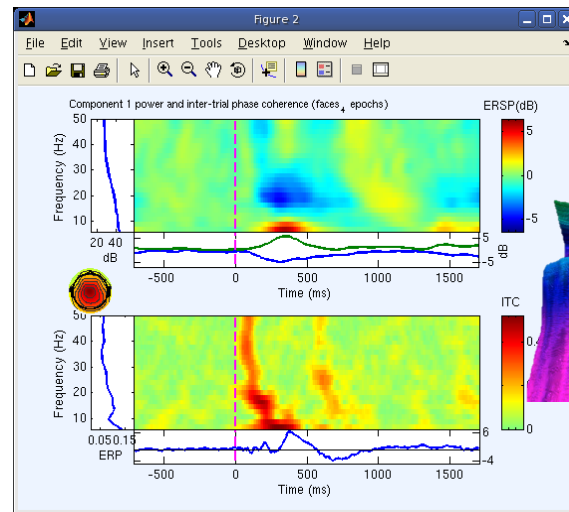


Time-Frequency Analysis of Biophysical Time series

Tim Mullen

(with many contributions from Arnaud Delorme)



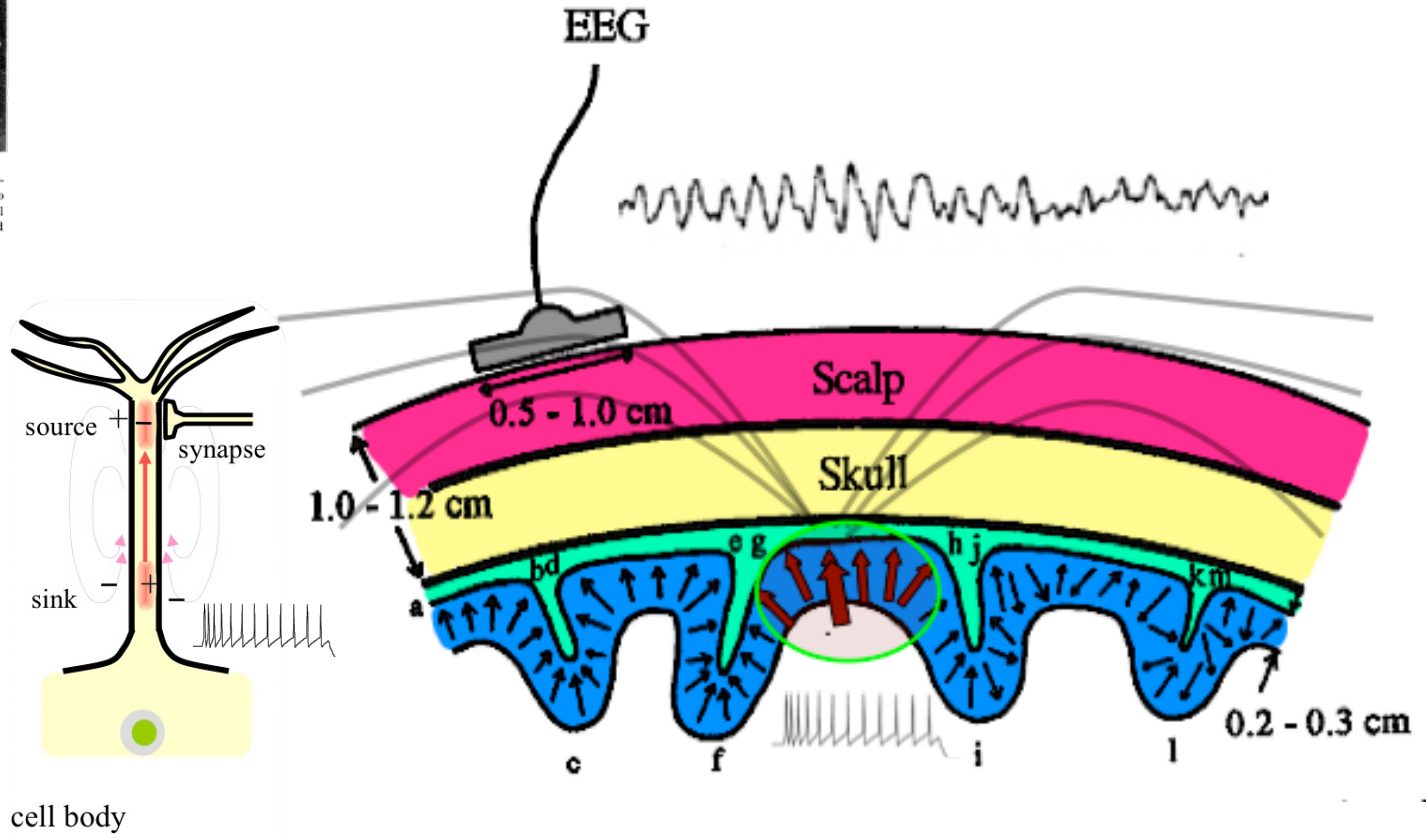
$$S(f) = \frac{1}{N} \sum_{t=0}^{N-1} x(t) e^{-2\pi i f t / N}$$



Biophysics of EEG



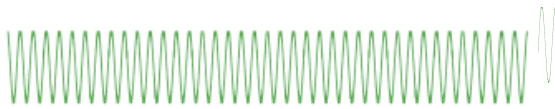
FIGURE 1-1.—Professor Hans Berger (1873-1941), neuro-psychiatrist, University of Jena, Jena, Germany, first to discover and describe in 1929 a unique kind of electrical activity recorded from the brain of man, which he named the electroencephalogram (Elektrenkephalogramm).



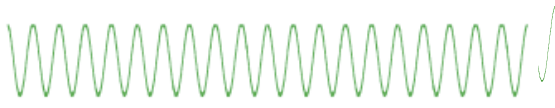
Synchronicity of cell excitation (due to recurrent cortico-cortical and cortico-thalamo-cortical projections) determines amplitude and rhythm of the EEG signal

Common Oscillatory Modes in EEG

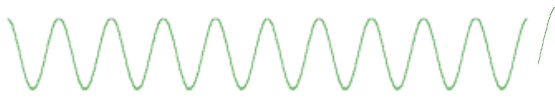
Simulated



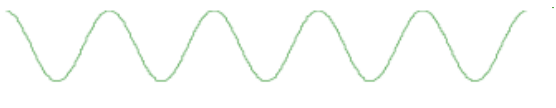
30-60 Hz Gamma



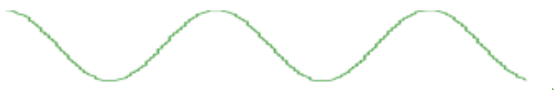
18-21 Hz Beta



9-11 Hz Alpha



4-7 Hz Theta

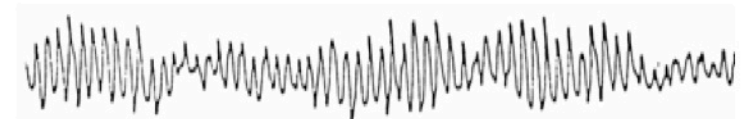


0.5-2 Hz Delta



1 second

Real



Sinusoids

phase shift

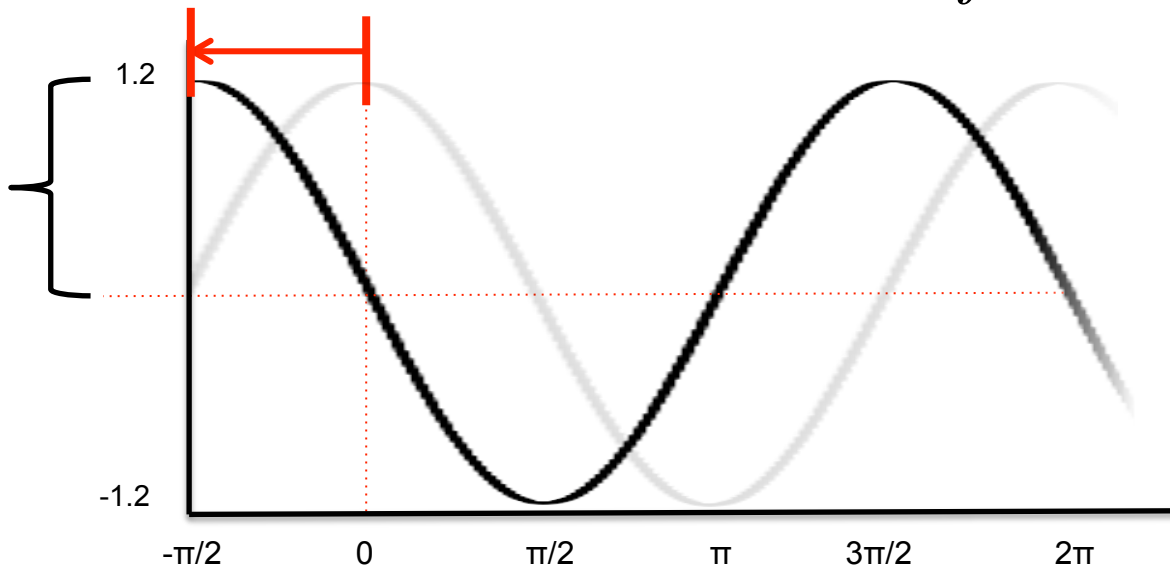
angular frequency

$$\theta = \pi / 2$$

$$\omega = 2\pi f = 2\pi \text{ rad/sec}$$

Amplitude

$$A=1.2$$



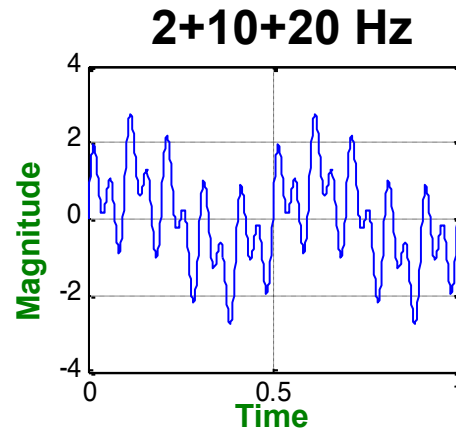
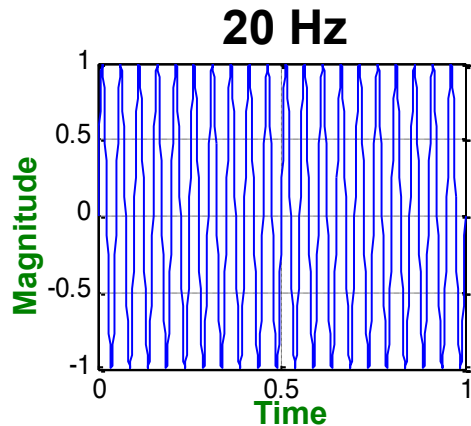
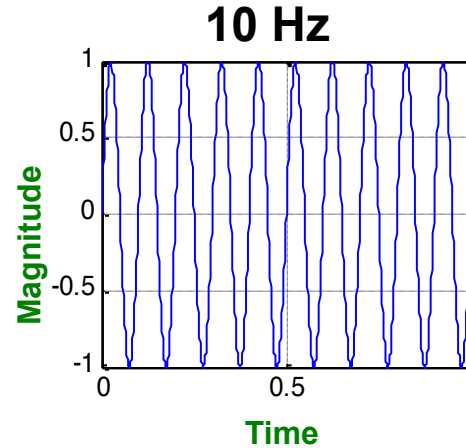
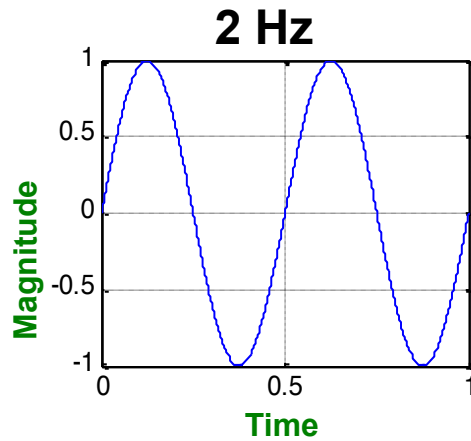
t

Wide-sense stationary signals

The first and second moments (mean and variance) of the data distribution do not depend on time.

Wide-sense stationary signals

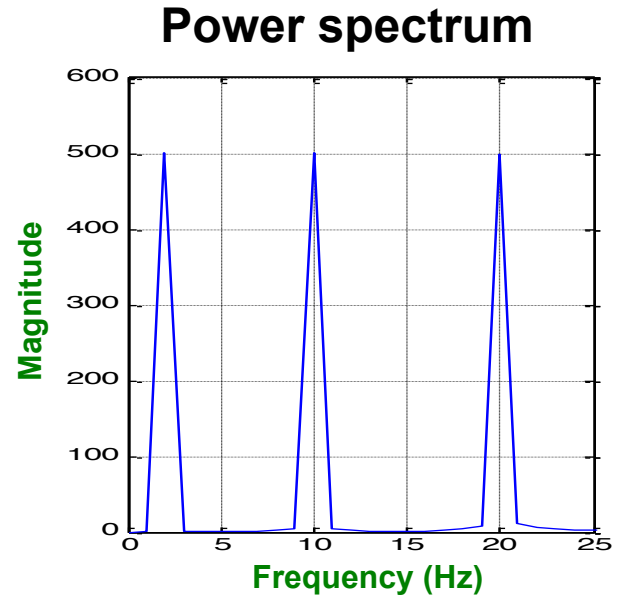
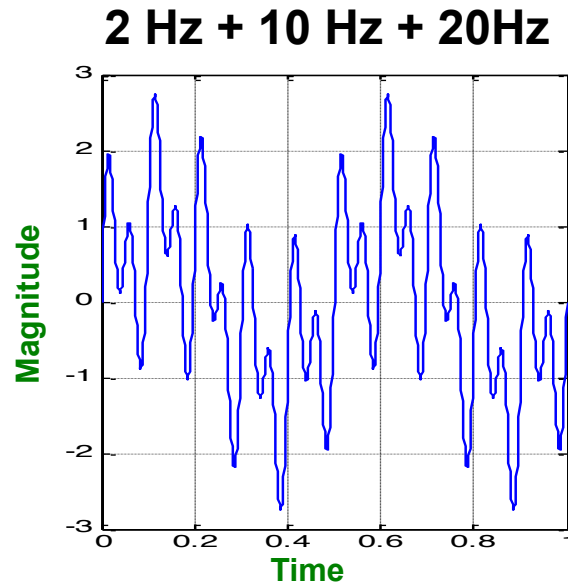
Cyclostationary signals



**Wide-sense
Stationary**

Wide-sense stationary signals

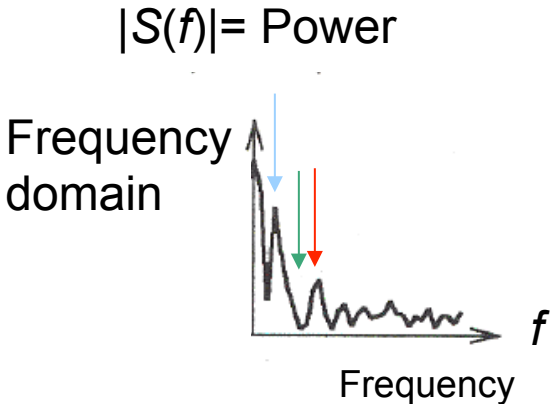
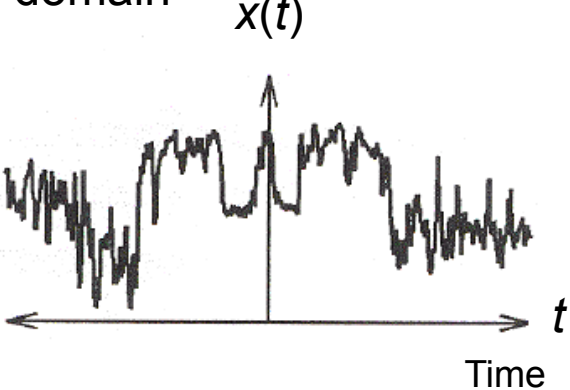
**Wide-sense
Stationary**



By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2, 10, 20Hz respectively).

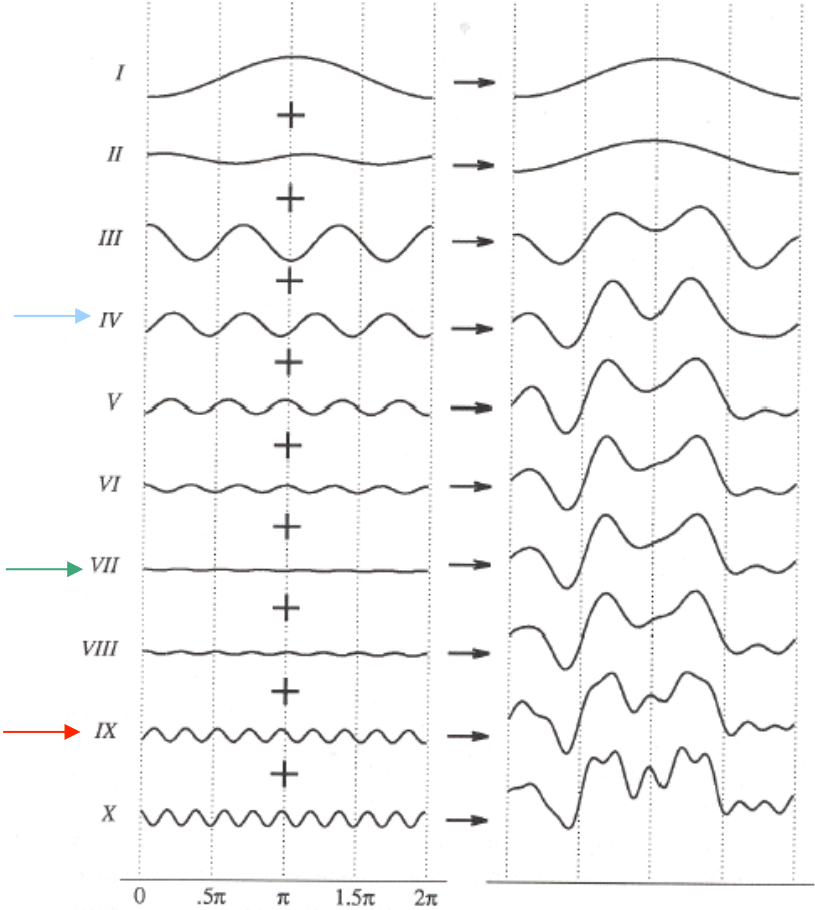
Fourier's Analysis

Time domain



Freq. decomp.

Sum of freq.



