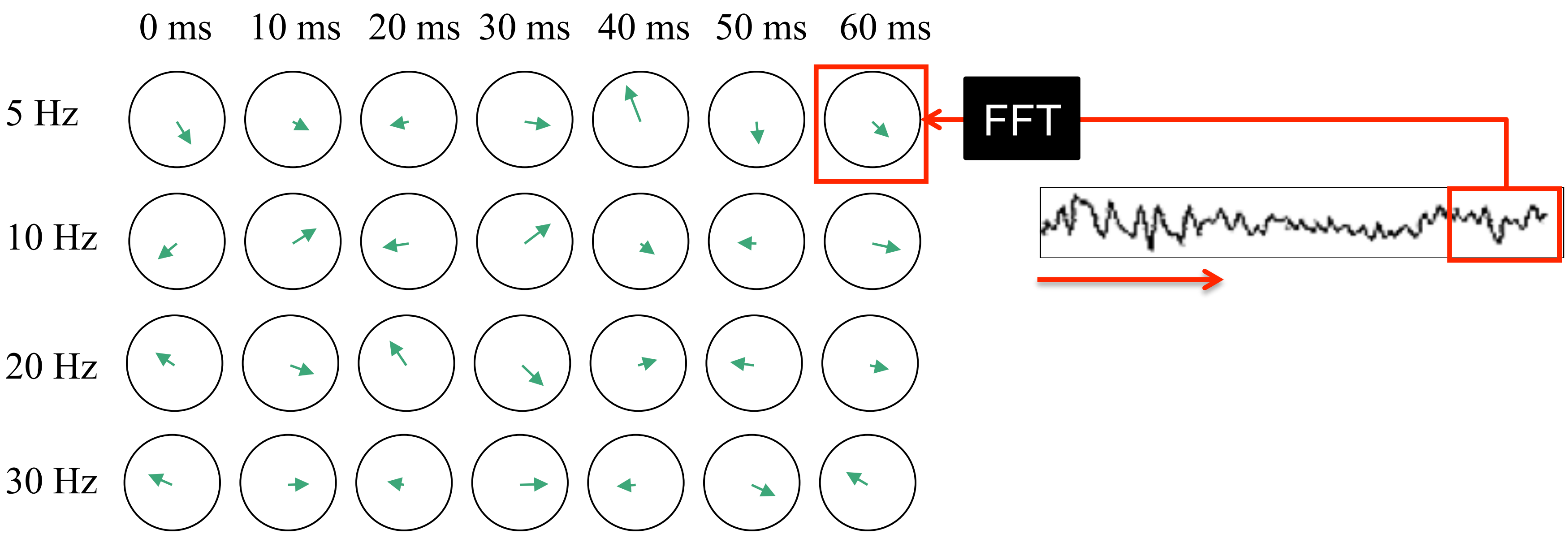
- Bursts, chirps, evoked potentials, …

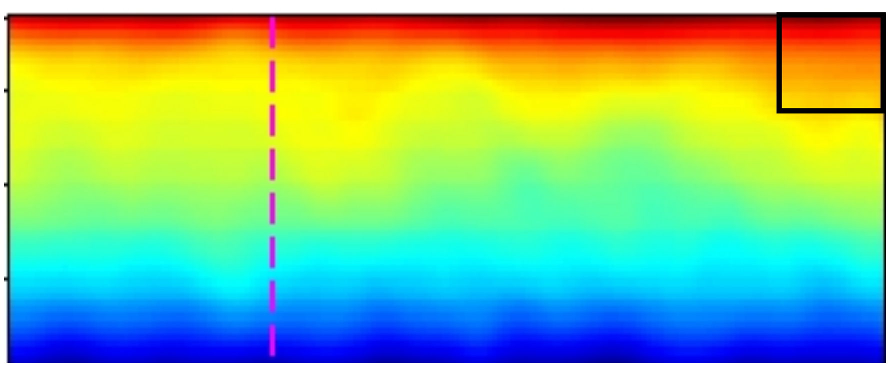


Spectrogram or ERSP

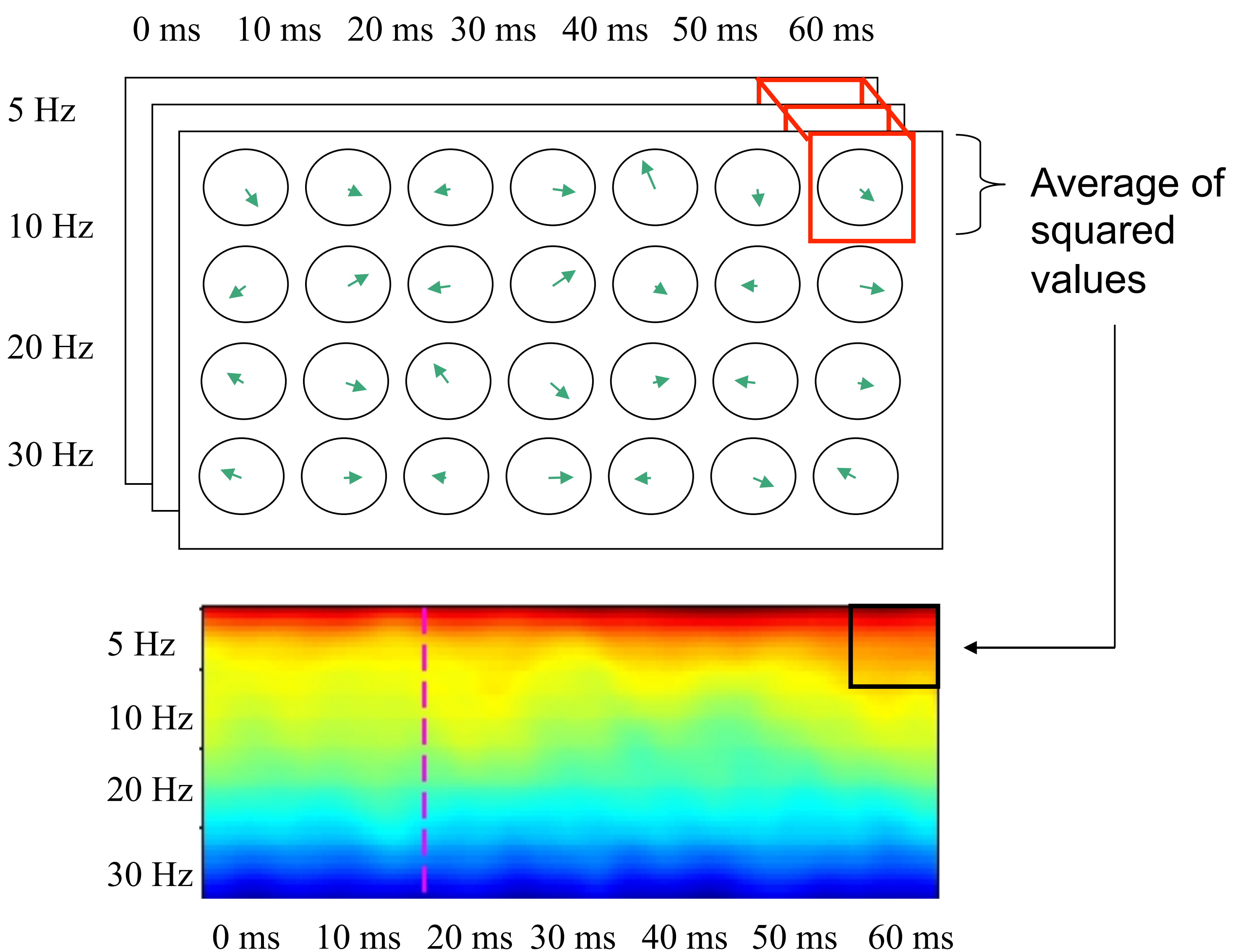


Spectrogram or ERSP





Spectrogram or ERSP



Power spectrum and event-related spectral (perturbation)

$$ERS(f,t) = \frac{1}{n}\sum_{k=1}^{n}\left|S_k(f,t)\right|^2$$

Ensemble average

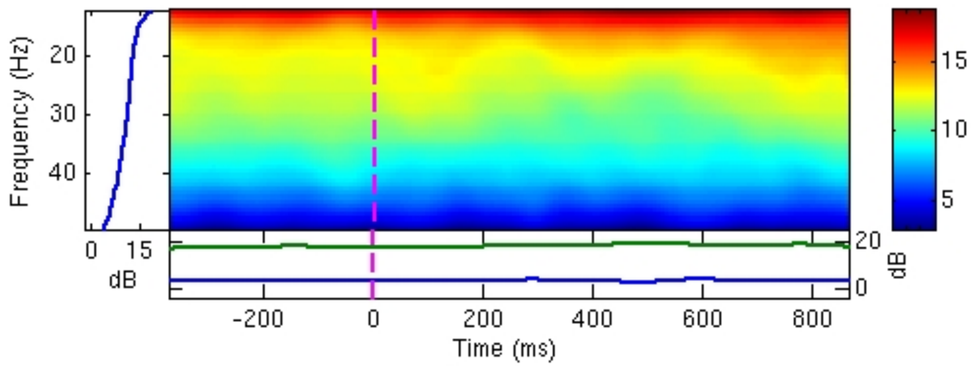
Complex number

Scaled to dB $10\mathrm{Log}_{10}$

Here, there are *n* trials
Each trial is time-locked to the same *event* (hence “event-related” spectrum)
The ERS is the average power across event-locked trials

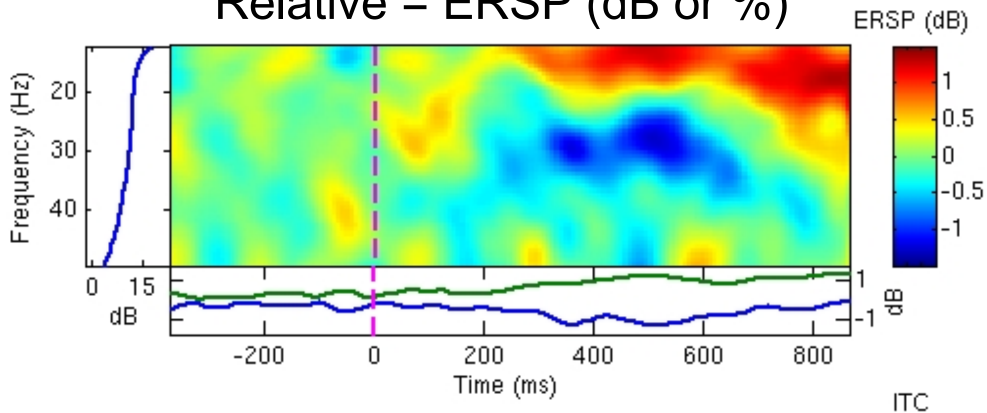
Absolute versus relative power

Absolute = ERS

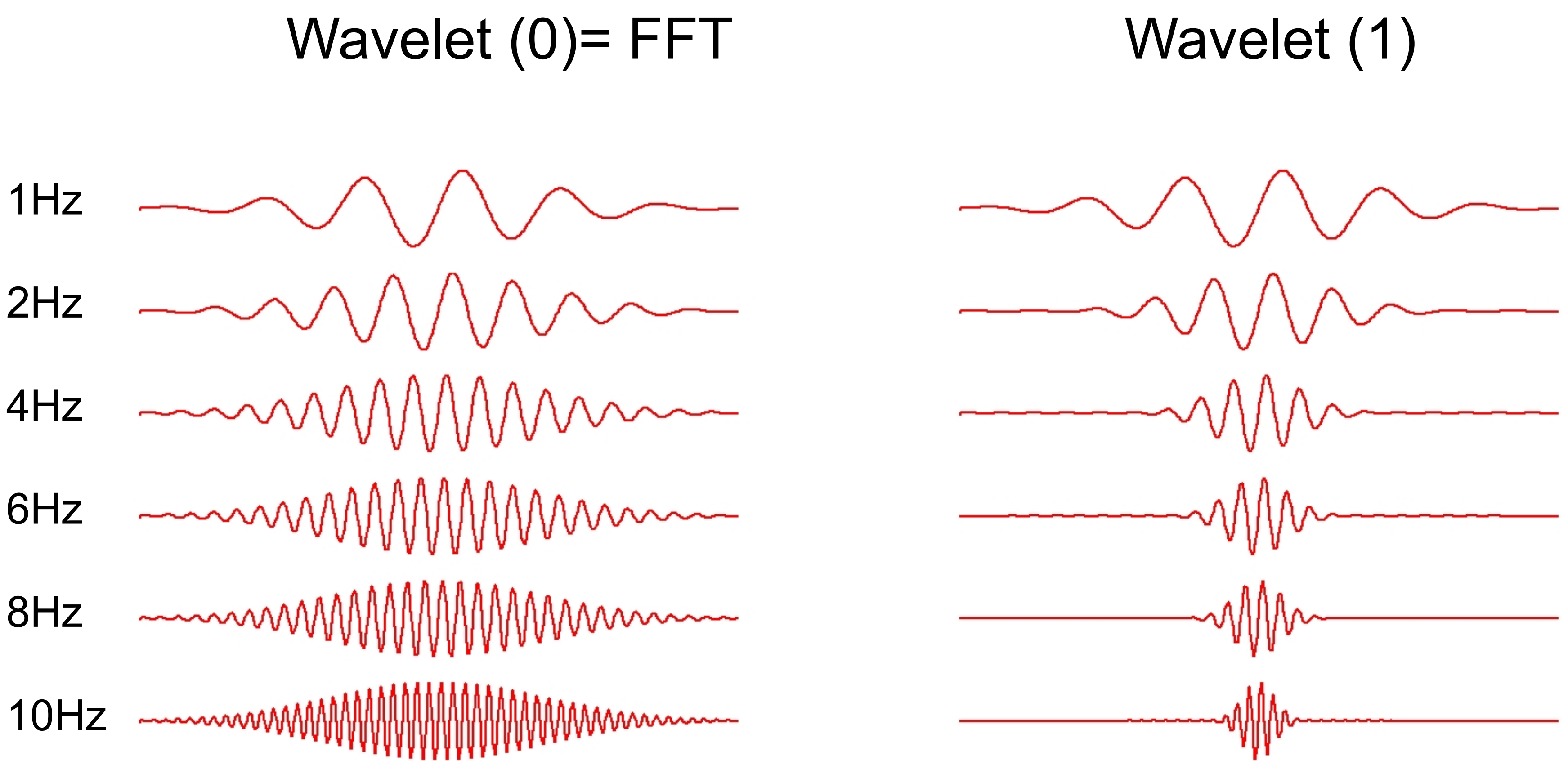


To compute the ERSP, we just subtract the pre-stimulus ERS from the whole trial

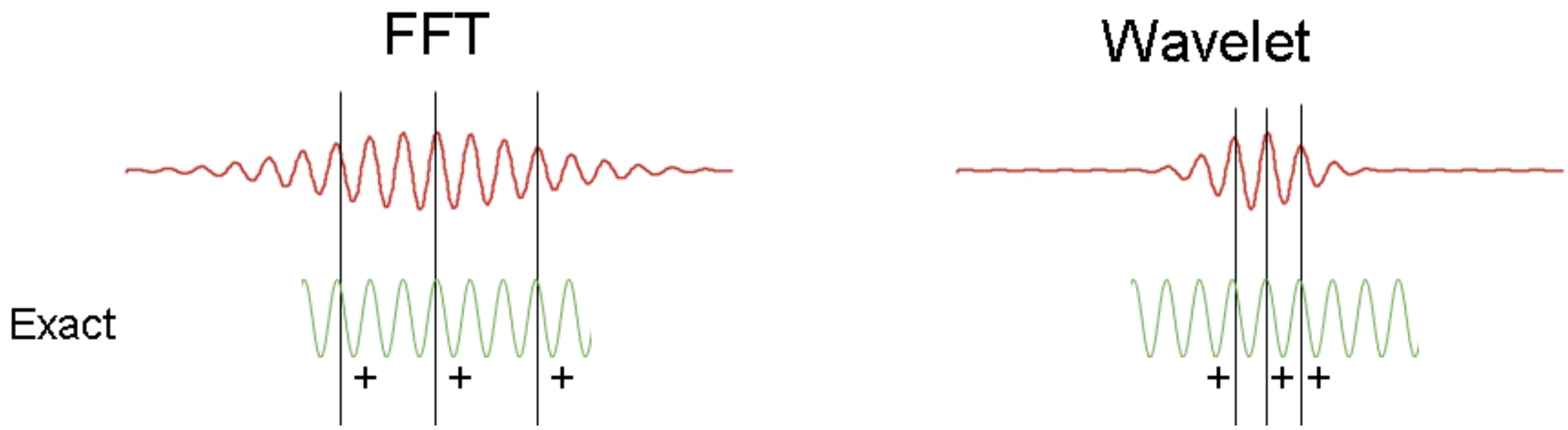
Relative = ERSP (dB or %)



