Time-frequency decomposition

Theory and Practice

EEGLAB Workshop XXI

Santa Margherita Ligure, Italy

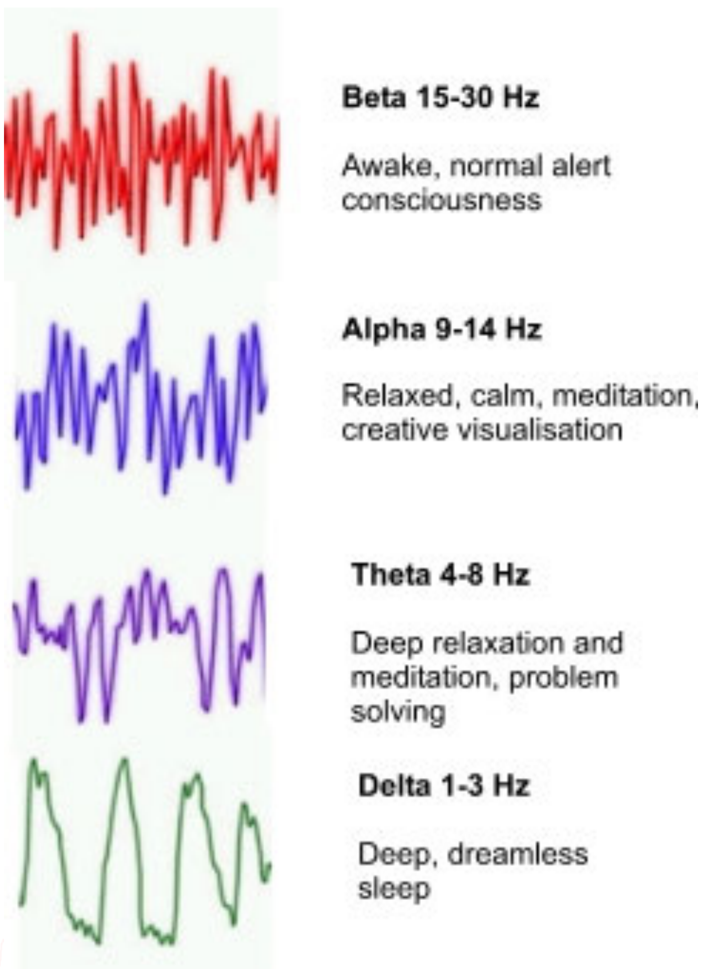
Day 1

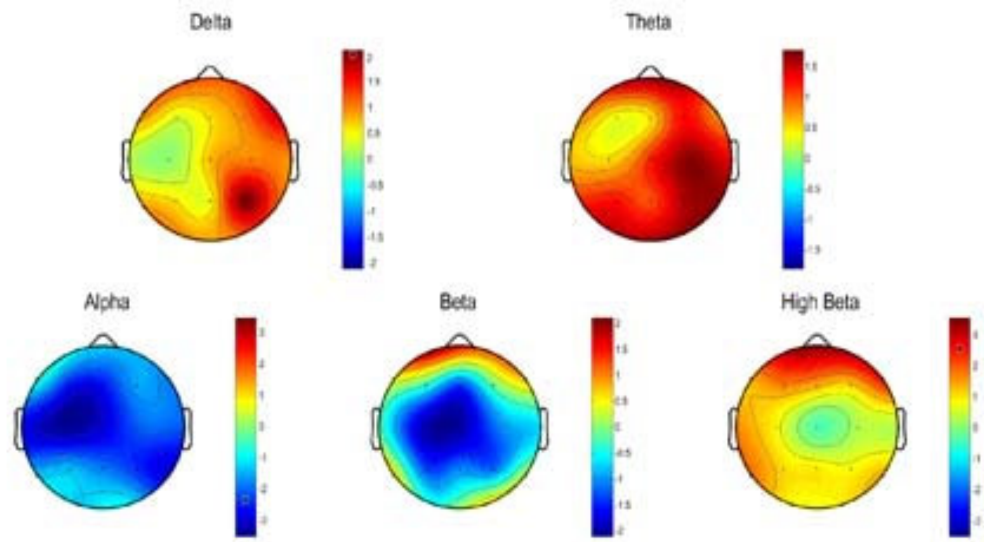
- Signals – EEG

- Goals
  - Describe dynamic characteristics of brain activity
  - Describe relation between different regions of brain

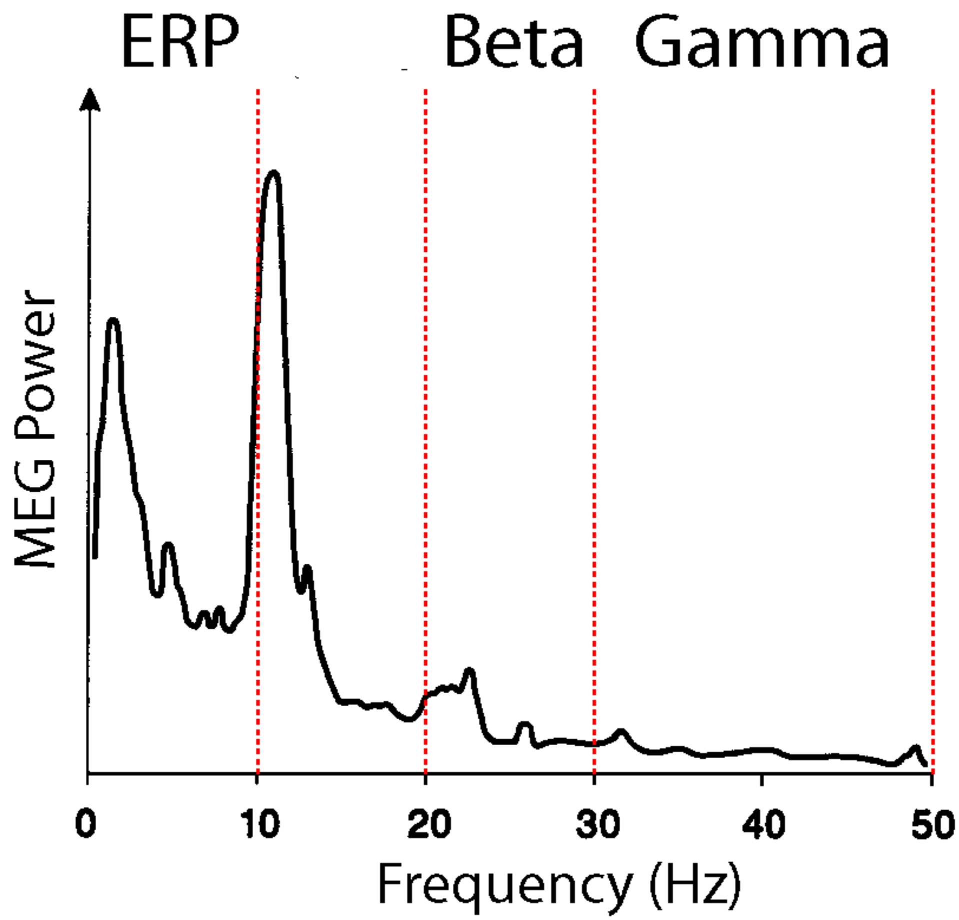
- Approaches
  - Time domain
  - Frequency domain
  - Time/Frequency

