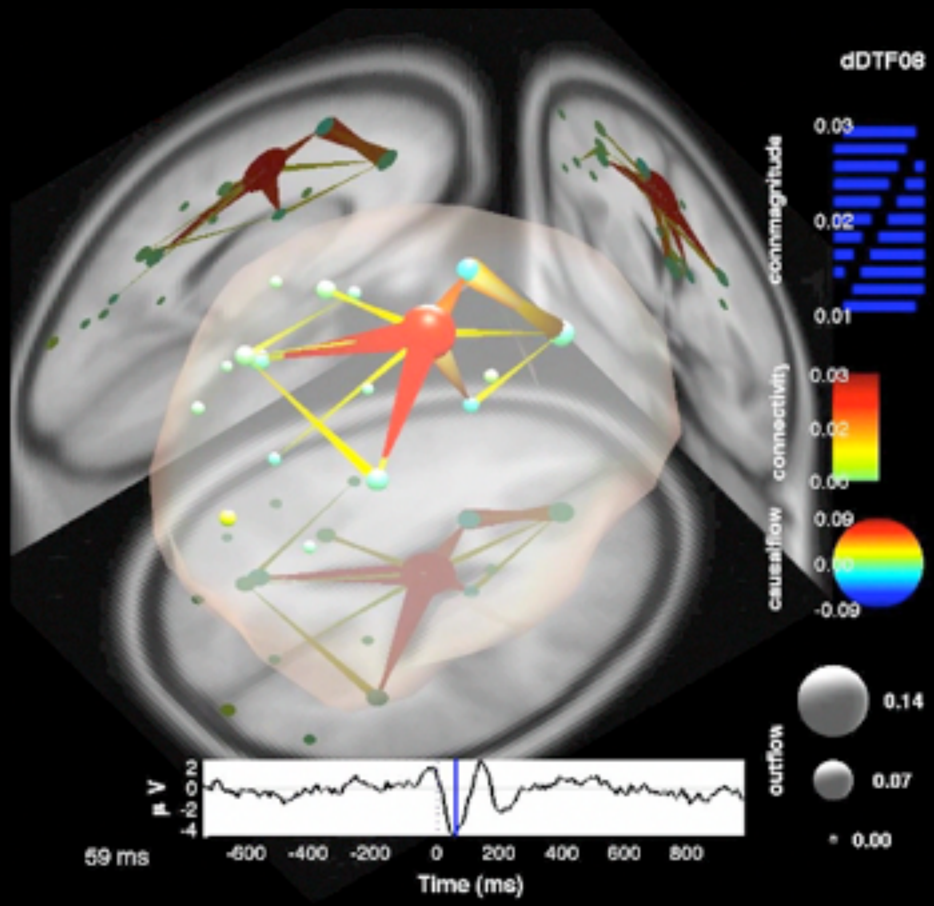
Analyzing Oscillatory EEG Source Dynamics and Interactions using SIFT

Tim Mullen

14th EEGLAB Workshop
Mallorca, Spain (ICON XI)
Sept 22-25, 2011

Swartz Center for Computational Neuroscience



Categorizations of Large-Scale Brain Connectivity Analysis

(Bullmore and Sporns, *Nature*, 2009)

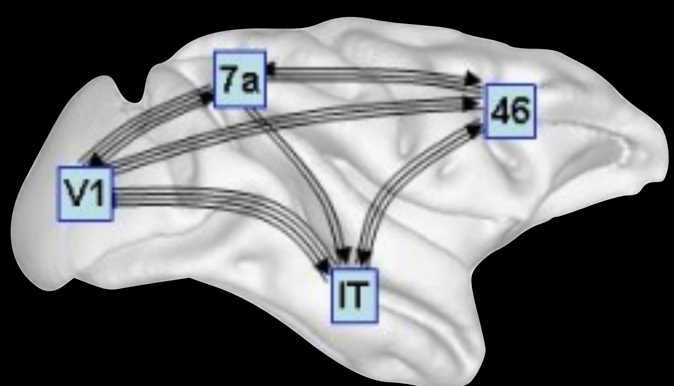
Hours-Years	**milliseconds-seconds**

Temporal Scale

Categorizations of Large-Scale Brain Connectivity Analysis

(Bullmore and Sporns, *Nature*, 2009)

Structural



state-invariant,
anatomical

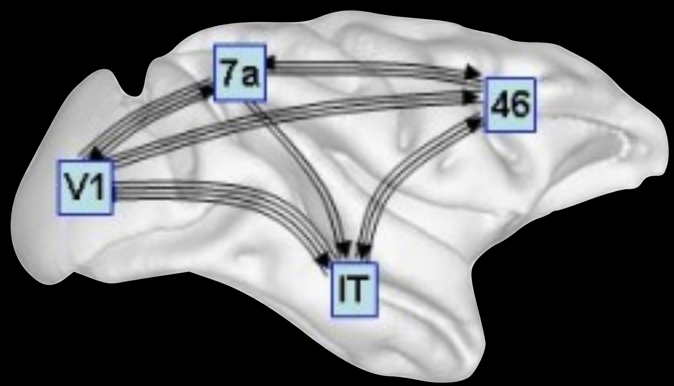
Hours-Years	**milliseconds-seconds**

Temporal Scale

Categorizations of Large-Scale Brain Connectivity Analysis

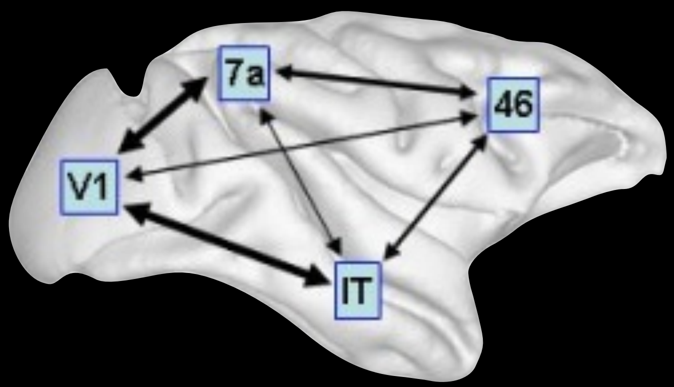
(Bullmore and Sporns, *Nature*, 2009)

Structural



state-invariant,
anatomical

Functional



dynamic, state-dependent,
correlative, symmetric

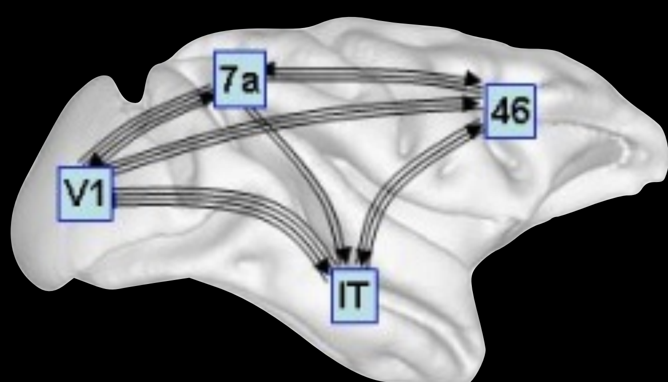
Hours-Years | **milliseconds-seconds**

Temporal Scale

Categorizations of Large-Scale Brain Connectivity Analysis

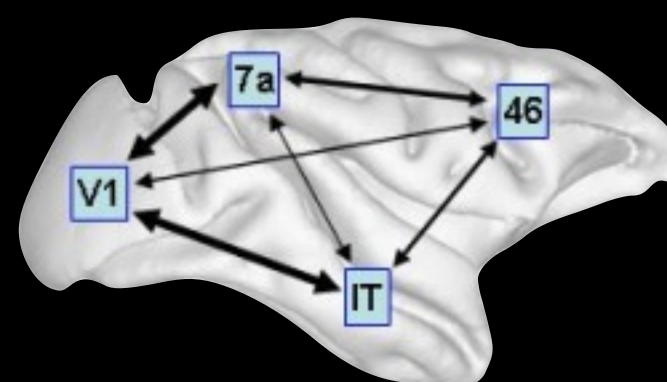
(Bullmore and Sporns, *Nature*, 2009)

Structural



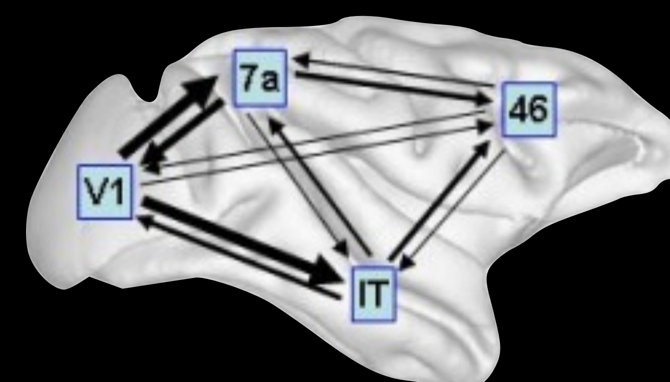
state-invariant, anatomical

Functional



dynamic, state-dependent, correlative, symmetric

Effective



dynamic, state-dependent, asymmetric, causal, information flow

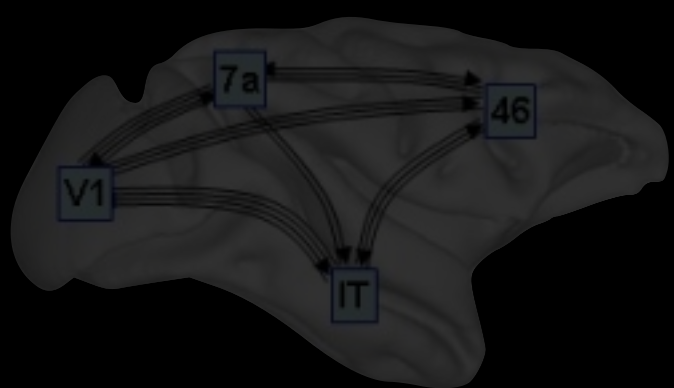
Hours-Years | **milliseconds-seconds**

Temporal Scale

Categorizations of Large-Scale Brain Connectivity Analysis

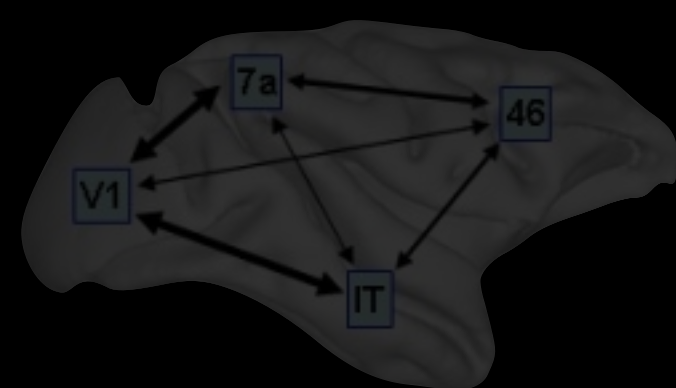
(Bullmore and Sporns, *Nature*, 2009)

Structural



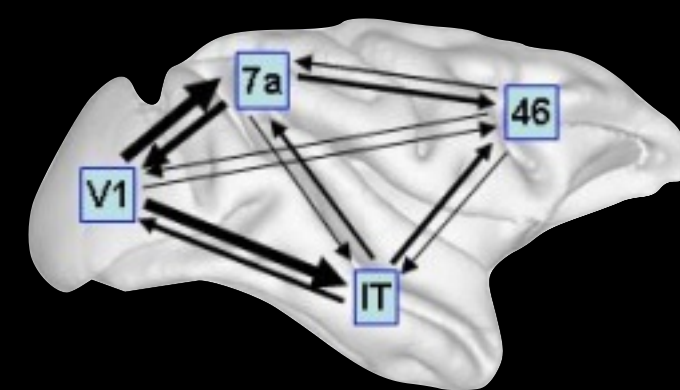
state-invariant, anatomical

Functional



dynamic, state-dependent, correlative, symmetric

Effective



dynamic, state-dependent, asymmetric, causal, information flow

Hours-Years | **milliseconds-seconds**

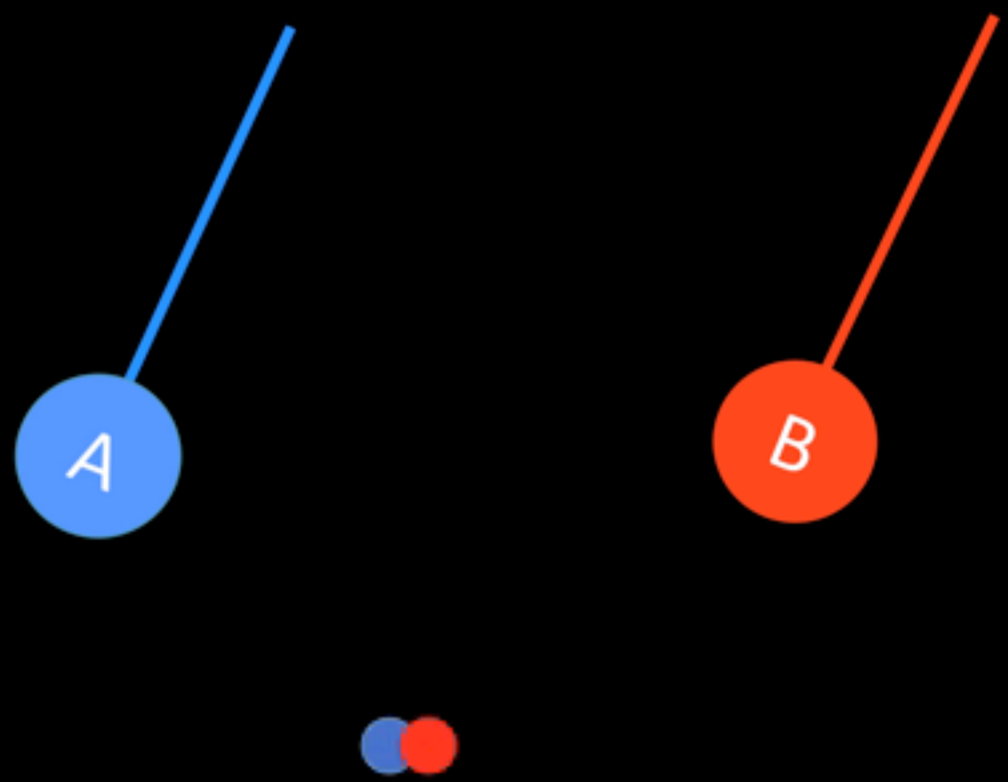
Temporal Scale

Estimating Functional Connectivity

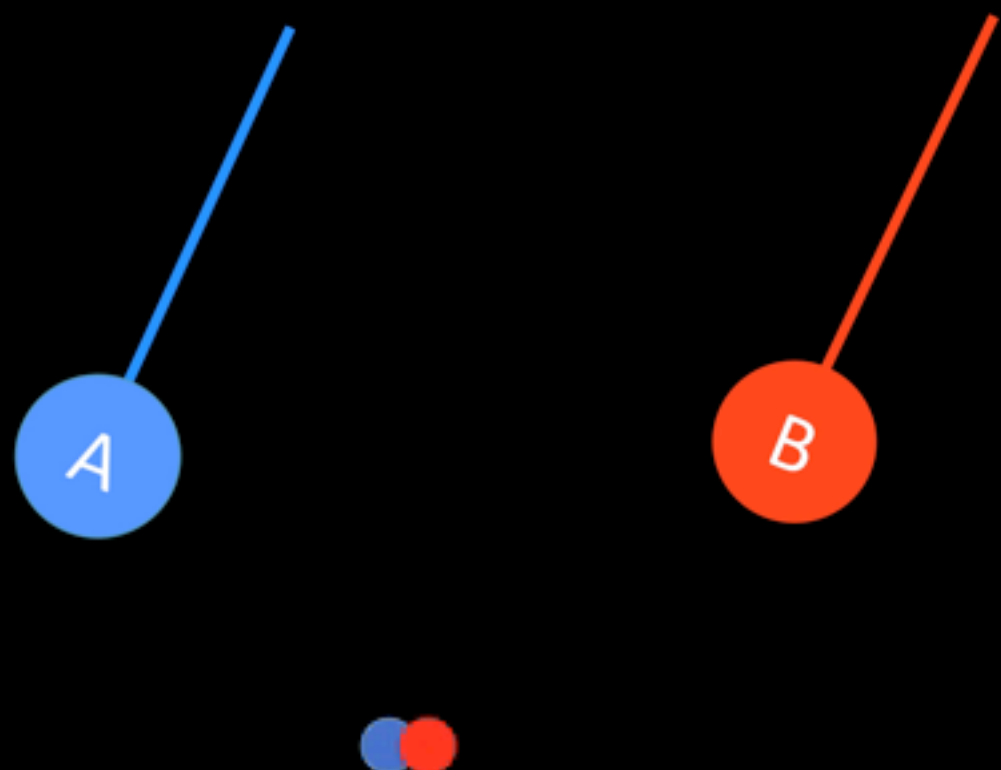
Correlative Measures
Cross-Correlation
Coherence
Phase-Locking Value
Phase-amplitude coupling
...

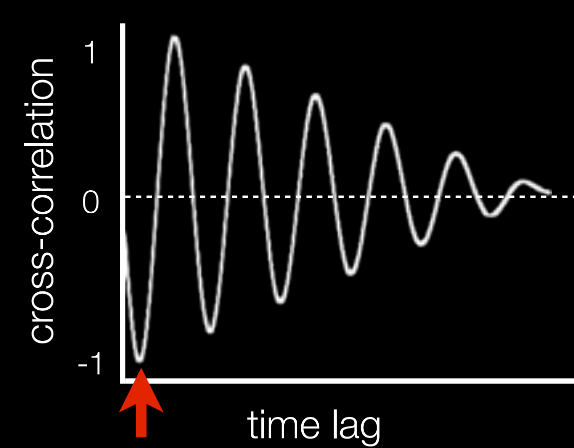
Cross-Correlation and Linear Coherence

Cross-Correlation and Linear Coherence



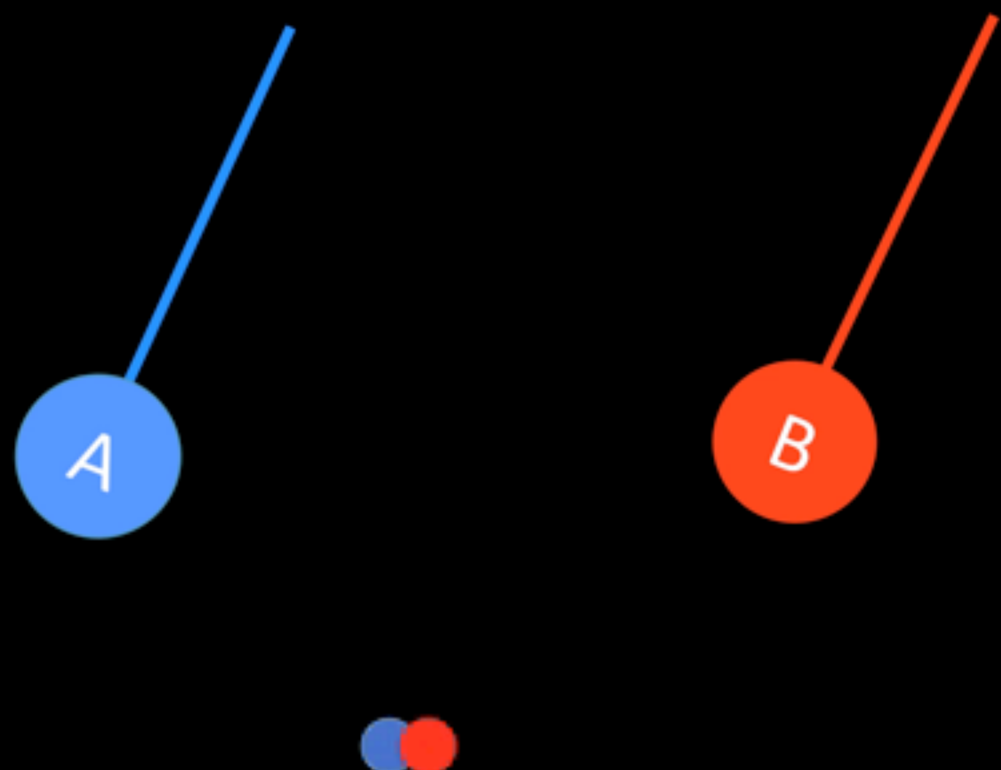
Cross-Correlation and Linear Coherence





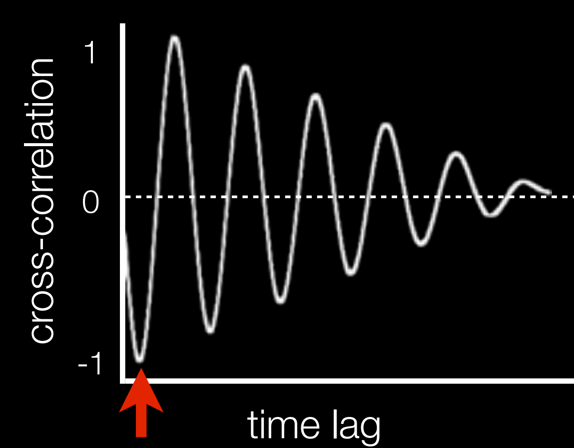
$$\rho_{AB}(k)$$

Cross-Correlation and Linear Coherence



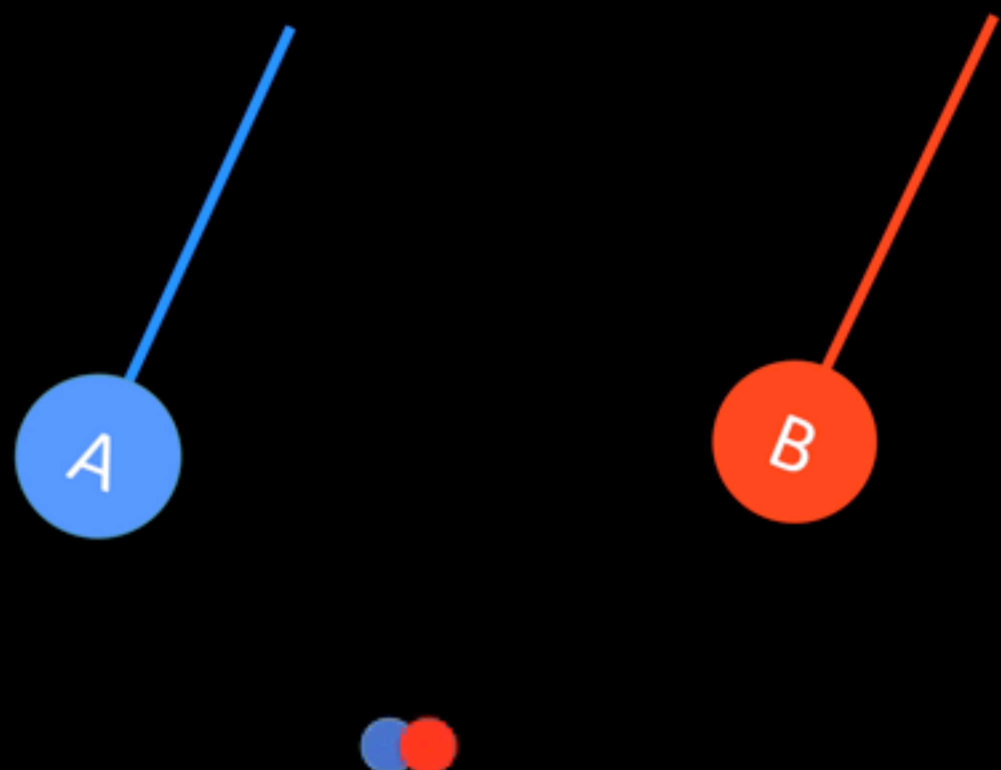
FFT

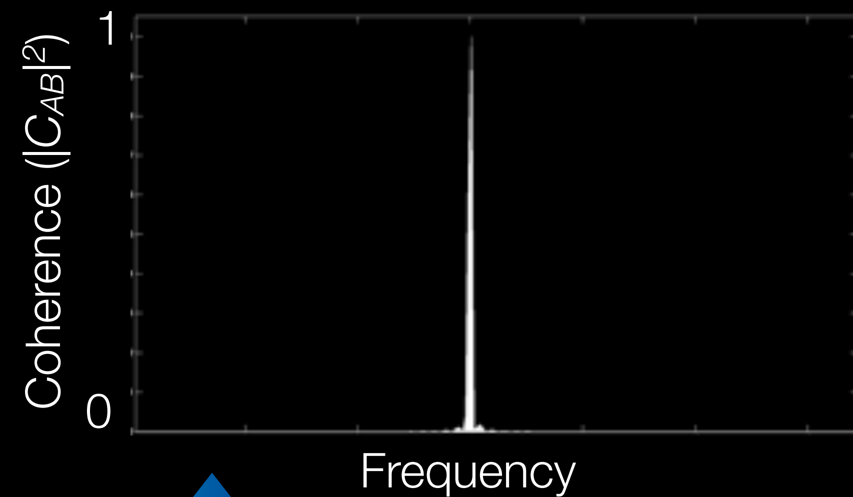
$$C_{AB}(f) = \sum_{k=0}^{p} \rho_{AB}(k) e^{-i2\pi fk}$$
$$= \frac{S_{AB}(f)}{\sqrt{S_A(f) S_B(f)}}$$



$\rho_{AB}(k)$

Cross-Correlation and Linear Coherence

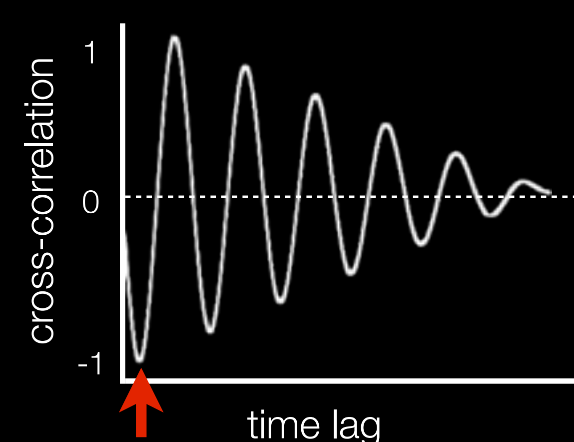




FFT

$$C_{AB}(f) = \sum_{k=0}^{p} \rho_{AB}(k) e^{-i2\pi fk}$$

$$= \frac{S_{AB}(f)}{\sqrt{S_A(f) S_B(f)}}$$



$$\rho_{AB}(k)$$

Theory: Euler's Formula

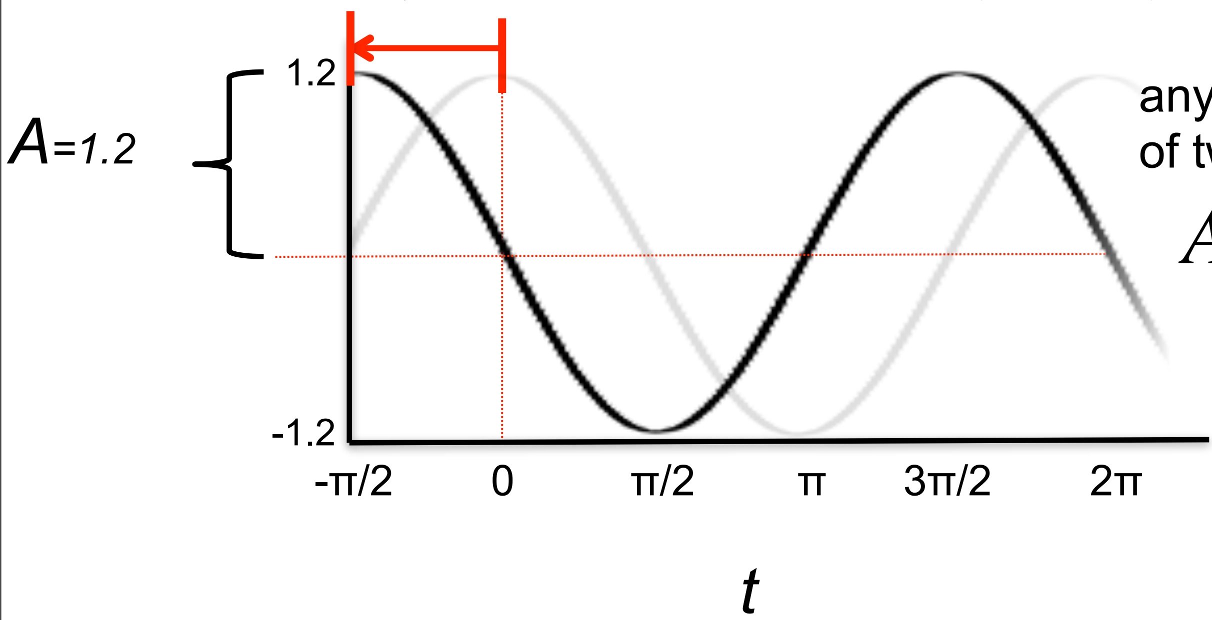
phase shift

$\phi = \pi / 2$

angular frequency

$\omega = 2\pi f = 2\pi$ rad/sec

instantaneous complex power (amplitude and phase)



any sinusoid can be expressed as the sum of two complex numbers...

$$A \cdot \cos(\omega t + \phi) = \frac{A}{2} e^{i(\omega t + \phi)} + \frac{A}{2} e^{-i(\omega t + \phi)}$$

$$= \mathrm{Re}\{A e^{i(\omega t + \phi)}\} = \mathrm{Re}\{S(\omega, t)\}$$

Theory: Euler's Formula

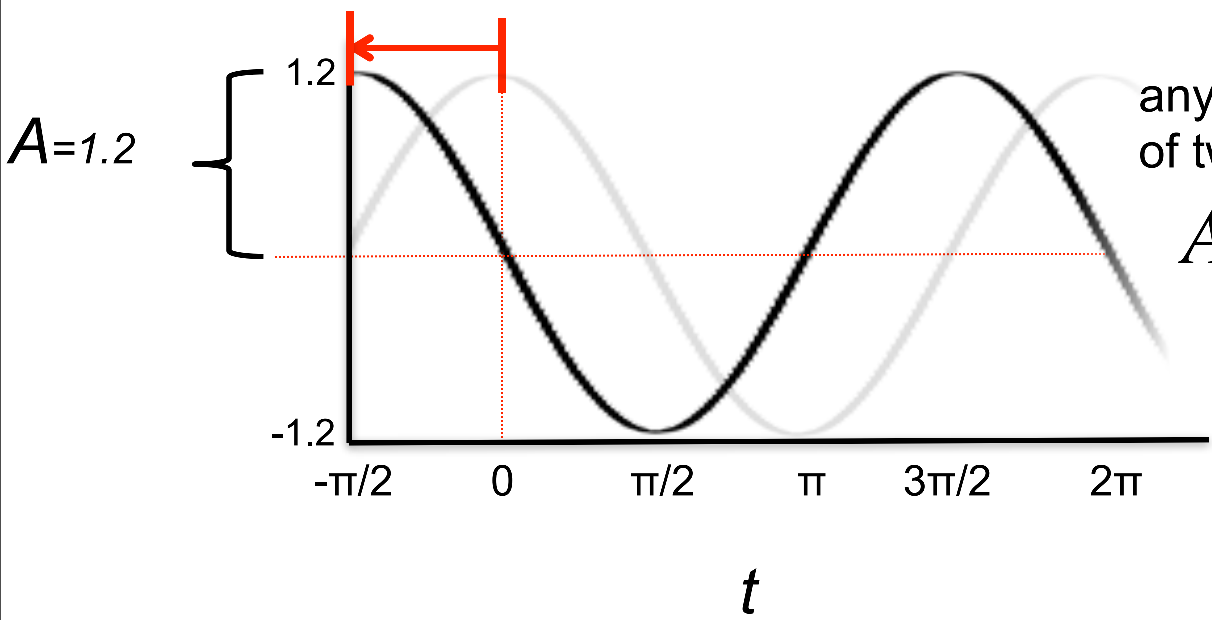
phase shift

$$\phi = \pi / 2$$

angular frequency

$$\omega = 2\pi f = 2\pi \text{ rad/sec}$$

instantaneous complex power (amplitude and phase)



any sinusoid can be expressed as the sum of two complex numbers...

$$A \cdot \cos(\omega t + \phi) = \frac{A}{2} e^{i(\omega t + \phi)} + \frac{A}{2} e^{-i(\omega t + \phi)}$$

$$= \mathrm{Re}\{A e^{i(\omega t + \phi)}\} = \mathrm{Re}\{S(\omega, t)\}$$

Theory: Euler's Formula

phase shift

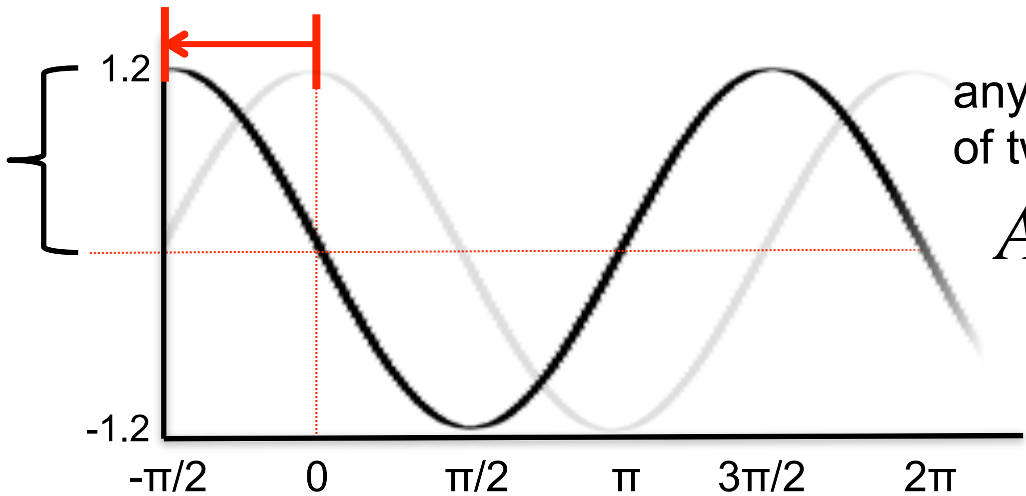
$\phi = \pi / 2$

angular frequency

$\omega = 2\pi f = 2\pi$ rad/sec

instantaneous complex power (amplitude and phase)

A=1.2



t

any sinusoid can be expressed as the sum of two complex numbers...

$$A \cdot \cos(\omega t + \phi) = \frac{A}{2} e^{i(\omega t + \phi)} + \frac{A}{2} e^{-i(\omega t + \phi)}$$

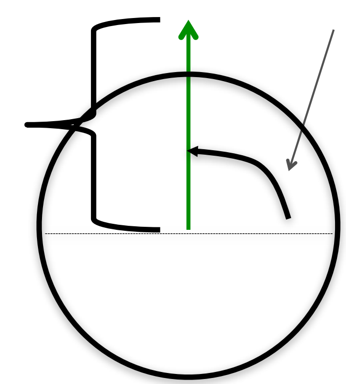
$$= \mathrm{Re}\{A e^{i(\omega t + \phi)}\} = \mathrm{Re}\{S(\omega, t)\}$$

$$\phi = \angle S(\omega, t)$$

$$= \pi / 2$$

$$|S(\omega, t)| = |A|$$

Spectral Power



Phasor!