Wavelets factor



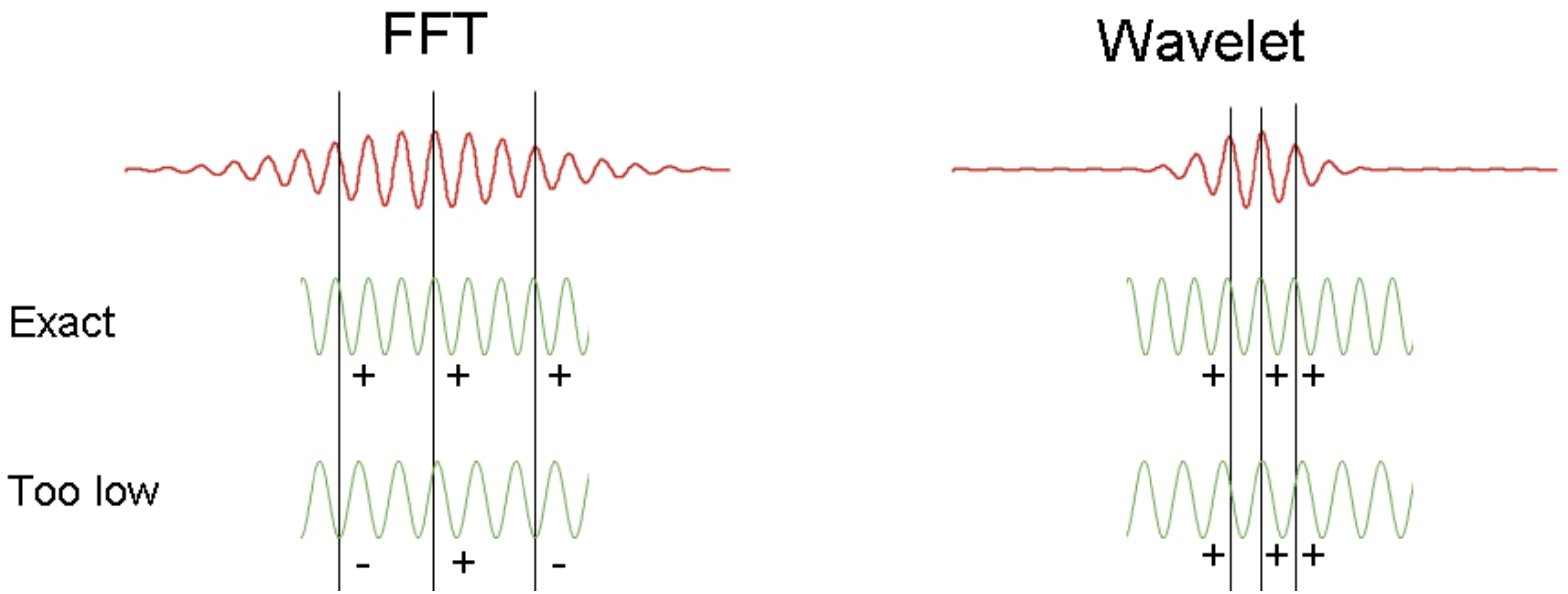
Time-frequency resolution trade off



High freq. resolution
low time-resolution

Low freq. resolution
high time-resolution

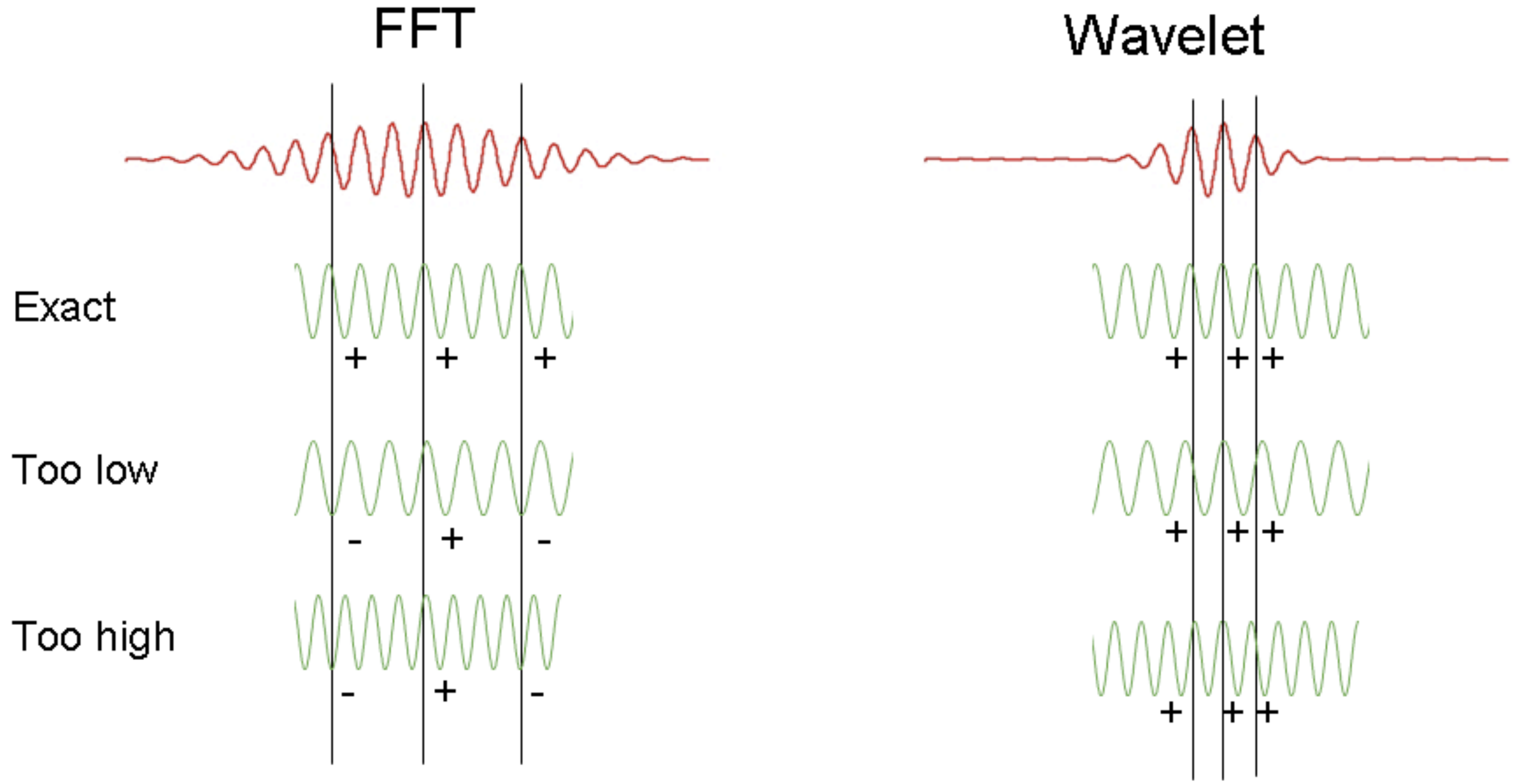
Time-frequency resolution trade off



High freq. resolution
low time-resolution

Low freq. resolution
high time-resolution

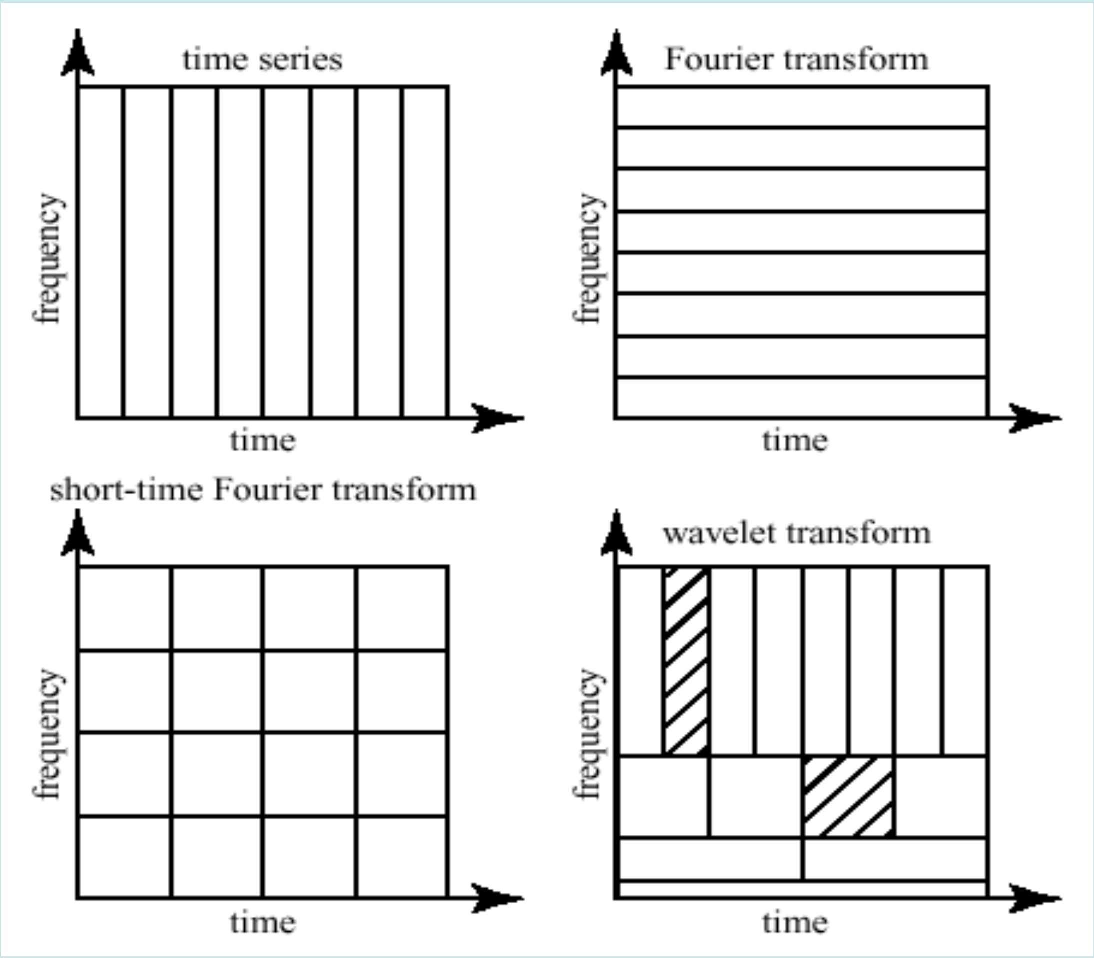
Time-frequency resolution trade off



High freq. resolution
low time-resolution

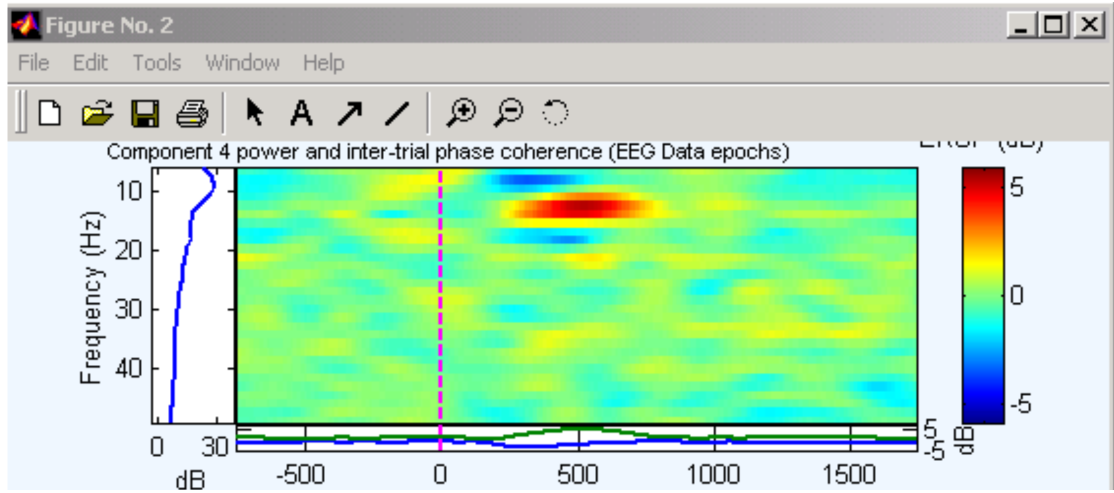
Low freq. resolution
high time-resolution

Difference between FFT and wavelets

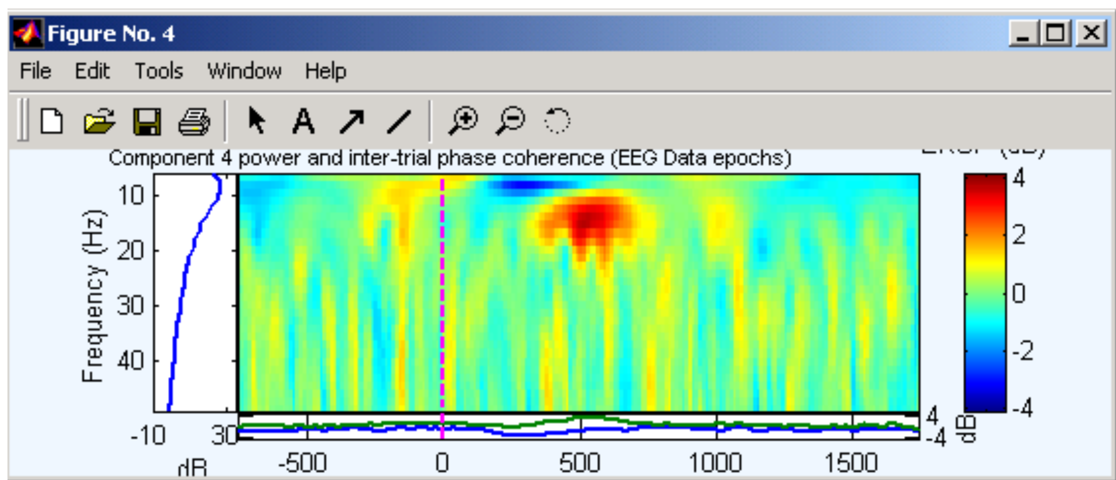


From http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10

FFT



Pure wavelet



The Uncertainty Principle

A signal <u>cannot</u> be localized arbitrarily well <u>both</u> in time/ position <u>and</u> in frequency/ momentum.