Different meanings traditionally given to different frequency bands

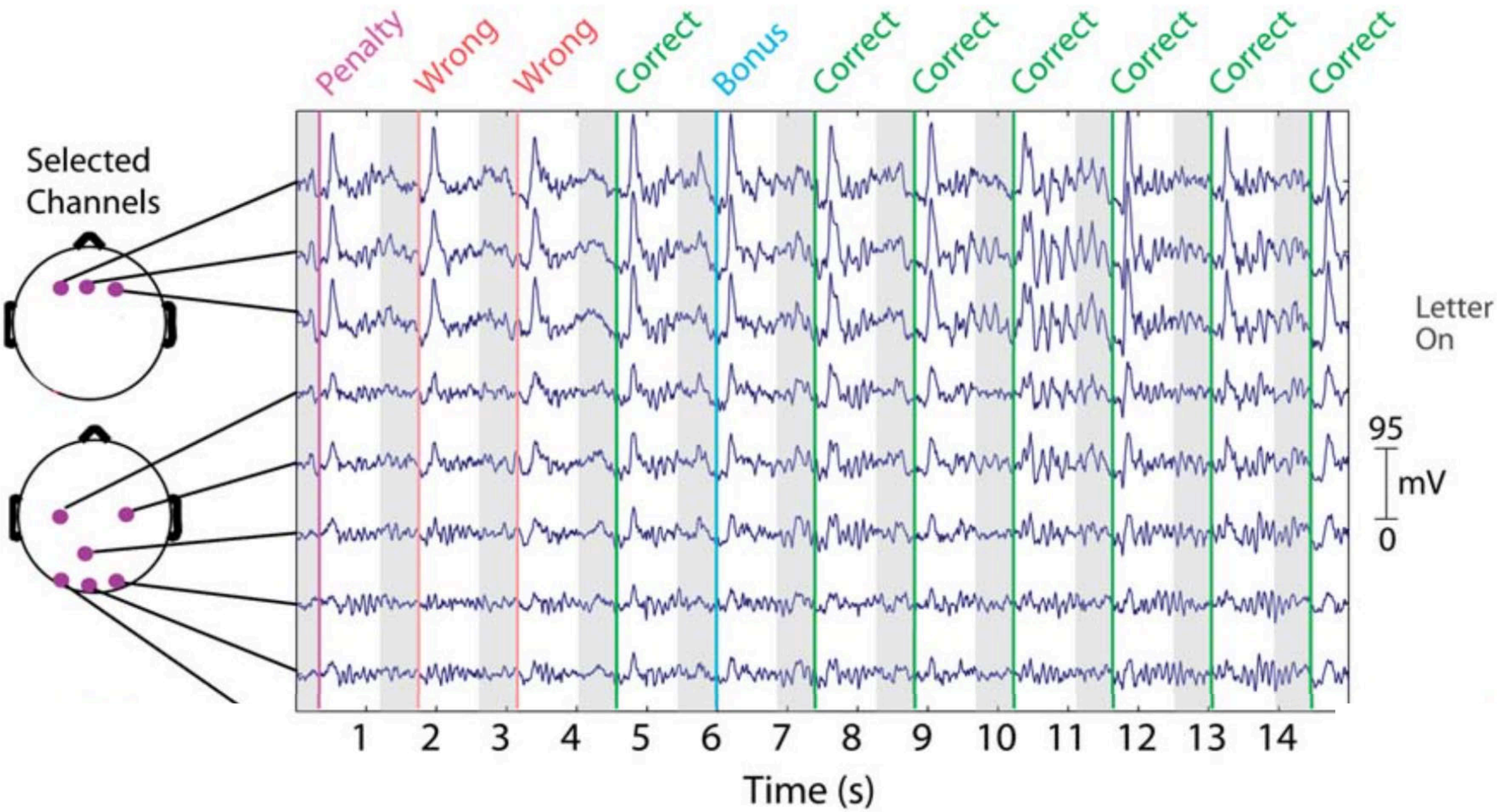




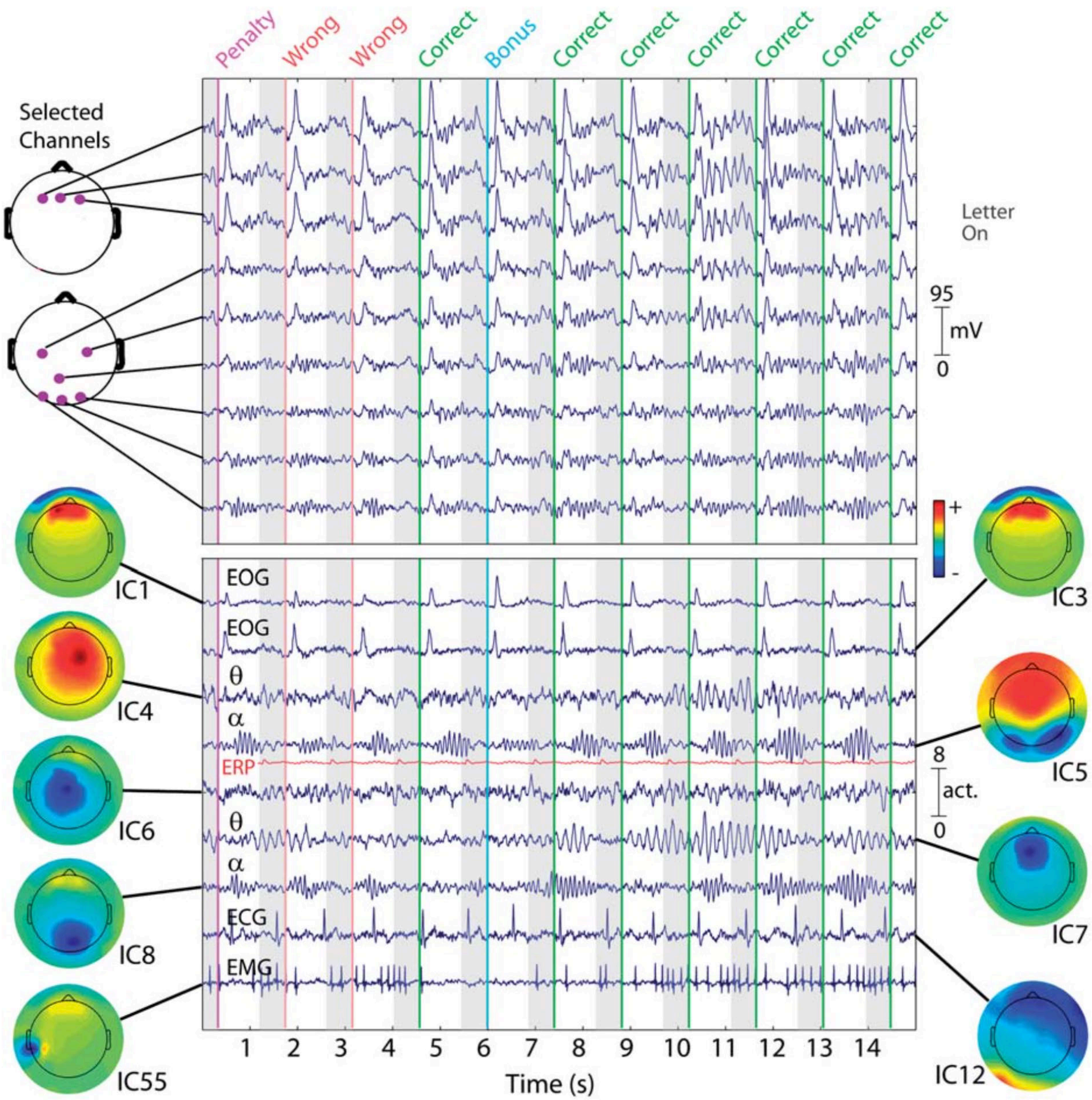
MEEG spectrum



Time varying frequency content

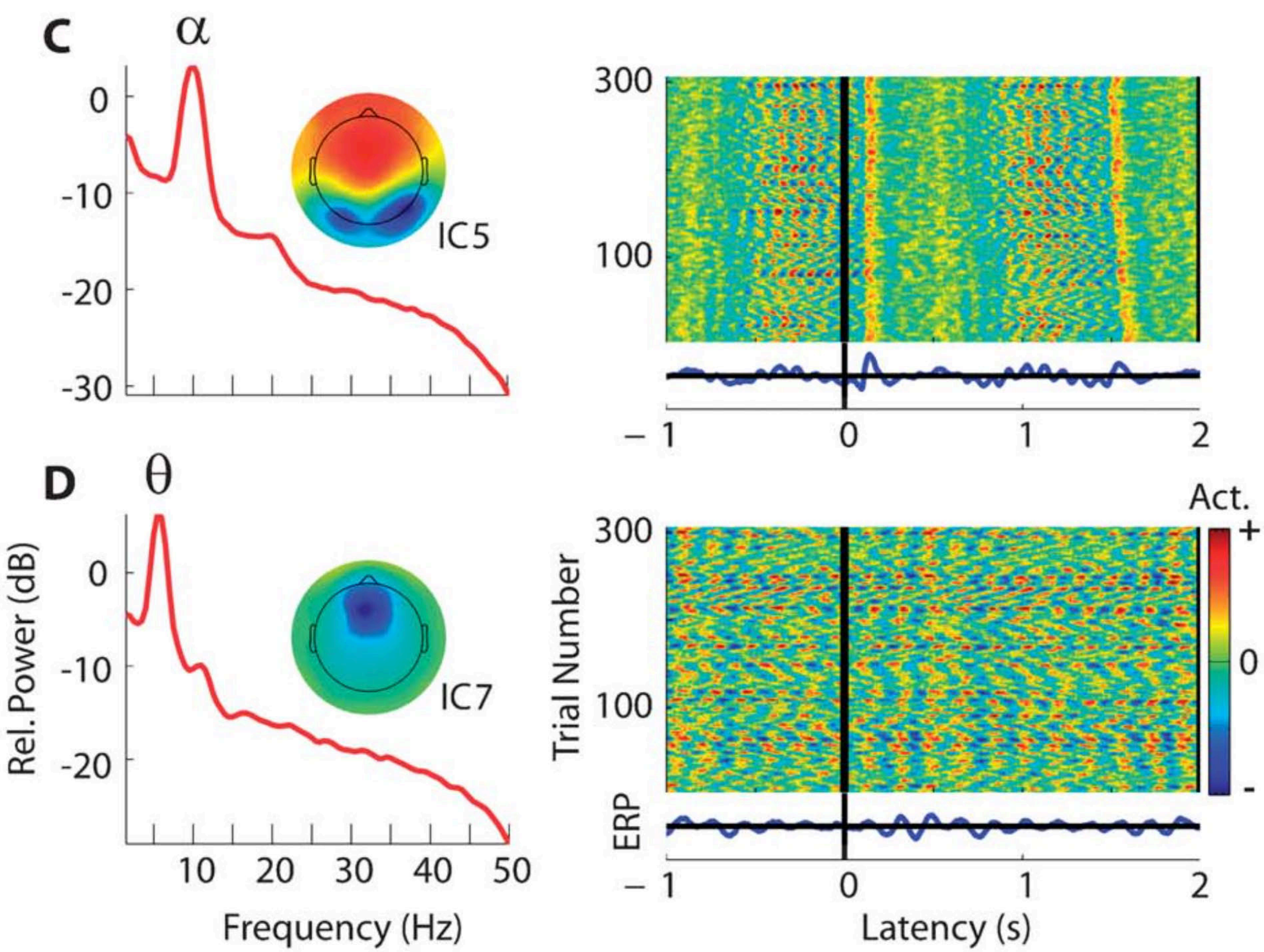


Time-varying frequency content

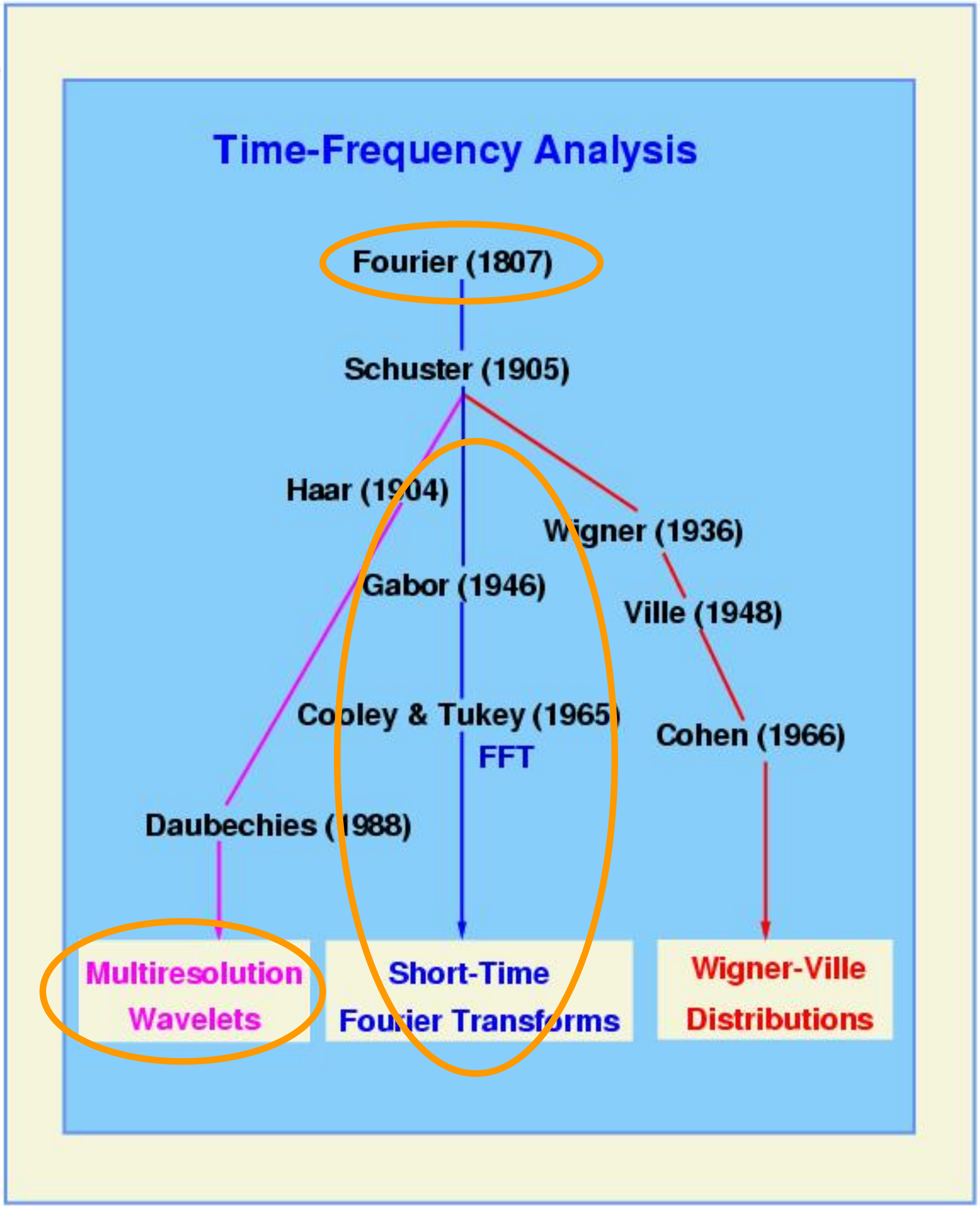


Onton & Makeig, 2006

Power Spectrum does not describe temporal variation



Onton & Makeig, 2006



S. Makeig, 2005

Plan

- **Part 1: Frequency Analysis**
  - Power Spectrum
    - Approaches
      - FFT
      - Welch's Method
    - Windowing

- **Part 2: Time-Frequency Analysis**
  - Short Time Fourier Transform
  - Wavelet Transform
  - ERSP

- **Part 3: Coherence Analysis**
  - Inter-Trial Coherence
  - Event-Related Coherence

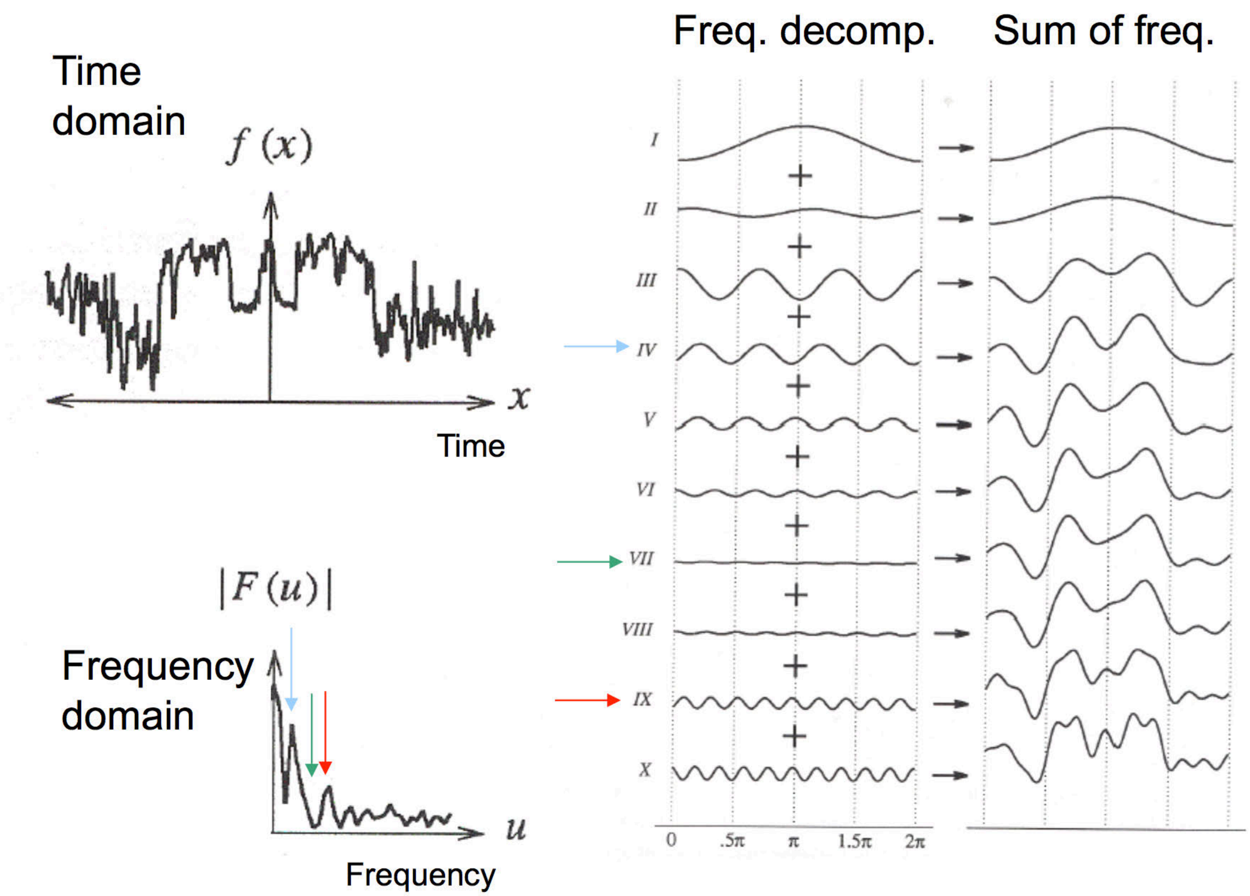
Part 1: Frequency Analysis

- Goal: What frequencies are present in signal?

- What is power at each frequency?

- Principle: Fourier Analysis

Fourier Analysis

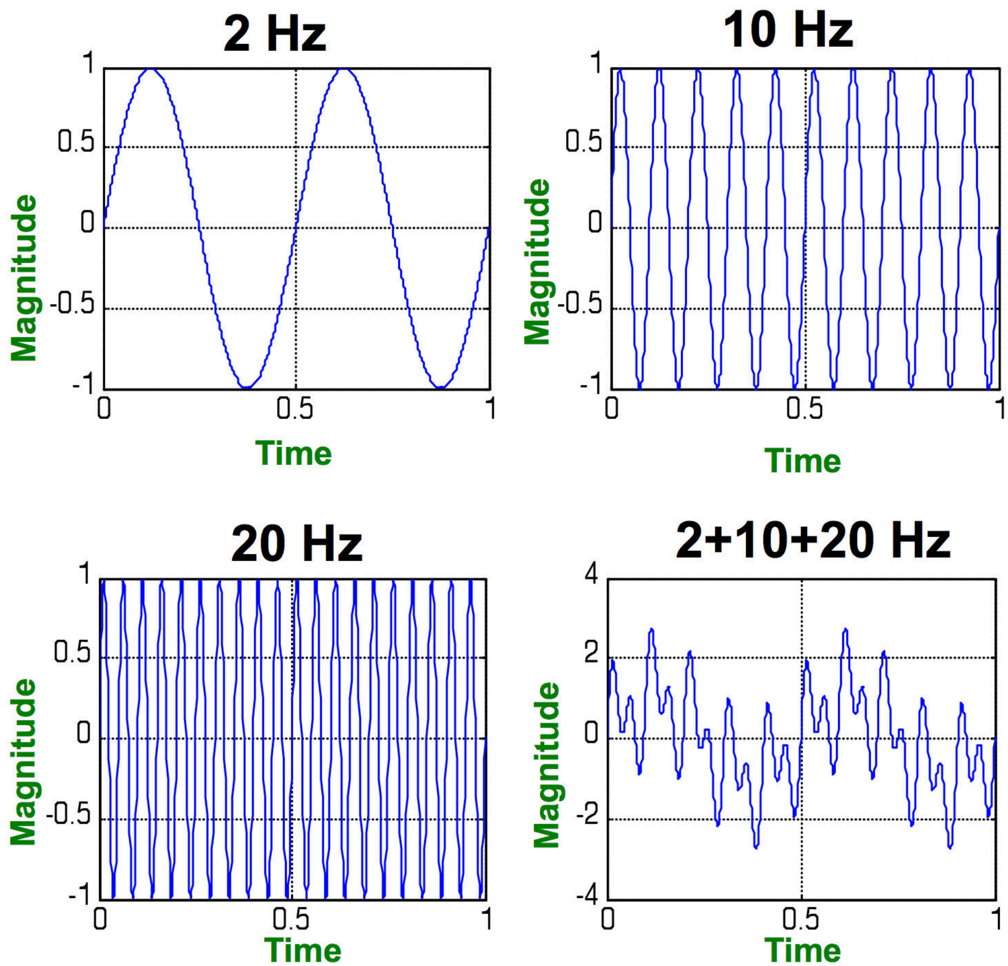


Forward transform
$$F(u) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi iux} dx$$

Inverse transform
$$f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} du$$

Figure, courtesy of Ravi Ramamoorthi & Wolberg

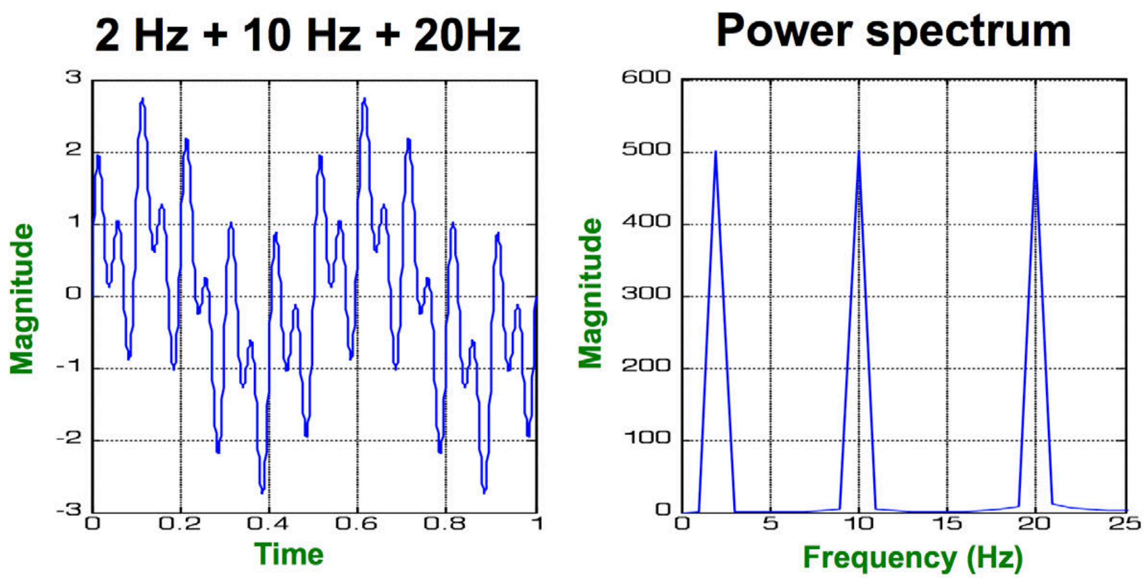
“Stationary” sinusoidal signals



Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Simplest case of frequency analysis

Stationary



By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2,10,20Hz respectively).

Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Power Spectrum. Approach 1: FFT

- Why not just take FFT of our signal of interest?
- Advantage – fine frequency resolution
  - $\Delta F = 1$ / signal duration (s)
  - E.g. 100s signal has 0.01 Hz resolution
  - But, do we really need this?

- Disadvantage – bias and variance
  - Solution: e.g. Welch's method

- Disadvantage – no temporal resolution
  - Solution 1: Short-Time Fourier Transform

Amplitude and phase

- Power spectra describe the *amount* of a given frequency present

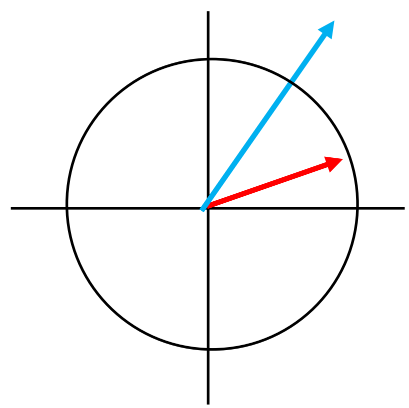
- NOT a complete description of a signal: We also must know the *phase* at each frequency

- FFT/STFT/Wavelet return an amplitude and phase at each time and frequency (represented as complex #).
- To find power, we compute the magnitude, which discards phase.

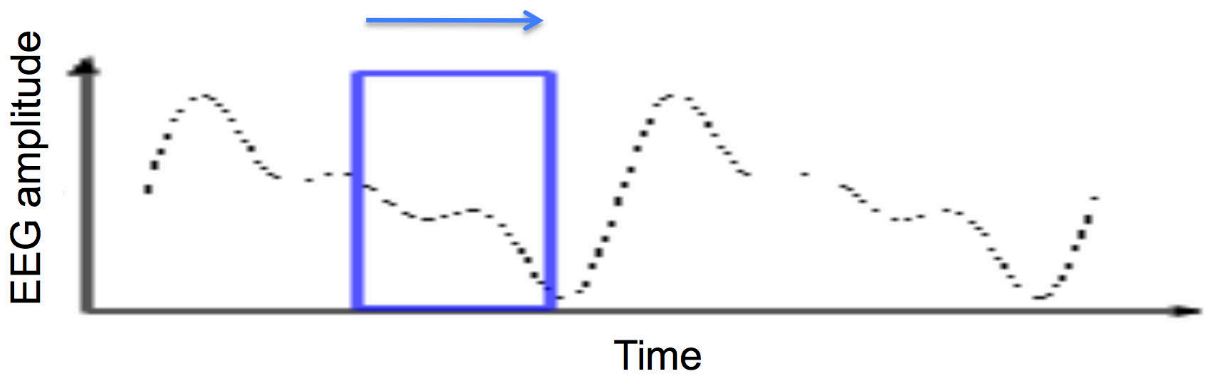
Phasor representation

- A complex number $x + yi$ can be expressed in terms of amplitude and phase: $ae^{i\theta}$

```
amplitude*exp(i*phase)
amplitude = sqrt(x^2 + y^2); phase = atan(y/x);
```



Approach 2: Welch's Method



Calculate power spectrum of short windows, average.

Advantage: Smoother estimate of power spectrum

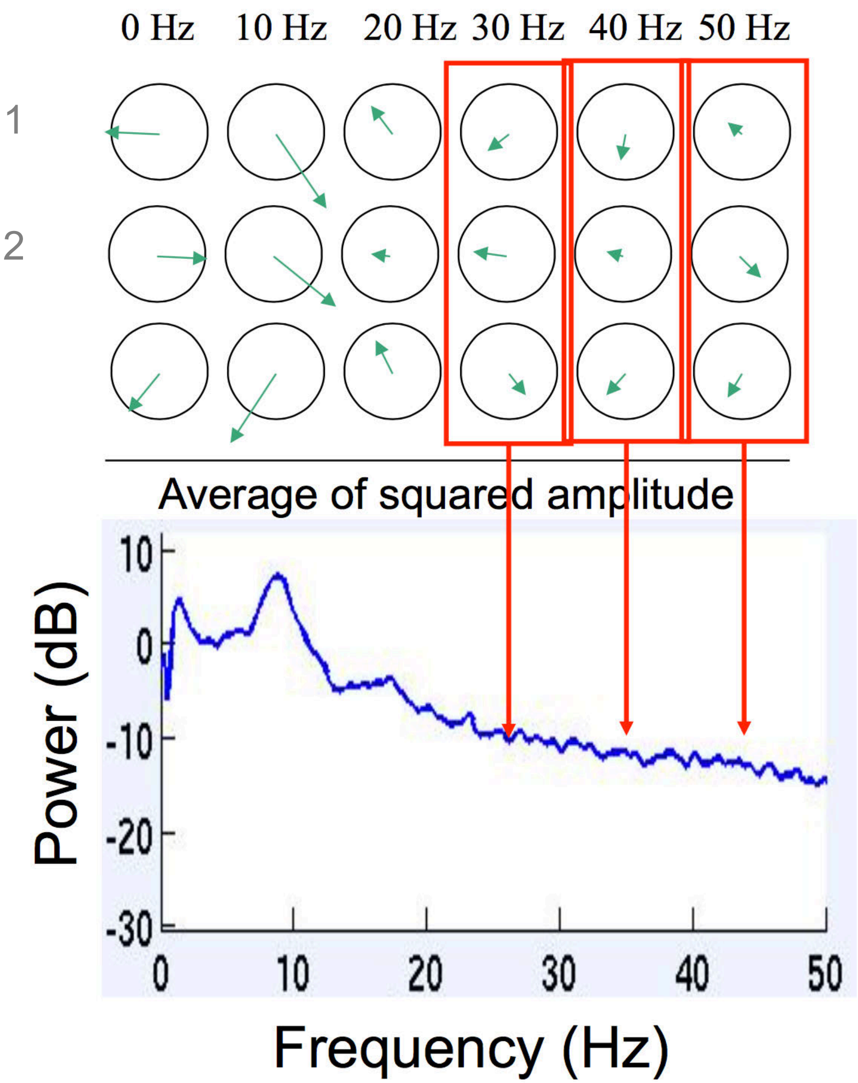
Frequency resolution set by window length

e.g. 1s window -> 1 Hz resolution

In practice: taper, don't use rectangular window

FFT of window 1

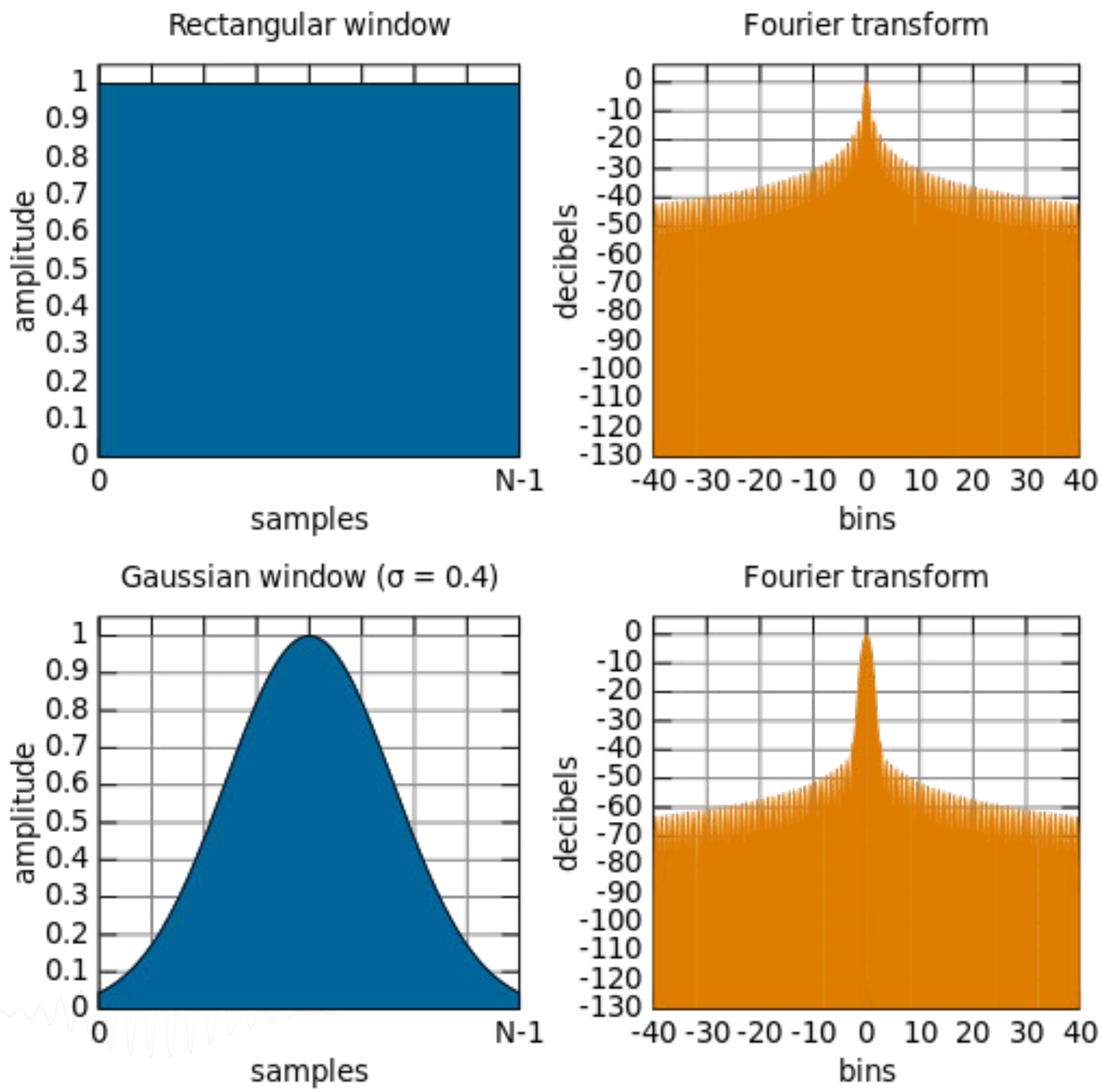
FFT of window 2



Windowing

- When we pick a short segment of signal, we typically window it with a smooth function.
- Windowing in time = convolving (filtering) the spectrum with the Fourier transform of the window
- No window (=rectangular window) results in the most smearing of the spectrum
- There are many other windows optimized for different purposes: Hamming, Gaussian…

Windows and their Fourier transforms



Narrowest main peak, but

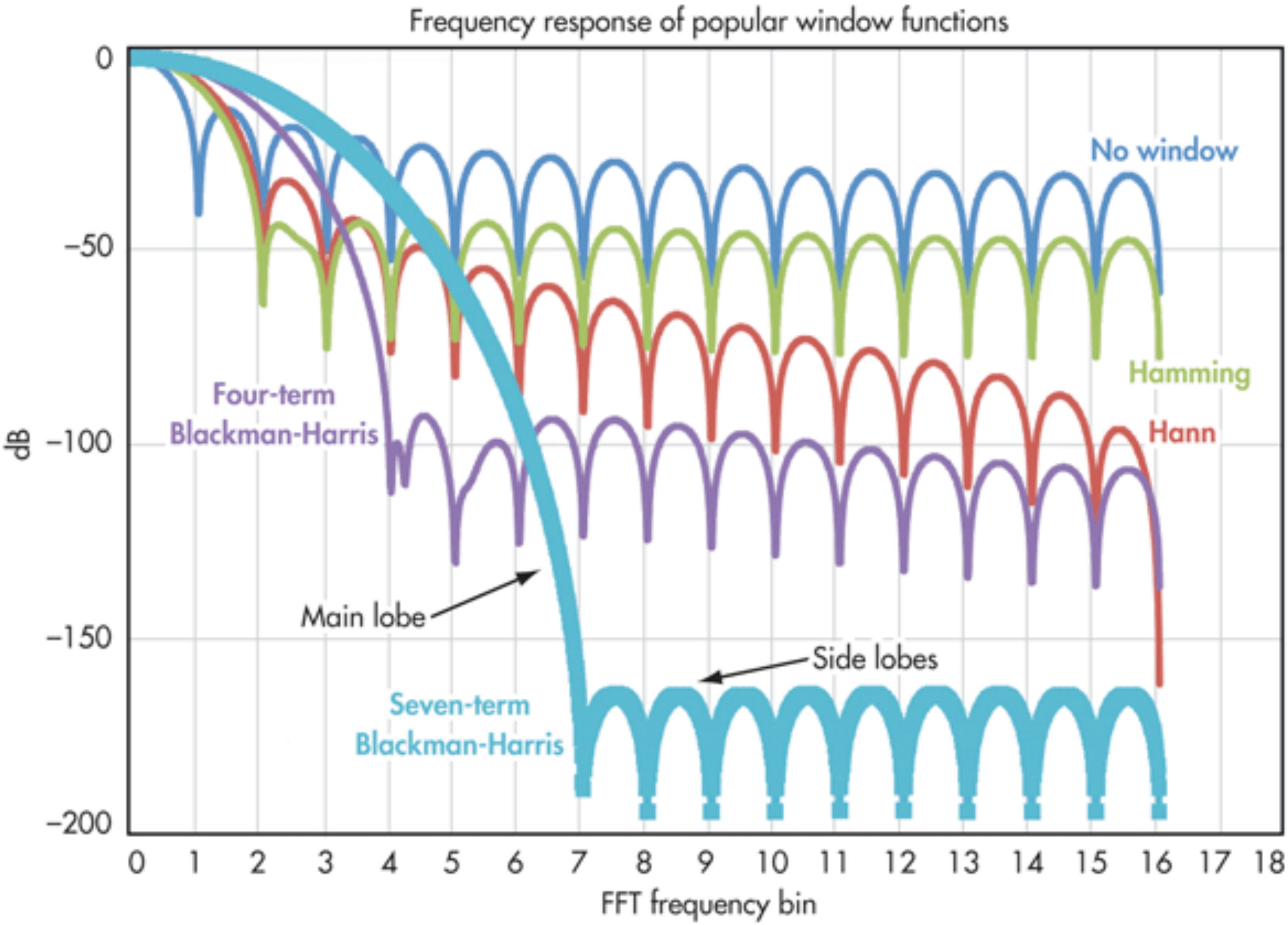
Highest side-lobes

Most spectral 'smearing'

Wider main peak, and

much lower side-lobes

Close-up view



Part 2: Time-Frequency Analysis

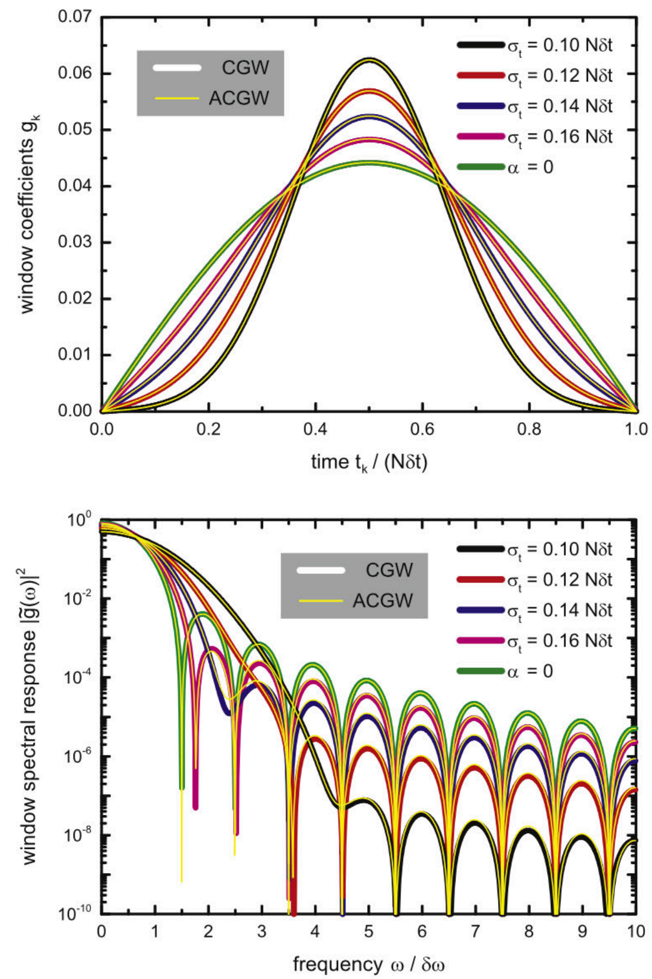
- Short-Time Fourier Transform
  - Find power spectrum of short windows
  - "Spectrogram"
- Advantage: Can visualize time-varying frequency content
- Disadvantage: Fixed temporal resolution is not optimal

Time-Frequency Uncertainty

- You cannot have both arbitrarily good temporal and frequency resolution!
  - **$\sigma_t * \sigma_f \geq 1/2$**

- If you want sharper temporal resolution, you will sacrifice frequency resolution, and vice versa.

