Time-frequency decomposition

Theory and Practice

EEGLAB Workshop XXIII

AIISH, Mysore, India

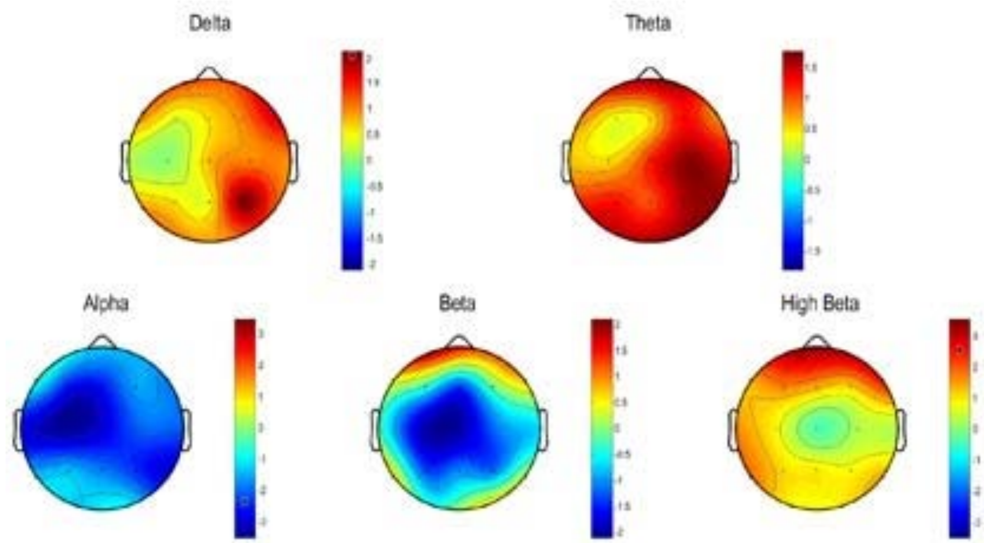
Day 1, 18:00

- Signals – EEG

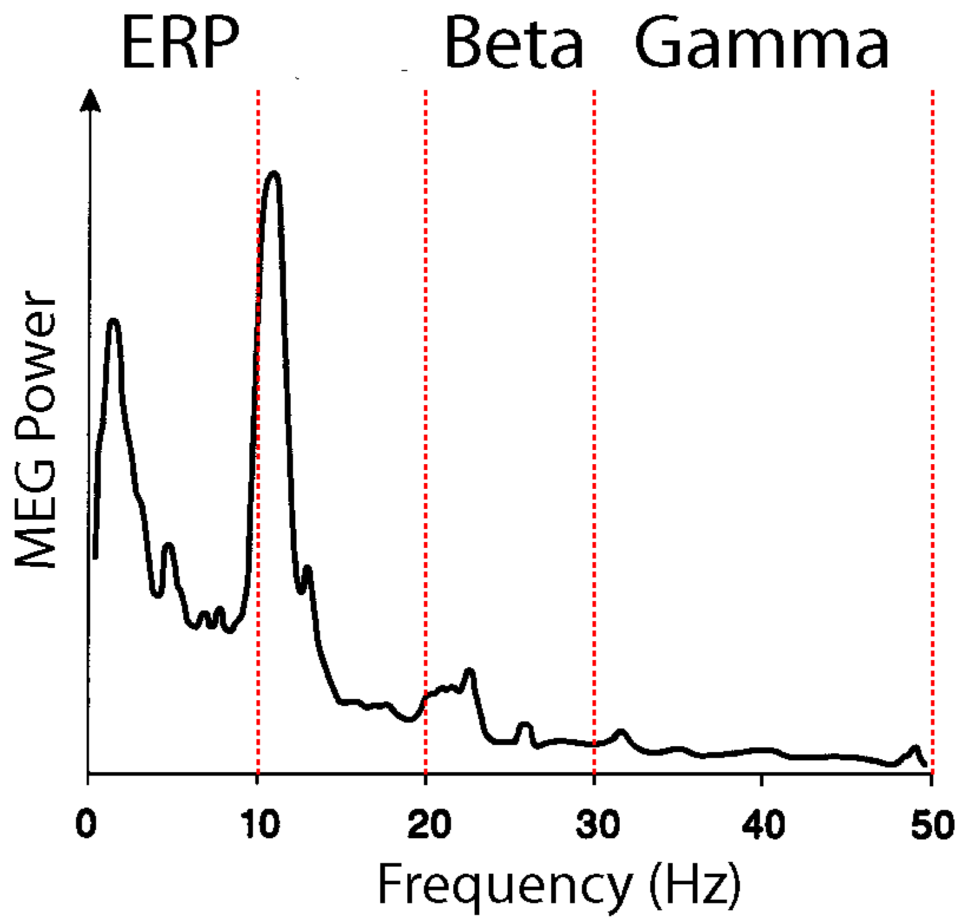
- Goals
  - Describe dynamic characteristics of brain activity
  - Describe relation between different regions of brain

- Approaches
  - Time domain
  - Frequency domain
  - Time/Frequency

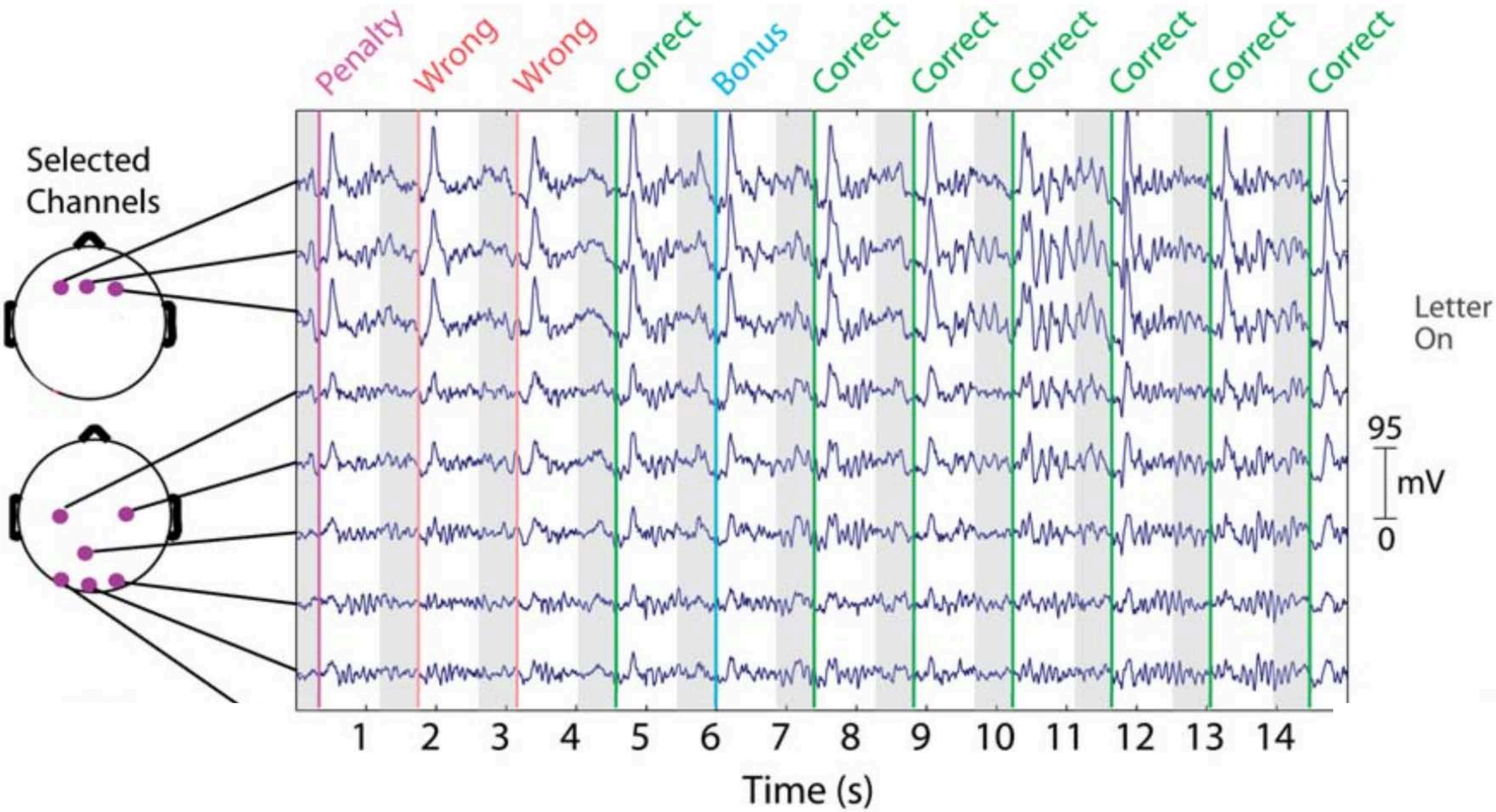
Different meanings traditionally given to different frequency bands