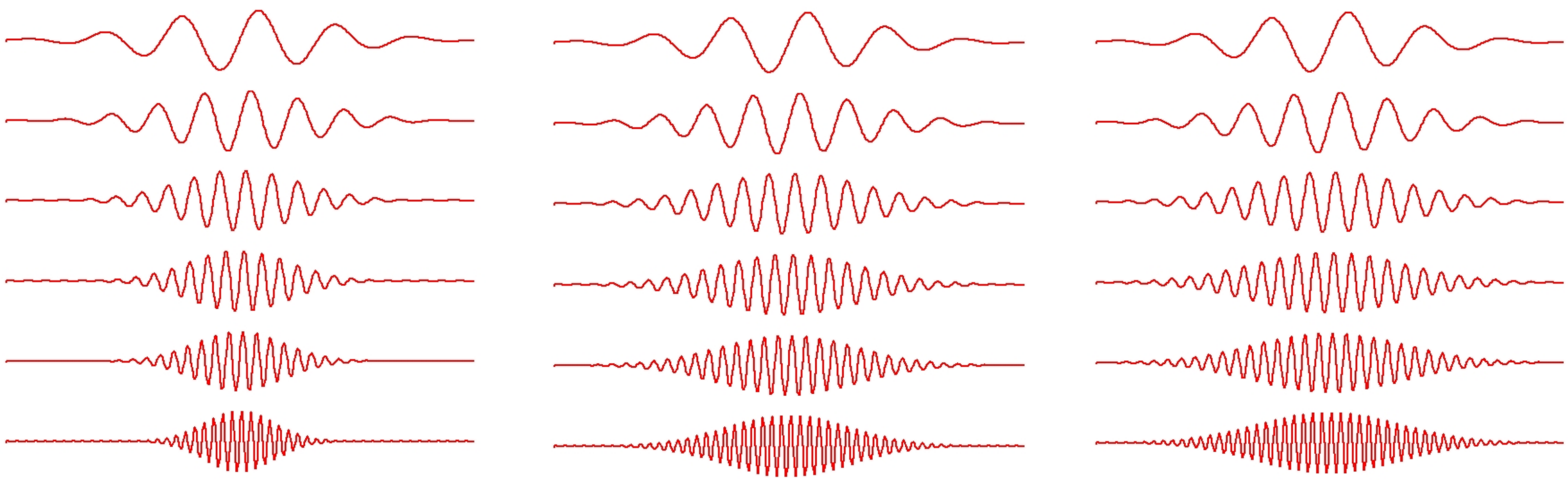
There exists a lower bound to the *Heisenberg product*:

$$\Delta t \, \Delta f \geq 1/(4\pi)$$

Δf = 1Hz, Δt = 80 msec or Δf = 2Hz, Δt = 40 msec

Modified wavelets

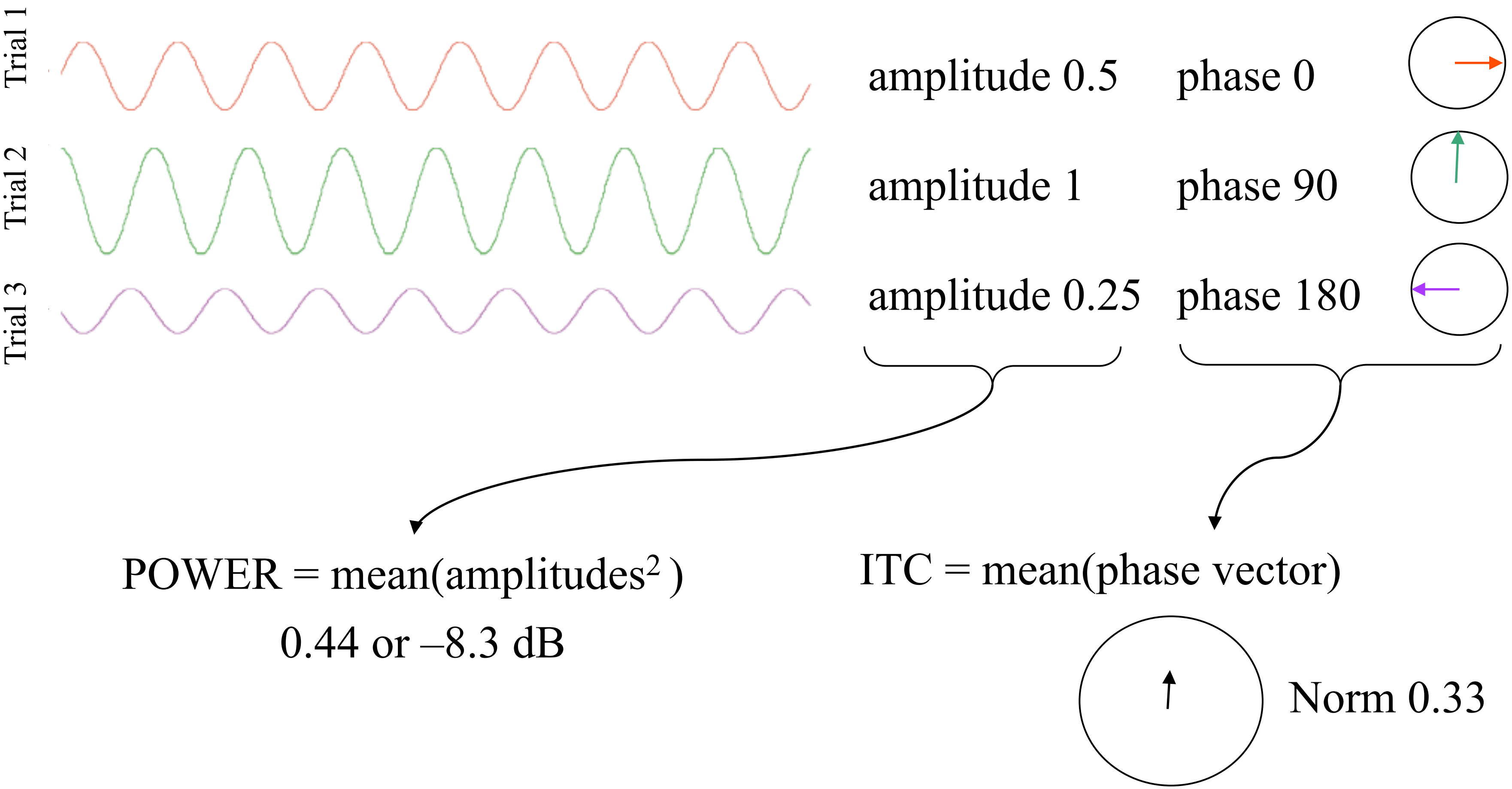
Wavelet (0.8) Wavelet (0.5) Wavelet (0.2)



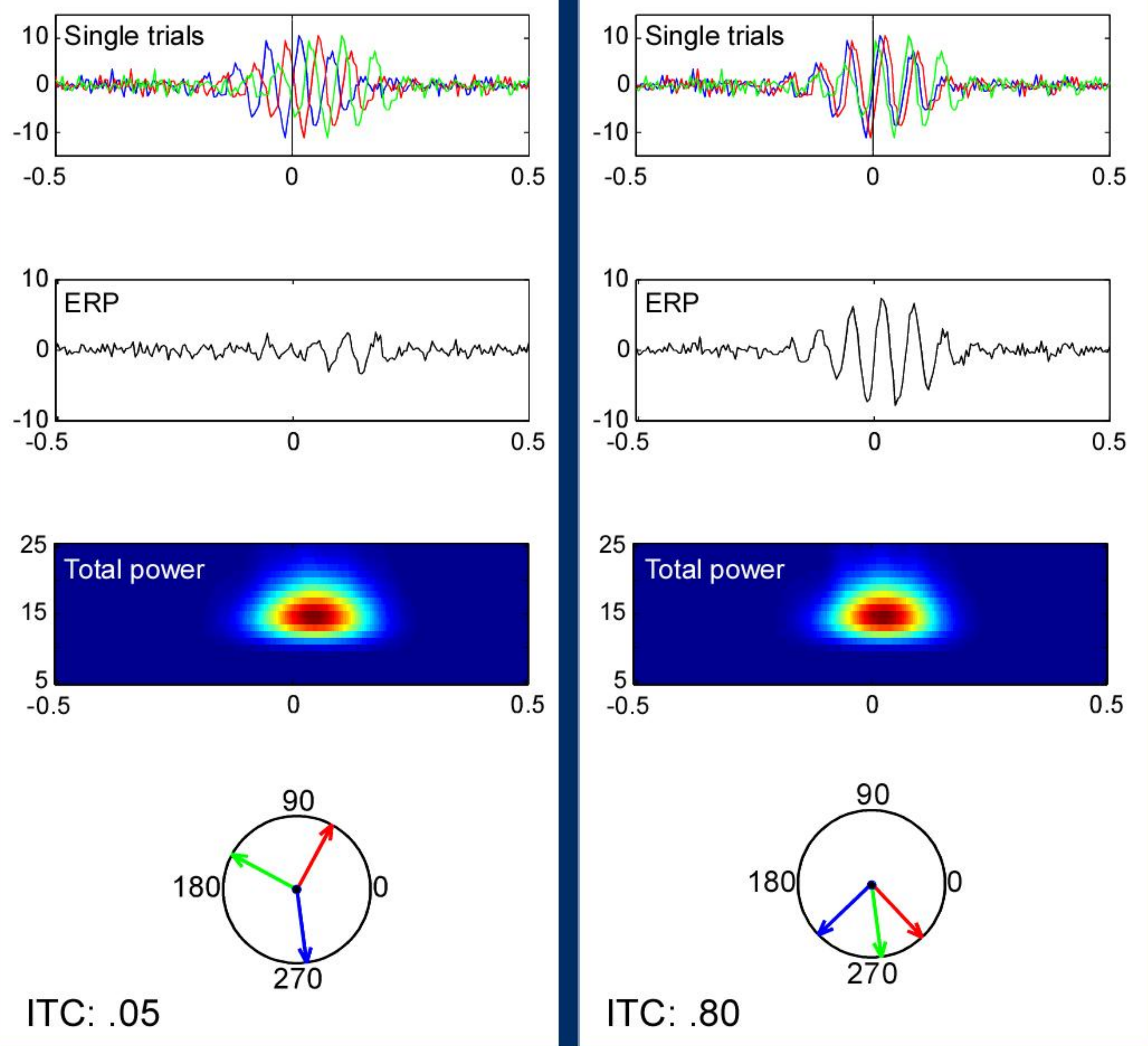
$$C_{fmax} = \frac{f_{max}}{f_{min}} C_{\min}(1-q)$$

Inter trial coherence (ITC)

same time, different trials



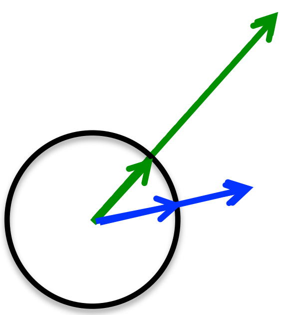
Intertrial Coherence (ITC)



Slide courtesy of Stefan Debener

Phase ITC

$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{\overbrace{S_k(f,t)}^{\text{complex numerator}}}{\underbrace{\left|S_k(f,t)\right|}_{\substack{\text{Normalized}\\ \text{(no amplitude information)}}}}$$



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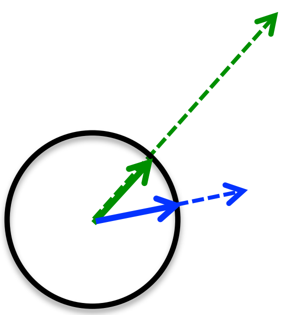
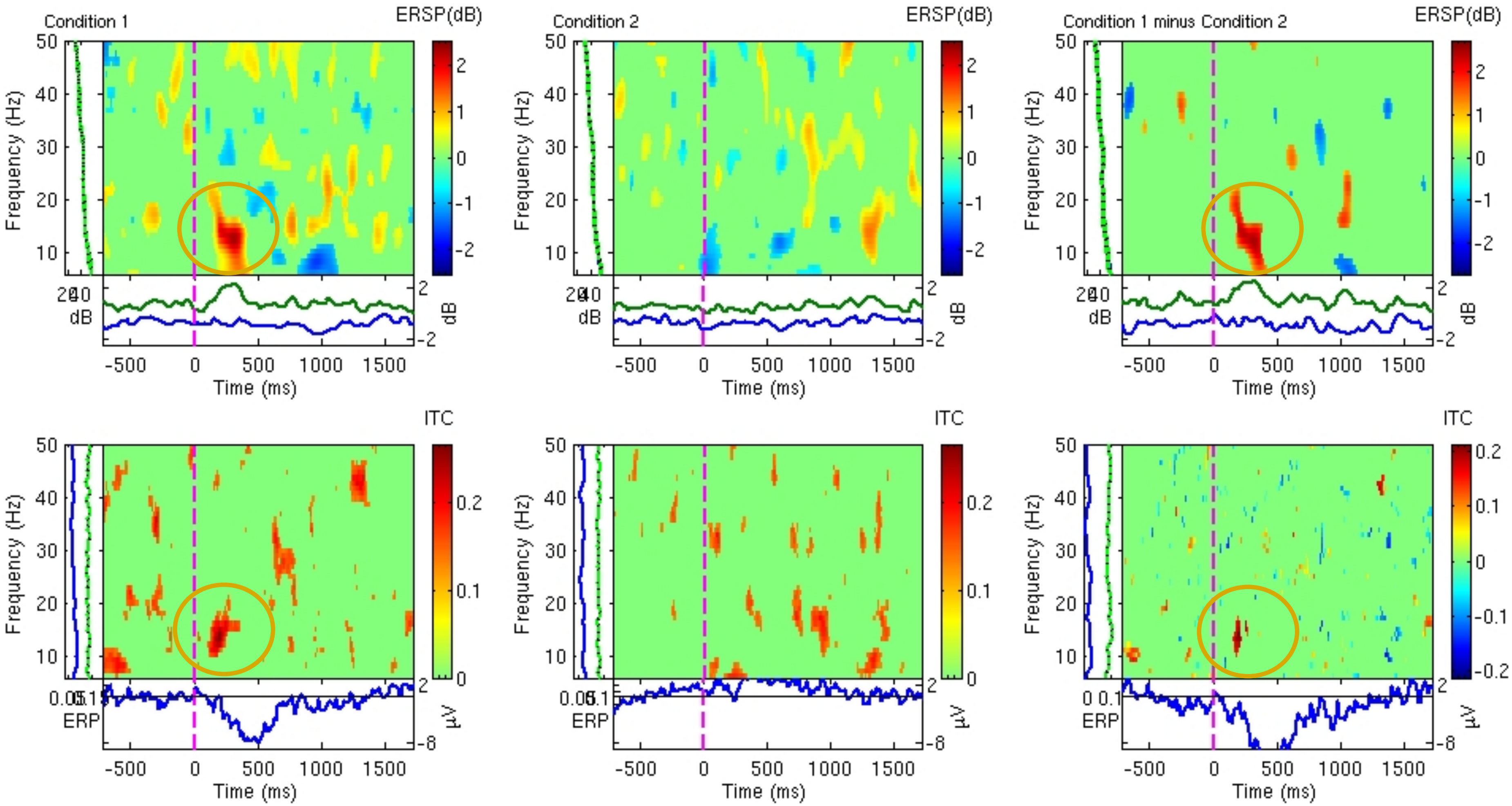