- (Optimal: Confined Gaussian)

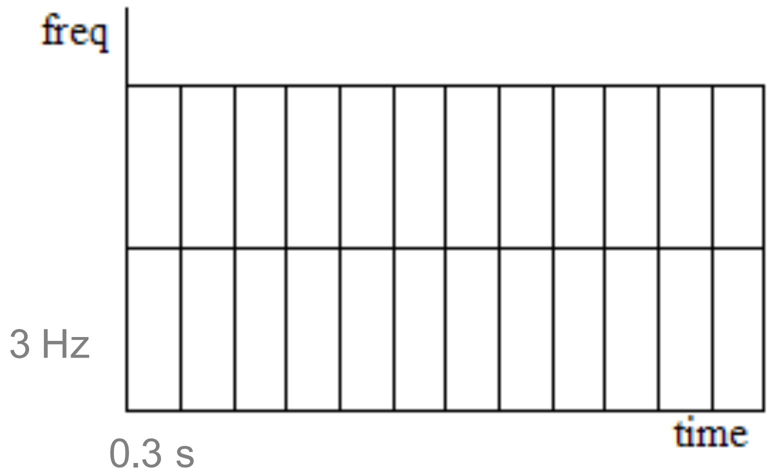


Starosielec S, Hägele D (2014) Discrete-time windows with minimal RMS bandwidth for given RMS temporal width. Signal Processing 102:240–6.

Consequence for STFT

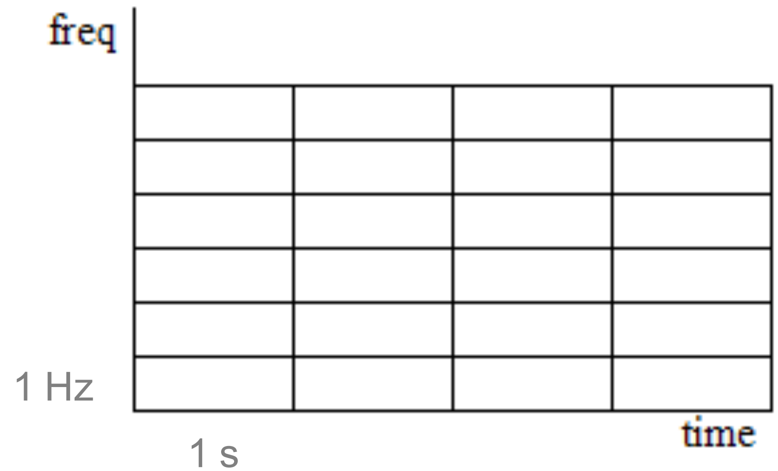
Shorter Windows

poorer frequency resolution



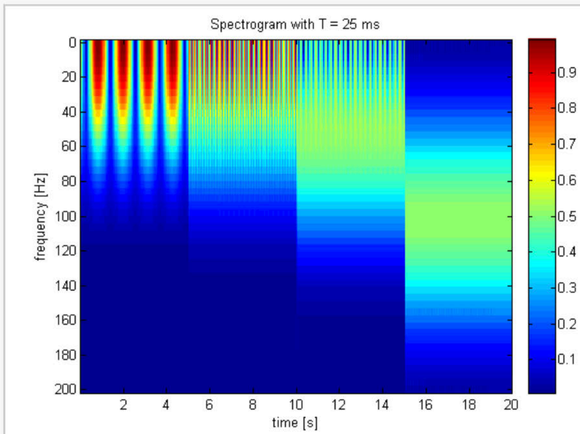
Longer Windows

finer frequency resolution

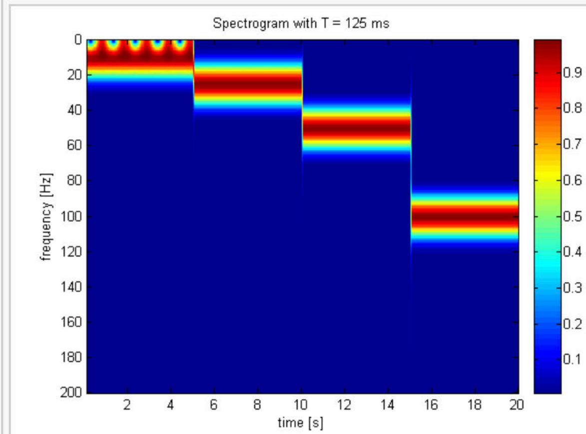


Time-Frequency Tradeoff

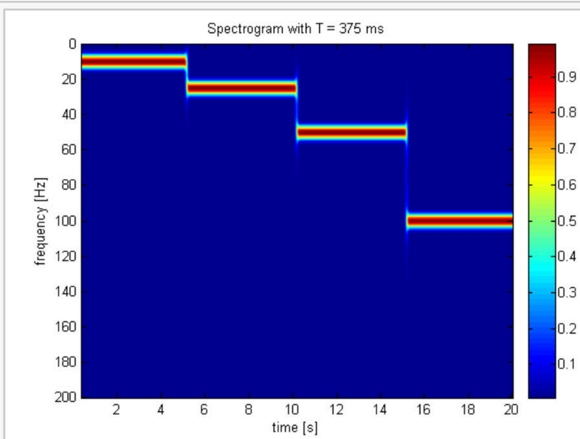
Signal: 10, 25, 50, 100 Hz



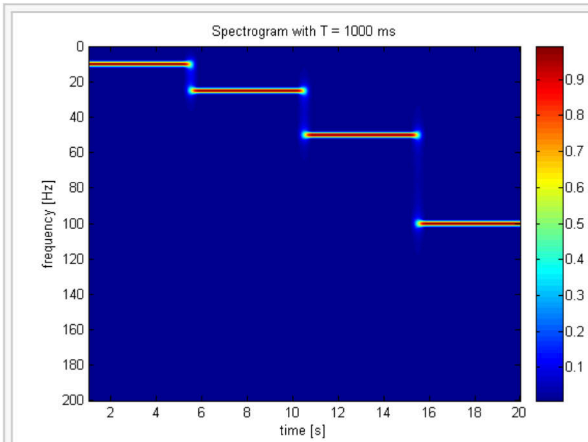
25 ms window



125 ms window



375 ms window

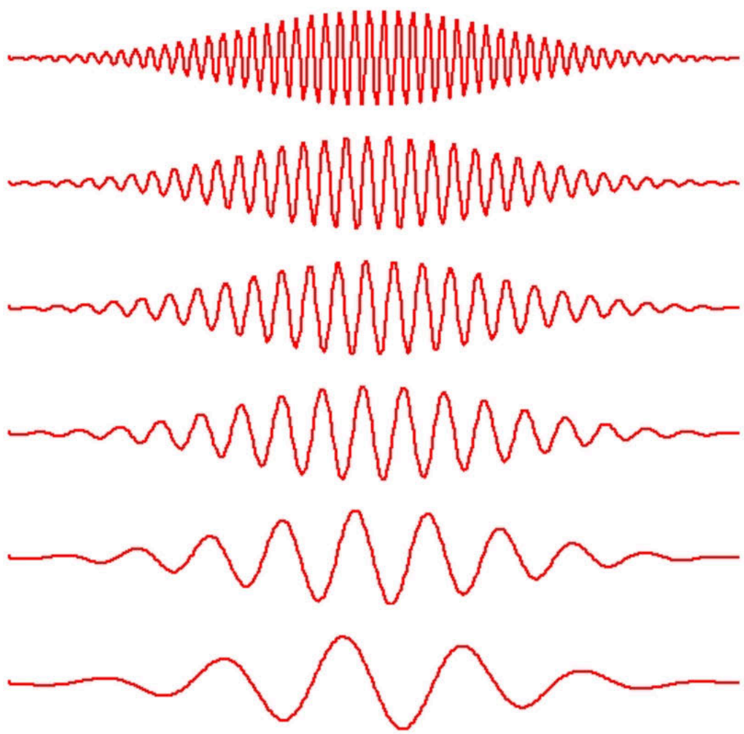


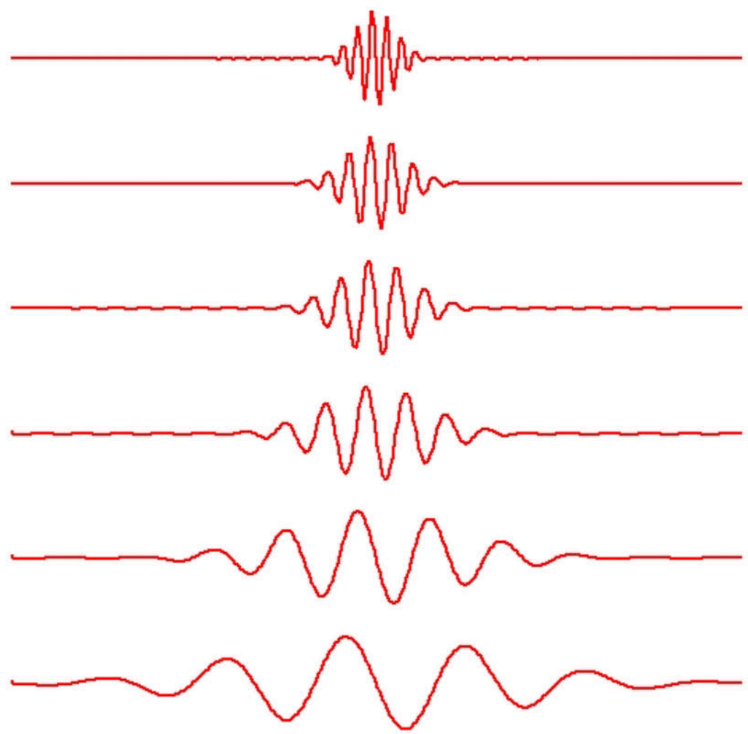
1000 ms window

A better way: Wavelet transform

- Wavelet transform is a ‘multi-resolution’ time-frequency decomposition.
- Intuition: Higher frequency signals have a faster time scale
- So, vary window length with frequency!
  - longer window at lower frequencies
  - shorter window at higher frequencies

Comparison of FFT & Wavelet





Scaled versions of one shape

Constant number of cycles*

FFT

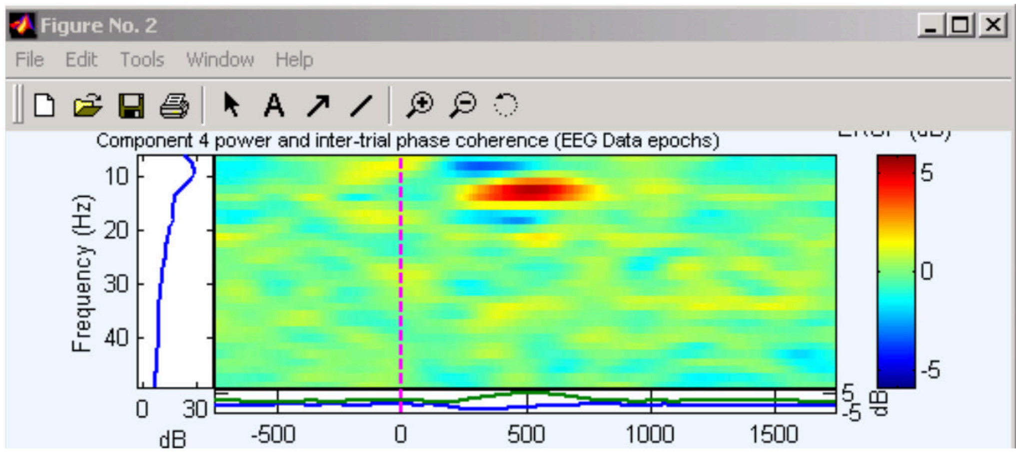


Figure No. 2
File Edit Tools Window Help
Component 4 power and inter-trial phase coherence (EEG Data epochs)
Frequency (Hz)
10
20
30
40
0 30
dB
-500 0 500 1000 1500
5
0
-5
dB