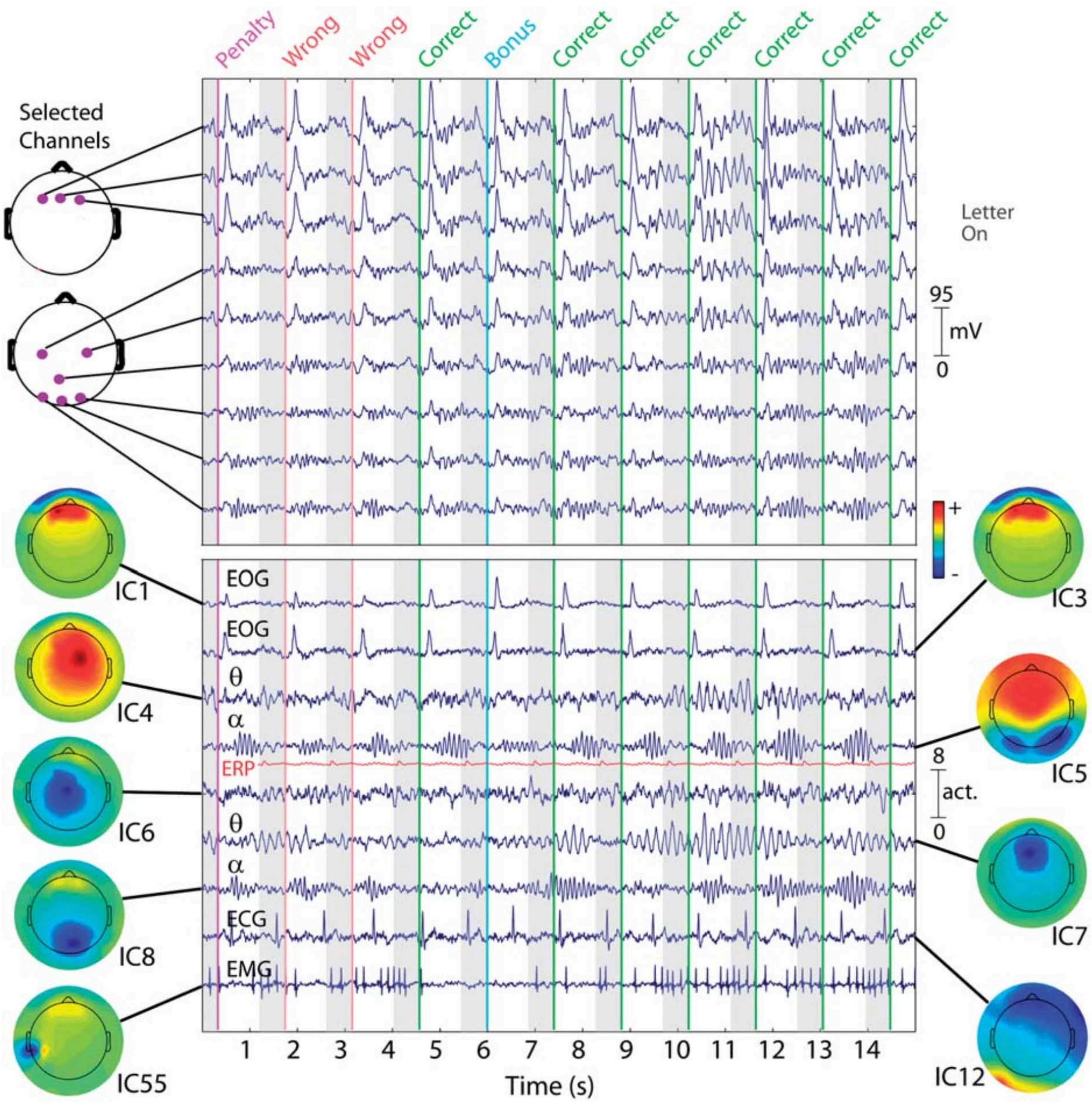
MEEG spectrum



Time varying frequency content

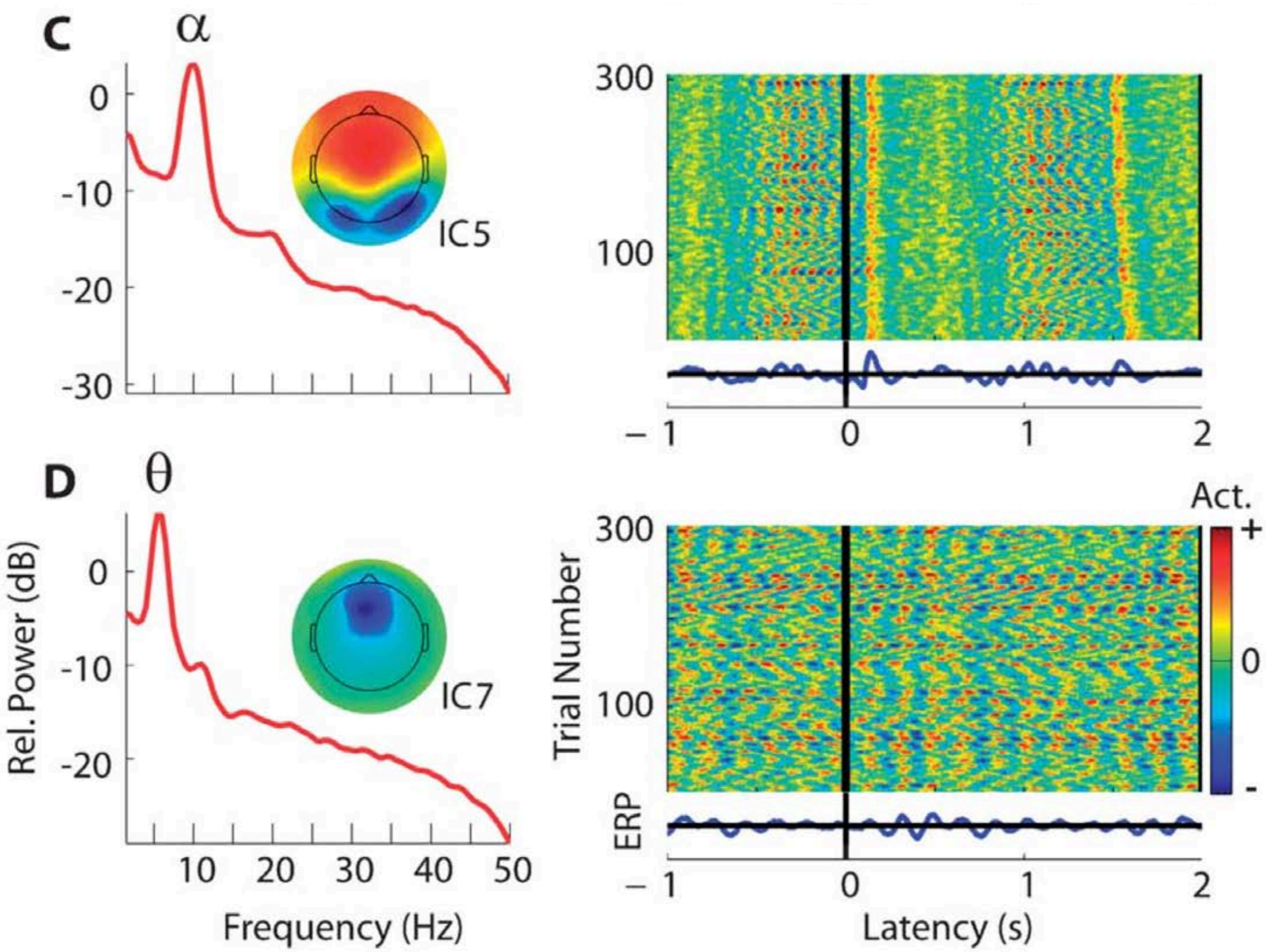


Time-varying frequency content

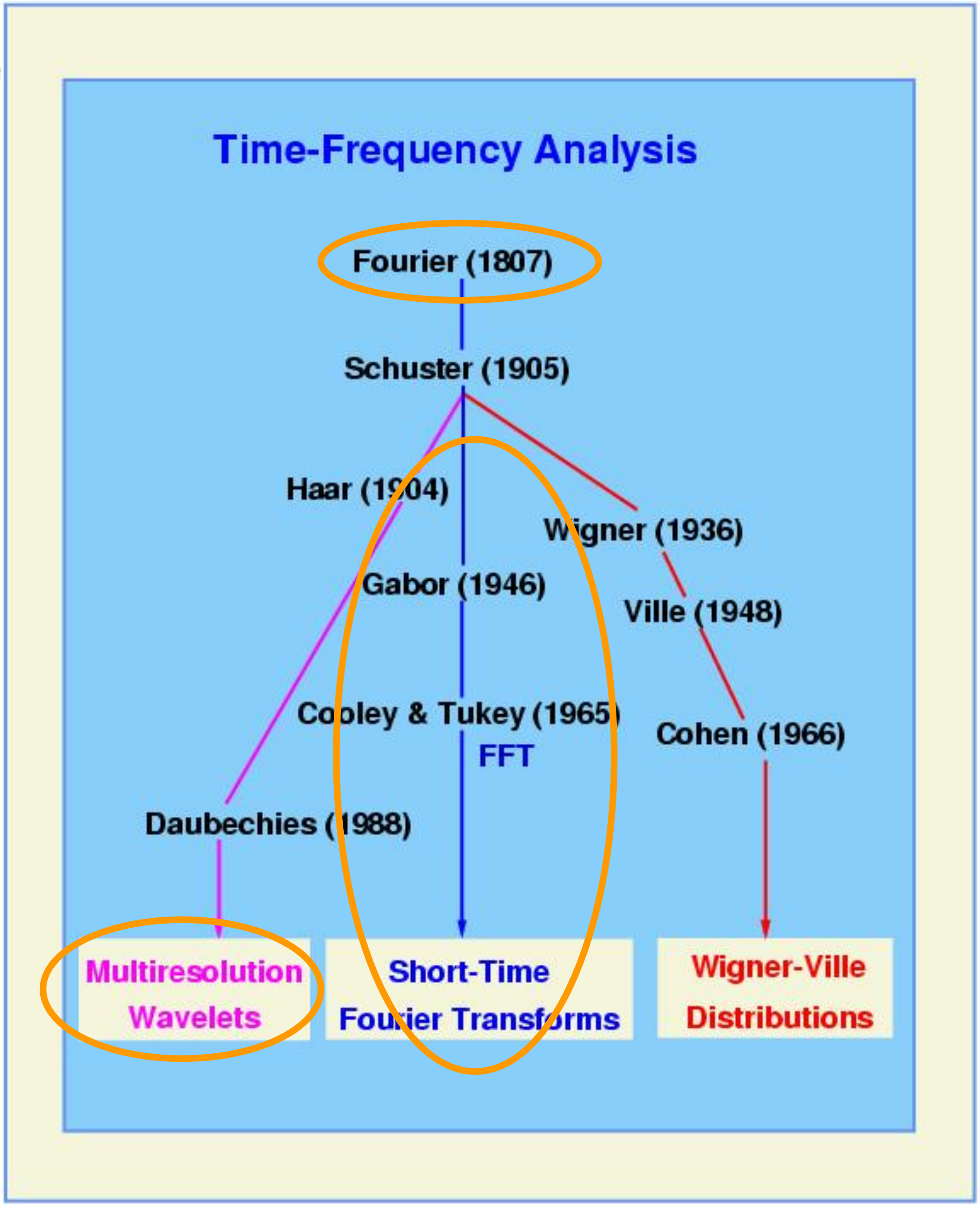


Onton & Makeig, 2006

Power Spectrum does not describe temporal variation



Onton & Makeig, 2006



S. Makeig, 2005

Plan

- **Part 1: Frequency Analysis**
  - Power Spectrum
    - Approaches
      - FFT
      - Welch's Method
    - Windowing

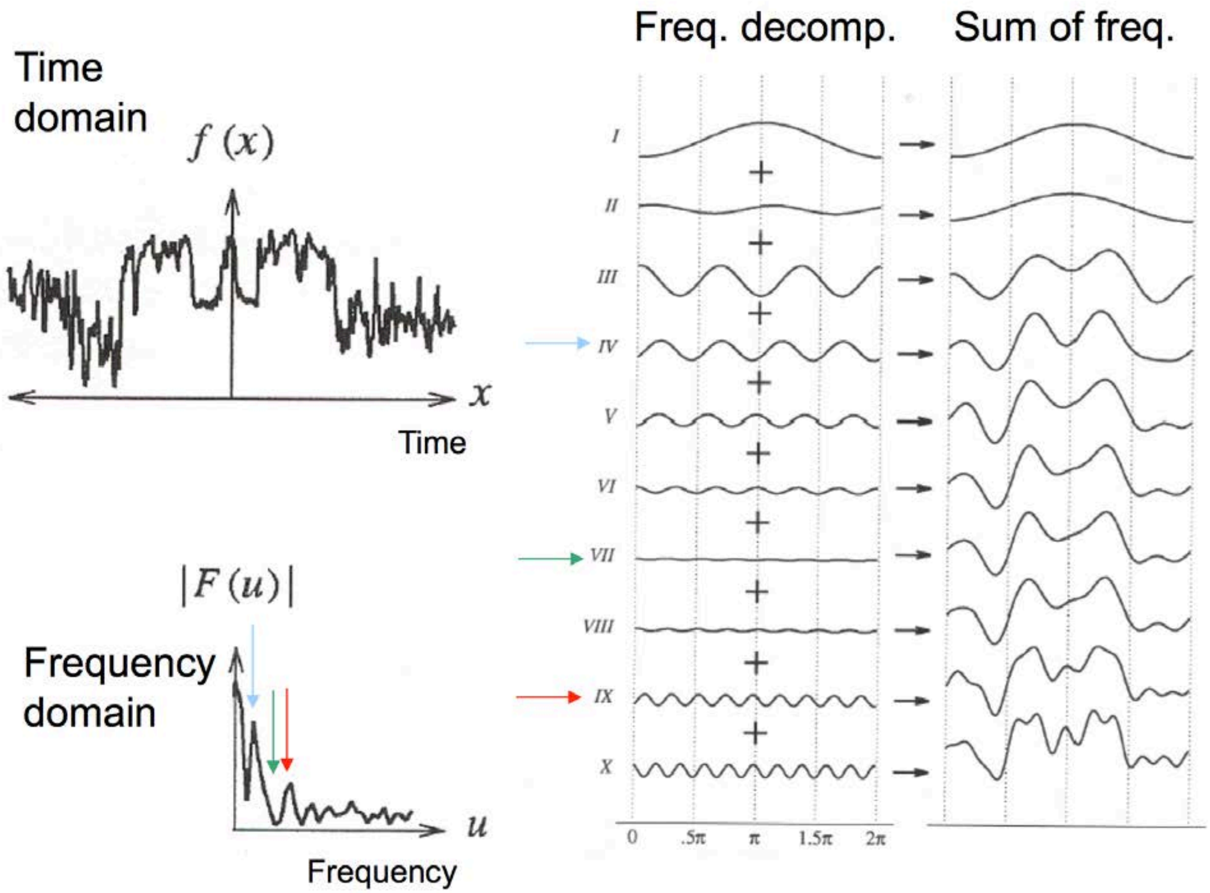
- **Part 2: Time-Frequency Analysis**
  - Short Time Fourier Transform
  - Wavelet Transform
  - ERSP

- **Part 3: Coherence Analysis**
  - Inter-Trial Coherence
  - Event-Related Coherence

Part 1: Frequency Analysis

- Goal: What frequencies are present in signal?
- What is power at each frequency?
- Principle: Fourier Analysis

Fourier Analysis



Forward transform $$F(u) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i u x} dx$$

Inverse transform $$f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi i u x} du$$

Figure, courtesy of Ravi Ramamoorthi & Wolberg

Power Spectrum. Approach 1: FFT

- Why not just take FFT of our signal of interest?
- Advantage – fine frequency resolution
  - $\Delta F = 1$ / signal duration (s)
  - E.g. 100s signal has 0.01 Hz resolution
  - But, do we really need this?

- Disadvantage 1 – high variance
  - Solution: e.g. Welch's method

- Disadvantage 2 – no temporal resolution
  - Solution 1: Short-Time Fourier Transform

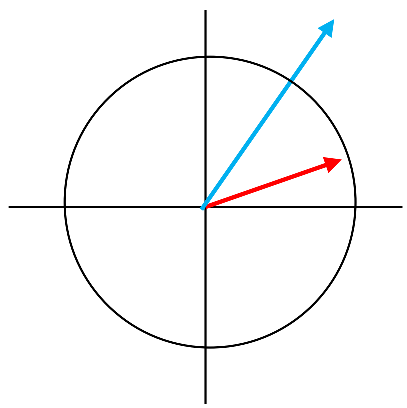
Amplitude and phase

- Power spectra describe the *amount* of a given frequency present
- NOT a complete description of a signal: We also must know the *phase* at each frequency
- FFT/STFT/Wavelet return an amplitude and phase at each time and frequency (represented as complex #).
- To find power, we compute the magnitude, which discards phase.

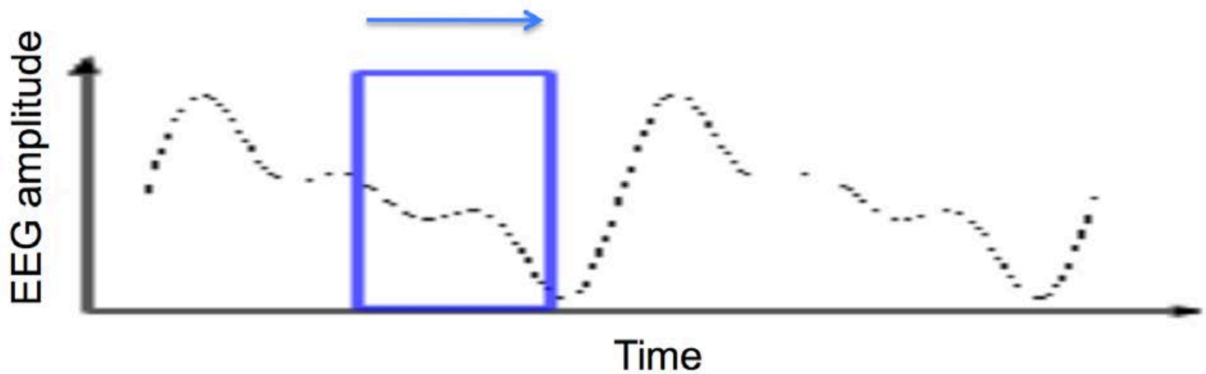
Phasor representation

- A complex number $x + yi$ can be expressed in terms of amplitude and phase: $ae^{i\theta}$

```
amplitude*exp(i*phase)
amplitude = sqrt(x^2 + y^2); phase = atan(y/x);
```



Approach 2: Welch's Method



Calculate power spectrum of short windows, average.

Advantage: Smoother estimate of power spectrum

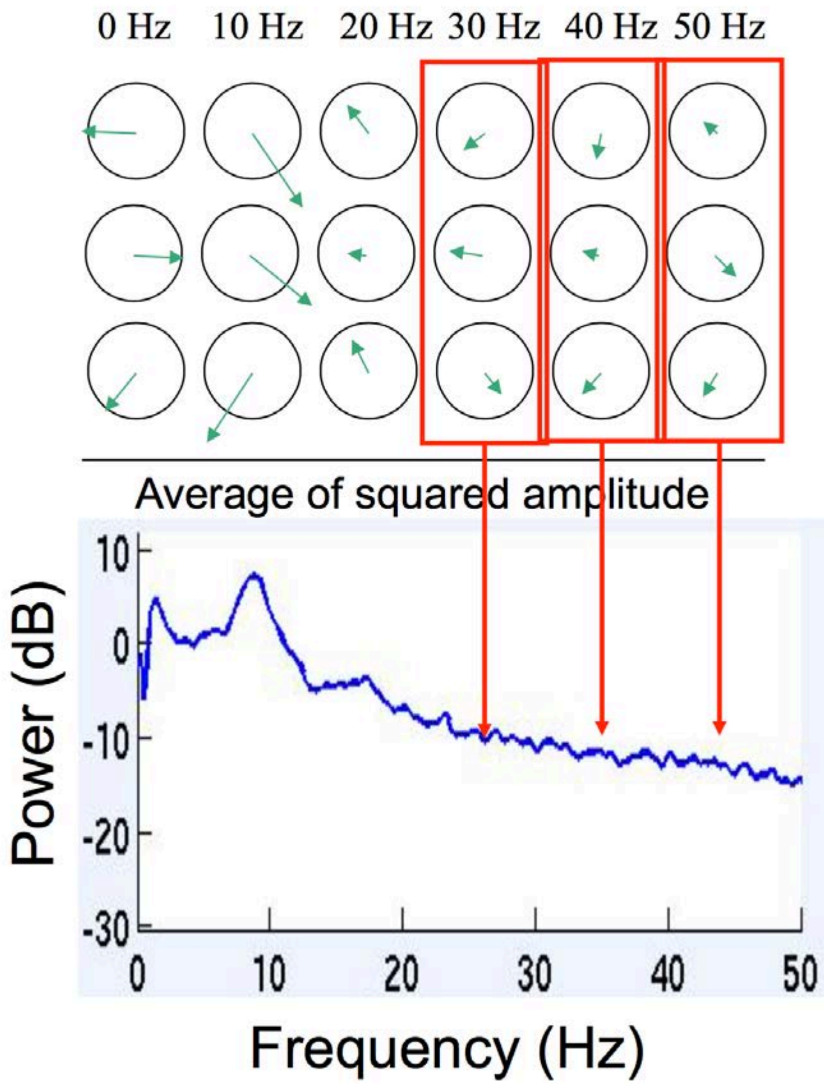
Frequency resolution set by window length

e.g. 1s window -> 1 Hz resolution

In practice: taper, don't use rectangular window

FFT of window 1

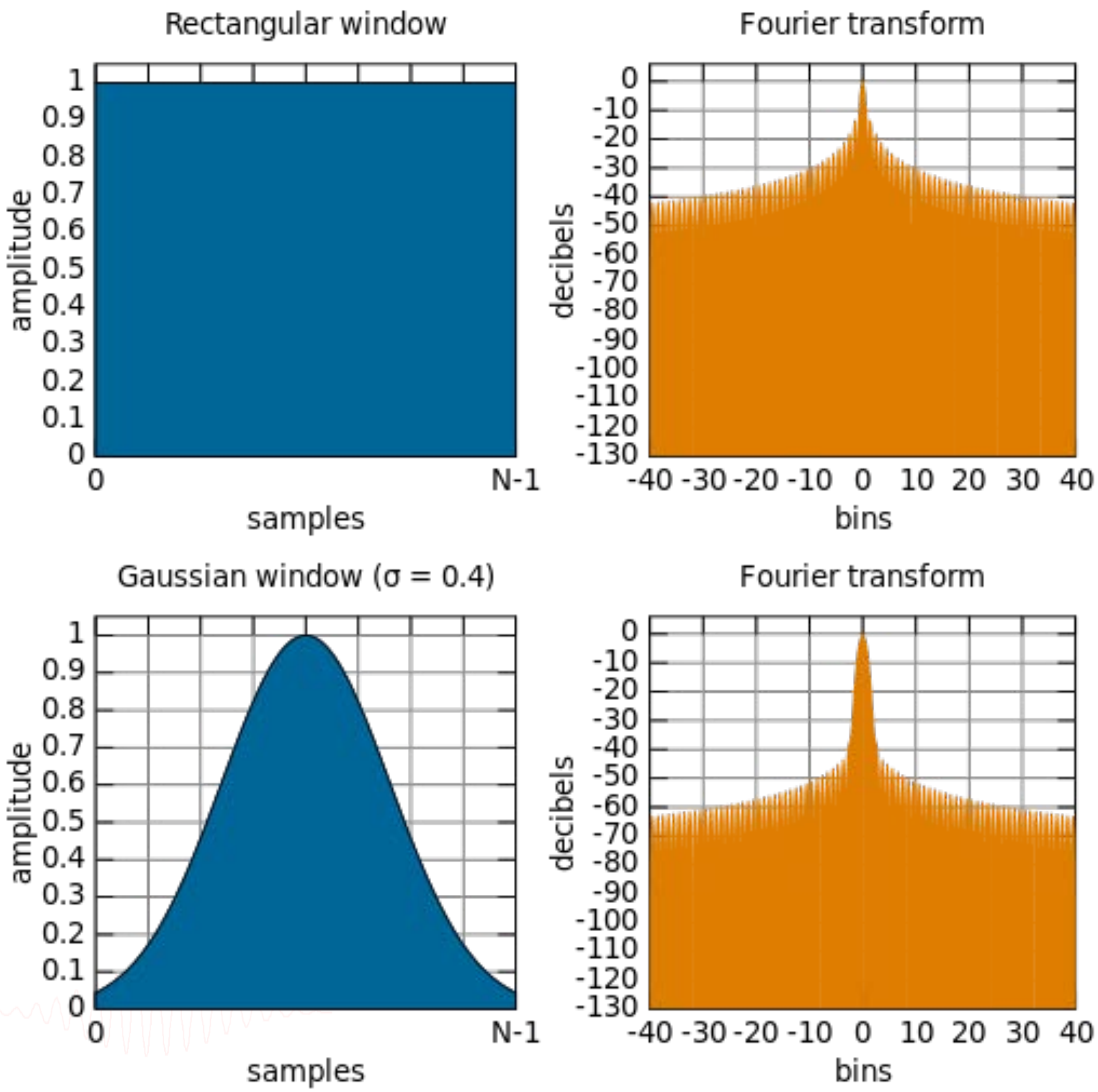
FFT of window 2



Windowing

- When we pick a short segment of signal, we typically window it with a smooth function.
- Windowing in time = convolving (filtering) the spectrum with the Fourier transform of the window
- No window (=rectangular window) results in the most smearing of the spectrum
- There are many other windows optimized for different purposes: Hamming, Gaussian…