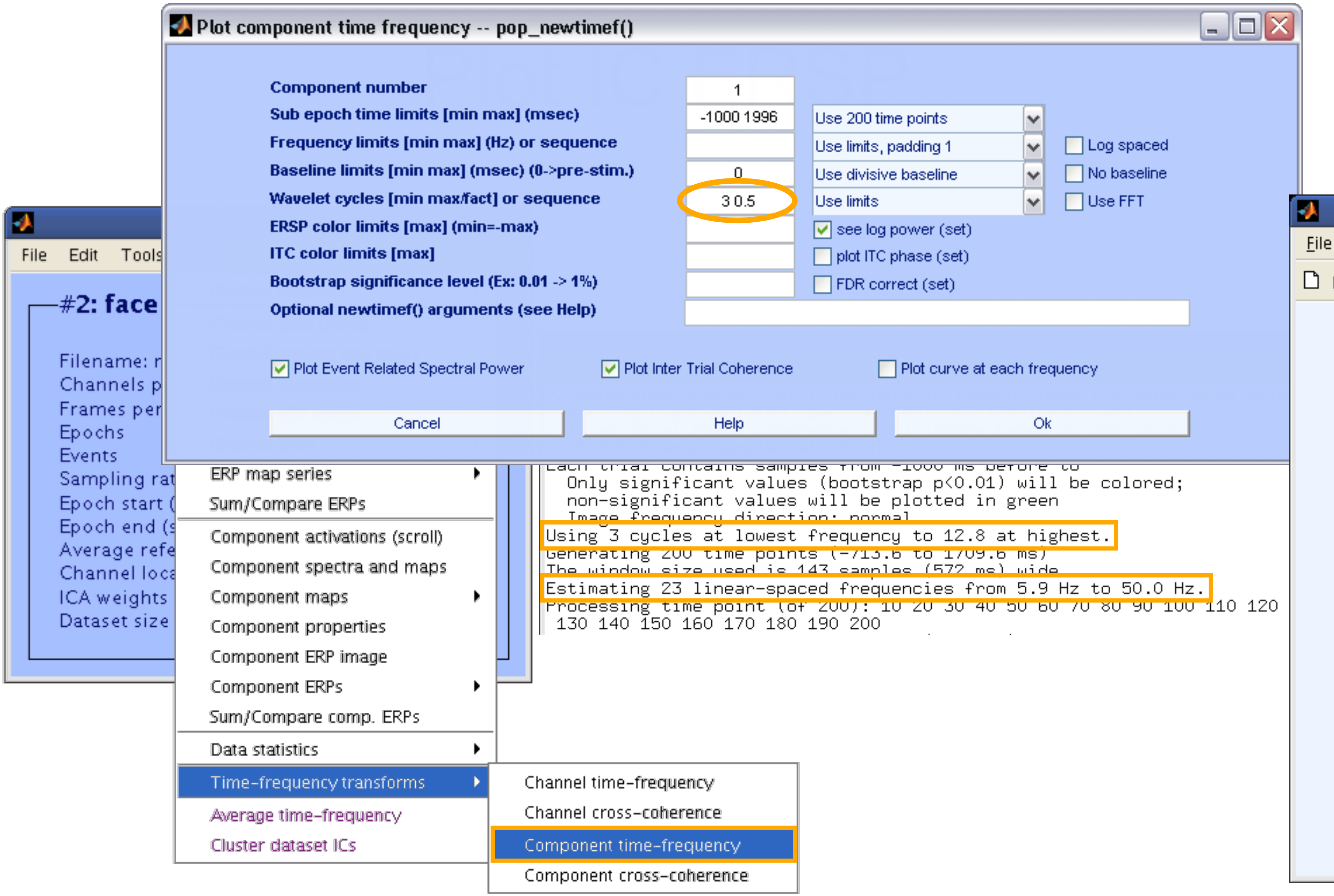
Power and inter trial coherence

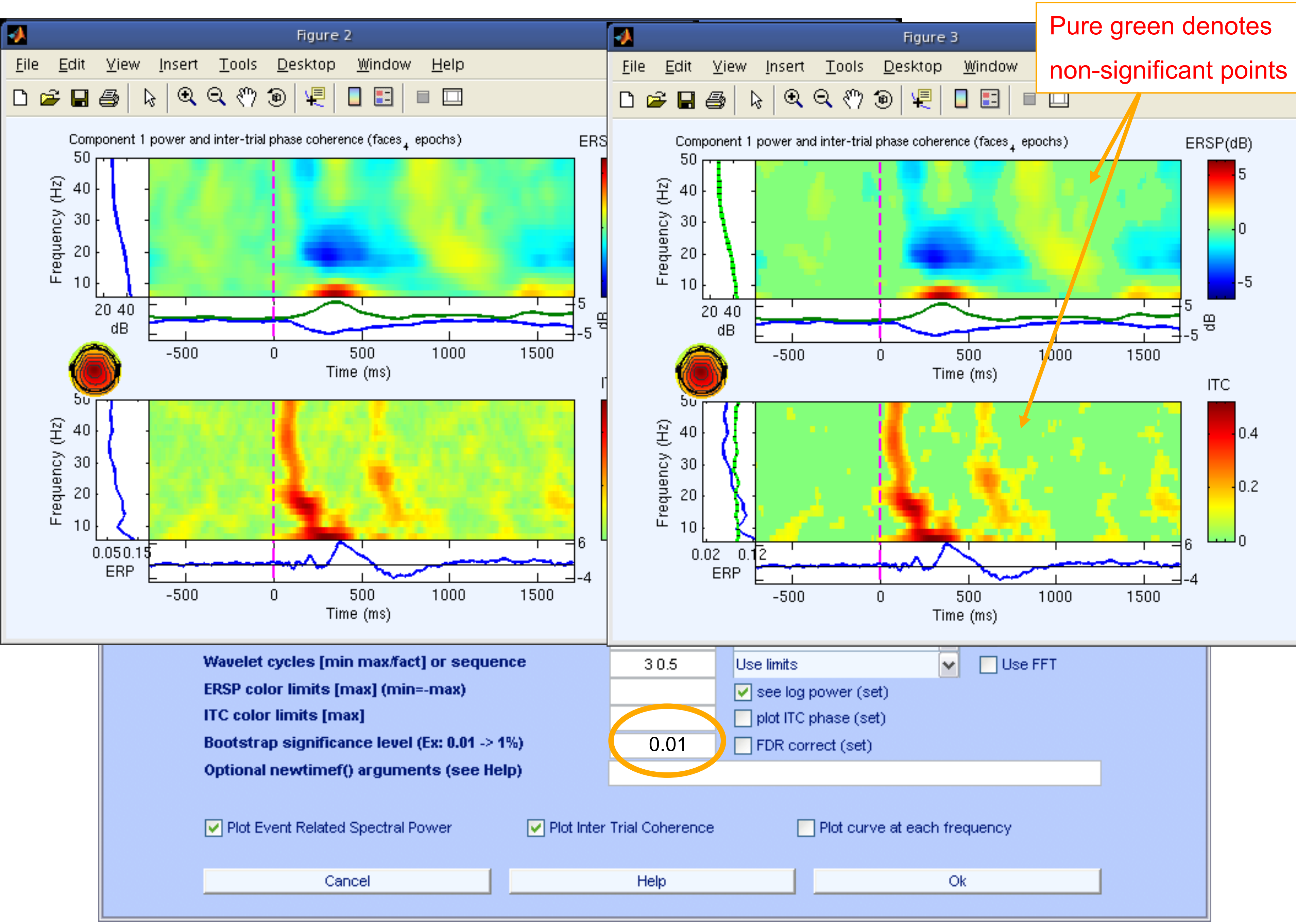
Attend left-stim left

Attend left-stim right

Difference




Plot component time frequency -- pop_newtimef()
Component number
1
Sub epoch time limits [min max] (msec)
-1000 1996
Use 200 time points
Frequency limits [min max] (Hz) or sequence
Use limits, padding 1
Log spaced
Baseline limits [min max] (msec) (0->pre-stim.)
0
Use divisive baseline
No baseline
Wavelet cycles [min max/fact] or sequence
3 0.5
Use limits
Use FFT
ERSP color limits [max] (min=-max)
see log power (set)
ITC color limits [max]
plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)
FDR correct (set)
Optional newtimef() arguments (see Help)
Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency
Cancel
Help
Ok
File
Edit
Tools
#2: face
Filename:
Channels p
Frames per
Epochs
Events
Sampling rat
Epoch start (
Epoch end (
Average refe
Channel loca
ICA weights
Dataset size
ERP map series
Sum/Compare ERPs
Component activations (scroll)
Component spectra and maps
Component maps
Component properties
Component ERP image
Component ERPs
Sum/Compare comp. ERPs
Data statistics
Time-frequency transforms
Average time-frequency
Cluster dataset ICs
Channel time-frequency
Channel cross-coherence
Component time-frequency
Component cross-coherence
Only significant values (bootstrap p<0.01) will be colored;
non-significant values will be plotted in green
Using 3 cycles at lowest frequency to 12.8 at highest.
Estimating 23 linear-spaced frequencies from 5.9 Hz to 50.0 Hz.
Processing time point (of 200): 10 20 30 40 50 60 70 80 90 100 110 120
130 140 150 160 170 180 190 200
File


Pure green denotes non-significant points
Figure 2
Figure 3
Component 1 power and inter-trial phase coherence (faces epochs)
ERSP(dB)
ITC
Frequency (Hz)
Time (ms)
ERP
Wavelet cycles [min max/fact] or sequence
3 0.5
Use limits
Use FFT
ERSP color limits [max] (min=-max)
see log power (set)
ITC color limits [max]
plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)
0.01
FDR correct (set)
Optional newtimef() arguments (see Help)
Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency
Cancel
Help
Ok

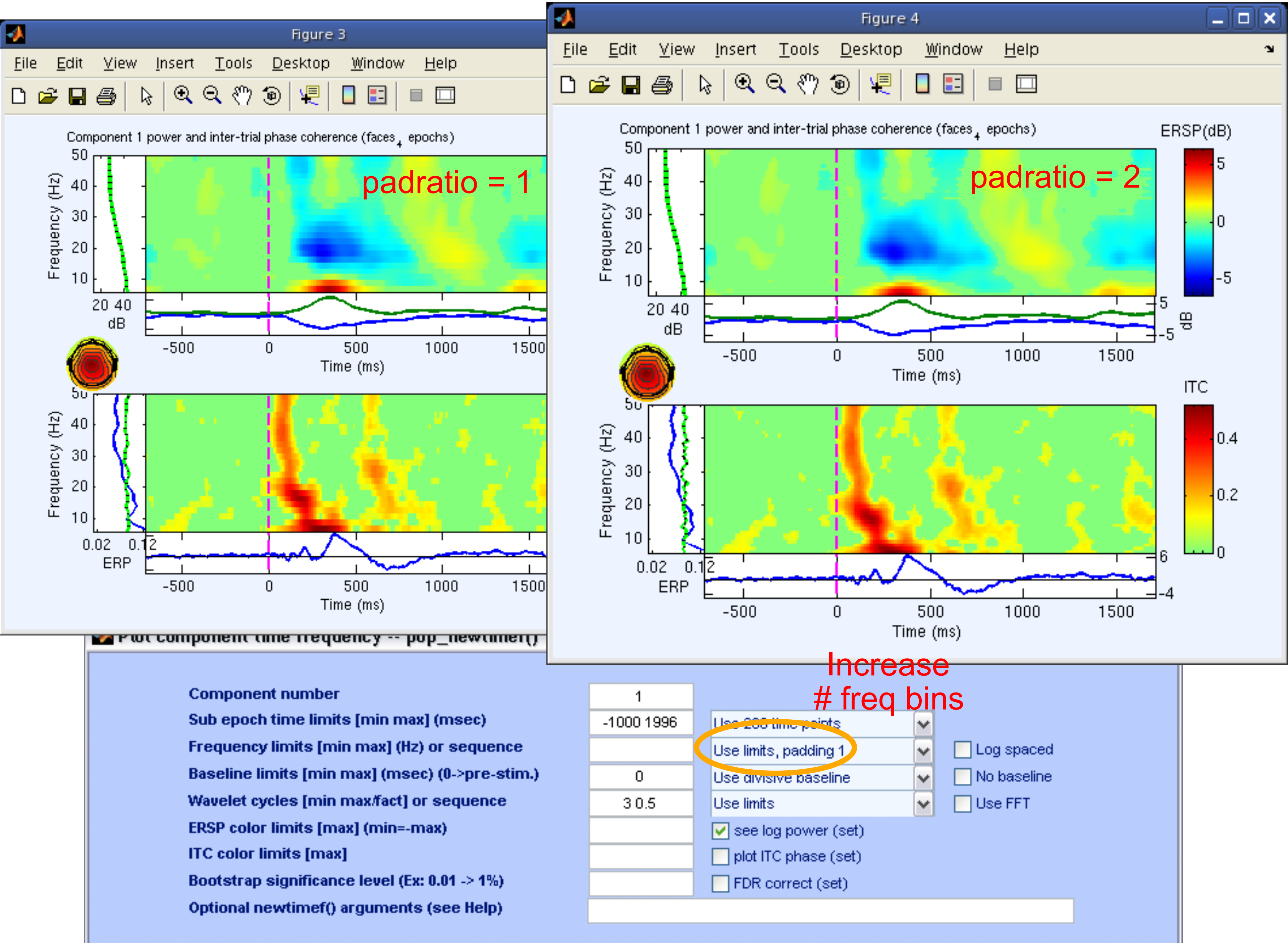
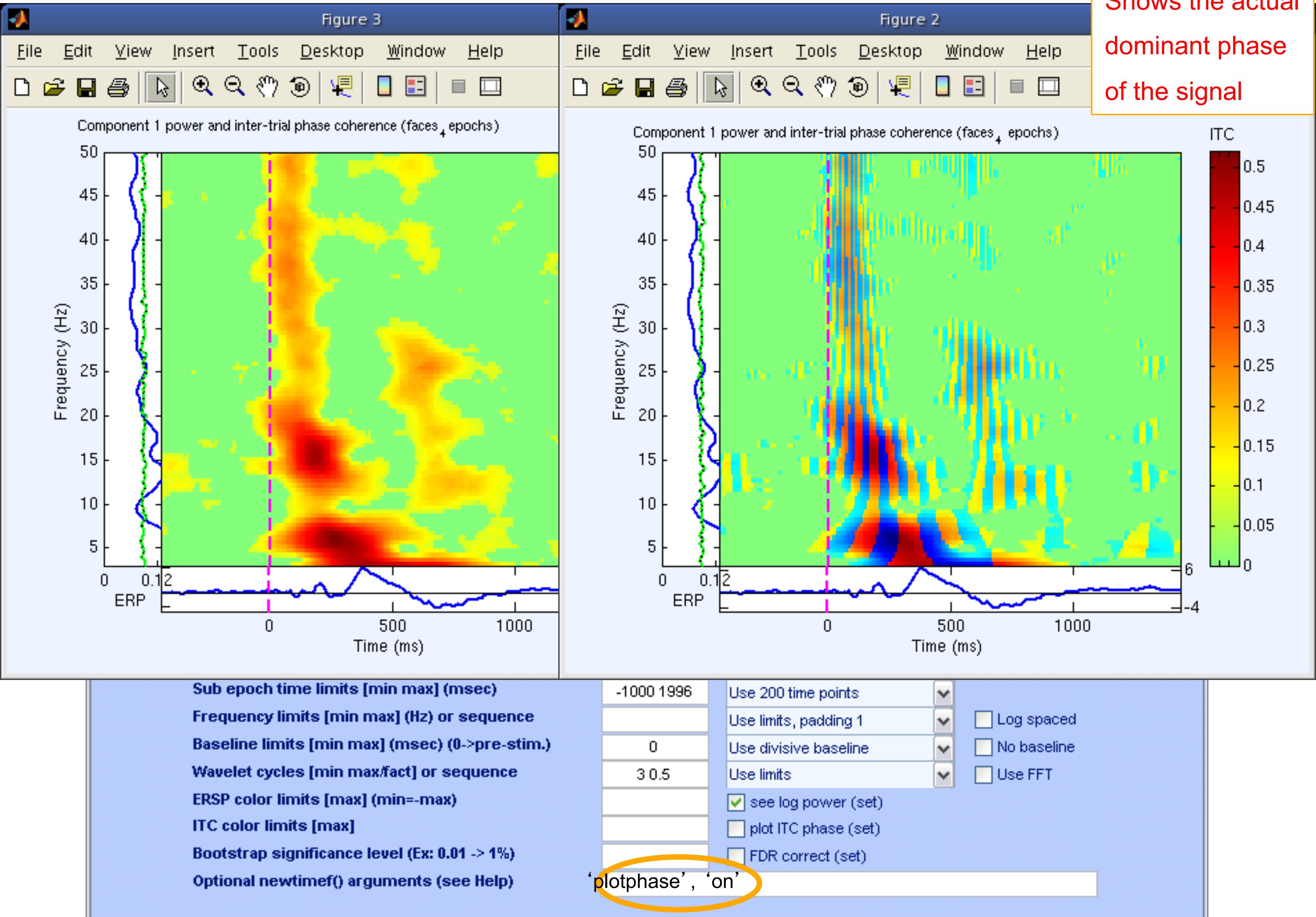


