Forward and inverse models

Localizing sources using DIPFIT

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DIPFIT: localizing dipoles

- Motivation
- Ingredients
  - Source model
  - Volume conductor model
    - Analytical (spherical model)
    - Numerical (realistic model)
  - Considerations regarding EEG
- Inverse modeling
  - Single and multiple dipole fitting
  - Distributed source models

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Motivation

- Why fit dipoles?
- Why measure EEG?
- Why do ICA?

- Get extra information about brain processes
  - Time course of activity ----> **EEG**
  - Location of activity ----> **fMRI**

Difference between EEG and fMRI

- EEG measures post-synaptic potentials
  - related to synchronized neuronal input
- fMRI measures BOLD
  - related to energy consumption

- Different characteristics in the time domain
- Different generators

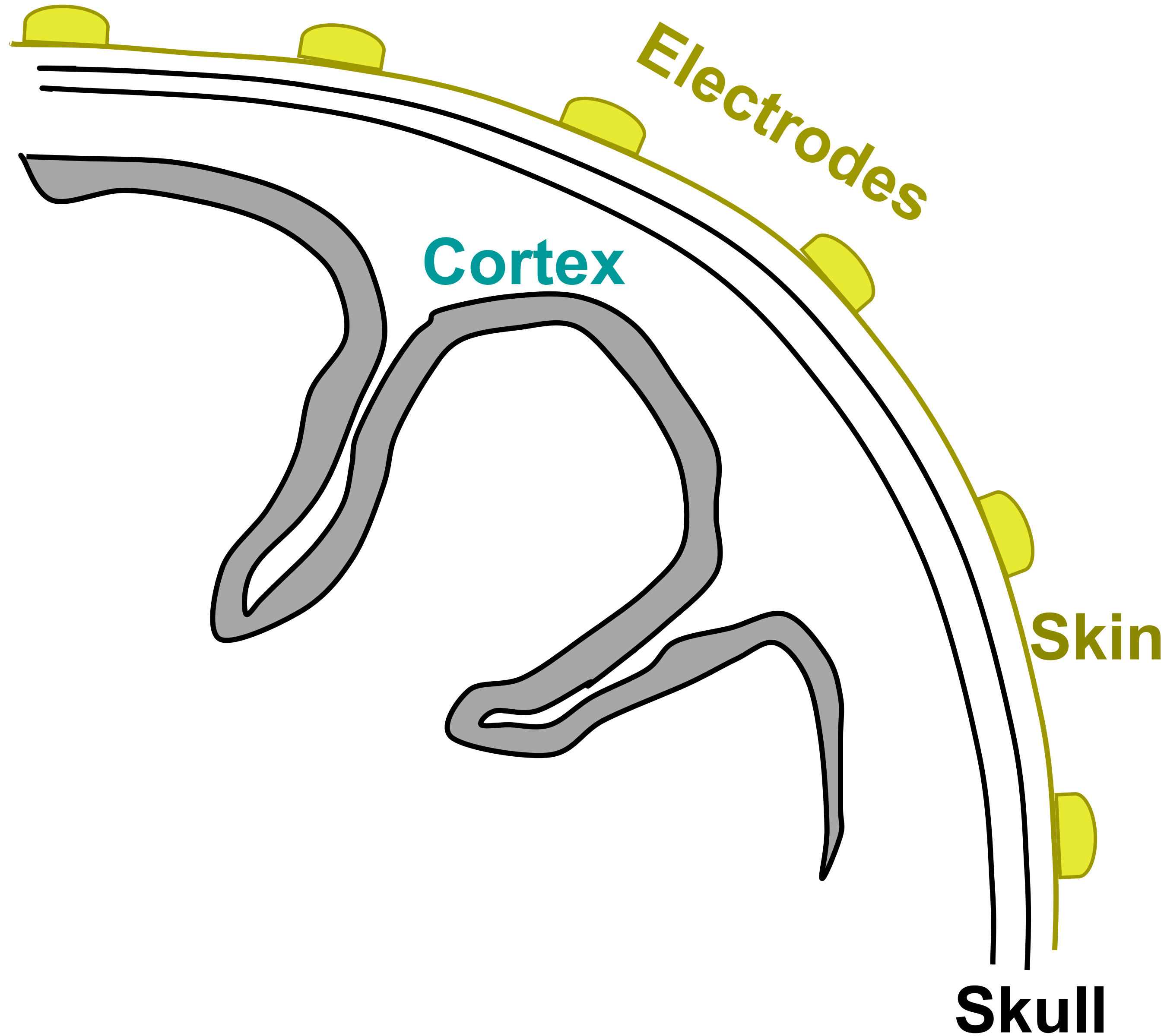
- Timecourse *and* location

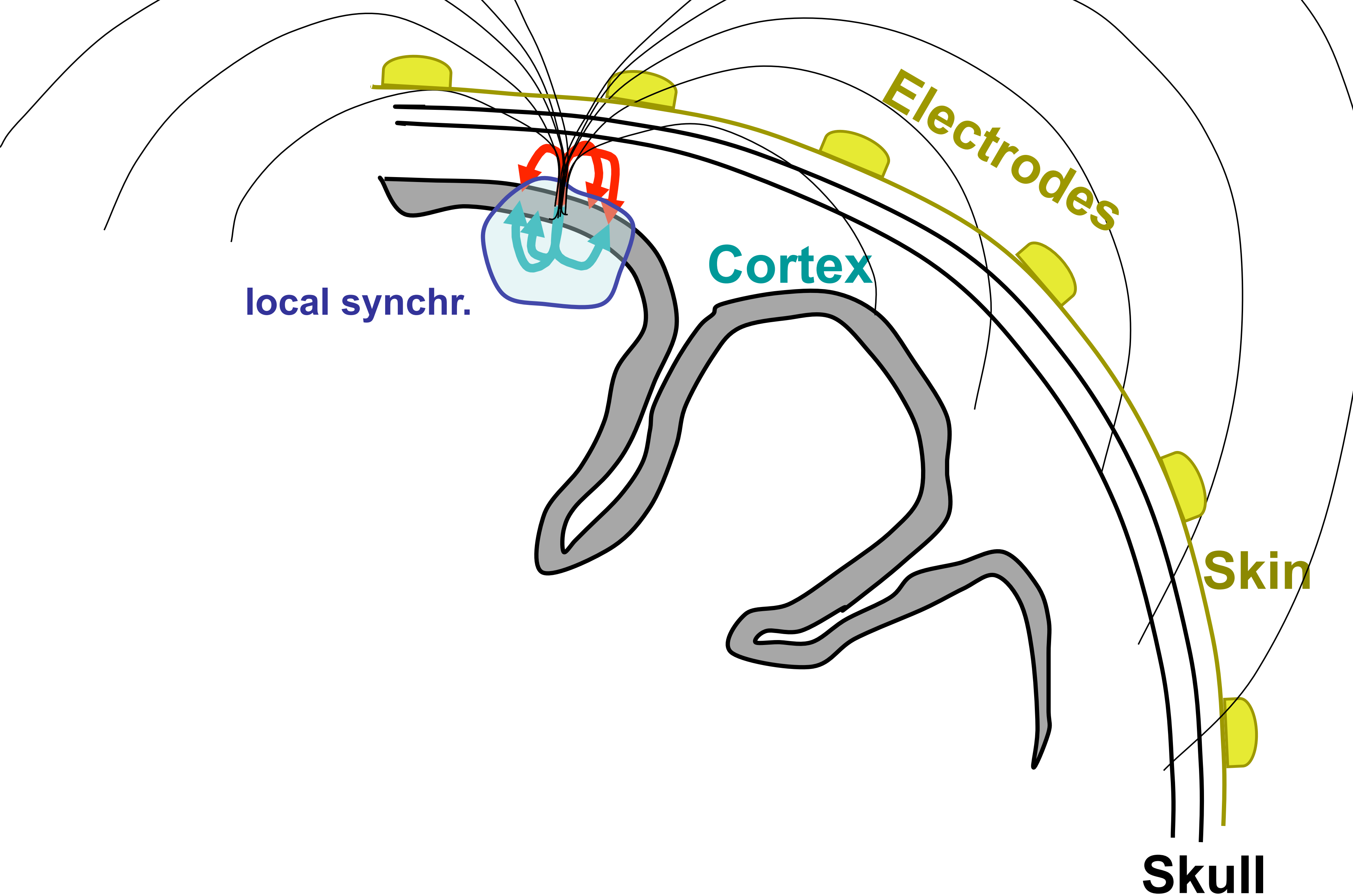
Why EEG: extra information

- Timecourse
  - ERSP
  - ERP

ICA

- Topography
  - Scalp distribution
  - Underlying generators


Electrodes
Cortex
Skin
Skull


Electrodes
Cortex
local synchr.
Skin
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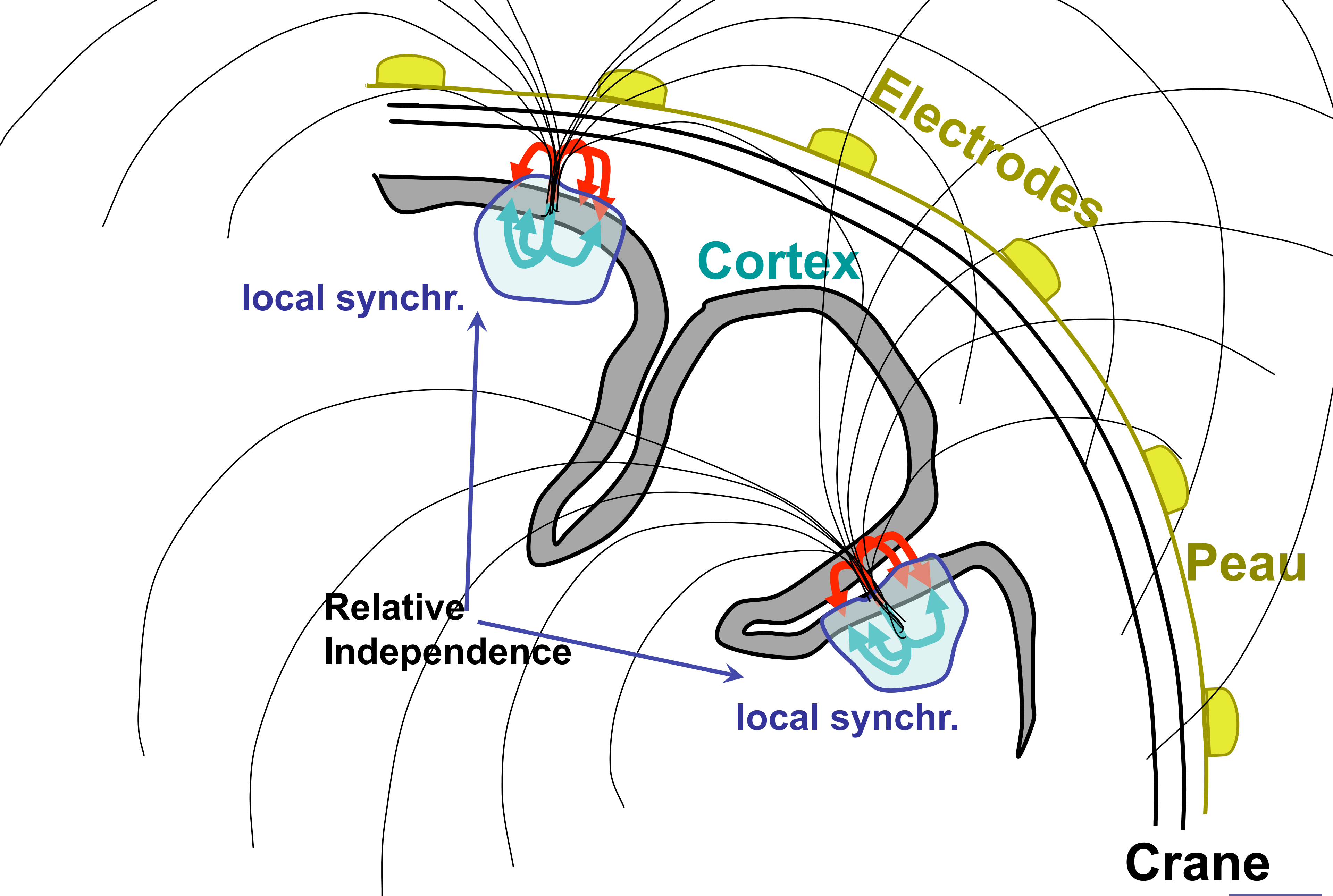
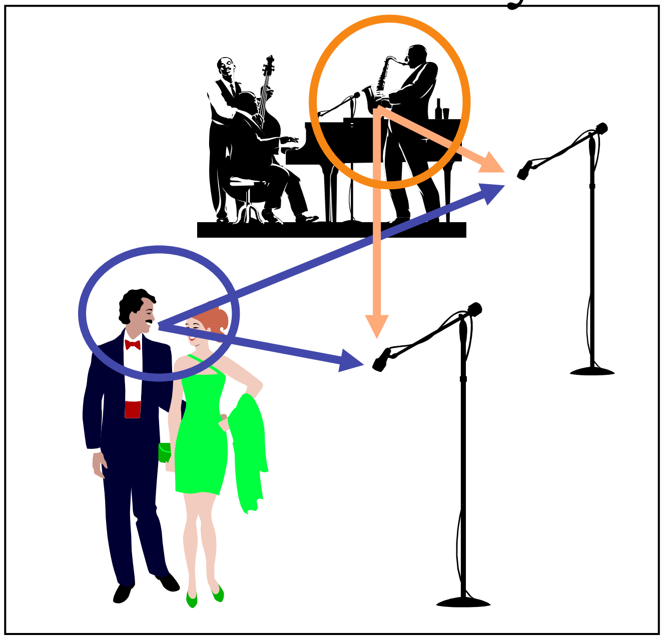
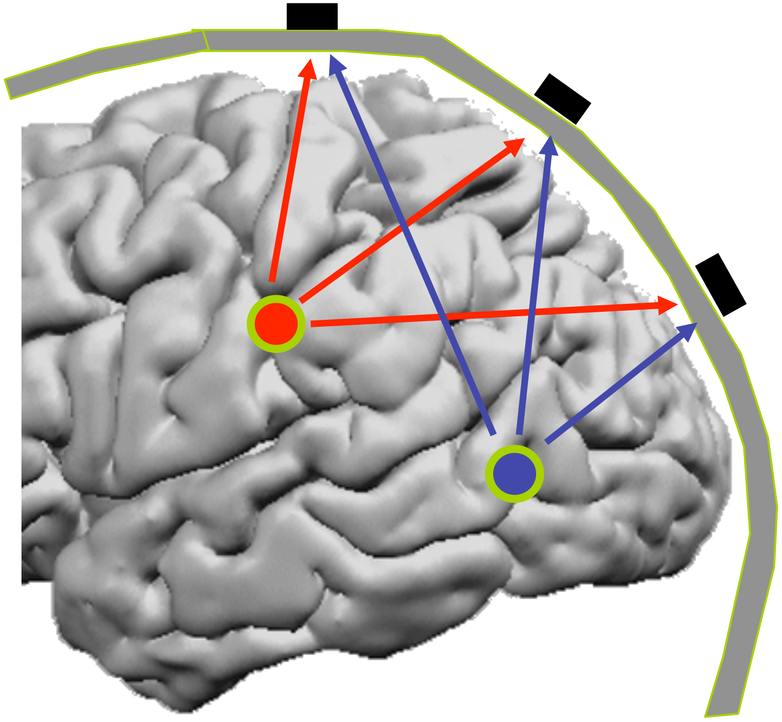