Theory: Euler's Formula

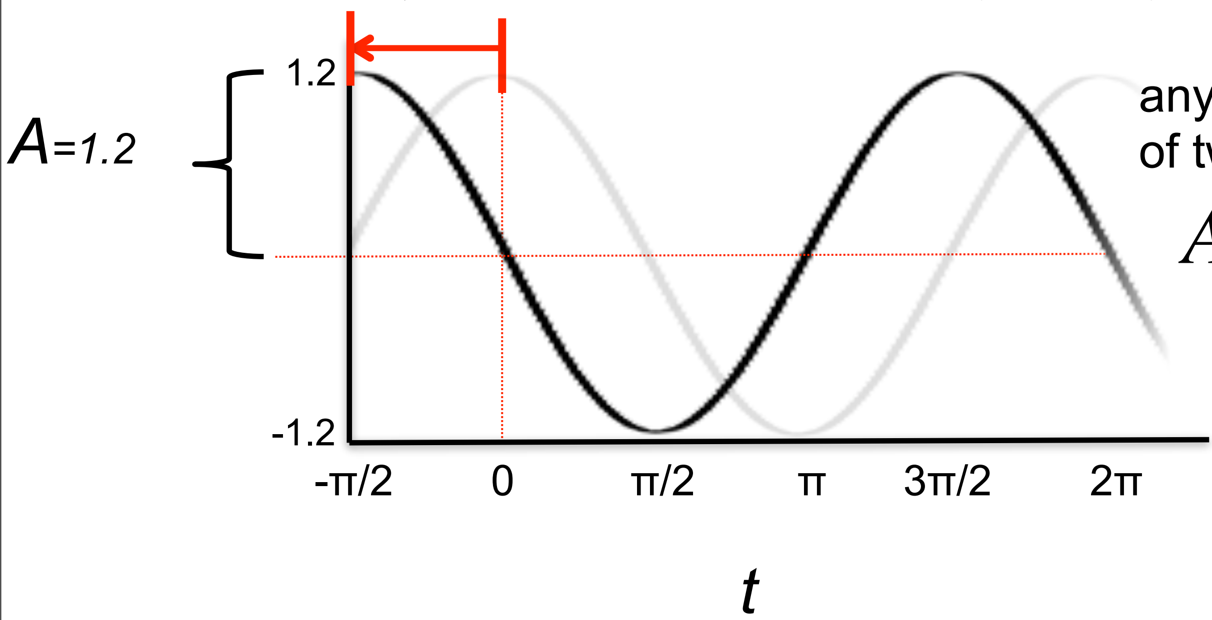
phase shift

$\phi = \pi / 2$

angular frequency

$\omega = 2\pi f = 2\pi$ rad/sec

instantaneous complex power (amplitude and phase)



any sinusoid can be expressed as the sum of two complex numbers...

$$A \cdot \cos(\omega t + \phi) = \frac{A}{2} e^{i(\omega t + \phi)} + \frac{A}{2} e^{-i(\omega t + \phi)}$$

$$= \mathrm{Re}\{A e^{i(\omega t + \phi)}\} = \mathrm{Re}\{S(\omega, t)\}$$

Another version:

$$e^{i(\omega t + \phi)} = \cos(\omega t + \phi) + i \sin(\omega t + \phi)$$

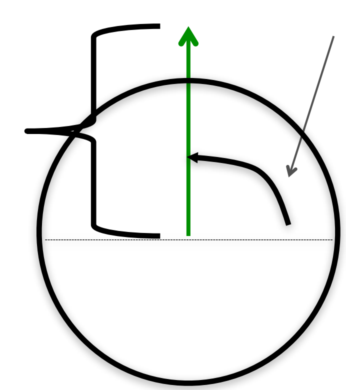
Real part
Cosine component

Imaginary part
Sine component

$$|S(\omega, t)| = |A|$$

Spectral Power

$$\phi = \angle S(\omega, t)$$
$$= \pi / 2$$



Phasor!

Phasers

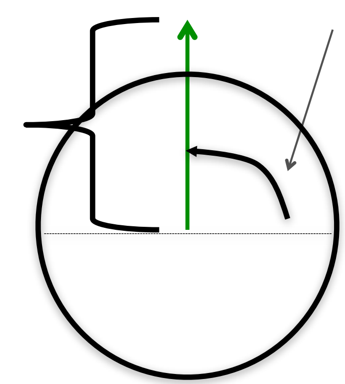
$$A \cdot \cos(\omega t + \phi) = \mathrm{Re}\{Ae^{i(\omega t + \phi)}\}$$

$$= \mathrm{Re}\{S(\omega, t)\}$$

$$\phi = \angle S(\omega, t)$$

$$= \pi / 2$$

$$|S(\omega, t)| = |A|$$



Phasor!

Phasors

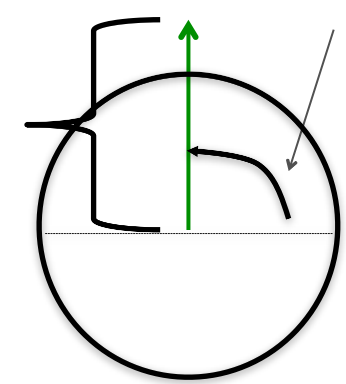
$$A \cdot \cos(\omega t + \phi) = \mathrm{Re}\{A e^{i(\omega t + \phi)}\}$$

$$= \mathrm{Re}\{S(\omega, t)\}$$

$$\phi = \angle S(\omega, t)$$

$$= \pi / 2$$

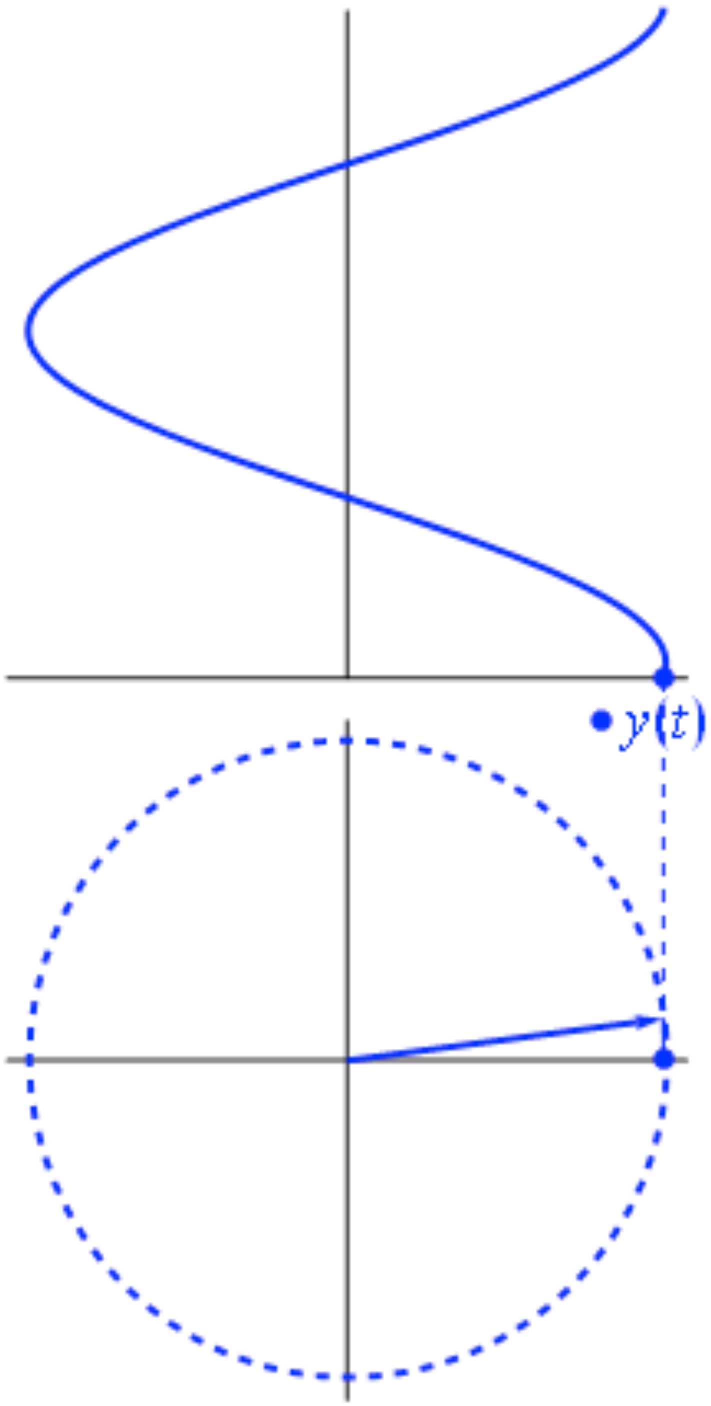
$$|S(\omega, t)| = |A|$$



Phasor!

Phasors

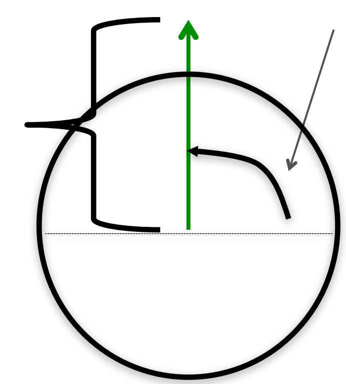
Rotation velocity (Rad/S; Hz)
= (angular) frequency (*w*; *f*)



$$A \cdot \cos(\omega t + \phi) = \mathrm{Re}\{Ae^{i(\omega t + \phi)}\}$$
$$= \mathrm{Re}\{S(\omega, t)\}$$

$$\phi = \angle S(\omega, t)$$
$$= \pi / 2$$

$$|S(\omega, t)| = |A|$$

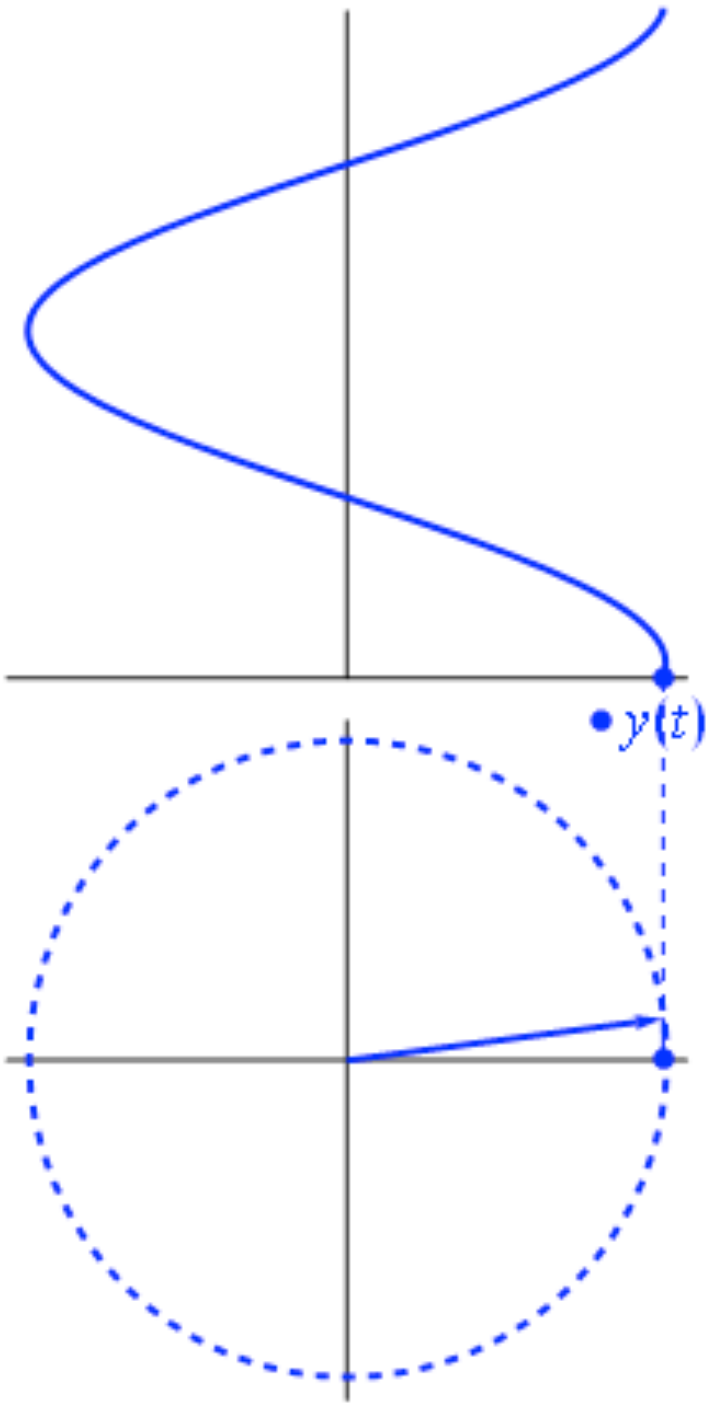


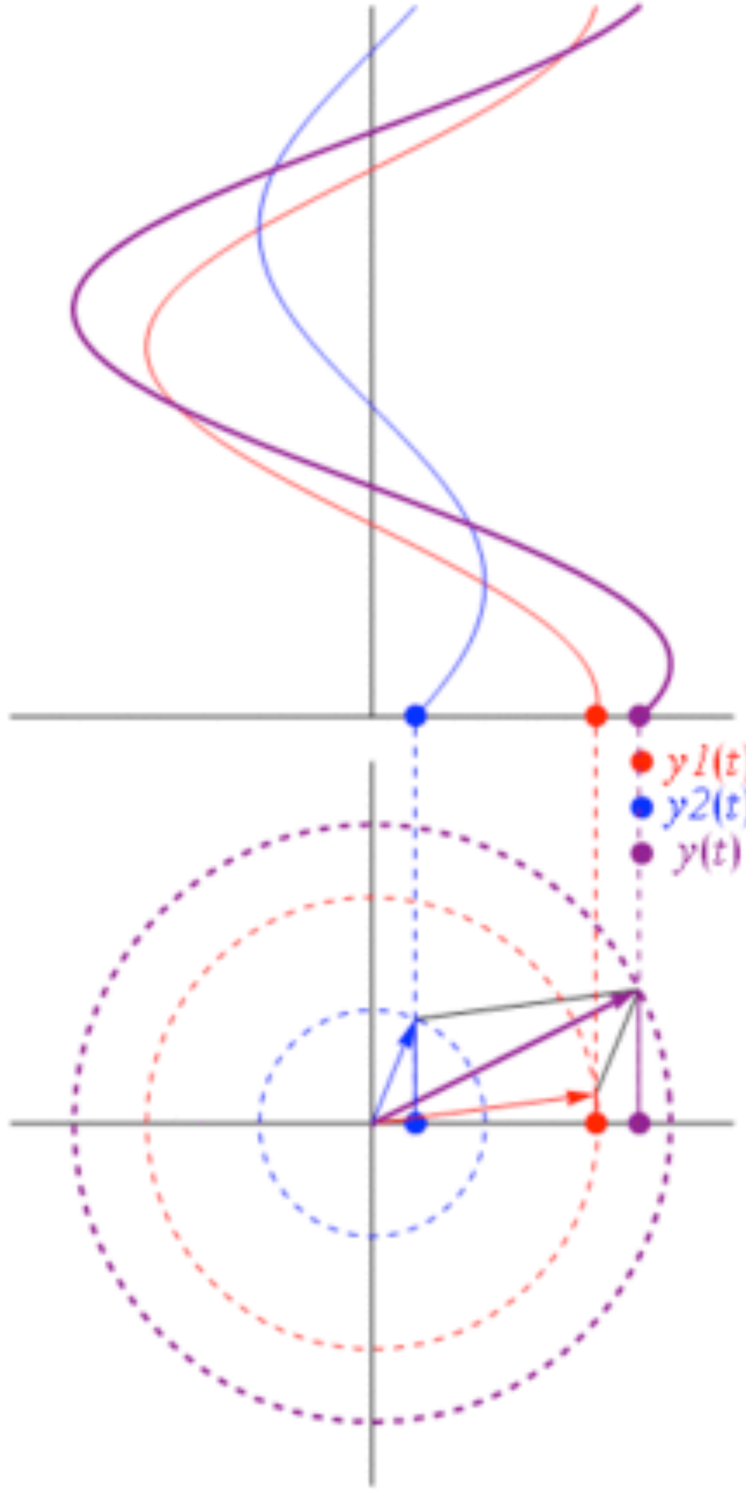
Phasor!

Polar animations courtesy Wikipedia

Phasors

Rotation velocity (Rad/S; Hz)
= (angular) frequency (w; f)

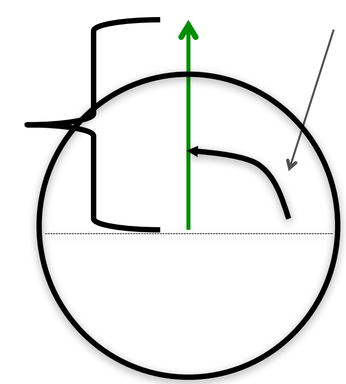




$$A \cdot \cos(\omega t + \phi) = \mathrm{Re}\{Ae^{i(\omega t + \phi)}\}$$
$$= \mathrm{Re}\{S(\omega, t)\}$$

$$\phi = \angle S(\omega, t)$$
$$= \pi / 2$$

$$|S(\omega, t)| = |A|$$

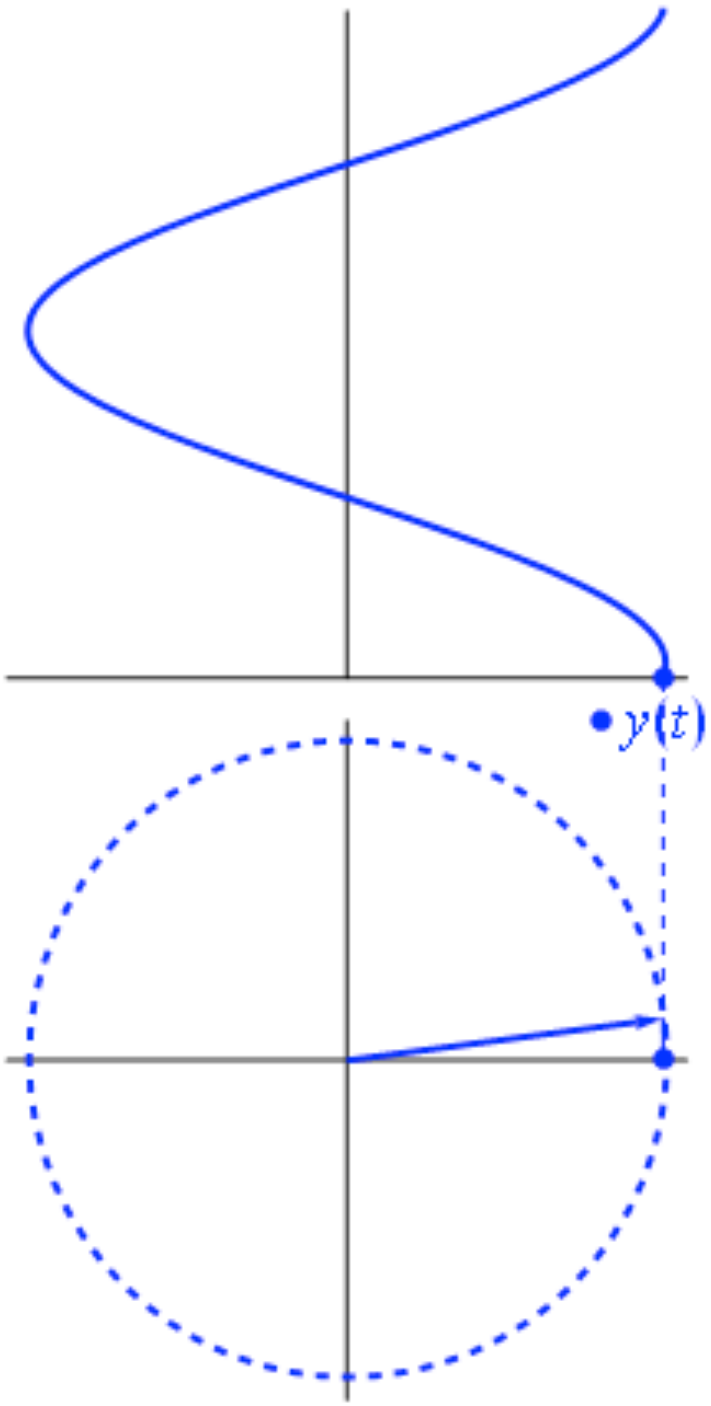


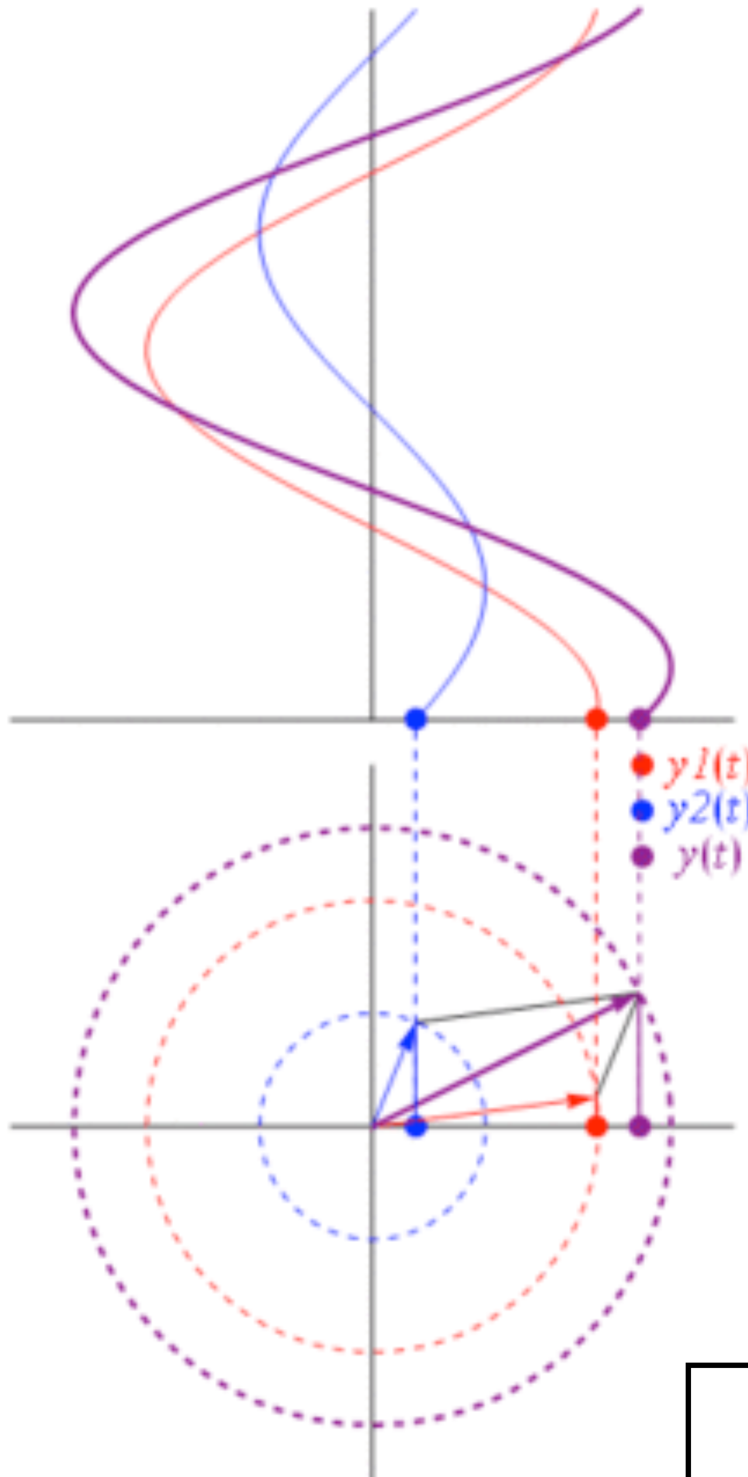
Phasor!

Polar animations courtesy Wikipedia

Rotation velocity (Rad/S; Hz)
= (angular) frequency (*w*; *f*)

Phasors

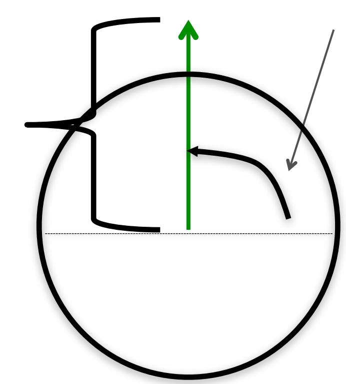




$$A \cdot \cos(\omega t + \phi) = \mathrm{Re}\{Ae^{i(\omega t+\phi)}\}$$
$$= \mathrm{Re}\{S(\omega,t)\}$$

$$\phi = \angle S(\omega,t)$$
$$= \pi / 2$$

$$|S(\omega,t)| = |A|$$



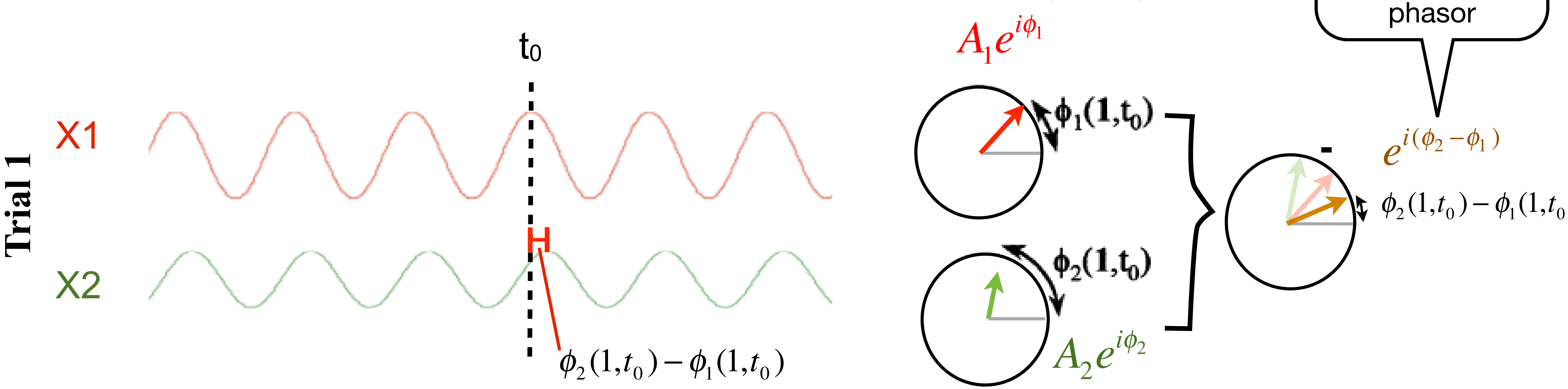
Shorthand phasor notation: $Ae^{i\phi}$

Phasor!

Polar animations courtesy Wikipedia

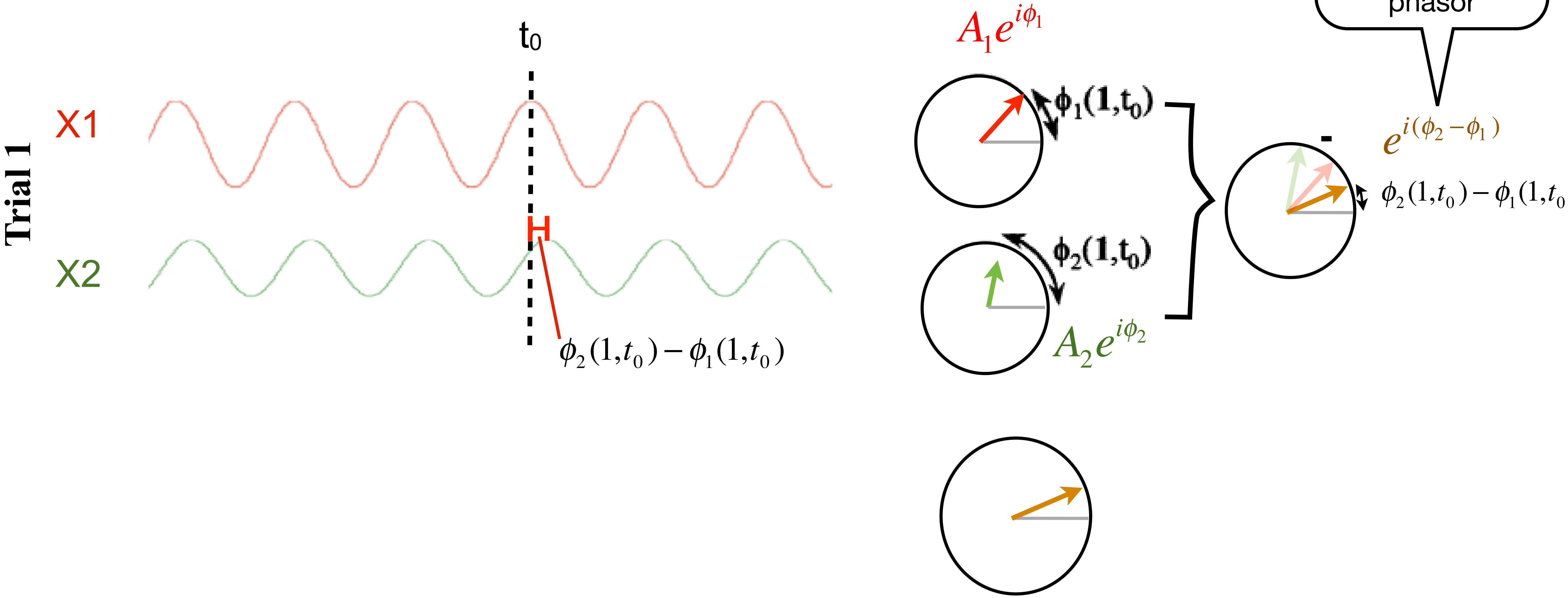
Phase-Locking Value (PLV)

Lachaux, J.P., et al (1999) *HBM*



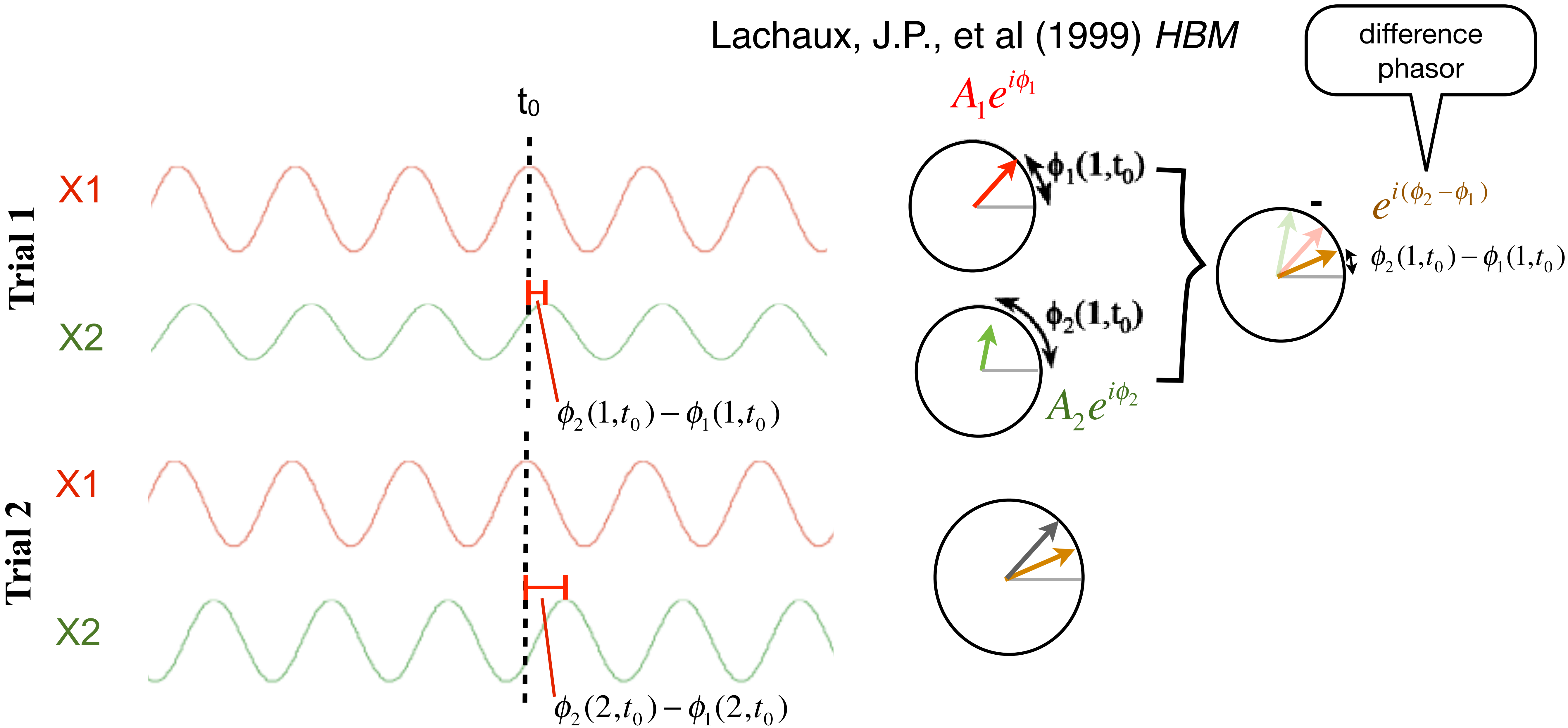
Phase-Locking Value (PLV)