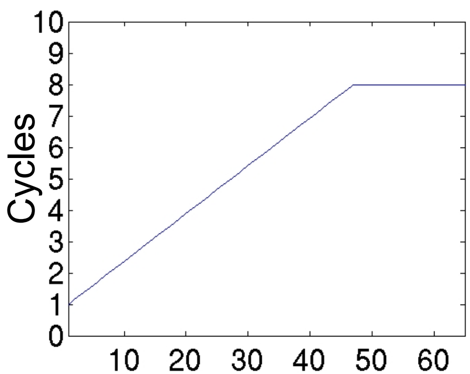
Figure 3
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
padratio = 1
Frequency (Hz)
20 40
dB
Time (ms)
0.02 0.12
ERP
Figure 4
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
ERSP(dB)
padratio = 2
ITC
Increase
freq bins
Component number
1
Sub epoch time limits [min max] (msec)
-1000 1996
Frequency limits [min max] (Hz) or sequence
Use limits, padding 1
Log spaced
Baseline limits [min max] (msec) (0->pre-stim.)
0
Use divisive baseline
No baseline
Wavelet cycles [min max/fact] or sequence
3 0.5
Use limits
Use FFT
ERSP color limits [max] (min=-max)
see log power (set)
ITC color limits [max]
plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)
FDR correct (set)
Optional newtimef() arguments (see Help)

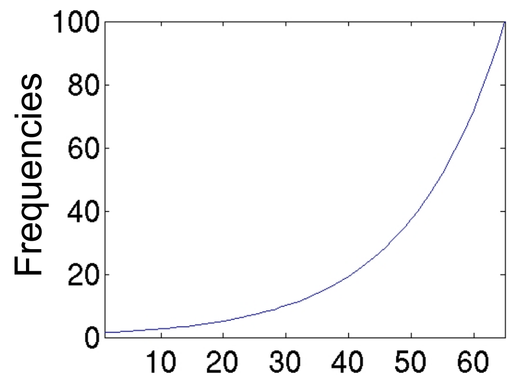
Shows the actual dominant phase of the signal

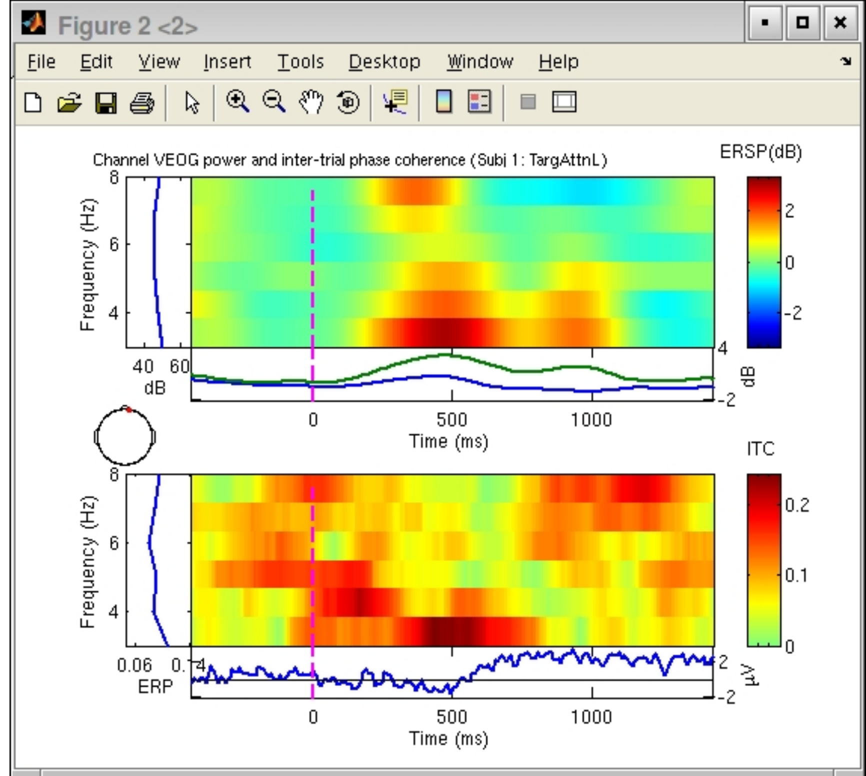


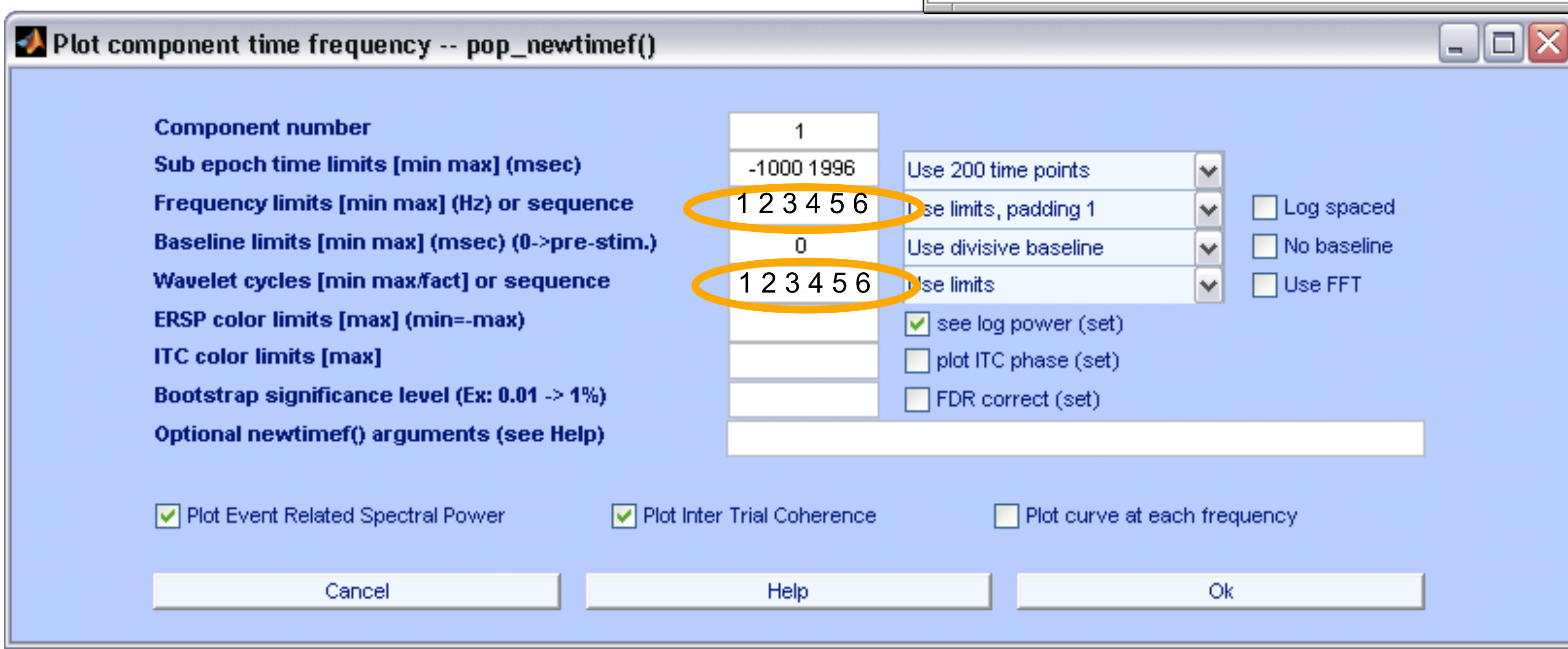
To visualize both low and high frequencies

```
freqs  = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
```







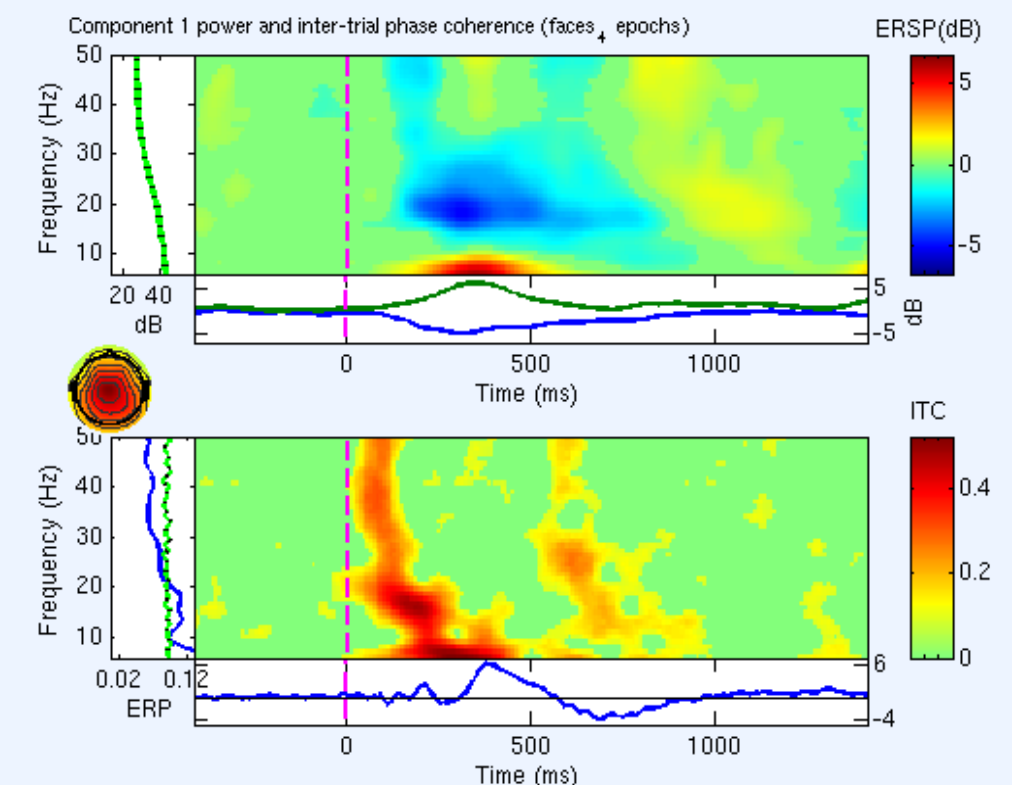


Evoked versus induced

- Evoked = ERSP of the average ERP
- Induced = usually standard ERSP
- Real induced

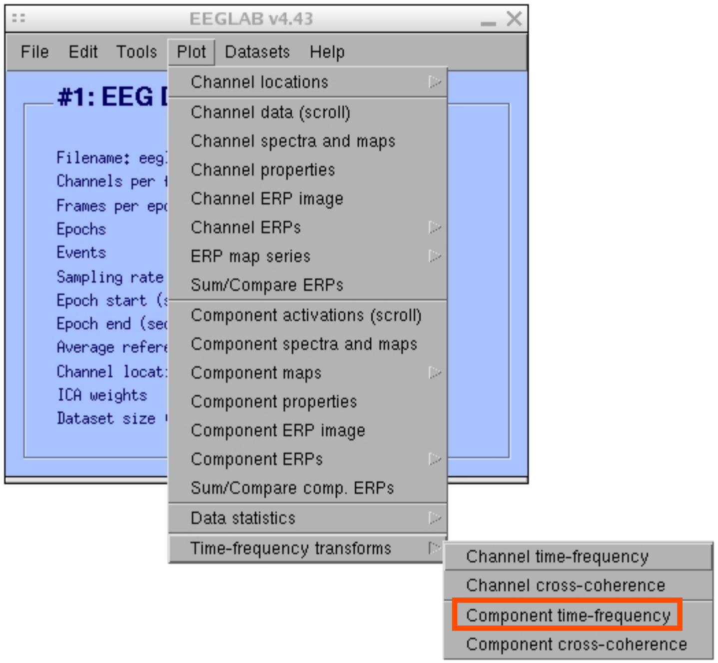
(1) standard ERSP with ERP regressed out of every trial