Electrodes
Cortex
local synchr.
Peau
Relative
Independence
local synchr.
Crane

Independent component analysis

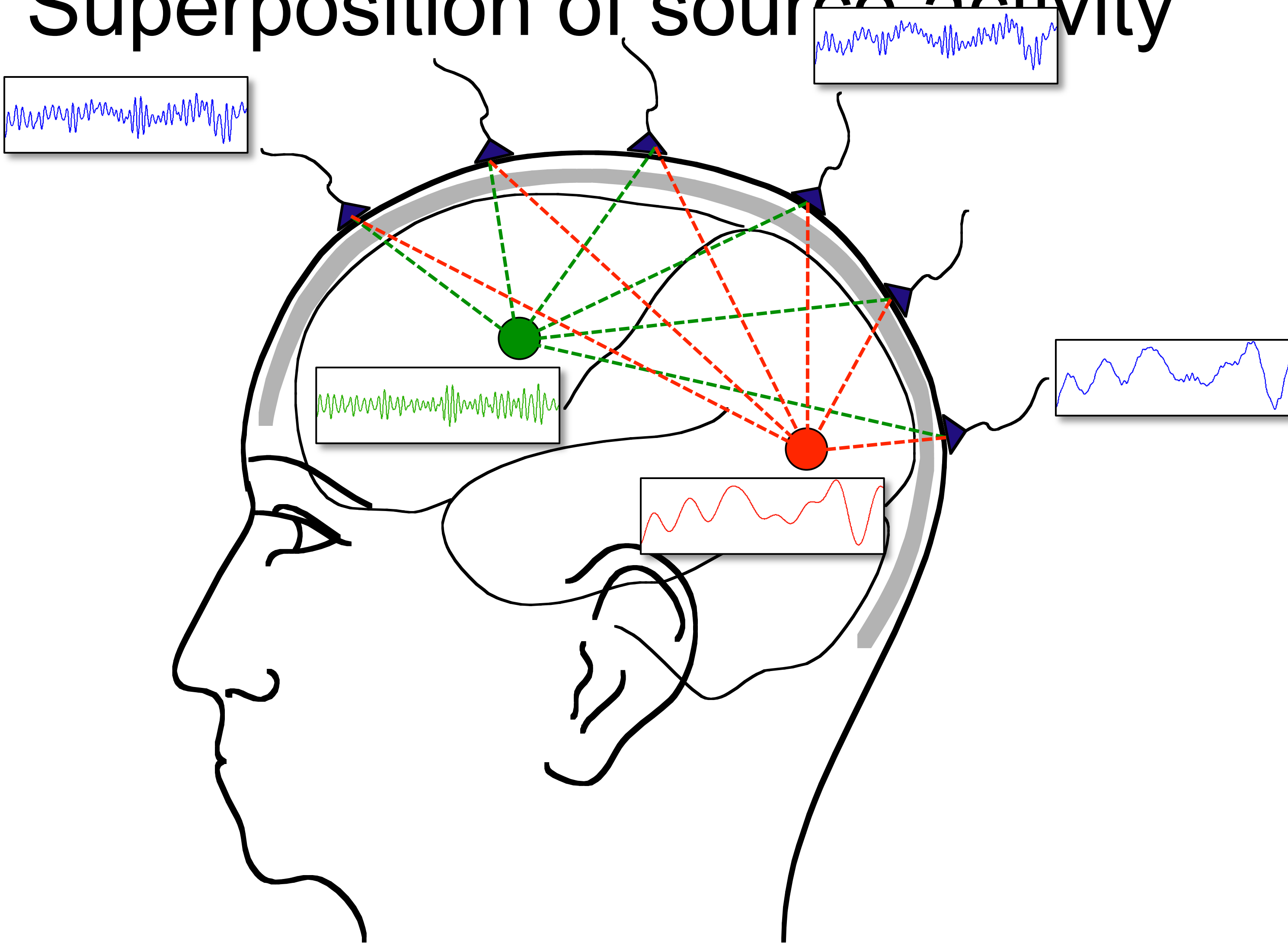
Cocktail Party

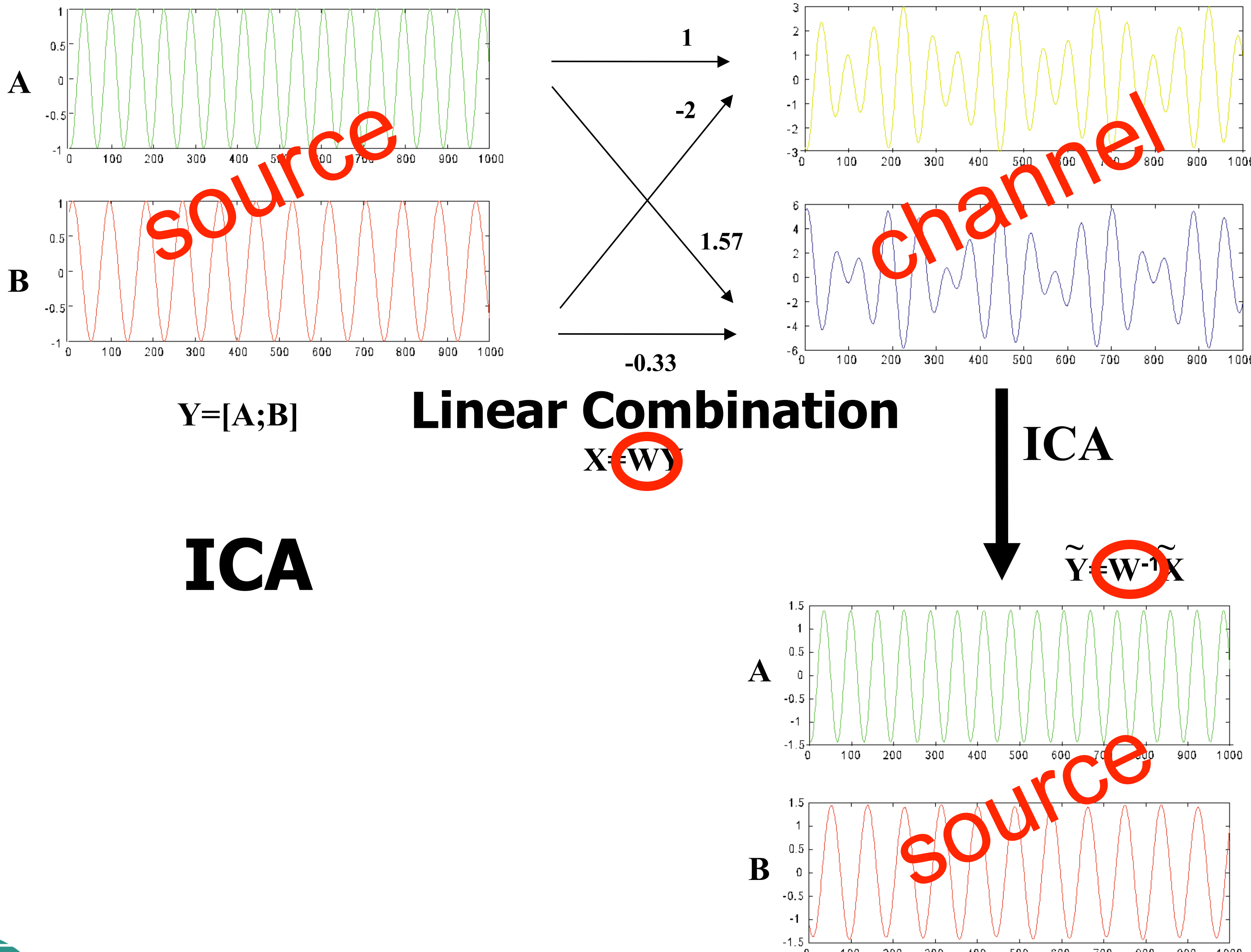


Mixture of Brain source activity

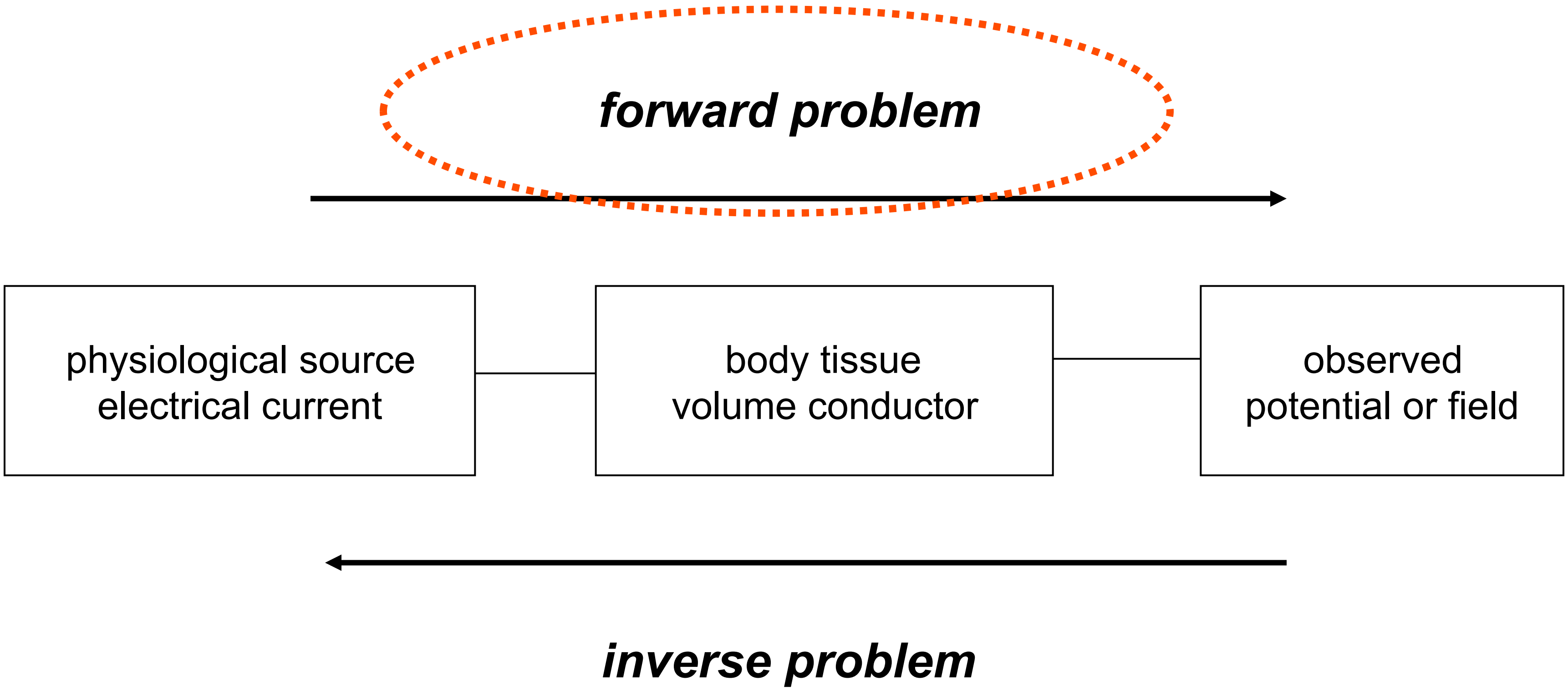