Windows and their Fourier transforms



Narrowest main peak, but

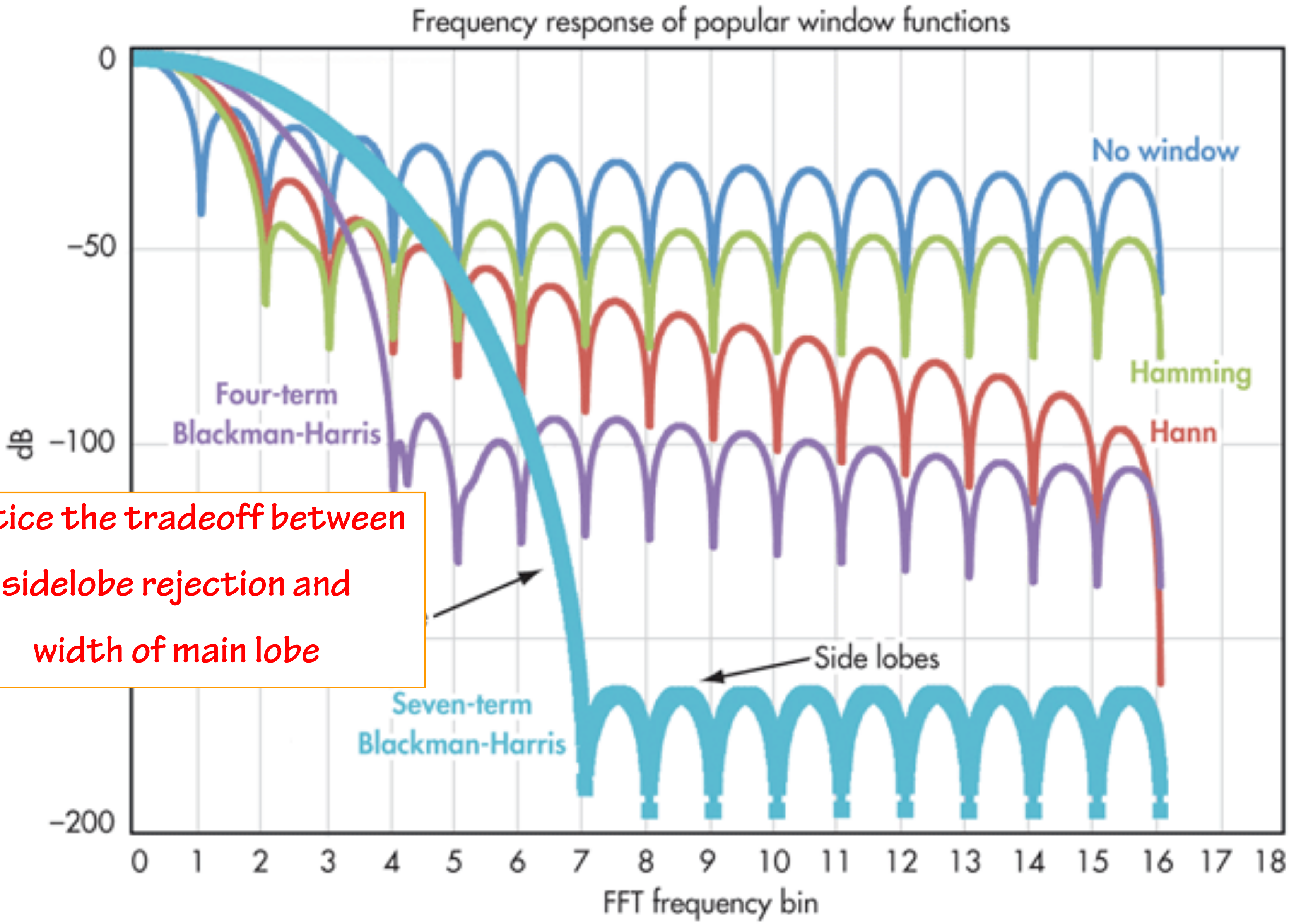
Highest side-lobes

Most spectral 'smearing'

Wider main peak, but

much lower side-lobes

Close-up view



Notice the tradeoff between sidelobe rejection and width of main lobe

Part 2: Time-Frequency Analysis

- Short-Time Fourier Transform
  - Find power spectrum of short windows
  - “Spectrogram”

- Advantage: Can visualize time-varying frequency content

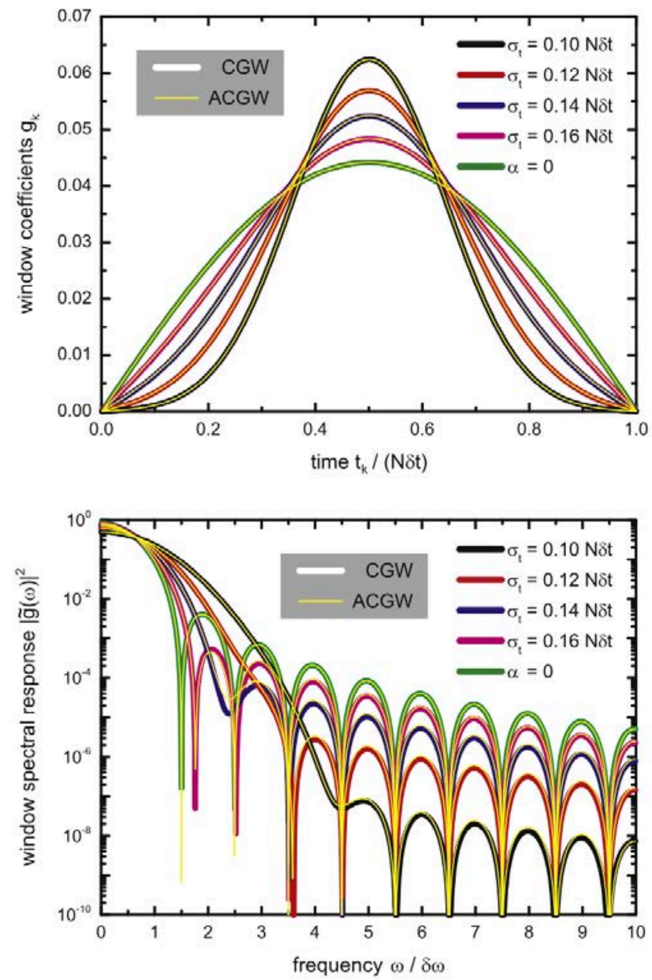
- Disadvantage: Fixed temporal resolution is not optimal

Time-Frequency Uncertainty

- You cannot have both arbitrarily good temporal and frequency resolution!
  - $\sigma_t * \sigma_f \geq 1/2$

- If you want sharper temporal resolution, you will sacrifice frequency resolution, and vice versa.

- (Optimal: Confined Gaussian)



Starosielec S, Hägele D (2014) Discrete-time windows with minimal RMS bandwidth for given RMS temporal width. Signal Processing 102:240–6.

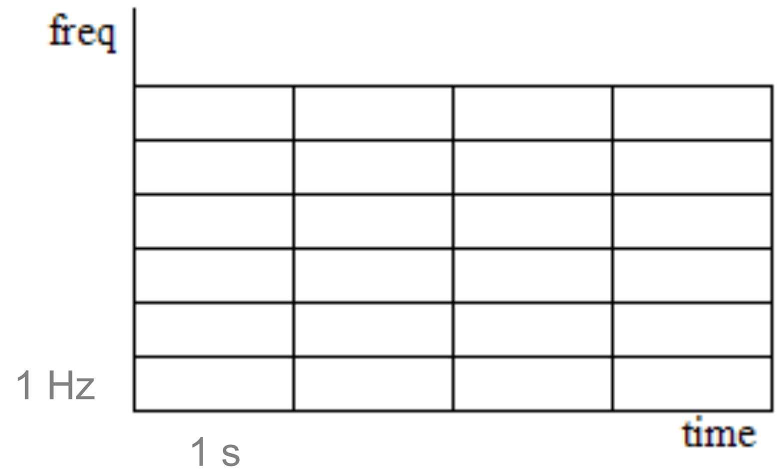
Consequence for STFT

Shorter Windows

poorer frequency resolution

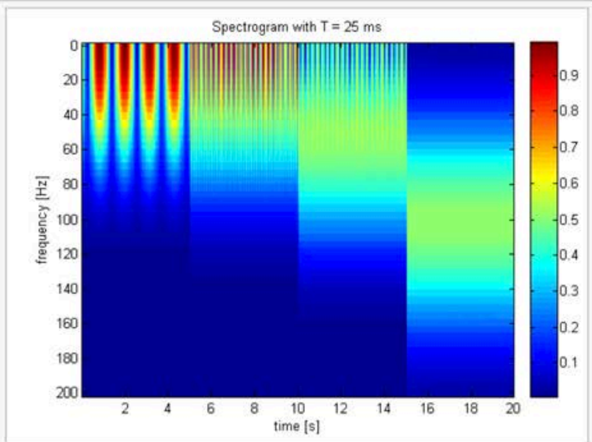
Longer Windows

finer frequency resolution

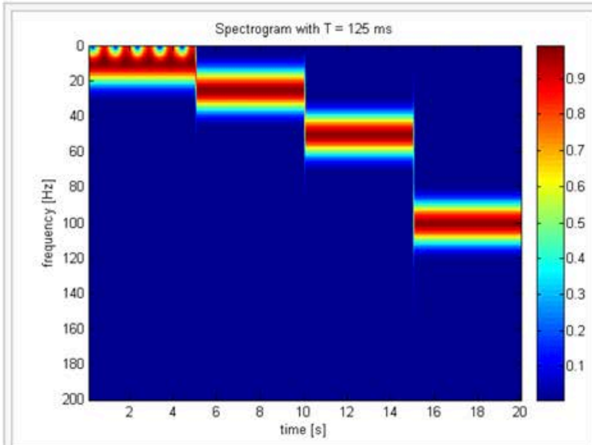


Time-Frequency Tradeoff

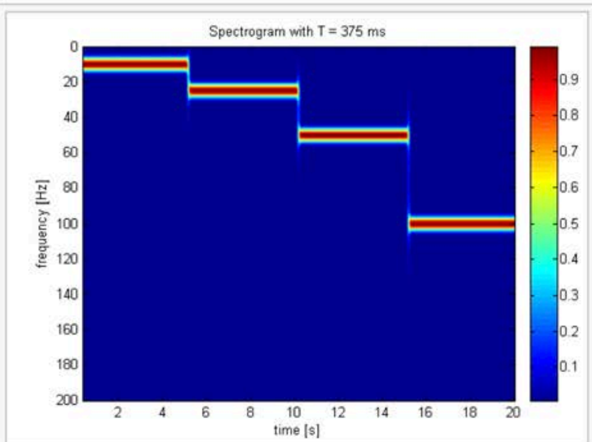
Signal: 10, 25, 50, 100 Hz



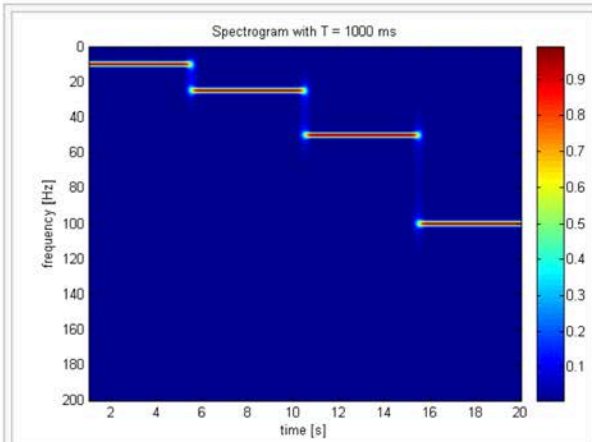
25 ms window



125 ms window



375 ms window

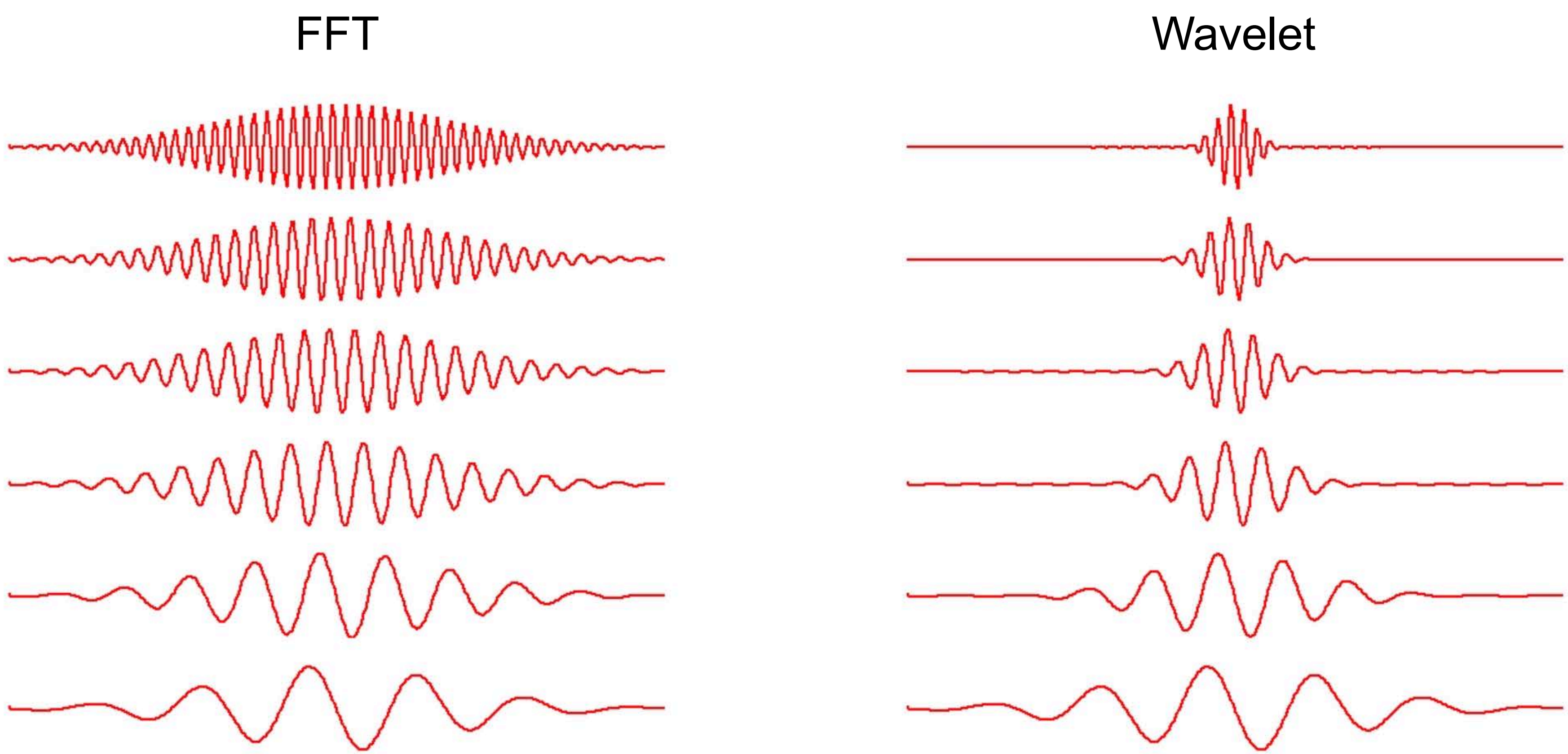


1000 ms window

A better way: Wavelet transform

- Wavelet transform is a ‘multi-resolution’ time-frequency decomposition.
- Intuition: Higher frequency signals have a faster time scale
- So, vary window length with frequency!
  - longer window at lower frequencies
  - shorter window at higher frequencies