Figure, courtesy of Ravi Ramamoorthi & Wolberg

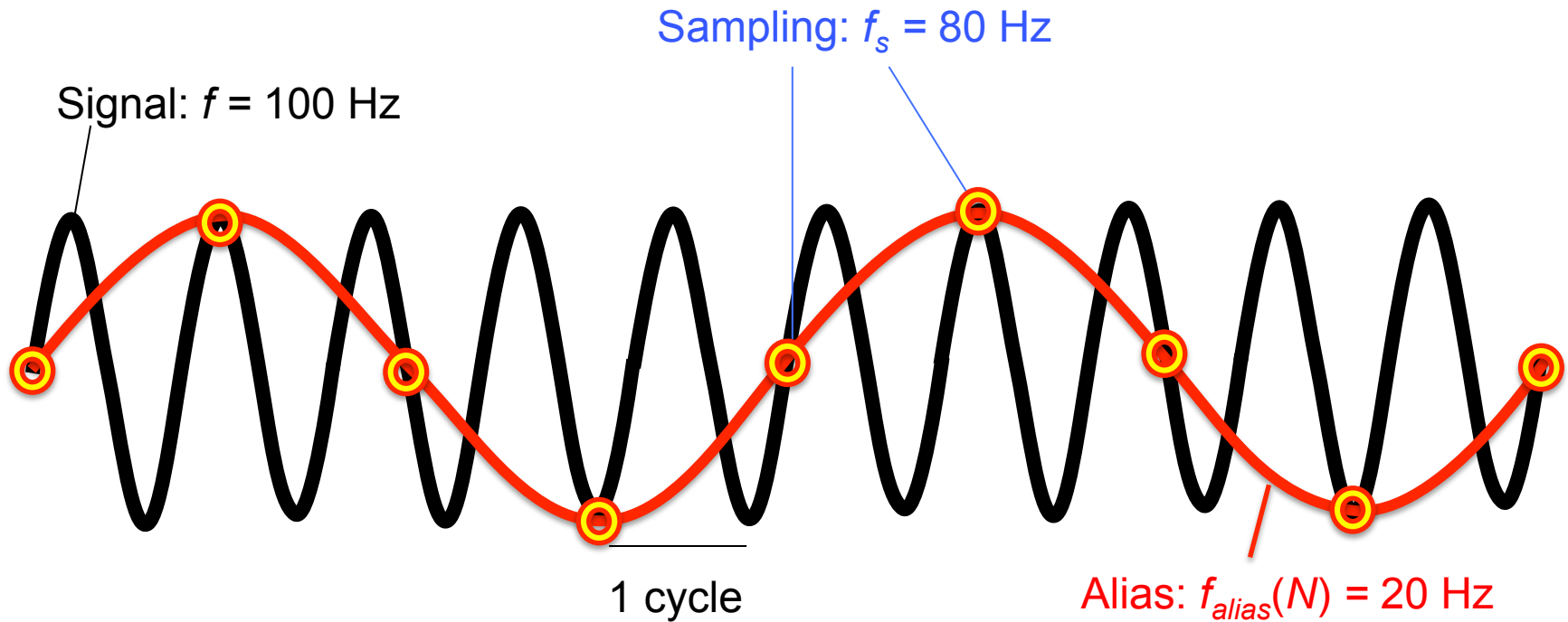
Fourier's Theorem

Any stationary, continuous process can be exactly described by an infinite sum of sinusoids of different amplitudes and phases.



Jean Baptiste Joseph Fourier
(1768 –1830)

Aliasing and the Nyquist Frequency



Nyquist Frequency:

The maximum frequency that can be uniquely recovered at a sampling rate of f_s

$$f_N = f_s / 2$$

$$f_{alias}(N) = |f - Nf_s|$$

$f_s =$ sampling rate

Quiz: What is N in the example above?

Euler's Formula

phase shift

$$\theta = \pi / 2$$

angular frequency

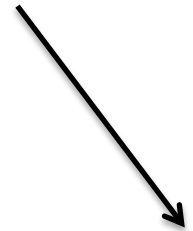
$$\omega = 2\pi f = 2\pi \text{ rad/sec}$$

any sinusoid can be expressed as the sum of two complex numbers...

$$A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t + \theta)} + \frac{A}{2} e^{-i(\omega t + \theta)}$$

$$= \text{Re}\{Ae^{i(\omega t + \theta)}\} = \text{Re}\{S(\omega, t)\}$$

instantaneous complex amplitude and phase

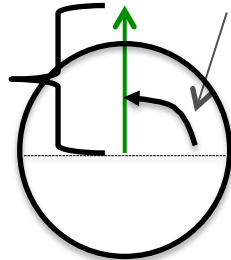


Phase

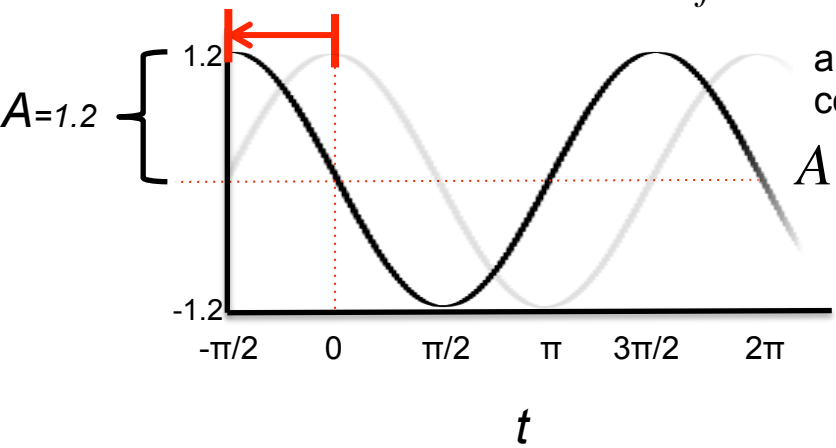
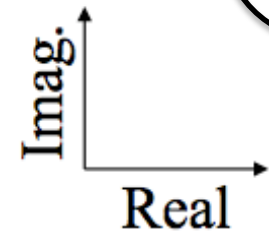
$$\theta = \angle S(\omega, t) = \pi / 2$$

Amplitude

$$|S(\omega, t)| = A$$



Phasor



Another version:

$$e^{i(\omega t + \theta)} = \cos(\omega t + \theta) + i \sin(\omega t + \theta)$$

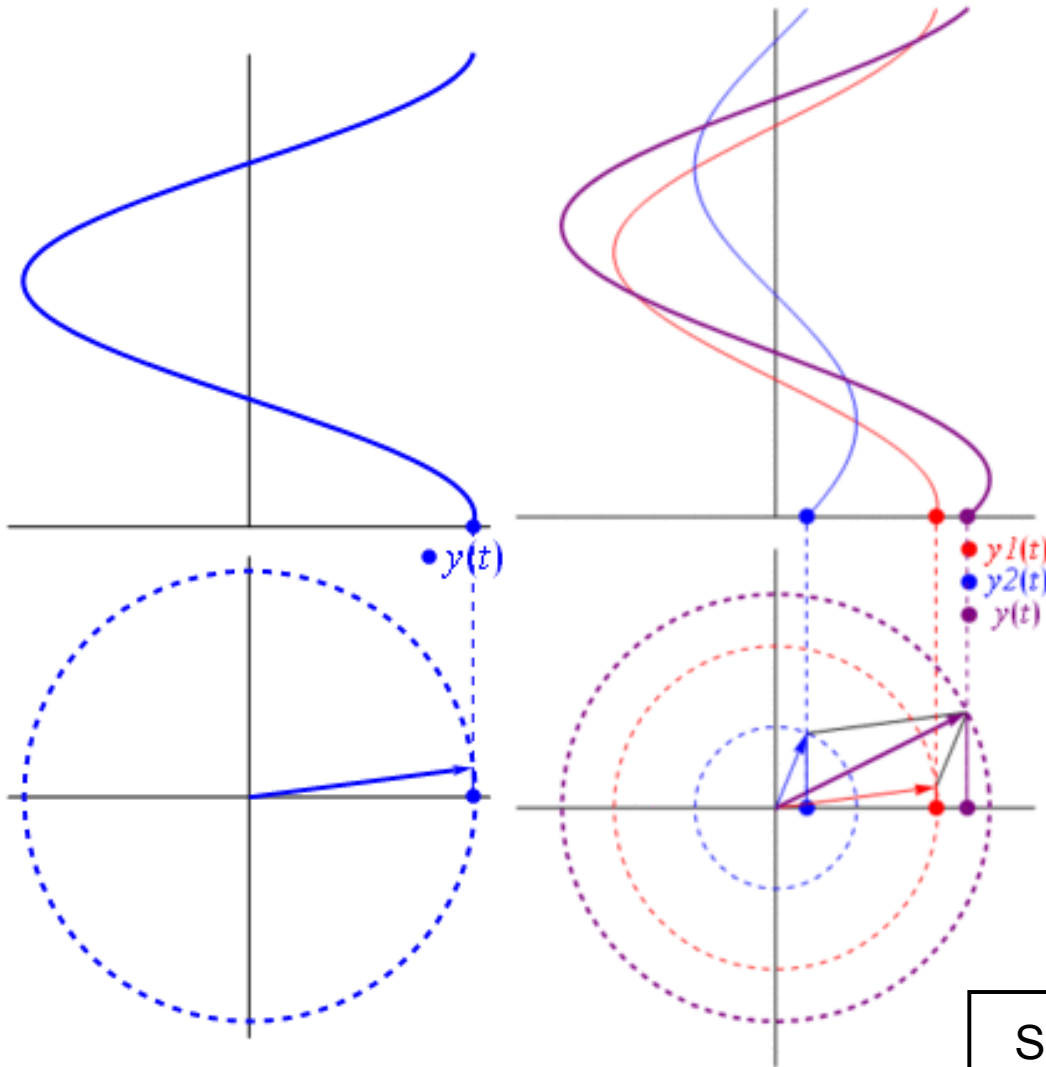
Real part
Cosine component

Imaginary part
Sine component



Phasors

Rotation velocity (Rad/S; Hz)
= (angular) frequency (ω ; f)

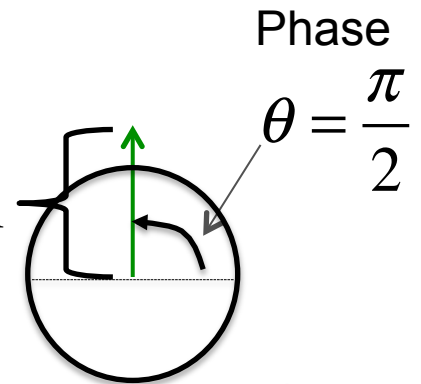


$$A \cdot \cos(\omega t + \theta) = \text{Re}\{Ae^{i(\omega t + \theta)}\}$$

$$= \text{Re}\{S(\omega, t)\}$$

Amplitude

$$|S(\omega, t)| = A$$

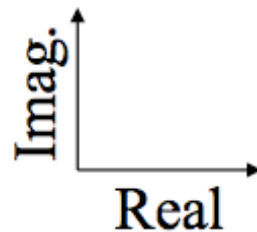


Phase

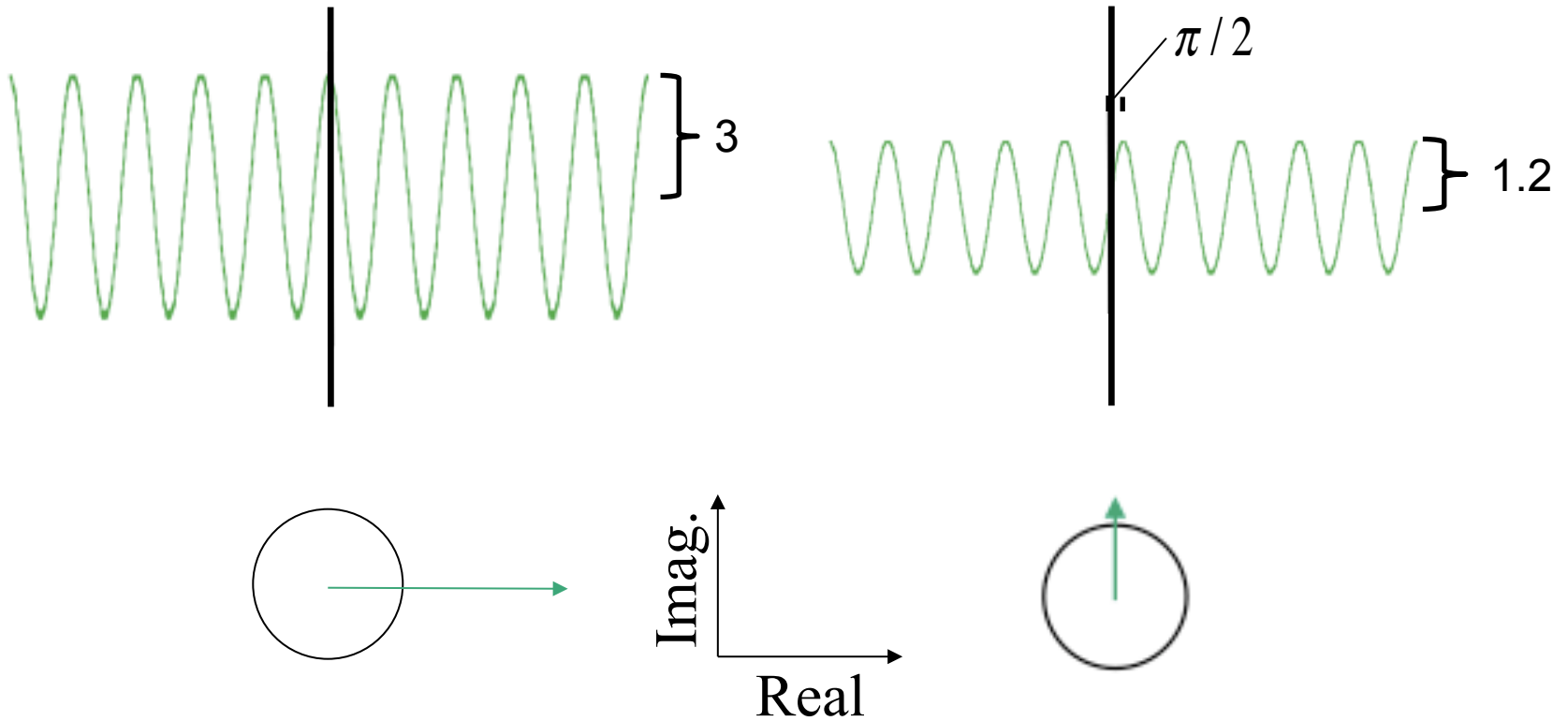
$$\theta = \frac{\pi}{2}$$

Shorthand
phasor notation:

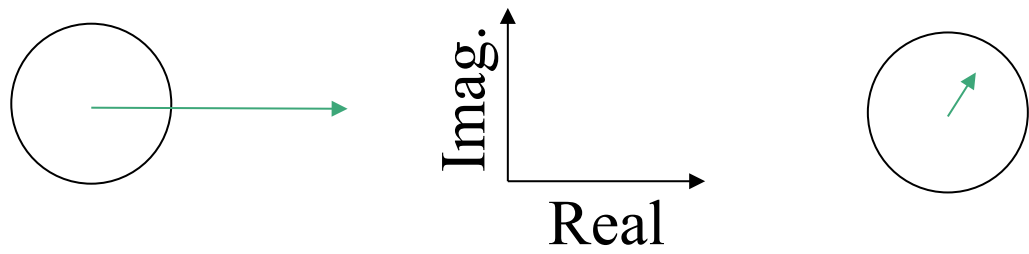
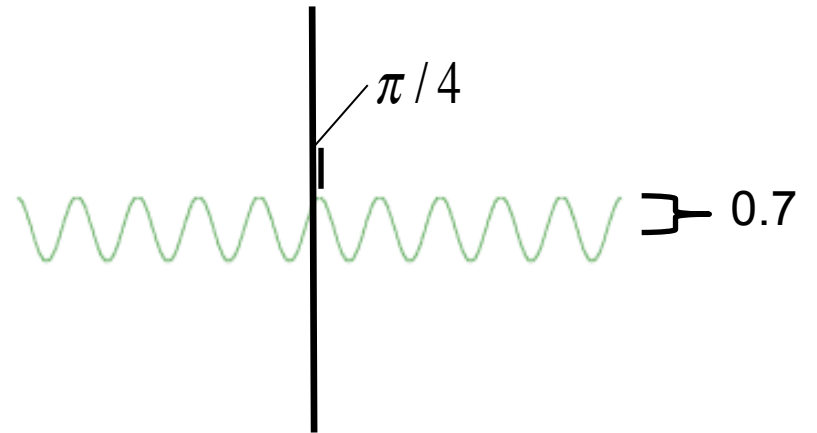
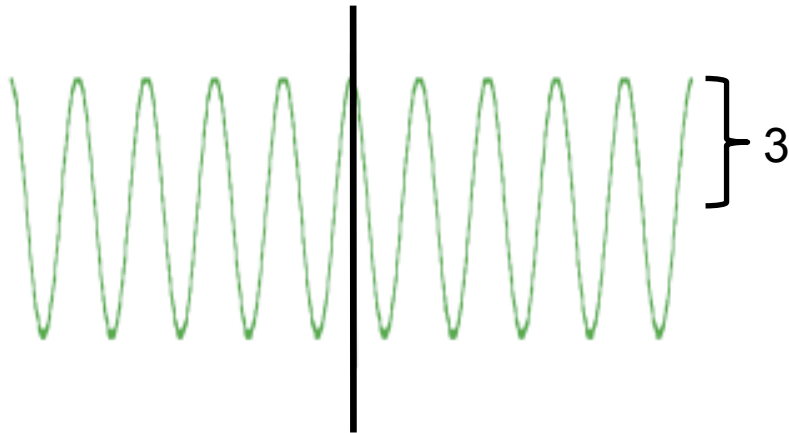
$$Ae^{i\phi}$$



Phasors: Example



Phasors: Example



Discrete Fourier Transform

Time \rightarrow Frequency

Frequency \rightarrow Time

Forward transform

$$S(f) = \frac{1}{N} \sum_{t=0}^{N-1} x(t) e^{-2\pi i f t / N}$$

Inverse transform

$$x(t) = \frac{1}{N} \sum_{f=0}^{N-1} S(f) e^{2\pi i f t / N}$$

N = number of samples

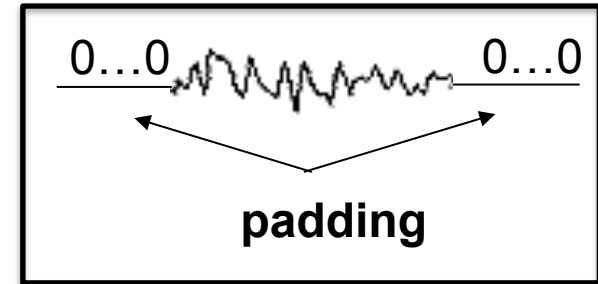
Fast Fourier Transform (FFT)

$$e^{\pm i(2\pi f t + \theta)} = \cos(2\pi f t + \theta) \pm i \sin(2\pi f t + \theta)$$

