Clustering of ICA components

Arnaud Delorme

(with Julie Onton, Romain Grandchamp, Nima Bigdely Shamlo, Scott Makeig)









Weight matrix W

ICA activity U



HISTORICAI Kemarks

- Herault & Jutten ("Space or time adaptive signal processing by neural network models", *Neural Nets for Computing Meeting*, Snowbird, Utah, 1986): Seminal paper, neural network
- Bell & Sejnowski (1995): Information Maximization
- Amari et al. (1996): Natural Gradient Learning
- Cardoso (1996): JADE
- Applications of ICA to biomedical signals
 - EEG/ERP analysis (Makeig, Bell, Jung & Sejnowski, 1996).
 - fMRI analysis (McKeown et al. 1998)

ICA Theory – Cost Functions

Family of BSS algorithms

- Information theory (Infomax)
- Bayesian probability theory (Maximum likelihood estimation)
- Negentropy maximization
- Nonlinear PCA
- Statistical signal processing (cumulant maximization, JADE)

A unifying Information-theoretic framework for ICA

- Pearlmutter & Parra showed that InfoMax, ML estimation are equivalent.
- Lee et al. (1999) showed negentropy has the equivalent property to InfoMax.
- Girolami & Fyfe showed nonlinear PCA can be viewed from

ICA and PCA

ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix,



While PCA simply decorrelates the outputs (using an orthogonal mixing matrix), ICA attempts to make the outputs statistically independent, while placing no constraints on the minxing matrix.

Central limit theorem





ICA Training Process

Entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x).$$



Fake dice (make a 6 half of the time): entropy 2.16 (base 2)



less random

$$H = 5\left(-\frac{1}{10}\log_2\left(\frac{1}{10}\right)\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 2.16$$

Entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x).$$

Joint entropy

$$H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y)\log_b p(x,y),$$

Mutual Information

$$H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$$

Shannon in his landmark 1948 paper ``A Mathematical Theory of Communication." From http://planetmath.org/encyclopedia/ShannonsTheoremEntropy.html Contingency table for stress and emotionality

	STR	RE					
	1	2	3	4	5	6	Total
EMOT= 1	19	4					23
2	11	63	64	3	1		142
3	2	16	18	20	2	2	60
4	1	4	1	9	6	2	23
5			1	2	4	3	10
6				1	1	1	3
Total	33	87	84	35	13	8	
M~~~							

From http://tecfa.unige.ch/~lemay/thesis/THX-Doctorat/node149.html

Contingency frequencies for stress and emotionality

	STRE					
	1	2	3	4	5	6
EMOT=1	0.07	0.02				
2	0.04	0.24	0.25	0.01		
3	0.01	0.06	0.07	0.08	0.01	0.01
4		0.02		0.03	0.02	0.01
5				0.01	0.02	0.01
6						

Joint entropy 3.46; exercise: compute mutual information $H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y) \log_b p(x,y)$

ICA learning rule

How to make the outputs statistical independent? Minimize their redundancy or mutual information. Consider the joint entropy of two components,

 $H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$

Maximizing $H(y_1, y_2) \Longrightarrow$ minimizing $I(y_1, y_2)$.

The learning rule:

=0 if the two variables

are independent

 $\Delta \mathbf{W} \propto \frac{\partial H(\mathbf{y})}{\partial \mathbf{W}} \mathbf{W}^T \mathbf{W}$

Natural gradient (Amari)

Entropy

extremum



Independent components of LEO/LIN



Steps of clustering

- Select ICA components for clustering
- Precompute measures of interest
- Cluster measures
- Plot clusters and edit them if necessary