Wavelet

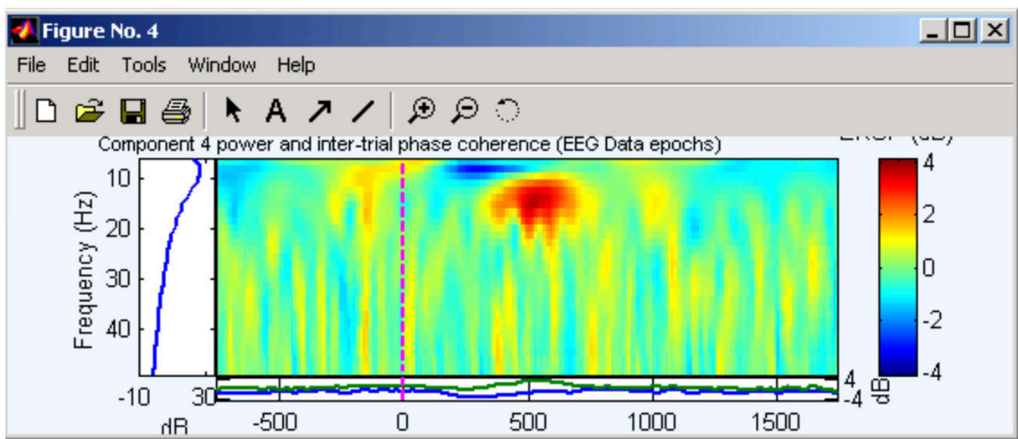
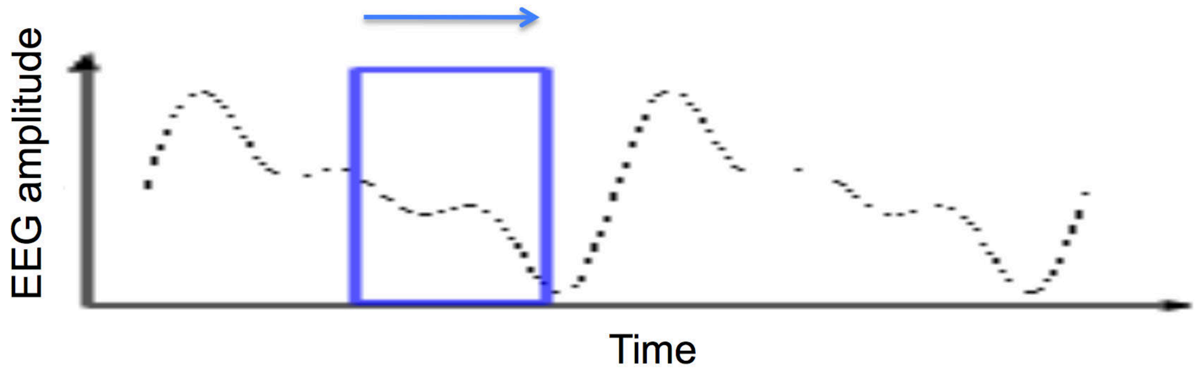
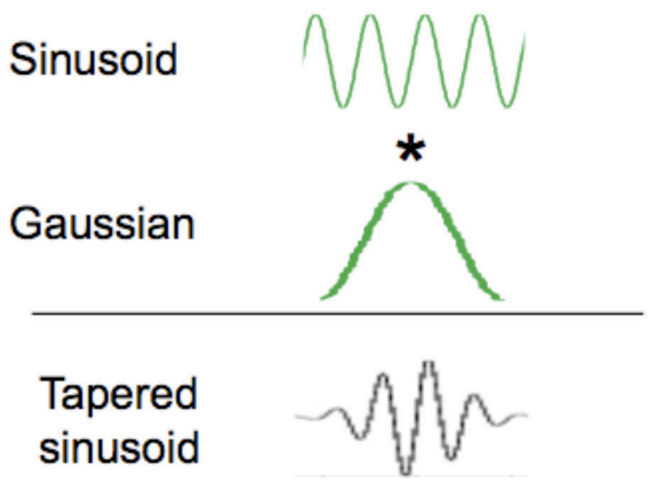


Figure No. 4
File Edit Tools Window Help
Component 4 power and inter-trial phase coherence (EEG Data epochs)
Frequency (Hz)
10
20
30
40
-10 30
-500 0 500 1000 1500
4
2
0
-2
-4
dB

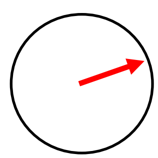




For each time window

Analyze signal using the wavelets

for different frequencies.



Exercise

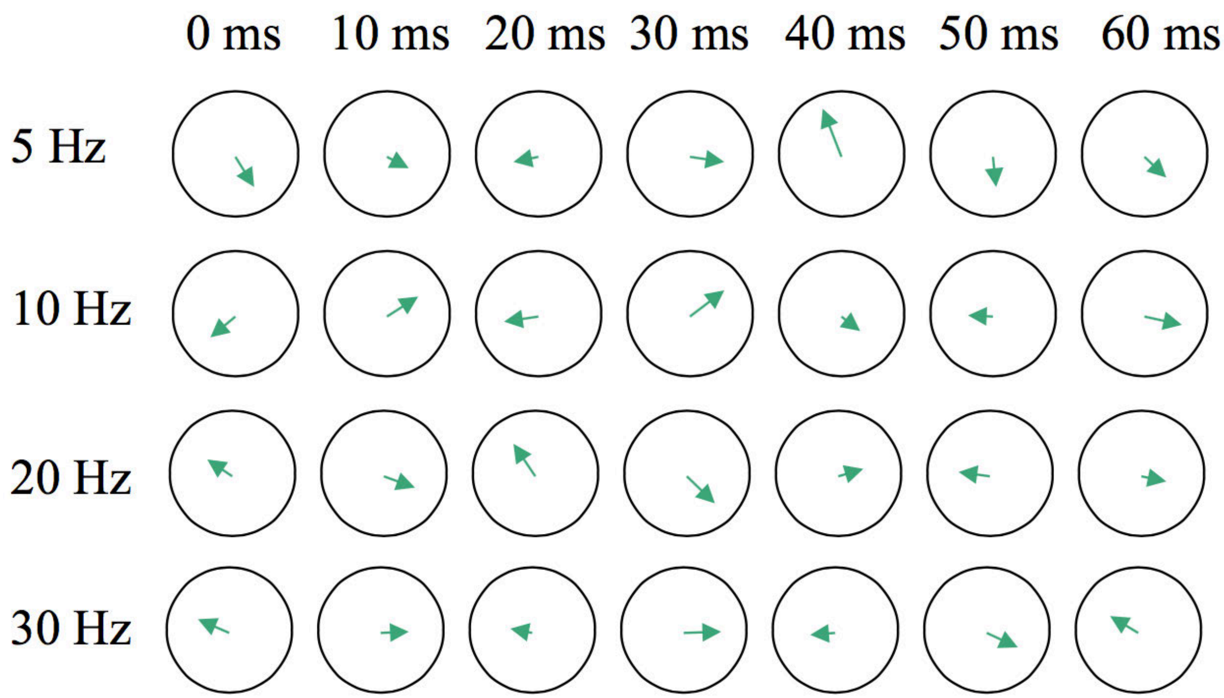
- Create a signal

```
>> t = 0:0.01:100;
>> x = sin(2*pi*10*t); plot(t,x)
```

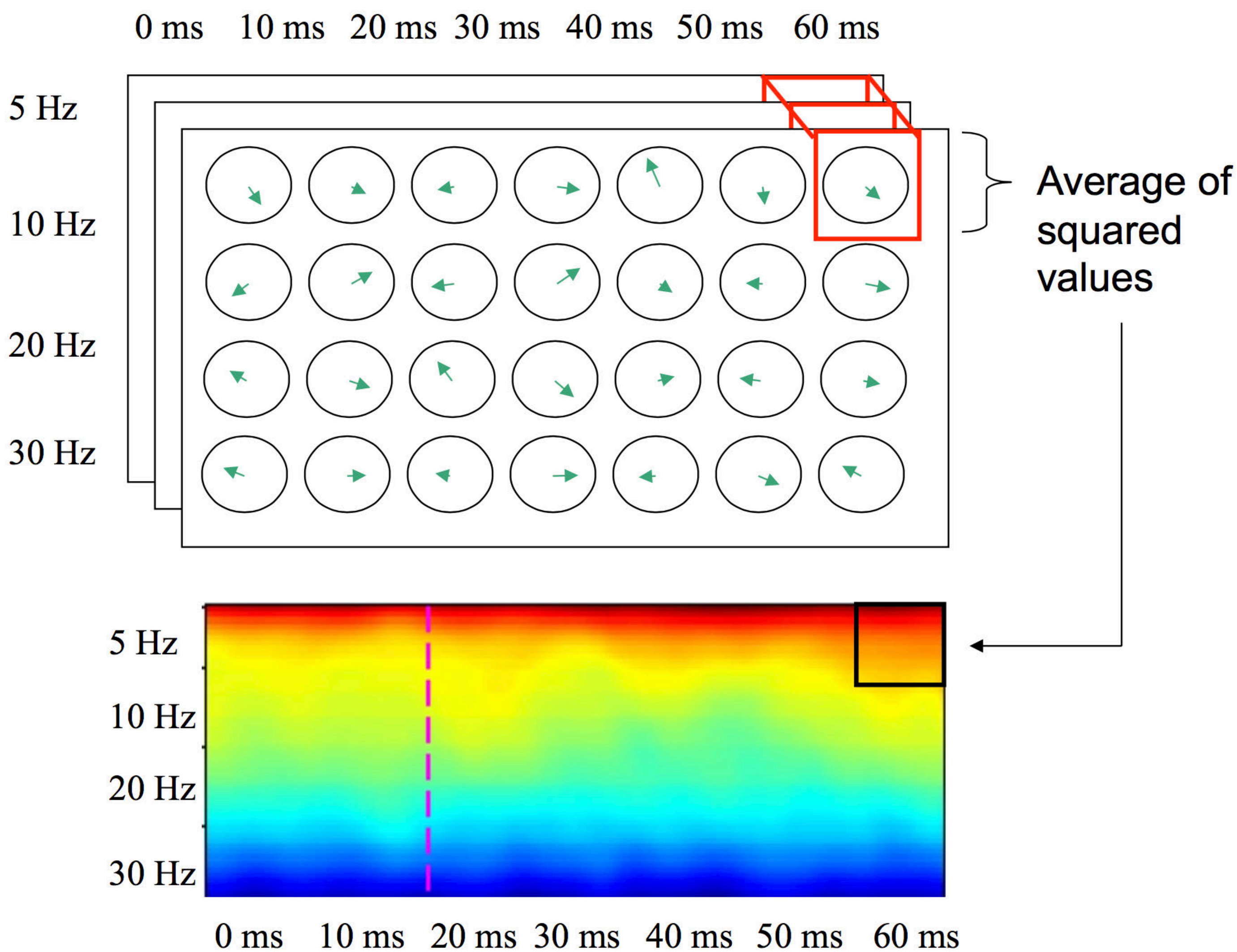
- Find FFT

```
>> F = fft(x);
>> F(1:3) %complex
>> power = F.*conj(F);
```

Spectrogram of one window of data

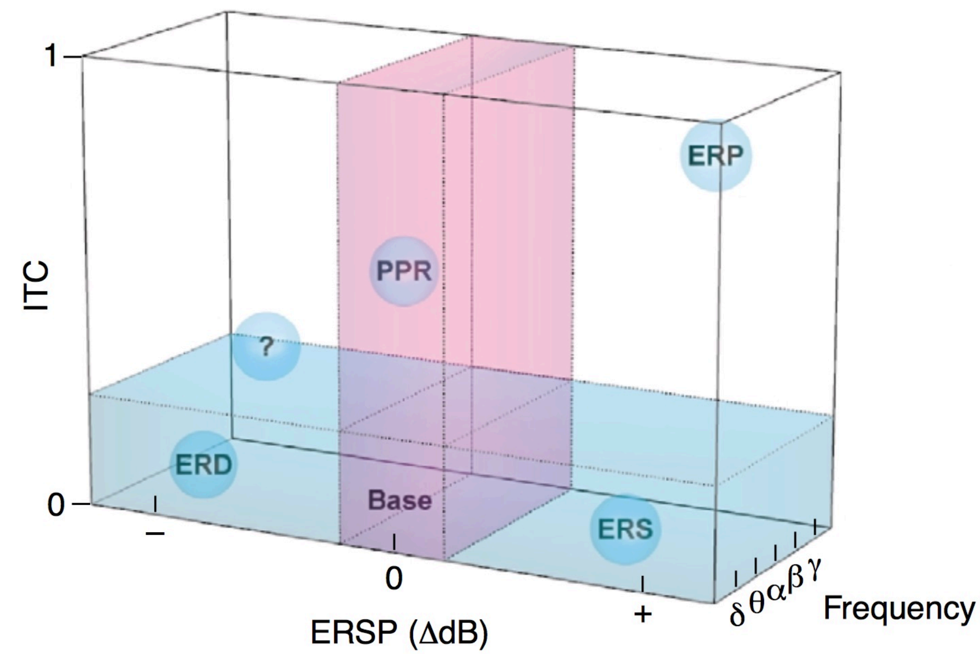


Computing Spectrogram Power

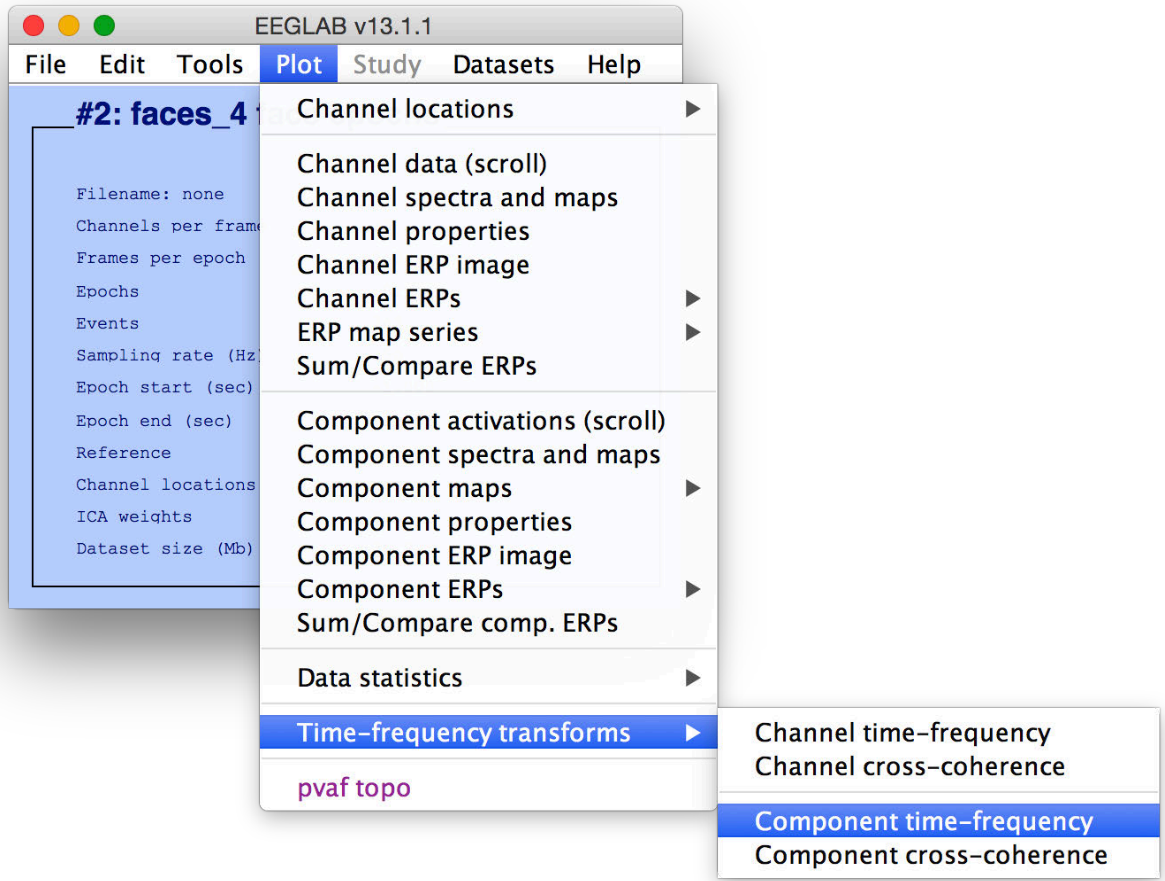


Definition: ERSP

- Event Related Spectral Perturbation
- Change in power in different frequency bands relative to a baseline. ERS , ERD