Power reflects the **covariance** between the original signal and a complex sinusoid at frequency f . Or you can think of it as the proportion of the signal variance explained by a sinusoid at frequency f

Zero-padding

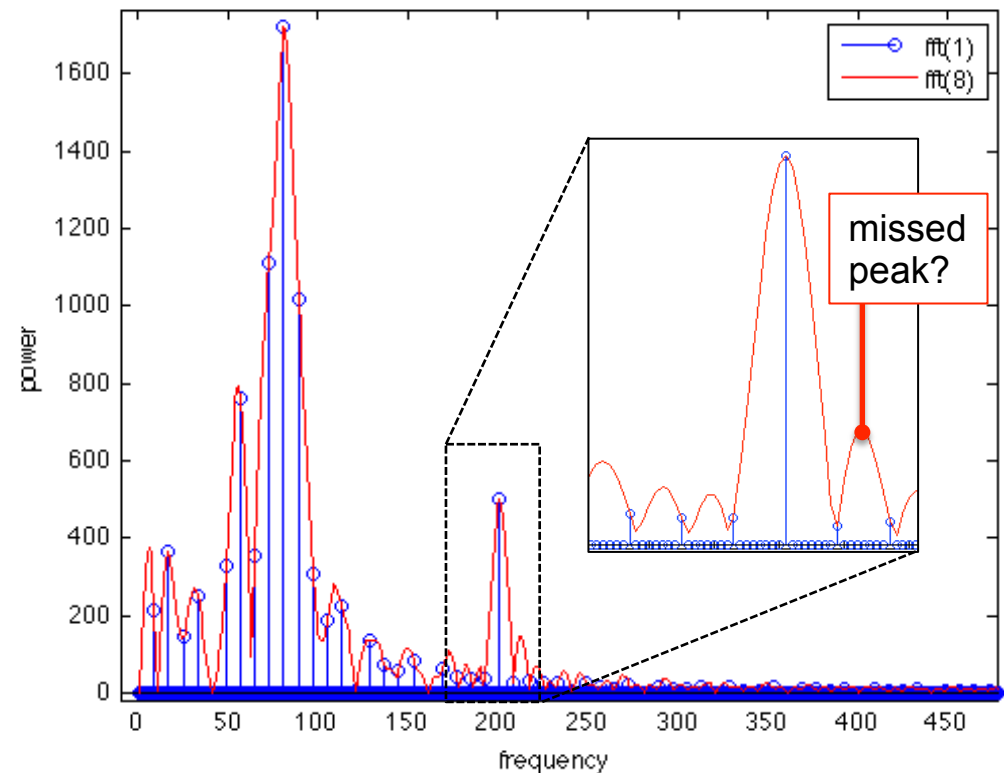
The DFT/FFT of a sequence of length N produces power estimates at N frequencies evenly distributed between 0 and the sampling rate (F_s) (or $\text{floor}(N/2+1)$ frequencies between 0 and the Nyquist rate, $F_n = F_s/2$).



Padding the signal with Q zeros achieves the following:

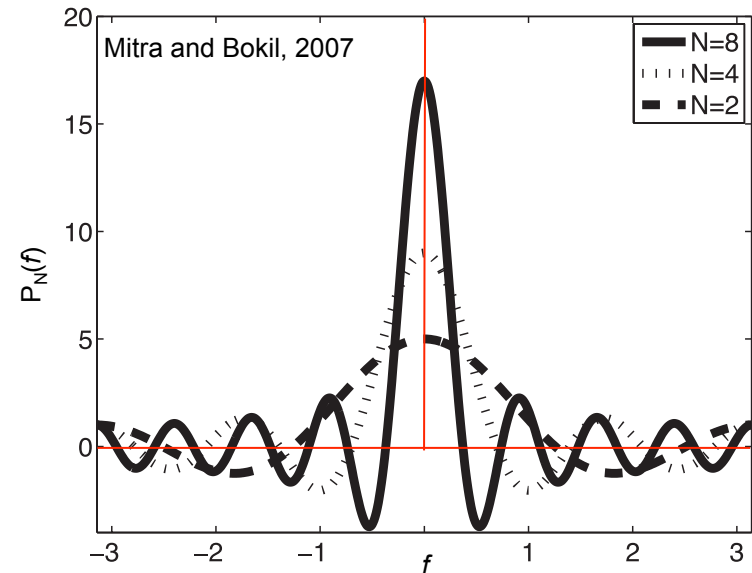
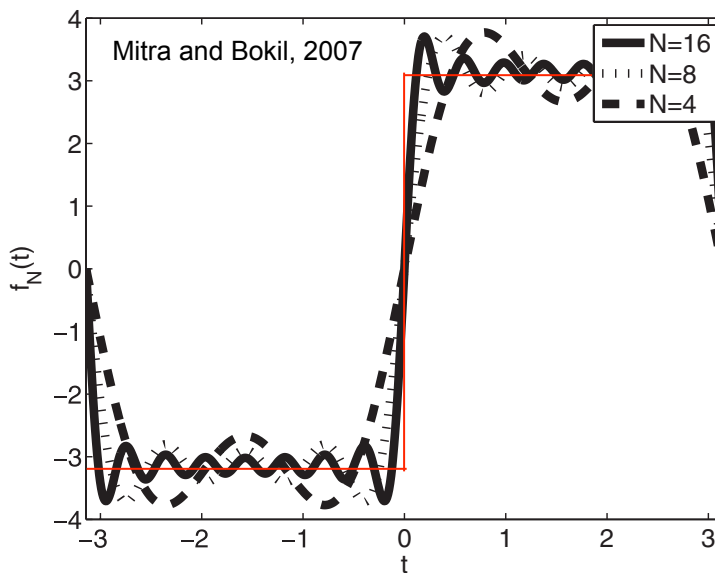
- 1) Allows enforcement of signal length as a power of two enabling FFT
- 2) Increases the number of frequency bins between 0 and F_s from N to $N+Q$ (intermediate points are sinc interpolates)

Zero-padding does not increase frequency resolution (number of independent degrees of freedom)



Tapering

Fourier's Theorem lets us exactly describe any continuous, stationary signal using an infinite weighted sum of sinusoids. Discontinuous functions can be infinitely *approximated*, but the approximation may be poor for finite sampling.



Gibbs Phenomenon

“Rippling” effect due to discontinuities in signal (e.g. edges of the truncated signal)

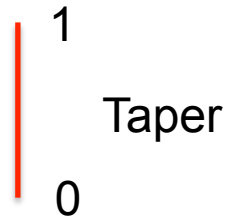
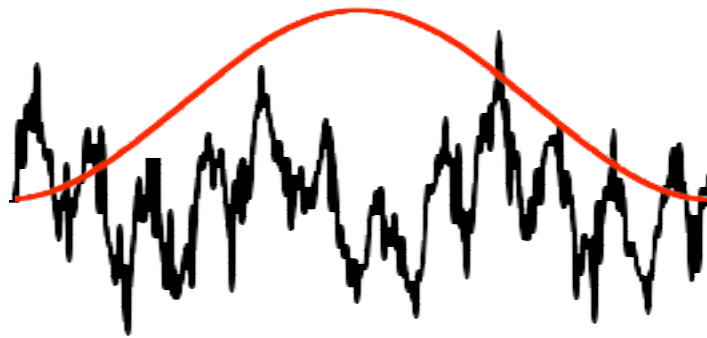
- Infinite number of frequencies required to approximate discontinuities
- This means infinite (or very large) number of samples required (not possible)

What can we do?

Tapering

Smoothly decay signal to zero at endpoints to smooth discontinuity

EEG



X

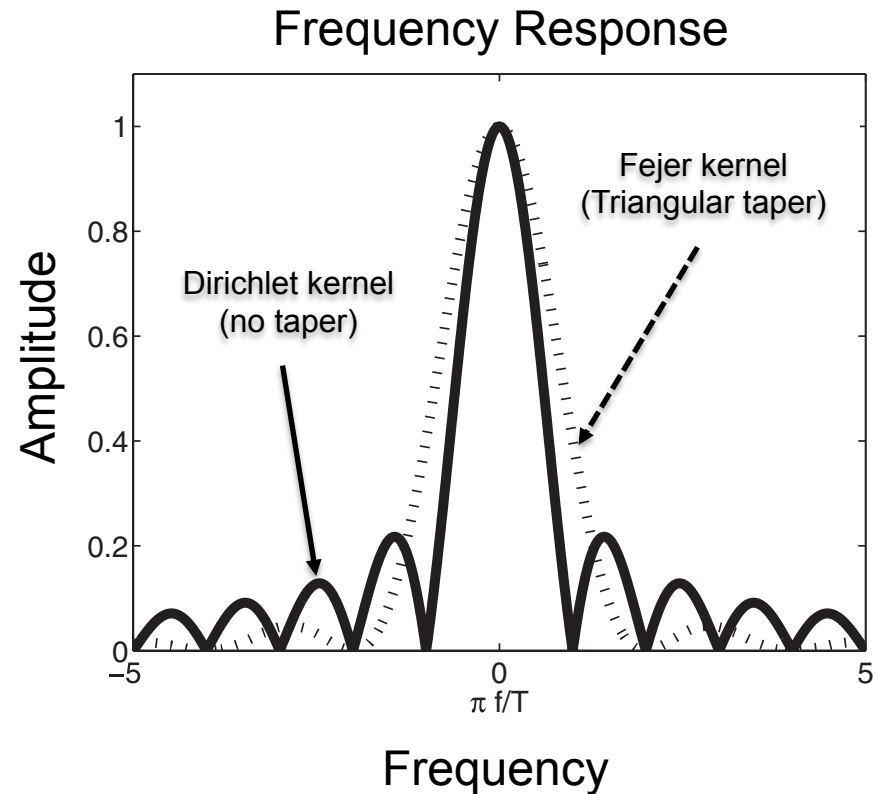
Tapered
EEG



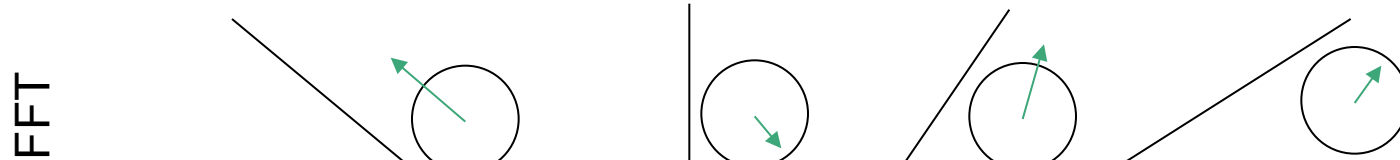
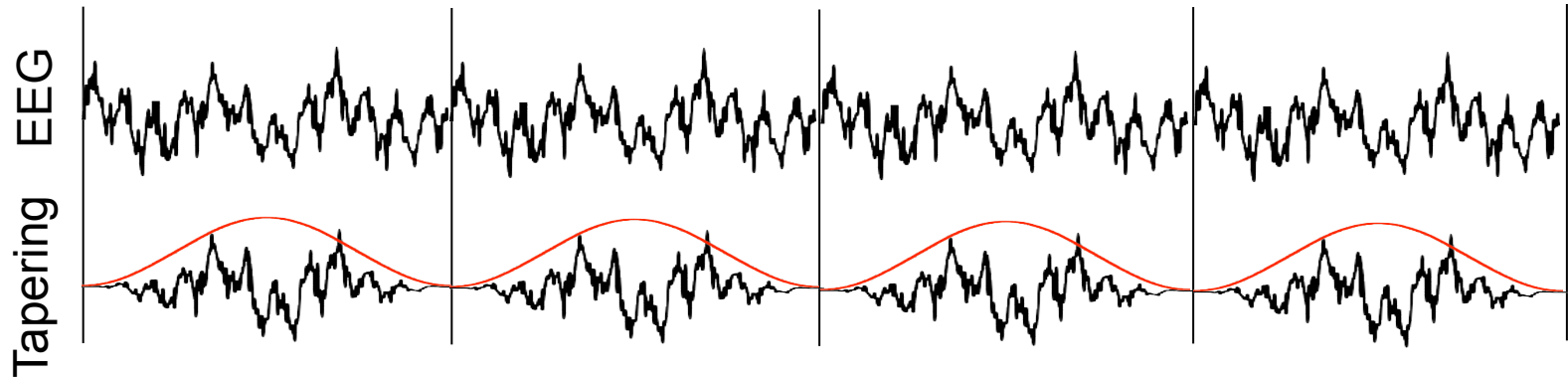
Tapering

Tapering reduces the effect of the Gibbs phenomenon making it easier to identify “true” peaks in the spectrum from spurious ripple peaks (minimized broadband bias or “spectral leakage”)

The cost is increased width of central peak (narrowband bias).



Spectral Estimation via Welch's Method

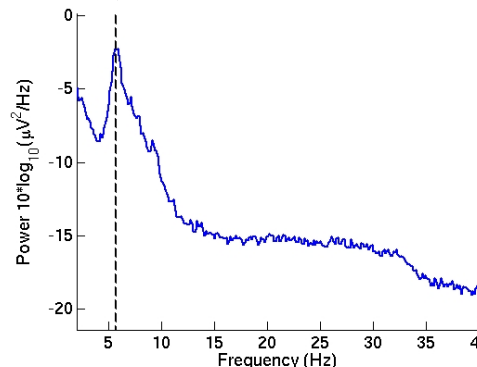


Average of squared absolute values

$$S_{Welch}(f) = \frac{1}{K} \sum_{k=1}^K |S_k(f)|^2$$

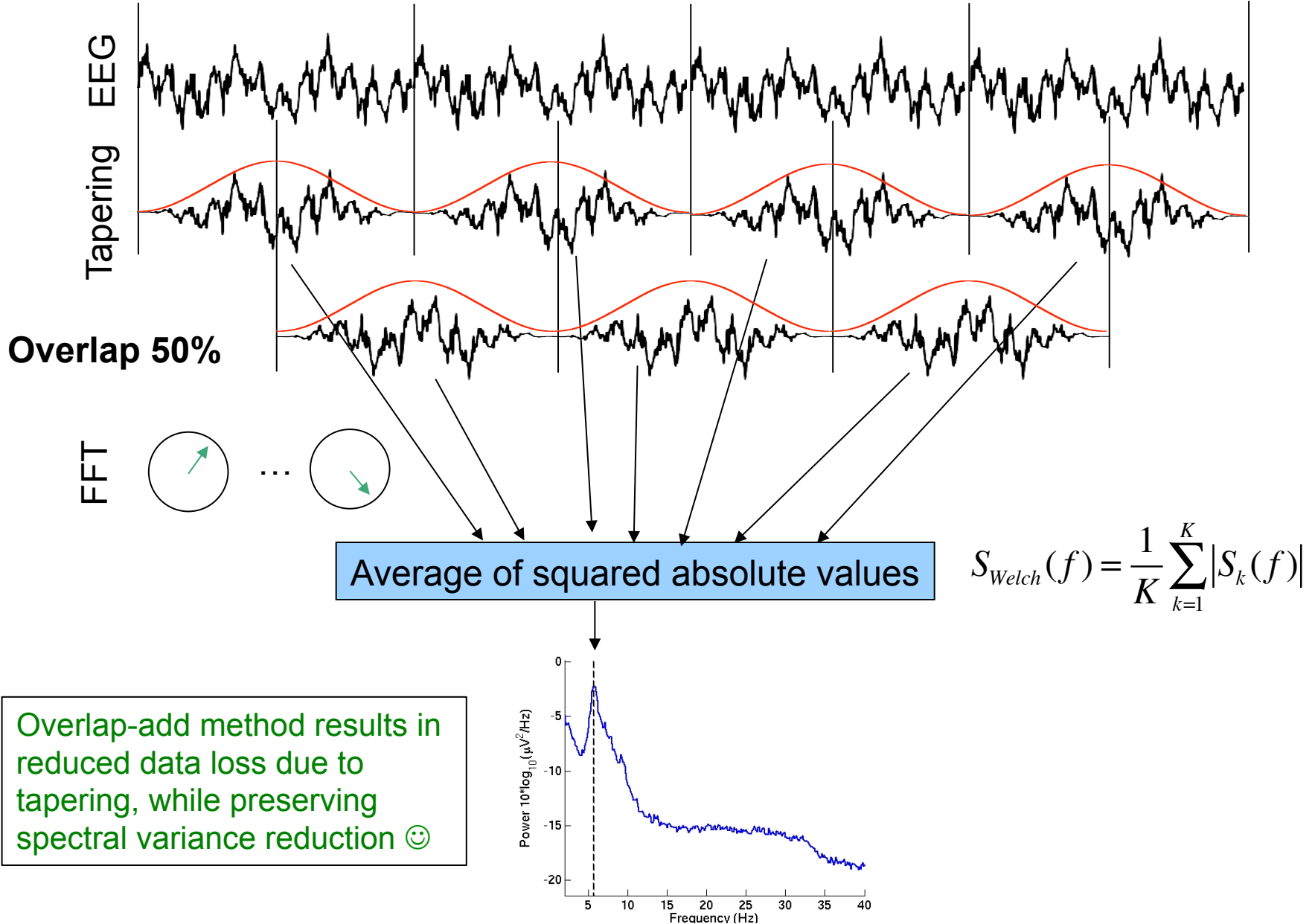
Given K windows:

- Variance is reduced by a factor of K
- Frequency resolution also reduced by a factor of K



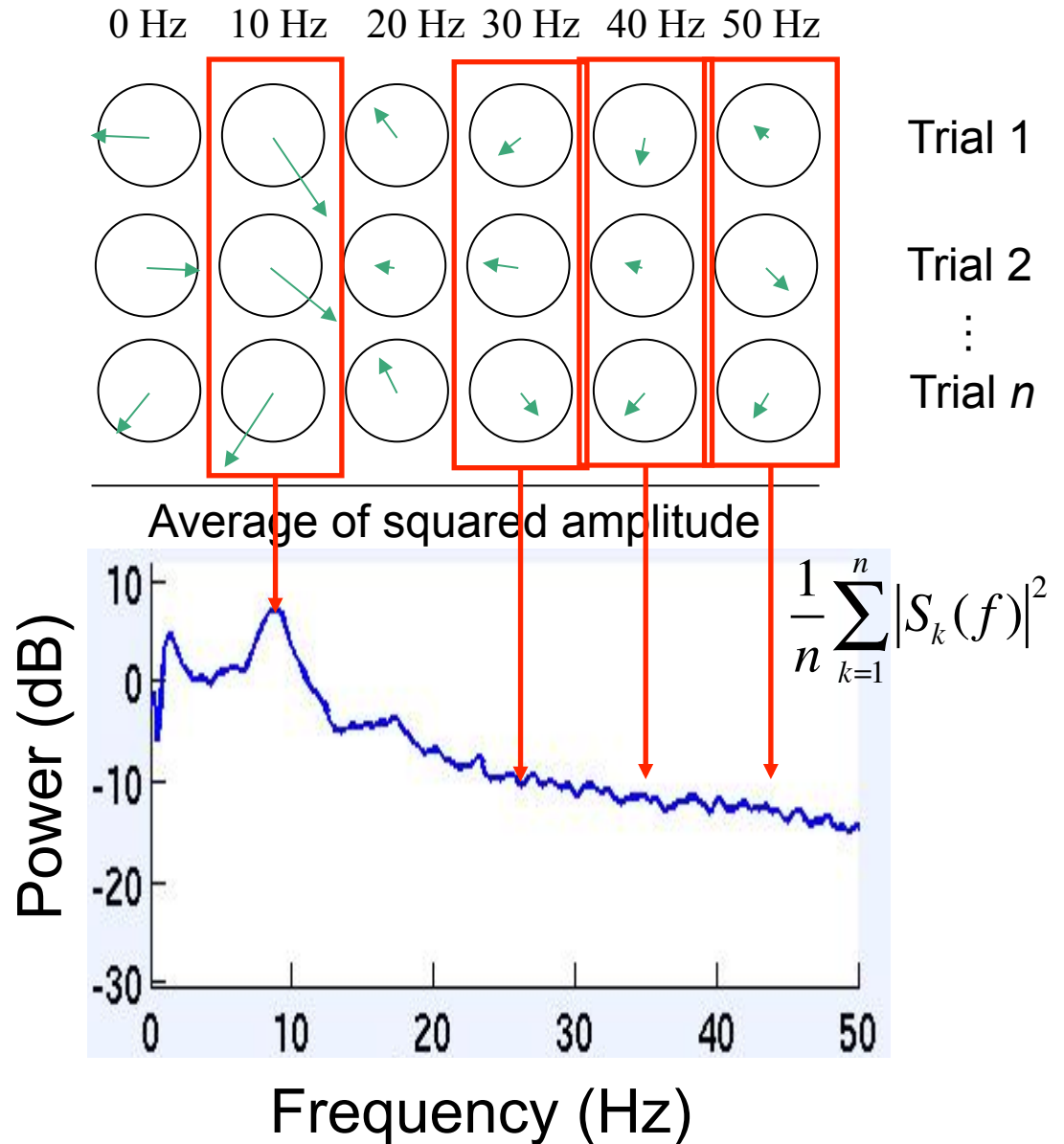
- Tapering also results in data loss → decreased frequency resolution (increased narrowband bias)
- Can we mitigate data loss?