Edit dataset info

				-		pop_stud	y(): Pre-select compon	ents		
					Enter maxir NOTE: This	num residu will delete	al (topo map – dipole any existing compon	e proj.) var. ent cluster	(in %) ′s!	
							15			
					Keep onl	y in-brain c	lipoles.			
Crea	ate a new STUDY set pop	_study()			Cano	cel	Help		Ok	
Edit	STUDY eat information - ror	oember to	eaue chang							_
Cuits	STUDY set name:	nember to	save chang	cs	Sternherg					
	STUDY set teak some:				Sternherg					
	STUDY set notes:				Sternberg					
	STOD T SEL HOLES.									
	dataset filename	browse	subject	session	condition	group	Select by r.v.			
1	C:\\Users\\julie\\Documents\\\	Vor	S01		memorize		Comp.: 3 5	Clear		
2	C:\\Users\\julie\\Documents\\\	Vor	S01		ignore		Comp.: 3 5	Clear		
3	C:\\Users\\julie\\Documents\\\	Vor	S01		probe		Comp.: 3 5	Clear		
4	C:\\Users\\julie\\Documents\\\	Vor	S02		memorize		Comp.: 5 6	Clear		
5	C:\\Users\\julie\\Documents\\\	Vor	S02		ignore		Comp.: 5 6	Clear		
6	C:\\Users\\julie\\Documents\\\	Vor	S02		probe		Comp.: 5 6	Clear		
7	C:\\Users\\julie\\Documents\\\	Vor	S03		memorize		Comp.: 6 7	Clear		
8	C:\\Users\\julie\\Documents\\	Vor	S03		ignore		Comp.: 6 7	Clear		
9	C:\\Users\\julie\\Documents\\\	Vor	S03		probe		Comp.: 6 7	Clear		
10	C:\\Users\\julie\\Documents\\	Vor	S04		memorize		Comp.: 1 2	Clear		
Impor	tant note: Removed datasets v	vill not be sa	ved before be	eing delete	d from EEGLAB n	emory				
			< 1	Page 1	>					
	Dataset info (condition, group) differs t	from study inf	o. [set] = 0	Overwrite dataset	info.				
	Delete cluster information (to a	allow loading	anew dataset	ts, set nev	v components for	clustering, e	etc.)			
	Help					C	Cancel C	Dk		

Computing residual variance (%)



$$r = \Sigma (x_i - \tilde{x}_i)^2 / \Sigma x_i^2$$



Clustering results example



RELICA: A method for estimating the reliability of independent components

Fiorenzo Artoni^{a,*}, Danilo Menicucci^b, Arnaud Delorme^{c,e,f}, Scott Makeig^c, Silvestro Micera^{a,d}

Within-cluster reliability

The distribution of dipolarity within the cluster helps assessing the **quality** and characteristics of Independent Components



Reliability criteria and the rv<15%

First justification why we should select an **r.v** <15% for components to include in further analyses: there is a forbidden region underlined in red, that indicates the absence of