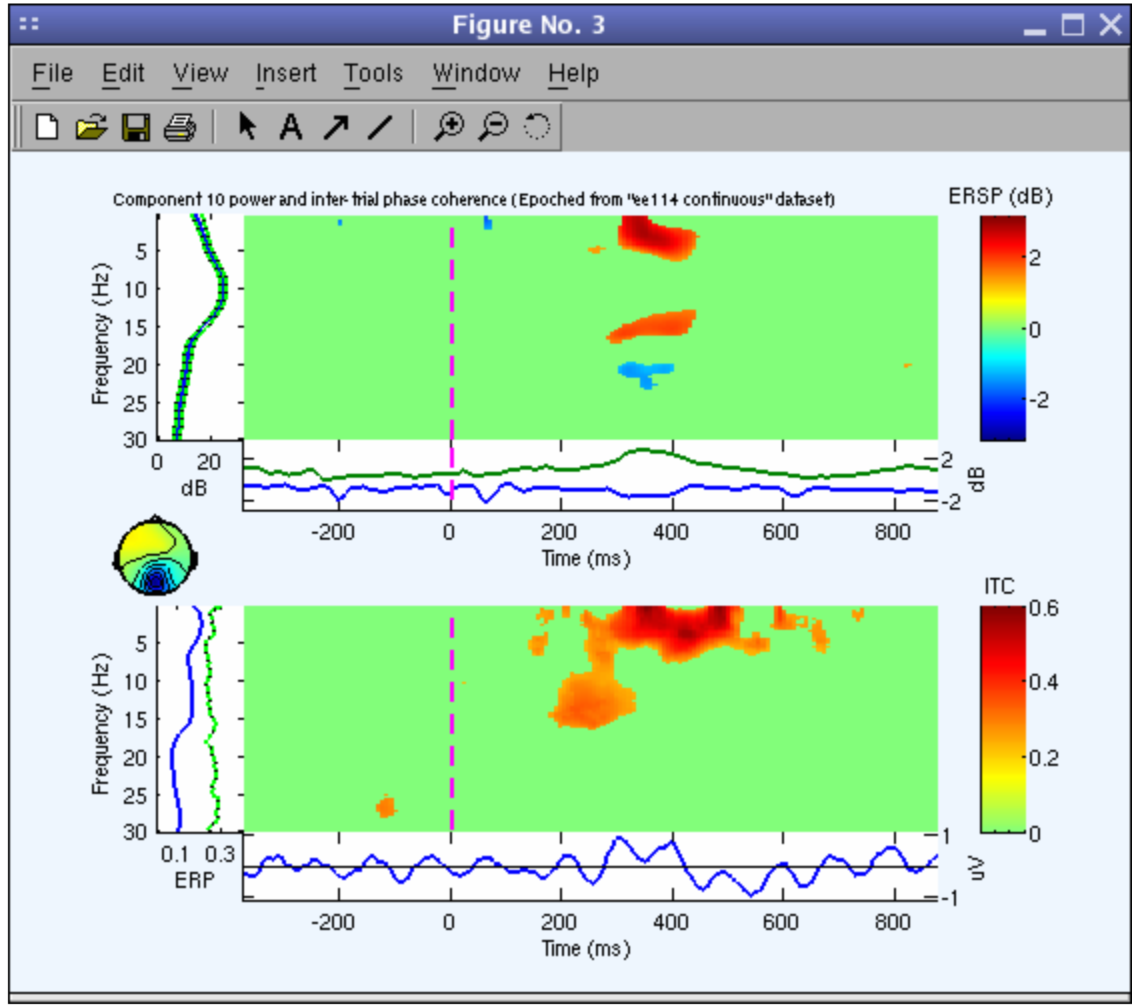
(2) standard ERSP minus ERSP of the average ERP scaled for averaging effect



In any case, looking at the ITC provides the amount of synchronization in the time-frequency decomposition that account for ERPs

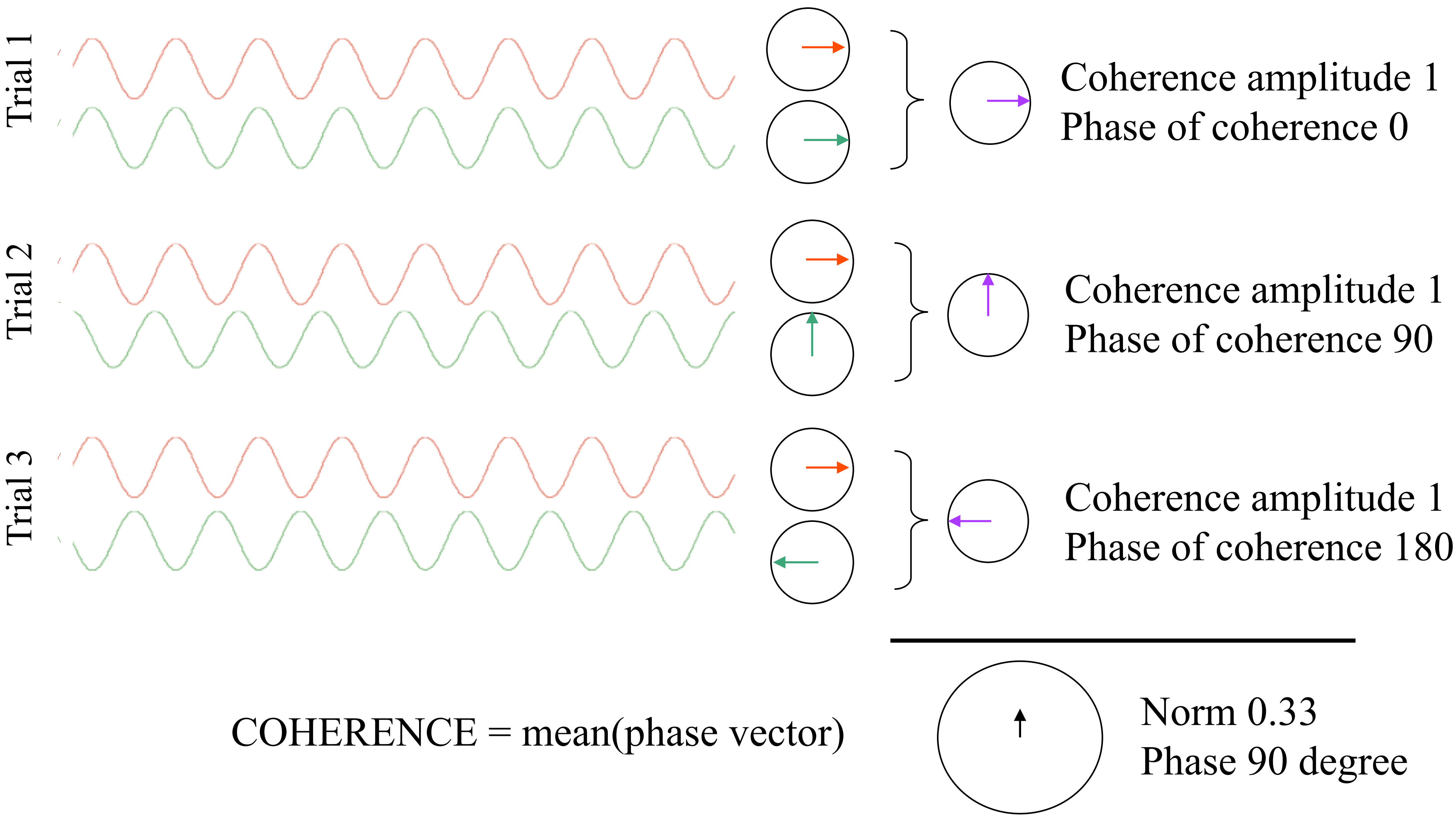
Component time-frequency





cross-coherence amplitude and phase

2 components, comparison on the same trials



COHERENCE = mean(phase vector)

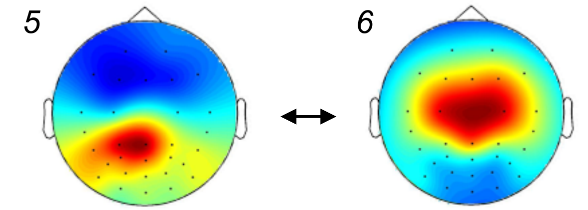
Event-related phase coherence

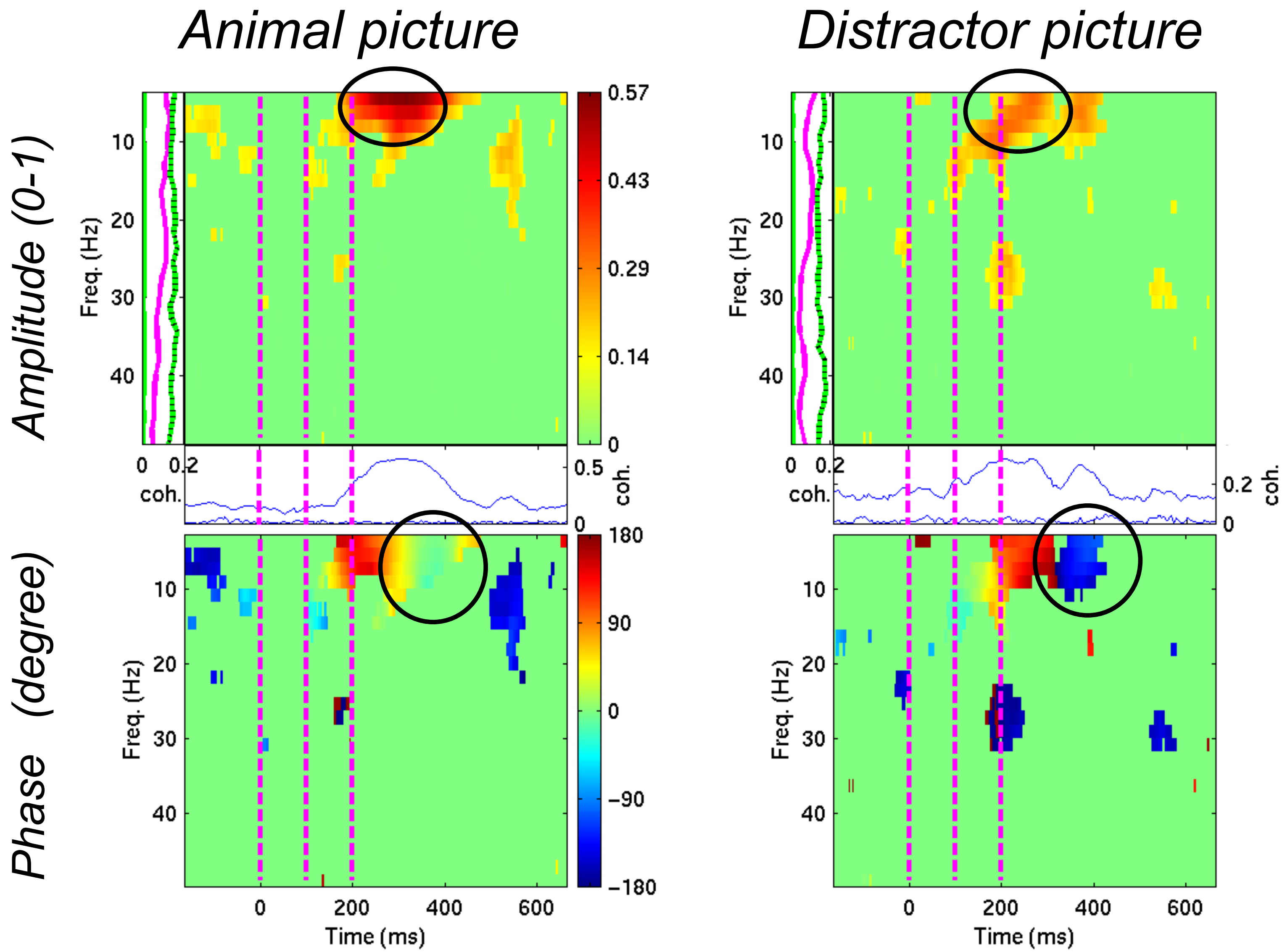
$$ERPCOH^{a,b}(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{S_k^a(f,t)S_k^b(f,t)^*}{\left|S_k^a(f,t)\right|\left|S_k^b(f,t)\right|}$$