Comparison of FFT & Wavelet

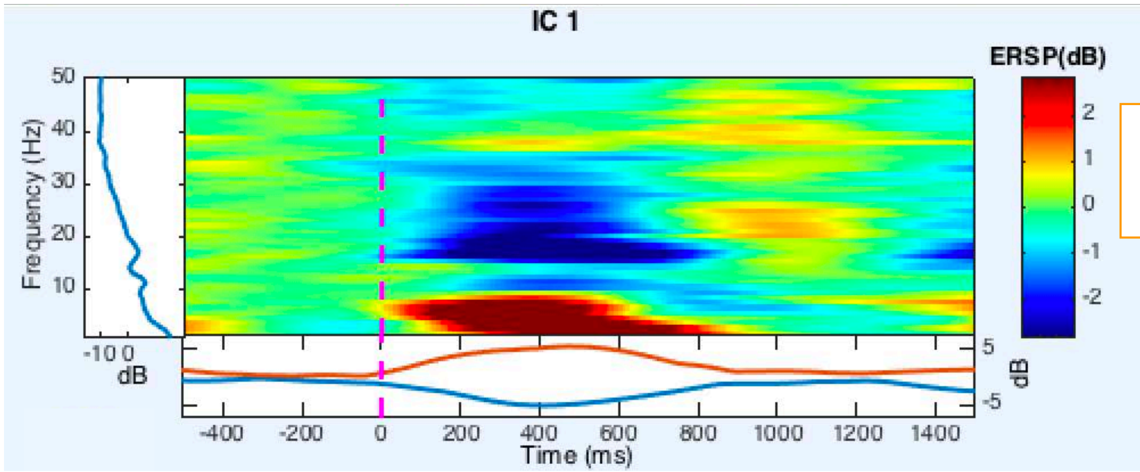


Scaled versions of one shape

Constant number of cycles

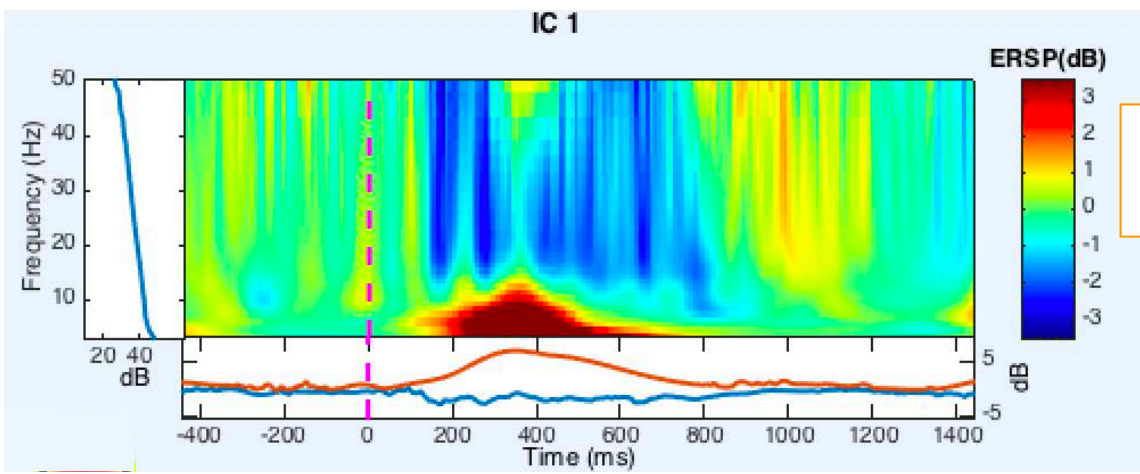
Comparison of FFT & Wavelet

FFT

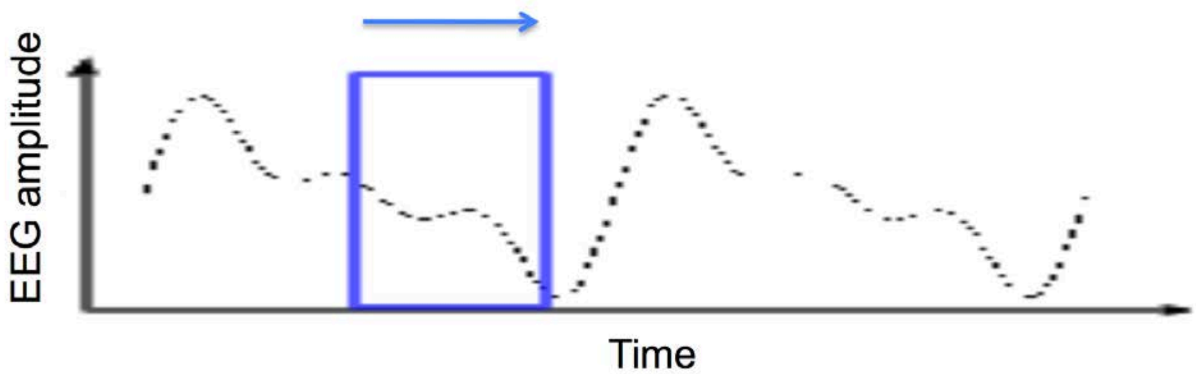


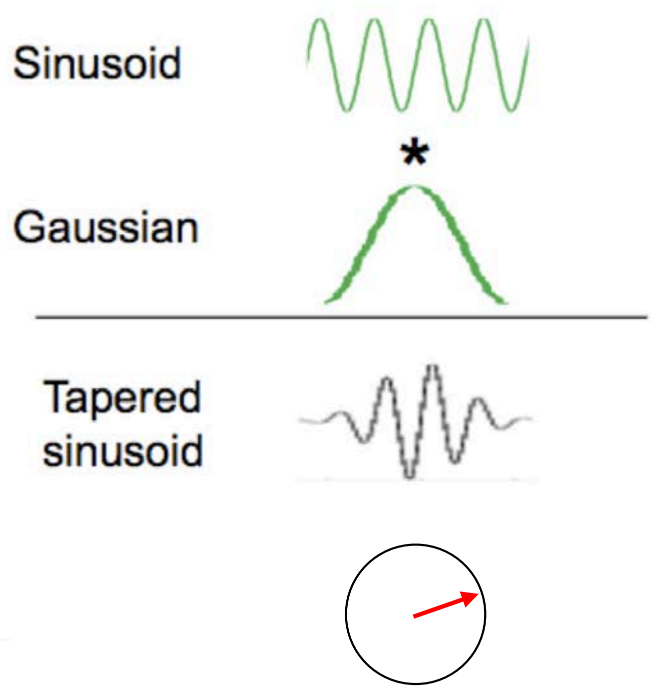
Similar time resolution across frequencies

Wavelet



Finer time resolution at high frequencies





For each time point

Analyze signal using the wavelets

for different frequencies.

Exercise

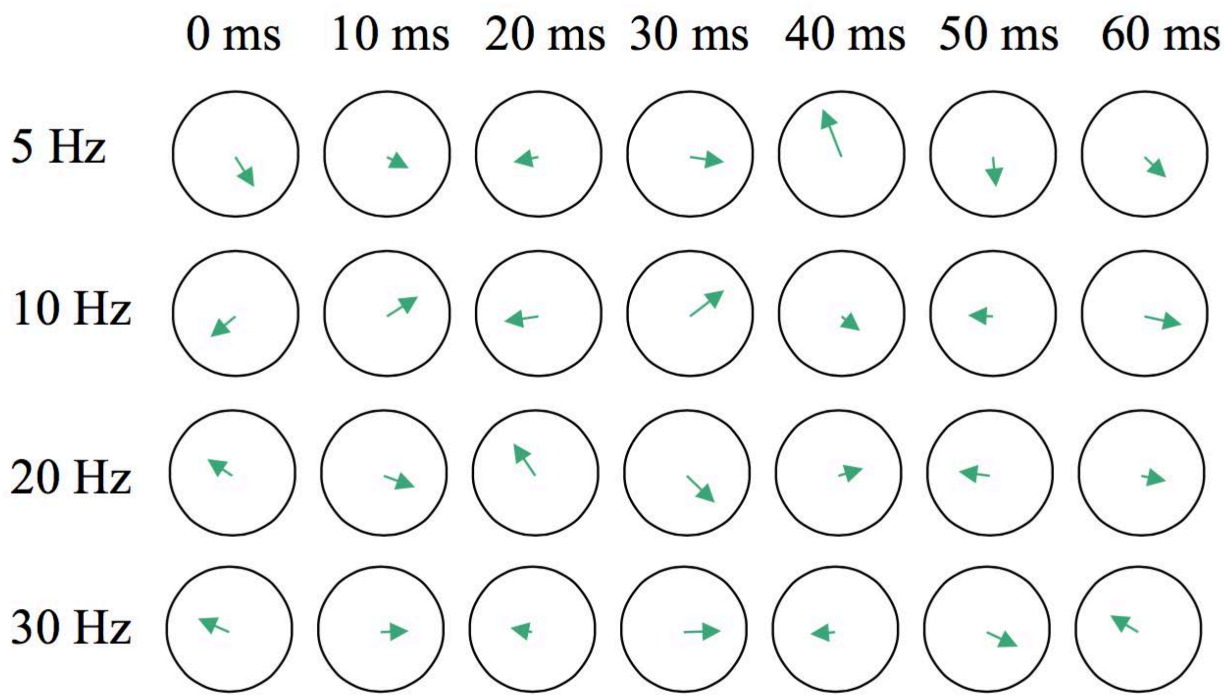
- Create a signal

```
>> t = 0:0.01:100;
>> x = sin(2*pi*10*t); plot(t,x)
```

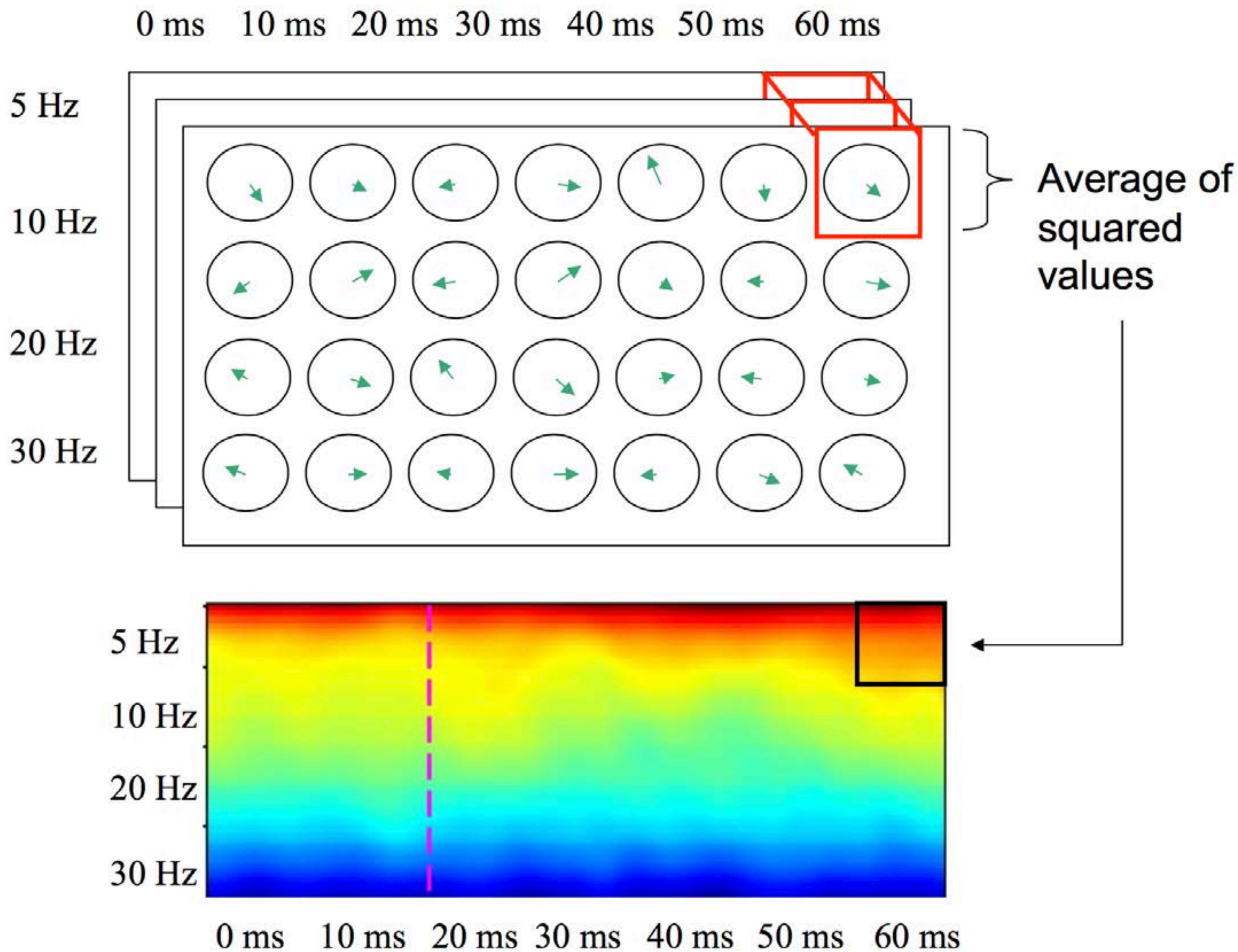
- Find FFT

```
>> F = fft(x);
>> F(1:3) %complex
>> power = F.*conj(F);
```