Spectral Estimation via Welch's Method



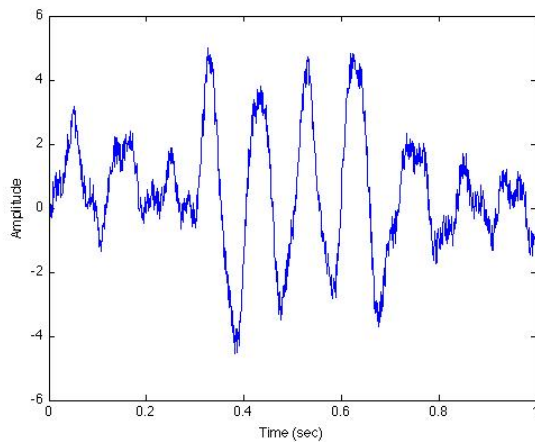
Trial Averaging

Averaging spectra over n independent trials leads to further reduction of variance by a factor of n

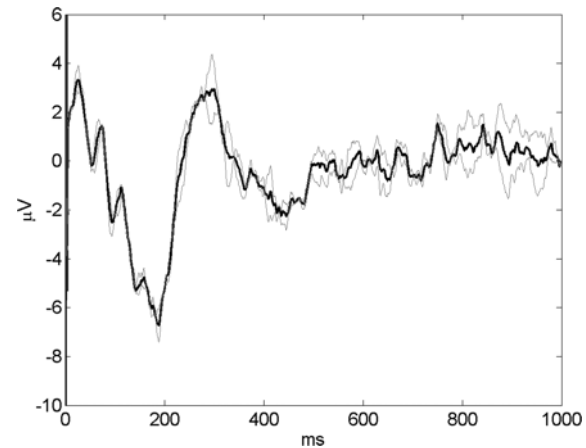


Non-Stationary Signals

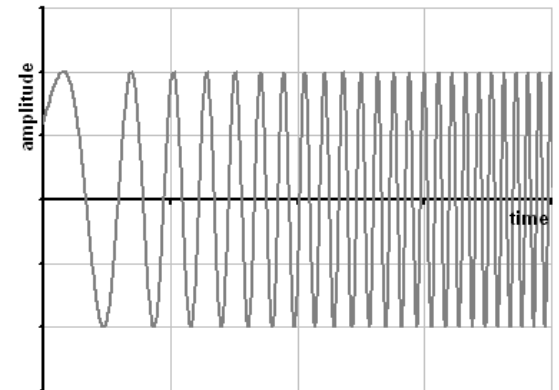
Non-stationary signals include bursts, chirps, evoked potentials, ...



Burst

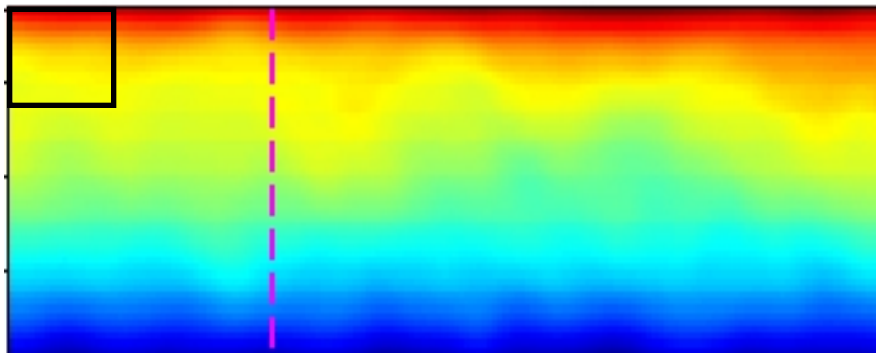
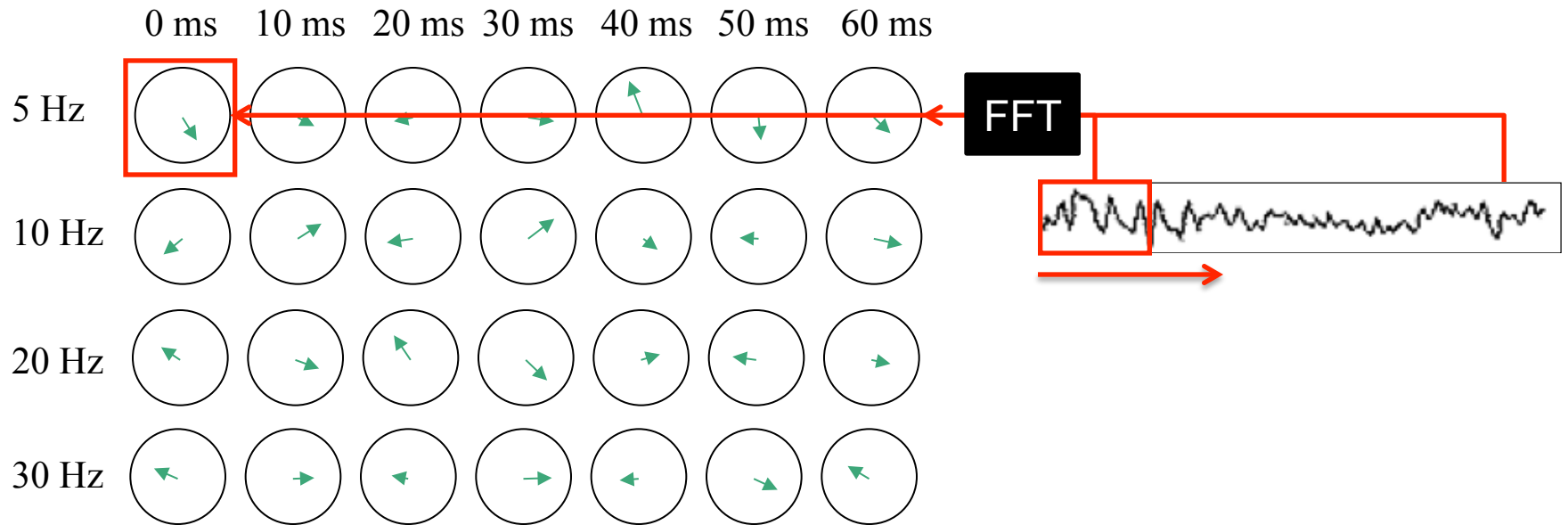


Evoked potential

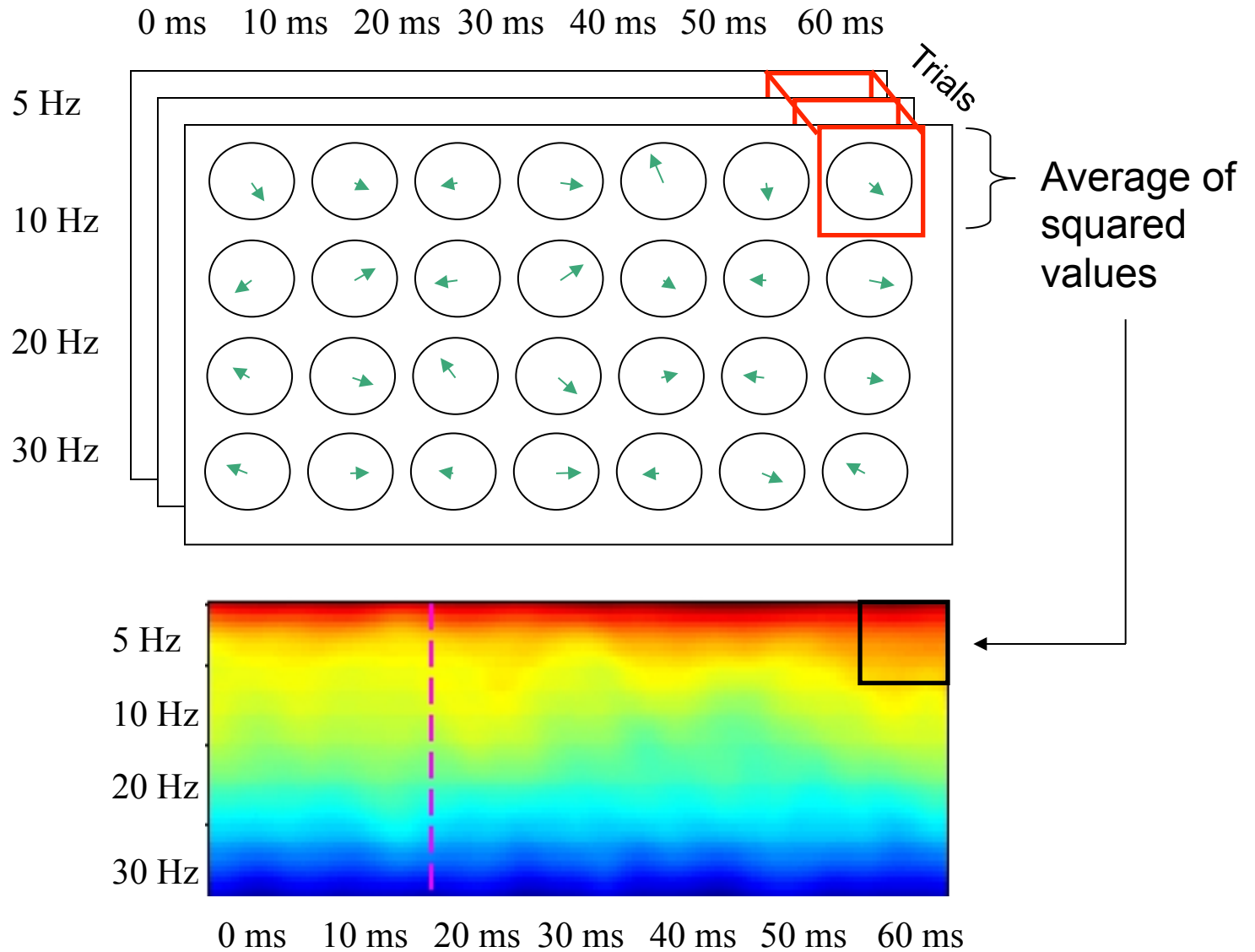


Chirp

Spectrogram or ERSP



Spectrogram or ERSP



Power spectrum and event-related spectral (perturbation)

$$ERS(f, t) = \frac{1}{n} \sum_{k=1}^n |S_k(f, t)|^2$$



Scaled to dB 10Log_{10}

Ensemble
average

Complex number

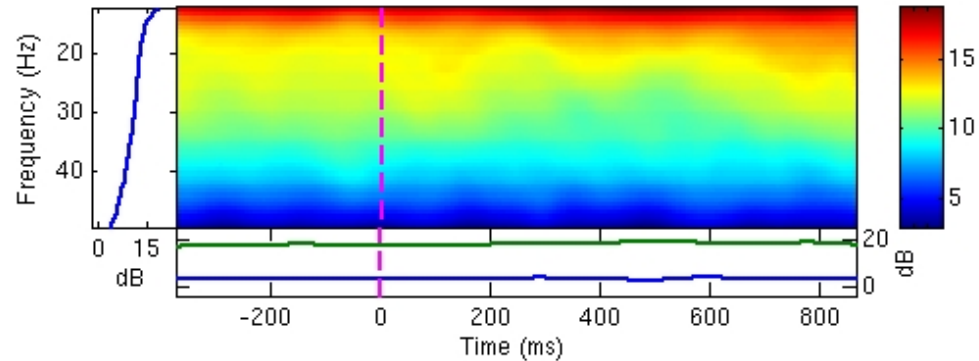
Here, there are n trials

Each trial is time-locked to the same *event* (hence “event-related” spectrum)

The ERS is the average power across event-locked trials

Absolute versus relative power

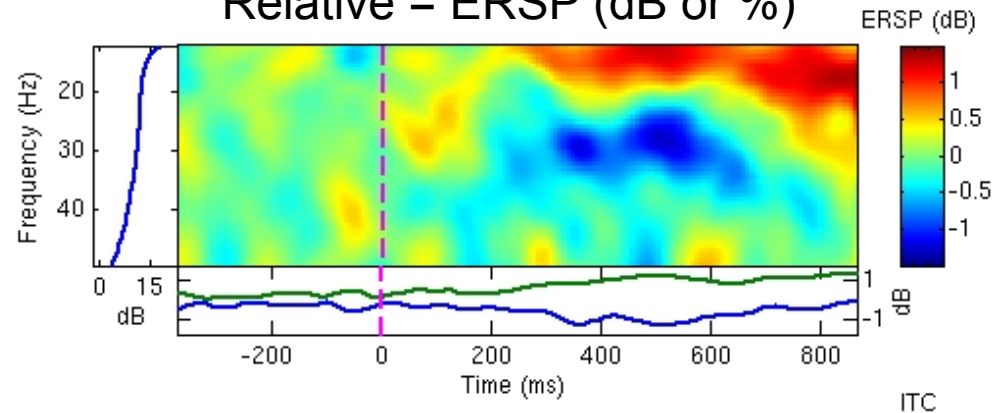
Absolute = ERS



To compute the ERSP, we just subtract the pre-stimulus ERS from the whole trial



Relative = ERSP (dB or %)



The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/position and in frequency/momentum.

There exists a lower bound to the Heisenberg product.

$$\Delta t \Delta f \geq 1/(4\pi)$$

or
$$\Delta f \geq 1/(4\pi\Delta t)$$

e.g. here are two possible $(\Delta f, \Delta t)$ pairs:

$$\Delta f = 1\text{Hz}, \Delta t = 80 \text{ msec} \quad \text{or}$$

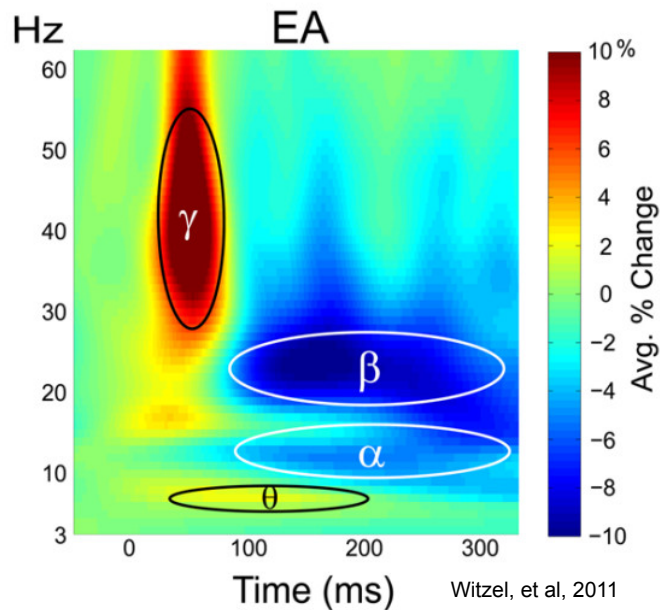
$$\Delta f = 2\text{Hz}, \Delta t = 40 \text{ msec}$$



Werner Karl Heisenberg
(1901 – 1976)

Time-Frequency Tradeoff

Natural biophysical processes may exhibit sustained changes in narrowband low-frequency oscillations along with rapidly-changing (e.g. “burst”) high-frequency oscillations.



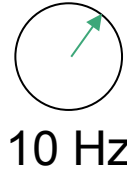
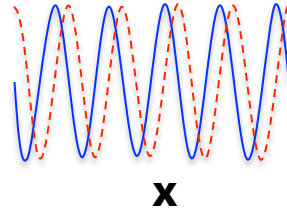
The Short-Time Fourier Transform has a constant temporal resolution for all frequencies.

Can we adapt the time-frequency resolution tradeoff for individual frequencies to improve spectral estimation?

Yes, we can!