Superposition of source activity




A
B
source
Y=[A;B]
1
-2
1.57
-0.33
channel
Linear Combination
X=WY
ICA
ICA
$\tilde{Y}=W^{-1}\tilde{X}$
A
B
source

Source modelling



Overview

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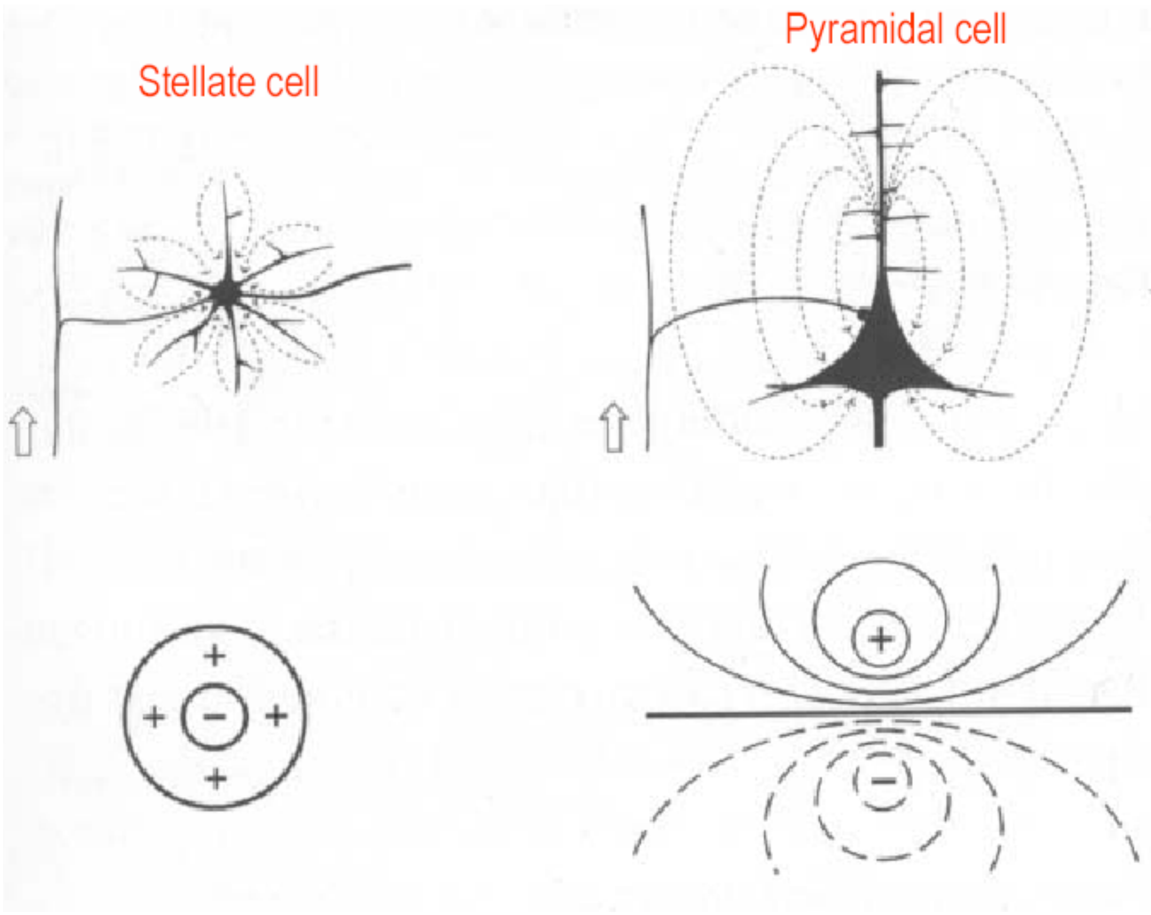
Neuronal currents