ICs to cluster

	STUDY set name	Ņ	(_ _ X	_	Sternberg			
	STUDY set task r	5	select components		E	Sternberg			
	dataset filename		ic 19 ic 20		sion	condition	group	Select by r.v.	
1	C:\\Users\\julie\\		ic 21 ic 22			memorize		Comp.: 3 5	Clea
2	C:\\Users\\julie\\		ic 23			ignore		Comp.: 3 5	Clea
3	C:\\Users\\julie\\		ic 24			probe		Comp.: 3 5	Clea
4	C:\\Users\\julie\\		ic 25 ic 26			memorize		Comp.: 5 6	Clea
5	C:\\Users\\julie\\		ic 27		П	ignore		Comp.: 5 6	Clea
6	C:\\Users\\julie\\		ic 28			probe		Comp.: 5 6	Clea
7	C:\\Users\\julie\\		ic 29		F.	memorize		Comp.: 6 7	Clea
8	C:\\Users\\julie\\		ic 31		F	ignore		Comp.: 6 7	Clea
9	C:\\Users\\julie\\		ic 32 Canaal II Ok		F.	probe		Comp.: 6 7	Clea
10	C:\\Users\\julie\\	-	Cancel OK		F.	memorize		Comp.: 1 2	Clea
Impor	tant note: Remove Dataset info (conc Delete cluster info	a uata dition,	group,) differs from stud	y info. [se	eletec 1 xt] = O t nevv	I from EEGLAB n	nemory info. clustering, etc.)		

~~~~

#### **Precompute data measures**

|      |                                               |                     | EE               | GLAB v6.           | Ob           |            |      |  |  |  |
|------|-----------------------------------------------|---------------------|------------------|--------------------|--------------|------------|------|--|--|--|
| File | e Edit                                        | Tools               | Plot             | Study              | Datasets     | Help       | Ŕ    |  |  |  |
|      | _стн                                          |                     | ά+ Δ1            | Edit               | t study info |            |      |  |  |  |
|      | 510                                           | 51 36               |                  | Pre                | compute ch   | annel meas | ures |  |  |  |
|      | Study                                         | filenar             | ne:              | Plot               | channel me   | easures    |      |  |  |  |
|      | Study task name Precompute component measures |                     |                  |                    |              |            |      |  |  |  |
|      | Nb of<br>Nb of                                | ring array          |                  |                    |              |            |      |  |  |  |
|      | Nbof                                          | sessio              | ns               | Cluster components |              |            |      |  |  |  |
|      | Nb of                                         | group               | s                | Edit/plot clusters |              |            |      |  |  |  |
|      | Epoch<br>Chan                                 | n consi:<br>nels ne | stency<br>r fram | / y∉<br>⊐e 3       | 25<br>1      |            |      |  |  |  |
|      | Chan                                          | nel loca            | ations           | Ves                |              |            |      |  |  |  |
|      | Clust                                         | ers                 |                  | 1                  |              |            |      |  |  |  |
|      | Statu                                         | s<br>               |                  | R                  | eady to pr   | ecluster   |      |  |  |  |
|      | lotal                                         | size (M             | (ai              | 3                  | 0.4          |            |      |  |  |  |
|      |                                               |                     |                  |                    |              |            |      |  |  |  |

### **Pre-compute measures**

| File Edit Tools Plot                                                                      | Study Datasets Help                                                                                            |
|-------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| STUDY set:                                                                                | Edit study info<br>Select/Edit study design(s)                                                                 |
| Study filename:s/dat<br>Study task name                                                   | Precompute channel measures<br>Plot channel measures                                                           |
| ND of subjects<br>ND of conditions<br>ND of sessions<br>ND of groups<br>Epoch consistency | Precompute component measures<br>Measure Product clustering<br>PCA clustering (original)<br>Edit/plot clusters |
| Channels per frame                                                                        | 61                                                                                                             |
| Channel locations                                                                         | ves                                                                                                            |
| Clusters                                                                                  | 1                                                                                                              |
| Status                                                                                    | Pre-clustered                                                                                                  |
| Total size (Mb)                                                                           | 8.2                                                                                                            |

#### ○ ○ ○ Select and compute component measures for later clustering -- pop\_precomp()

| Pre-  | compute component measures for STUDY 'Sternberg' - 'STUDY.design 1'                              |  |
|-------|--------------------------------------------------------------------------------------------------|--|
| ☑     | Compute ERP/spectrum/ERSP only for components selected by RV (set) or for all components (unset) |  |
| List  | of measures to precompute                                                                        |  |
|       | ERPs Recoling ((min maylin ma)                                                                   |  |
|       | Power se citrum Spectropo parameters 'specmode', 'fit' (Test)                                    |  |
|       | ERSPs                                                                                            |  |
|       | Time/freq, parameters                                                                            |  |
|       | Scalp maps                                                                                       |  |
|       |                                                                                                  |  |
| - 📃 🤅 | Save single-trial measures for single-trial statistics - requires disk space                     |  |
|       | Recompute even if present on disk                                                                |  |
|       |                                                                                                  |  |
|       | Help Cancel Ok                                                                                   |  |

| File | Edit     | Tools     | Plot   | Study           | Datasets                   | Help        |          |   |
|------|----------|-----------|--------|-----------------|----------------------------|-------------|----------|---|
| [!   | STUD     | Y set:    |        | Edit s<br>Selec | study info<br>t/Edit study | / design(s  | )        |   |
|      | Study fi | lename: . | s/data | Preco           | mpute cha                  | nnel mea:   | sures    |   |
|      | Study ta | isk name  |        | Plot o          | hannel me                  | asures      |          |   |
| 1    | Nb of su | bjects    |        | Duese           |                            |             |          |   |
| 1    | Nb of co | nditions  |        | Preco           | ompute con                 | iponent n   | neasures |   |