Spectrogram of one epoch of data



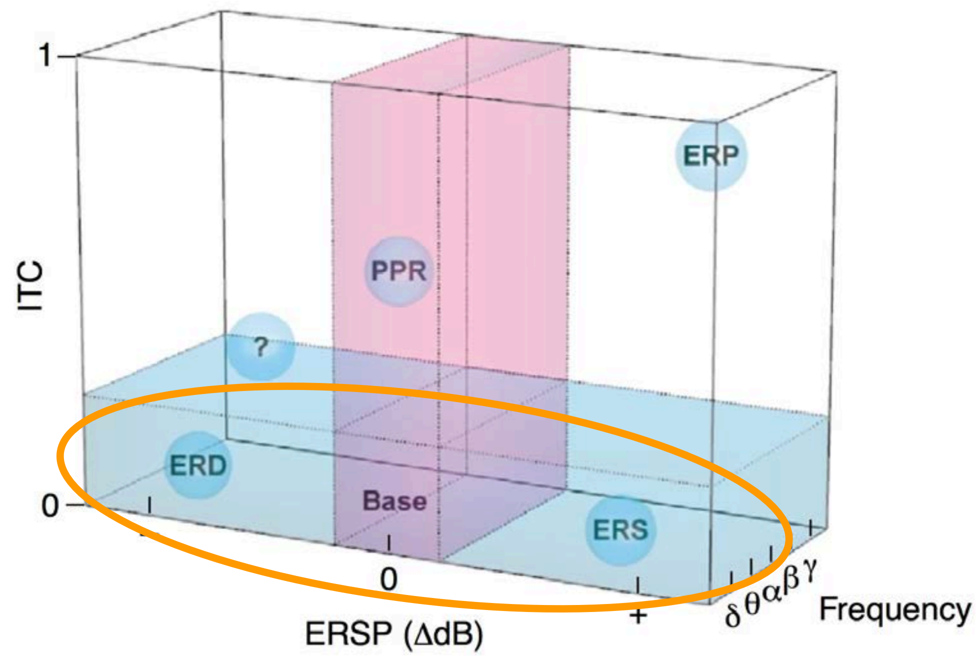
Computing Spectrogram Power



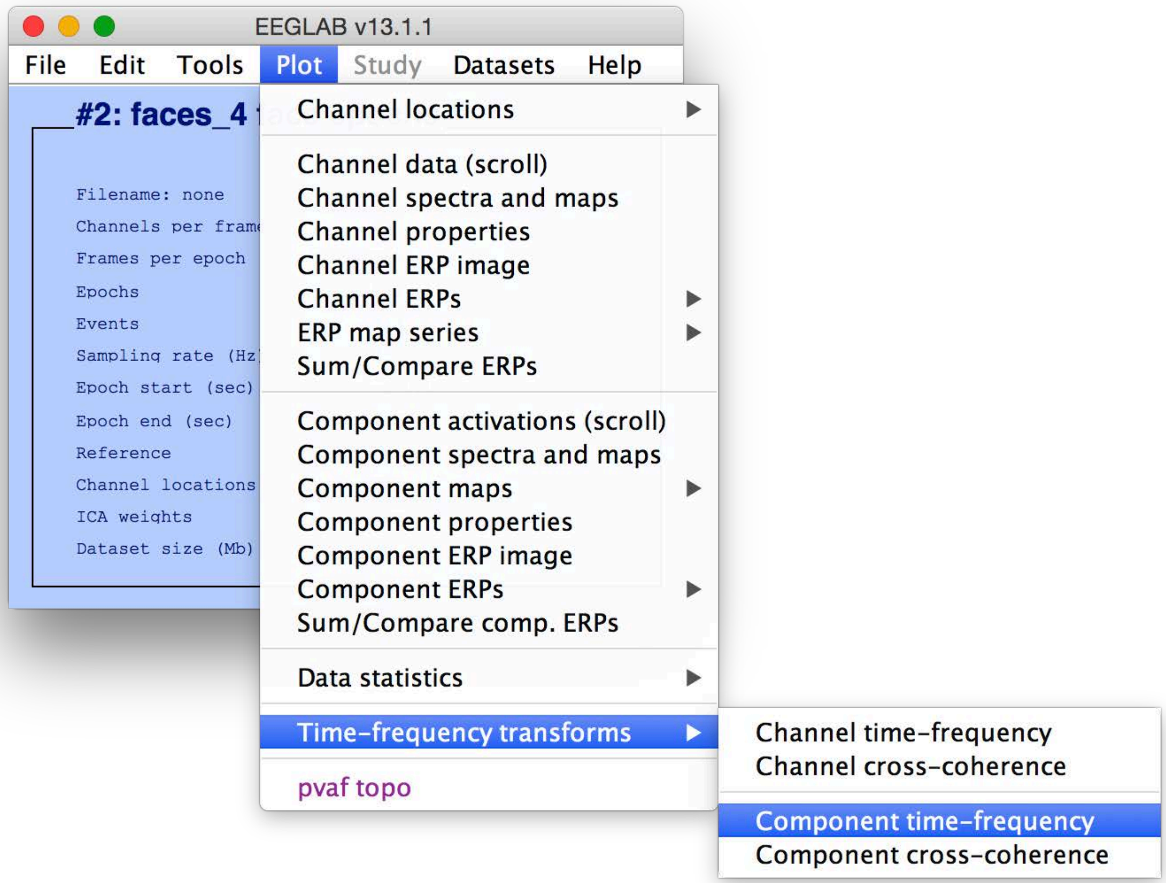
Definition: ERSP

- Event Related Spectral Perturbation

- Change in power in different frequency bands relative to a baseline. ERS (Event-Related *Synchronization*), ERD (Event-Related *Desynchronization*)



Try it out



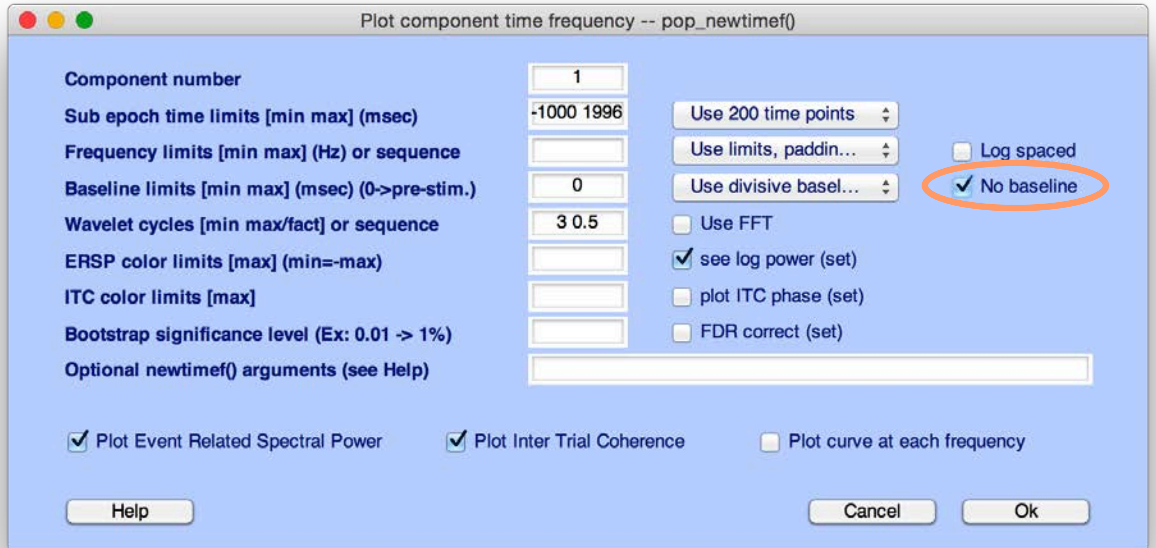
(Load faces_4.set

Epoch on 'face' event)

Display ERS vs. ERSP

Event-related

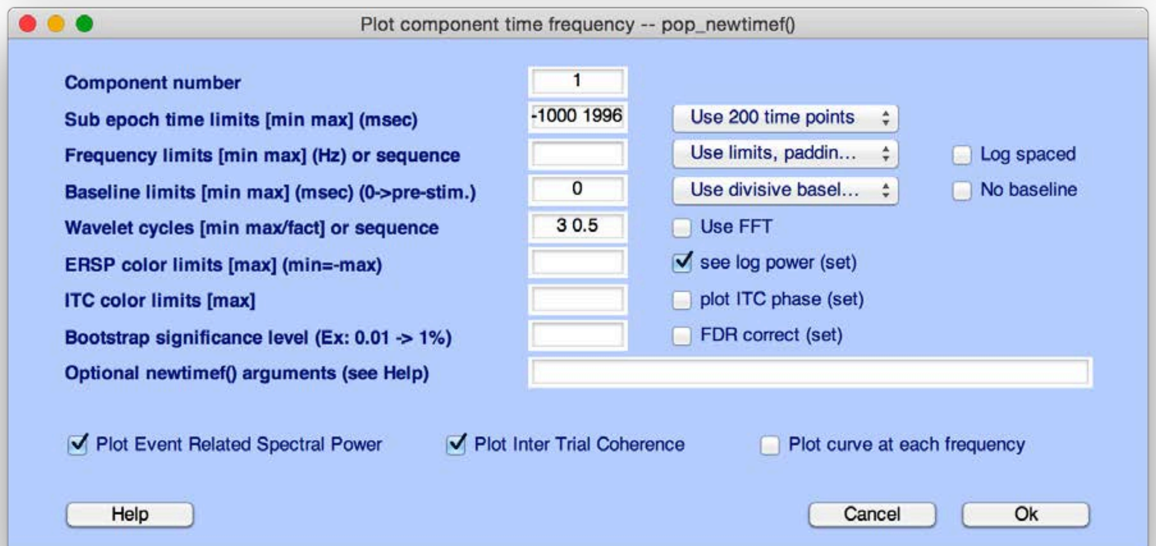
Spectrogram



Event-Related

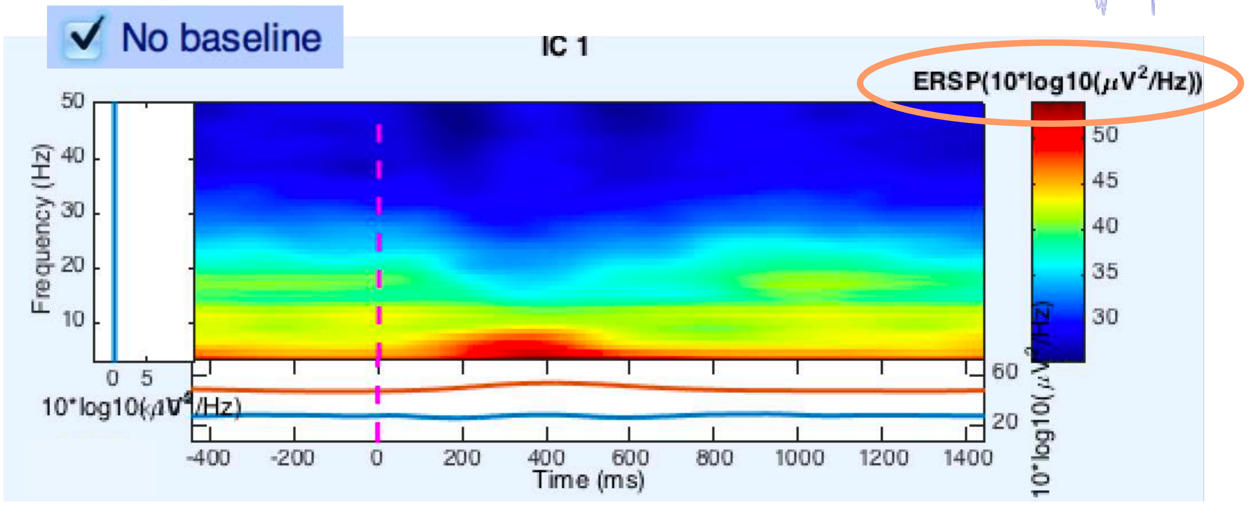
Spectral Perturbation

(ERSP)



Event-related

Spectrogram

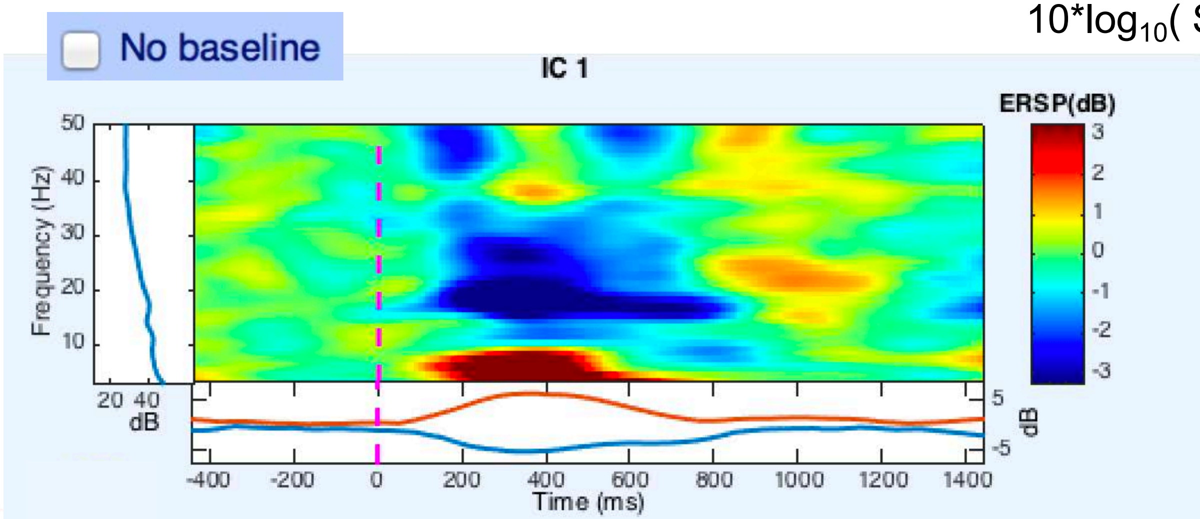


$10*\log_{10}$(SG(t,f) / baseline(f))

Event-Related

Spectral Perturbation

(ERSP)

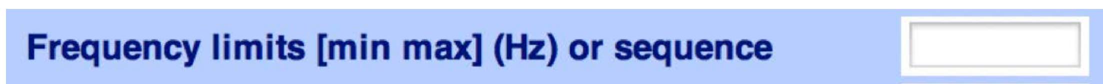


Exercises

- Try different wavelet specifications

  - Default: 3 0.5
    - 3 cycles. Try 2. How do the time limits of the plot change?
    - What is the 0.5? Try 0. Try 1…what do you observe?

- Try different low-frequency limit



  - what is the effect on the time limits of the ERSP?

- Try different baseline methods
  - divisive
  - standard deviation (express spectral perturbations in #sd relative to baseline sd)

Wavelet Specification

Wavelet cycles [min max/fact] or sequence | 3 0.5

Answer: The first #cycles controls the basic duration of the wavelet in cycles.

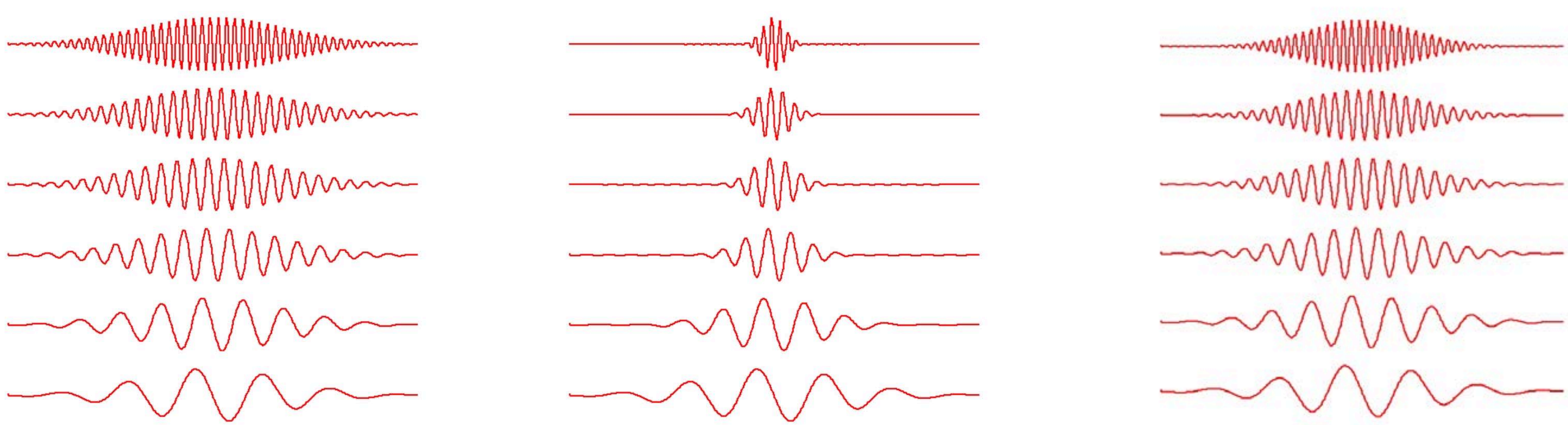
The second factor controls the degree of shortening of time windows as frequency increases

0 = no shortening = FFT (duration remains constant with frequency)

1 = pure wavelet (#cycles remains constant with frequency)

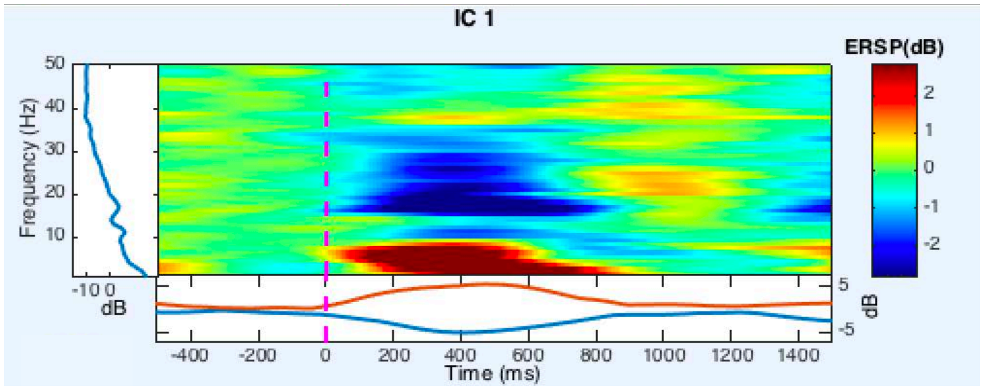
0.5 = intermediate, a compromise that reduces HF time resolution to gain more frequency resolution.