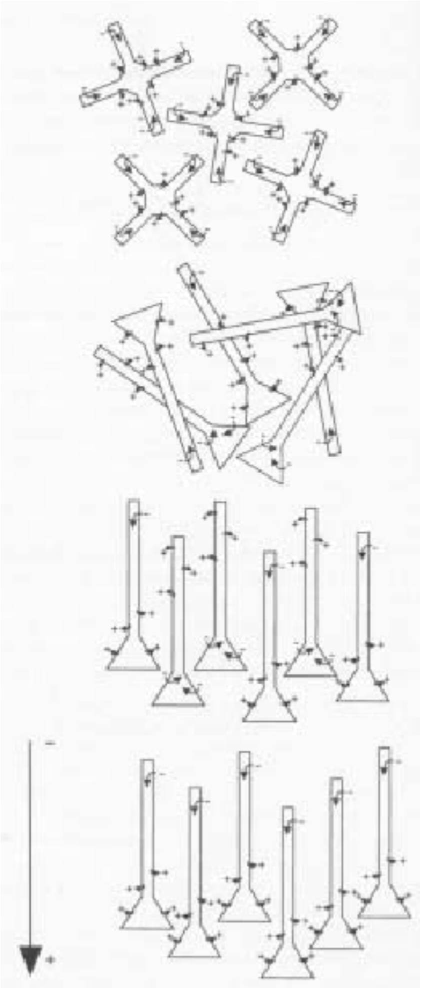
Symmetry, orientation and activation

radial symmetric

random oriented

asynchronously activated

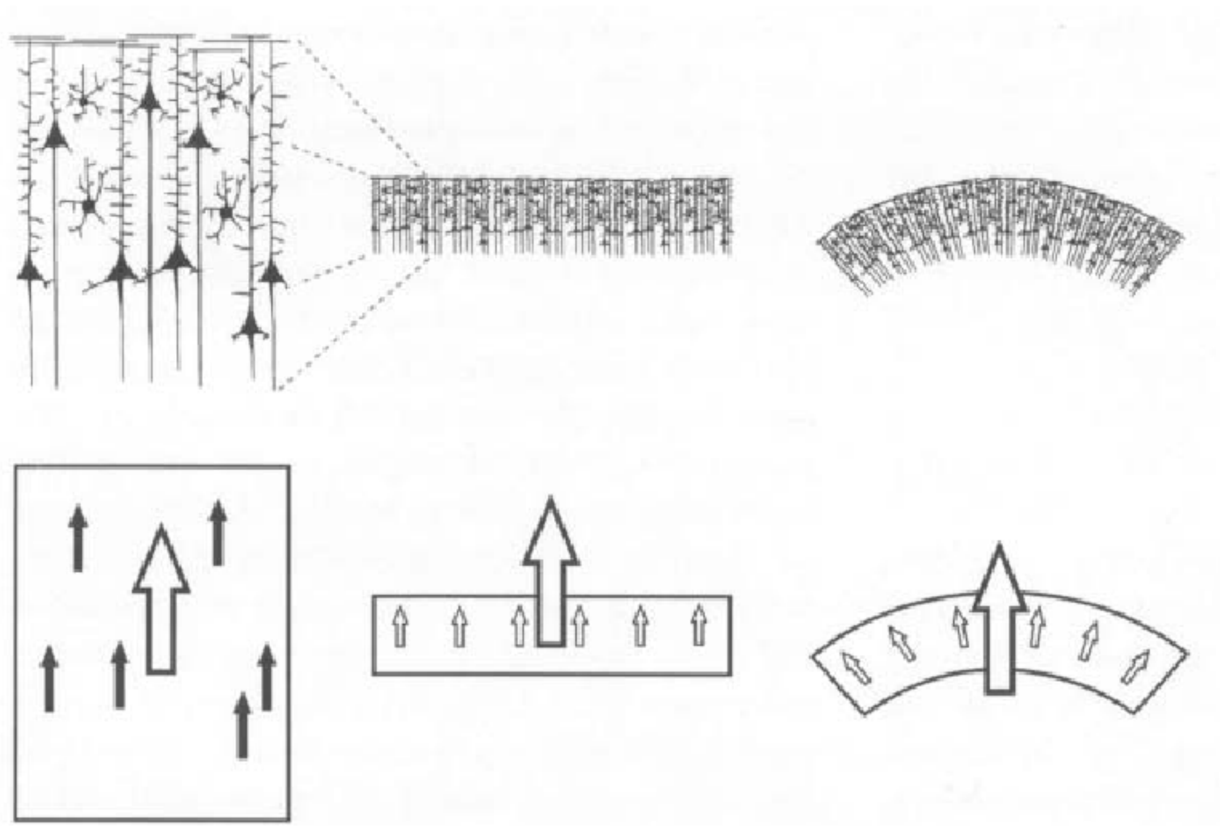
synchronously activated
parallel oriented



Motivation for current dipoles

- Neurophysiological motivation

Equivalent current dipoles



Motivation for current dipoles

- Neurophysiological motivation
- Physical/mathematical motivation
  - Any current distribution can be written as a multipole expansion
  - First term: monopole (must be zero)
  - Second term: dipole
  - Higher order terms: quadrupole, octupole

Motivation for current dipoles

- Neurophysiological motivation
- Physical/mathematical motivation
  - Any current distribution can be written as a multipole expansion
  - First term: monopole (must be zero)
  - Second term: dipole
  - Higher order terms: quadrupole, octupole
- Convenience
  - dipoles can be used as building block in distributed source models

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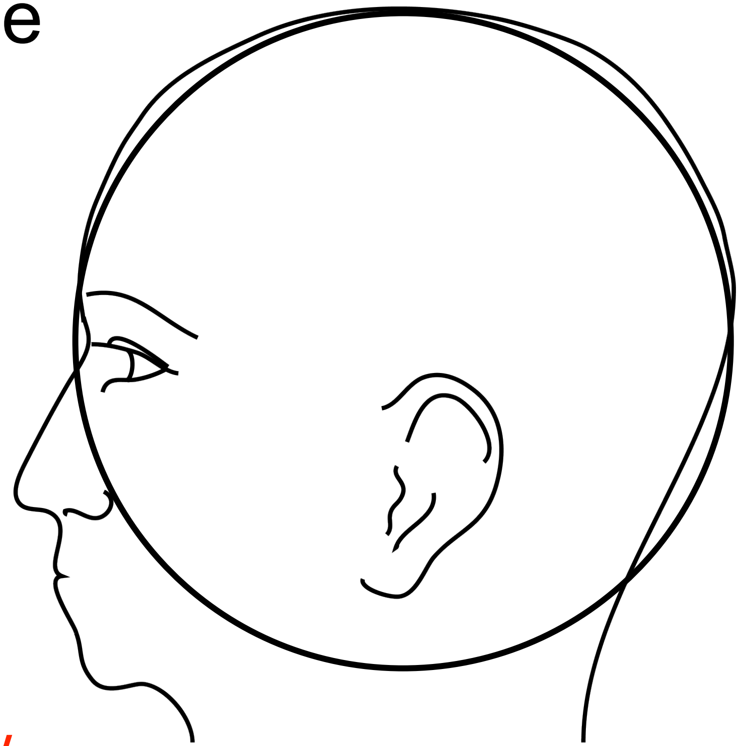
Volume conductor

- electrical properties of tissue
- geometrical description

- spherical model
- realistic shaped model



→ Describes how the currents flow,
not where they originate from

Volume conductor

- Advantages spherical model
  - mathematically accurate
  - reasonably accurate
  - computationally fast
  - easy to use

- Disadvantages spherical model
  - inacurate, esp. in some regions
  - difficult alignment with anatomy

Volume conductor

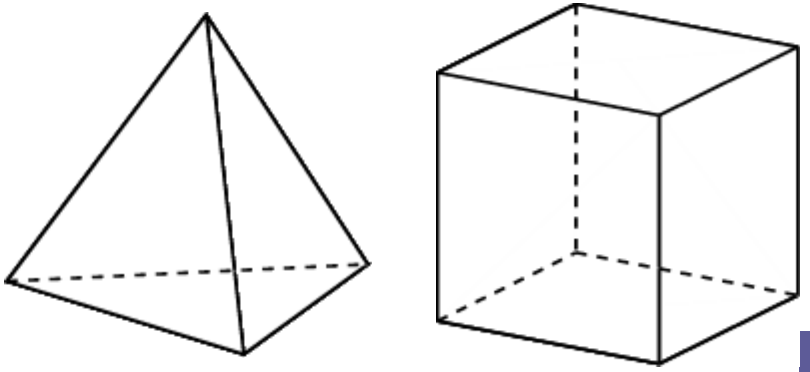
- Advantages realistic model
  - accurate solution for EEG
- Disadvantages realistic model
  - more work
  - individual anatomical MRI required
  - computationally slow(er)
  - numerically instable
  - difficult in interindividual comparison

→The pragmatic solution is to use a standard realistic headmodel for EEG

Realistic volume conductor

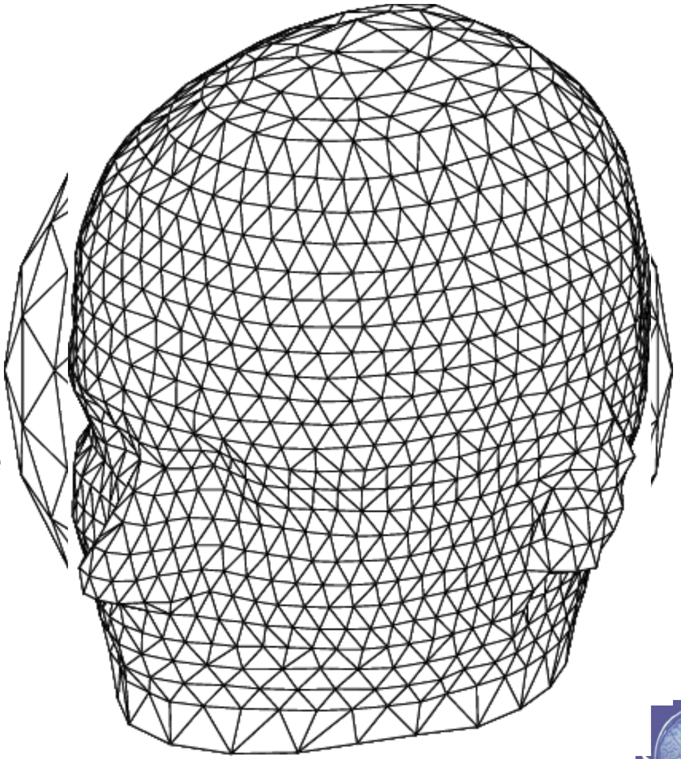
- Computational methods for volume conduction problem that allow realistic geometries
  - Boundary Element Method (BEM)
  - Finite Element Method (FEM)