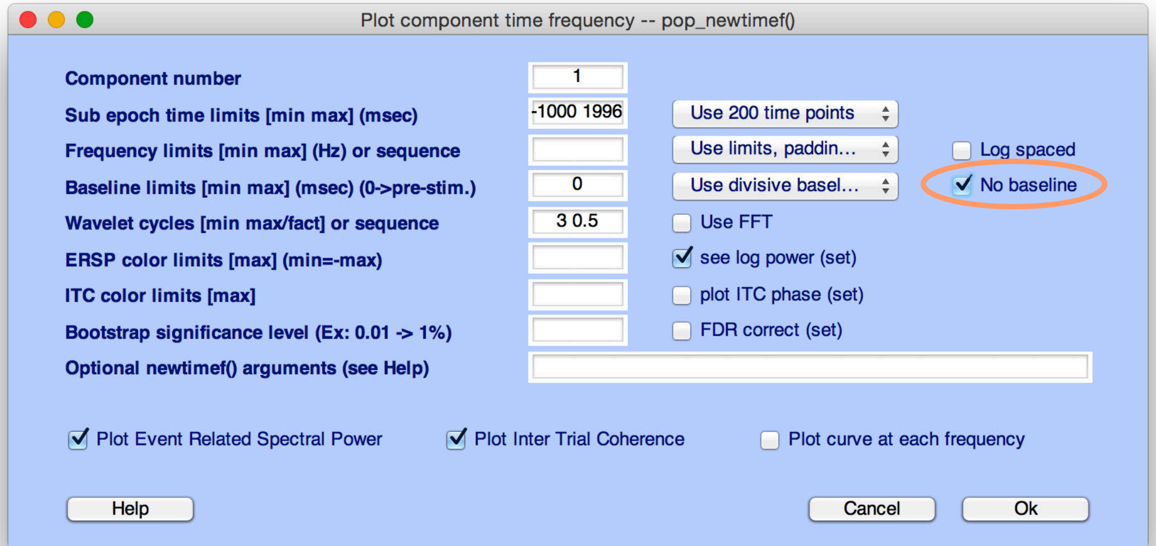


Try it out (faces_4.set)

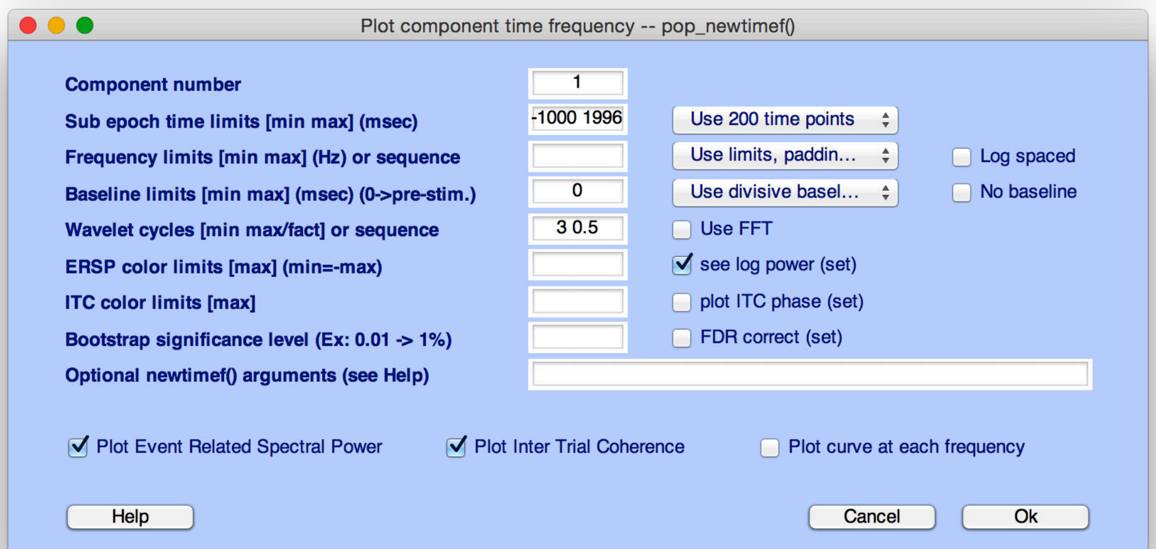


Display ERS vs. ERSP

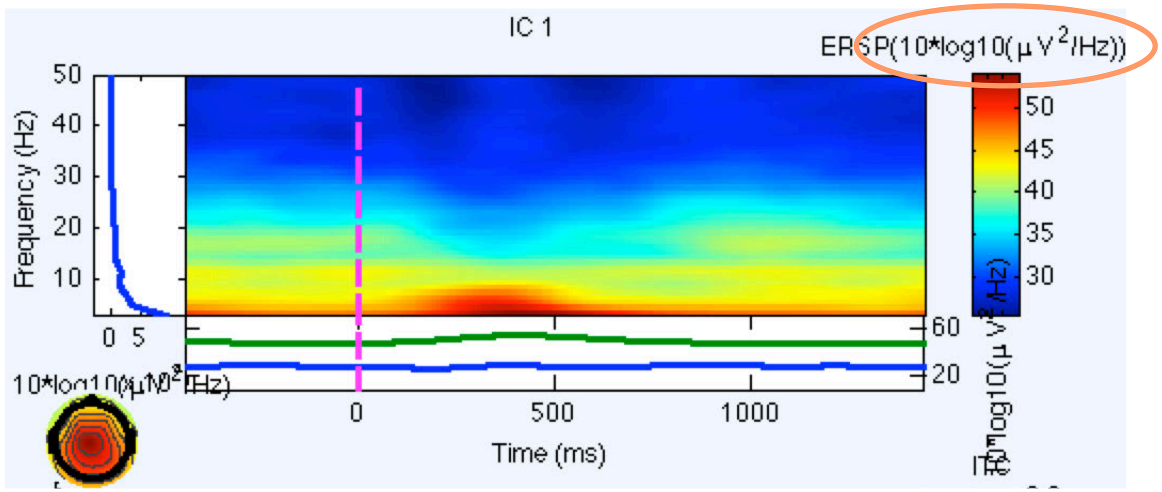
Event-related Spectrogram (ERS)



Event-Related Spectral Perturbation (ERSP)

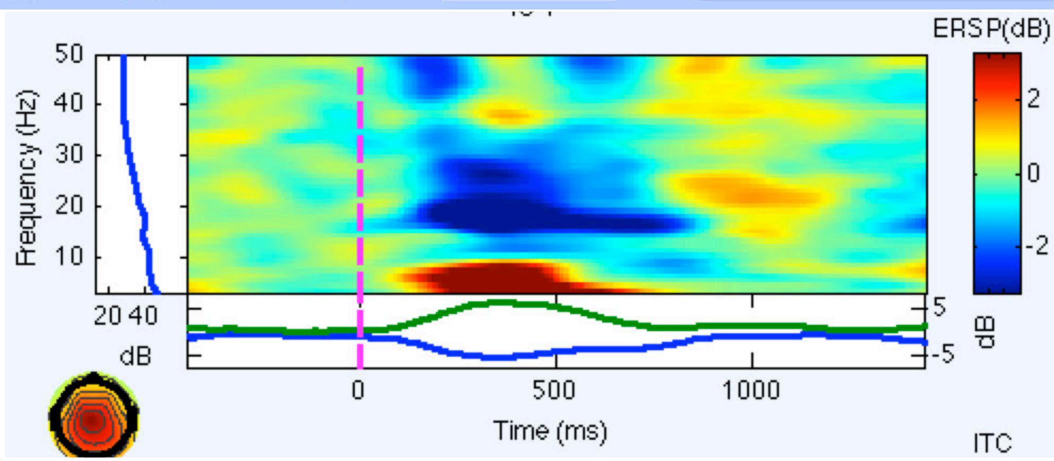


Event-related Spectrogram (ERS)



Baseline limits [min max] (msec) (0->pre-stim.) | 0 | Use divisive basel... | No baseline

Event-Related Spectral Perturbation (ERSP)



$10*\log_{10}(\ SG(t,f) / \text{baseline}(f)\ )$

Exercises

- Try different baseline methods
  - divisive
  - standard deviation (express spectral perturbations in #sd relative to baseline sd)

- Try different wavelet specifications

  Wavelet cycles [min max/fact] or sequence 3 0.5

  - Default: 3 0.5
    - 3 cycles
    - What is the 0.5? Try 0. Try 1…

Wavelet Specification

Wavelet cycles [min max/fact] or sequence 3 0.5

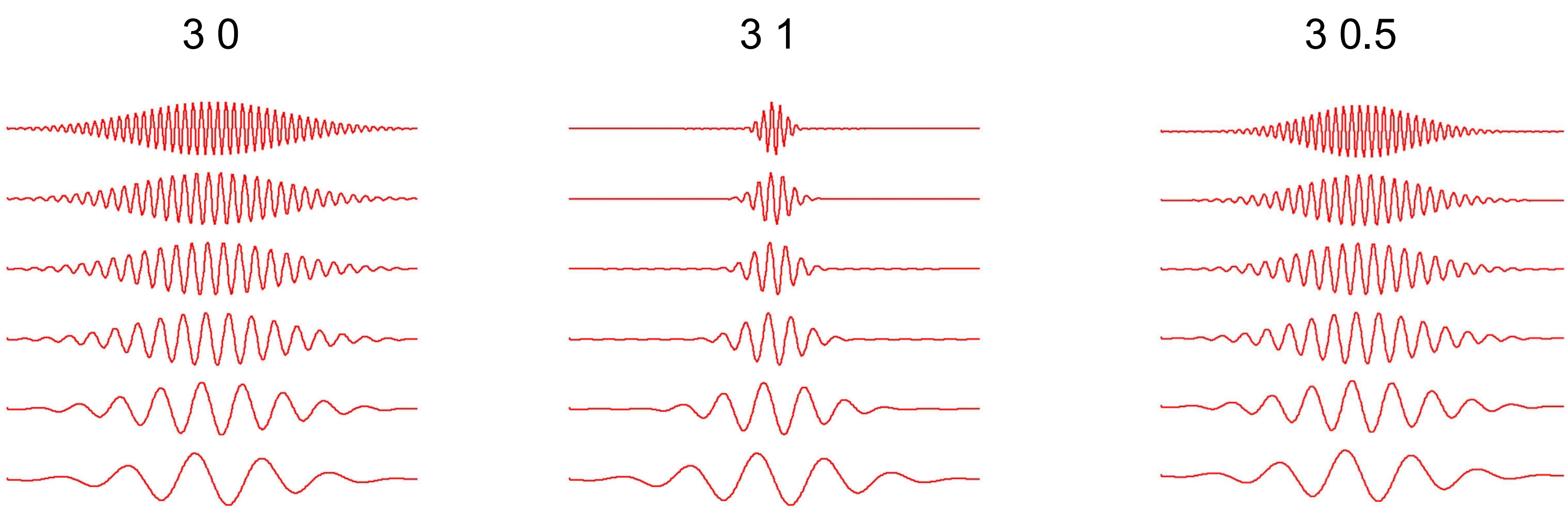
Answer: The first #cycles controls the basic duration of the wavelet in cycles.

The second factor controls the degree of shortening of time windows as frequency increases

0 = no shortening = FFT (duration remains constant with frequency)

1 = pure wavelet (#cycles remains constant with frequency)

0.5 = intermediate, a compromise that reduces HF time resolution to gain more frequency resolution



Part 3: Coherence Analysis

- Goal: How much do two signals resemble each other

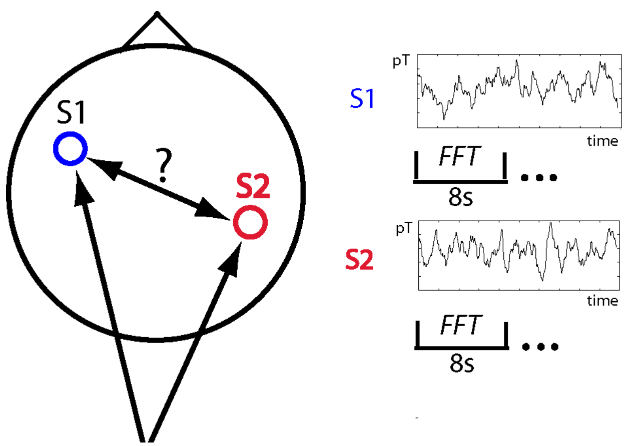
- Coherence = complex version of correlation: how similar are power and phase at each frequency?

- Variant: phase coherence (phase locking, etc.) considers only phase similarity, ignoring power
  - Regular coherence is simply a power-weighted phase coherence

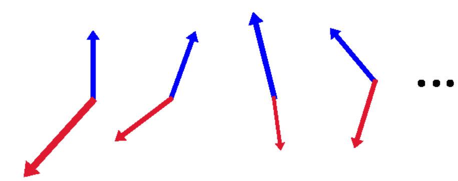
Coherence

$$C(f,t) \propto \sum_{k=trials} F1_k(f,t)\overline{F2_k(f,t)}$$

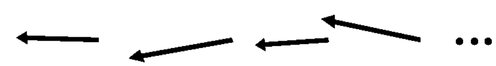
$$a_1 e^{i\theta_1} a_2 e^{-i\theta_2} \propto e^{i(\theta_1-\theta_2)}$$

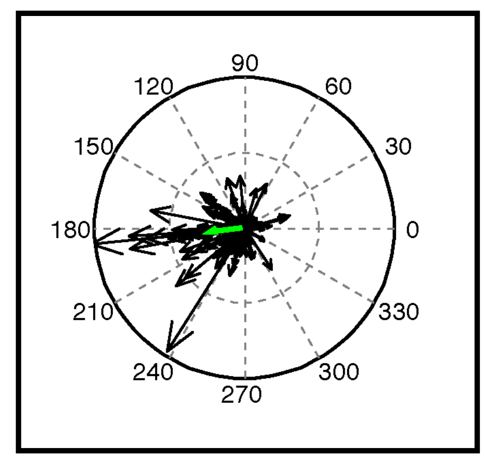


Fourier time series F_{S1} and F_{S2}



Phase difference between S1 and S2,





Part 3a: Inter-Trial Coherence

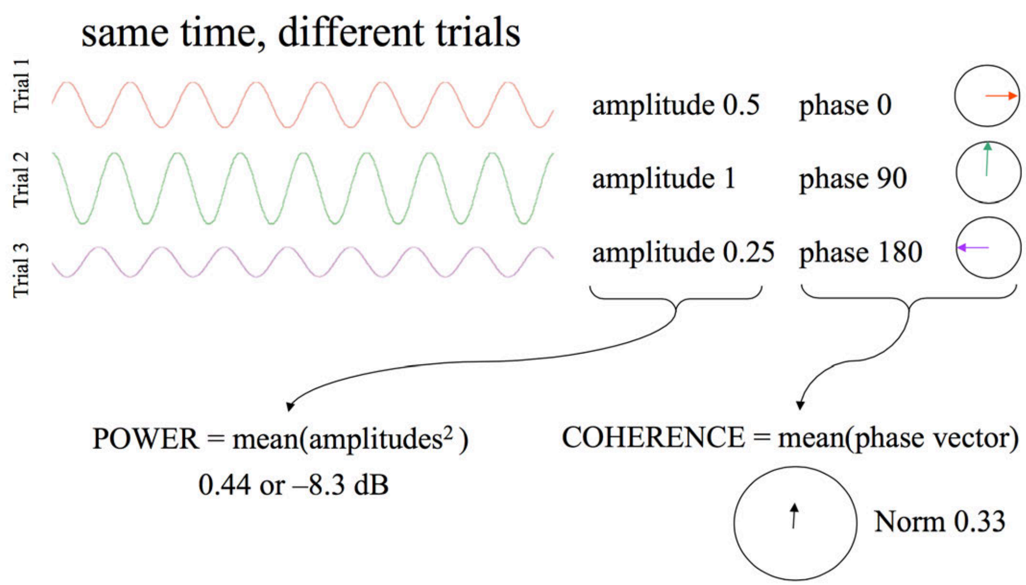
- Goal: How much do different trials resemble each other?
- Phase coherence not between two processes, but between multiple trials of the same process
- Defined over a (generally) narrow frequency range