Lachaux, J.P., et al (1999) *HBM*



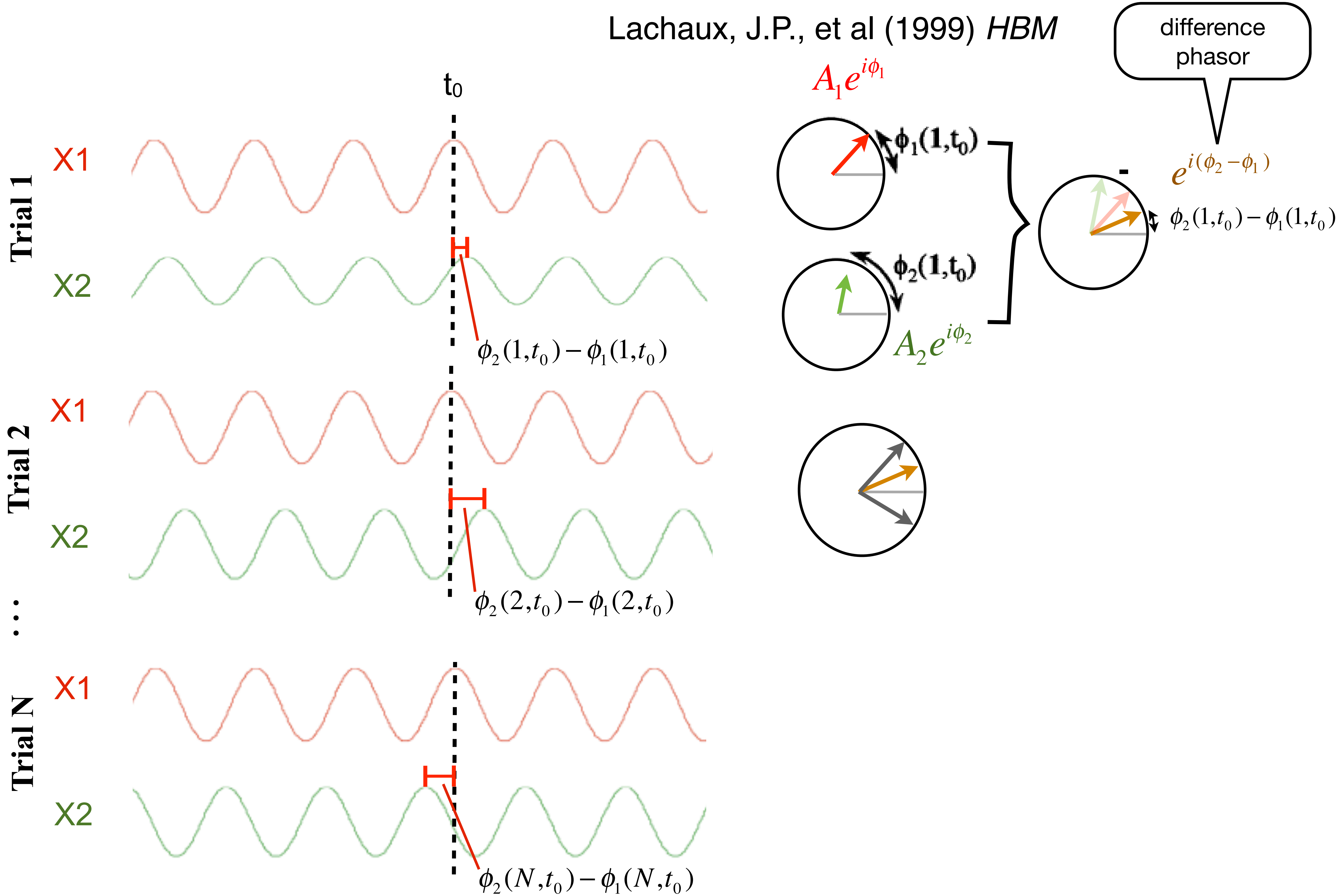
Phase-Locking Value (PLV)

Lachaux, J.P., et al (1999) *HBM*



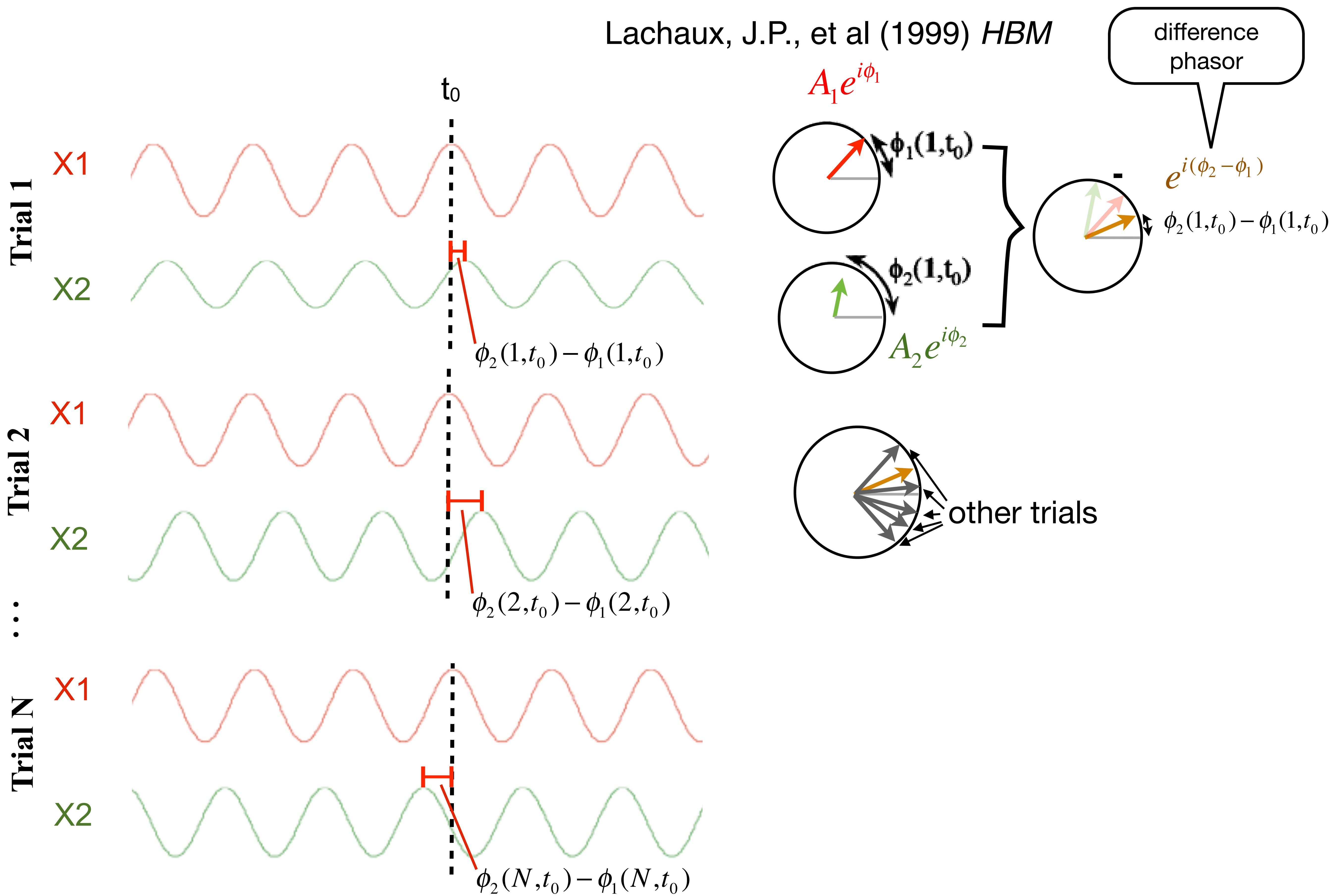
Phase-Locking Value (PLV)

Lachaux, J.P., et al (1999) *HBM*



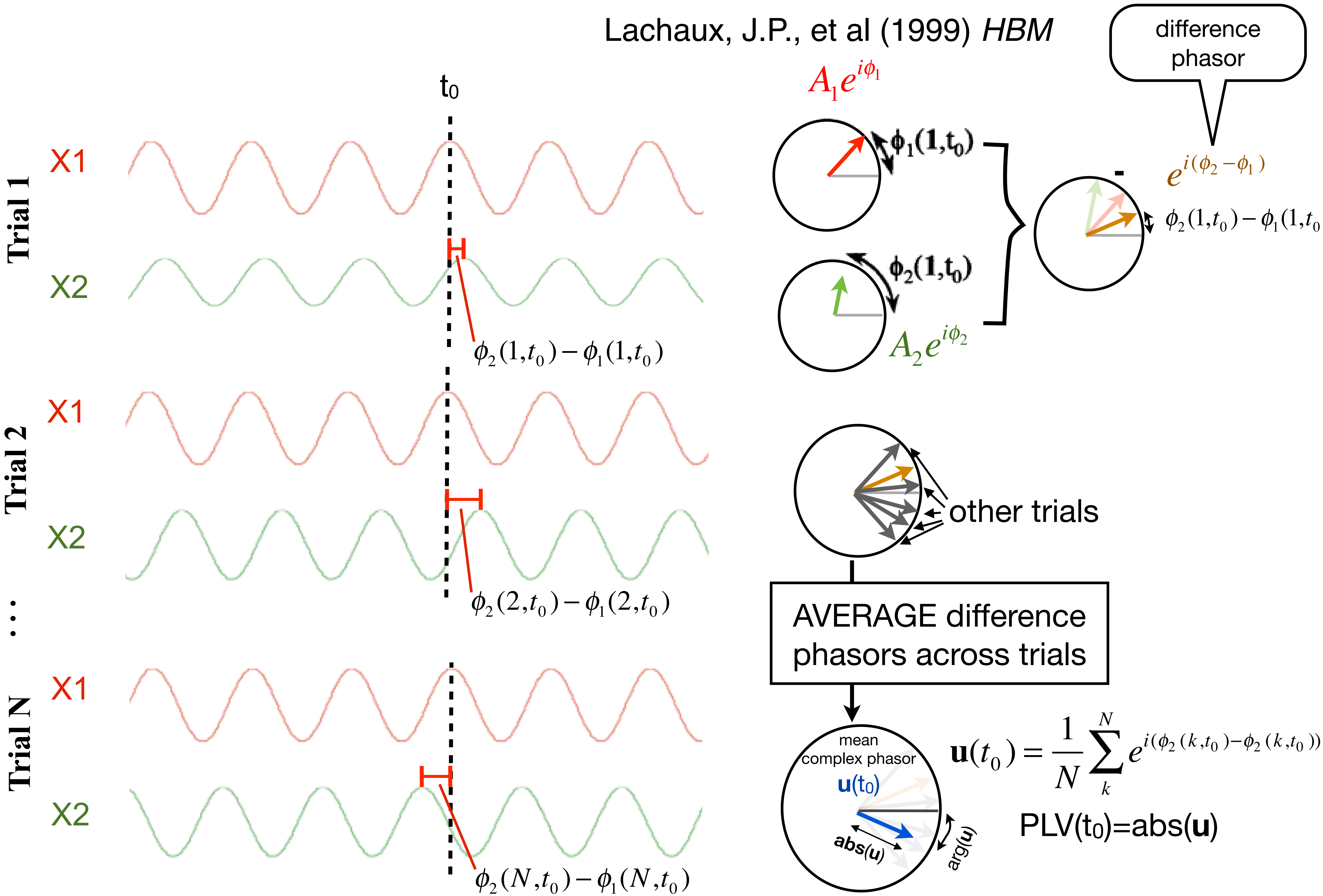
Phase-Locking Value (PLV)

Lachaux, J.P., et al (1999) *HBM*



Phase-Locking Value (PLV)

Lachaux, J.P., et al (1999) *HBM*



$$\mathbf{u}(t_0) = \frac{1}{N}\sum_{k}^{N} e^{i(\phi_2(k,t_0) - \phi_2(k,t_0))}$$

PLV(t_0)=abs(**u**)

Phase-Locking Value (PLV)

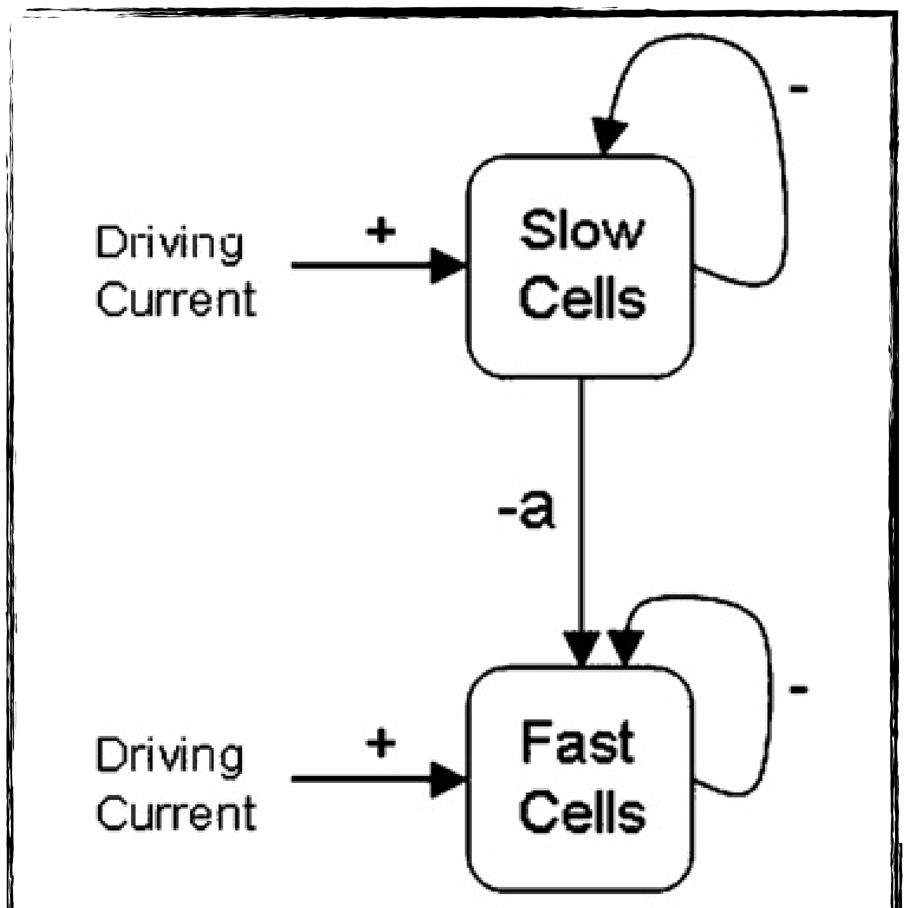
Lachaux, J.P., et al (1999) *HBM*

Computing PLV (“phase coherence”) in EEGLAB:

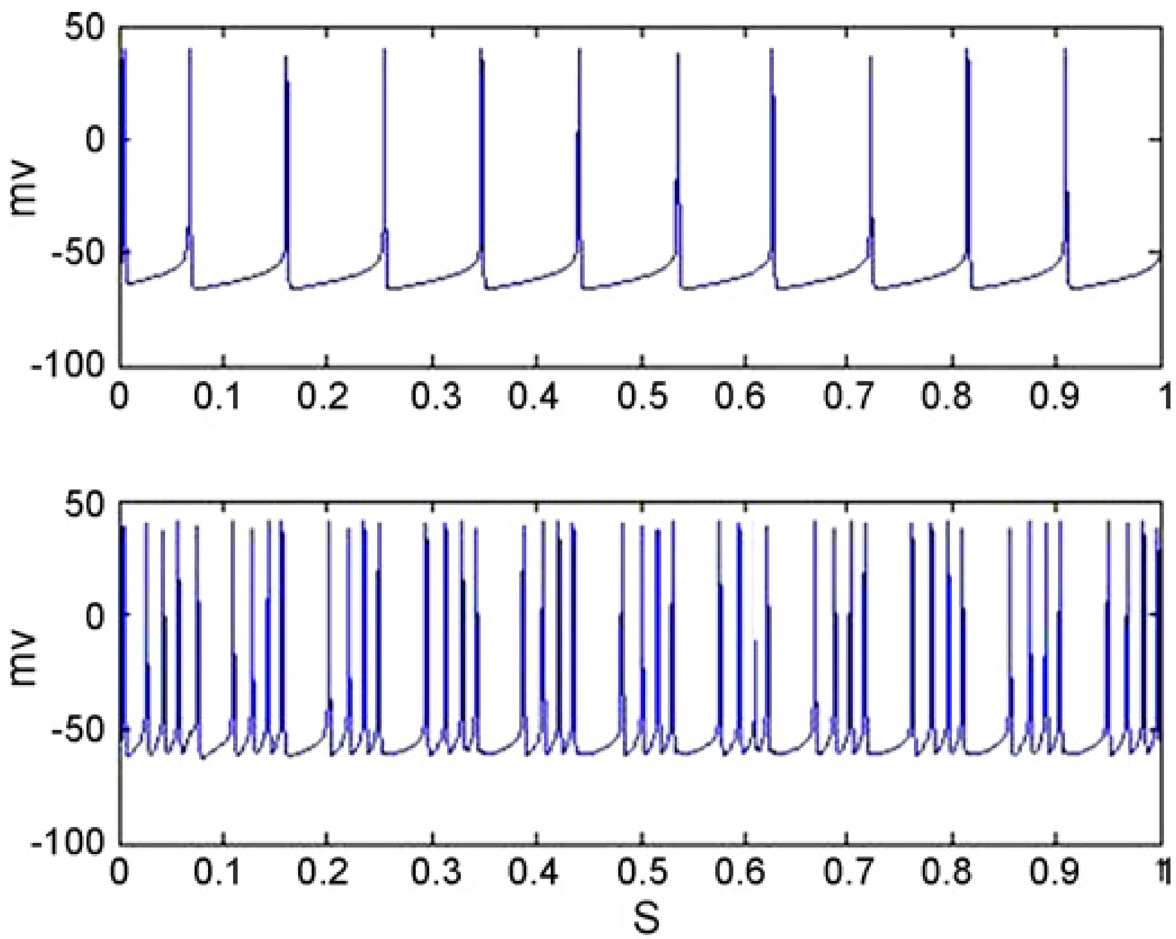
```
pop_newcrossf(....,‘type’,‘phase’)
```

Phase-Amplitude Coupling

'burst-suppress' oscillators

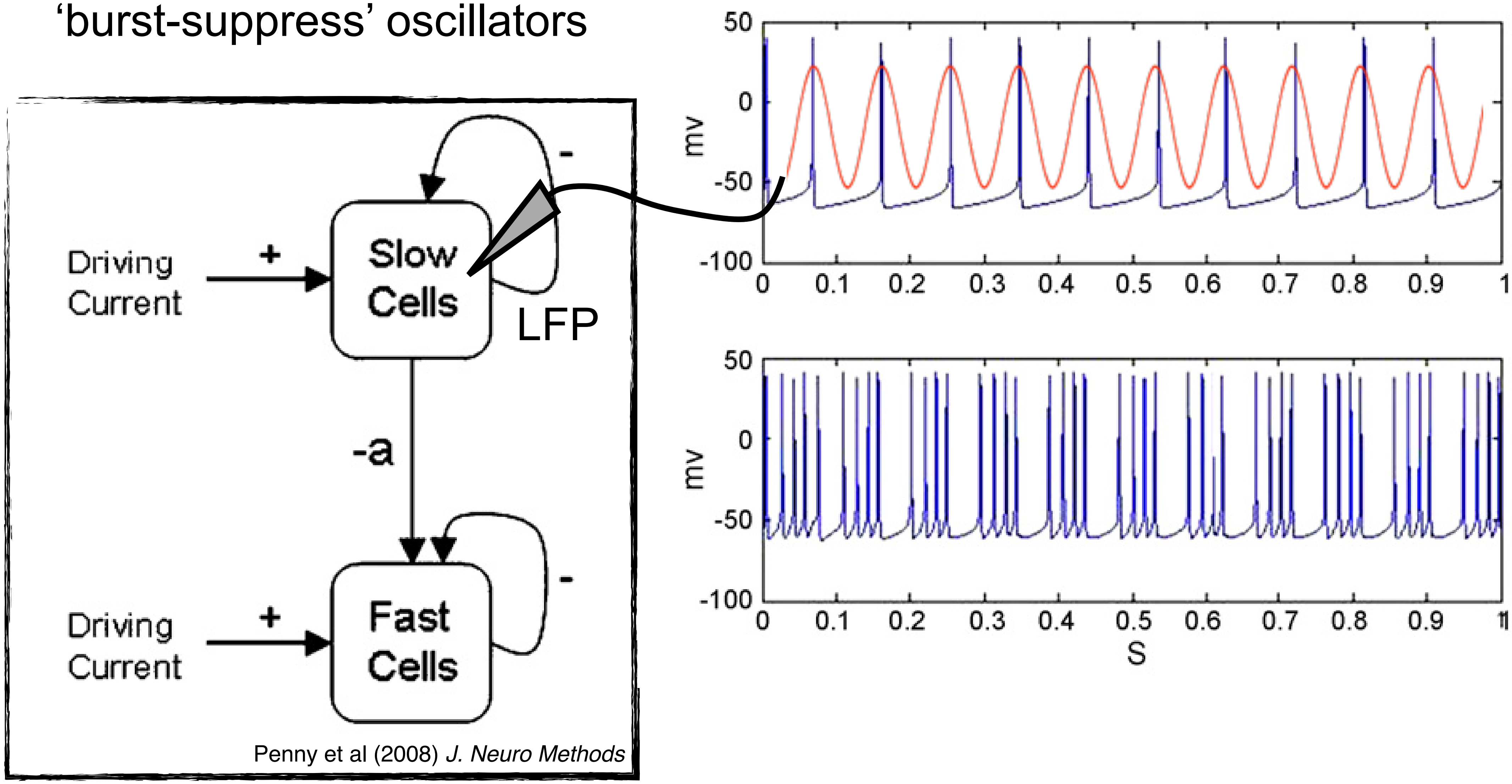


Penny et al (2008) *J. Neuro Methods*



Phase-Amplitude Coupling

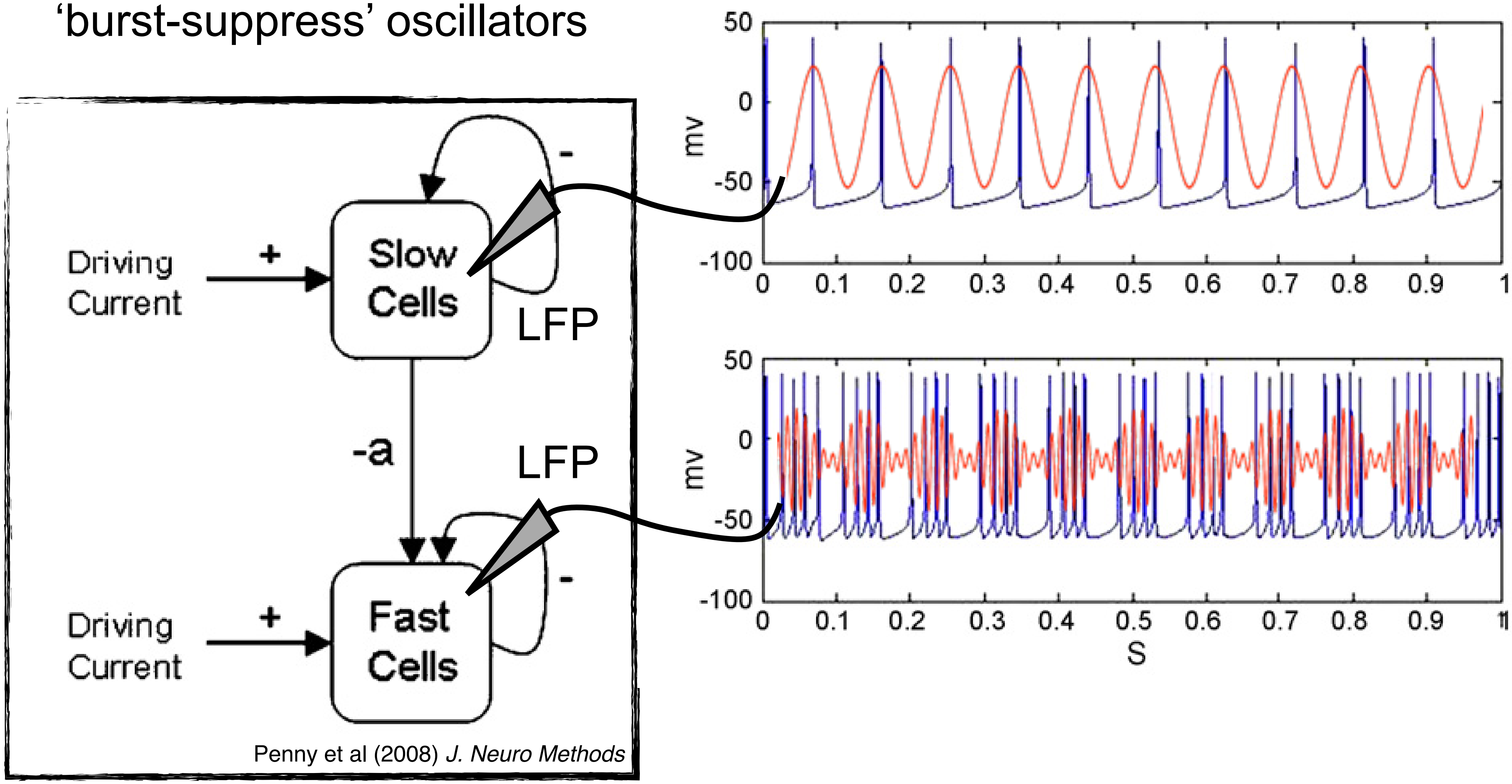
'burst-suppress' oscillators



Penny et al (2008) *J. Neuro Methods*

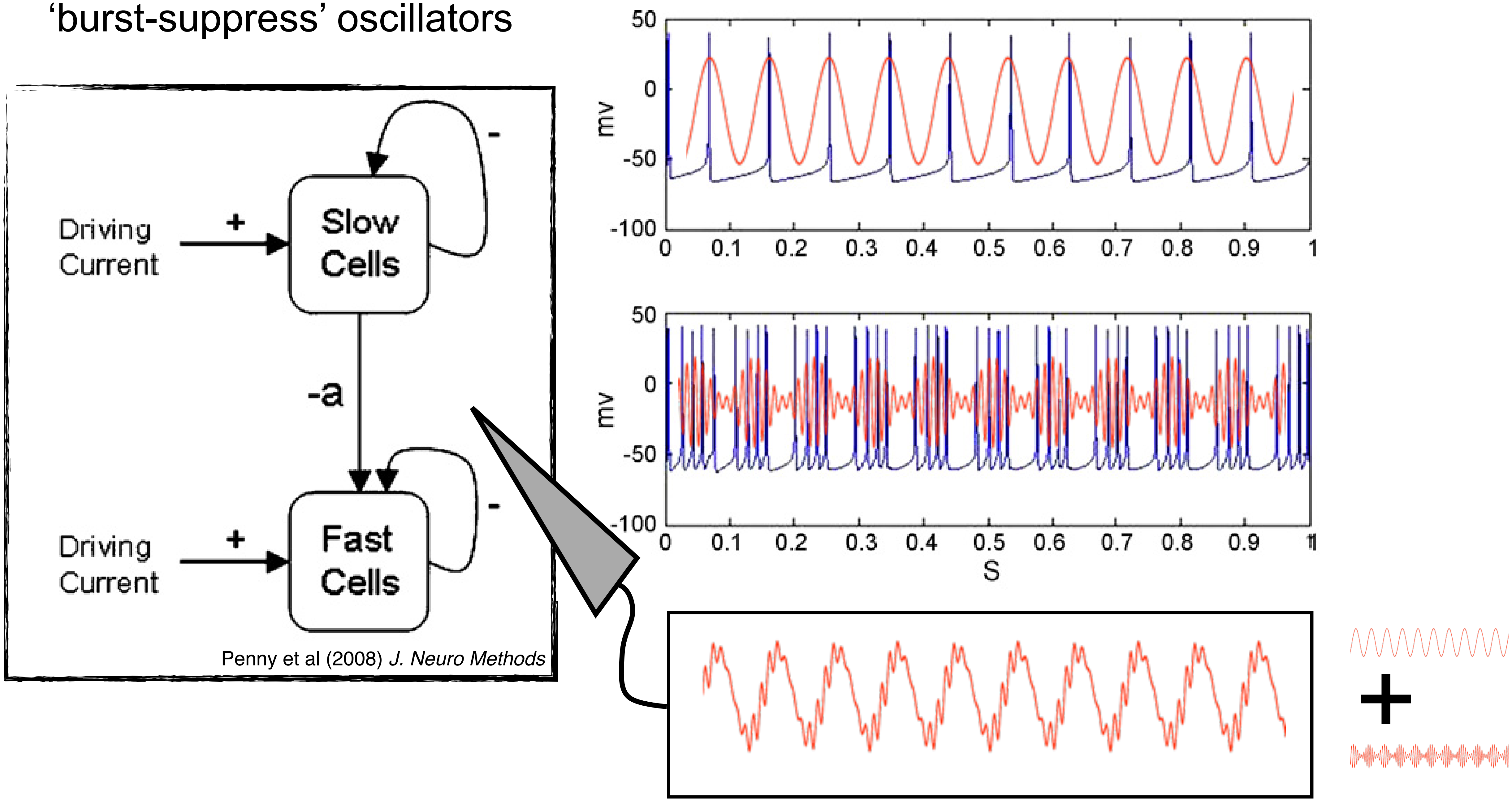
Phase-Amplitude Coupling

'burst-suppress' oscillators



Phase-Amplitude Coupling

'burst-suppress' oscillators



Local Field Potential (Slow + Fast cells)

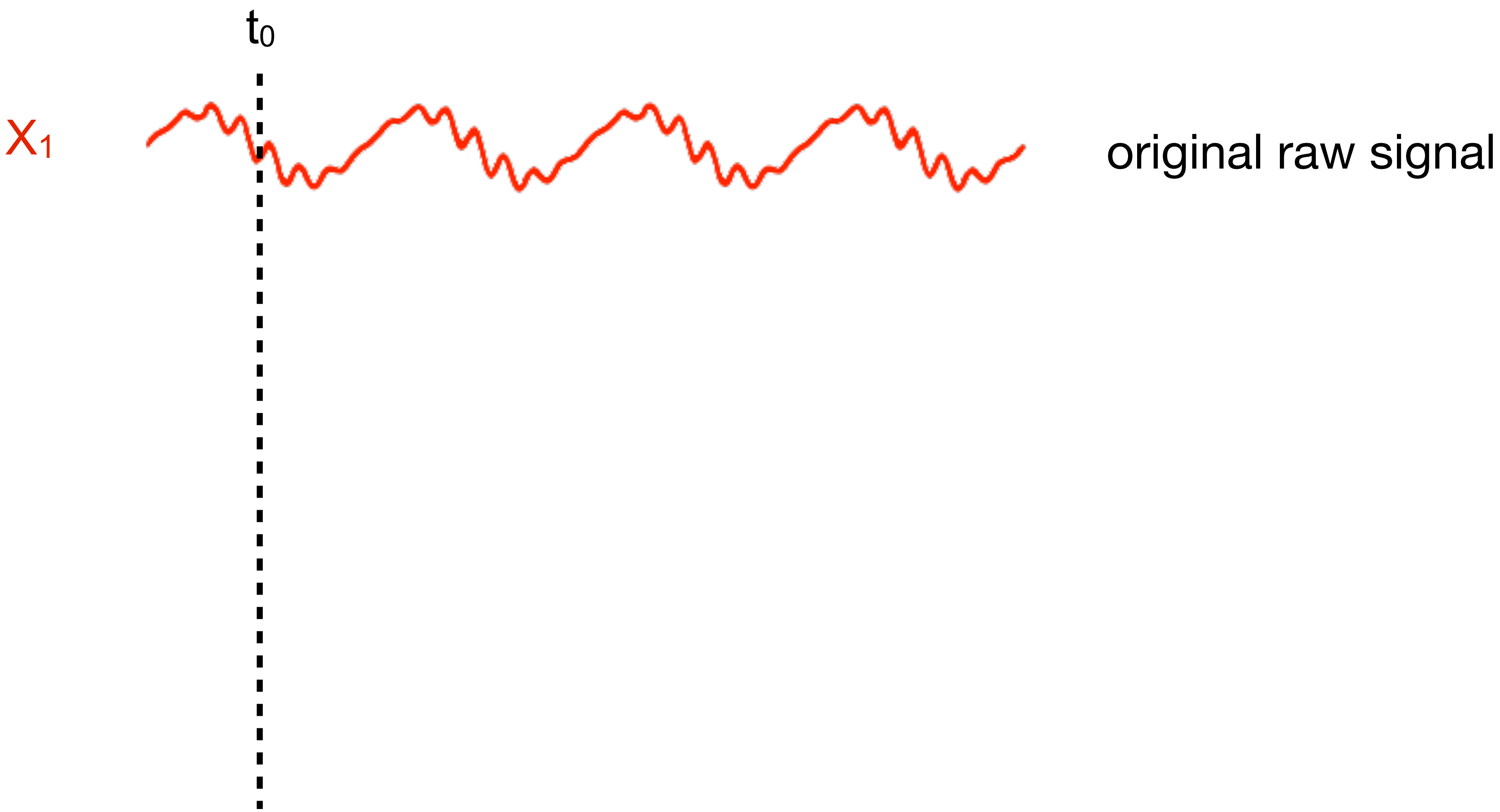
Phase-Amplitude Coupling

- May present a functional role in execution of cognitive functions (Axmacher et al. 2010; Cohen et al. 2009a,b; Lakatos et al. 2008; Tort et al. 2008, 2009).