3 0 | 3 1 | 3 0.5

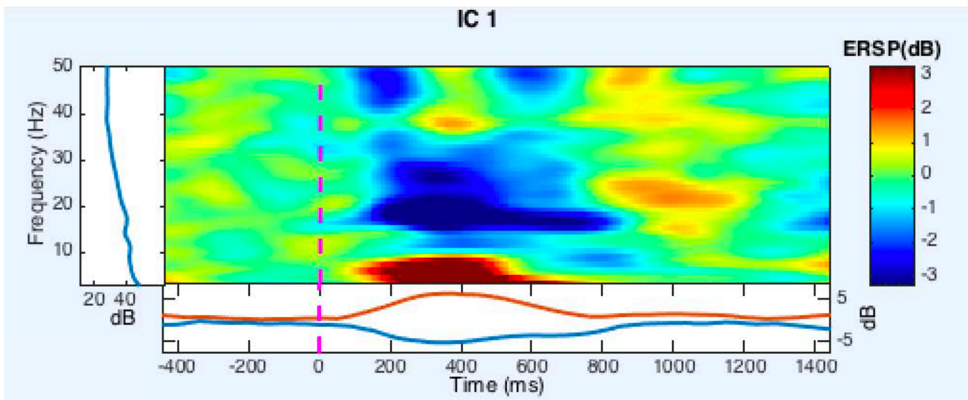


Comparison of FFT & Wavelet

[3 0] (FFT)



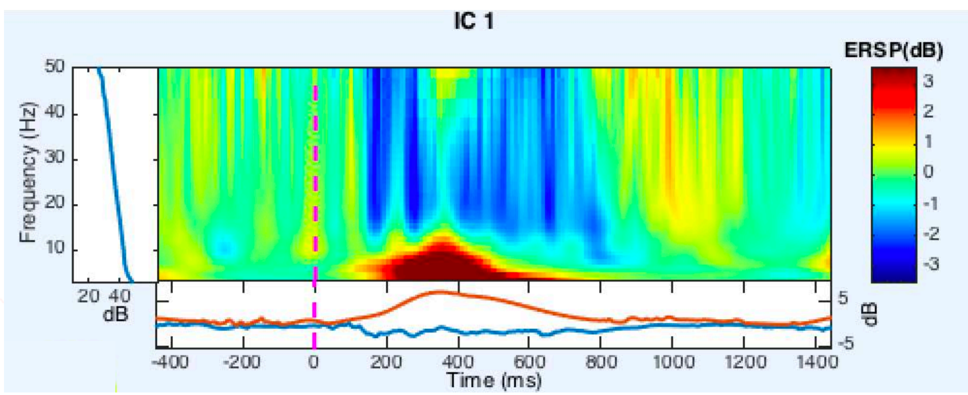
[3 0.5] Wavelet



This is the EEGLAB default

Notice: features have similar time and frequency resolution

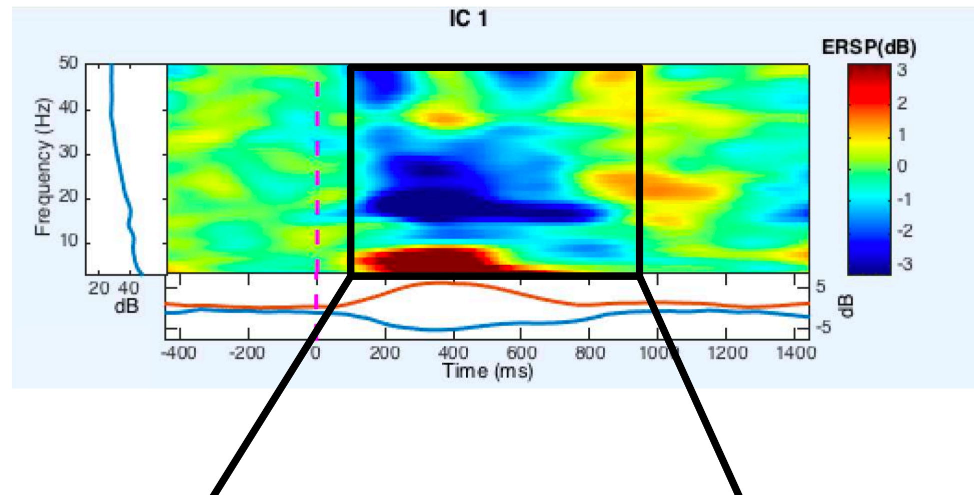
[3 1] Wavelet



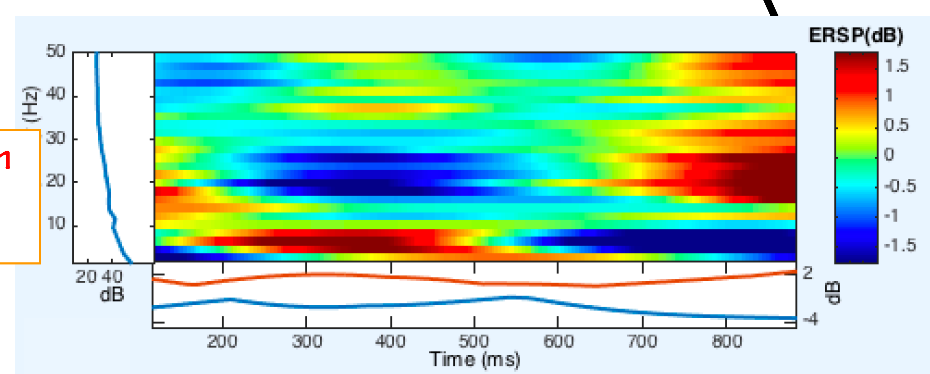
Time loss at edge of ERSP

- Settings for wavelet cycles and lowest frequency impact the time limits of analysis

MIN FREQ: 3 Hz



MIN FREQ: 1 Hz



more wavelet cycles, or a lower minimum frequency loses time at edges of epoch

Solution: If you need low frequencies, be sure to extract longer epochs to counteract this. Barring this, try reducing the number of wavelet cycles.

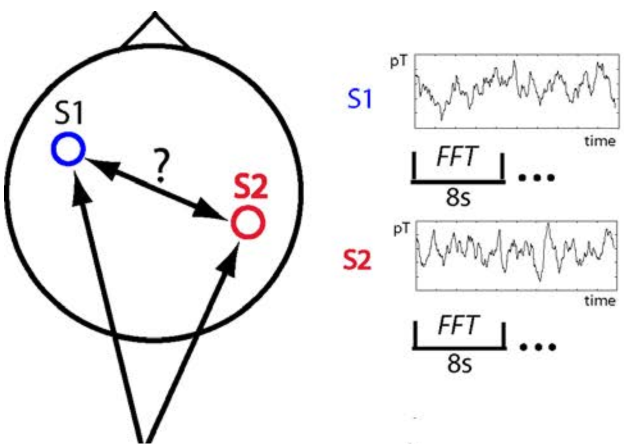
Part 3: Coherence Analysis

- Goal: How much do two signals resemble each other?
- Coherence = complex version of correlation: how similar are power and phase at each frequency?
- Variant: phase coherence (phase locking, etc.) considers only phase similarity, ignoring power
  - Regular coherence is simply a power-weighted phase coherence

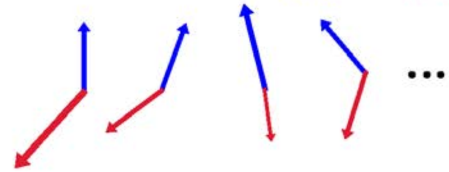
Coherence

$$C(f,t) \propto \sum_{k=trials} F1_k(f,t)\overline{F2_k(f,t)}$$

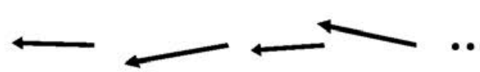
$$a_1 e^{i\theta_1} a_2 e^{-i\theta_2} \propto e^{i(\theta_1-\theta_2)}$$

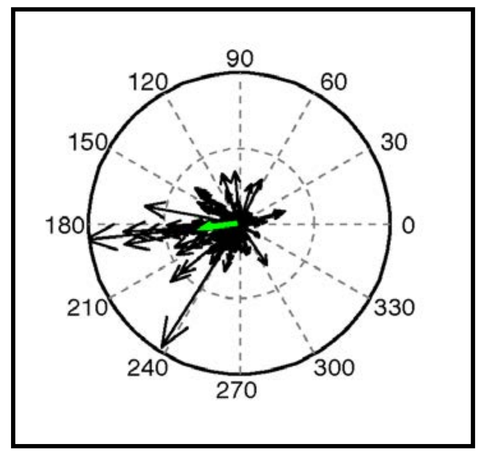


Fourier time series F_{S1} and F_{S2}



Phase difference between S1 and S2,





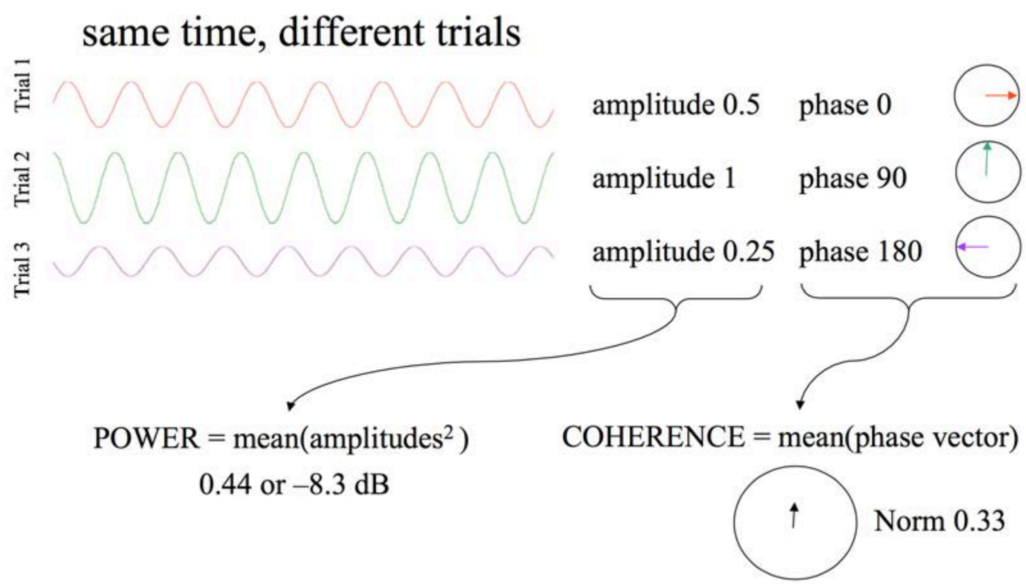
Part 3a: Inter-Trial Coherence

- Goal: How much do different trials resemble each other?
- Phase coherence not between two processes, but between multiple trials of the same process
- Defined over a (generally) narrow frequency range

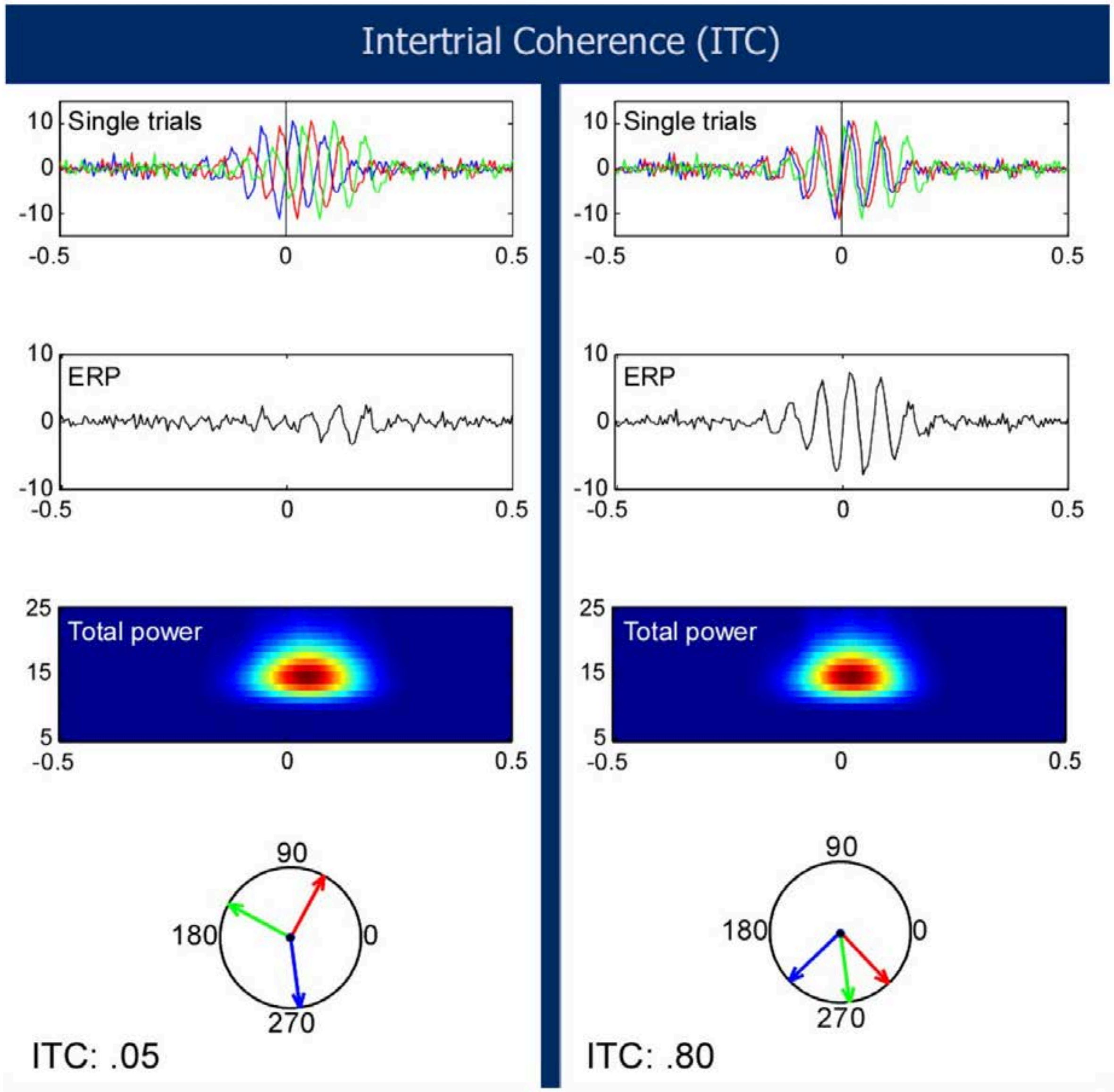
EEGLAB's Inter-Trial Coherence is *phase* ITC

Phase ITC

$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\underbrace{\frac{F_k(f,t)}{|F_k(f,t)|}}_{\substack{\text{Normalized} \\ \text{(no amplitude information)}}}$$



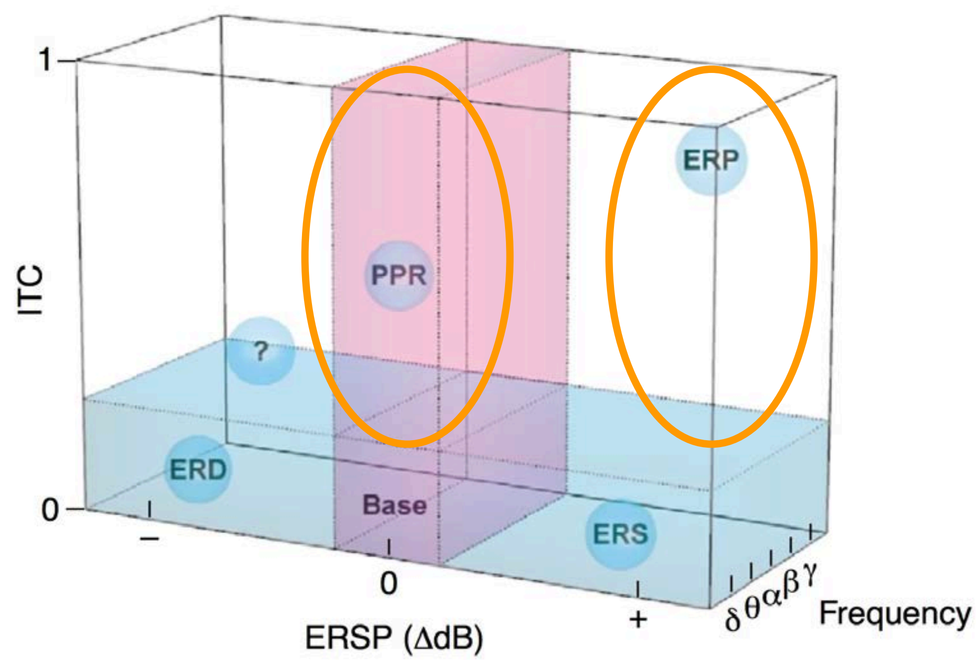
ITC Example (3 trials)