EEGLAB's Inter-Trial Coherence is *phase* ITC

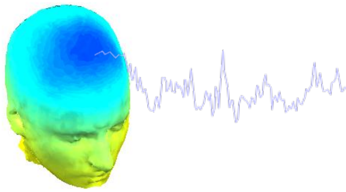
Phase ITC

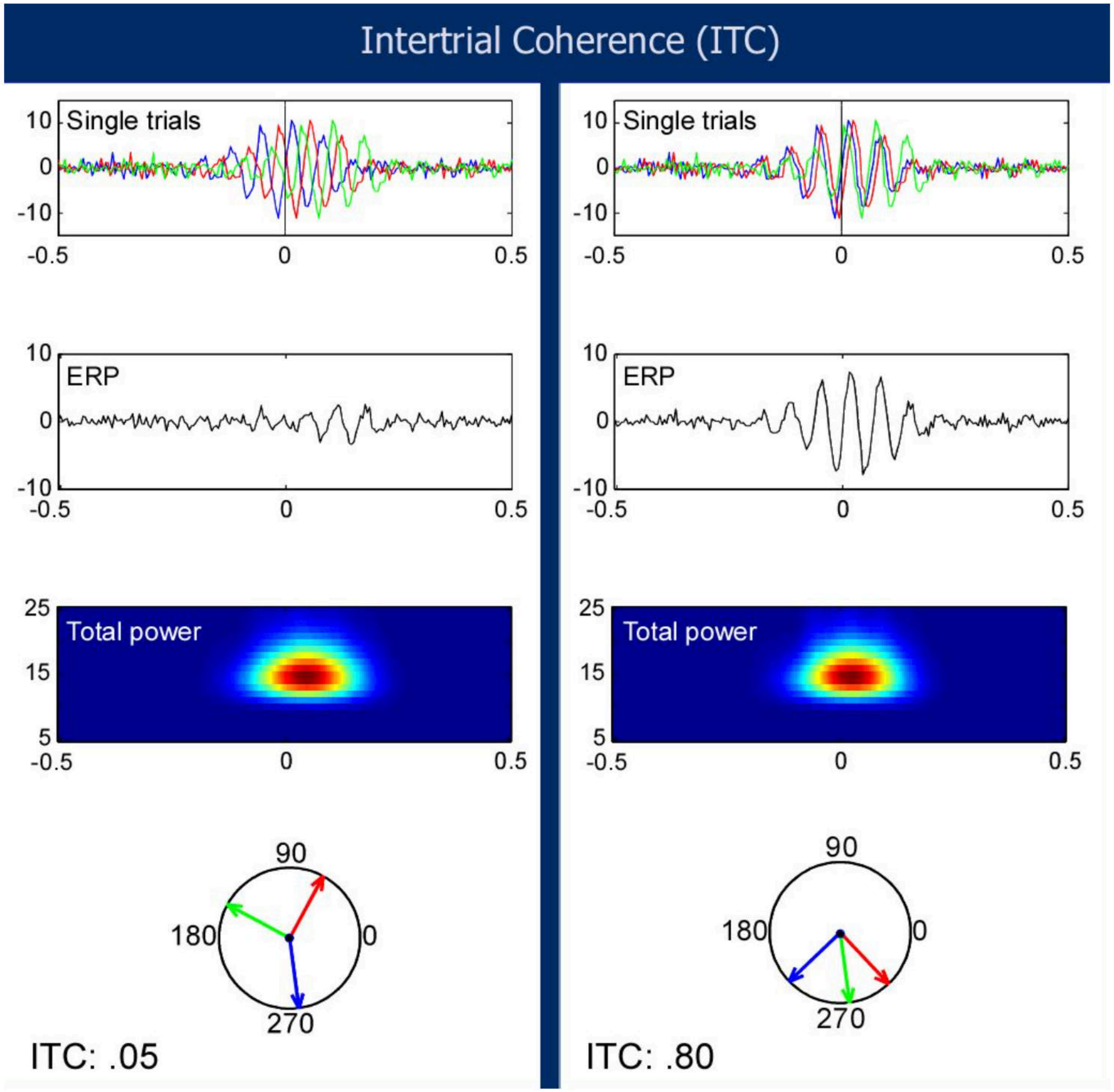
$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k(f,t)}{\left|F_k(f,t)\right|}$$

Normalized
(no amplitude information)

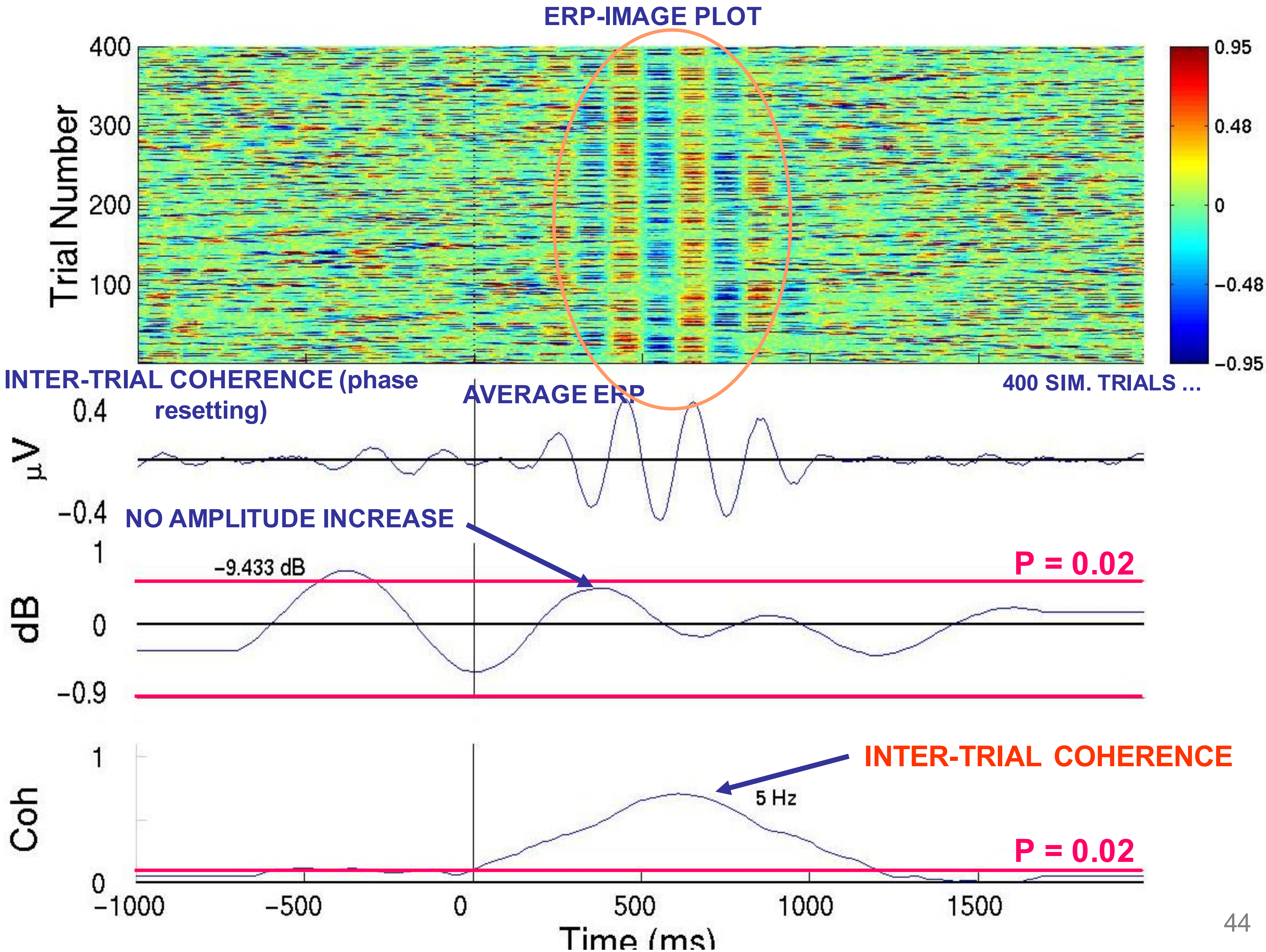


ITC Example (3 trials)

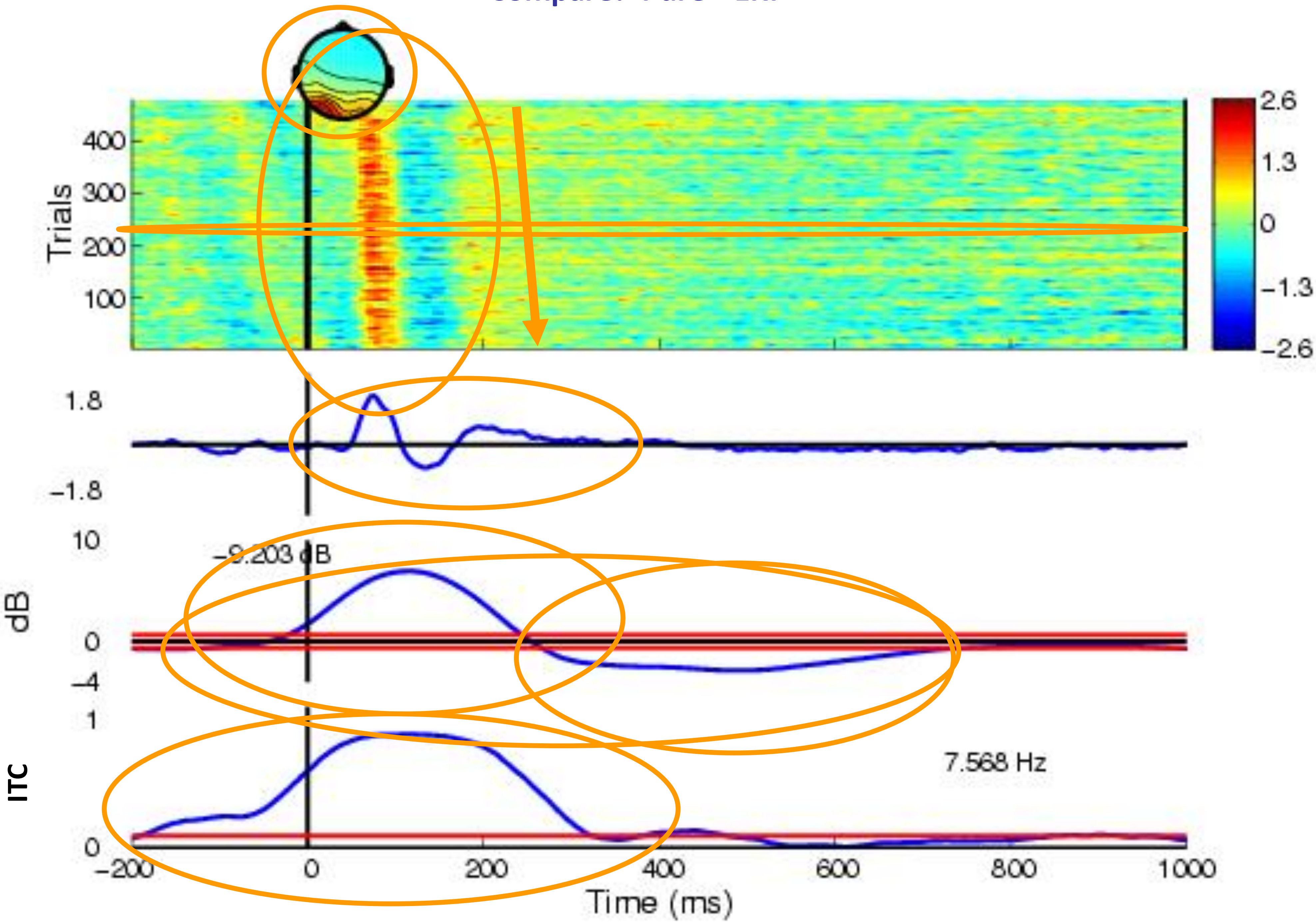




Slide courtesy of Stefan Debener

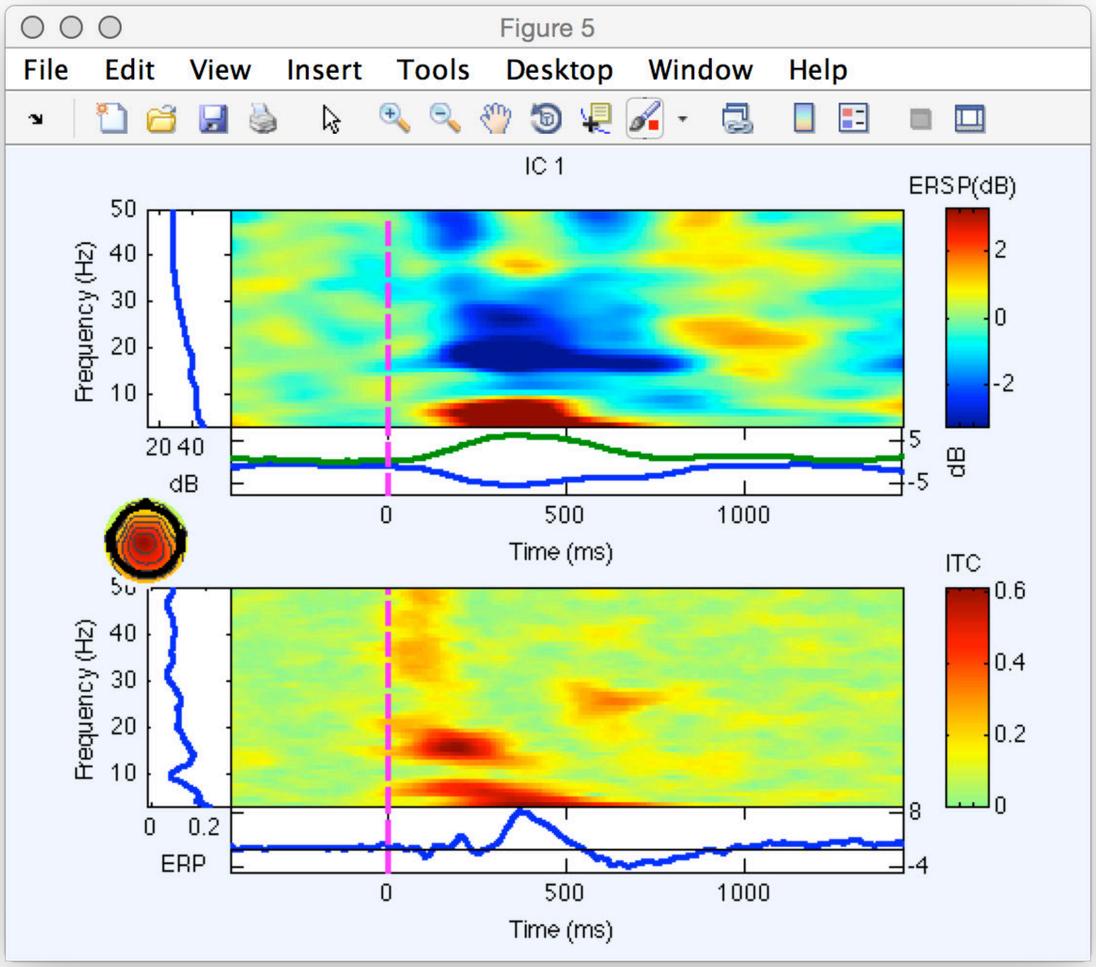

ERP-IMAGE PLOT
Trial Number
400
300
200
100
0.95
0.48
0
−0.48
−0.95
INTER-TRIAL COHERENCE (phase resetting)
AVERAGE ERP
400 SIM. TRIALS ...
μV
0.4
−0.4
NO AMPLITUDE INCREASE
dB
1
0
−0.9
−9.433 dB
P = 0.02
INTER-TRIAL COHERENCE
Coh
1
0
5 Hz
P = 0.02
−1000
−500
0
500
1000
1500
Time (ms)