- Suggested involvement in **sensory signal detection** (Handel and Haarmeier 2009), **attentional selection** (Schroeder and Lakatos 2009), and **memory processes** (Axmacher et al. 2010; Tort et al. 2009)

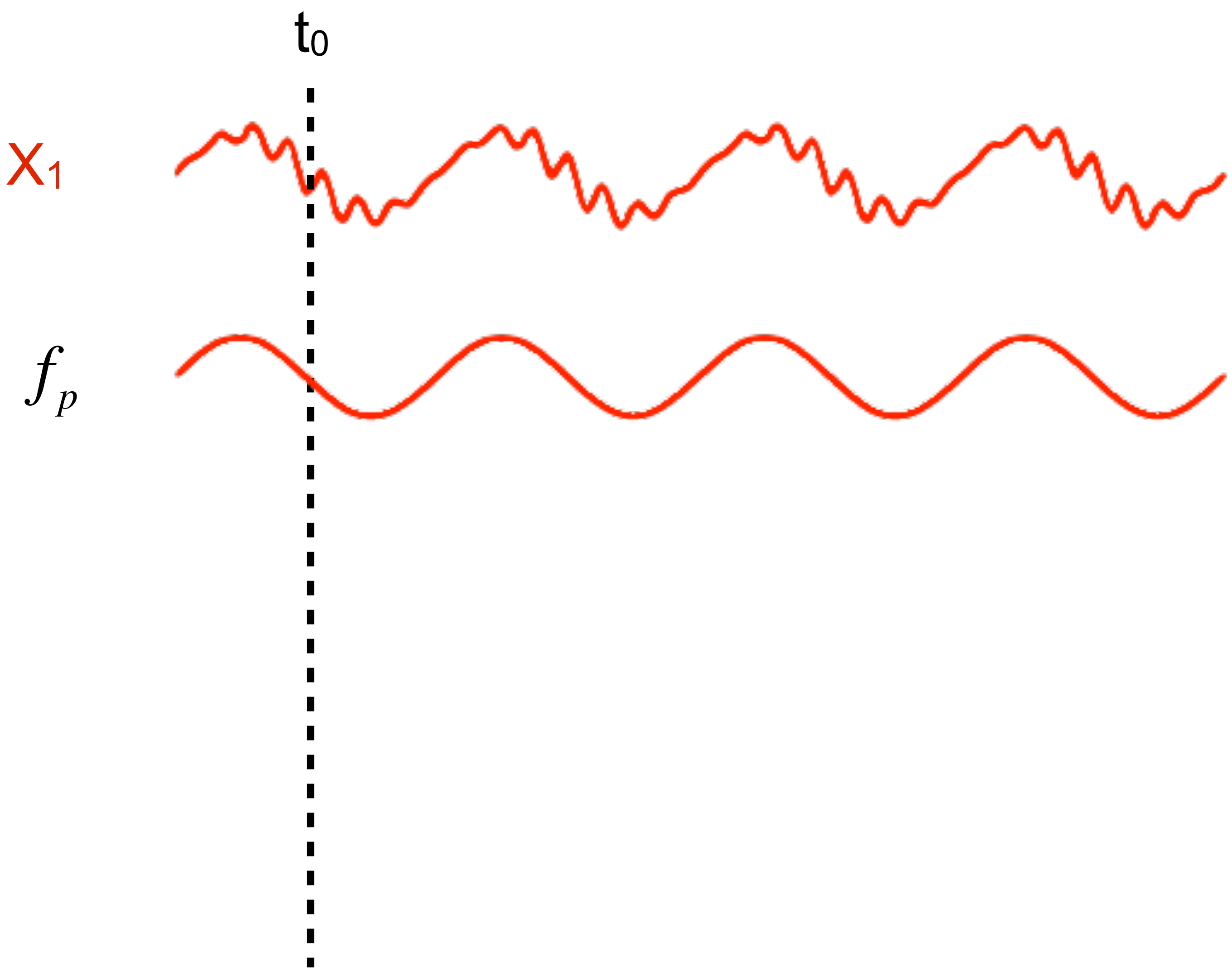
Phase-Amplitude Coupling: PLV Method

Vanhatalo, S et al (2004) *PNAS*



Phase-Amplitude Coupling:
PLV Method

Vanhatalo, S et al (2004) *PNAS*

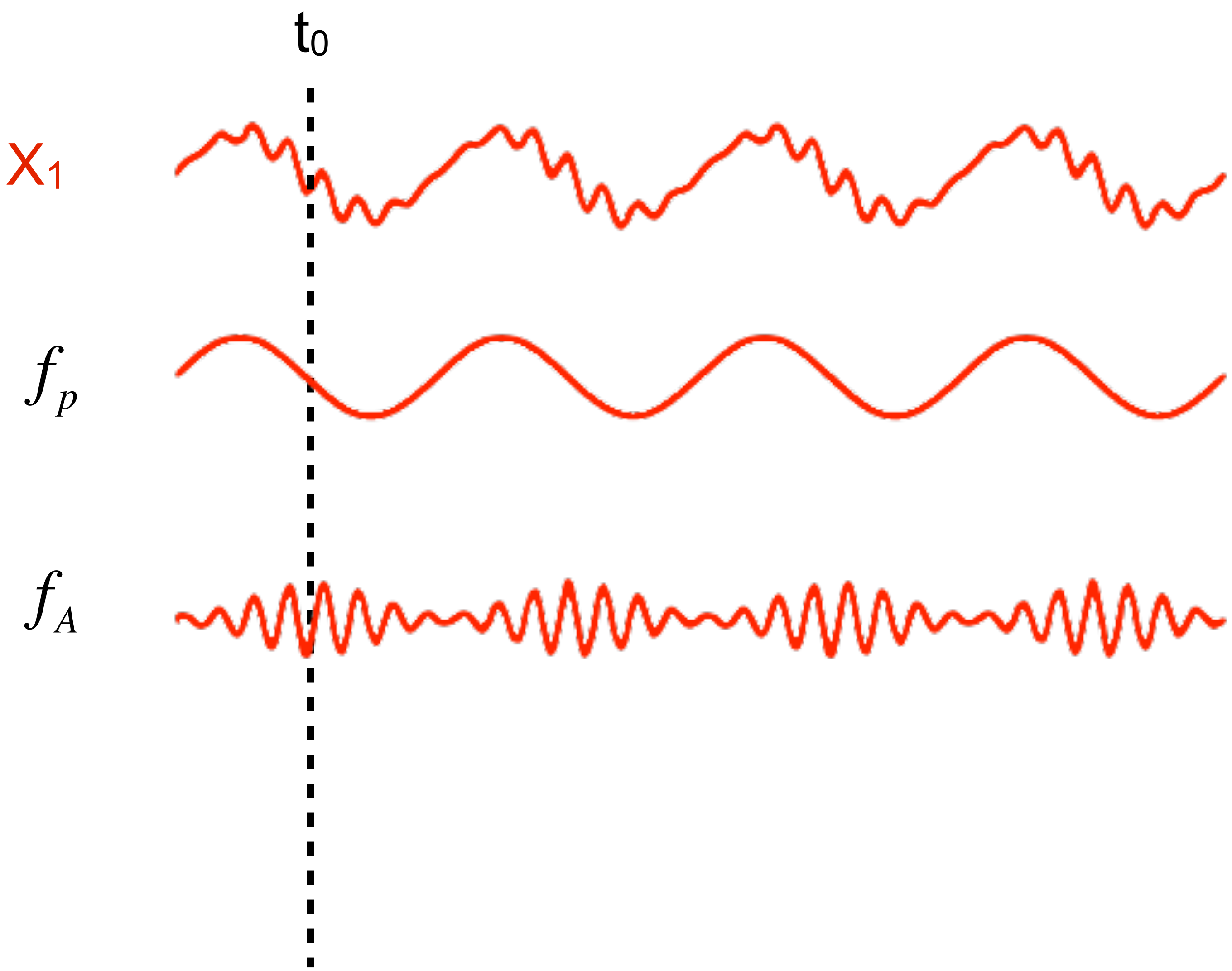


original raw signal

filter X_1 at phase-modulation band (e.g. theta)

Phase-Amplitude Coupling: PLV Method

Vanhatalo, S et al (2004) *PNAS*



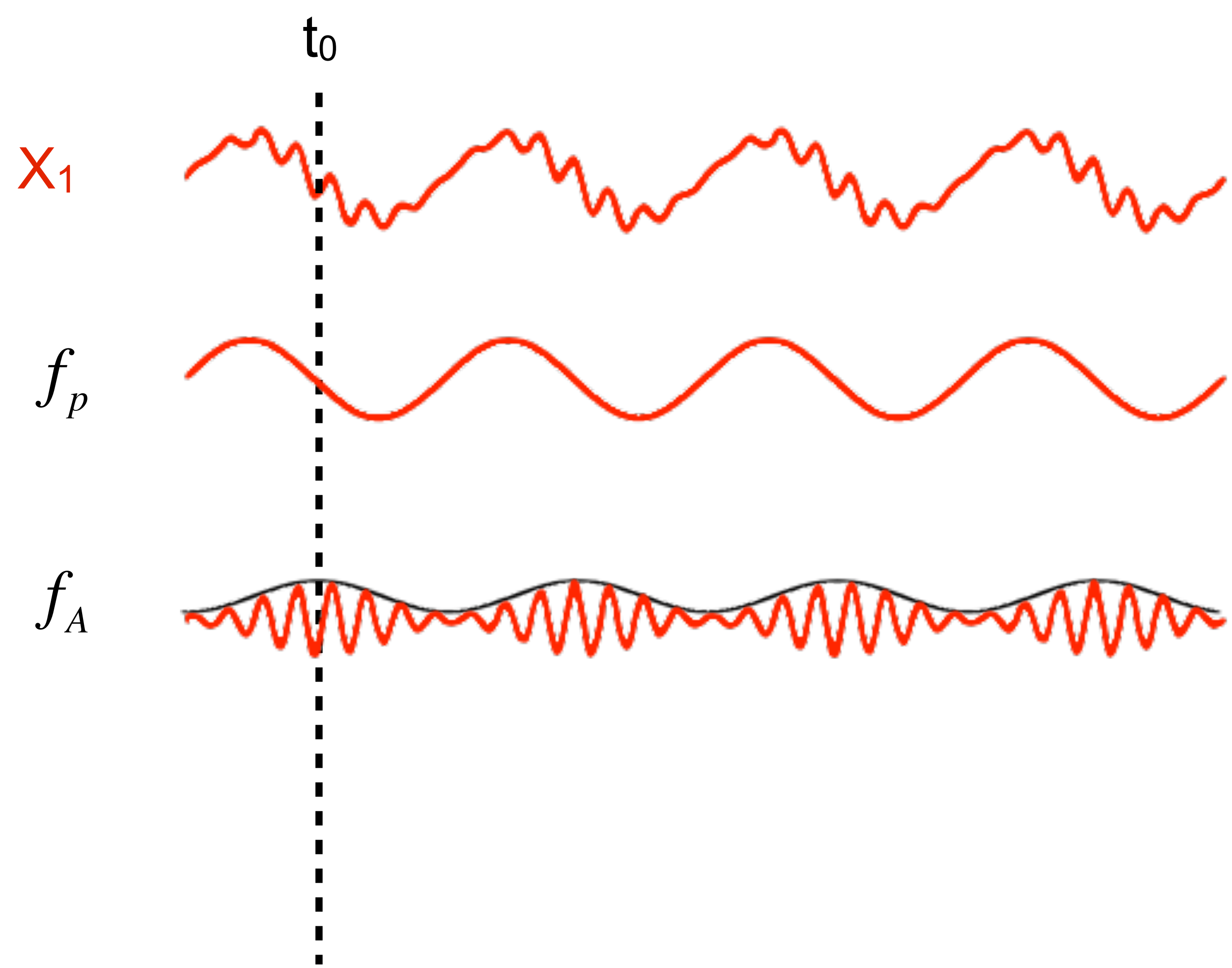
original raw signal

filter X_1 at phase-modulation band
(e.g. theta)

filter X_1 at amp-modulation band
(e.g. gamma)

Phase-Amplitude Coupling: PLV Method

Vanhatalo, S et al (2004) *PNAS*



original raw signal

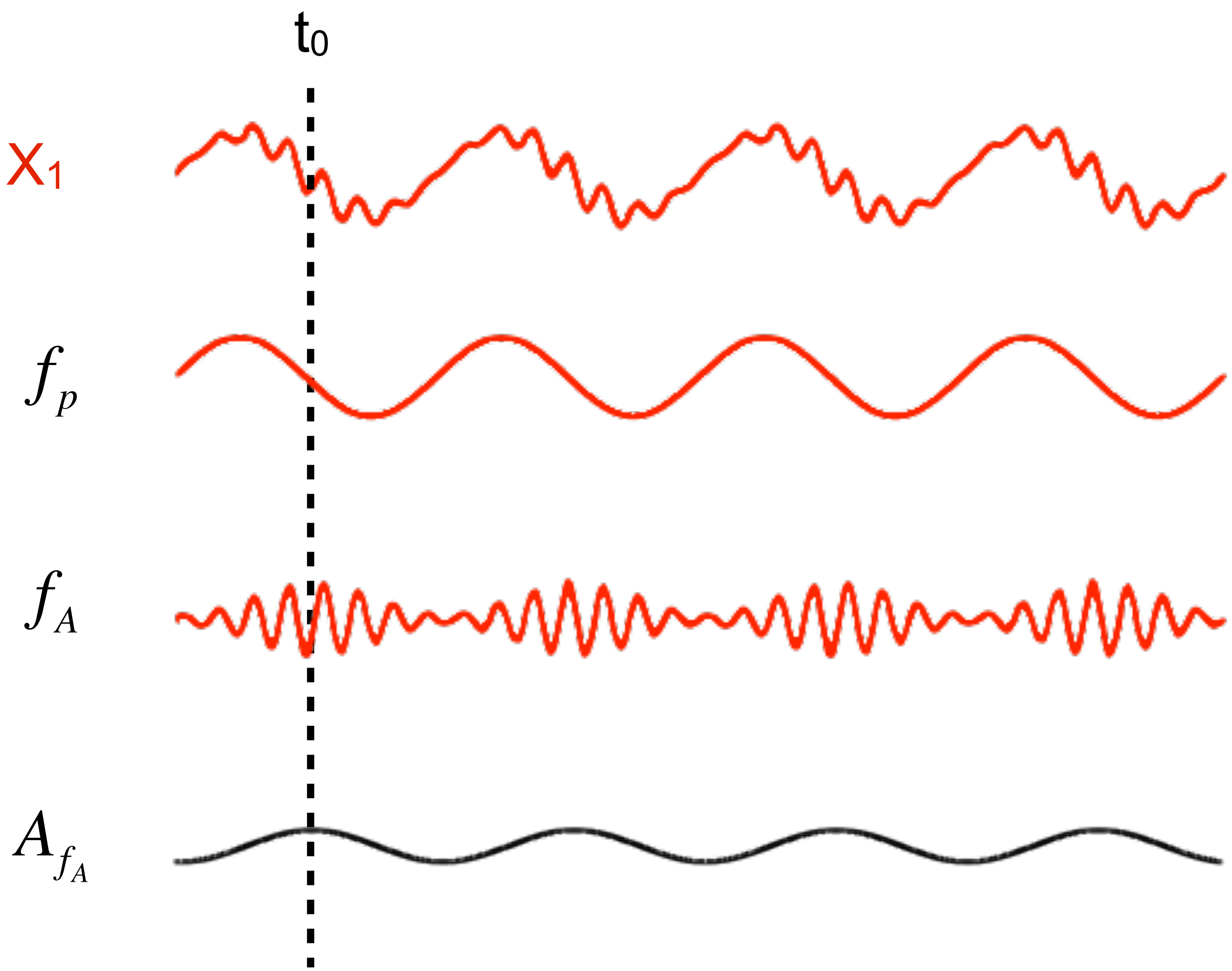
filter X_1 at phase-modulation band (e.g. theta)

filter X_1 at amp-modulation band (e.g. gamma)

get amplitude envelope of filtered signal

Phase-Amplitude Coupling: PLV Method

Vanhatalo, S et al (2004) *PNAS*



original raw signal

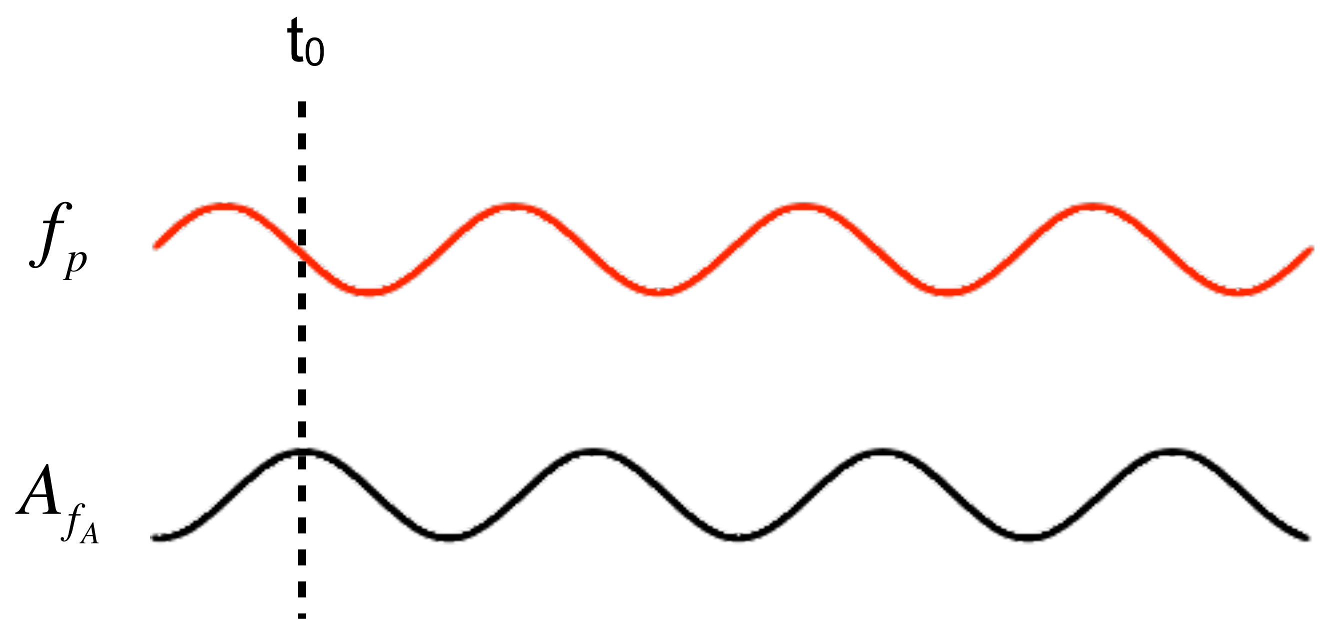
filter X_1 at phase-modulation band (e.g. theta)

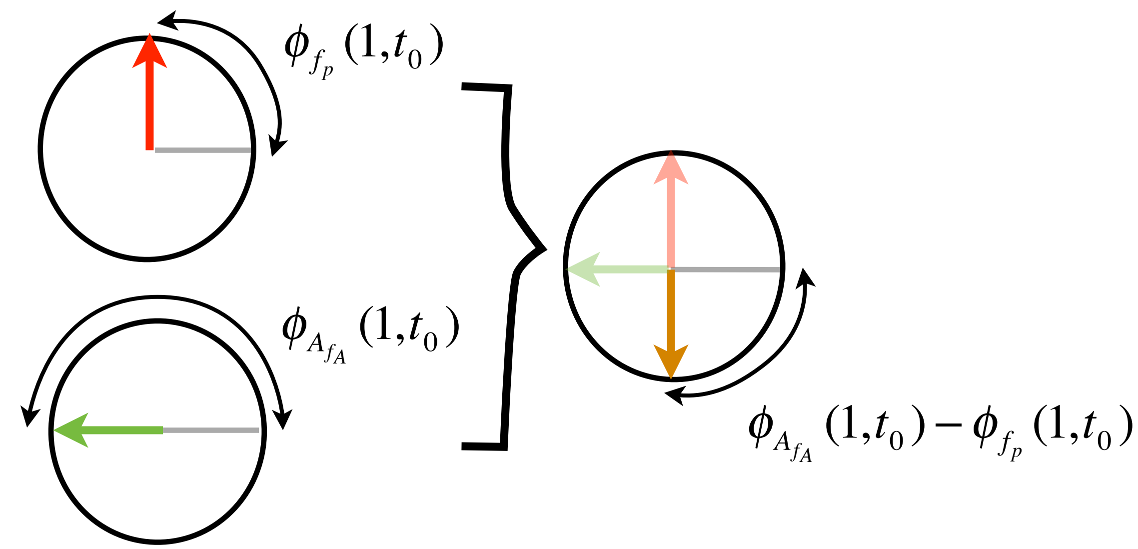
filter X_1 at amp-modulation band (e.g. gamma)

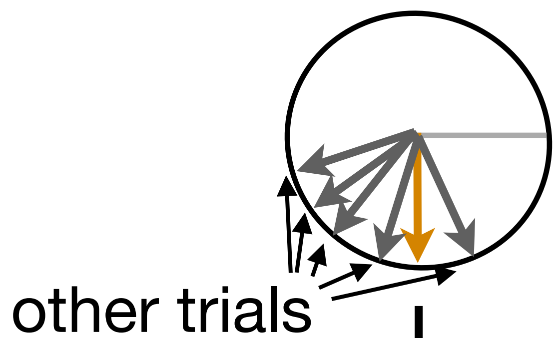
get amplitude envelope of filtered signal

Phase-Amplitude Coupling: PLV Method

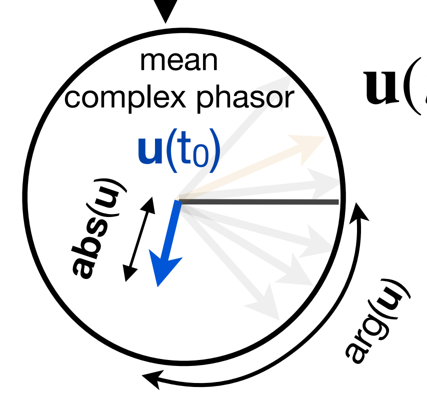
Vanhatalo, S et al (2004) *PNAS*







AVERAGE difference phasors across trials



$$\mathbf{u}(t_0) = \frac{1}{N}\sum_{k}^{N} e^{i(\phi_{A_{f_A}}(k,t_0) - \phi_{f_p}(k,t_0))}$$

PLV(t_0)=abs(**u**)

Compute PLV between phase-modulation time-series (f_p) and amplitude envelope of amplitude modulation time-series (A_{fA}). Significant PLV indicates that the central frequency of f_p modulates the amplitude of the central frequency of f_A

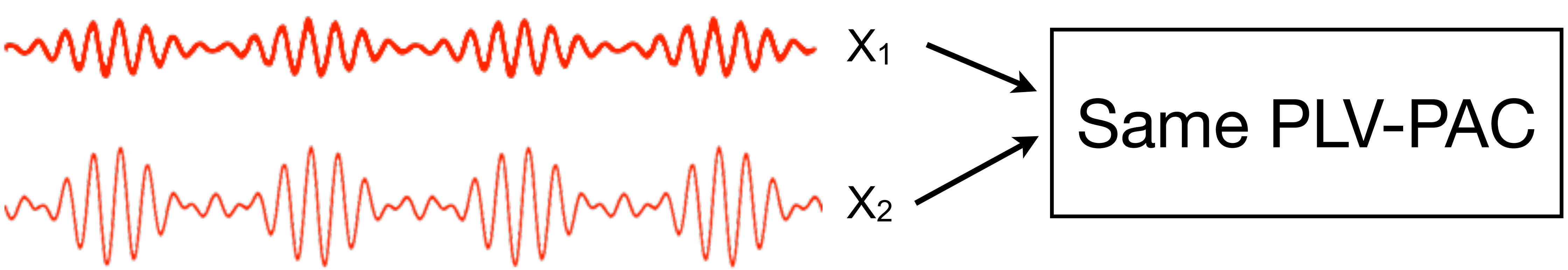
Phase-Amplitude Coupling: PLV Method

Vanhatalo, S et al (2004) *PNAS*

Problem:

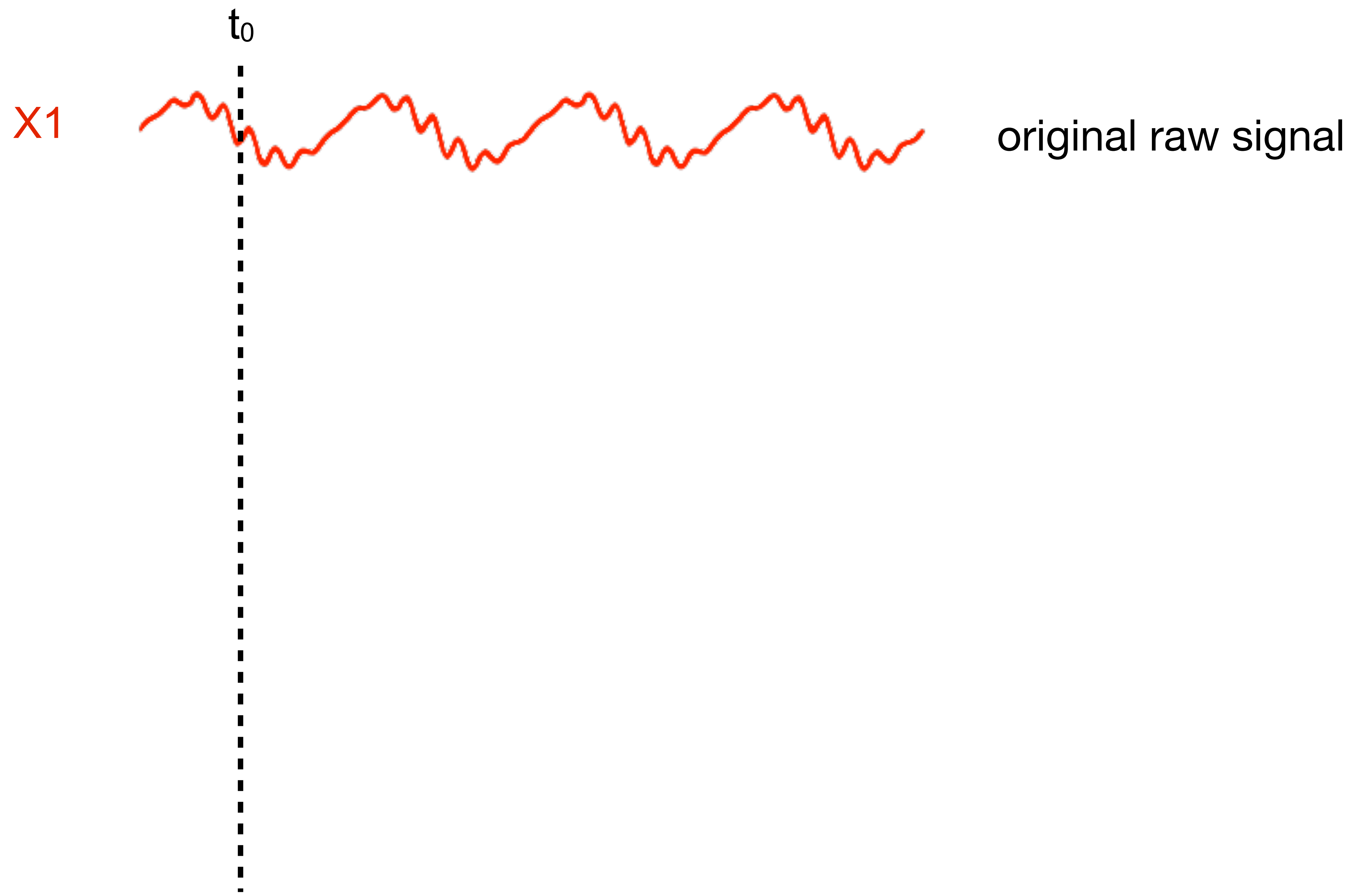
PLV is invariant to differences in amplitude between the two time-series (it only considers phase). Thus PLV-PAC doesn't take into account the *amplitude* of the co-modulation.

In the example below, X_1 and X_2 both would produce the same PAC, even though the high-frequency amplitude of X_2 clearly is more strongly modulated by the low-frequency rhythm.



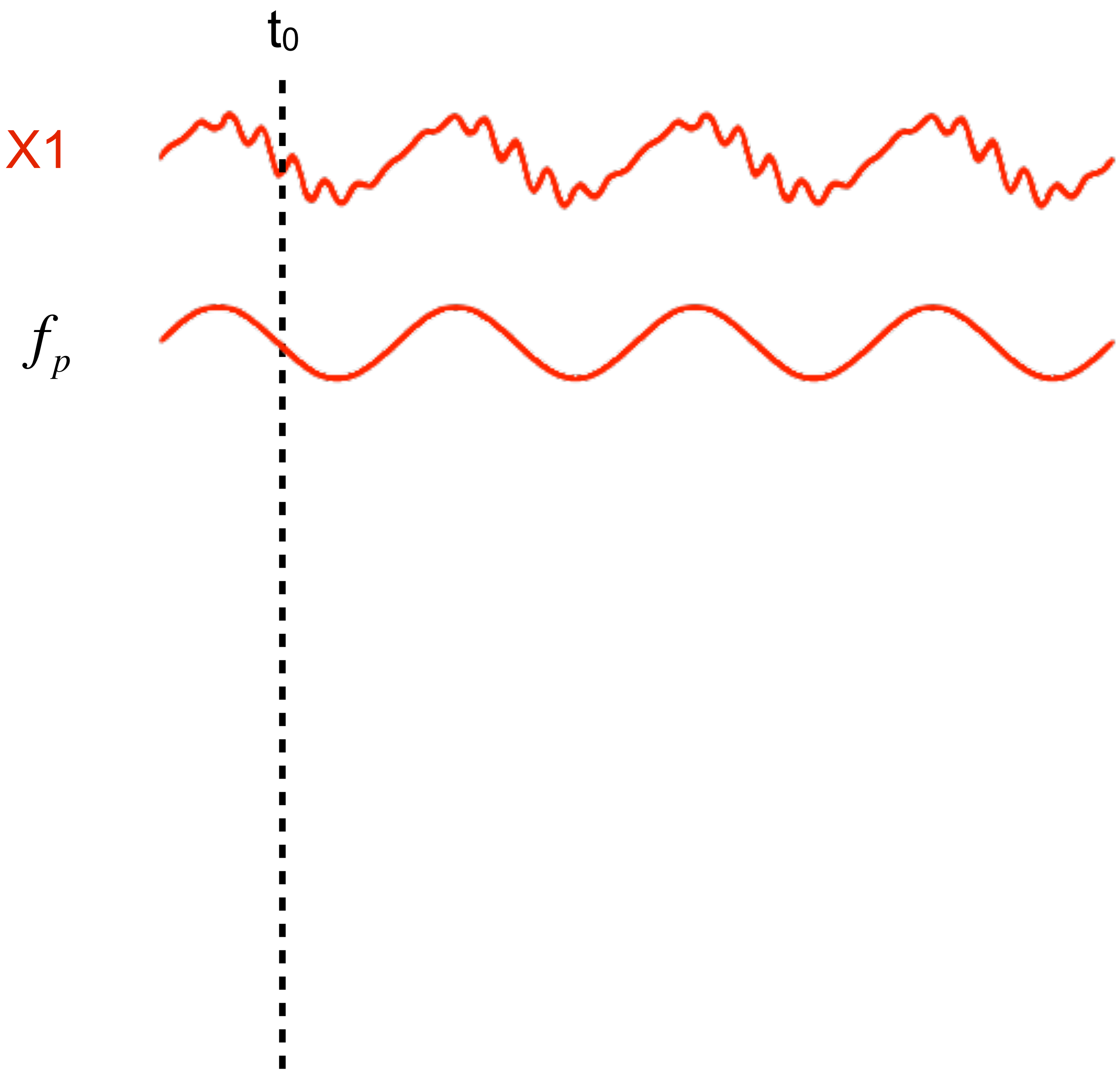
Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*

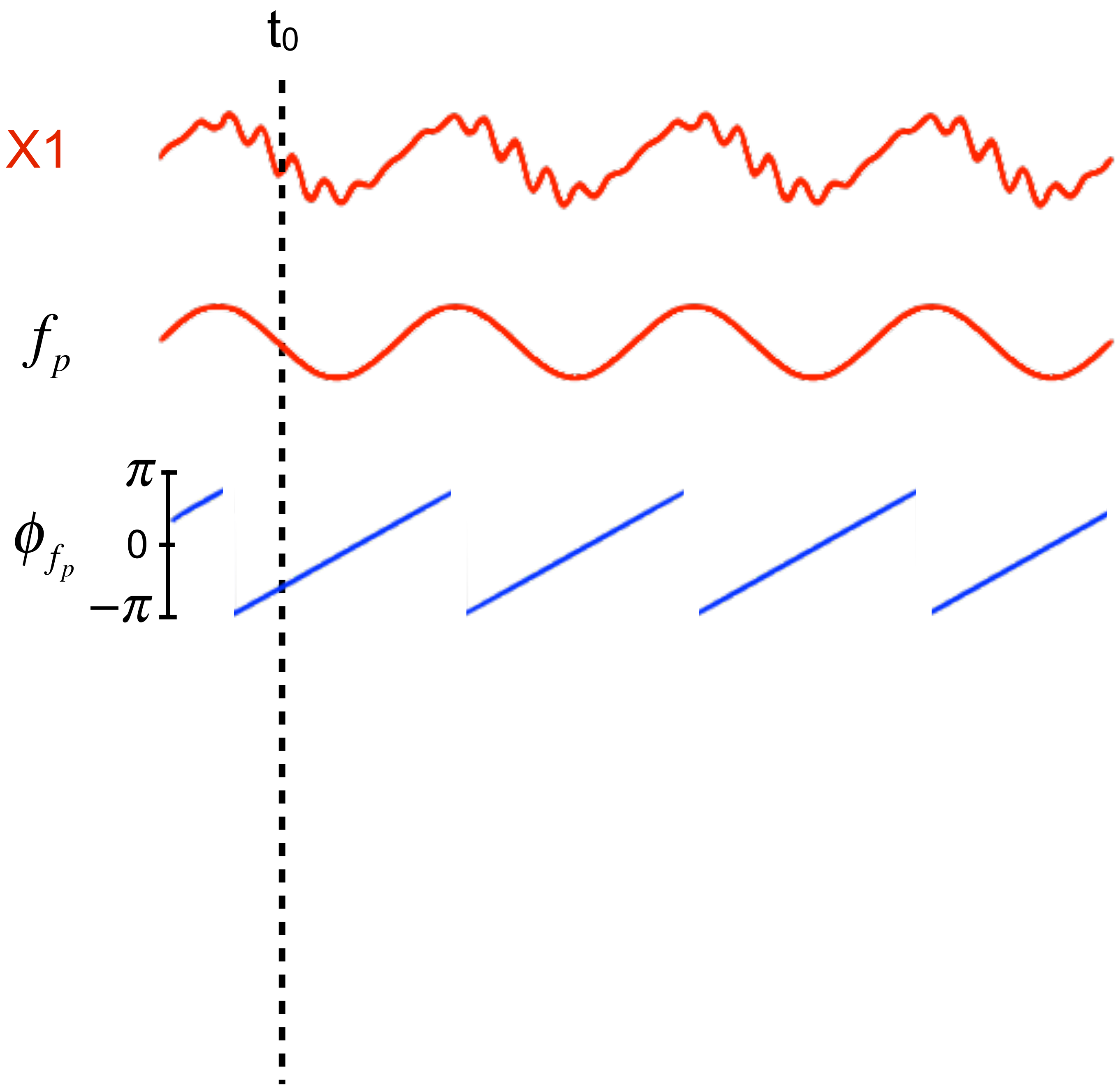


original raw signal

filter X_1 at phase-modulation band (e.g. theta)

Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



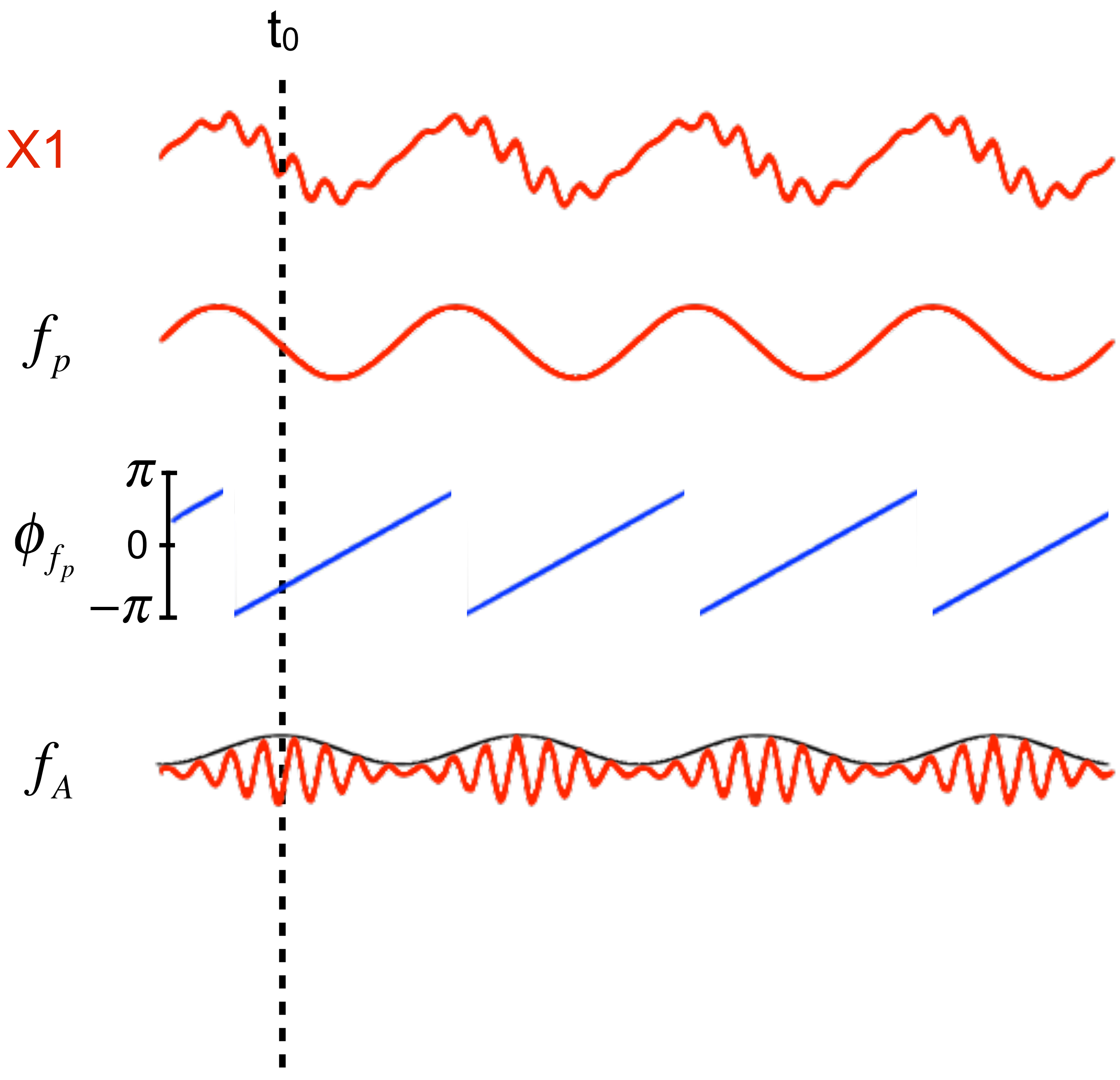
original raw signal

filter X_1 at phase-modulation band (e.g. theta)

extract the instantaneous phase of f_p

Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



original raw signal

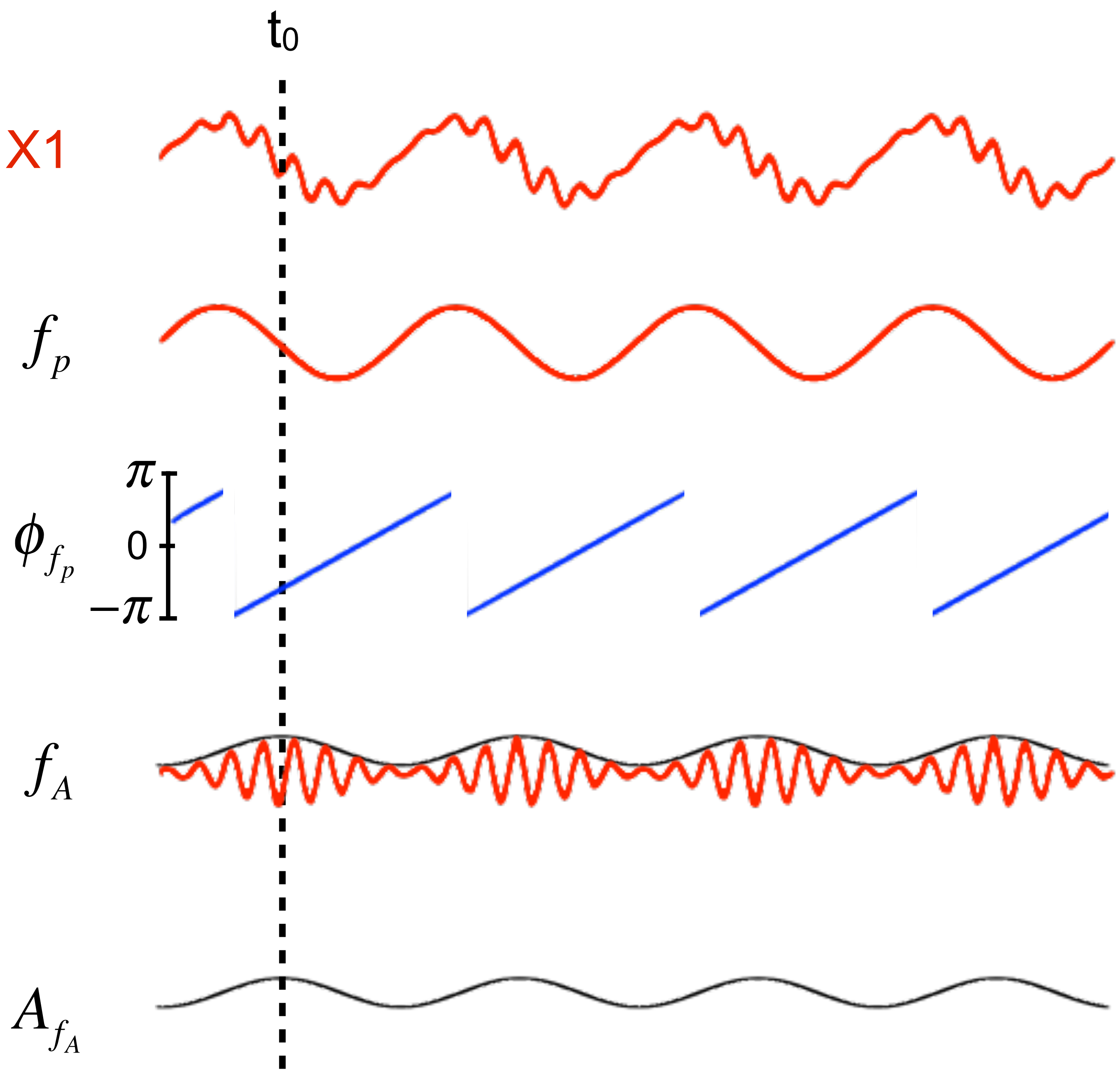
filter X_1 at phase-modulation band (e.g. theta)

extract the instantaneous phase of f_p

filter X_1 at amp-modulation band (e.g. gamma)

Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



original raw signal

filter X_1 at phase-modulation band (e.g. theta)

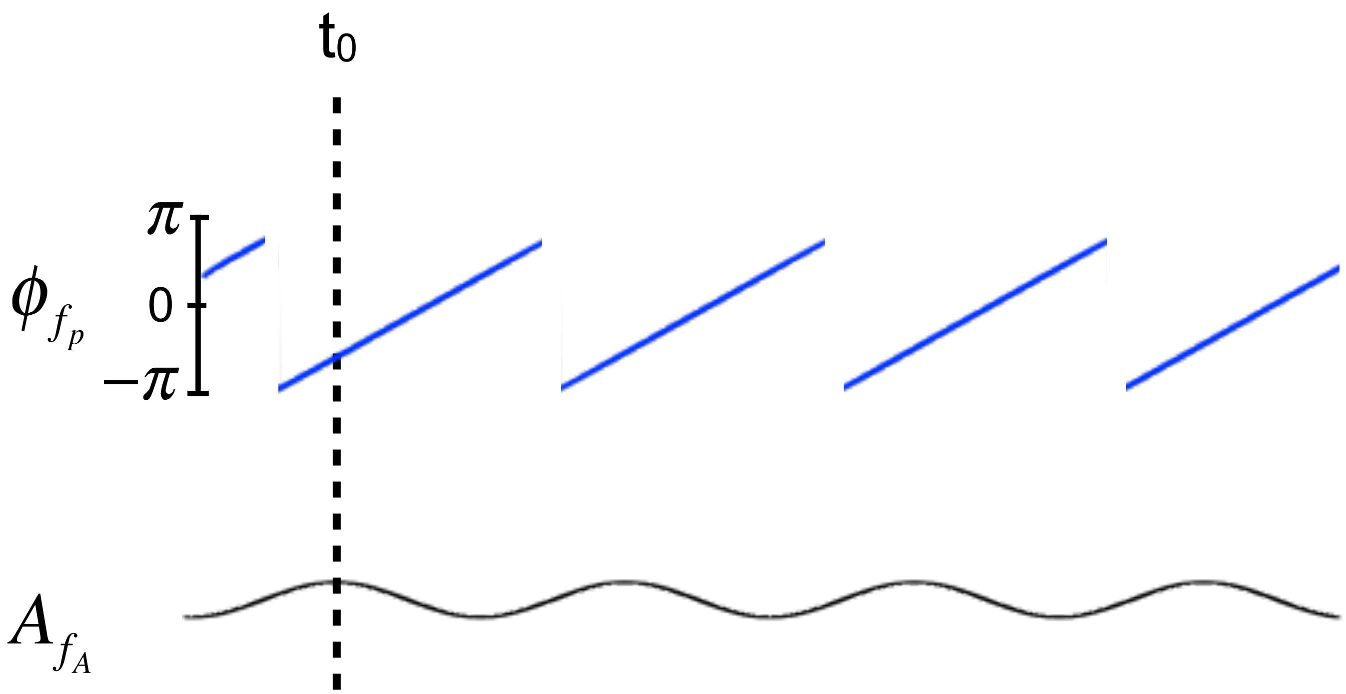
extract the instantaneous phase of f_p

filter X_1 at amp-modulation band (e.g. gamma)

get amplitude envelope of filtered signal

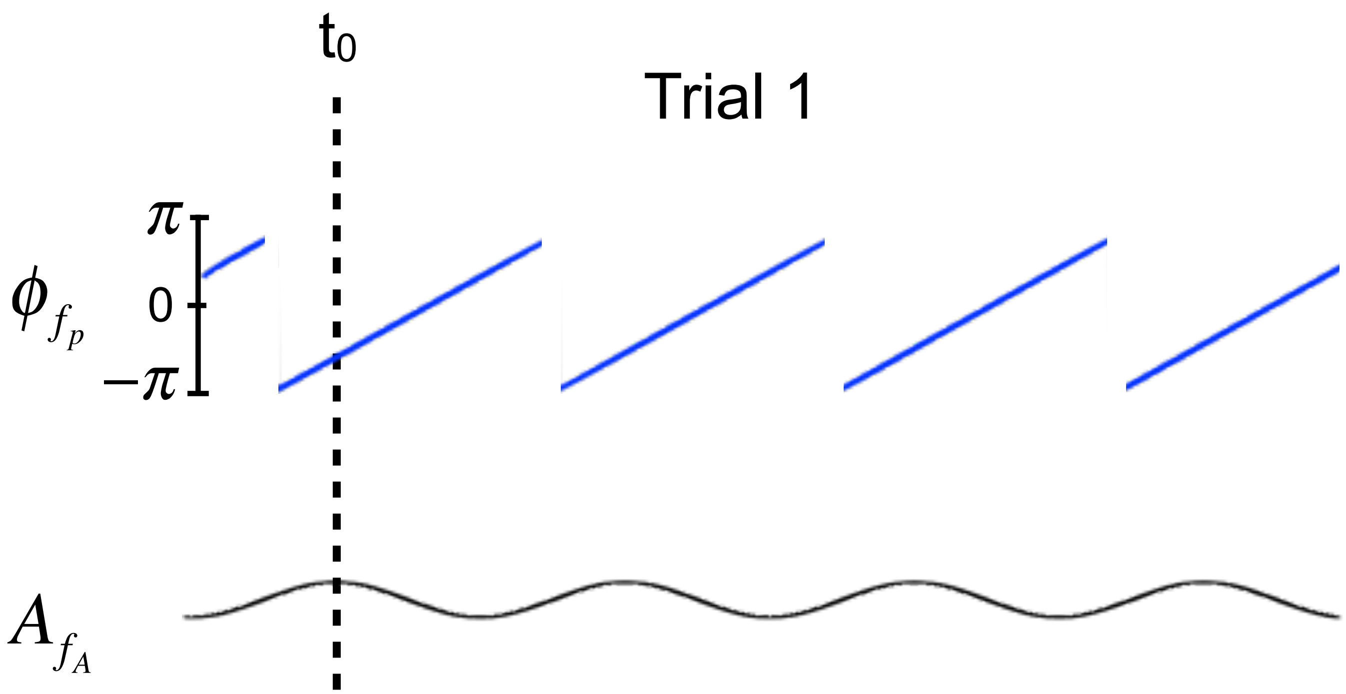
Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



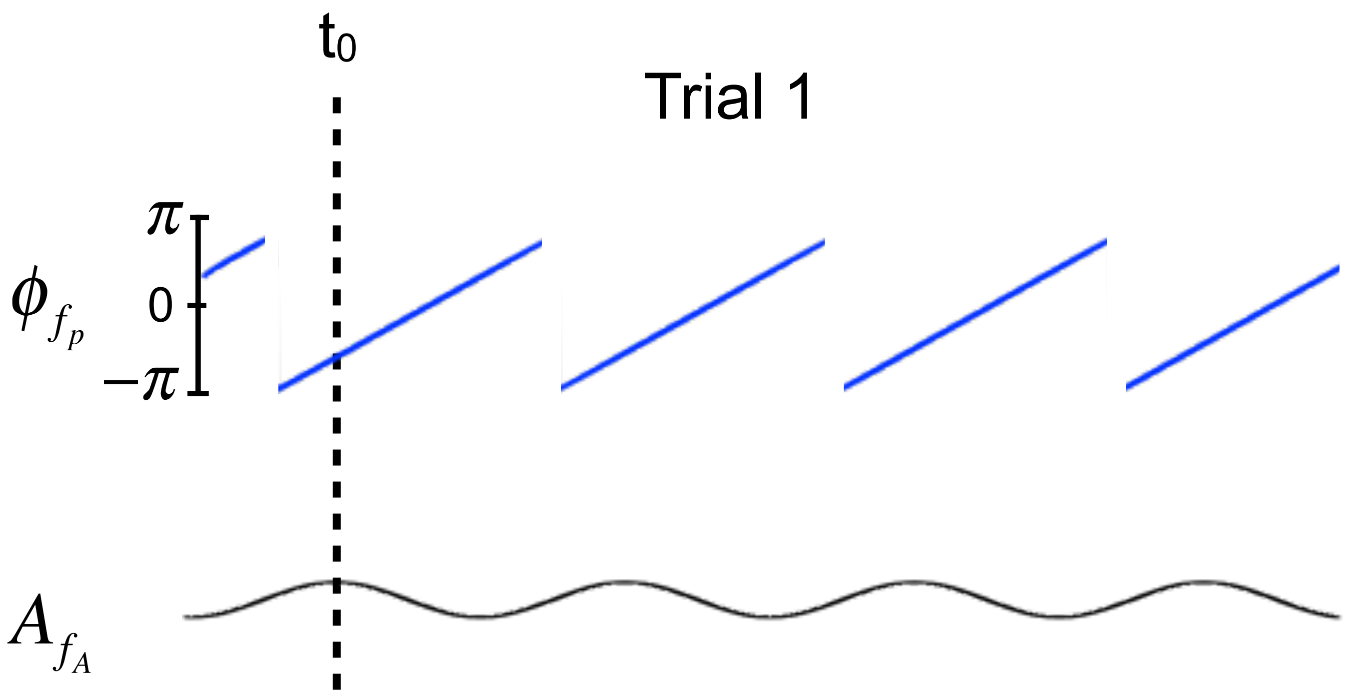
Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



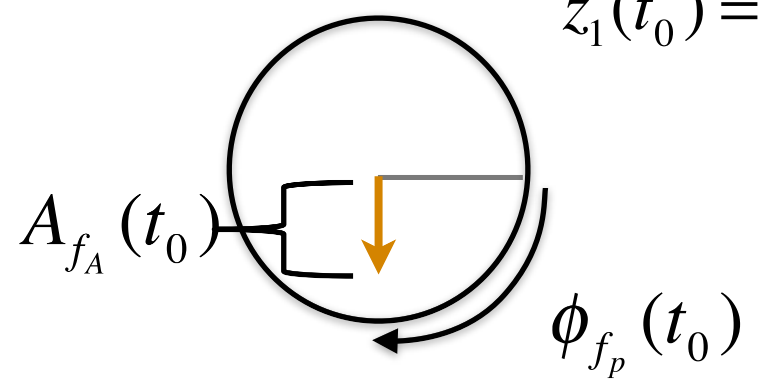
Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*



build complex phasor with instantaneous amplitude and phase

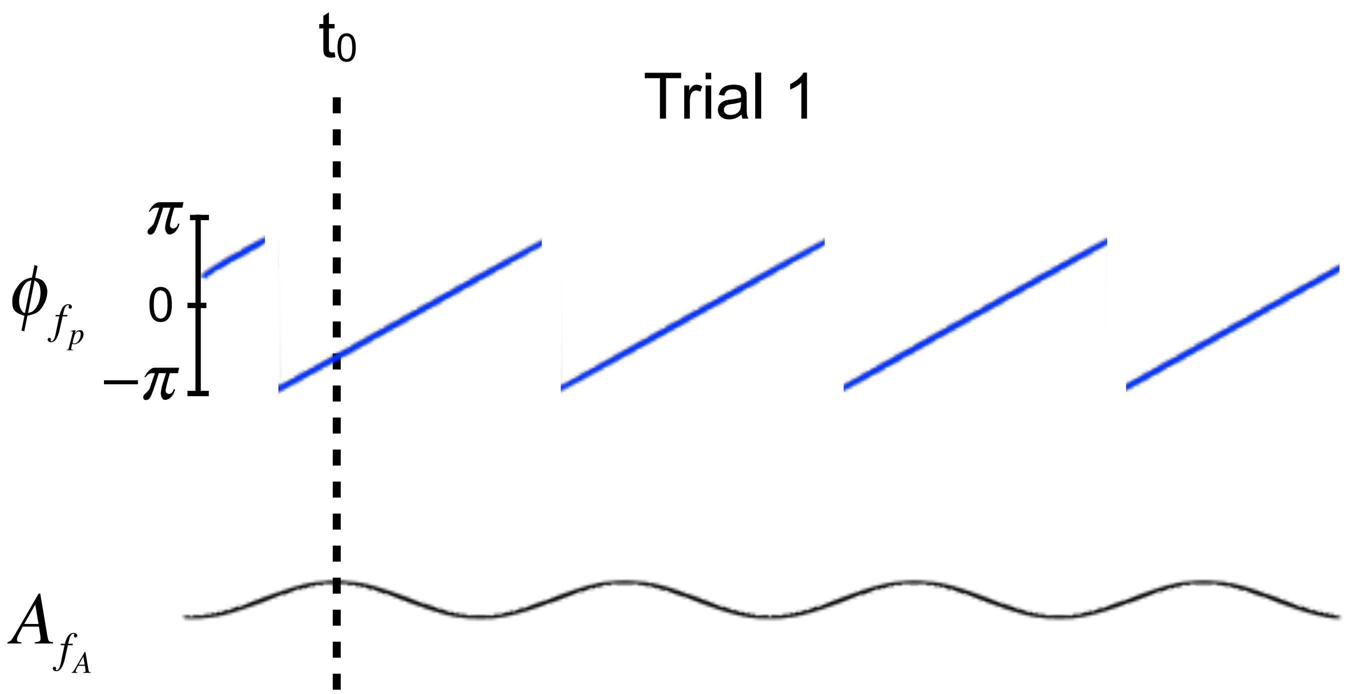
$$z_1(t_0) = A_{f_A} e^{i\phi_{f_p}}$$

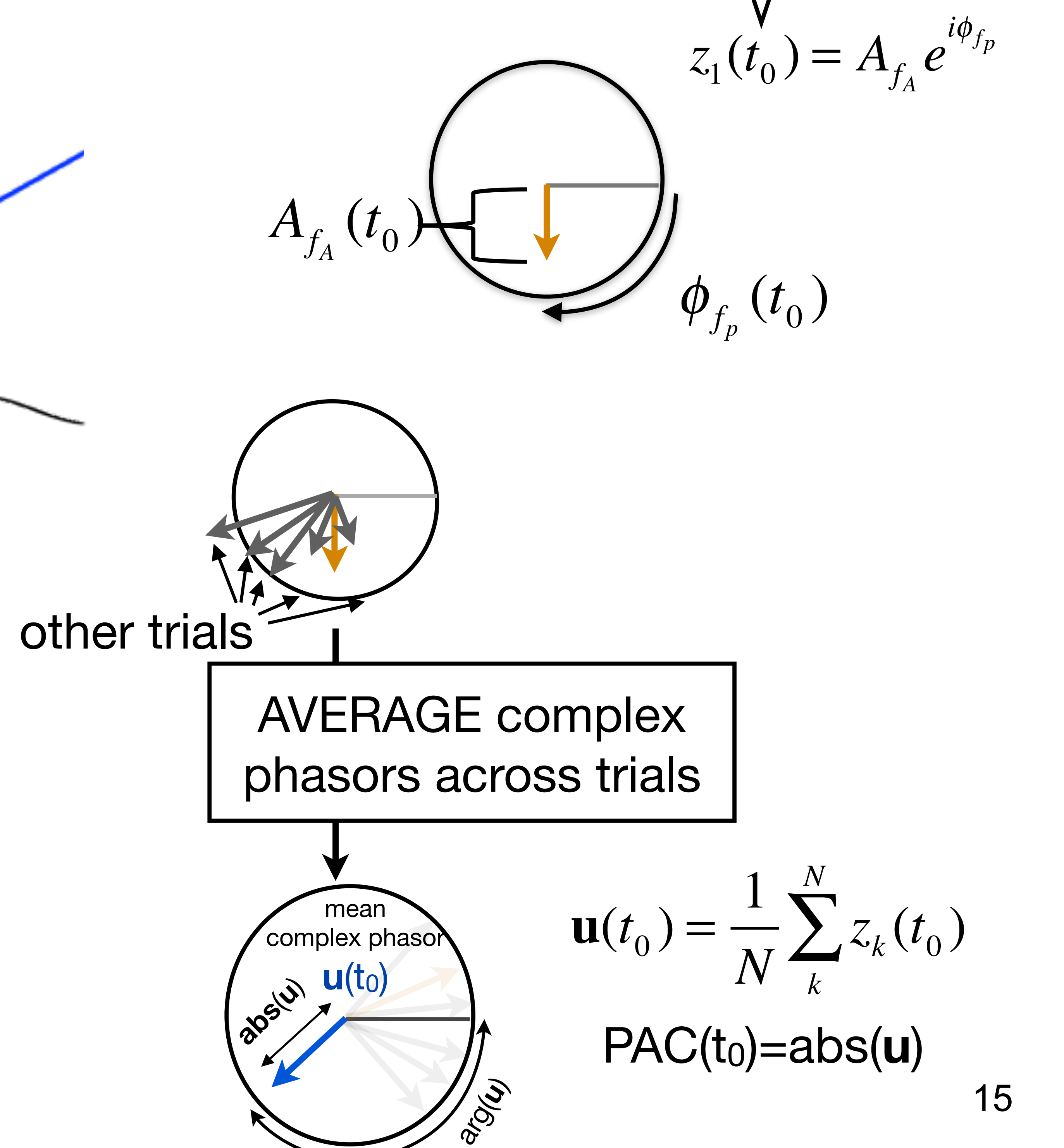


Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*

build complex phasor with instantaneous amplitude and phase





$$\mathbf{u}(t_0) = \frac{1}{N}\sum_{k}^{N} z_k(t_0)$$

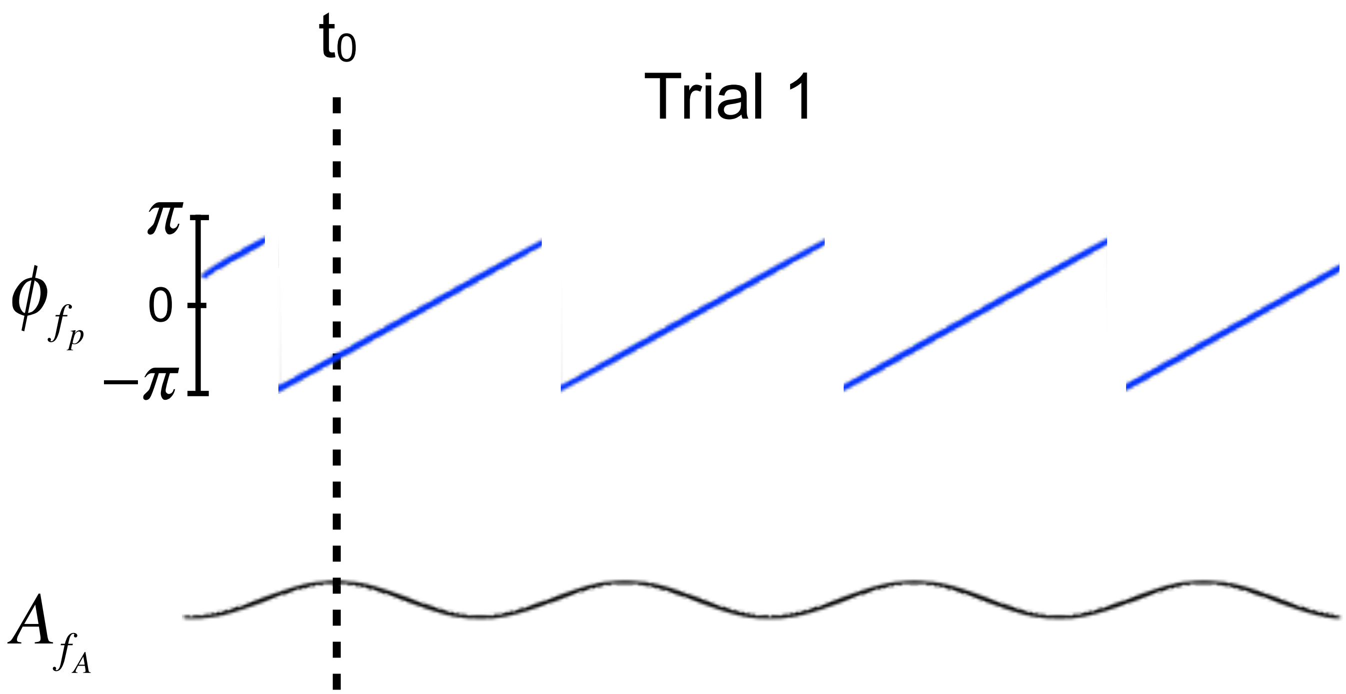
PAC(t_0)=abs(**u**)

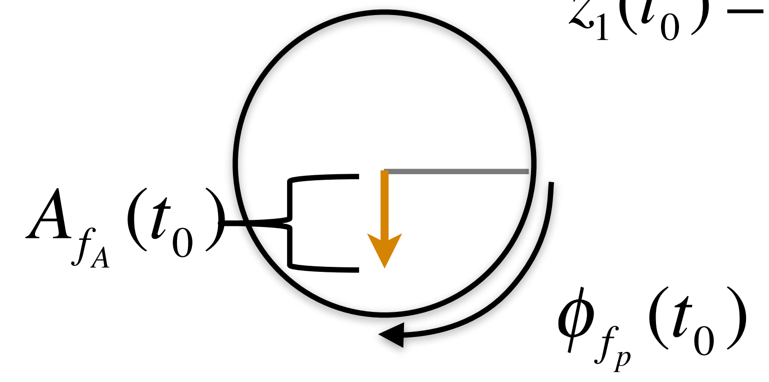
Phase-Amplitude Coupling: Modulation Index Method

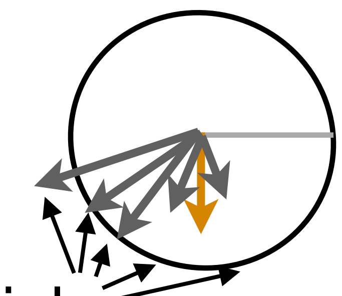
Canolty et al, (2006) *Science*

build complex phasor with instantaneous amplitude and phase

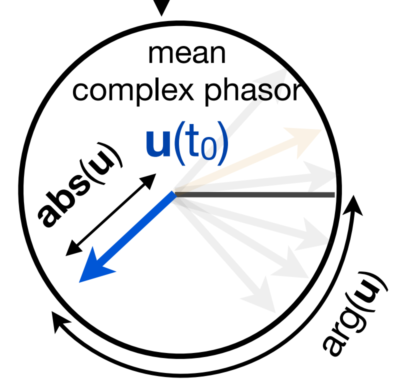
$$z_1(t_0) = A_{f_A} e^{i\phi_{f_p}}$$







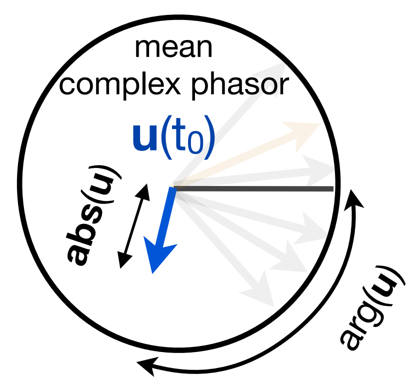
AVERAGE complex phasors across trials



$$\mathbf{u}(t_0) = \frac{1}{N}\sum_{k}^{N} z_k(t_0)$$

PAC(t_0)=abs(**u**)

Comparison:
PLV-PAC



Phase-Amplitude Coupling: Modulation Index Method

Canolty et al, (2006) *Science*

Computing PAC in EEGLAB:

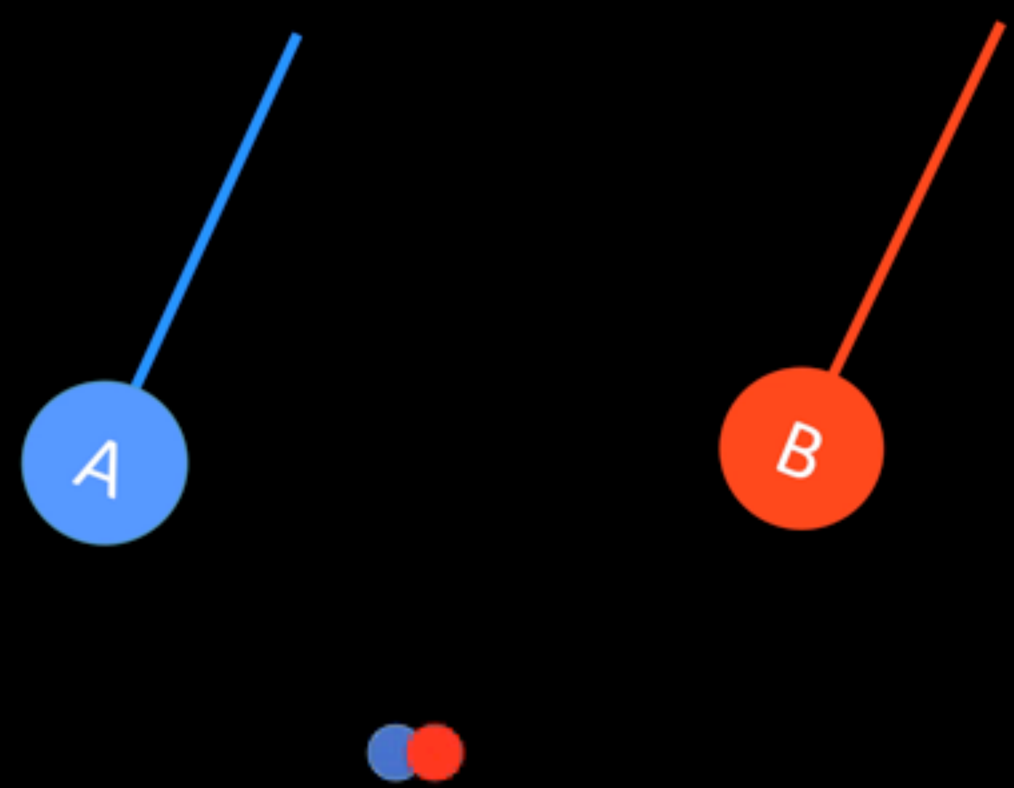
```
pac(IC1,IC2,...,'method','mod')
```

PAC can also be applied *between* sources/channels (e.g. determine whether the phase of oscillation at freq. w_p in IC1 modulates the amplitude of oscillation at freq. w_A *in IC2.* This leads to a measure of cross-frequency (non-linear) functional connectivity.