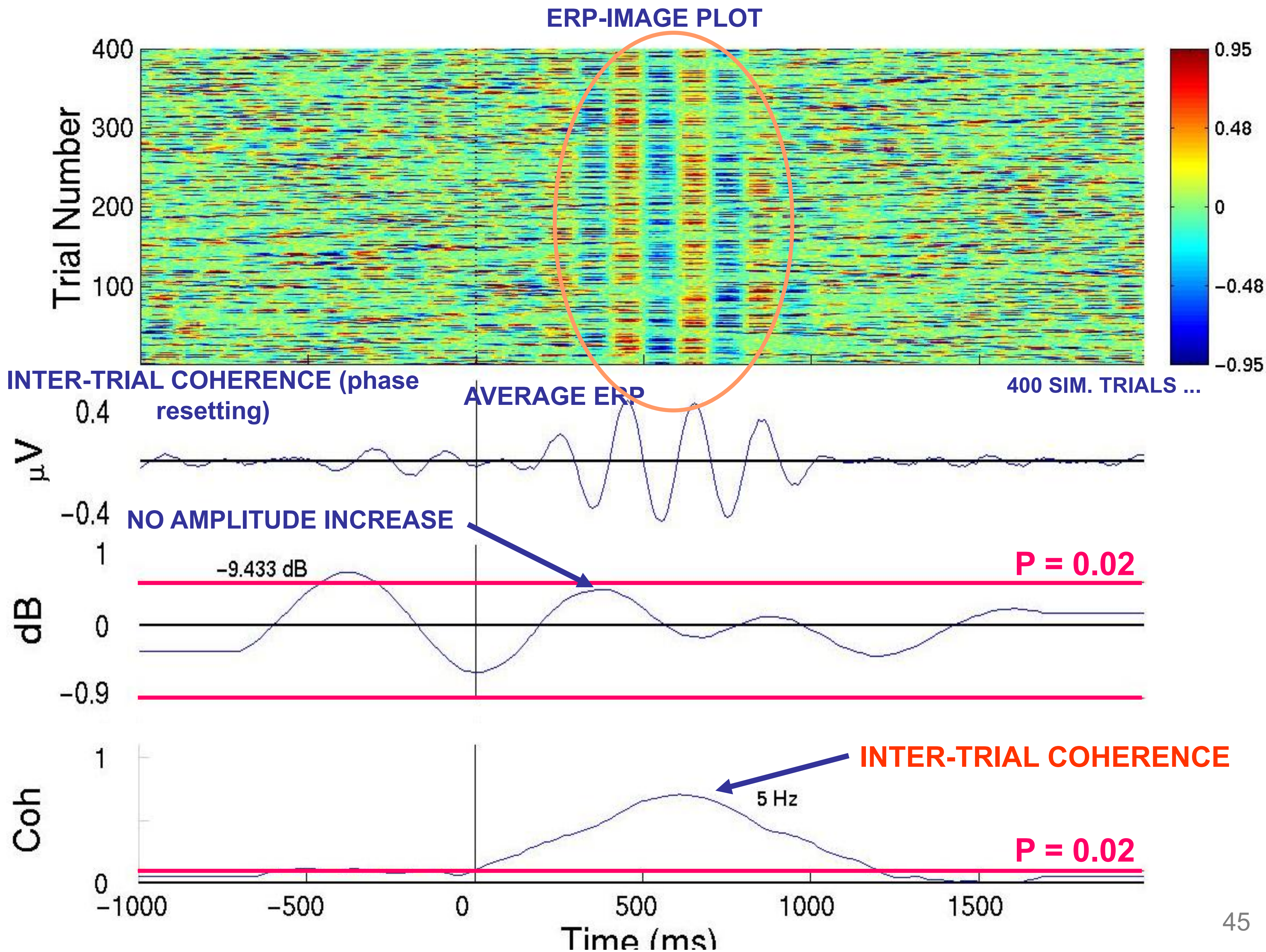


Slide courtesy of Stefan Debener

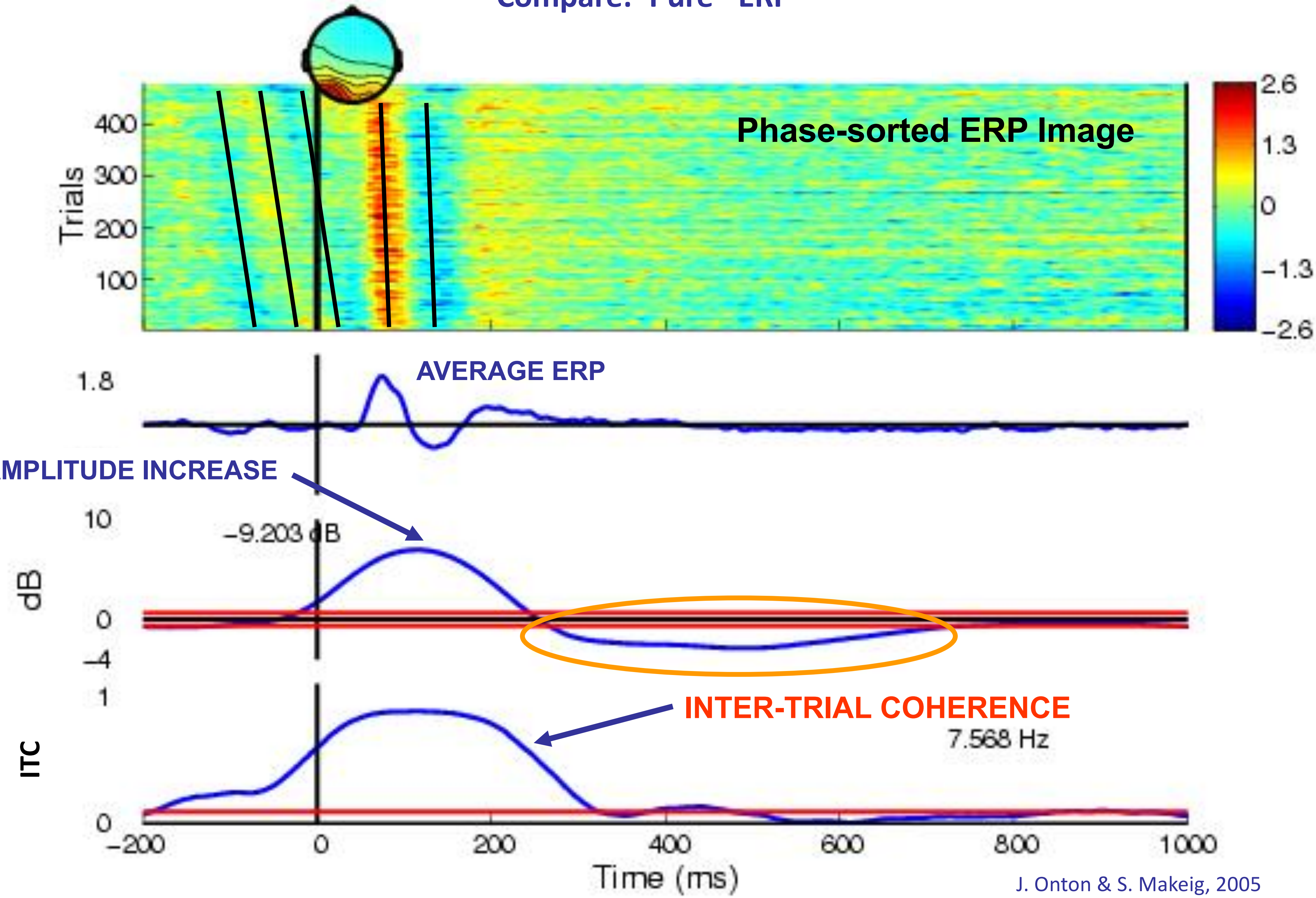
Several possible origins of an ERP

- Event Related Potential can result from
  - ITC increase (with no change in power)
  - ITC & Power change



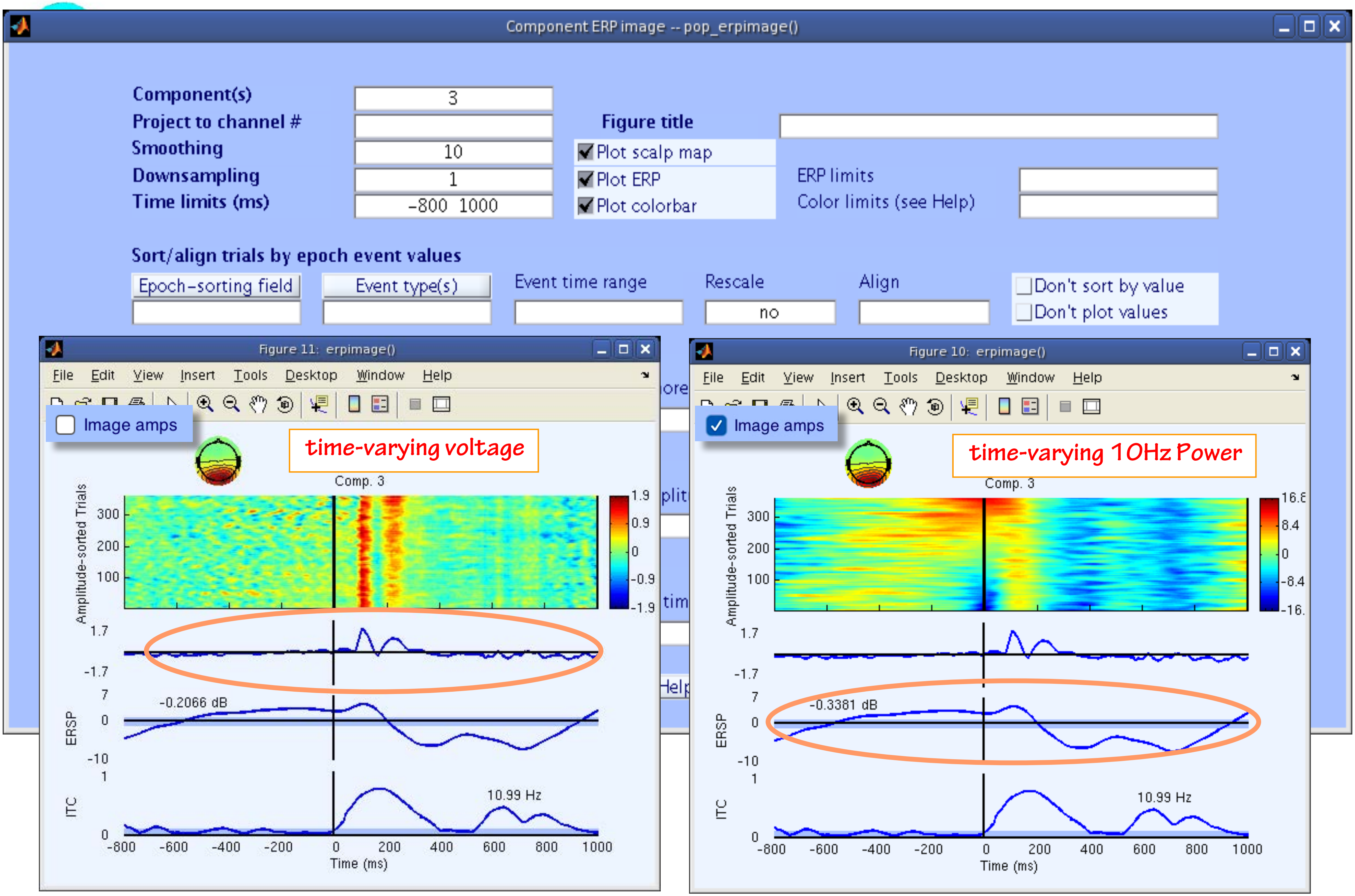

ERP-IMAGE PLOT
Trial Number
400
300
200
100
0.95
0.48
0
−0.48
−0.95
INTER-TRIAL COHERENCE (phase resetting)
AVERAGE ERP
400 SIM. TRIALS ...
0.4
μV
−0.4
NO AMPLITUDE INCREASE
1
−9.433 dB
P = 0.02
dB
0
−0.9
INTER-TRIAL COHERENCE
1
5 Hz
Coh
P = 0.02
0
−1000
−500
0
500
1000
1500
Time (ms)

Compare: 'Pure' ERP

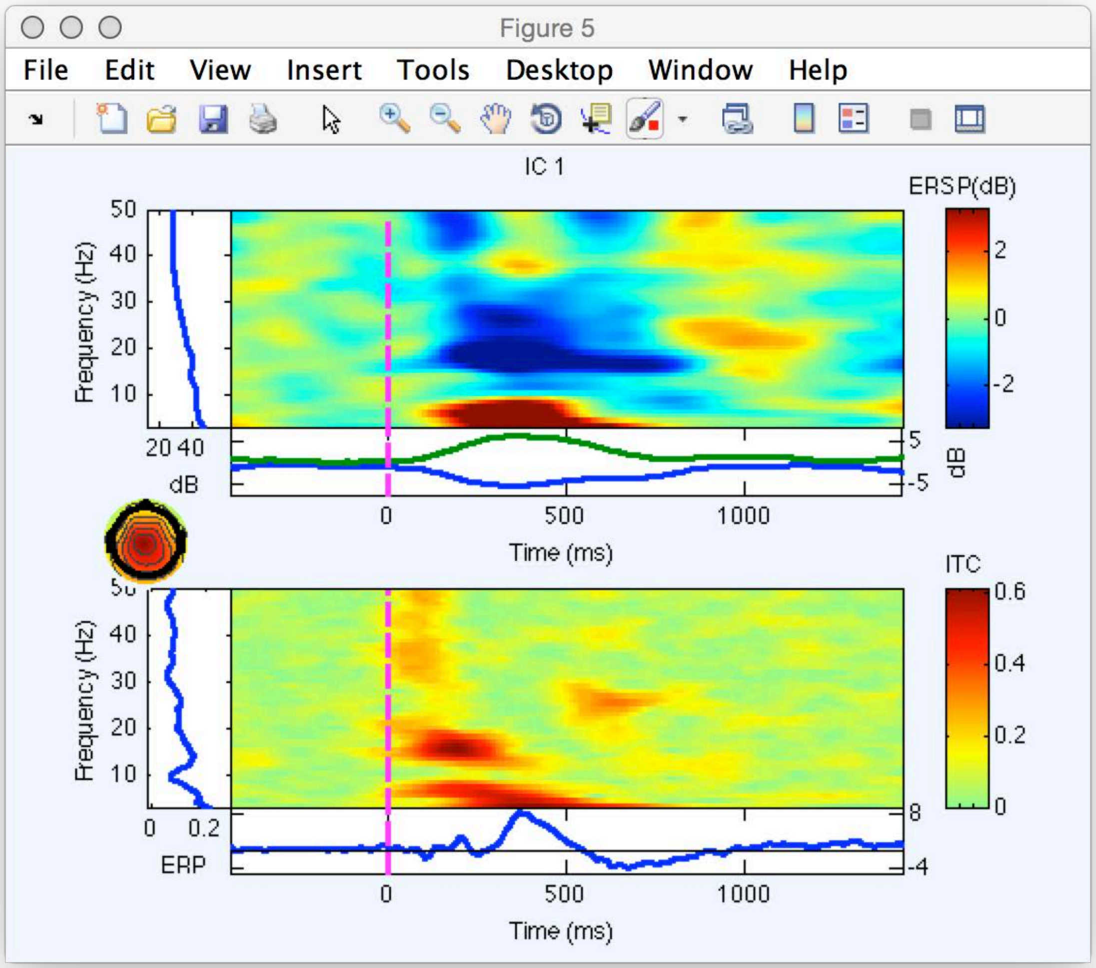


J. Onton & S. Makeig, 2005

Component ERP Image: Activation vs. Amplitude



Putting it all together



Exercise

All: Compute ERSP/ITC for a component of your choice

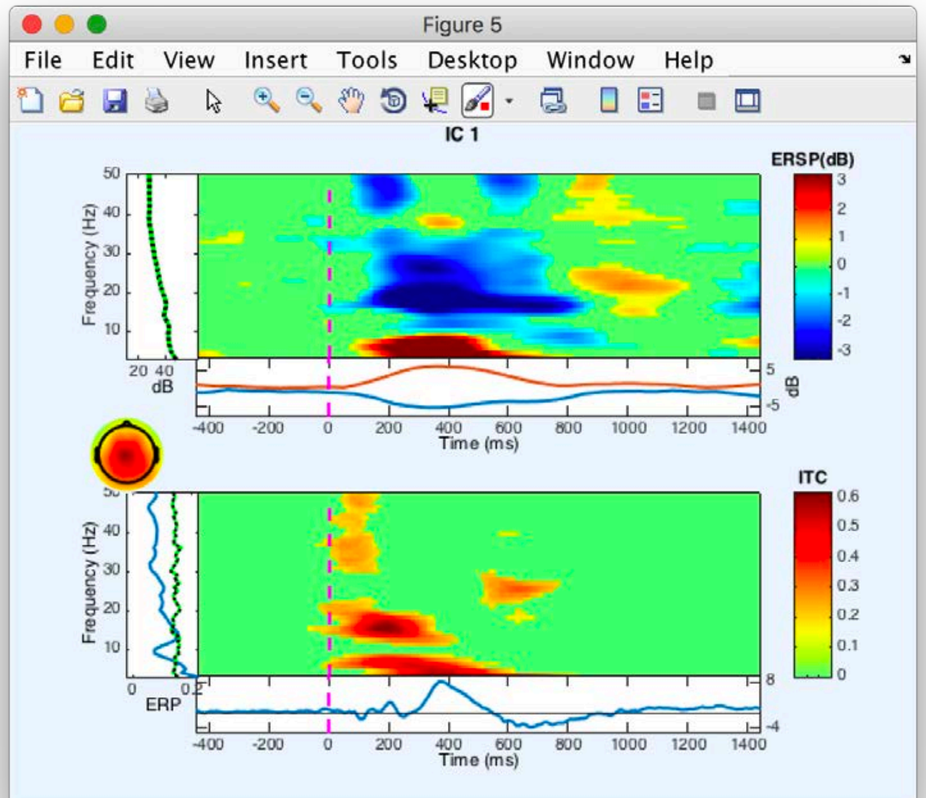
Compute ERP Image (with ERSP and ITC displayed*)

Use all of this information to explain the origin of the Evoked Response

Question: Which changes are significant? Use the options in ERP Image and ERSP dialogs to set significance threshold e.g. 0.01. Do the results survive?