- Geometrical description
  - triangles
  - tetraeders/voxels



Volume conductor: BEM

- Boundary Element Method
  - description of geometry by compartments
  - each compartment is
    - homogenous
    - isotropic
  - important tissues
    - skin
    - skull
    - brain
    - (CSF)
  - triangulated surfaces as boundarie
  - surfaces should be closed



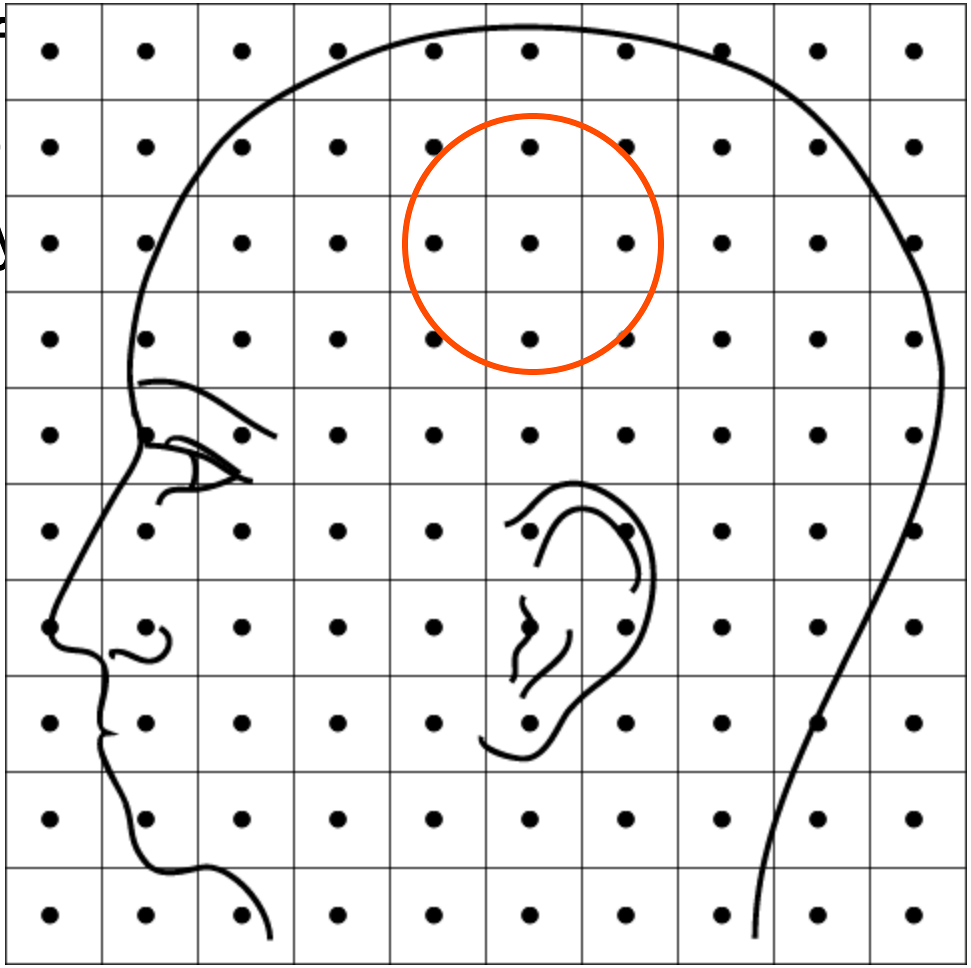
Volume conductor: FEM

- Tesselation of 3D volume in tetraeders
- Large number of elements
- Each tetraeder can have its own conductivity

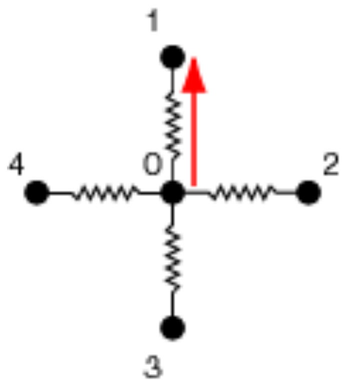
- FEM is most accurate numerical method
- Computationally expensive
- Accurate conductivities are not (well) known

Volume conductor: FDM

- Finite Dif
  - easy to
  - not very



Volume conductor: FDM



$$I_1 + I_2 + I_3 + I_4 = 0$$
$$V = I*R$$

$$\Delta V_1/R_1 + \Delta V_2/R_2 + \Delta V_3/R_3 + \Delta V_4/R_4 = 0$$

$$(V_1-V_0)/R_1 + (V_2-V_0)/R_2 + (V_3-V_0)/R_3 + (V_4-V_0)/R_4 = 0$$

Volume conductor: FDM

- unknown potential Vi at each node
- linear equation for each node
  - approx. 100x100x100 = 1.000.000 linear equations
  - just as many unknown potentials

- add a source/sink
  - sum of currents is zero for all nodes, except
  - sum of current is I+ for a certain node
  - sum of current is I- for another node

- solve system of linear equations for the potential

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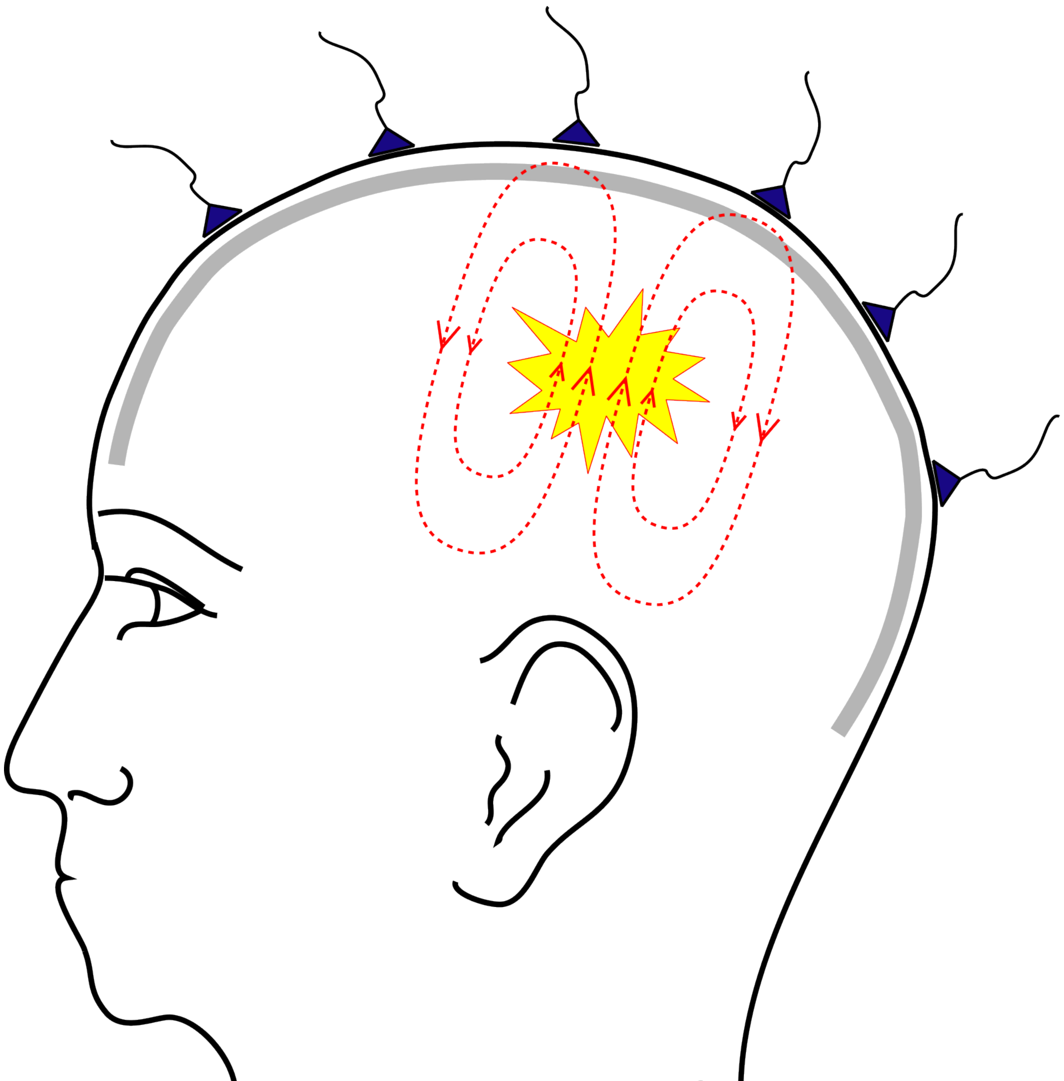
EEG volume conduction