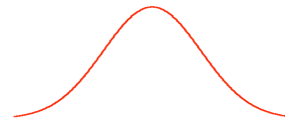
Wavelet Analysis

Complex Sinusoid



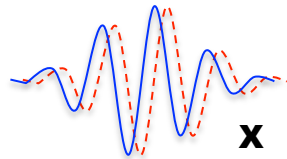
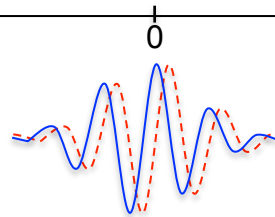
— Real Part (cosine)
- - - Imaginary Part (sine)

Window (Taper)

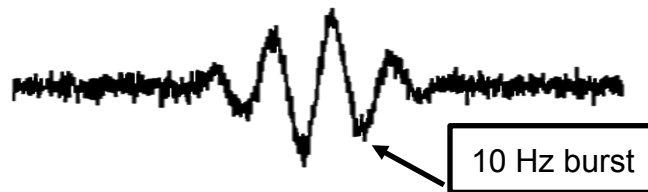


time ->

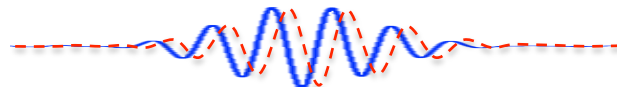
Tapered sinusoid (wavelet)



EEG



Covariance



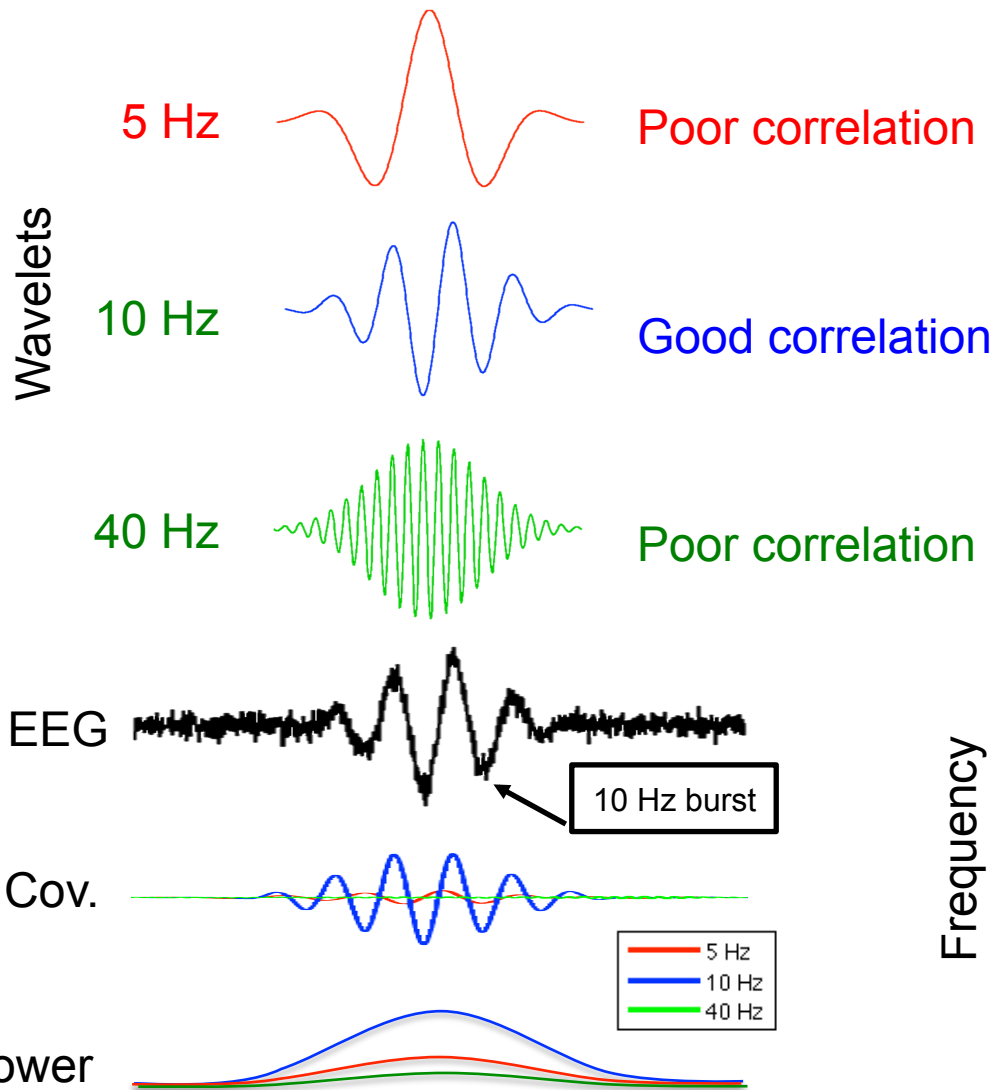
Power



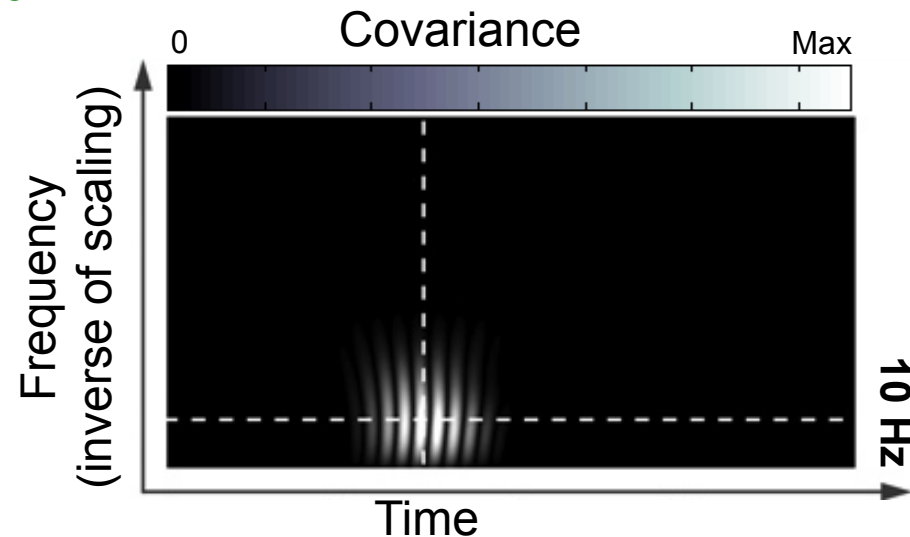
$$|S(f,t)| = |\text{Re} + \text{Im}|$$

We estimate the time-varying power at 10 Hz by convolving EEG signal with a tapered 10 Hz complex sinusoid (Morlet wavelet)

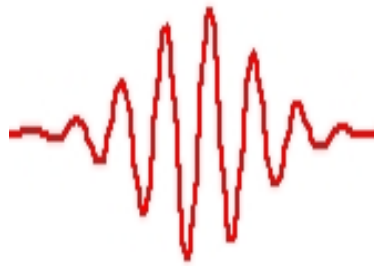
Wavelet Time-Frequency Image



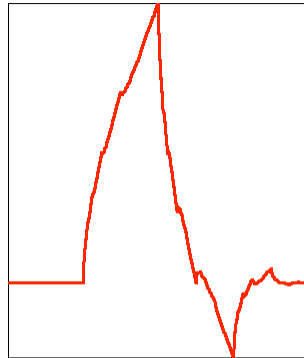
By convolving stretched and scaled versions of the "mother" wavelet with the EEG signal, we determine the time-frequency distribution of power



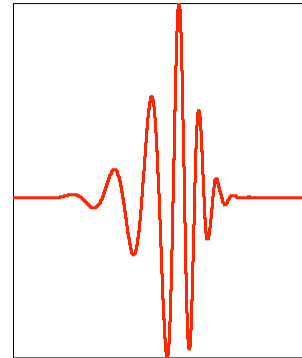
Some Wavelet Families



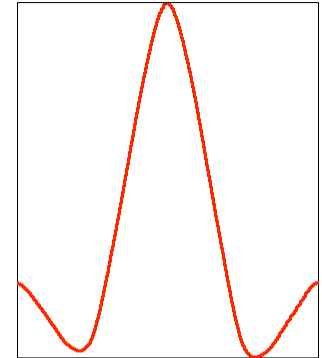
Morlet



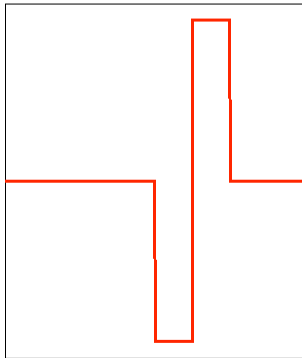
Daubechies_4



Daubechies_20



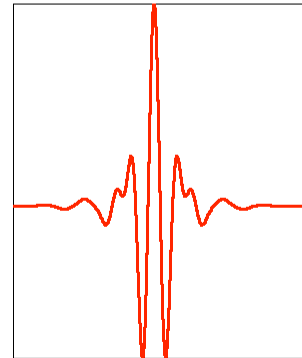
Coiflet_3



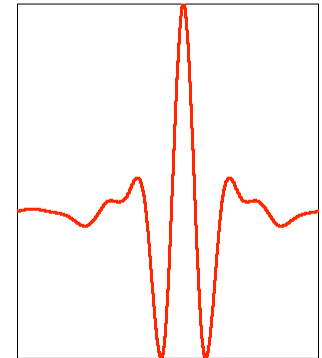
Haar_4



Symmlet_4



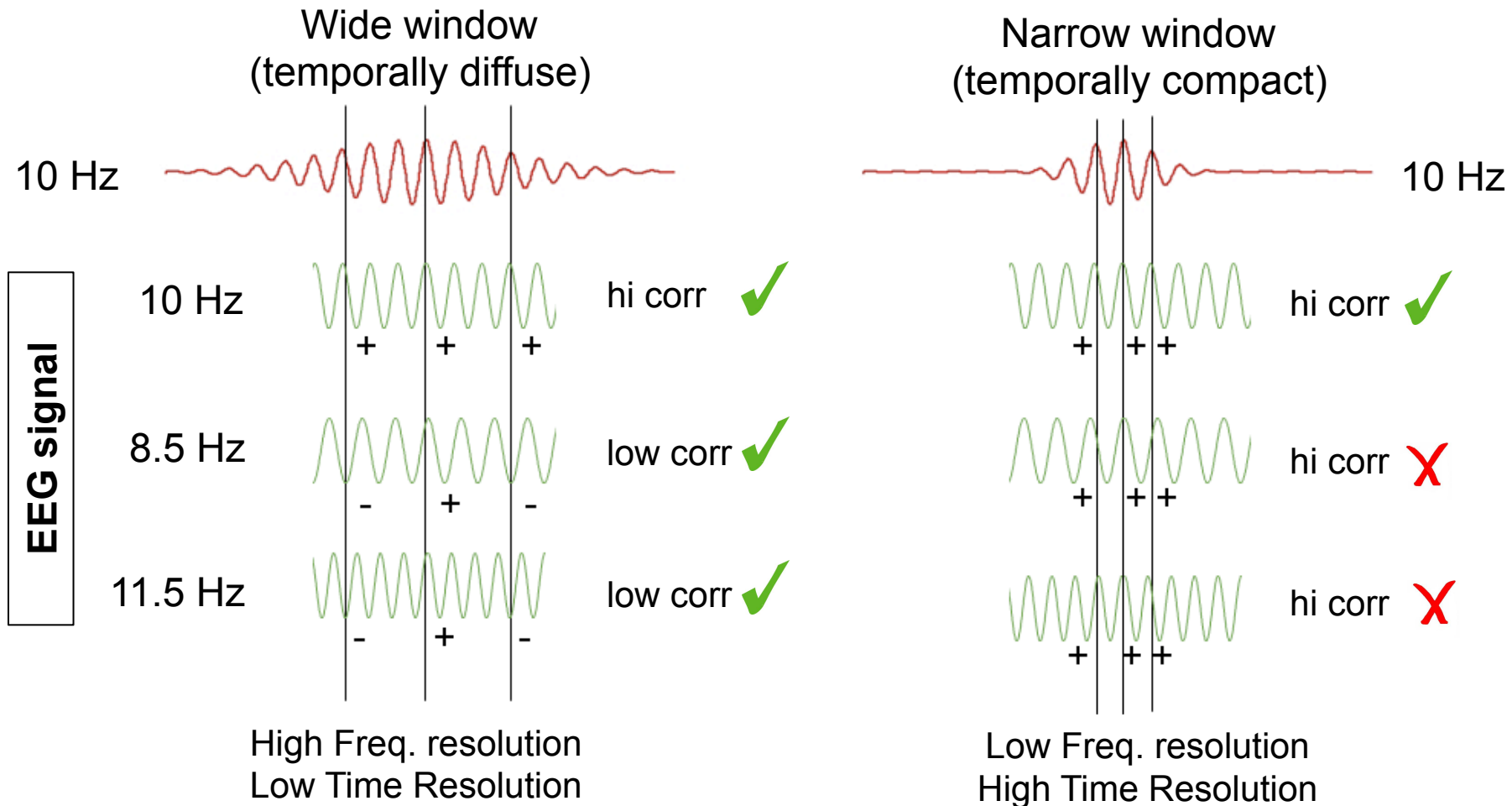
Meyer_2



Battle_3

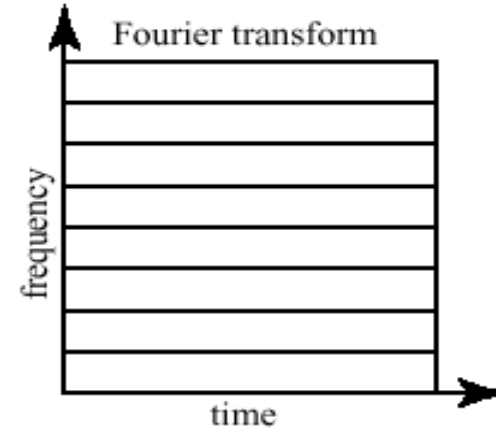
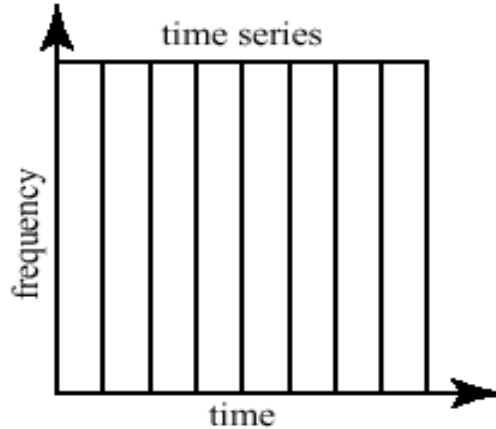
Trading Frequency for Time

(and vice versa)



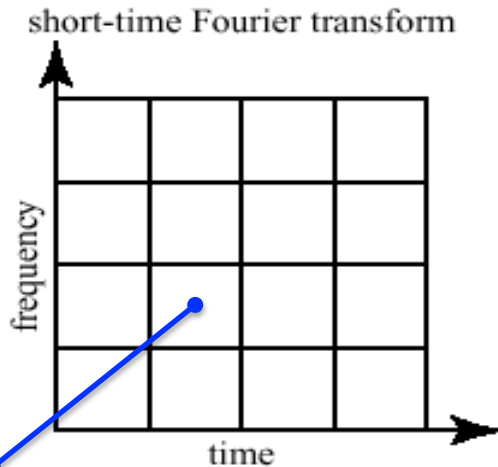
FFT versus Wavelets

fixed time res
no freq res

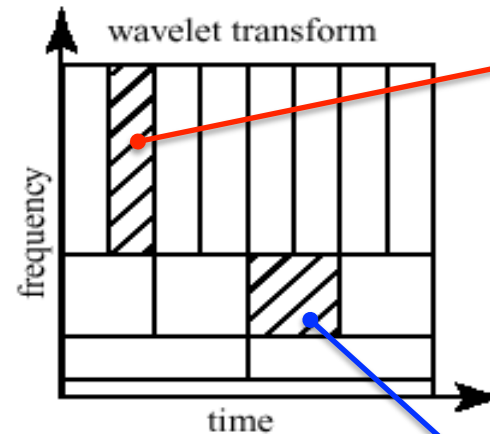


fixed freq res
no time res

fixed time res
fixed freq res



equal time
and freq
resolution



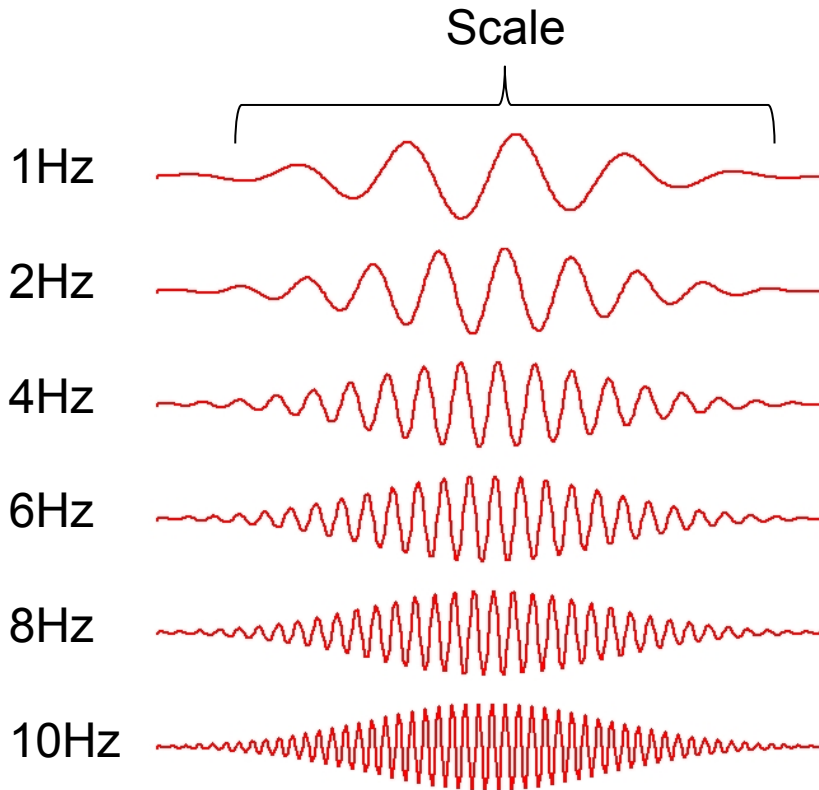
high time res
low freq res

variable time res
variable freq res

equal time
and freq
resolution

Wavelet scale

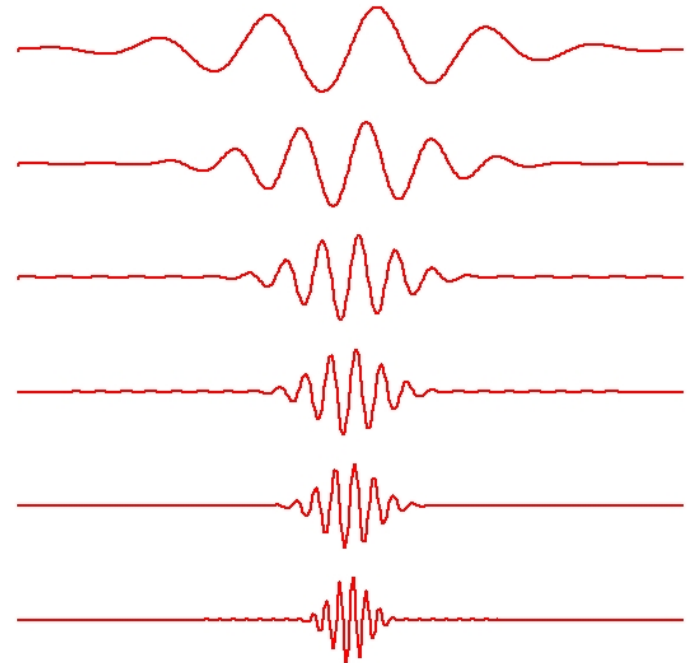
Wavelet (0)



constant window size (time resolution) for increasing frequency \rightarrow increasing # cycles with frequency.