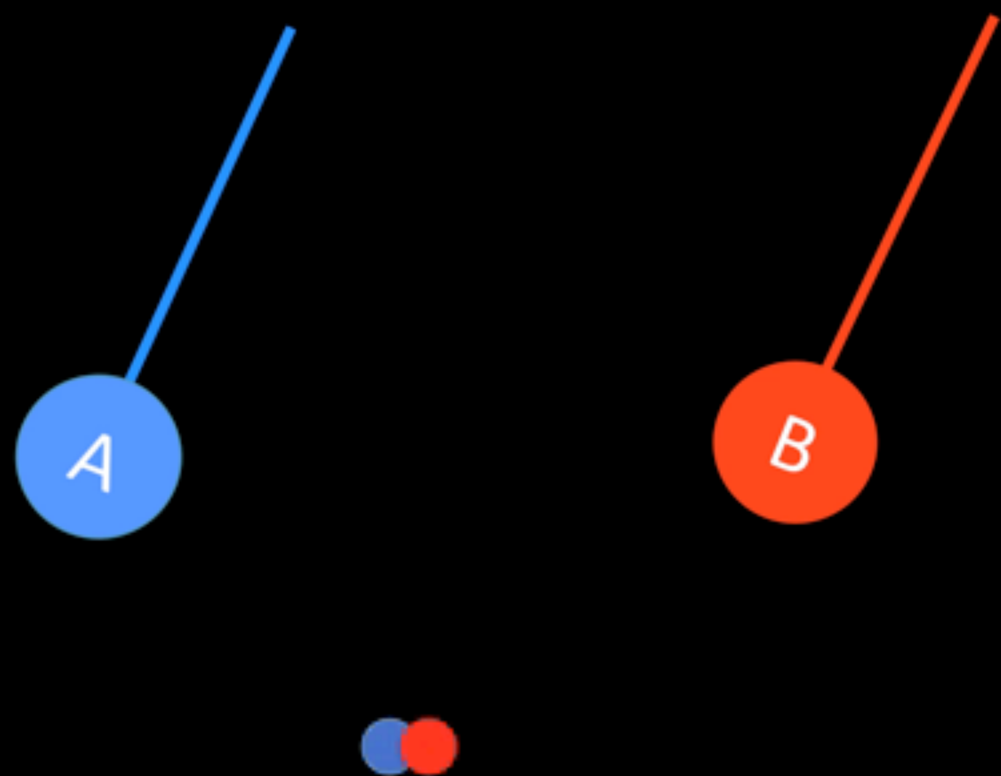
For Modulation Index method (other modes also available)

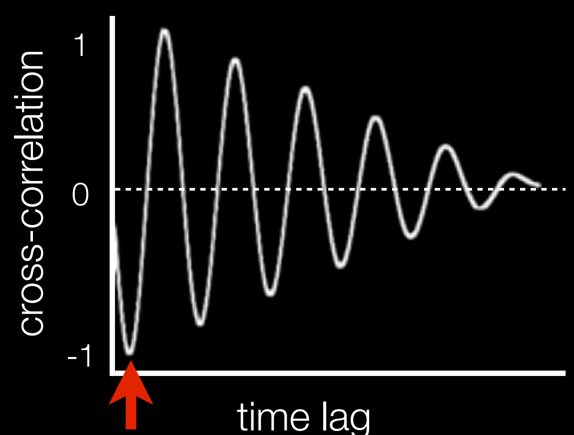
(Cross)-Correlation ≠ Causation

(Cross)-Correlation ≠ Causation

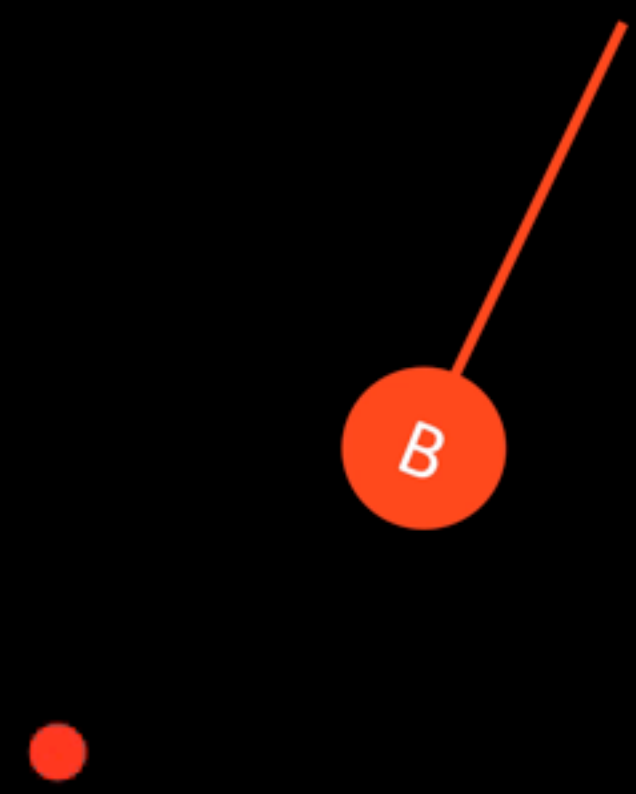


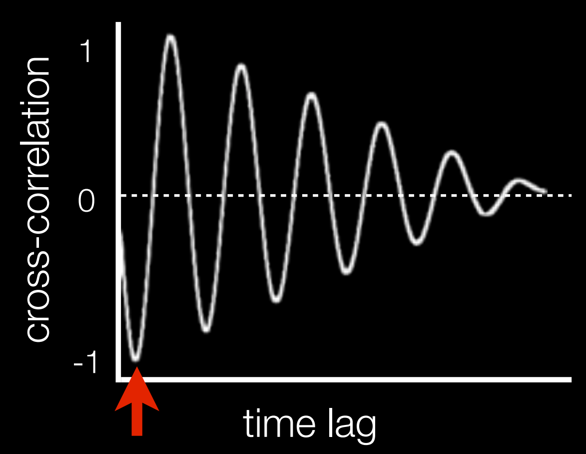
(Cross)-Correlation ≠ Causation



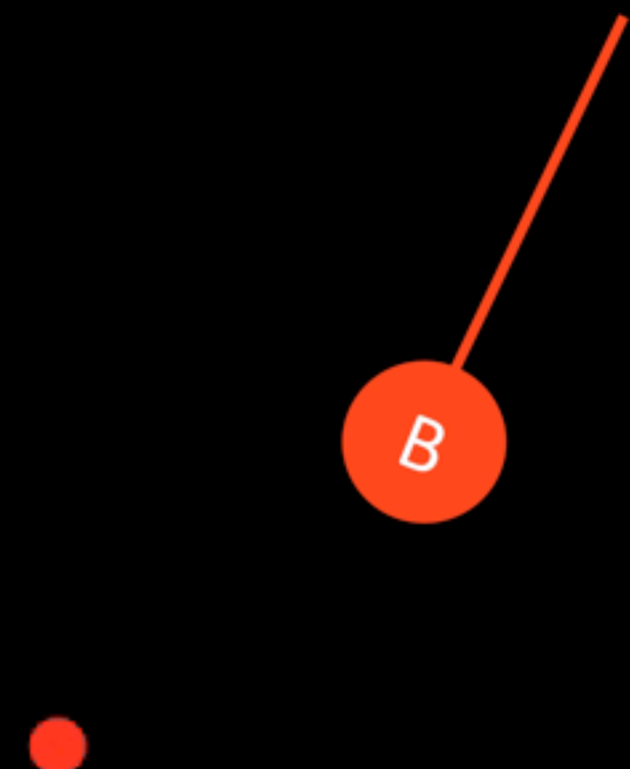


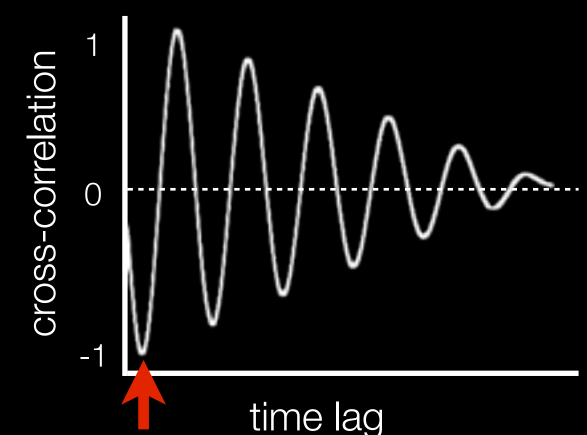
(Cross)-Correlation ≠ Causation





(Cross)-Correlation ≠ Causation





Coherence/CC/PLV/PAC indicate ***functional***,
but not ***effective*** connectivity

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful
- Scalable (Valdes-Sosa, 2005)

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful
- Scalable (Valdes-Sosa, 2005)
- Extendable to nonlinear and/or non-stationary systems (Freiwald, 1999; Ding, 2001; Chen, 2004; Ge, 2009)

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful
- Scalable (Valdes-Sosa, 2005)
- Extendable to nonlinear and/or non-stationary systems (Freiwald, 1999; Ding, 2001; Chen, 2004; Ge, 2009)
- Extendable to non-parametric representations (Dhamala, 2009a,b)

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful
- Scalable (Valdes-Sosa, 2005)
- Extendable to nonlinear and/or non-stationary systems (Freiwald, 1999; Ding, 2001; Chen, 2004; Ge, 2009)
- Extendable to non-parametric representations (Dhamala, 2009a,b)
- Can be (partially) controlled for (unobserved) exogenous causes (Guo, 2008a,b; Ge, 2009)

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful
- Scalable (Valdes-Sosa, 2005)
- Extendable to nonlinear and/or non-stationary systems (Freiwald, 1999; Ding, 2001; Chen, 2004; Ge, 2009)
- Extendable to non-parametric representations (Dhamala, 2009a,b)
- Can be (partially) controlled for (unobserved) exogenous causes (Guo, 2008a,b; Ge, 2009)
- Equivalent to Transfer Entropy for Gaussian Variables (Seth, 2009)

Estimating Effective Connectivity

Non-Invasive

- *Post-hoc* analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Simple, but powerful
- Scalable (Valdes-Sosa, 2005)
- Extendable to nonlinear and/or non-stationary systems (Freiwald, 1999; Ding, 2001; Chen, 2004; Ge, 2009)
- Extendable to non-parametric representations (Dhamala, 2009a,b)
- Can be (partially) controlled for (unobserved) exogenous causes (Guo, 2008a,b; Ge, 2009)
- Equivalent to Transfer Entropy for Gaussian Variables (Seth, 2009)
- Flexibly allows us to examine **time-varying** (dynamic) multivariate causal relationships in either the **time** or **frequency** domain

Granger Causality

- First introduced by Wiener (1958). Later reformulated by Granger (1969) in the context of linear stochastic autoregressive models
- Relies on two assumptions:

Granger Causality

- First introduced by Wiener (1958). Later reformulated by Granger (1969) in the context of linear stochastic autoregressive models
- Relies on two assumptions:

Granger Causality Axioms

Granger Causality

- First introduced by Wiener (1958). Later reformulated by Granger (1969) in the context of linear stochastic autoregressive models
- Relies on two assumptions:

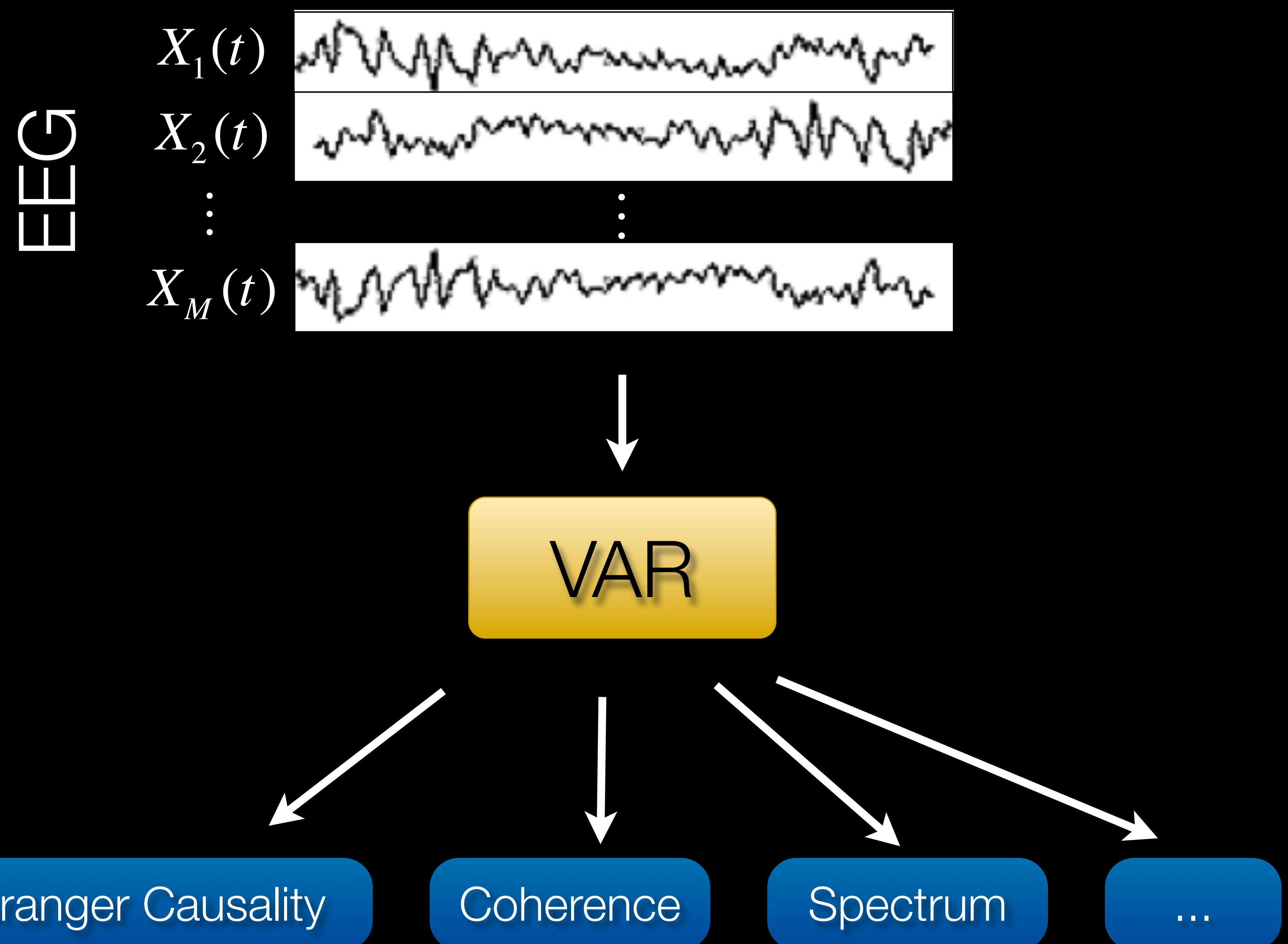
Granger Causality Axioms
1. Causes should precede their effects in time (Temporal Precedence)

Granger Causality

- First introduced by Wiener (1958). Later reformulated by Granger (1969) in the context of linear stochastic autoregressive models
- Relies on two assumptions:

Granger Causality Axioms
1. Causes should precede their effects in time (Temporal Precedence)
2. Information in a cause's past should improve the prediction of the effect, above and beyond the information contained in past of the effect (and other measured variables)

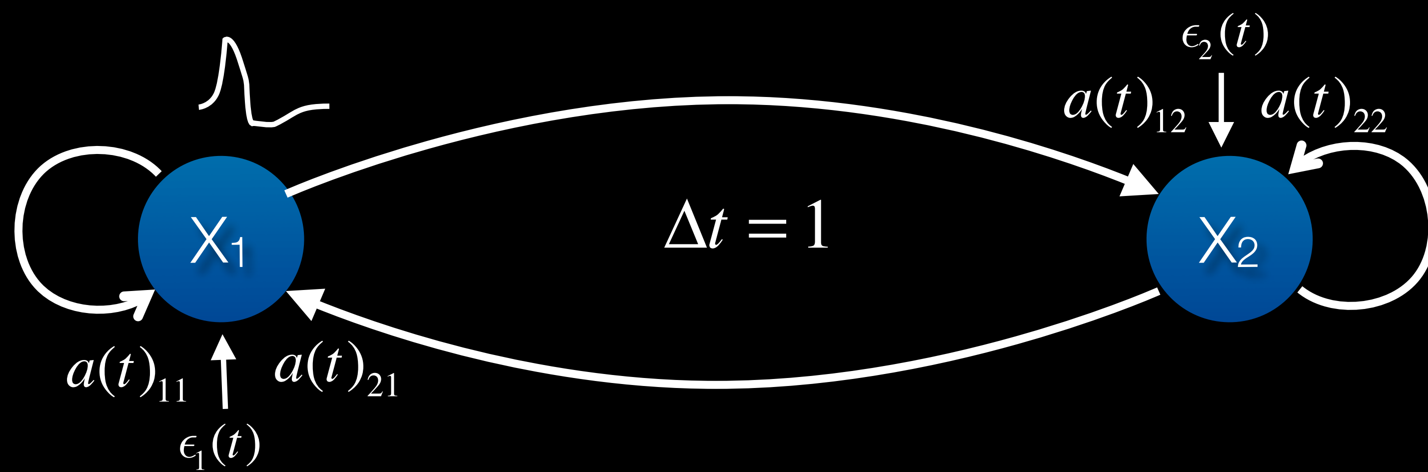
Multivariate (Vector) Autoregressive (VAR) Modeling



The VAR Process

Stochastic Linear Dynamical System

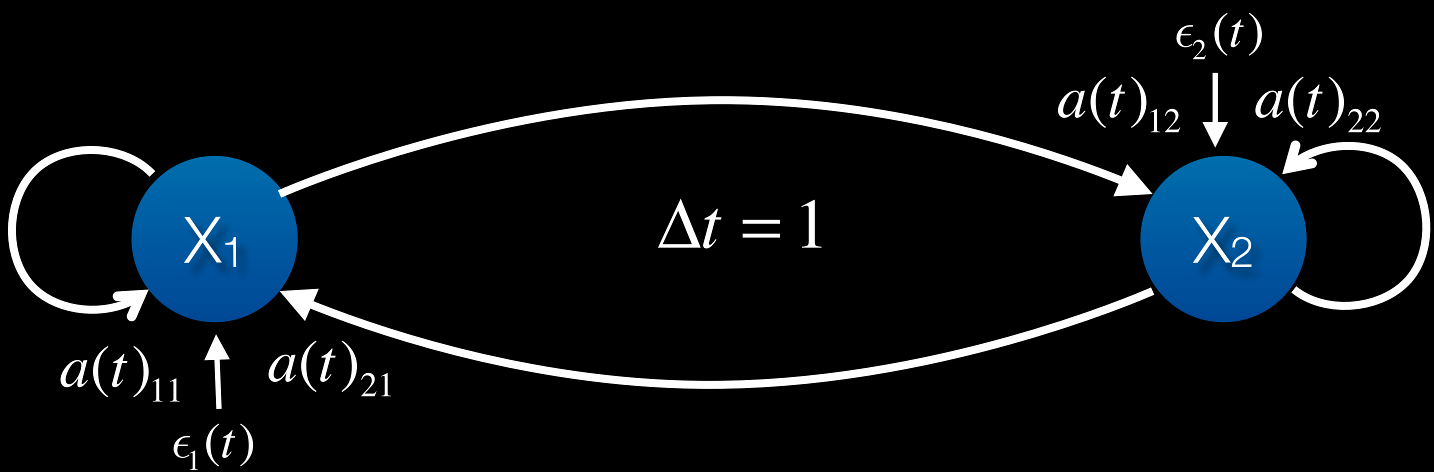
$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$



The VAR Process

Stochastic Linear Dynamical System

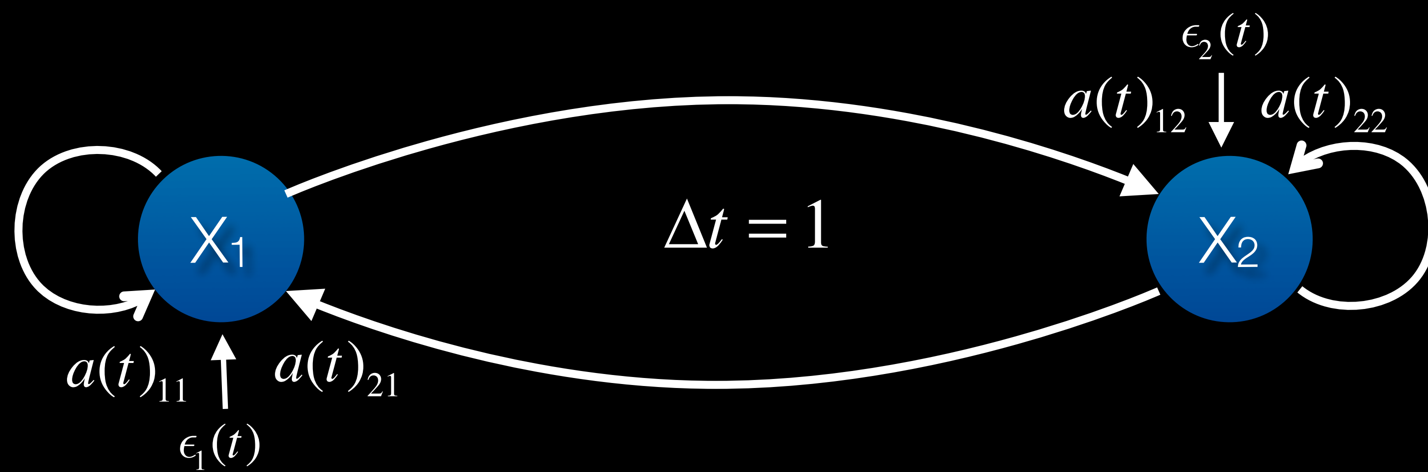
$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$