Only phase information component b

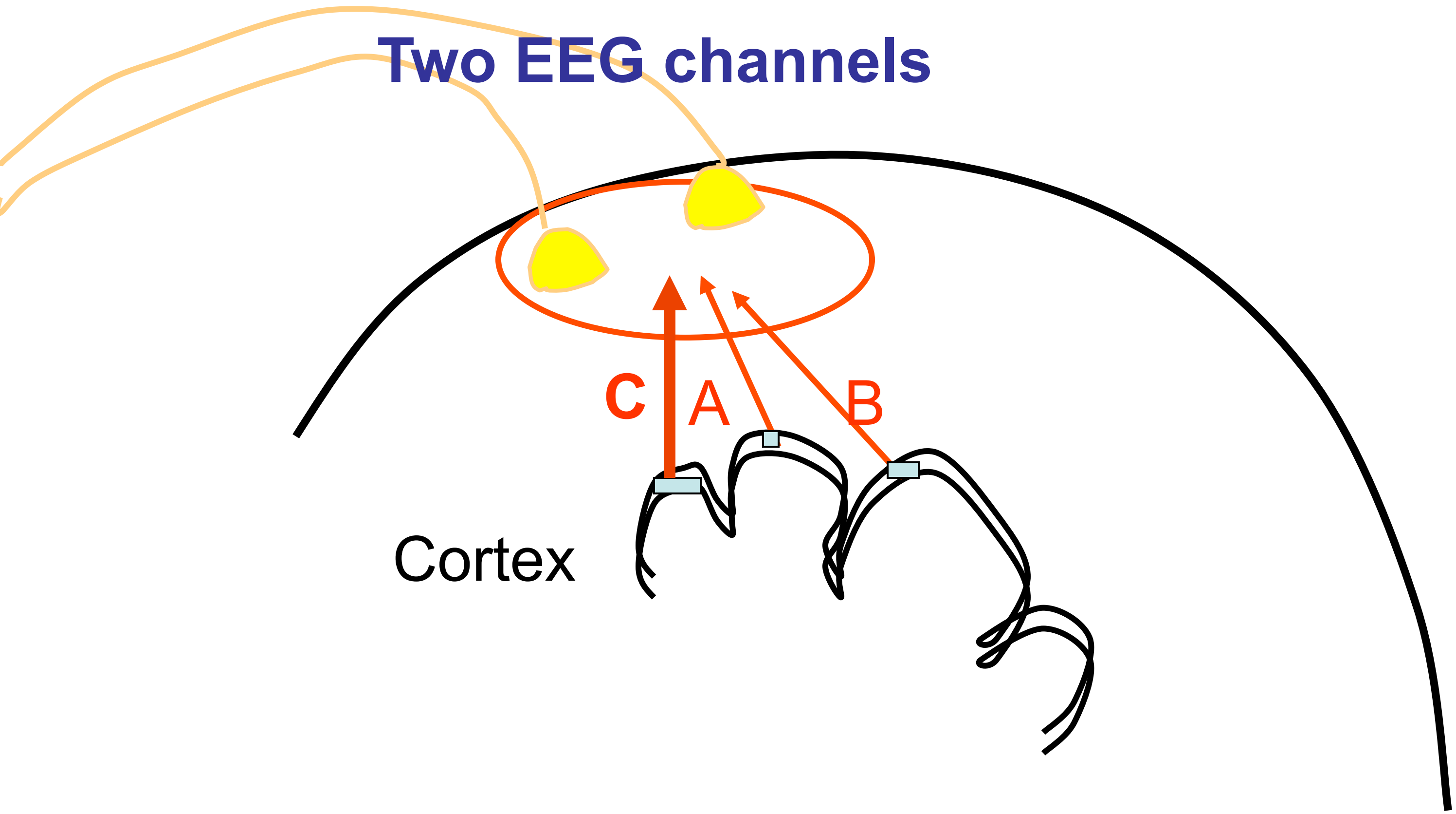
Only phase information component a

Cross-coherence amplitude and phase

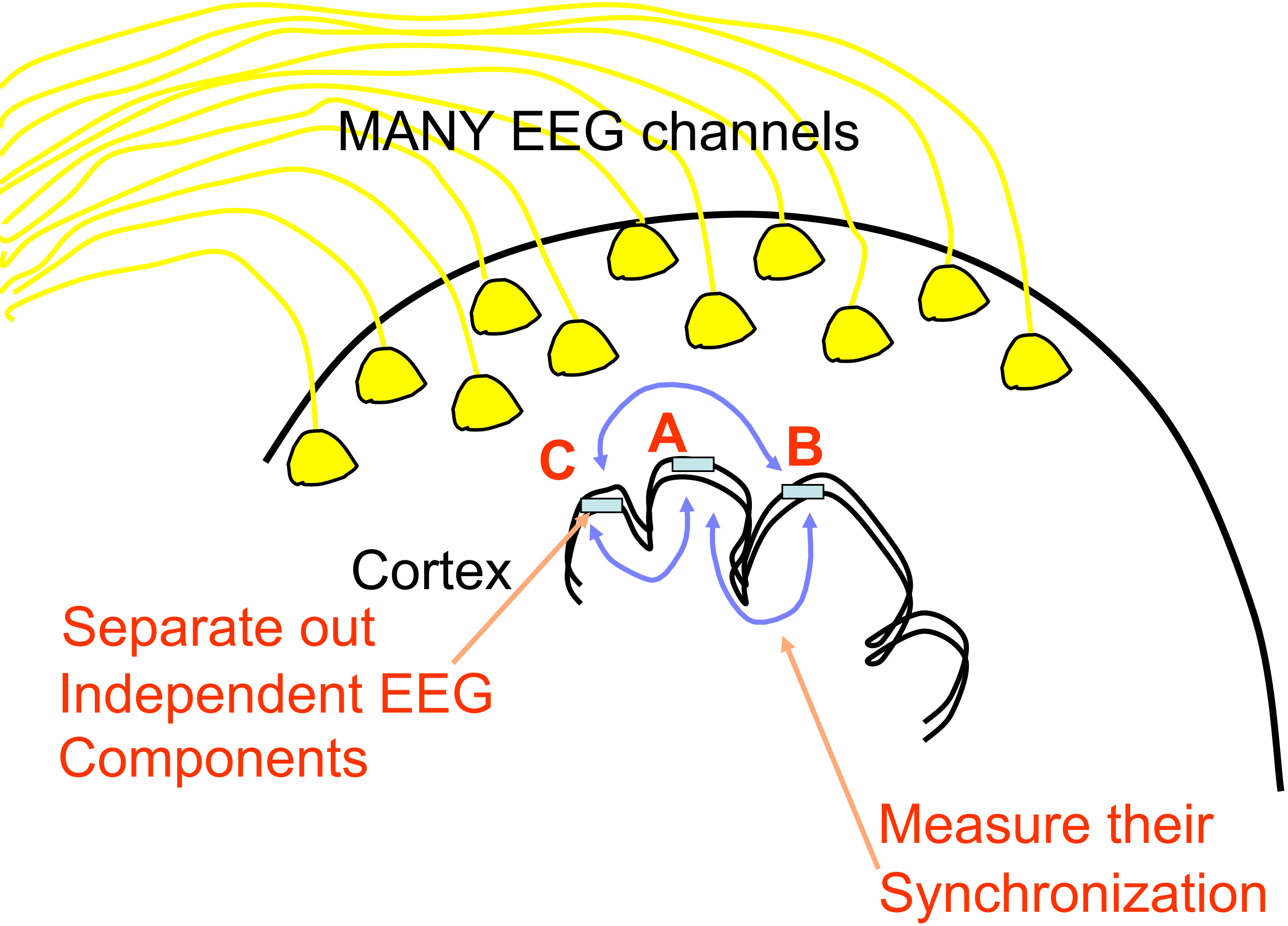




Two EEG channels

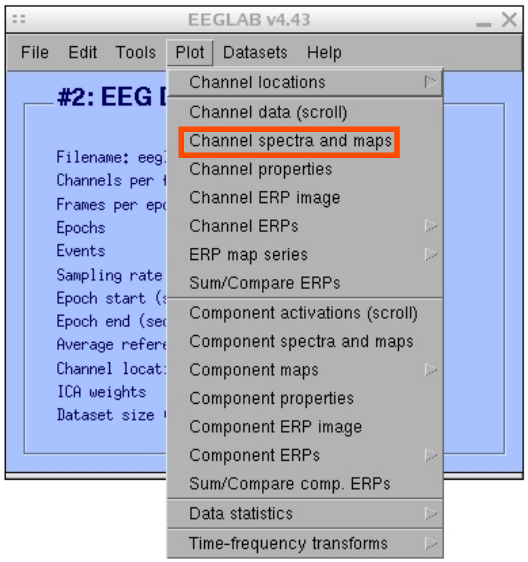


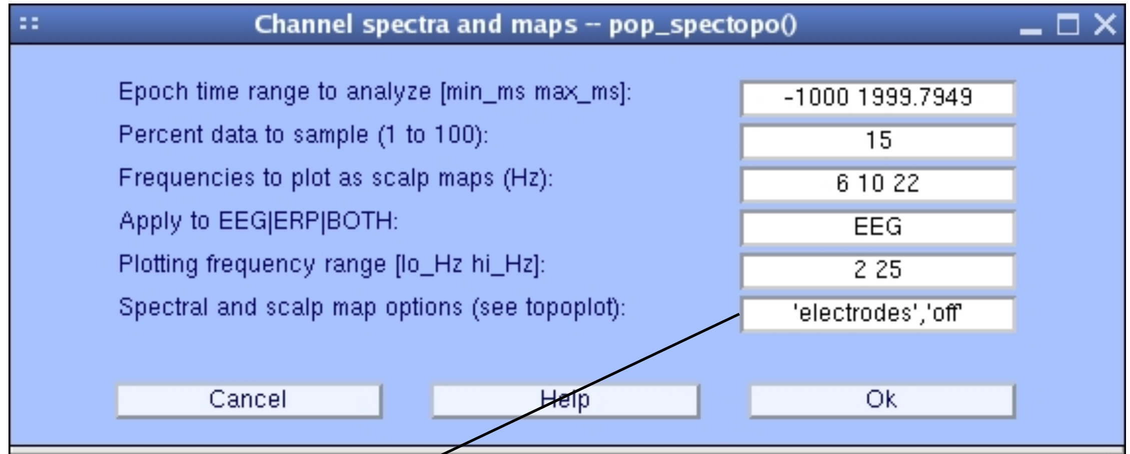
Scalp channel coherence → source confounds!

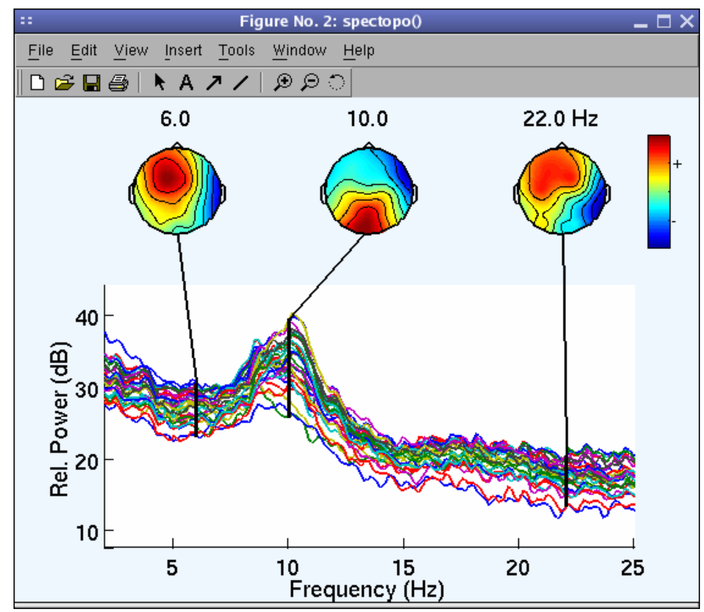


source dynamics!

Plot data spectrum using EEGLAB







'winsize' , 256 (change FFT window length)
'nfft' , 256 (change FFT padding)
'overlap' , 128 (change window overlap)

Exercise

- **ALL**

  Start EEGLAB, from the menu load *sample_data/ eeglab_data_epochs_ica.set*

  or your own data (epoch, reject noise if not done already)

- **Novice**

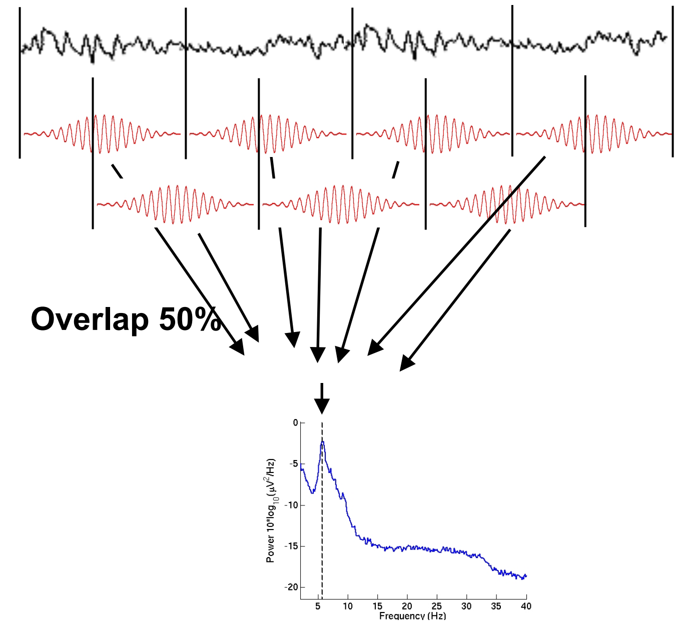
  From the GUI, Plot spectral decomposition with 100% data and 50% overlap ('overlap'). Try reducing window length ('winsize') and FFT length ('nfft')

- **Intermediate**

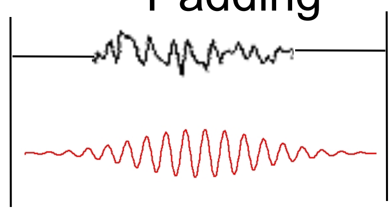
  Same as novice but using a command line call to the *pop_spectopo()* function. Use GUI then history to see a standard call ("eegh").

- **Advanced**

  Same as novice but using a command line call to the *spectopo()* function.



Padding



Exercise - newtimef

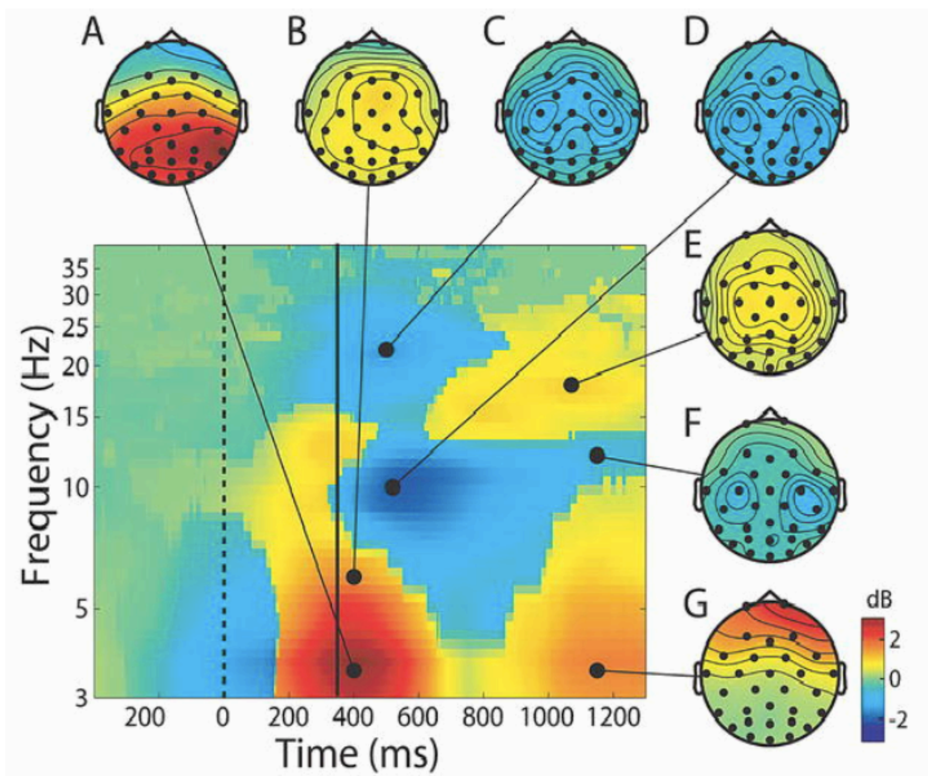
- **Novice**
  From the GUI, pick an interesting IC and plot component ERSP. Try changing parameters window size, number of wavelet cycles, padratio,

- **Intermediate**
  From the command line, use newtimef() to tailor your time/ frequency output to your liking. Look up the help to try not to remove the baseline, change baseline length and plot in log scale. Enter custom frequencies and cycles (2 slides back).

- **Advanced**
  Compare FFT, the different wavelet methods (see help), and multi-taper methods (use timef function not newtimef). Enter custom frequencies and cycles. Look up newtimef help to compare conditions. Visualize single-trial timef-frequency power using erpimage.

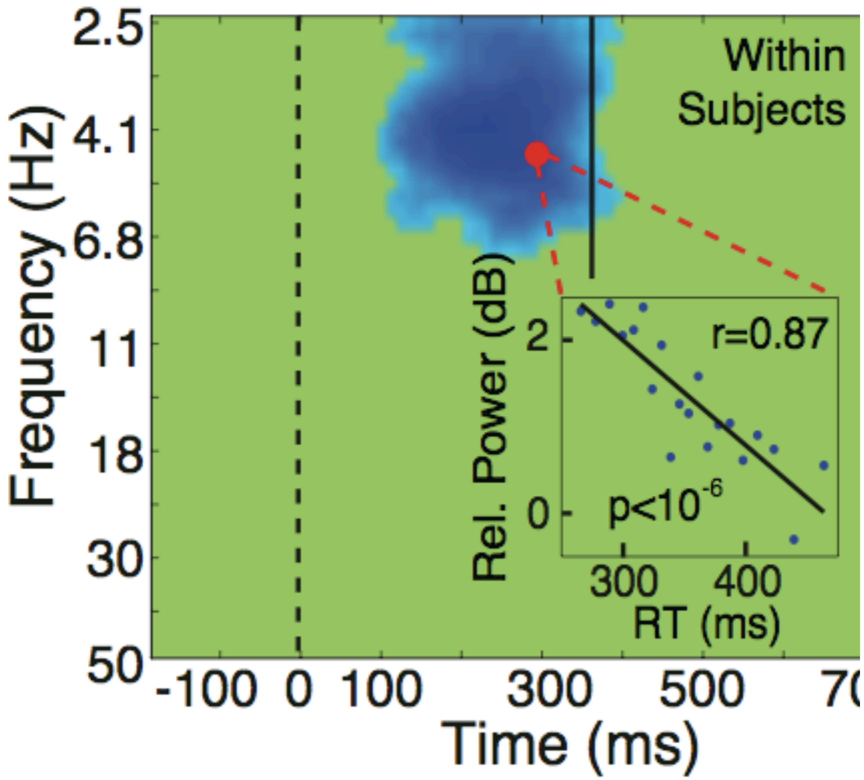
Advanced time-frequency functions

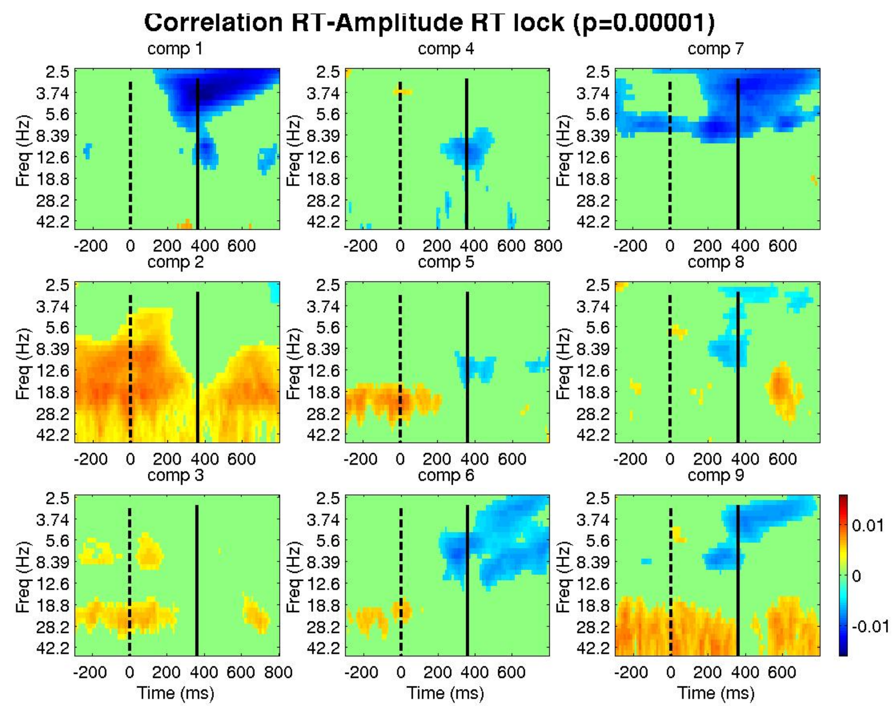
- Tftopo(): allow visualizing time-frequency power distribution over the scalp