Wavelet (1)

scale expansion factor (q)



window size decreases by a factor of 2 for each octave (power of 2) \rightarrow constant # of cycles at each frequency

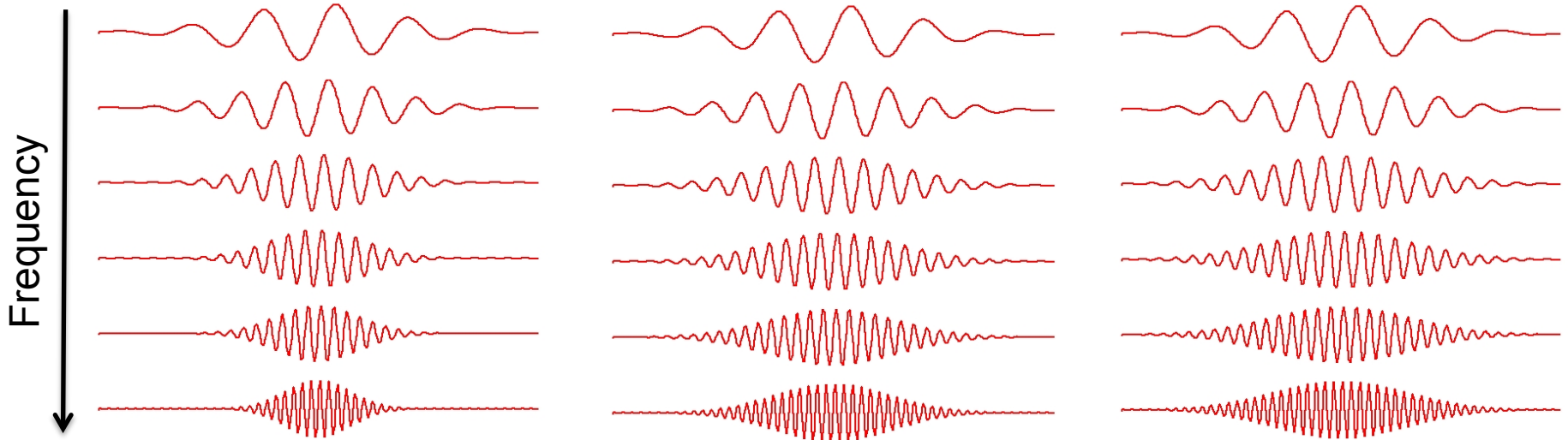
Wavelet scale expansion factor

Larger expansion factor produces larger scale decrements (increased time resolution, lower frequency resolution) for increasing frequency

Wavelet (0.8)

Wavelet (0.5)

Wavelet (0.2)

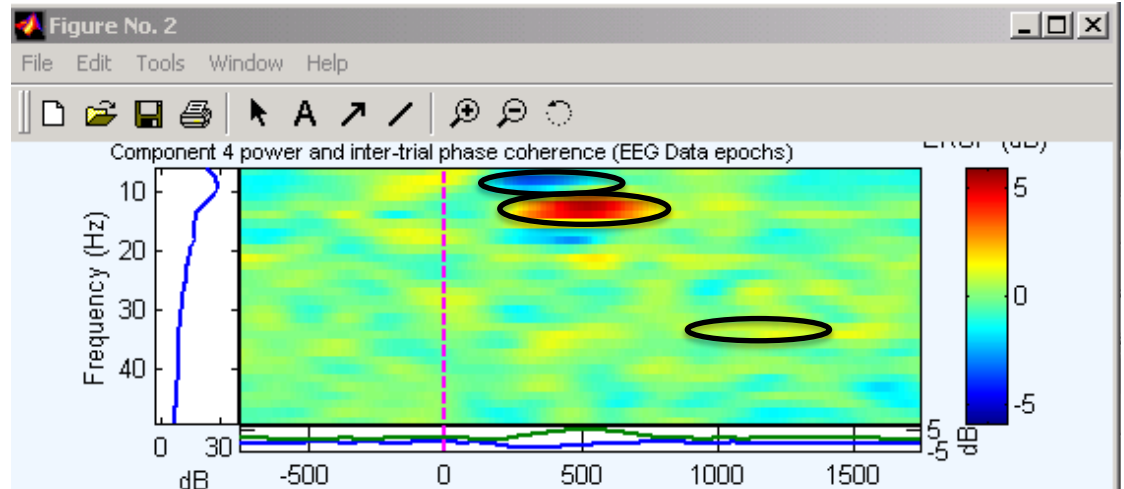


Number of cycles at highest frequency for an expansion factor of q :

$$C_{f_{\max}} = \frac{f_{\max}}{f_{\min}} C_{f_{\min}} (1 - q)$$

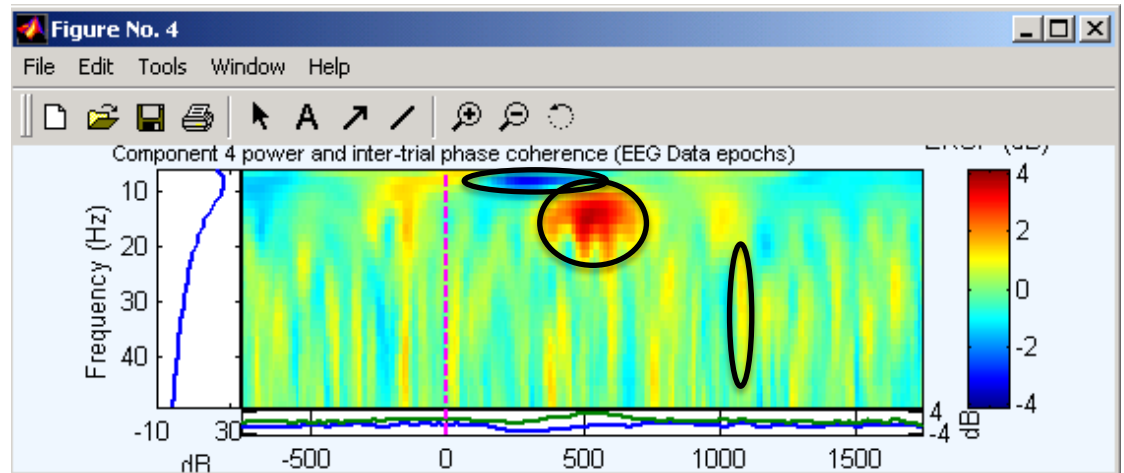
FFT versus Wavelets

FFT
(Wavelet(0))

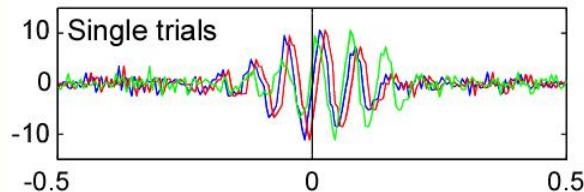
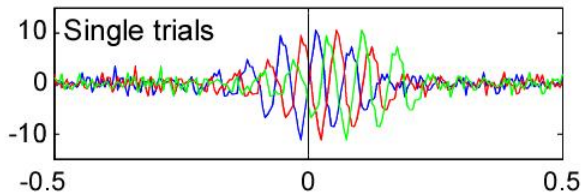


Wavelet (1)

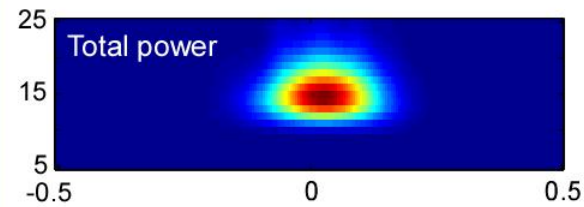
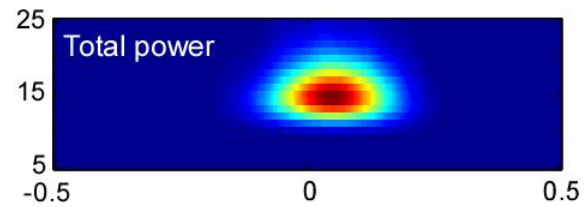
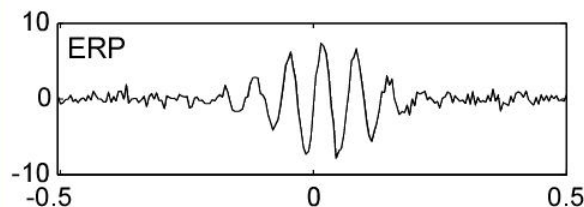
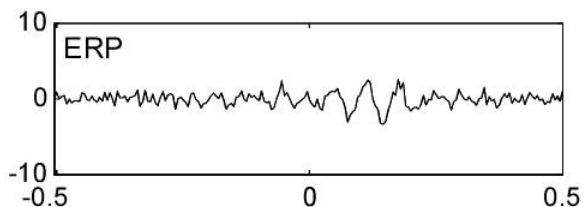
scale expansion factor (q)



Intertrial Coherence (ITC)

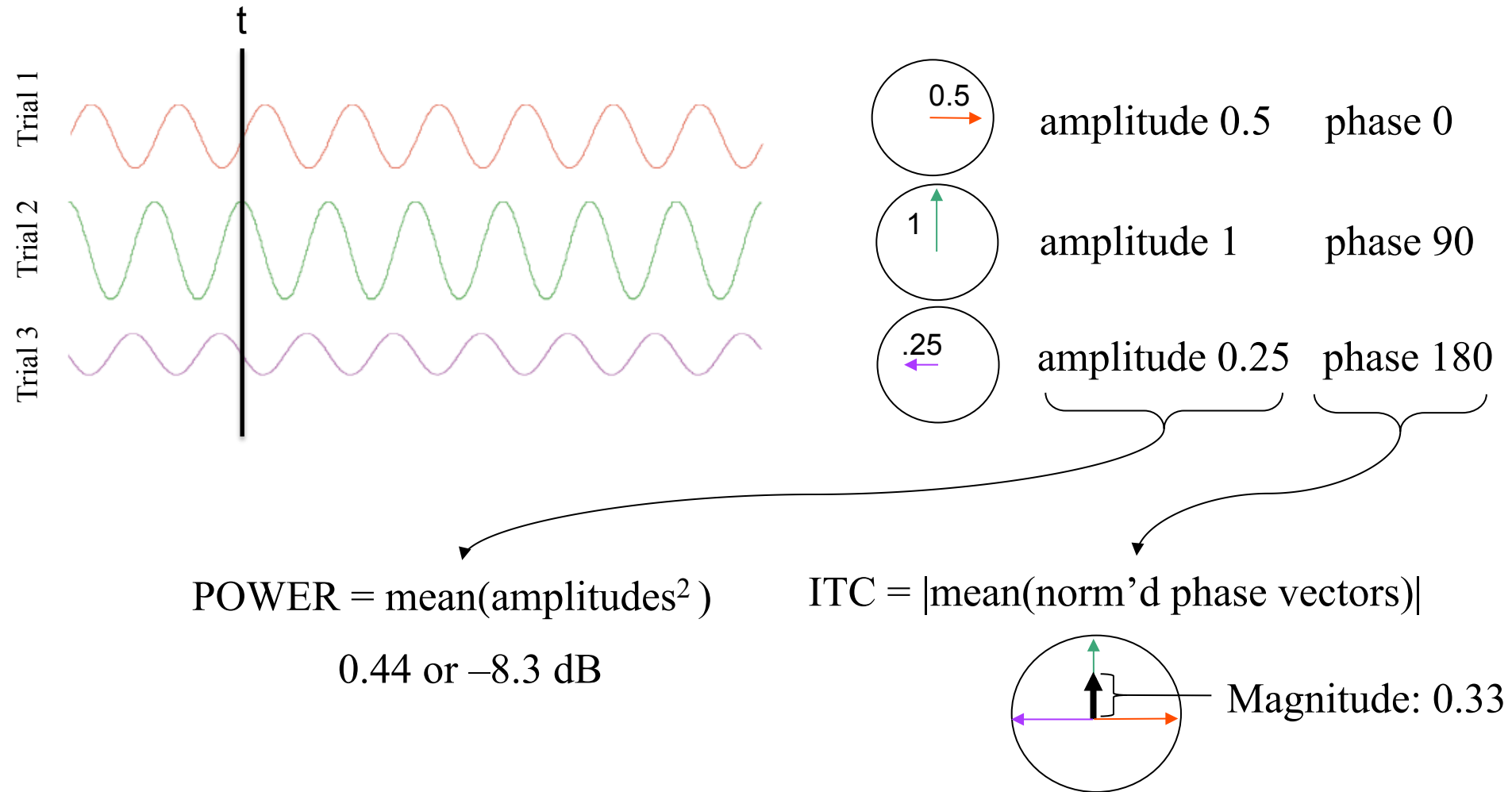


Phase
Resetting



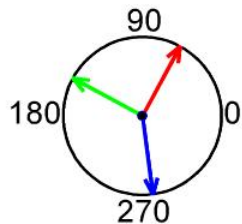
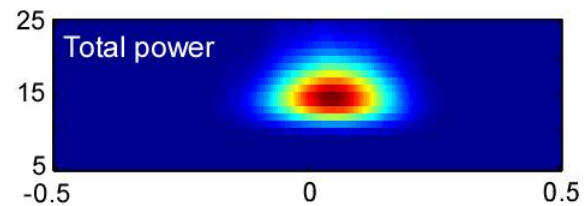
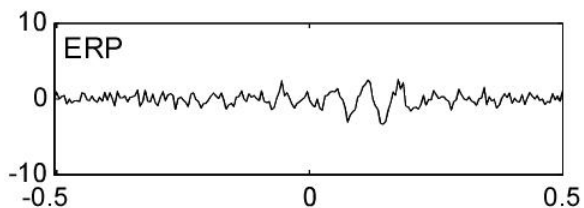
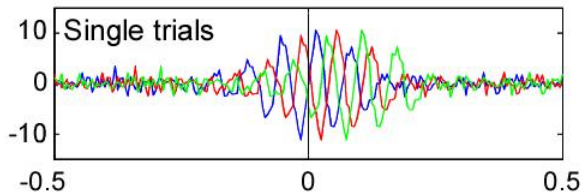
Inter-Trial Coherence (ITC)

Tallon-Baudry, et al, 1996

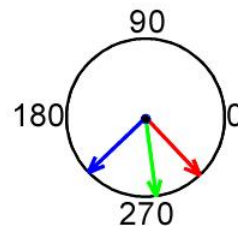
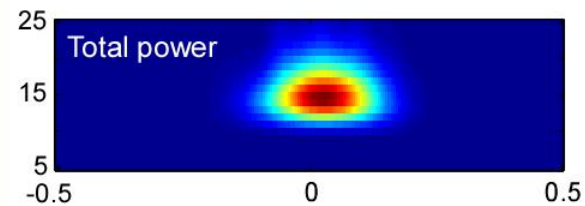
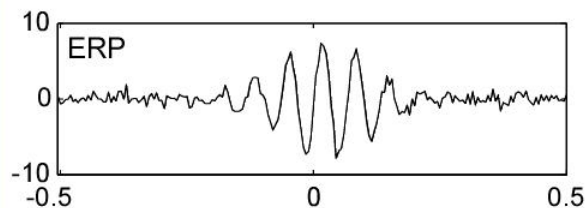
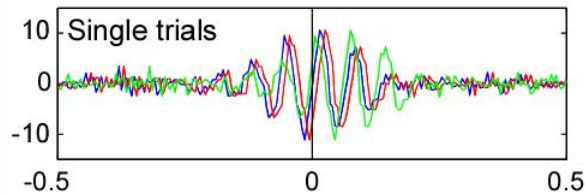


```
>> EEG = pop_newtimef(EEG, ..., 'plotitc', 'on');
```

Intertrial Coherence (ITC)



ITC: .05



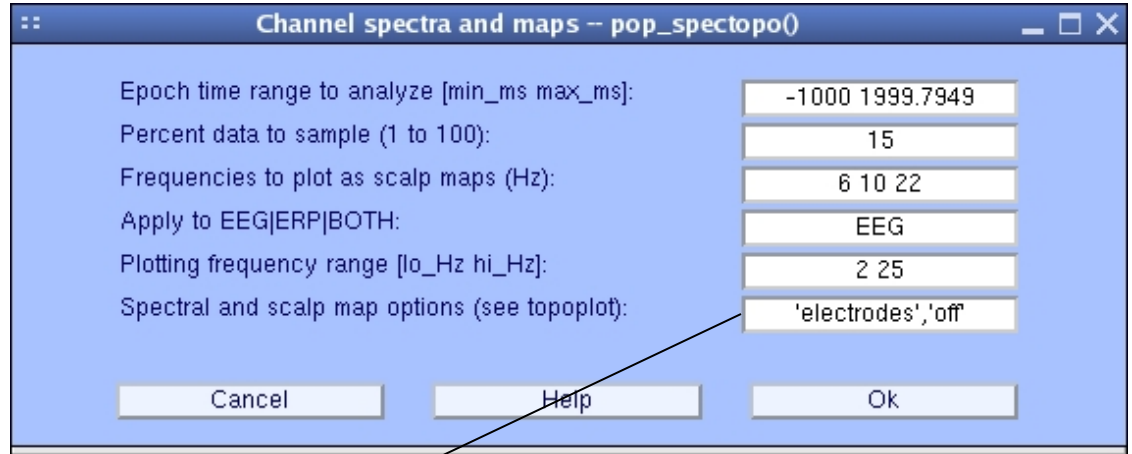
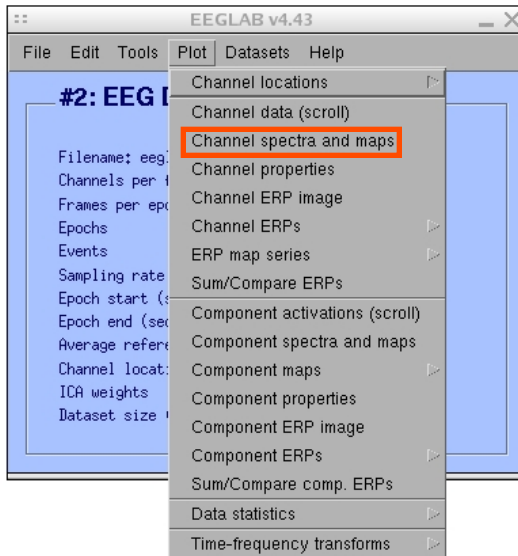
ITC: .80