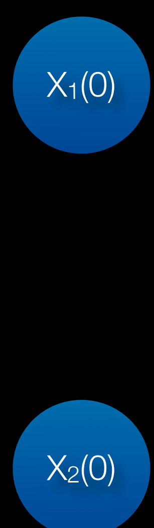


The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$





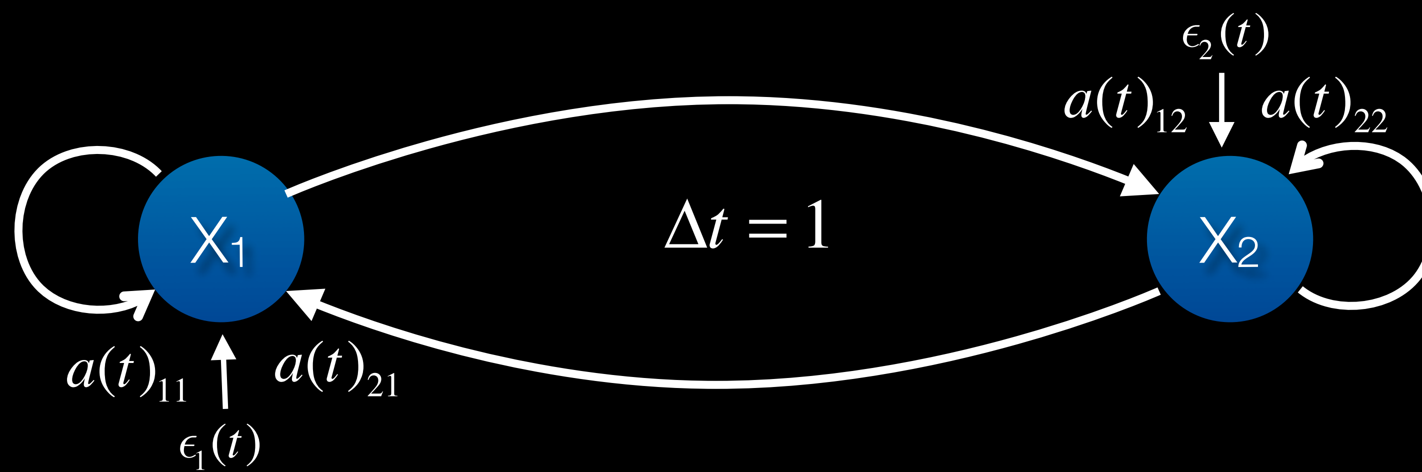
t=0

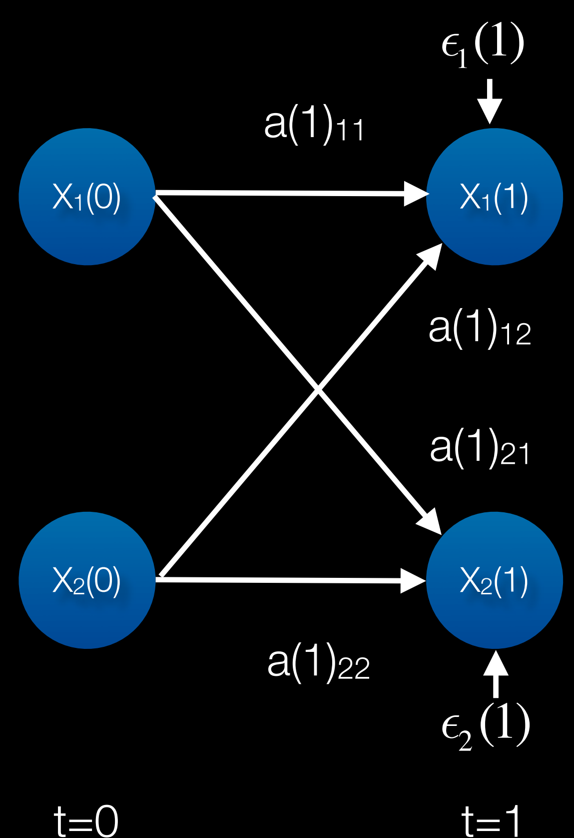
time step

The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$



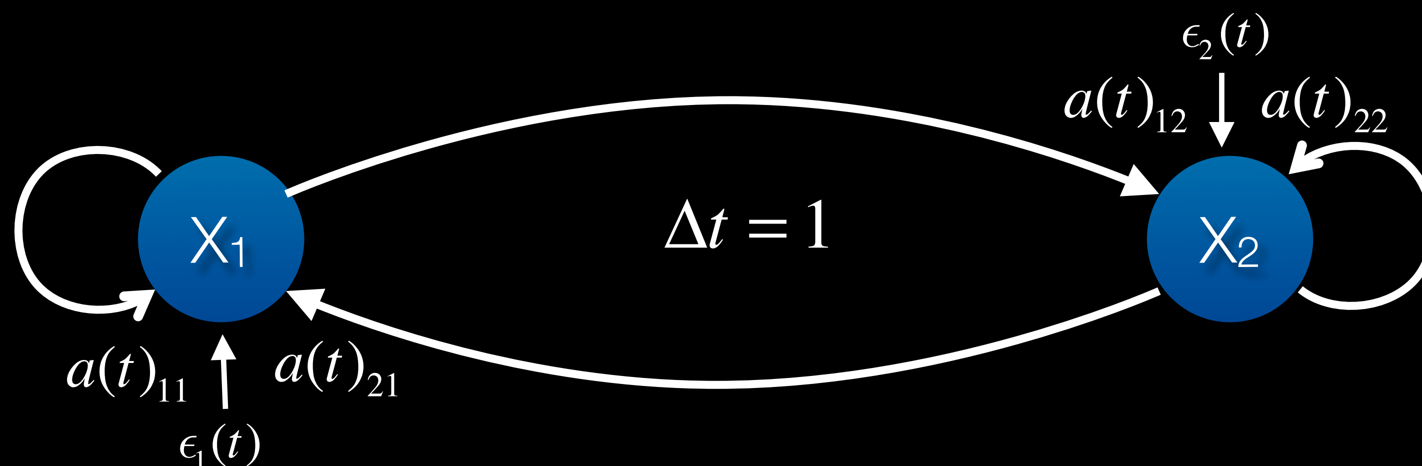


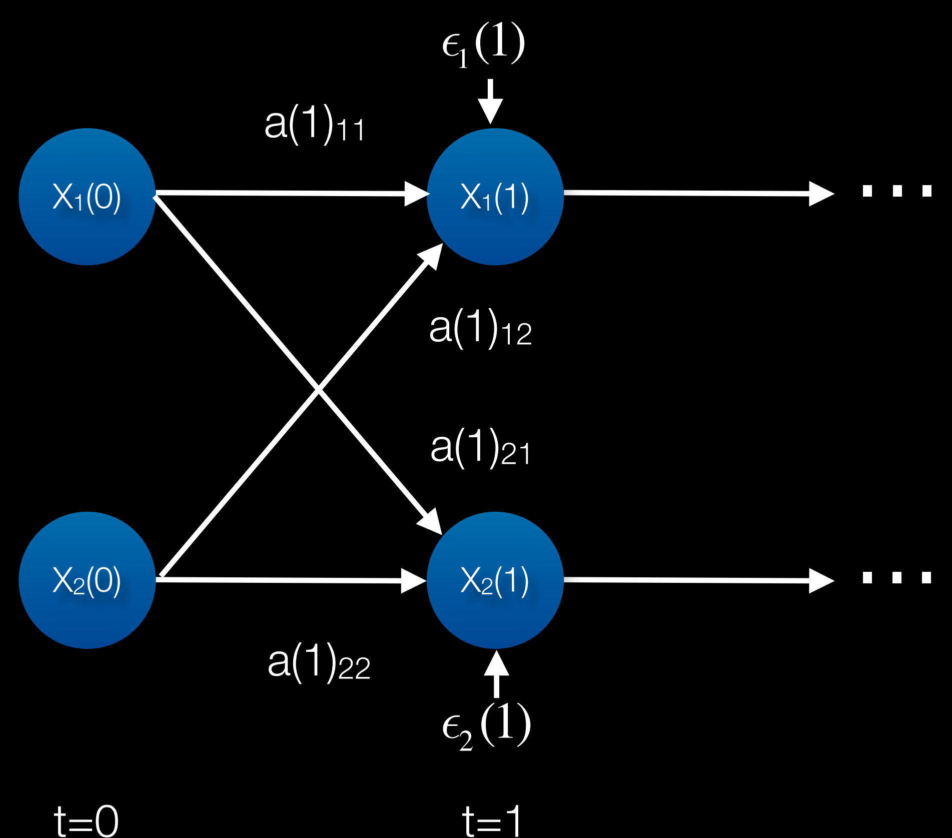
time step

The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$



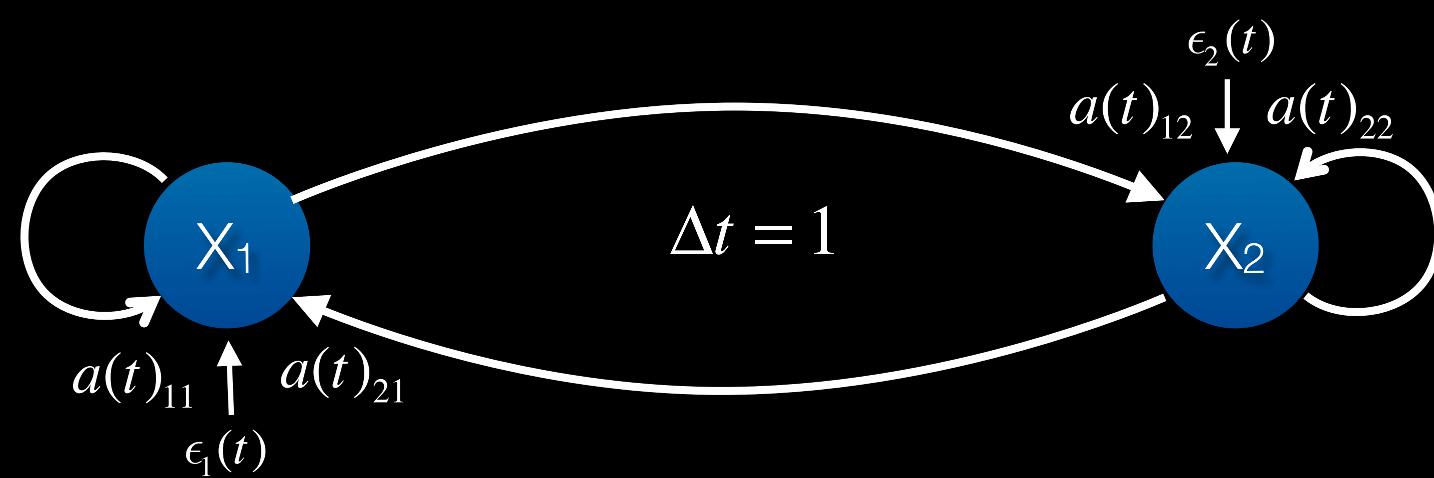


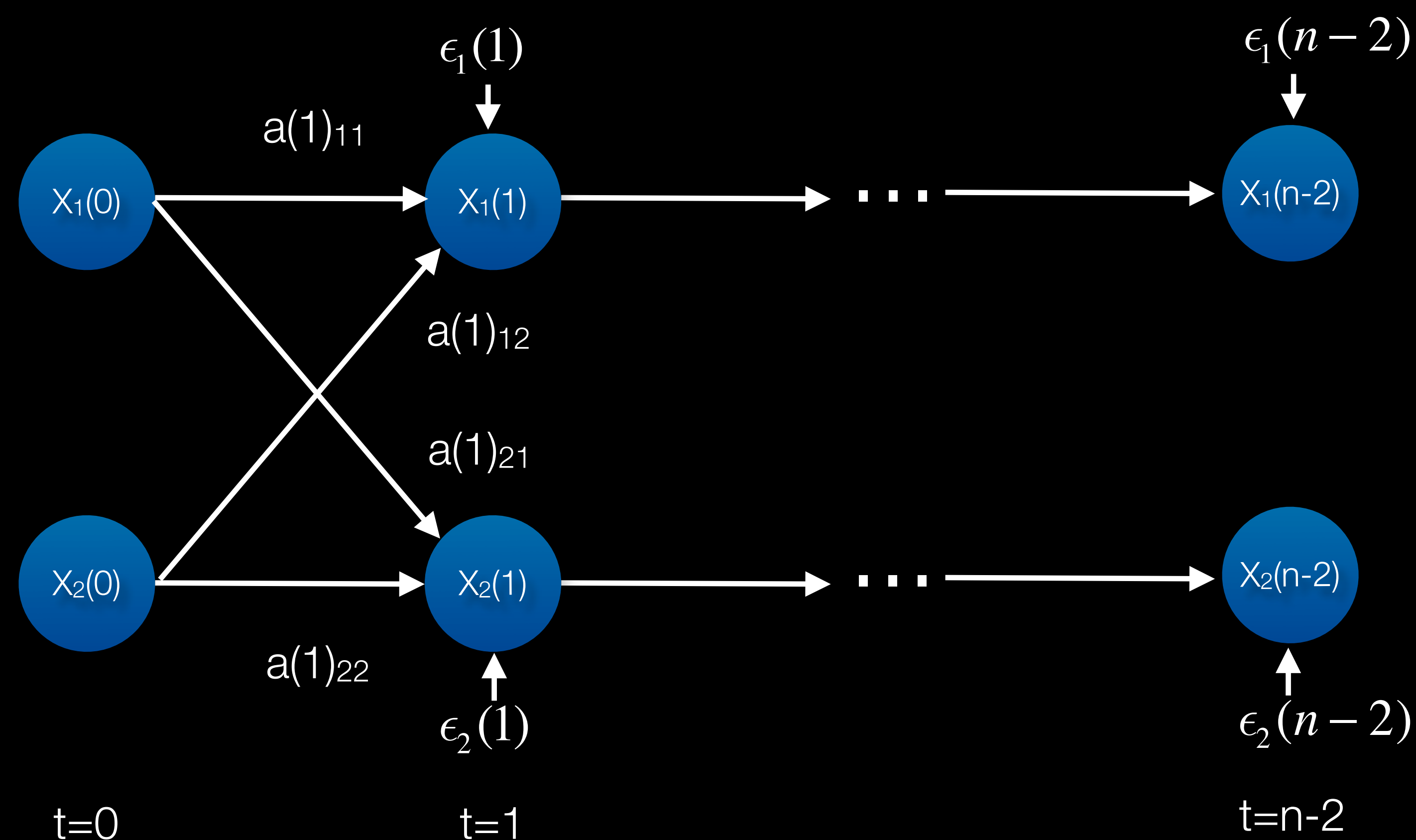
time step

The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$



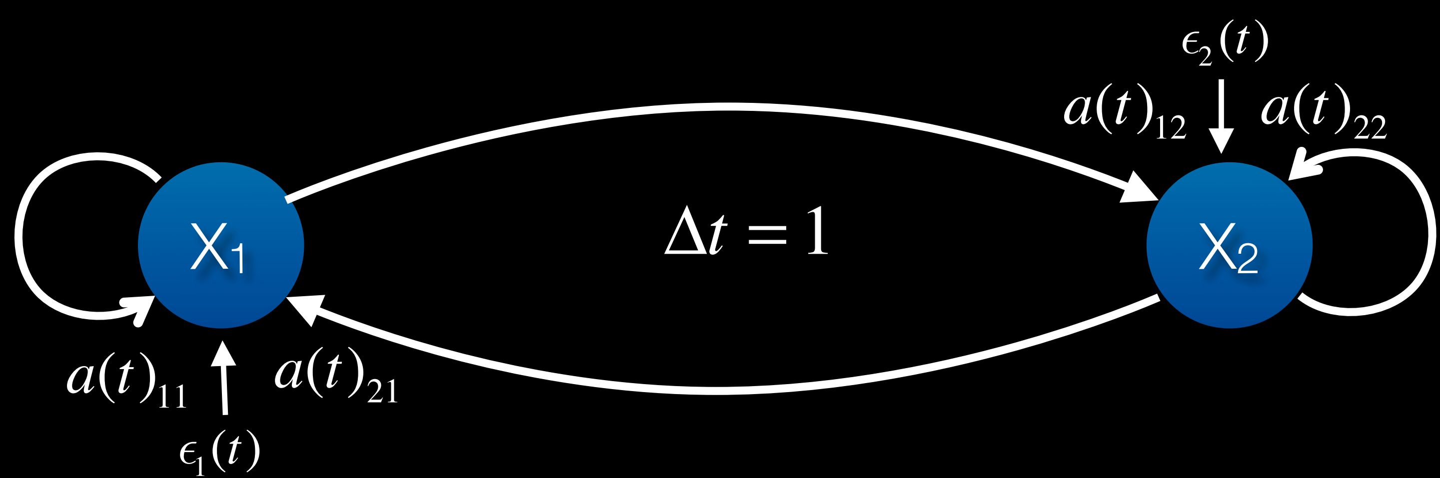


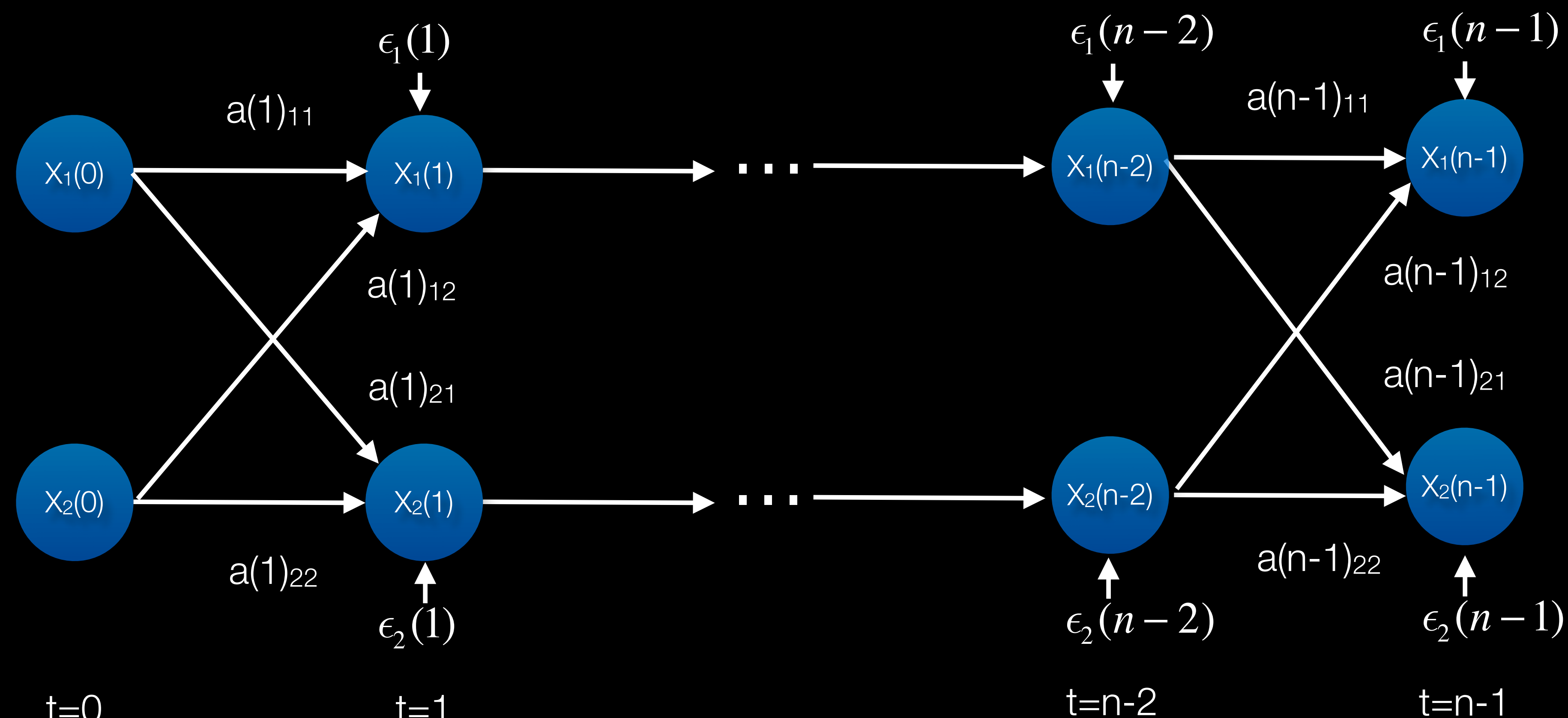
time step

The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$



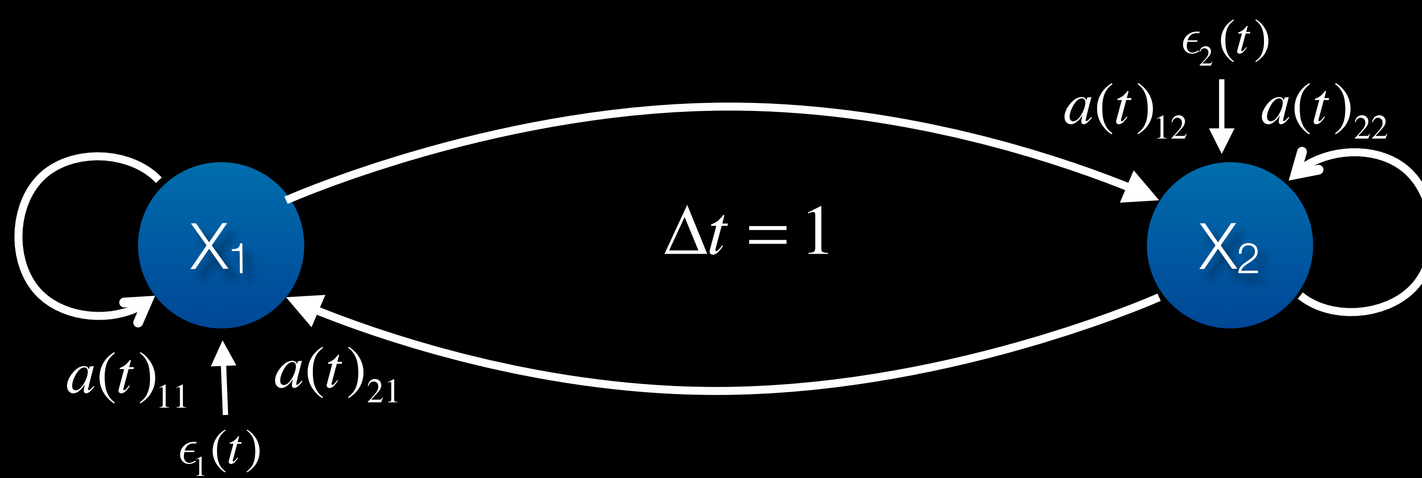


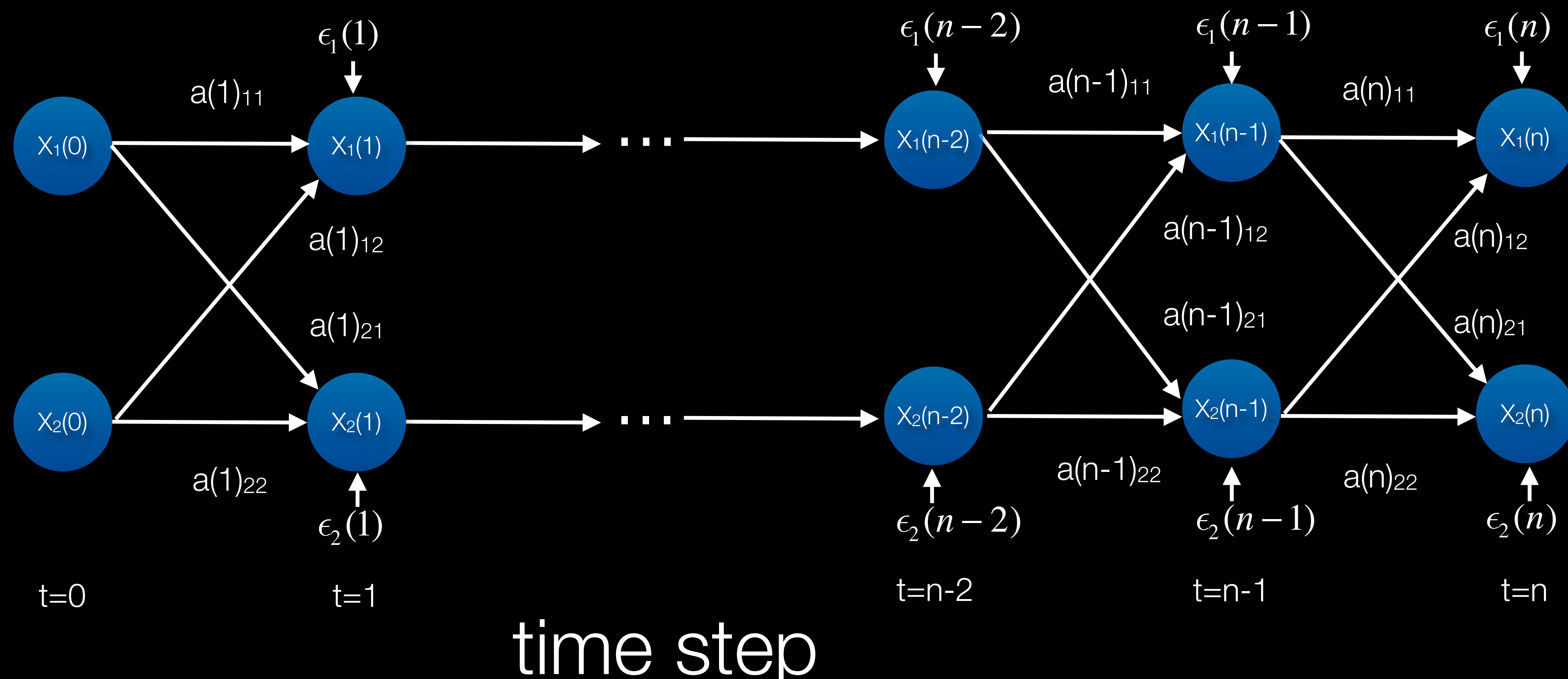
time step

The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$

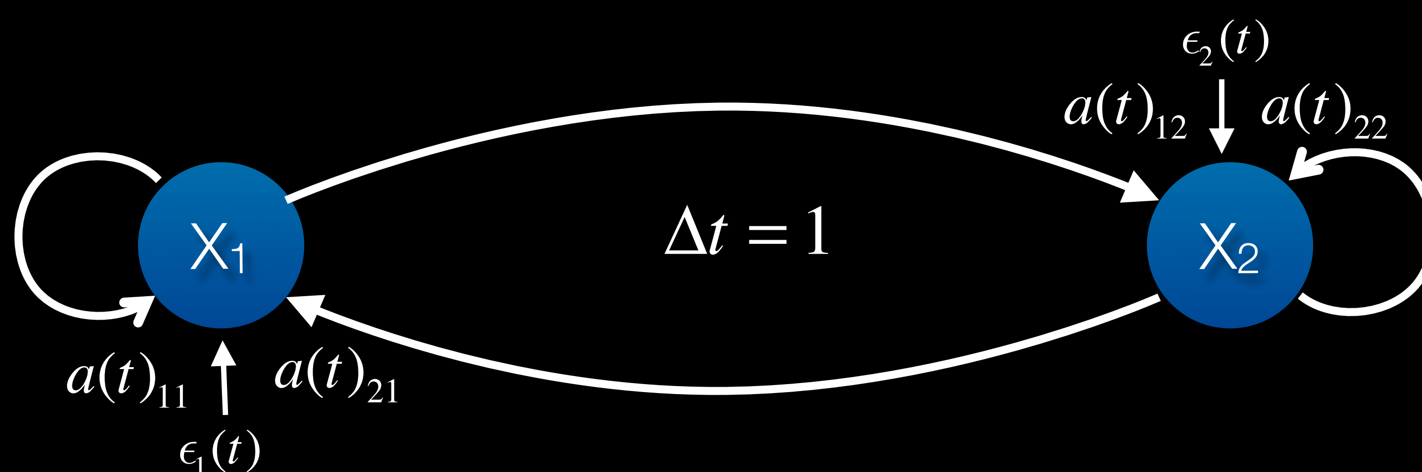




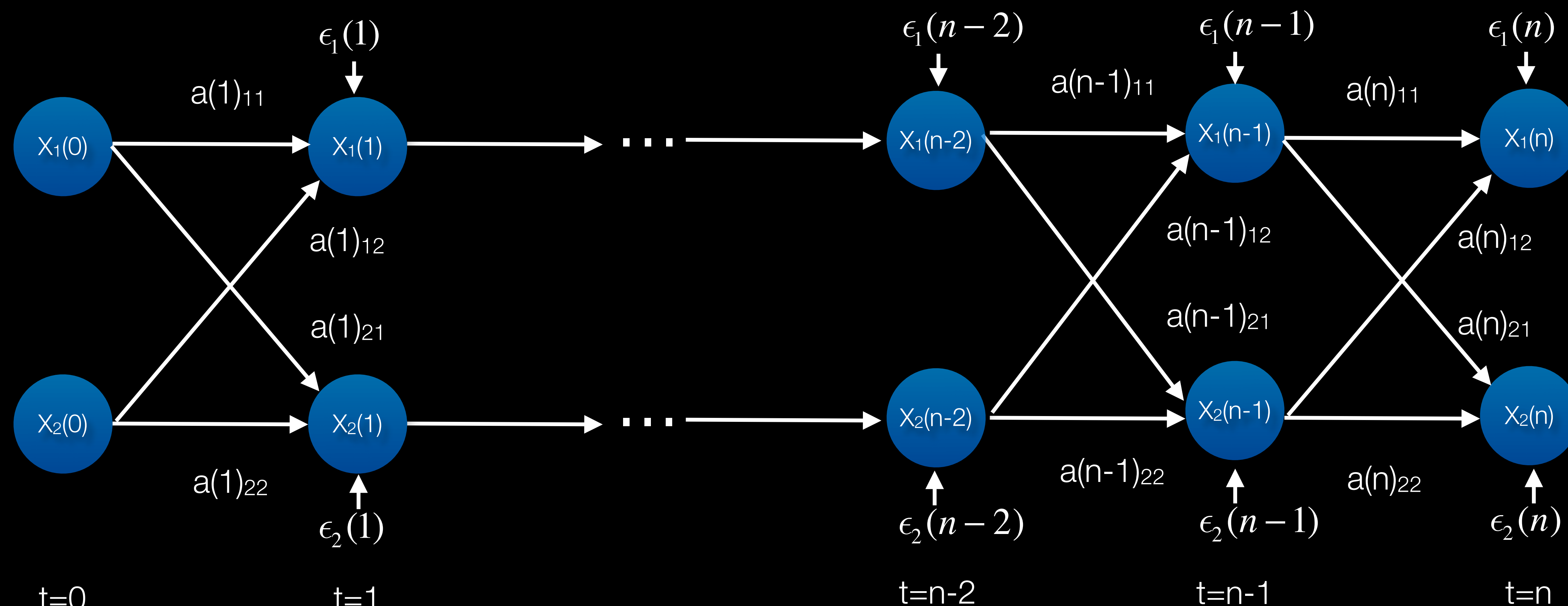
The VAR Process

Stochastic Linear Dynamical System

$$X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t)$$
$$X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t)$$



Order 1 VAR Process (VAR[1])

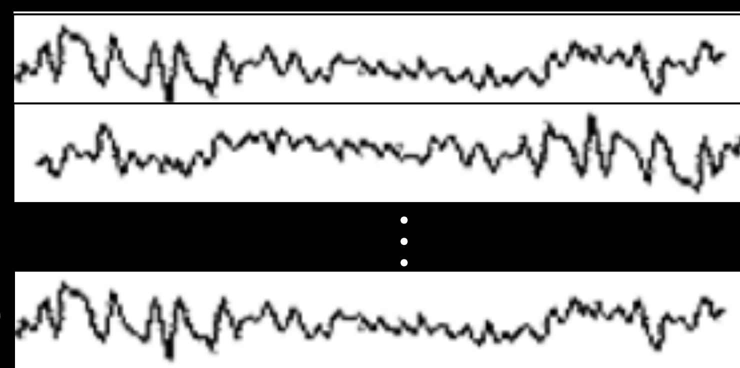


time step

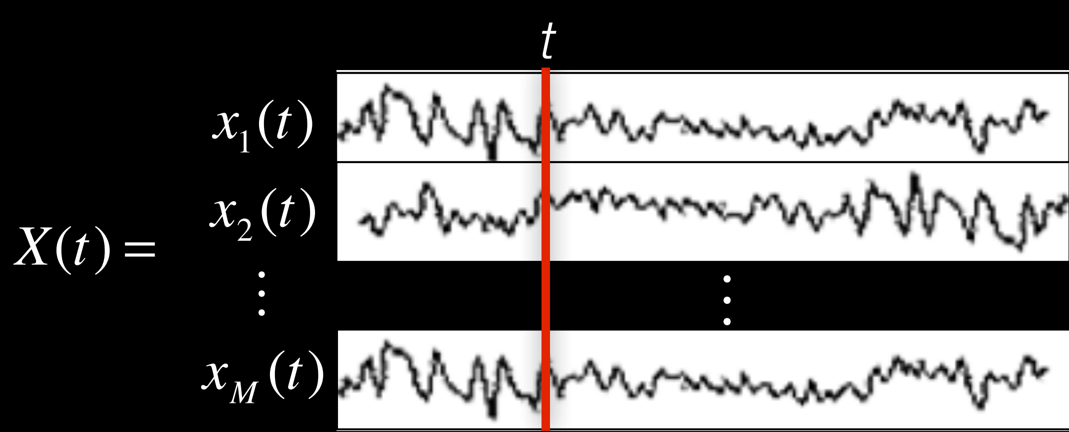
The VAR Model

The VAR Model

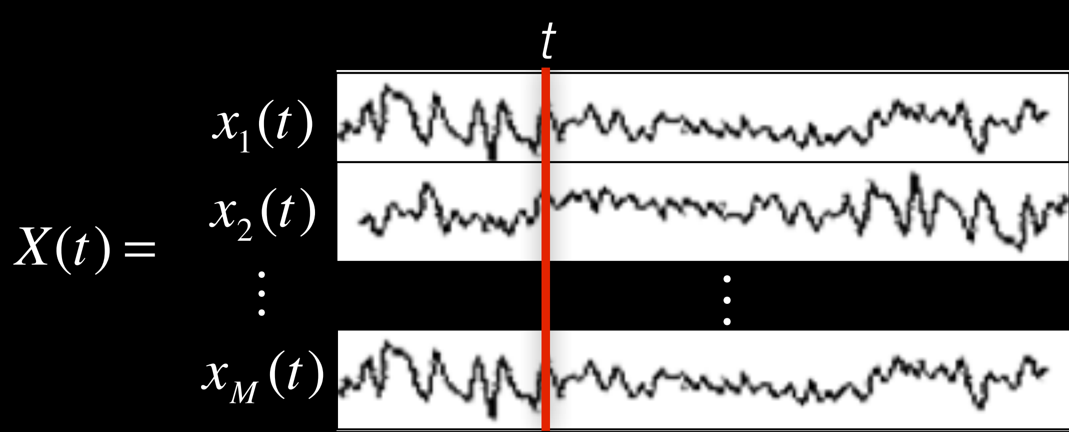
$$X(t) = \begin{matrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{matrix}$$



The VAR Model



The VAR Model

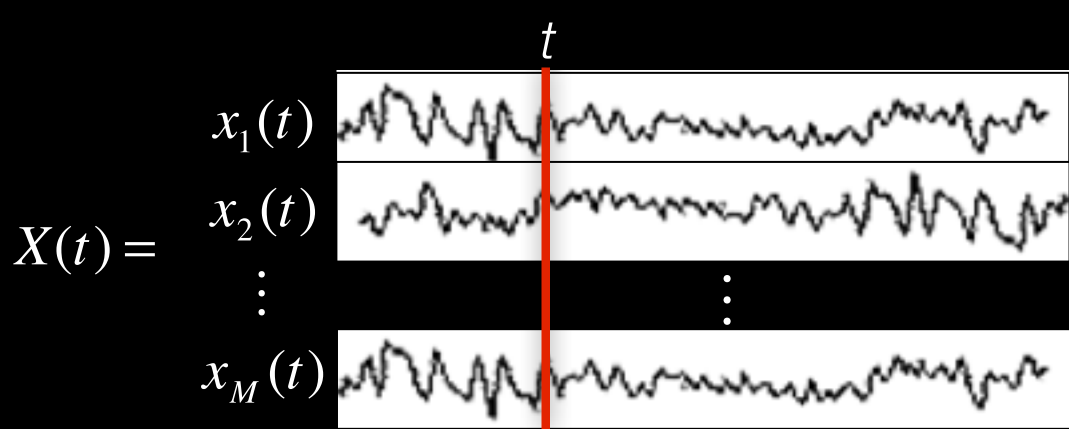


VAR[*p*] model

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

M-channel data vector
at current time *t*

The VAR Model



VAR[*p*] model

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

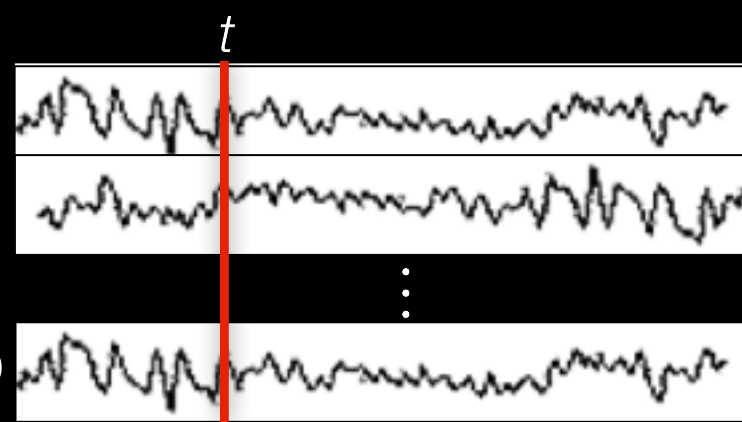
M-channel data vector at current time *t*

M x M matrix of (time-varying) model coefficients indicating variable dependencies at lag *k*

$$\mathbf{A}^{(k)}(t) = \begin{pmatrix} a_{11}^{(k)}(t) & \cdots & a_{1M}^{(k)}(t) \\ \vdots & \ddots & \vdots \\ a_{M1}^{(k)}(t) & \cdots & a_{MM}^{(k)}(t) \end{pmatrix}$$

The VAR Model

$X(t) =$ $x_1(t)$, $x_2(t)$, $\vdots$, $x_M(t)$



VAR[p] model

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

M-channel data vector at current time t

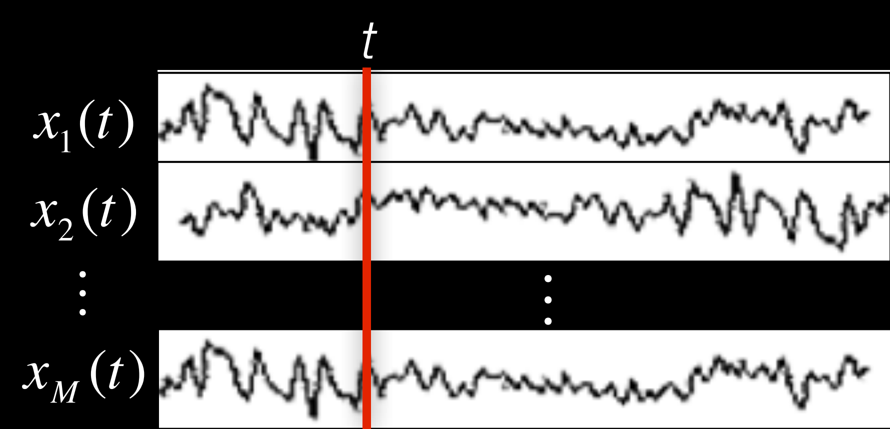
M x M matrix of (time-varying) model coefficients indicating variable dependencies at lag k

multichannel data k samples in the past

$$\mathbf{A}^{(k)}(t) = \begin{pmatrix} a_{11}^{(k)}(t) & \cdots & a_{1M}^{(k)}(t) \\ \vdots & \ddots & \vdots \\ a_{M1}^{(k)}(t) & \cdots & a_{MM}^{(k)}(t) \end{pmatrix}$$

The VAR Model

$X(t) =$



VAR[*p*] model

model order

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

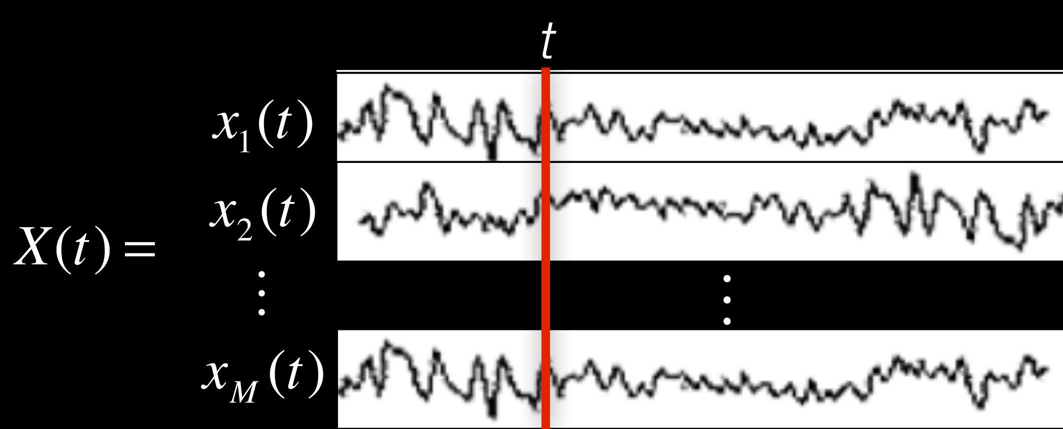
M-channel data vector at current time *t*

M x M matrix of (time-varying) model coefficients indicating variable dependencies at lag *k*

multichannel data *k* samples in the past

$$\mathbf{A}^{(k)}(t) = \begin{pmatrix} a_{11}^{(k)}(t) & \cdots & a_{1M}^{(k)}(t) \\ \vdots & \ddots & \vdots \\ a_{M1}^{(k)}(t) & \cdots & a_{MM}^{(k)}(t) \end{pmatrix}$$

The VAR Model



VAR[p] model

model order

random noise process

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

M-channel data vector at current time *t*

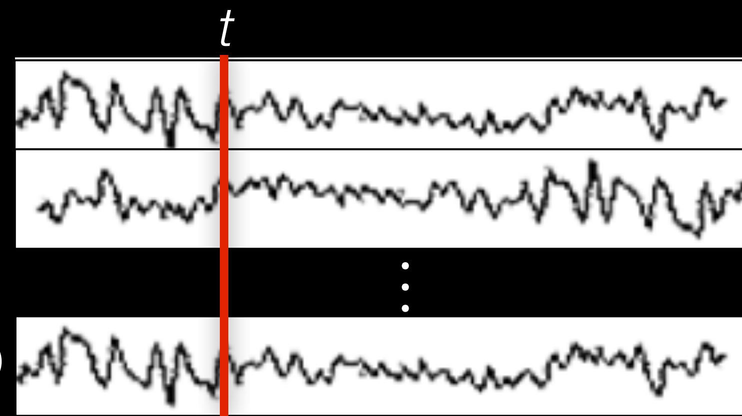
M x M matrix of (time-varying) model coefficients indicating variable dependencies at lag *k*

multichannel data *k* samples in the past

$$\mathbf{A}^{(k)}(t) = \begin{pmatrix} a_{11}^{(k)}(t) & \cdots & a_{1M}^{(k)}(t) \\ \vdots & \ddots & \vdots \\ a_{M1}^{(k)}(t) & \cdots & a_{MM}^{(k)}(t) \end{pmatrix} \qquad \mathbf{E}(t) = N(0, \mathbf{V})$$

The VAR Model

$$X(t) = \begin{matrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{matrix}$$



Ordinary Least-Squares
Lattice Filters
Bayesian Estimation
...