Significance Testing

- Keep in mind: "is this significant?"



Method: Bootstrap

Green areas are not significant.

Scale of ERSP & ITC vales also give a clue:

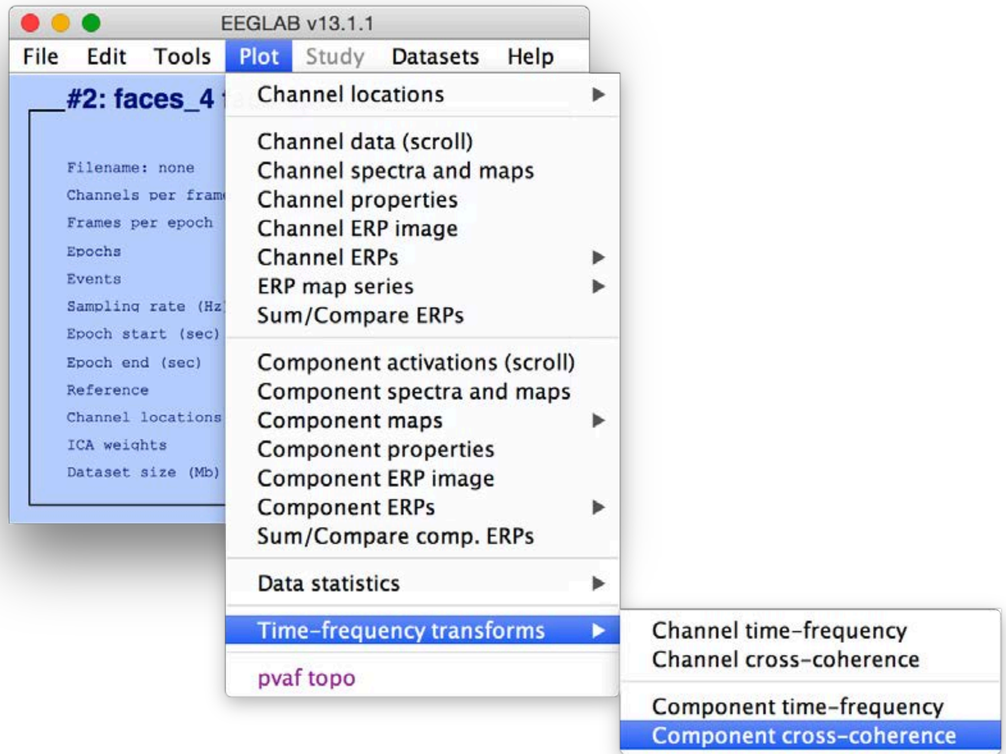
Large values are often encouraging of a significant effect

(Large ≈ > 1dB for ERSP; > 0.5 for ITC)

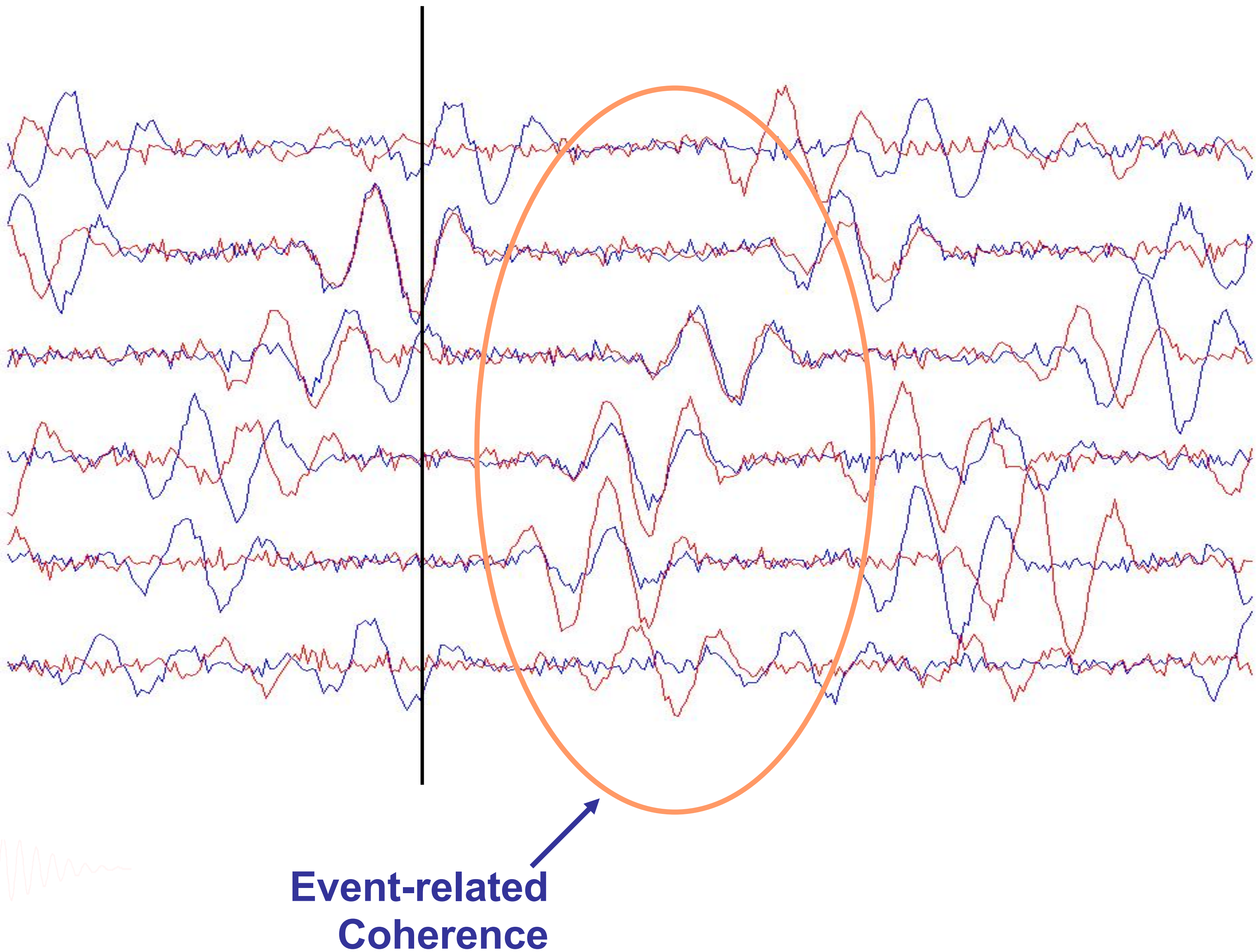
For exploratory purposes, can try 0.01 without FDR correction

Part 3b: Event Related Coherence

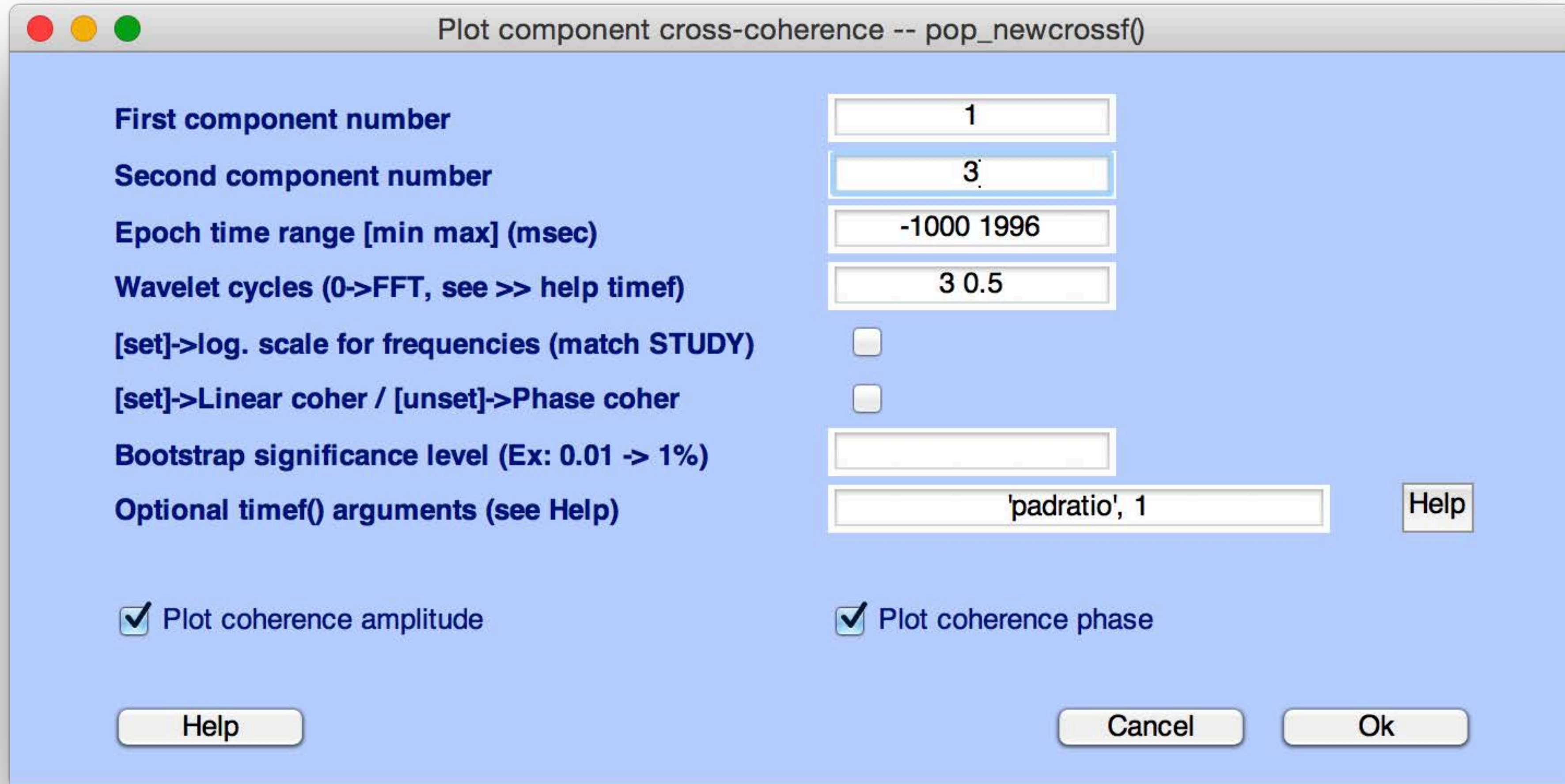
- Goal: How similar is the event-related response of two signals?
  - Between channels (problematic due to volume conduction)
  - Between ICs
  - Useful to quickly begin to understand relationships between components



TWO SIMULATED THETA PROCESSES

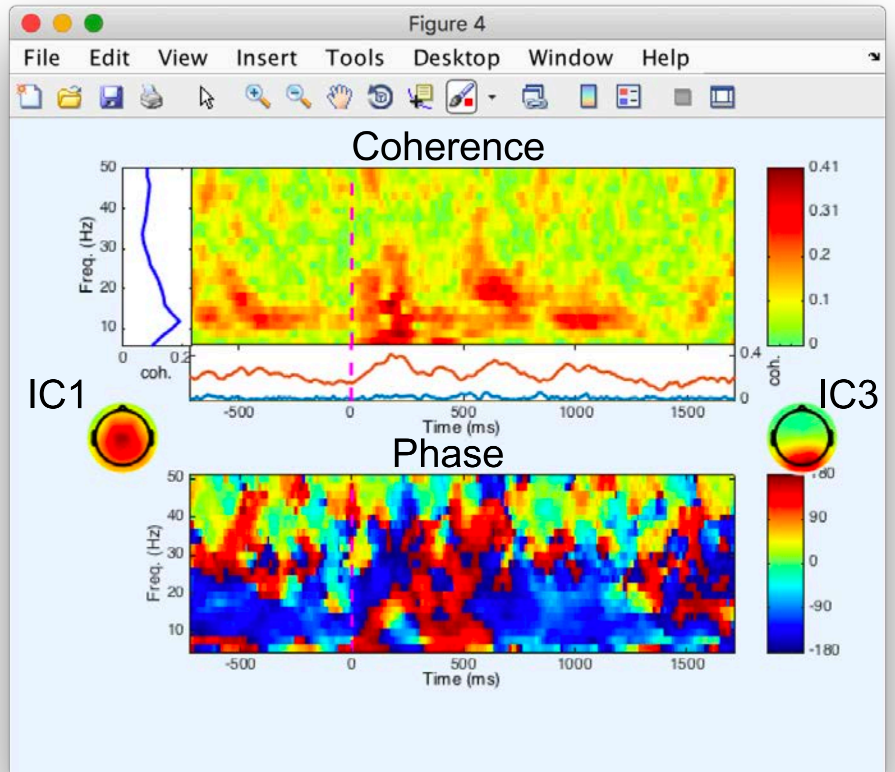


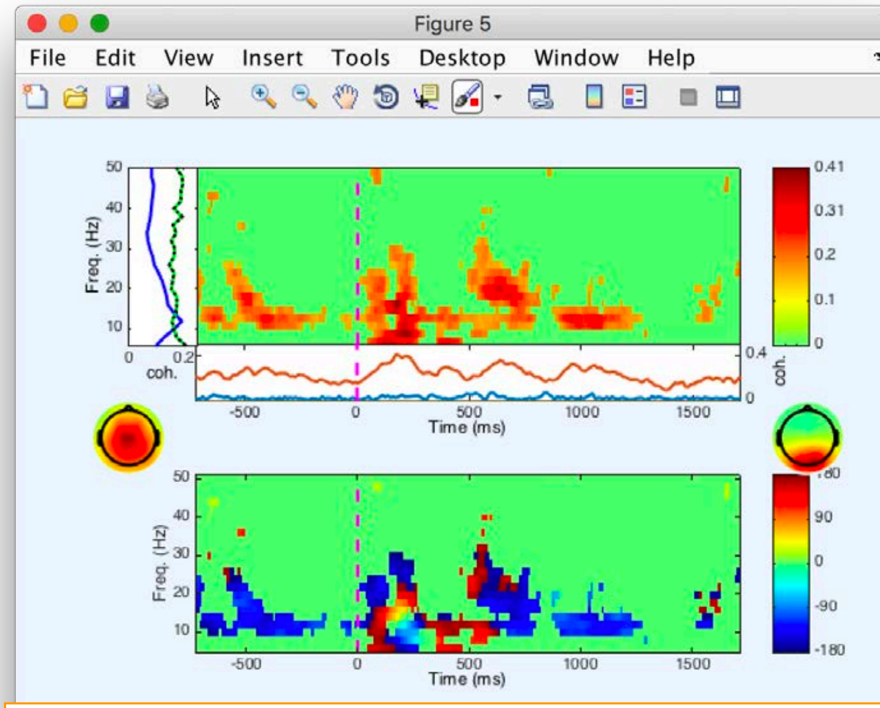
Try it!


Plot component cross-coherence -- pop_newcrossf()
First component number
1
Second component number
3
Epoch time range [min max] (msec)
-1000 1996
Wavelet cycles (0->FFT, see >> help timef)
3 0.5
[set]->log. scale for frequencies (match STUDY)
[set]->Linear coher / [unset]->Phase coher
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional timef() arguments (see Help)
'padratio', 1
Help
Plot coherence amplitude
Plot coherence phase
Help
Cancel
Ok

Cross coherence between IC 1 and IC 3

$\alpha = 0.01$





Significant event-related coherence (as well as tonic coherence) in alpha/beta bands

IC 1 tonically leads IC 3 (negative phase), but phase relationships are changed post-stimulus

More advanced, directional, measures of effective connectivity are present in the SIFT toolbox (a later lecture).

Event-Related Coherence Exercise

- Examine event-related coherence between two ICs
  - Which pair did you pick, and why? What do you predict?
  - What did you learn?

- Explore other options:
  - Significance threshold
  - Figure out how to subtract a baseline
  - Phase vs. Linear Coherence