Compare: Pure ERP



J. Onton & S. Makeig, 2005

Putting it all together



Exercise

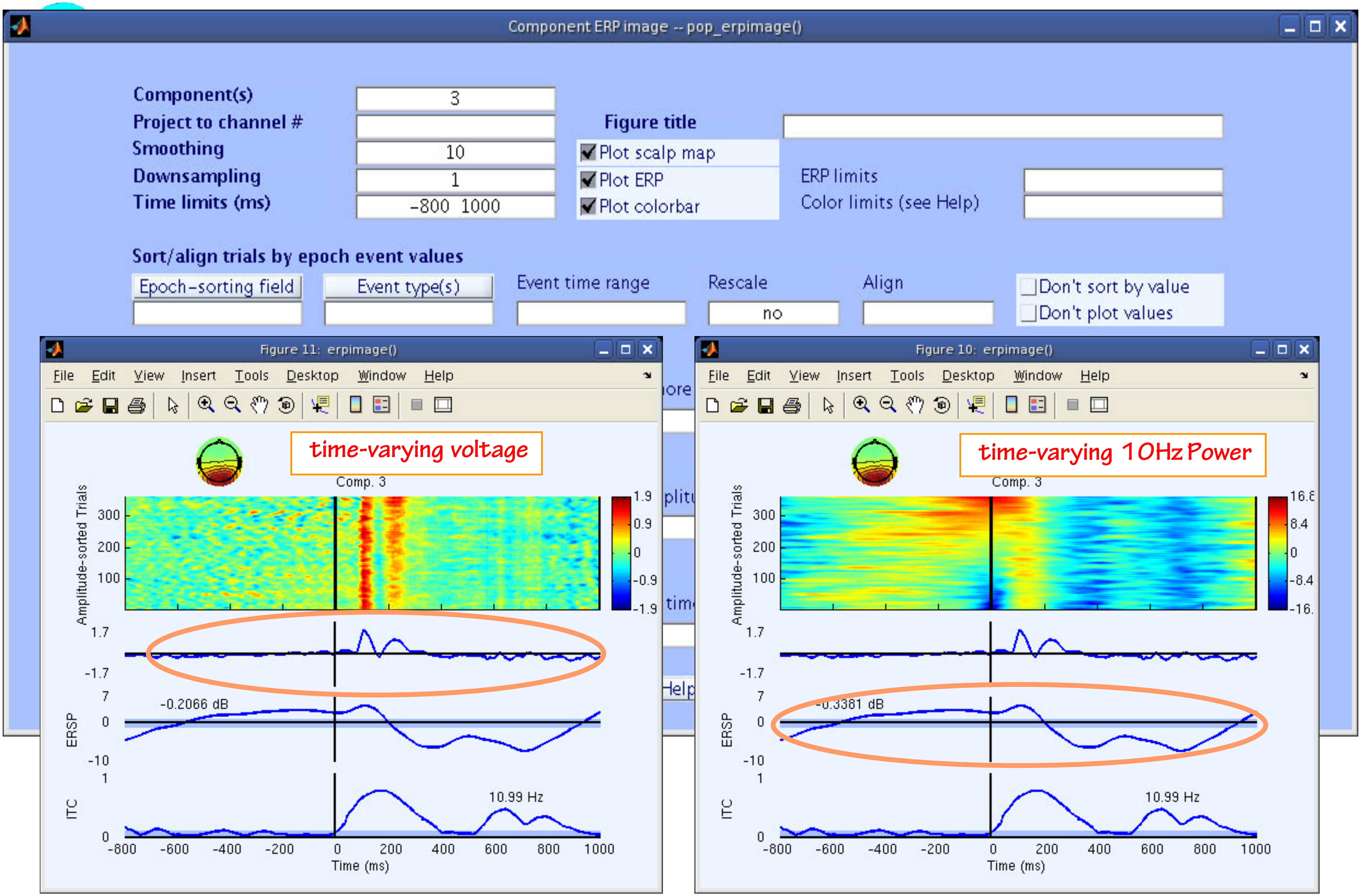
All: Compute ERSP/ITC for a component of your choice

Compute ERP Image (with ERSP and ITC displayed*)

Use all of this information to explain the origin of the Evoked Response

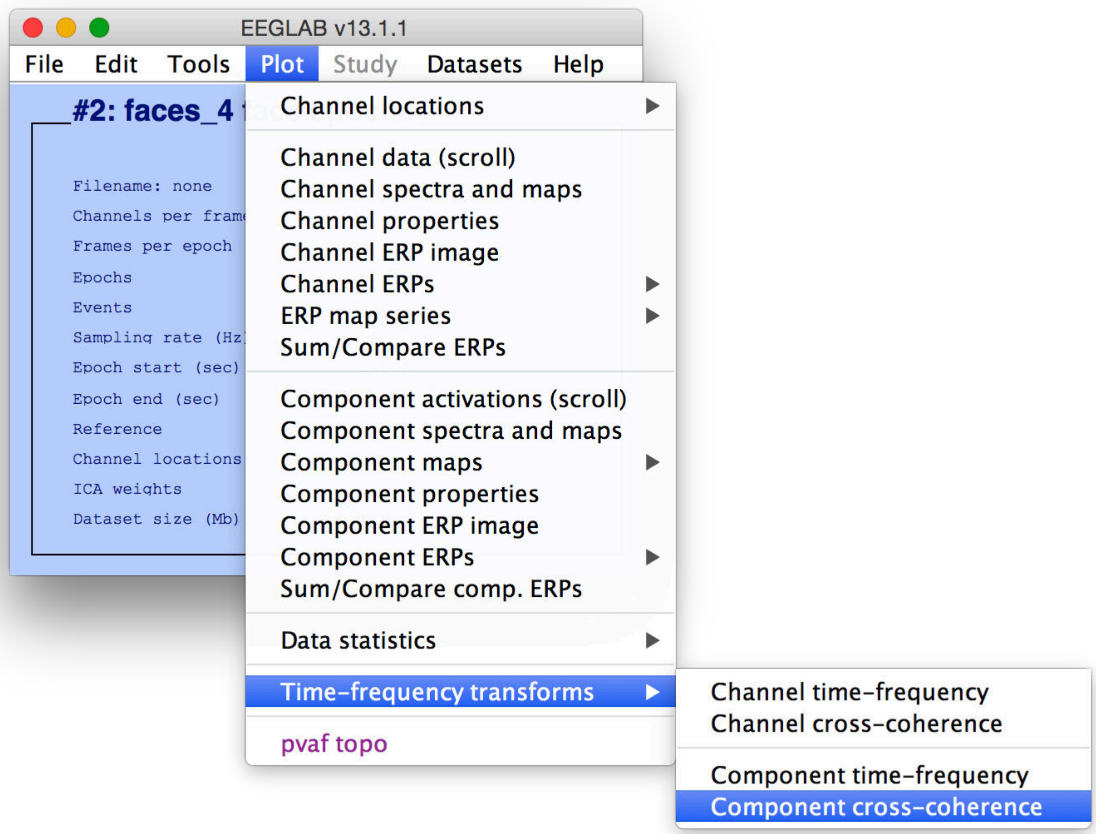
Question: Which changes are significant? Use the options in ERP Image and ERSP dialogs to set significance threshold e.g. 0.01. Do the results survive?

Component ERP Image: Activation vs. Amplitude

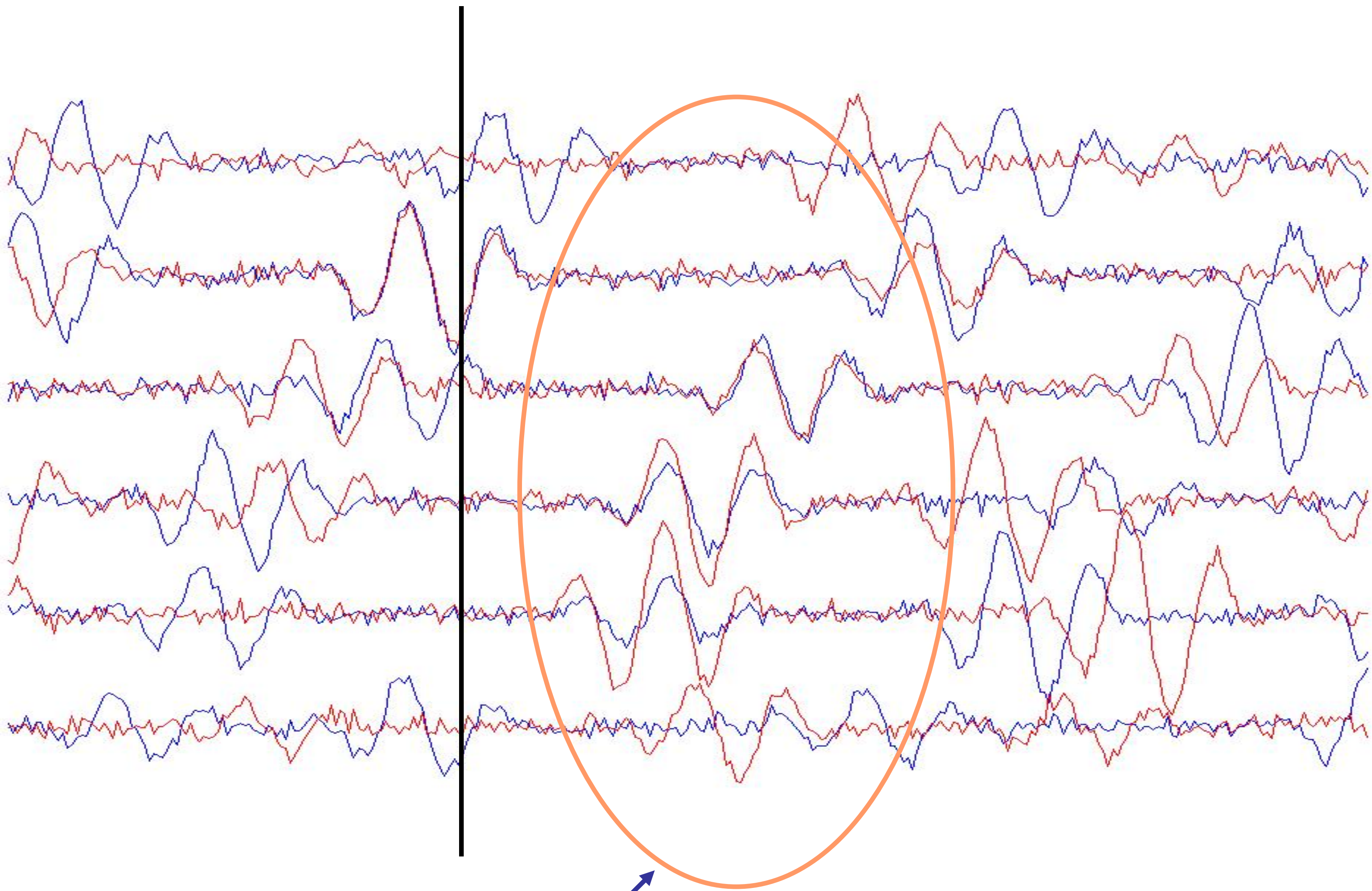


Part 3b: Event Related Coherence

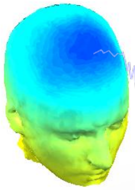
- Goal: How similar is the event-related response of two signals
  - Typically between channels (problematic due to volume conduction)
  - or between ICs

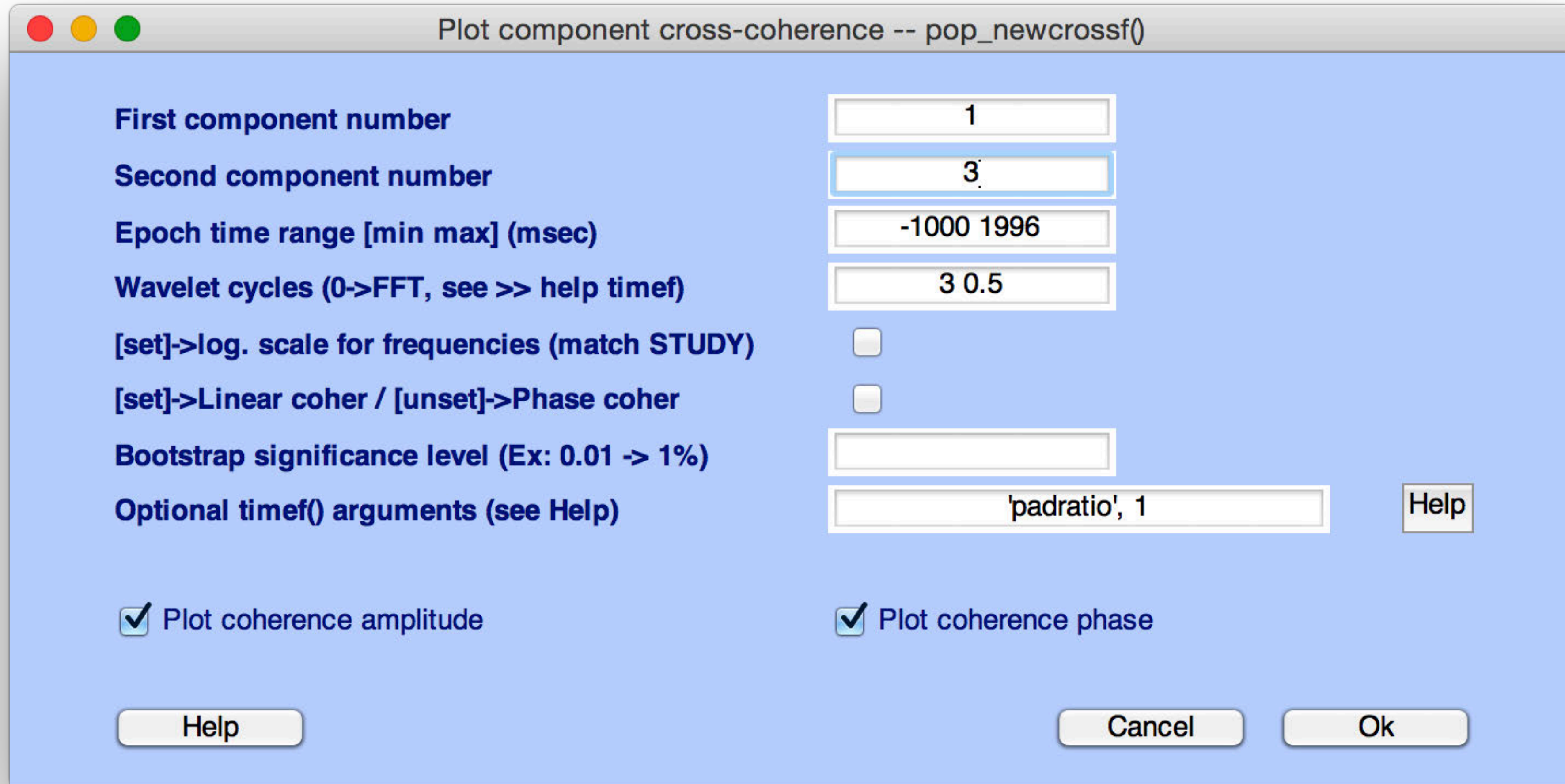


TWO SIMULATED THETA PROCESSES



Try it!




Plot component cross-coherence -- pop_newcrossf()
First component number
1
Second component number
3
Epoch time range [min max] (msec)
-1000 1996
Wavelet cycles (0->FFT, see >> help timef)
3 0.5
[set]->log. scale for frequencies (match STUDY)
[set]->Linear coher / [unset]->Phase coher
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional timef() arguments (see Help)
'padratio', 1
Help
Plot coherence amplitude
Plot coherence phase
Help
Cancel
Ok

Event-Related Coherence Exercise

- Examine event-related coherence between two ICs
  - Which pair did you pick, and why? What do you predict?
  - What did you learn?

- Explore other options:
  - Significance threshold
  - Figure out how to subtract a baseline
  - Phase vs. Linear Coherence