Phase
Resetting

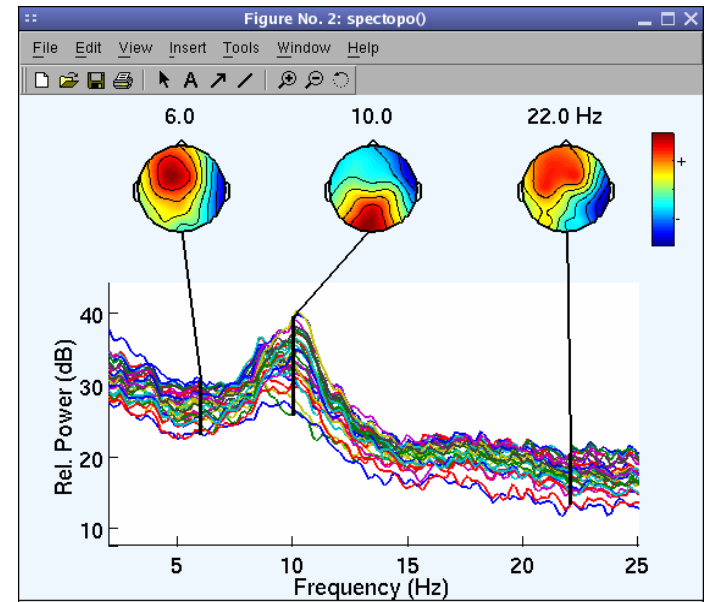
Time-Frequency Analysis of Biophysical Time series:

Practicum

Plot periodogram (spectrum) using Welch's method



'winsize', 256 (change FFT window length)
'nfft', 256 (change FFT padding)
'overlap', 128 (change window overlap)



Plot component time frequency -- pop_newtimef()

Component number: 1

Sub epoch time limits [min max] (msec): -1000 1996

Frequency limits [min max] (Hz) or sequence: [empty]

Baseline limits [min max] (msec) (0->pre-stim.): 0

Wavelet cycles [min max/fact] or sequence: 3 0.5

ERSP color limits [max] (min=-max): [empty]

ITC color limits [max]: [empty]

Bootstrap significance level (Ex: 0.05): [empty]

Optional newtimef() arguments (see help): [empty]

Plot Event Related Spectral Power

Buttons: Cancel

File Edit Tools

#2: face

Filename: r

Channels p

Frames per

Epochs

Events

Sampling rat

Epoch start (

Epoch end (s

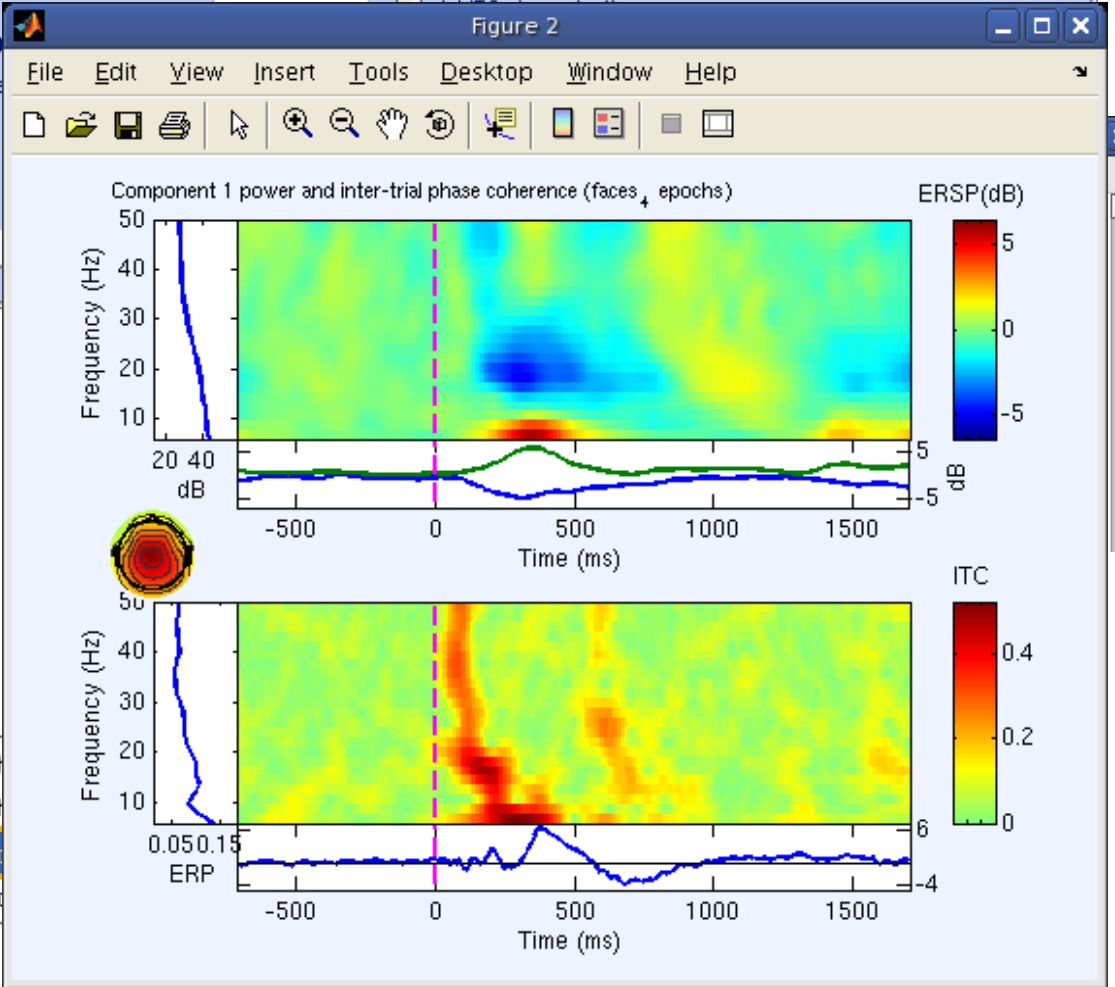
Average refe

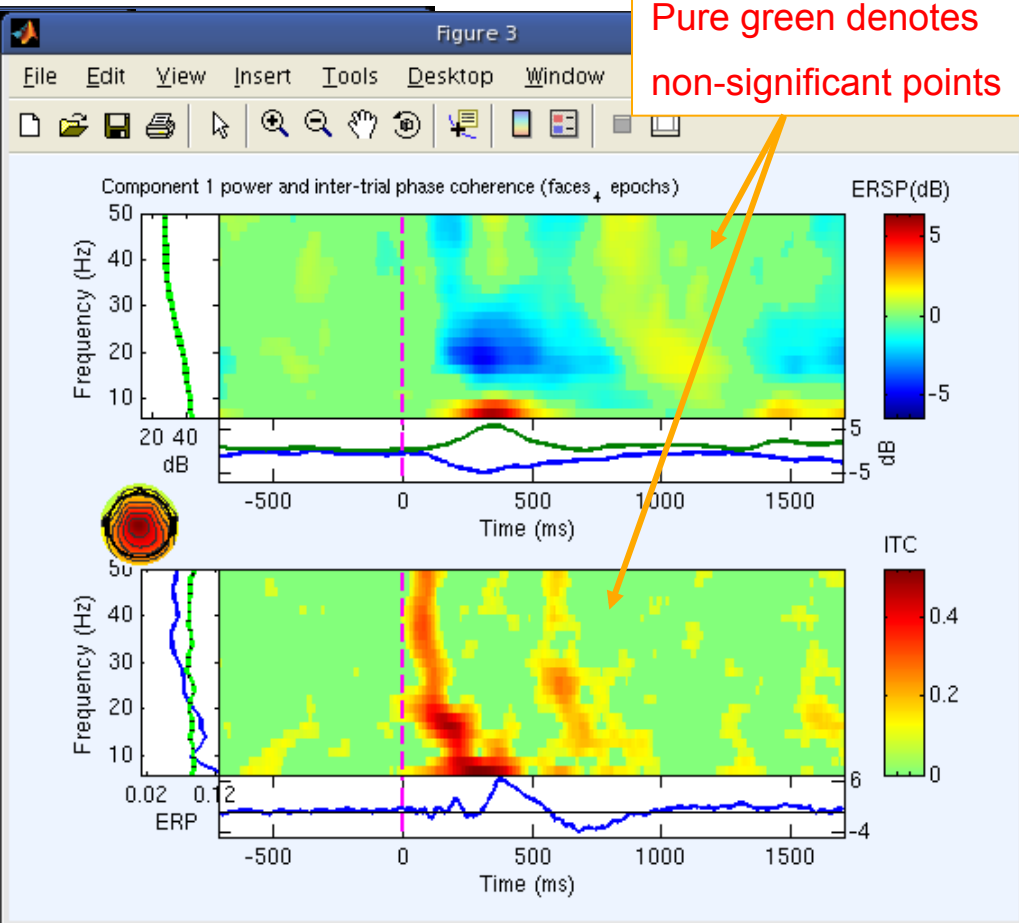
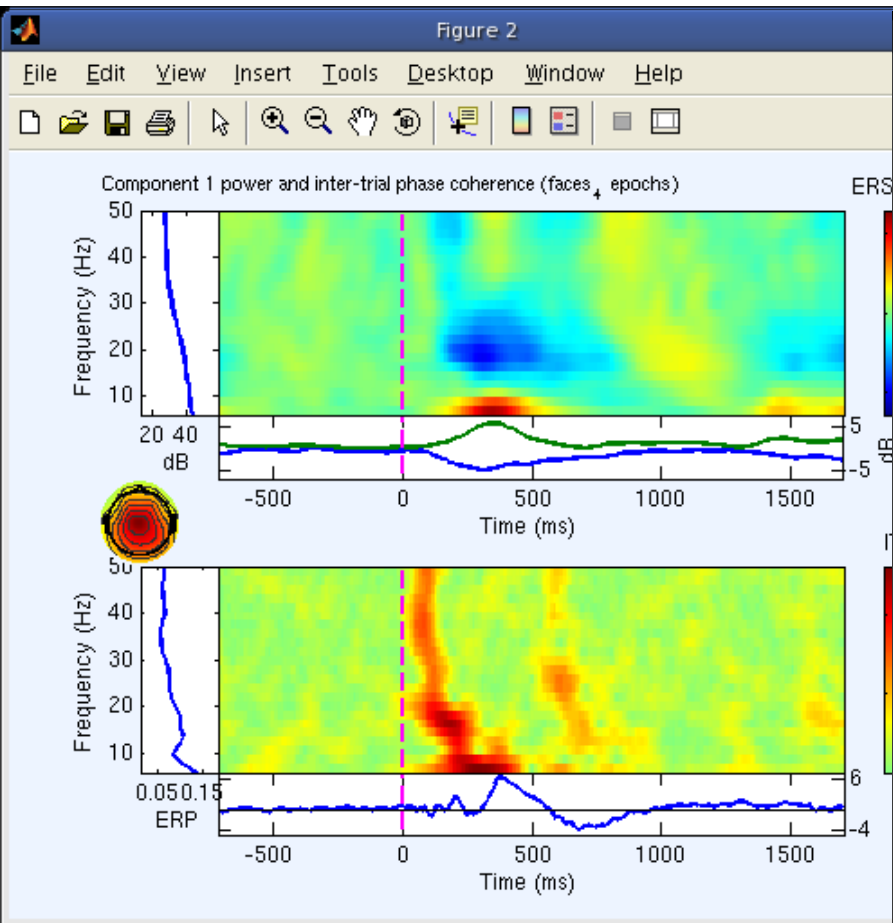
Channel loca

ICA weights

Dataset size

- ERP map series
- Sum/Compare ERPs
- Component activations (scroll)
- Component spectra and maps
- Component maps
- Component properties
- Component ERP image
- Component ERPs
- Sum/Compare comp. ERPs
- Data statistics
- Time-frequency transforms**
- Average time-frequency
- Cluster dataset ICs





Wavelet cycles [min max/fact] or sequence

ERSP color limits [max] (min=-max)

ITC color limits [max]

Bootstrap significance level (Ex: 0.01 -> 1%)

Optional newtimef() arguments (see Help)

3 0.5

Use limits Use FFT

see log power (set)

plot ITC phase (set)

FDR correct (set)

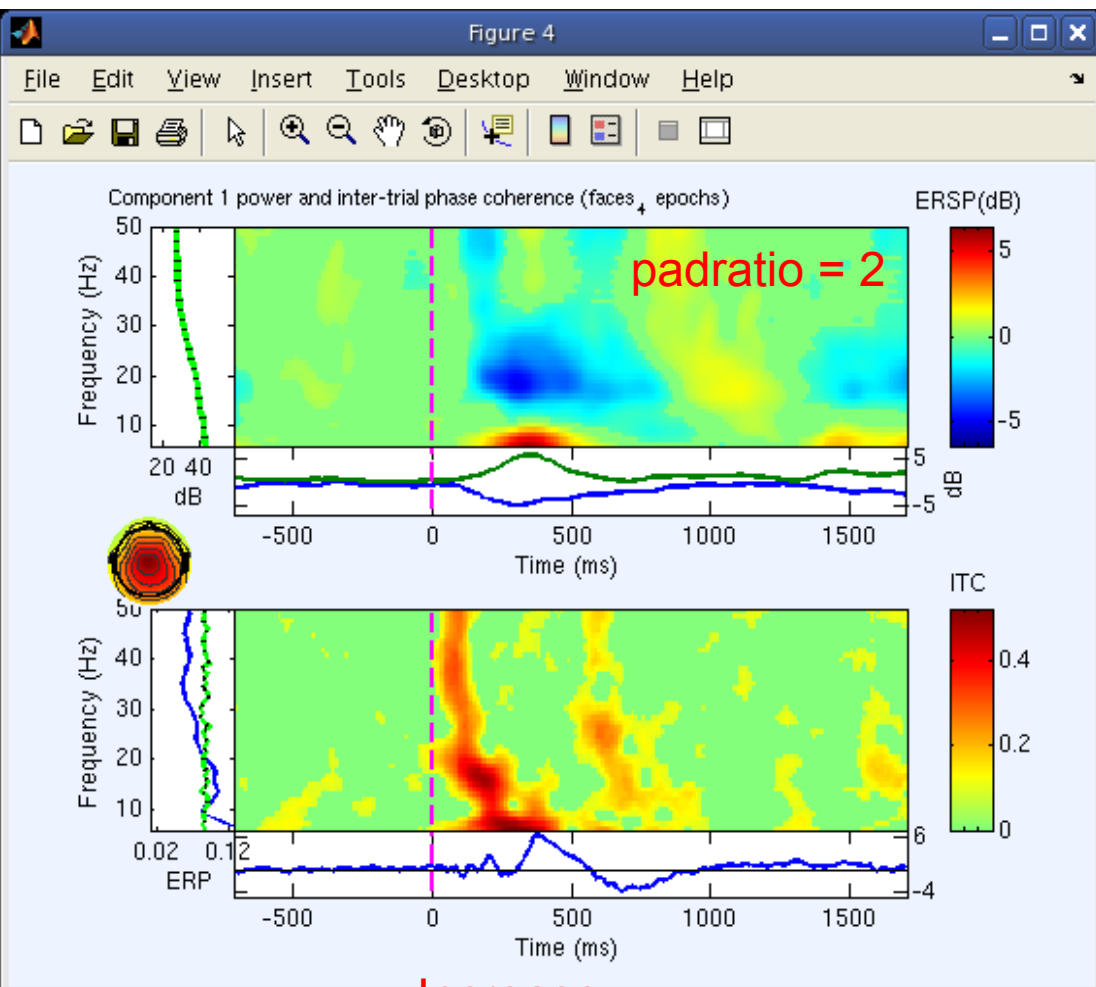
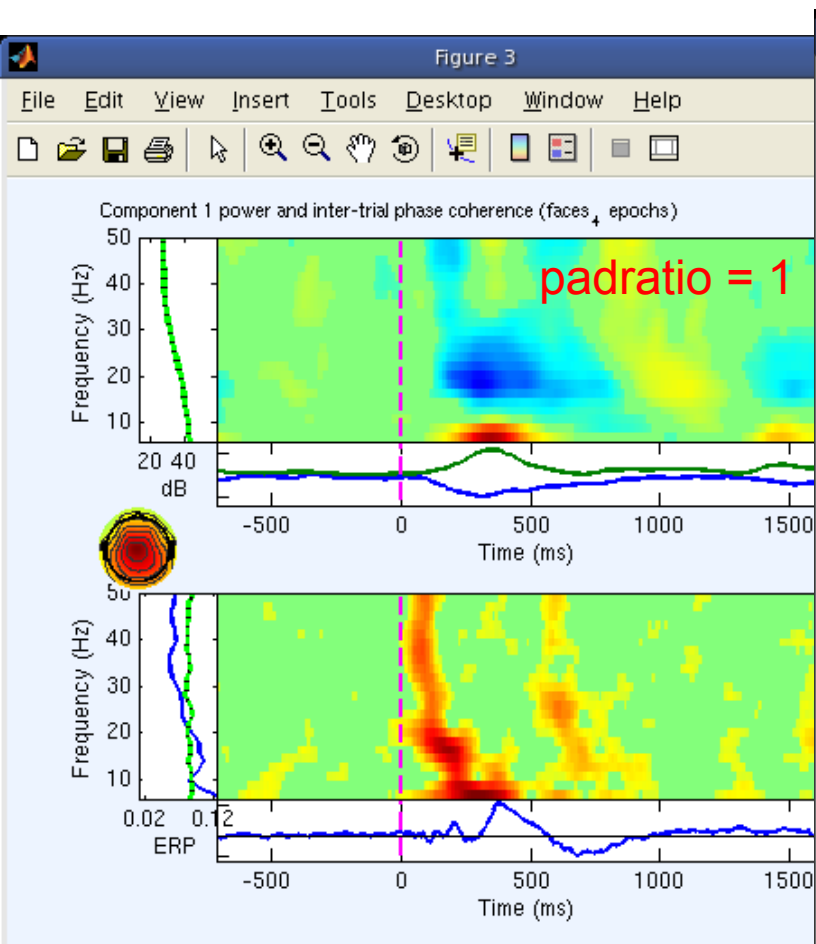
0.01