| 1    | Nb of se | ssions    |        | Meas            | ure Produc                 | t clusterin | ig       | ► |
| 1    | Nb of gr | oups      |        | PCA (           | clustering (               | original)   |          | ► |
| 1    | Epoch co | nsistency |        | Edit/           | plot cluster               | s           |          |   |
| (    | Channels | per fram  | е      | 61              |                            |             |          |   |
| (    | Channel  | locations |        | yes             |                            |             |          |   |
| (    | Clusters |           |        | 1               |                            |             |          |   |
|      | Status   |           |        | Pre-cl          | ustered                    |             |          |   |
|      | Total si | ze (Mb)   |        | 8.2             |                            |             |          |   |

Select and compute component measures for later clustering -- pop\_precomp()

Pre-compute channel measures for STUDY 'Sternberg' - 'STUDY.design 1'

| Cha<br>✓ | nnel list (default:all)<br>Spherical interpolatio<br>Remove ICA artifactu | n of missing channels (p<br>al components pre-tagg | performed afte<br>ed in each da | er optional ICA removal belo<br>taset<br>ParentCluster 1 | w)       |
|----------|---------------------------------------------------------------------------|----------------------------------------------------|---------------------------------|----------------------------------------------------------|----------|
|          | Remove artifactual IC                                                     | A cluster or clusters (ho                          | ld shift key)                   | Cls 2<br>Cls 3<br>Cls 4                                  | <u> </u> |
| List     | of measures to prece                                                      | ompute                                             |                                 |                                                          |          |
|          | ERPs                                                                      | Baseline ([min m                                   | ax] in ms)                      |                                                          |          |
|          | Power spectrum                                                            | Spectopo parame                                    | eters                           | 'specmode', 'fft'                                        | Test     |
|          | ERSPs                                                                     | Time/freq, parameter                               | s <sup>i</sup> cycl             | es', [3 0.5], 'nfreqs', 100                              | Test     |
|          | Save single-trial measu<br>Recompute even if pre                          | ures for single-trial statis<br>sent on disk       | tics - requires                 | disk space                                               |          |
|          | Help                                                                      |                                                    |                                 | Cancel                                                   | Ok       |

#### **Precompute data measures**

#### TIP: Compute all measures so you can

#### test different combinations for clustering

|   | Select and compute comp                                           | oonent measures for later clu | stering - | pop_precomp()                  | • ×            |                   |
|---|-------------------------------------------------------------------|-------------------------------|-----------|--------------------------------|----------------|-------------------|
|   | Pre-compute component                                             | measures for STUDY 'Stern     | berg'     |                                |                |                   |
| l | Compute ERP/spectrum                                              | NERSP only for components sel | ected by  | R∨ (set) or for all components | (unset)        |                   |
| L | List of measures to prec                                          | ompute                        |           |                                |                |                   |
| L | 🔽 ERPs                                                            | Baseline ([min max] in m      | s)        | [-200 0]                       |                |                   |
| L | Power spectrum                                                    | Spectopo parameters           |           |                                | Test           |                   |
|   | ERSPs     ITCs                                                    | Time/freq.parameters          | 'cyc      | les', [3 0.5], 'nfreqs', 100 🔻 | Test           |                   |
|   | <ul> <li>Scalp maps</li> <li>Recompute even if present</li> </ul> | nt on disk                    |           |                                | Time-<br>optic | -frequency<br>ons |
|   | Help                                                              |                               |           | Cancel                         | Ok             |                   |

-----

## **Cluster components**

| Studv filename:<br>Studv task name     | Precompute channel measures<br>Plot channel measures |         |                       |              |            |                      |                      |                                     |
|----------------------------------------|------------------------------------------------------|---------|-----------------------|--------------|------------|----------------------|----------------------|-------------------------------------|
| Ib of subjects                         | Precompute component measures                        | _       |                       |              |            |                      |                      |                                     |
| b of sessions                          | PCA clustering (original)                            | ▶ Builc | l preclustering array |              |            |                      |                      |                                     |
| lb of aroups                           | Edit/plot clusters                                   | Clus    | ter components        |              |            |                      |                      |                                     |
| boch consistency<br>Channels per frame | ves 69,70,71                                         |         |                       |              |            |                      |                      |                                     |
| Channel locations                      | ves                                                  |         |                       |              |            |                      |                      |                                     |
| lusters                                |                                                      |         | Select and            | compute o    | component  | measures for later   | <sup>-</sup> cluster | ing pop_preclust()                  |
| otal size (Mb)                         | 229.3                                                |         |                       |              |            |                      |                      |                                     |
|                                        |                                                      |         | Build pre-clustering  | matrix for S | STUDY set: | Sternberg            |                      |                                     |
|                                        |                                                      |         | Only measures that ha | ave been pro | ecomputed  | may be used for clu  | stering              |                                     |