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EEG volume conduction

- Potential difference between electrodes corresponds to current flowing through skin
- Only tiny fraction of current passes through skull
- Therefore the model should describe skull and skin as accurately as possible
- Challenges with skull
  - Not visible in anatomical MRI
  - Thickness varies
  - Conductivity is not homogeneous
  - Complex geometry at base of skull

MEG volume conduction

- Only tiny fraction of current passes through the poorly conductive skull
- Therefore skull and skin can be neglected in the MEG model

- However
  - difficulties with head movements
  - not all sources are equally visible
  - expensive and not widely available

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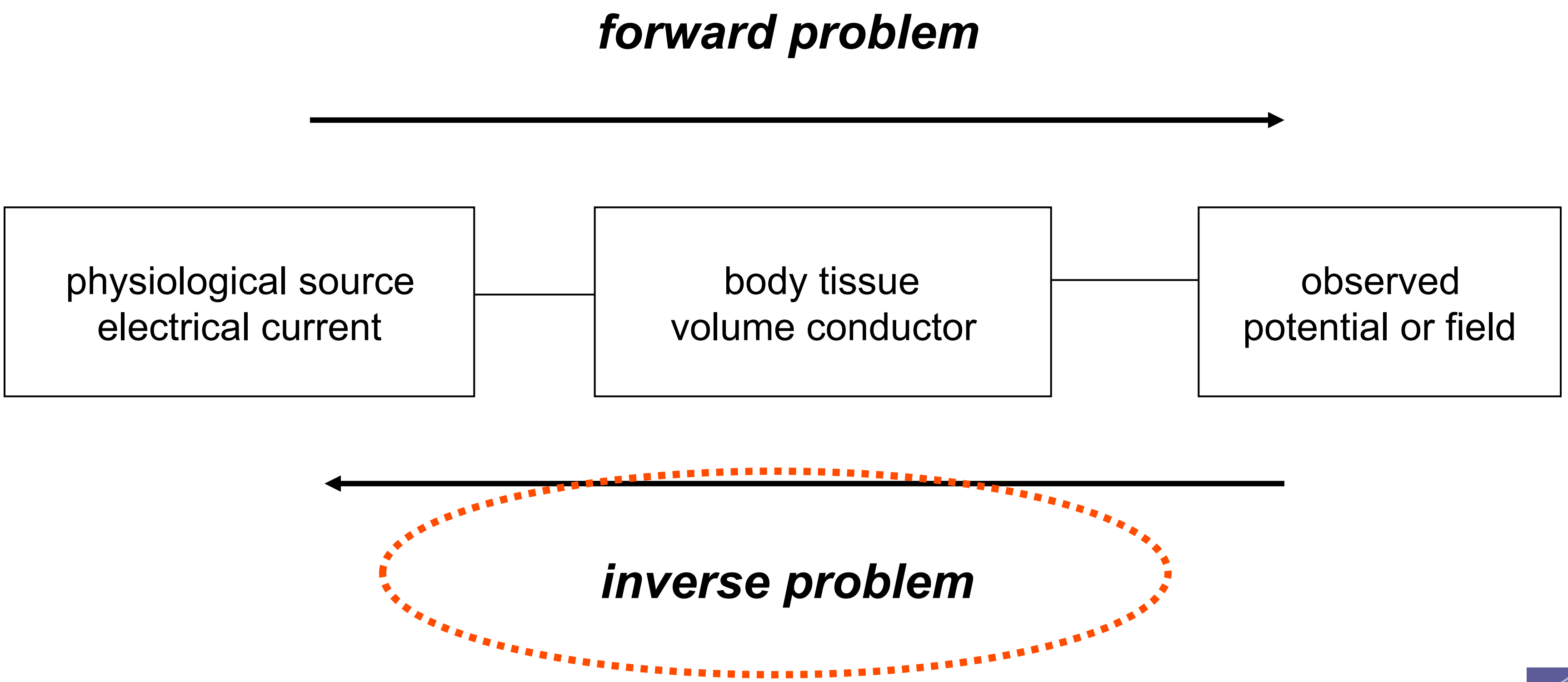
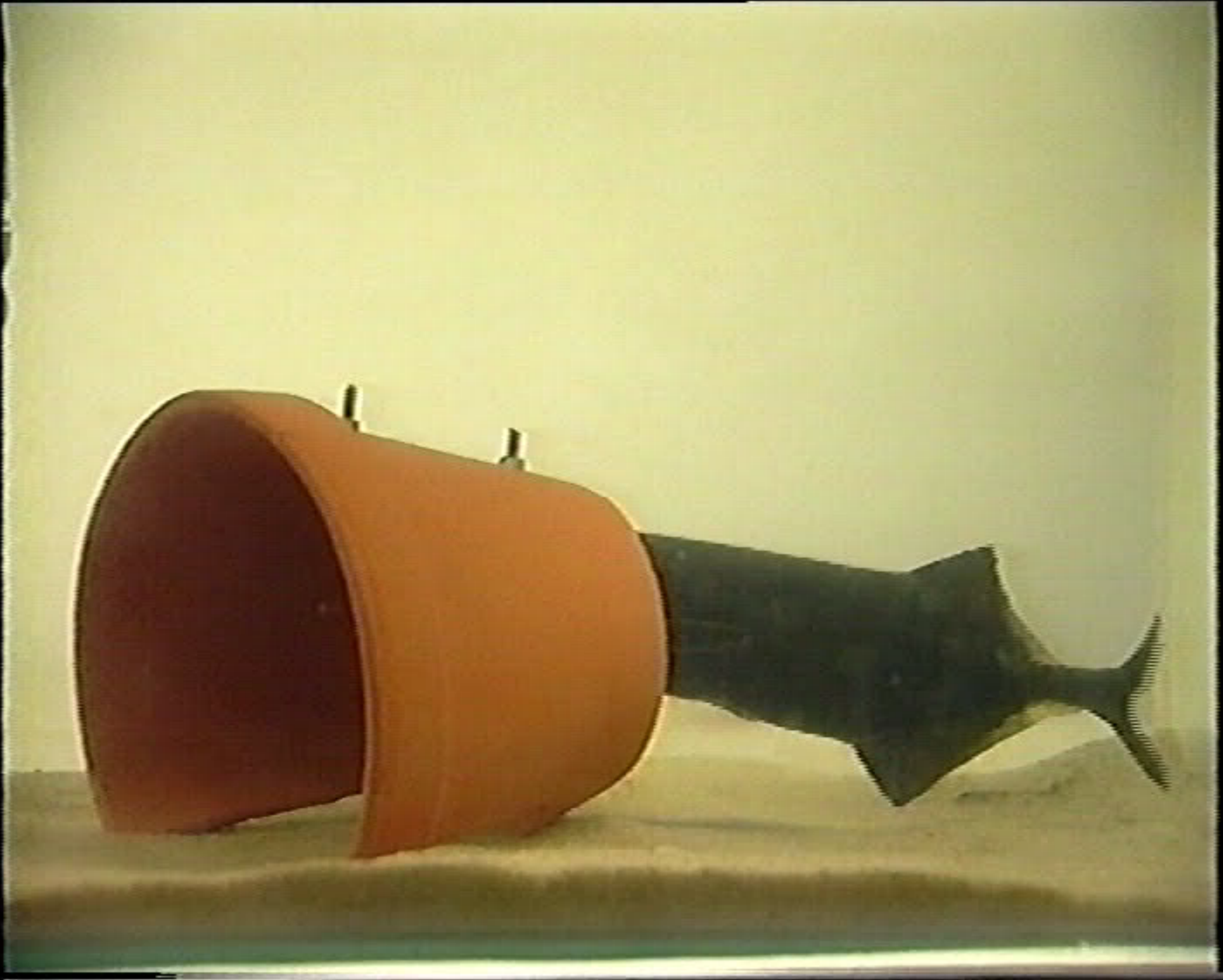
Source modelling