Plot Event Related Spectral Power

Plot Inter Trial Coherence

Plot curve at each frequency

Cancel Help Ok

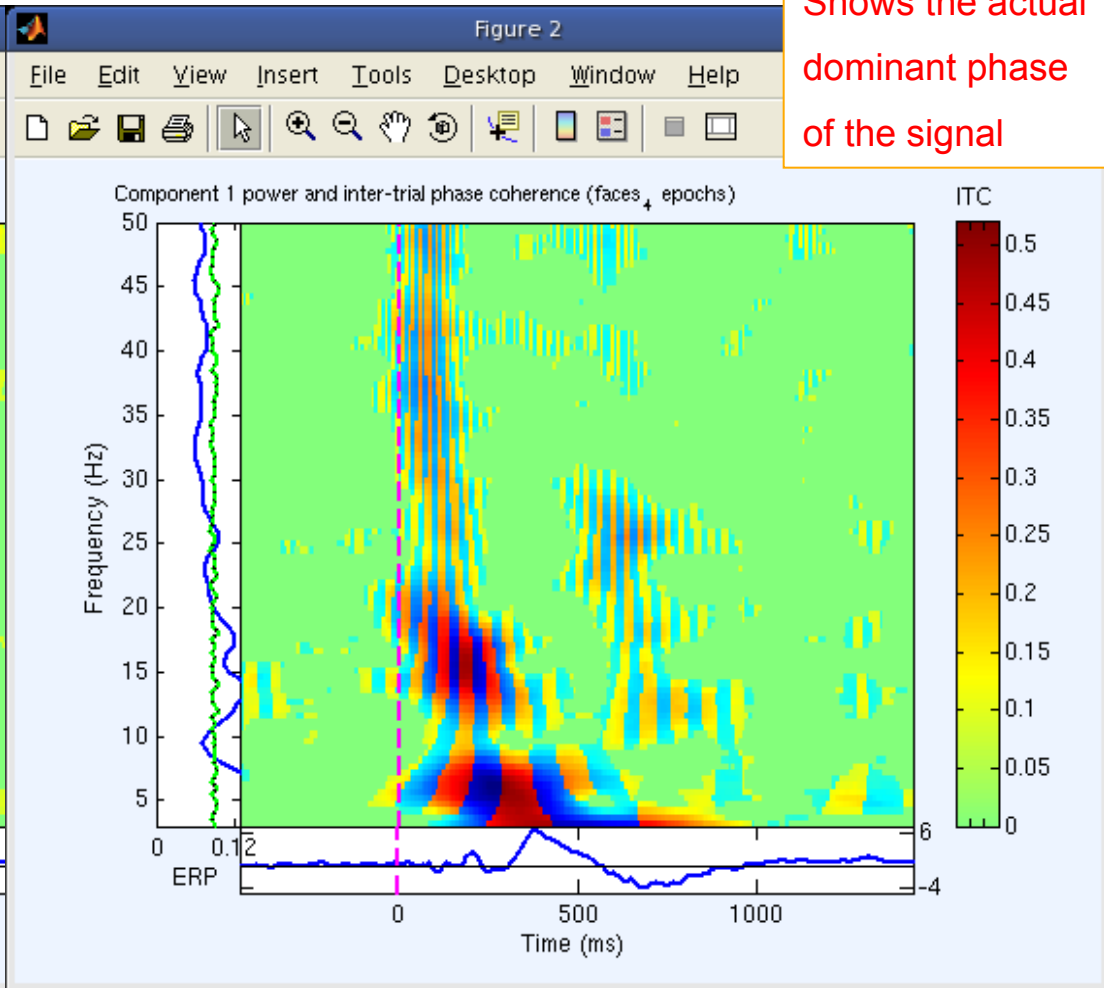
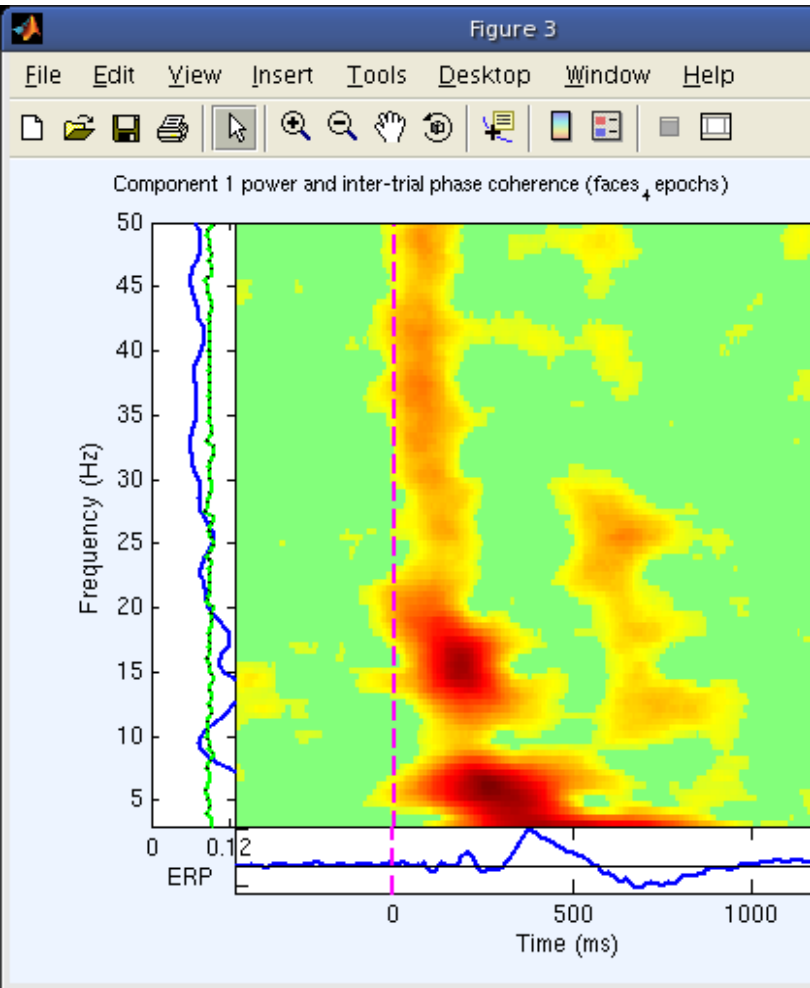


Increase # freq bins

- Component number
- Sub epoch time limits [min max] (msec)
- Frequency limits [min max] (Hz) or sequence
- Baseline limits [min max] (msec) (0->pre-stim.)
- Wavelet cycles [min max/fact] or sequence
- ERSP color limits [max] (min=-max)
- ITC color limits [max]
- Bootstrap significance level (Ex: 0.01 -> 1%)
- Optional newtimef() arguments (see Help)

1	Use 256 time points	<input type="checkbox"/> Log spaced
-1000 1996	Use limits, padding 1	<input type="checkbox"/> No baseline
	Use divisive baseline	<input type="checkbox"/> Use FFT
0	Use limits	
3 0.5	<input checked="" type="checkbox"/> see log power (set)	
	<input type="checkbox"/> plot ITC phase (set)	
	<input type="checkbox"/> FDR correct (set)	

Shows the actual dominant phase of the signal



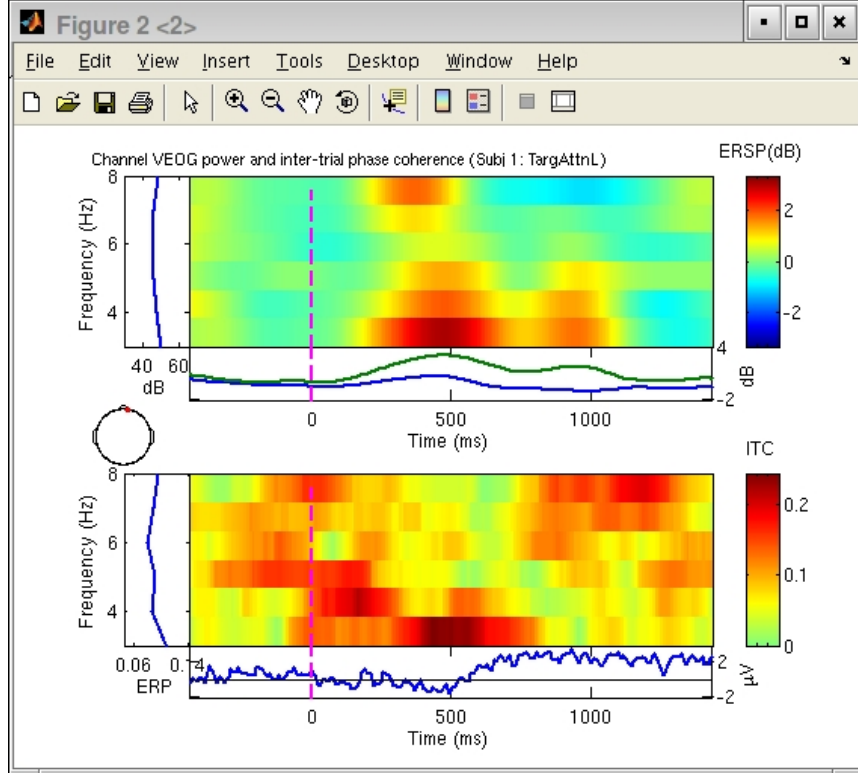
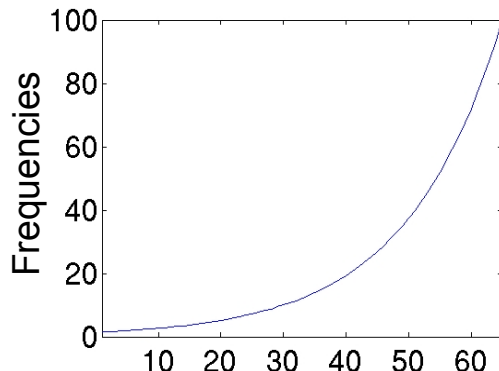
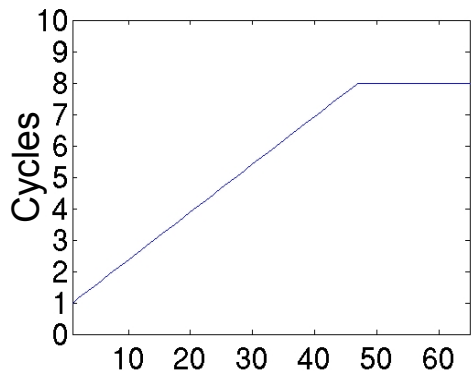
Sub epoch time limits [min max] (msec)
 Frequency limits [min max] (Hz) or sequence
 Baseline limits [min max] (msec) (0->pre-stim.)
 Wavelet cycles [min max/fact] or sequence
 ERSP color limits [max] (min=-max)
 ITC color limits [max]
 Bootstrap significance level (Ex: 0.01 -> 1%)
 Optional newtimef() arguments (see Help)

-1000 1996 Use 200 time points
 Use limits, padding 1 Log spaced
 Use divisive baseline No baseline
 Use limits Use FFT
 see log power (set)
 plot ITC phase (set)
 FDR correct (set)

'plotphase', 'on'

To visualize both low and high frequencies

```
freqs = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
```



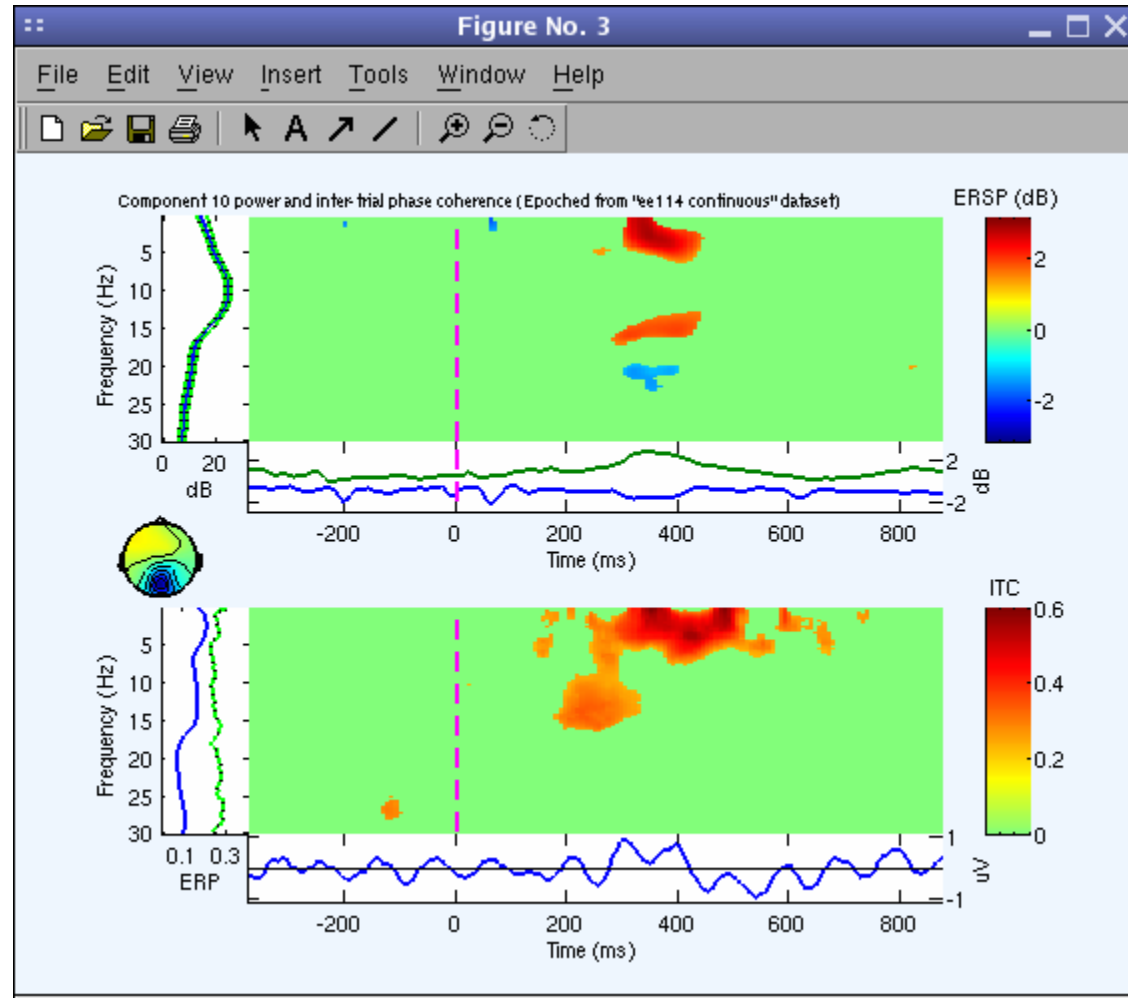
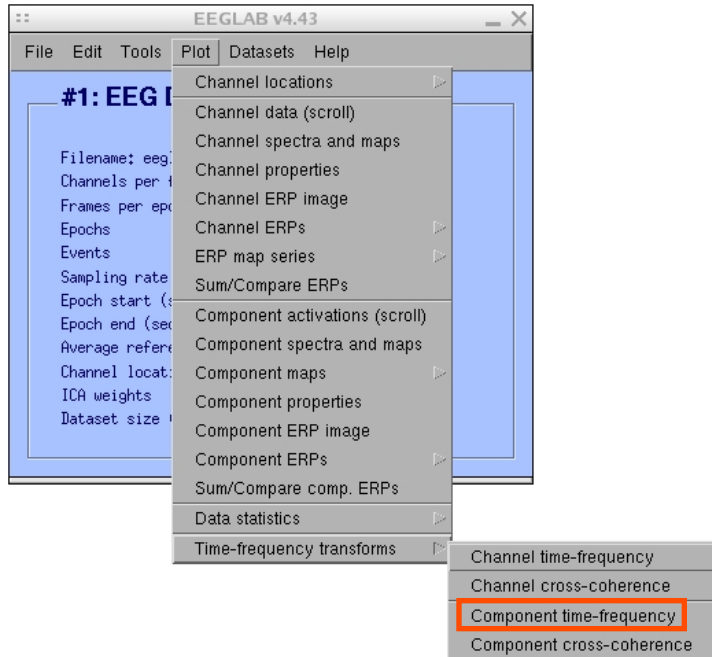
Plot component time frequency -- pop_newtimef()

Component number	1	
Sub epoch time limits [min max] (msec)	-1000 1996	Use 200 time points
Frequency limits [min max] (Hz) or sequence	1 2 3 4 5 6	Use limits, padding 1
Baseline limits [min max] (msec) (0->pre-stim.)	0	Use divisive baseline
Wavelet cycles [min max/fact] or sequence	1 2 3 4 5 6	Use limits
ERSP color limits [max] (min=-max)		<input checked="" type="checkbox"/> see log power (set)
ITC color limits [max]		<input type="checkbox"/> plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)		<input type="checkbox"/> FDR correct (set)
Optional newtimef() arguments (see Help)		

Plot Event Related Spectral Power Plot Inter Trial Coherence Plot curve at each frequency

Cancel Help Ok

Component time-frequency



Exercise

- **ALL**

Start EEGLAB, from the menu:

```
load <eeglab_root>/sample_data/  
eeglab_data_epochs_ica.set
```

or your own data

- **Novice**

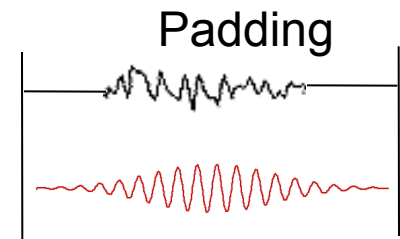
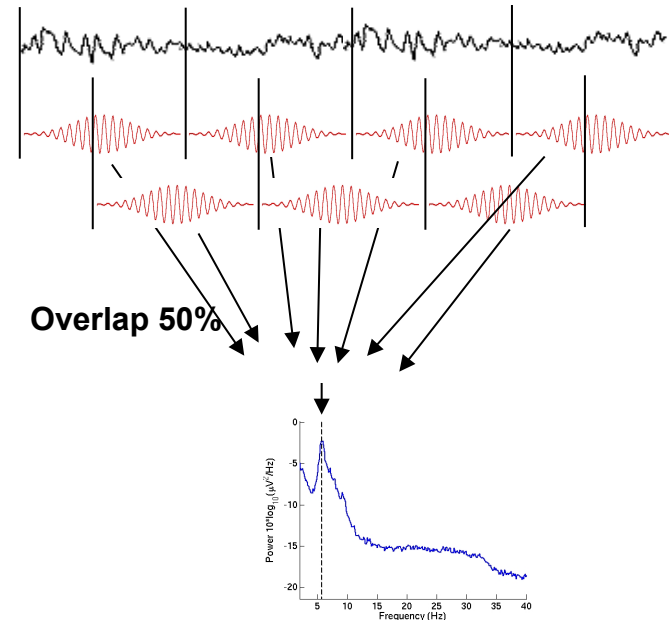
From the GUI, Plot spectral decomposition with 100% data and 50% overlap (‘overlap’). Try reducing window length (‘winsize’) and FFT length (‘nfft’)

- **Intermediate**

Same as novice but using a command line call to the *pop_spectopo()* function. Use GUI then history to see a standard call (“eegh”).

- **Advanced**

Same as novice but using a command line call to the *spectopo()* function.



Exercise - newtimef

- **Novice**

From the GUI, pick an interesting IC and plot component ERSP. Try changing parameters window size, number of wavelet cycles, padratio,

- **Intermediate**

From the command line, use `newtimef()` to tailor your time/frequency output to your liking. Look up the help to try not to remove the baseline, change baseline length and plot in log scale. Enter custom frequencies and cycles (2 slides back).

- **Advanced**

Compare FFT, the different wavelet methods (see help), and multi-taper methods (use `timef` function not `newtimef`). Enter custom frequencies and cycles. Look up `newtimef` help to compare conditions. Visualize single-trial time-frequency power using `erpimage`.