|                                        |                                                      |         | Mixing time-based and | d location-t | based meas | ures to cluster migh | it result i          | in Help                             |
|                                        |                                                      |         |                       |              |            |                      | it roouit i          |                                     |
|                                        |                                                      |         | lime-based info       | PCA          | Weight     |                      |                      |                                     |
|                                        |                                                      |         | spectra               | 10           | 1          | Freq.range [Hz]      | 3 25                 |                                     |
|                                        |                                                      |         | ERPs                  | 10           | 1          | Time range [ms]      |                      |                                     |
|                                        |                                                      |         | ERSPs                 | 10           | 1          | Time range [ms]      |                      | Freq. range [Hz]                    |
|                                        |                                                      |         | ITCs                  | 10           | 1          | Time range [ms]      |                      | Freq. range [Hz]                    |
|                                        |                                                      |         | Location-based info   | PCA          | Weight     |                      |                      |                                     |
|                                        |                                                      |         | dipole locations      | 3            | 1          |                      |                      |                                     |
|                                        |                                                      |         | dipole orient.        | 3            | 1          | Amplitude & pola     | rity is ig           | nored                               |
|                                        |                                                      |         | scalp maps            | 10           | 1          | Use channel v.       |                      | <ul> <li>Absolute values</li> </ul> |
|                                        |                                                      |         |                       |              |            |                      |                      |                                     |
|                                        |                                                      |         |                       |              |            |                      |                      |                                     |



-----





#### **Precluster: Use singular values from PCA**





# **Classical KMean**



#### **Cluster components**

| File        | Edit                          | Tools                             | Plot   | Study              | Datasets                   | Help                 |          |
|-------------|-------------------------------|-----------------------------------|--------|--------------------|----------------------------|----------------------|----------|
| {           | STUD                          | Y set: S                          | ternbe | rg Edit s<br>Selec | study info<br>t/Edit stud  | y design(            | s)       |
| 5           | Studv f<br>Studv t            | filename:<br>Lask name            |        | Preco<br>Plot o    | ompute cha<br>channel me   | nnel mea<br>asures   | sures    |
| N<br>N<br>N | Ib of s<br>Ib of a<br>Ib of s | subiects<br>condition<br>sessions | S      | Preco<br>PCA (     | ompute con<br>clustering ( | nponent<br>original) | measures |
| N           | lb of d                       | aroups                            |        | Edit/              | plot cluster               | s                    |          |
| E           | lpoch d                       | consisten                         | CV     | ves                |                            |                      |          |
| C           | Channel                       | ls per fr                         | ame    | 69.70              | ,71                        |                      |          |
| C           | Channel                       | l locatio                         | ns     | ves                |                            |                      |          |
| C           | Cluster                       | cs                                |        | 7                  |                            |                      |          |
| S           | Status                        |                                   |        | Pre-c              | lustered                   |                      |          |
| Γ           | otal s                        | size (Mb)                         |        | 229.3              |                            |                      |          |



### Choosing data measures

What measure(s) should you use?

It depends on your final cluster criteria...

- If for example, your priority is dipole location, then cluster only based on dipole location...

But consider:

- What is the difference between these two components?



#### **Choosing data measures**



#### **Subject differences?**





#### **Subject differences?**



statistics within subject and binomial probability between subjects (p < 0.01)

between the two clusters by bootstrap statistics (p < 0.001)

### **Results (Cluster 1)**



### **Results (Cluster 2)**



### **Results (Cluster 8)**



#### **Results (Cluster 13)**



#### 63.64% Sessions contribute







#### **Results (Cluster 14)**



### View and edit clusters

| EEGLA                                                | B v15.x (dev)                                              |
|------------------------------------------------------|------------------------------------------------------------|
| File Edit Tools Plot                                 | Study Datasets Help                                        |
| STUDY set: Sternk                                    | Edit study info<br>Select/Edit study design(s)             |
| Studv filename:<br>Studv task name                   | Precompute channel measures<br>Plot channel measures       |
| Nb of subjects<br>Nb of conditions<br>Nb of sessions | Precompute component measures<br>PCA clustering (original) |
| ND OF Groups<br>Epoch consistencv                    | ves                                                        |
| Channels per frame<br>Channel locations              | 69.70.71<br>ves                                            |
| Clusters                                             | 7<br>Pre-clustered                                         |
| Total size (Mb)                                      | 229.3                                                      |
|                                                      |                                                            |

#### **Plot/edit clusters**









#### MAKE SURE THAT SIFT PATH

···/eeglab/plugins/sift\_1.5/utils/

#### IS AT THE BOTTOM (EDIT PATH)



#### **Plot cluster ERP**



# Exercise

- Load the STUDY stern.study
- Precompute spectrum, ERP and scalp maps for components
- Precluster and cluster components using dipole locations and dipole moments (affinity clustering)
- Look at your cluster. Identify frontal midline theta cluster
   and occipital alpha cluster
- Remove outliers if any
- Plot significant difference (parametric statistics) for one component cluster spectrum between the two conditions ignore vs memorize