Inverse methods

- Single and multiple dipole models
  - Minimize error between model and measured potential/field
- Distributed dipole models
  - Perfect fit of model to the measured potential/field
  - Minimize additional constraint on sources
  - LORETA (smoothness)
  - Minimum Norm (L2)
  - Minimum Current (L1)
- Spatial filtering
  - Scan whole brain with single dipole and compute the filter output at every location
  - MUSIC
  - Beamforming (e.g. LCMV, SAM, DICS)

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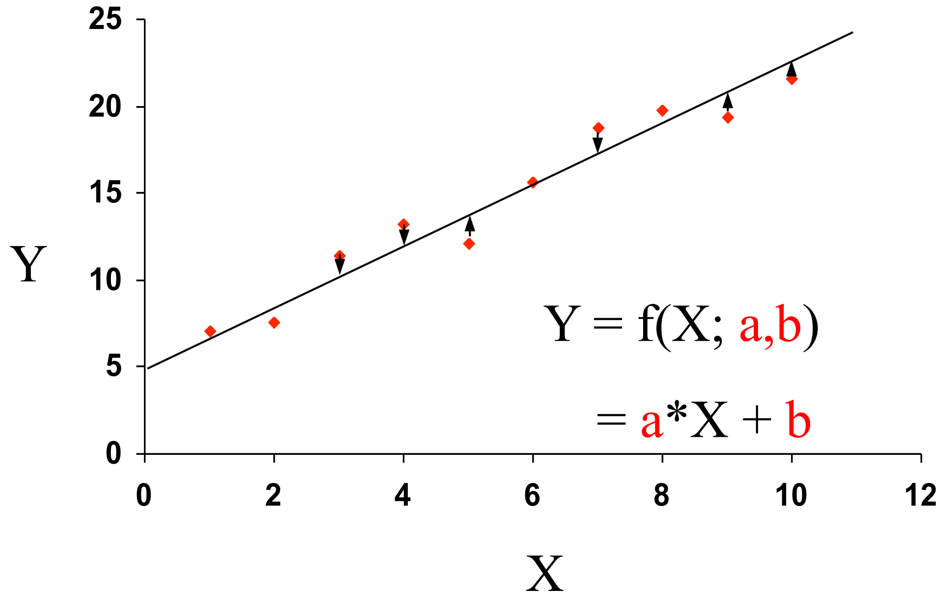
Single or multiple dipole models

- Manipulate source parameters to minimize error between measured and model data
  - Location of each source
  - Orientation of each source
  - Strength of each source
- Orientation and strength together correspond to the "dipole moment" and can be estimated linearly
- Position is estimated non-linearly
- Source parameter estimation

Parameter estimation



$\zeta = a,b,c,\ldots$

Parameter estimation: model

measured potential

$$V_i = V(\vec{r}_i) + \text{noise}$$

forward model for the data

$$Y_i = Y(r_i;\zeta) + \text{noise}$$

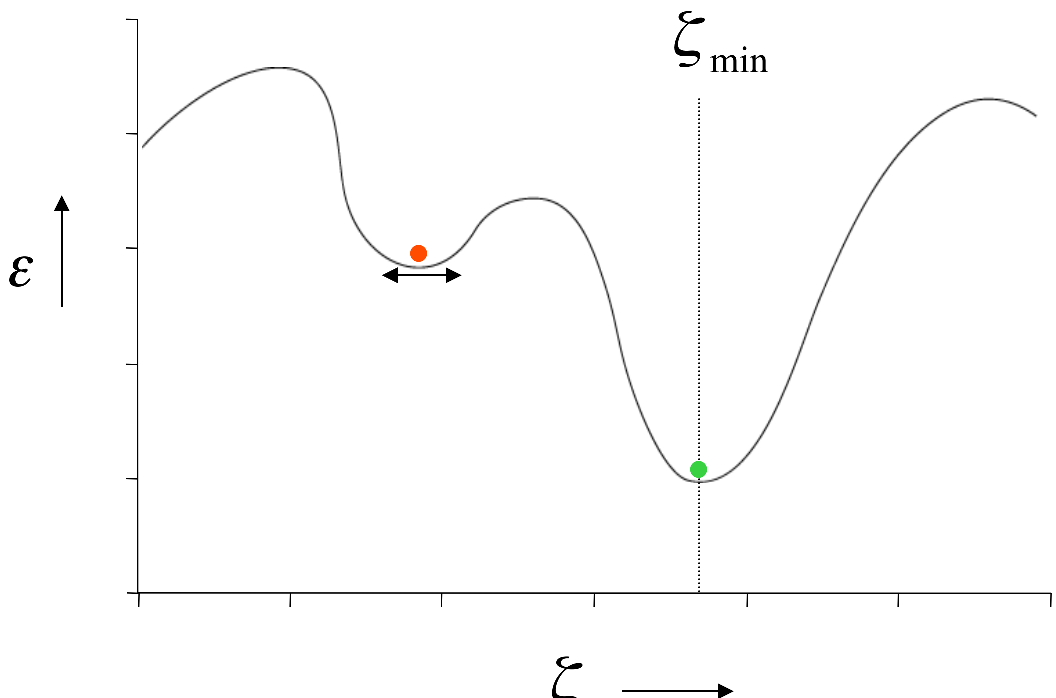
select "optimal" model

$$\min_{\zeta}\left\{ \sum_{i=1}^{N} \left(Y(r_i;\zeta) - V(r_i)\right)^2 \right\}$$

Select optimal model

$$error(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(error(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



Dipole *scanning*: grid search

- define grid with allowed dipole locations
- compute optimal dipole moment for each location
- compute value of goal-function
- plot value of goal-function on grid

- number of evaluations:
  - single dipole, 1 cm grid: ~4 000
  - single dipole, ½ cm grid: ~32 000
  - two dipoles, 1cm grid: ~16 000 000

Dipole *fitting*: nonlinear search

- start with an initial guess
- evaluate the local derivative of goal-function
- “walk down hill” to the most optimal solution

- number of evaluations: ~100

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Distributed source model

- Position of the source is not estimated as such
  - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
  - In principle easy to solve, however…
  - More “unknowns” (parameters) than “knowns” (measurements)
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem

Distributed source model

- Linear estimation

$$\vec{\Psi} = q_1\vec{\Psi}_1 + q_2\vec{\Psi}_2 + \ldots \quad = \begin{bmatrix} \Psi_{1,1} & \Psi_{2,1} & \cdots \\ \Psi_{1,2} & \Psi_{2,2} & \cdots \\ \vdots & \vdots & \ddots \\ \Psi_{1,N} & \Psi_{2,N} & \cdots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} \quad = \mathrm{L} \cdot \vec{q}$$

$$\vec{q} = \mathrm{L}^{-1} \cdot \vec{\Psi}$$

Distributed source model

$$V = L \cdot q + Noise$$

$$\min_q \{\| V - L \cdot q \|^2\} = 0 \quad !!$$

- Regularized linear estimation:

$$\min_q \{\| V - L \cdot q \|^2 + \lambda^2 \cdot \| D \cdot q \|^2\}$$

- Constrained linear estimation:

$$\min_q \{q^T \cdot W \cdot q\} \quad \text{while} \quad \| V - L \cdot q \|^2 = 0$$

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Spatial filtering

- position of the source as such is not estimated
- scanning the whole brain
  - single dipole as source
  - estimate activity at each grid location
    - that explains a part of the data
    - that supresses other activity
- various methods
  - multiple signal classification (MUSIC)
  - beamforming
  - LCMV, SAM, DICS, …
- not a distributed source model, but a distributed representation of the single dipole estimate

Spatial filtering

- unmixing of data into “signal” and “noise” sources
- requires assumptions on
  - temporal relation between sources and
  - biophysical model

- with ICA we have already separated the timeseries and we only have spatial topographies that need to be explained

- hence spatial filtering (beamforming etc.) are incompatible with ICA

Summary 1

- Forward modelling
  - Required for the interpretation of scalp topographies
  - Interpretation of scalp topography *is* "source estimation"
  - Mathematical techniques are available that aid in interpreting scalp topographies -> inverse modeling

Summary 2

- Inverse modeling
  - Model assumption for volume conductor
  - Model assumption for source (I.e. dipole)
  - Additional assumptions on source
    - Single point-like source
    - Multiple point-like sources
    - Distributed source
  - Different mathematical solutions
    - Dipole fitting (linear and nonlinear)
    - Linear estimation (regularized)
    - Spatial filtering (e.g. beamforming)