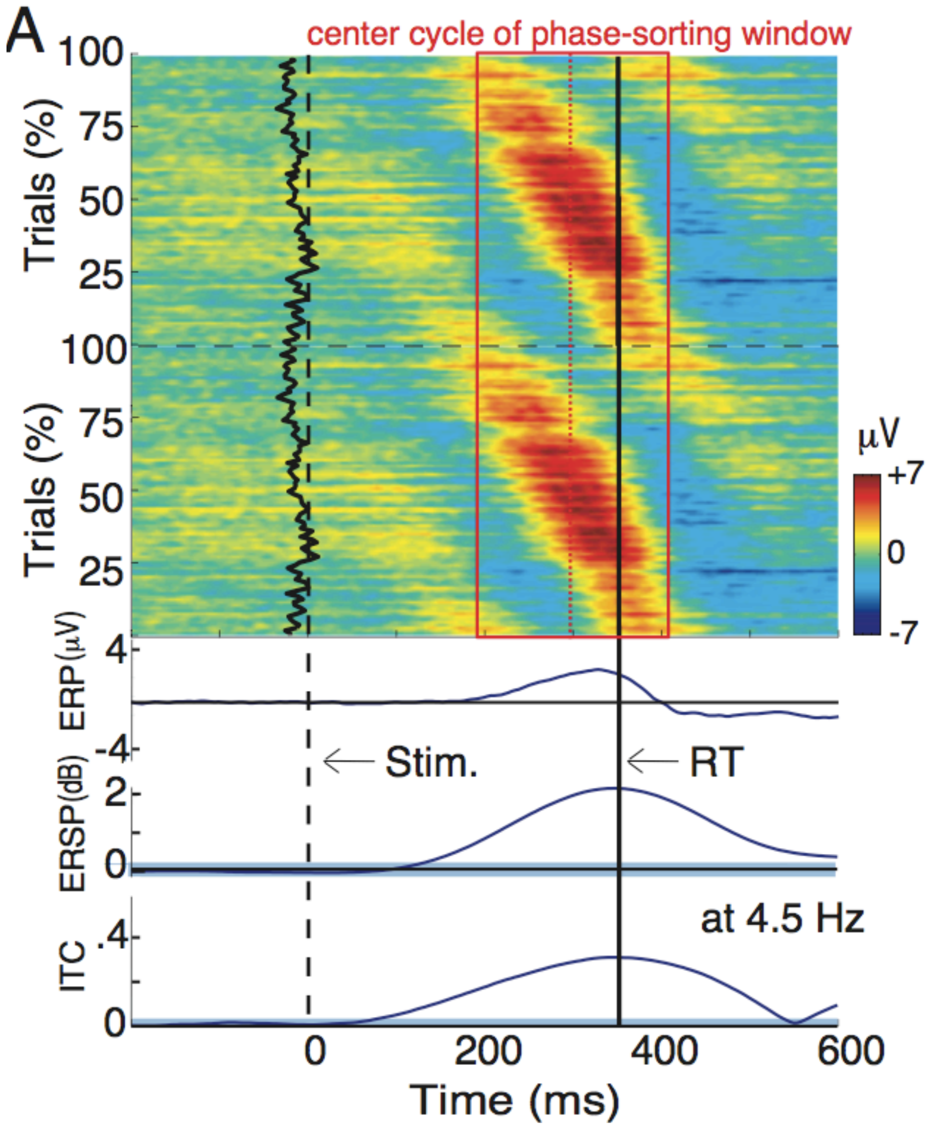
Advanced time-frequency functions

- ERPimage: allow visualizing time-frequency power or phase in single trials

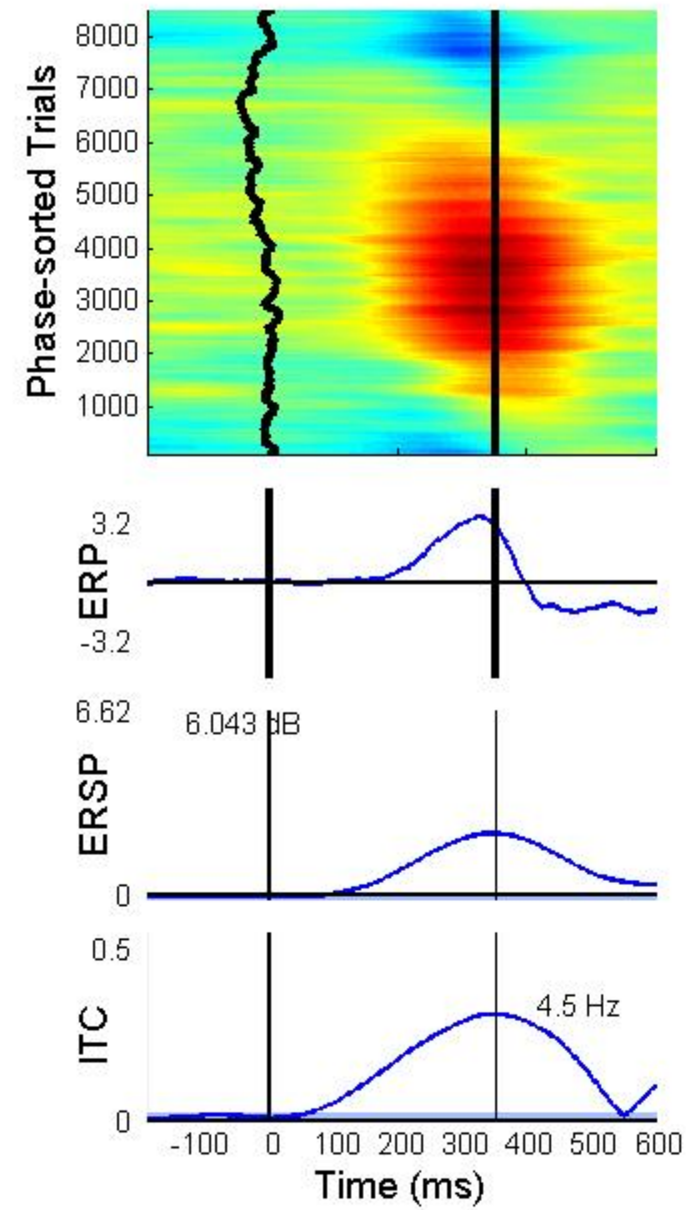




ERPimage

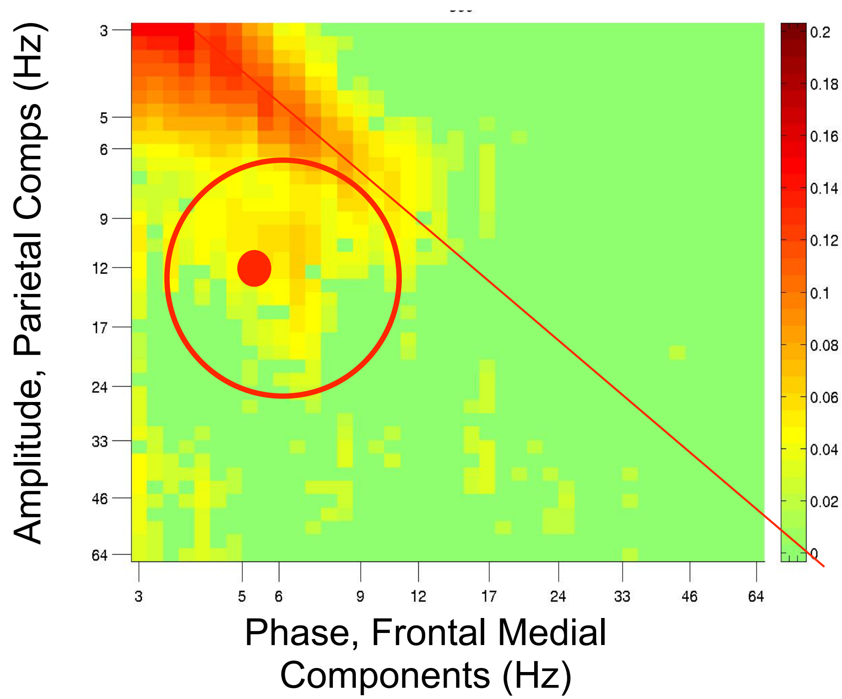


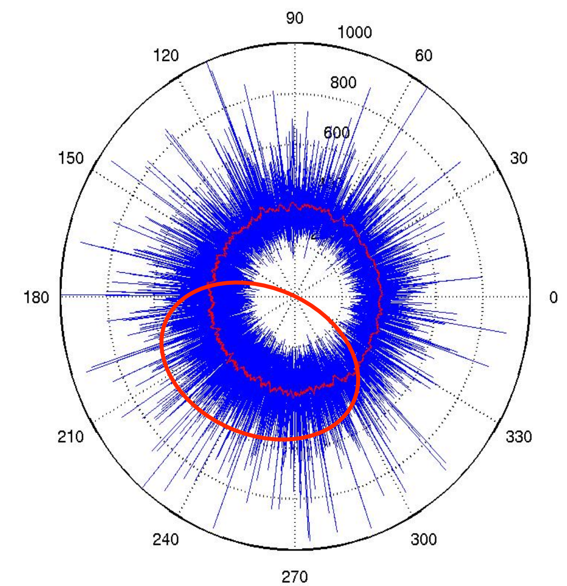
A
center cycle of phase-sorting window
Trials (%)
100
75
50
25
μV
+7
0
-7
ERP(μV)
4
-4
ERSP(dB)
2
0
ITC
.4
Stim.
RT
at 4.5 Hz
0
200
400
600
Time (ms)



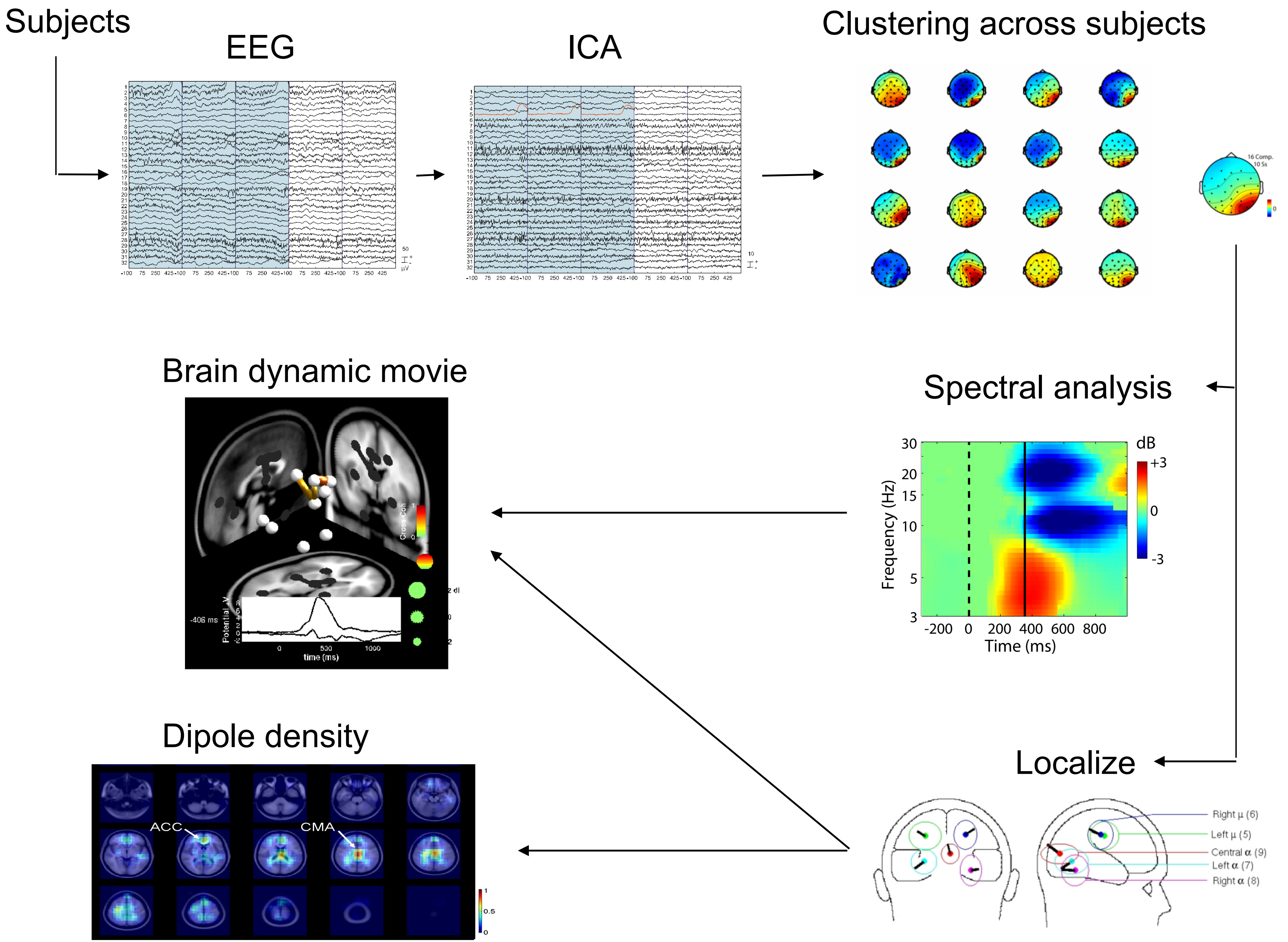
Phase-sorted Trials
8000
7000
6000
5000
4000
3000
2000
1000
ERP
3.2
-3.2
ERSP
6.62
6.043 dB
0
ITC
0.5
4.5 Hz
-100
0
100
200
300
400
500
600
Time (ms)

Across frequency study





$$APCOH^{a,b}(f_1,f_2,t)=\sum_{k=1}^{n}\left|F_k^a(f_1,t)\right|\frac{F_k^b(f_2,t)}{\left|F_k^b(f_2,t)\right|}\Bigg/\sqrt{n\sum_{k=1}^{n}\left|F_k^a(f_1,t)\right|^2}$$



Subjects
EEG
ICA
Clustering across subjects
16 Comp.
10 Ss
Spectral analysis
dB
+3
0
-3
Frequency (Hz)
Time (ms)
Brain dynamic movie
time (ms)
Localize
Right μ (6)
Left μ (5)
Central α (9)
Left α (7)
Right α (8)
Dipole density
ACC
CMA

Dynamical brain movies

5 Hz