VAR[*p*] model

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

model order

random noise process

M-channel data vector at current time *t*

M x M matrix of (time-varying) model coefficients indicating variable dependencies at lag *k*

multichannel data *k* samples in the past

$$\mathbf{A}^{(k)}(t) = \begin{pmatrix} a_{11}^{(k)}(t) & \cdots & a_{1M}^{(k)}(t) \\ \vdots & \ddots & \vdots \\ a_{M1}^{(k)}(t) & \cdots & a_{MM}^{(k)}(t) \end{pmatrix} \qquad \mathbf{E}(t) = N(0, \mathbf{V})$$

Selecting a VAR Model Order

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

$$AIC(p) = 2\log(\det(\mathbf{V})) + M^2p/N$$

Selecting a VAR Model Order

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

$$AIC(p) = 2\log(\det(\mathbf{V})) + M^2p/N$$

entropy rate (amount of prediction error)

Selecting a VAR Model Order

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

$$AIC(p) = 2\log(\det(\mathbf{V})) + M^2p/N$$

Penalizes high model orders (parsimony)

entropy rate (amount of prediction error)

Selecting a VAR Model Order

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

$$AIC(p) = 2\log(\det(\mathbf{V})) + M^2p/N$$

Penalizes high model orders (parsimony)

entropy rate (amount of prediction error)

Selecting a VAR Model Order

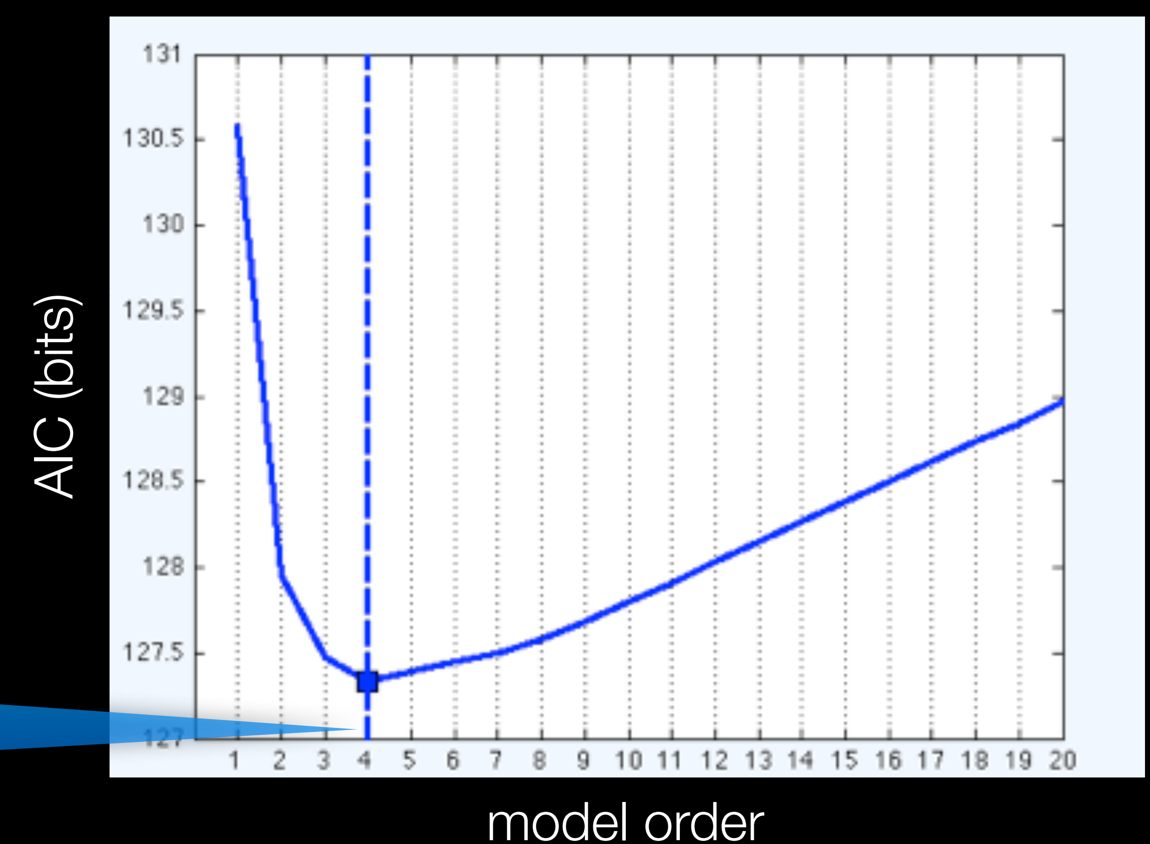
- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

$$AIC(p) = 2\log(\det(\mathbf{V})) + M^2p/N$$

Penalizes high model orders (parsimony)

entropy rate (amount of prediction error)

optimal order



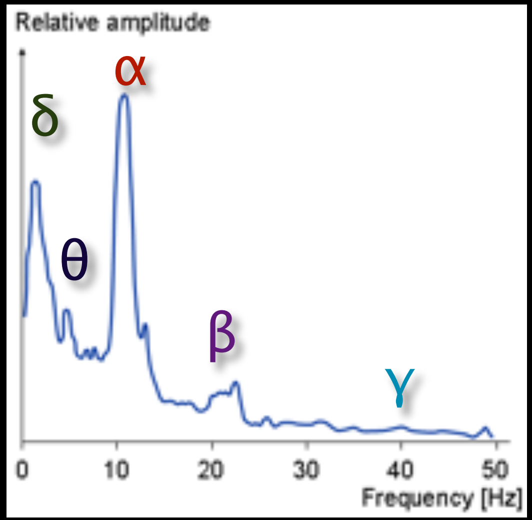
Selecting a VAR Model Order

- Other considerations:
  - A M-dimensional VAR model of order p has at most $Mp/2$ spectral peaks distributed amongst the M variables. This means we can observe at most $p/2$ peaks in each variables' spectrum (or in the causal spectrum between two each pair of variables)
  - Optimal model order depends on sampling rate (higher sampling rate often requires higher model orders)

Selecting a VAR Model Order

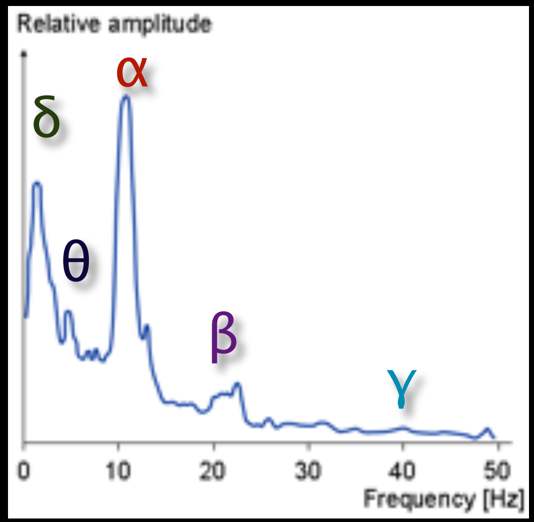
Selecting a VAR Model Order

- Jansen (1981) and Florian and Pfurtscheller (1995) demonstrated that a model order of 10 was generally quite adequate for describing EEG spectra



Selecting a VAR Model Order

- Jansen (1981) and Florian and Pfurtscheller (1995) demonstrated that a model order of 10 was generally quite adequate for describing EEG spectra
- VAR model is an "all-pole" filter well-suited for modeling oscillatory processes with "peaky" spectra (like EEG!)



VAR Modeling: Assumptions

VAR Modeling: Assumptions

- **“Weak” stationarity of the data**

VAR Modeling: Assumptions

- **“Weak” stationarity of the data**
  - mean and variance do not change with time

VAR Modeling: Assumptions

- **“Weak” stationarity of the data**
  - mean and variance do not change with time
  - An EEG trace containing prominent evoked potentials is a classic example of a non-stationary time-series

VAR Modeling: Assumptions

- **"Weak" stationarity of the data**
  - mean and variance do not change with time
  - An EEG trace containing prominent evoked potentials is a classic example of a non-stationary time-series
- **Stability**

VAR Modeling: Assumptions

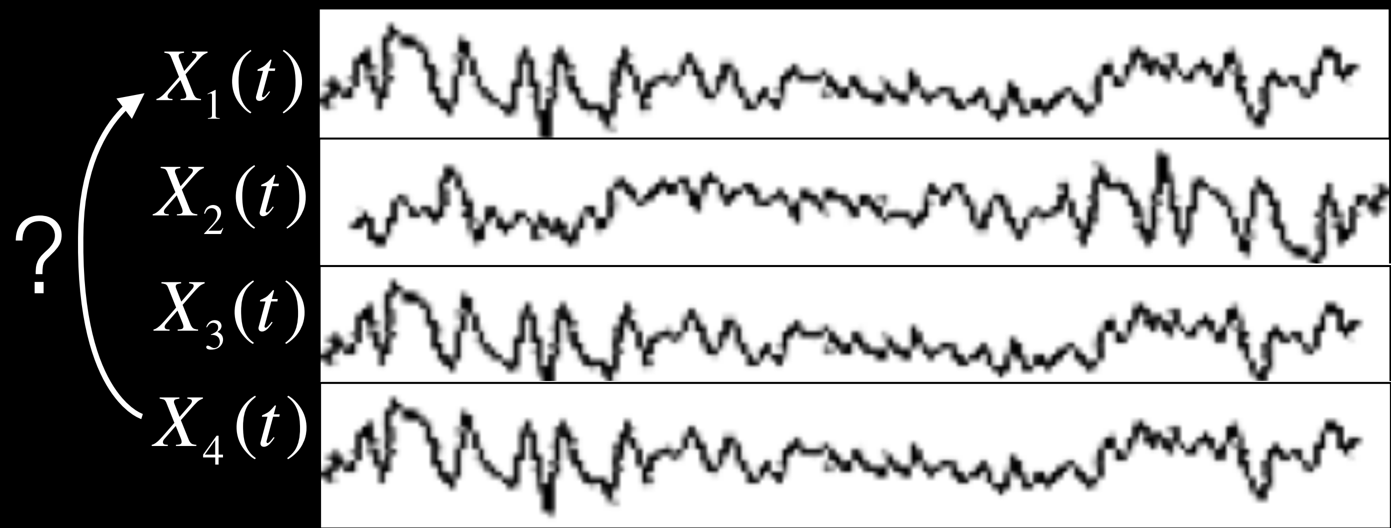
- **"Weak" stationarity of the data**
  - mean and variance do not change with time
  - An EEG trace containing prominent evoked potentials is a classic example of a non-stationary time-series
- **Stability**
  - A stable process will not "blow up" (diverge to infinity)

VAR Modeling: Assumptions

- **"Weak" stationarity of the data**
  - mean and variance do not change with time
  - An EEG trace containing prominent evoked potentials is a classic example of a non-stationary time-series
- **Stability**
  - A stable process will not "blow up" (diverge to infinity)
  - Importantly, stability implies stationarity and SIFT provides you techniques for verifying the stability

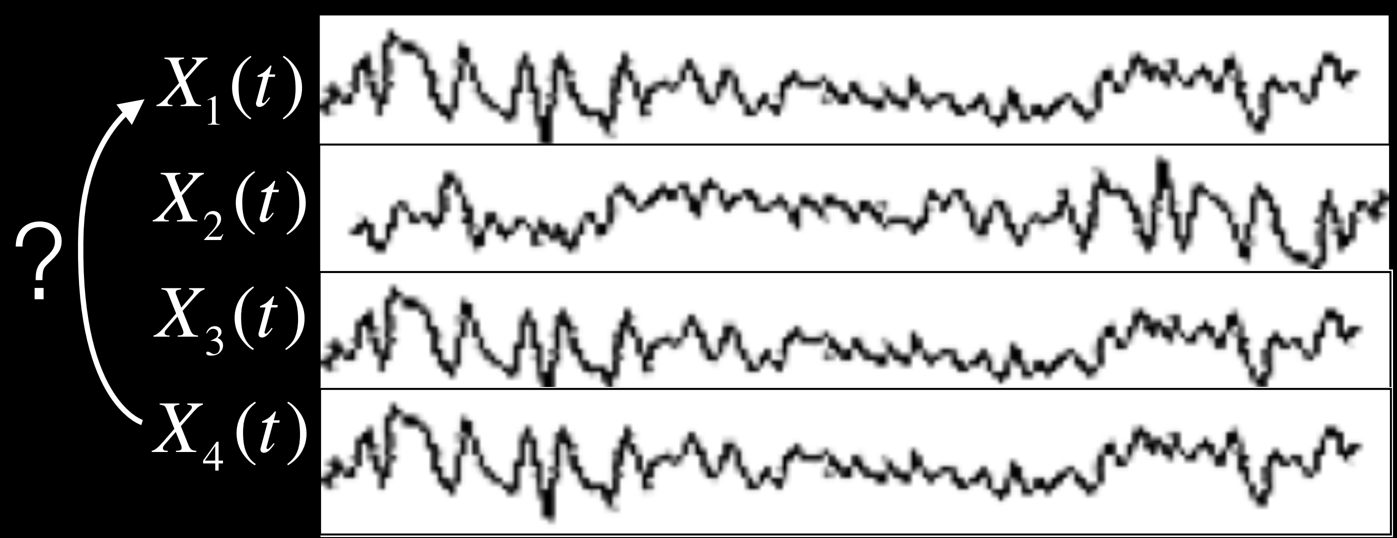
Granger Causality

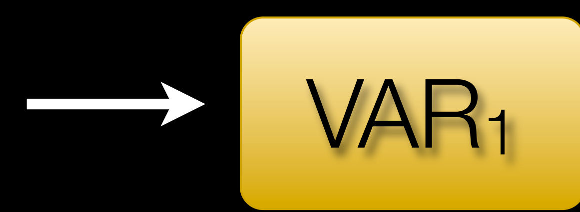
Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?
(conditioned on $\mathbf{X}_2$, $\mathbf{X}_3$)



Granger Causality

Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?
(conditioned on $\mathbf{X}_2$, $\mathbf{X}_3$)

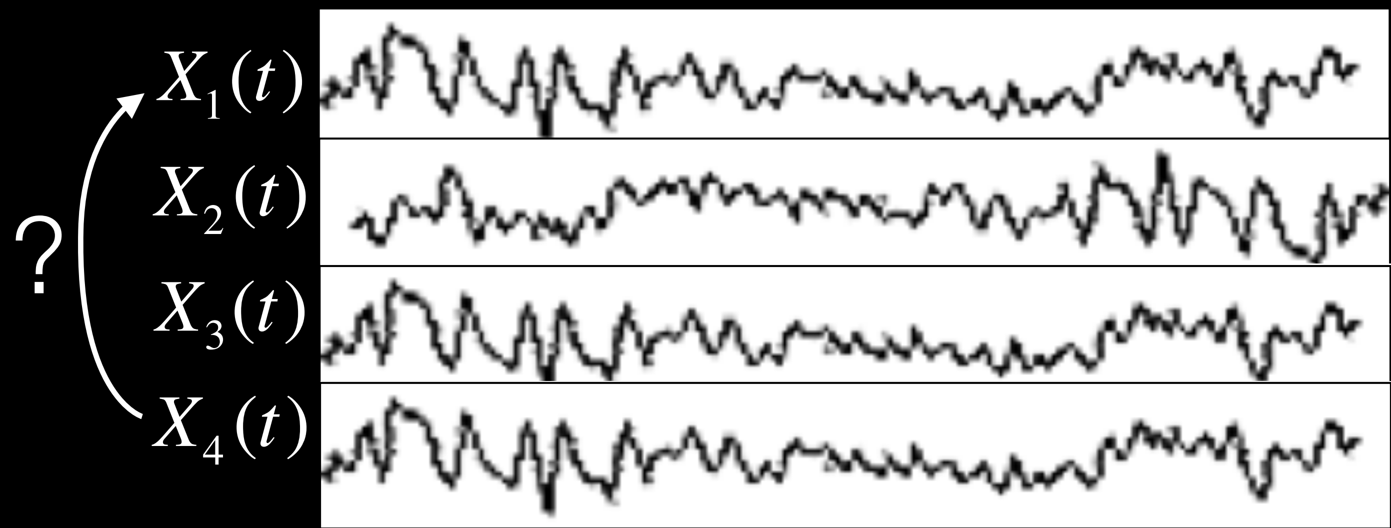


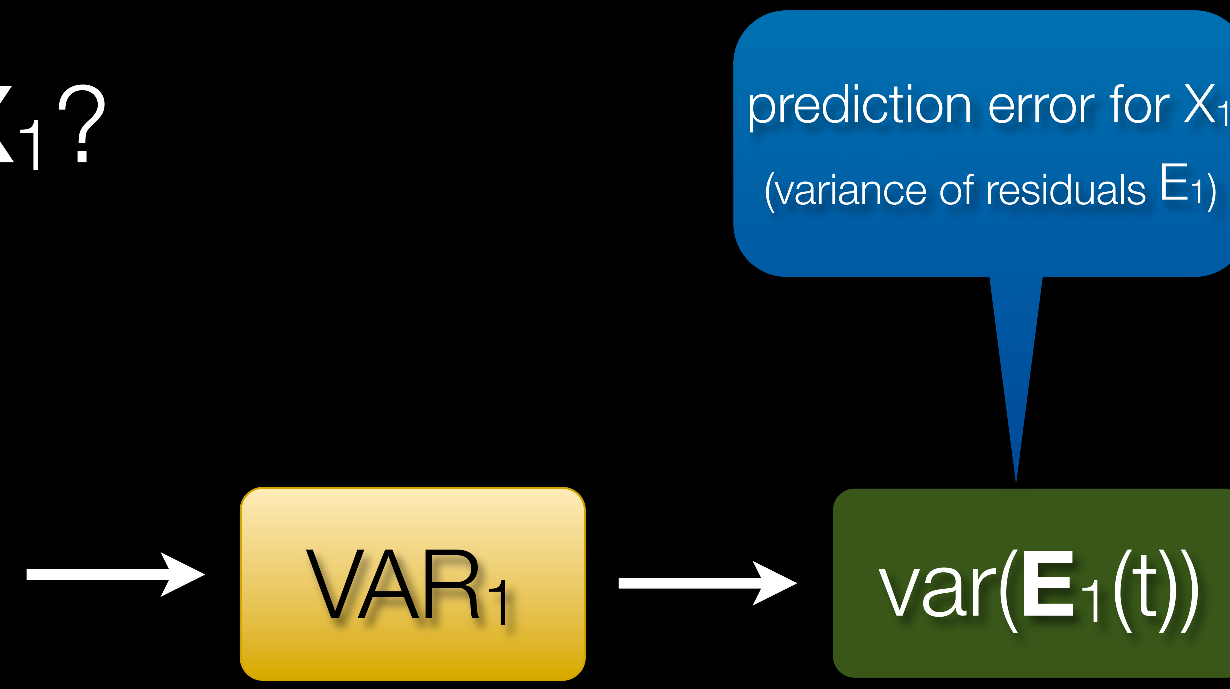


$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

Granger Causality

Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?
(conditioned on $\mathbf{X}_2$, $\mathbf{X}_3$)

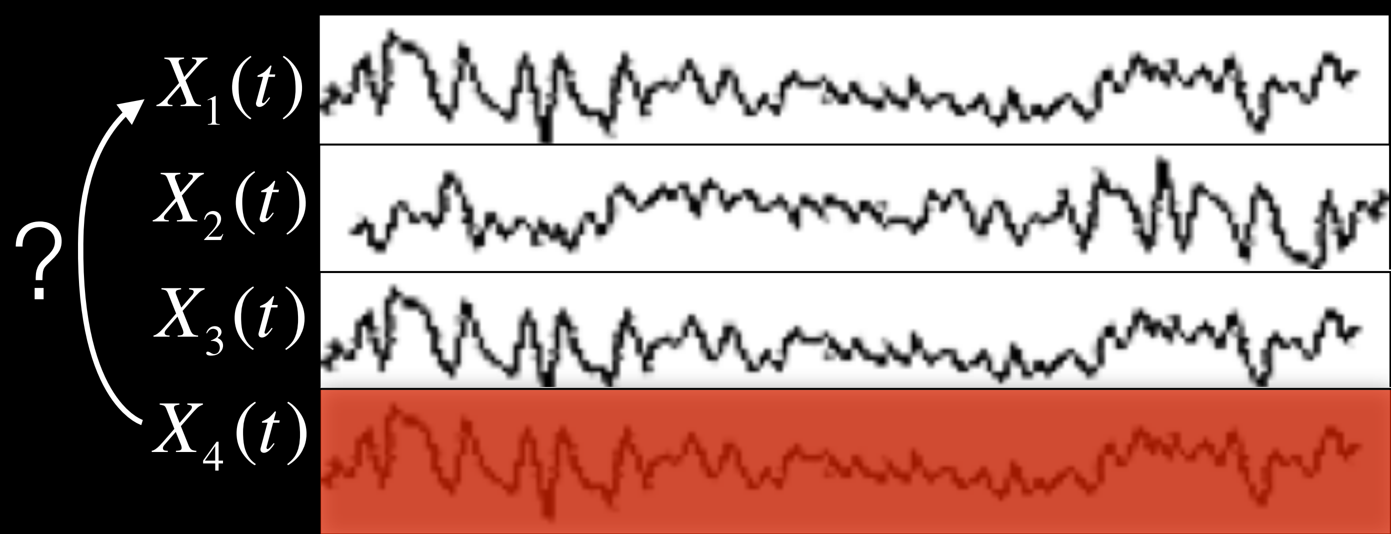


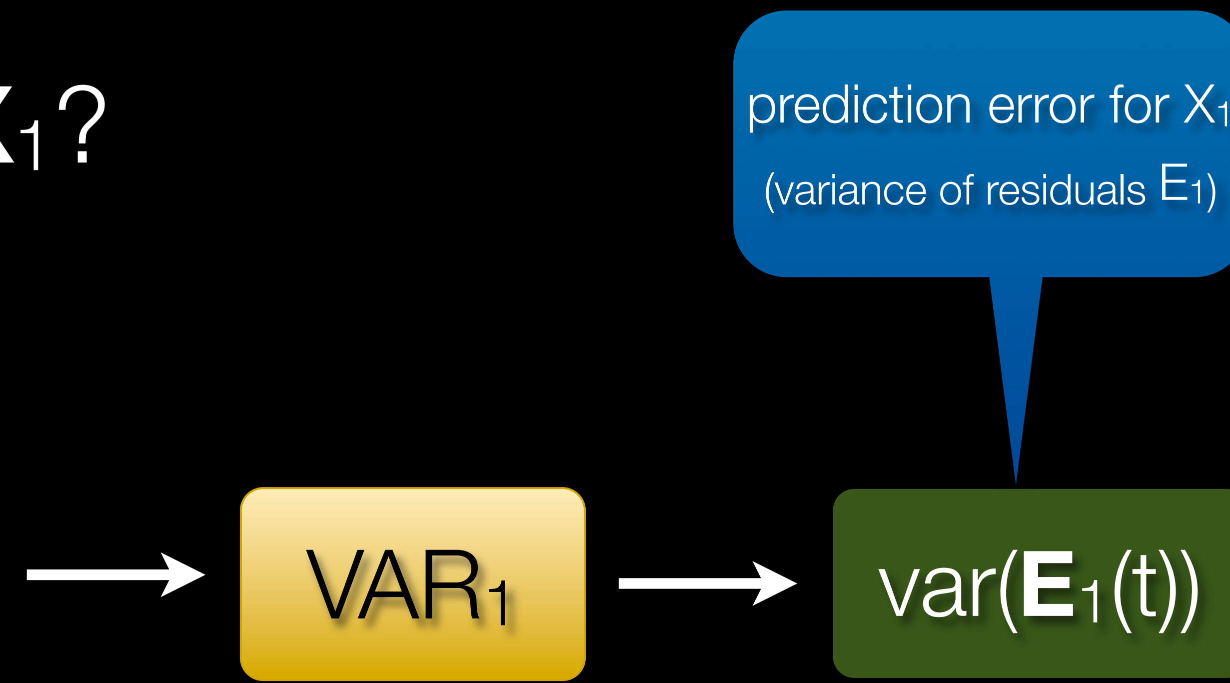


$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

Granger Causality

Does **X_4** granger-cause **X_1**?
(conditioned on **X_2**, **X_3**)

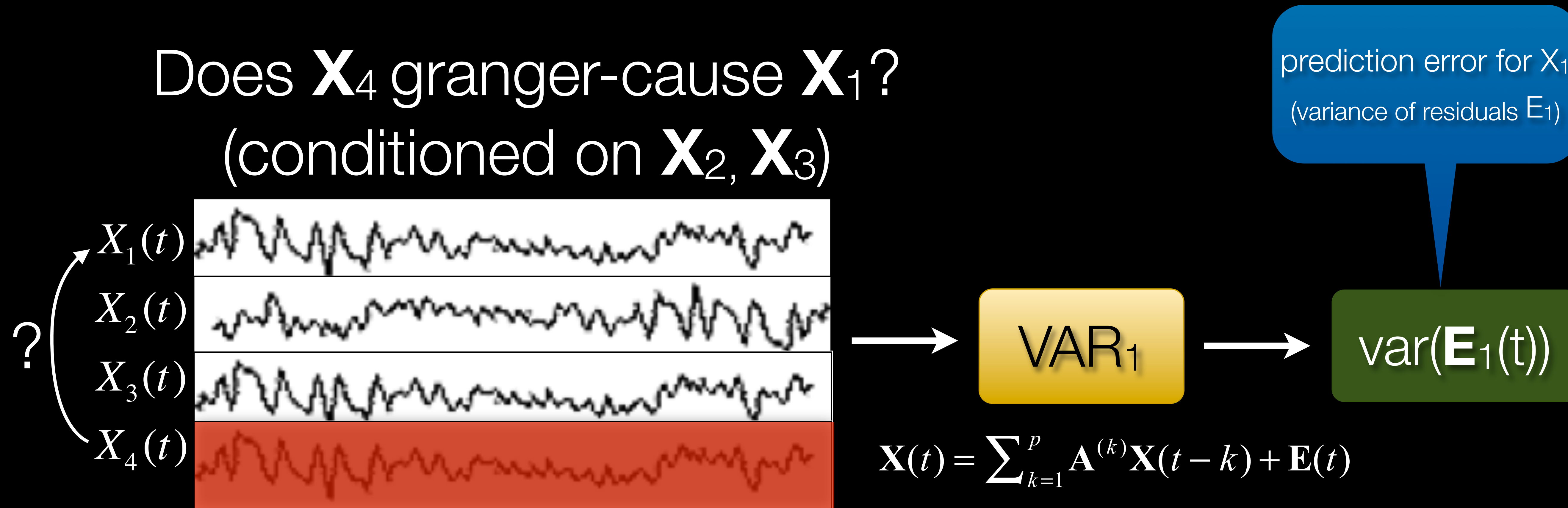




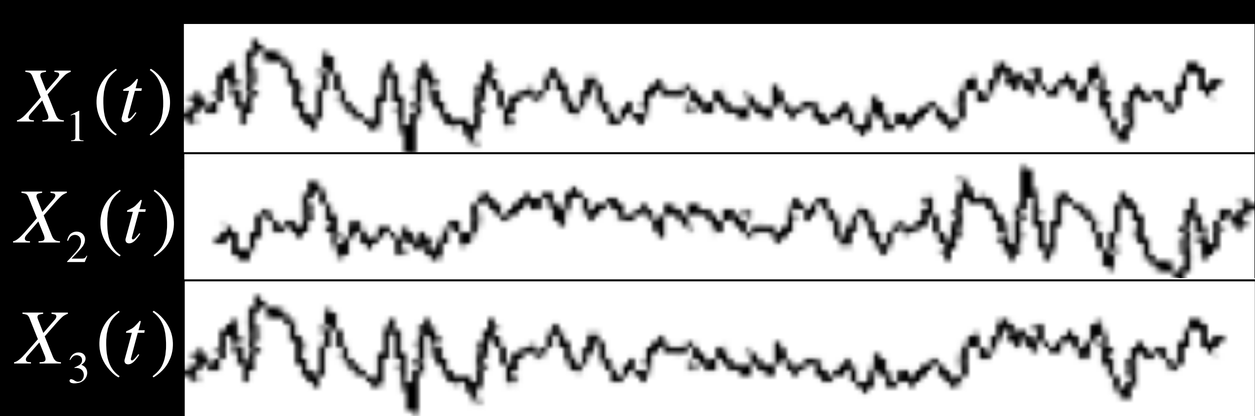
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

Granger Causality

Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?
(conditioned on $\mathbf{X}_2$, $\mathbf{X}_3$)

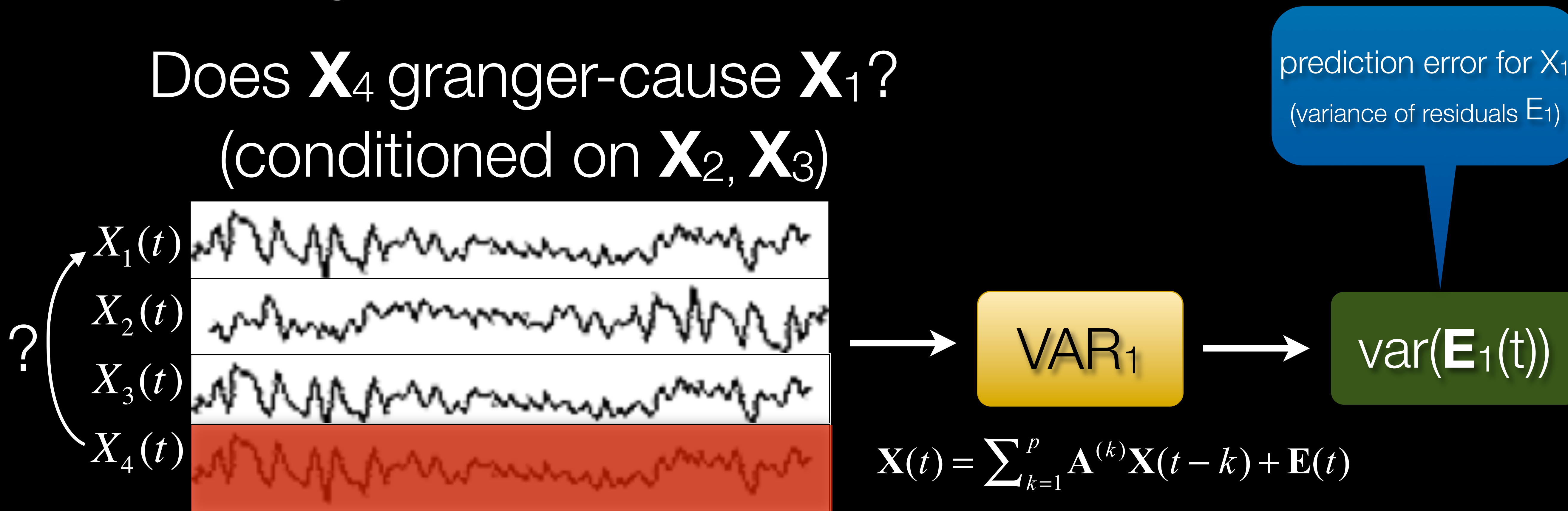


$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

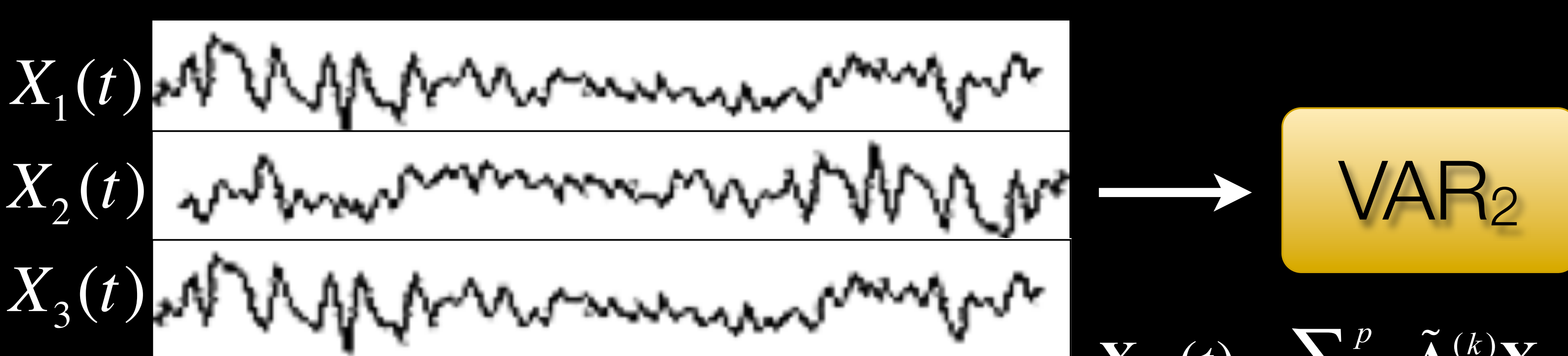


Granger Causality

Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?
(conditioned on $\mathbf{X}_2$, $\mathbf{X}_3$)



$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

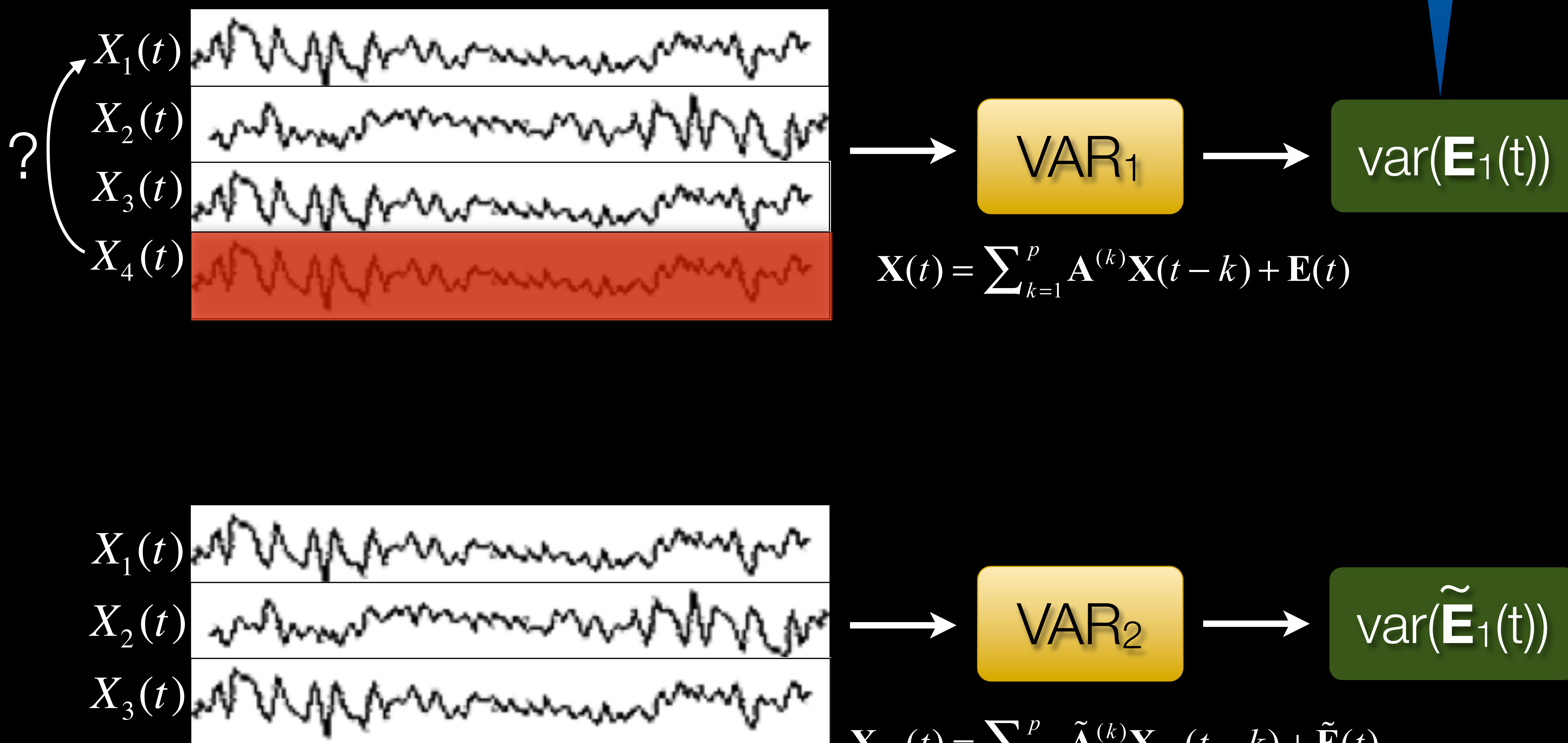


$$\mathbf{X}_{-4}(t) = \sum_{k=1}^{p} \tilde{\mathbf{A}}^{(k)} \mathbf{X}_{-4}(t-k) + \tilde{\mathbf{E}}(t)$$

Granger Causality

Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?
(conditioned on $\mathbf{X}_2$, $\mathbf{X}_3$)

prediction error for X_1
(variance of residuals E_1)

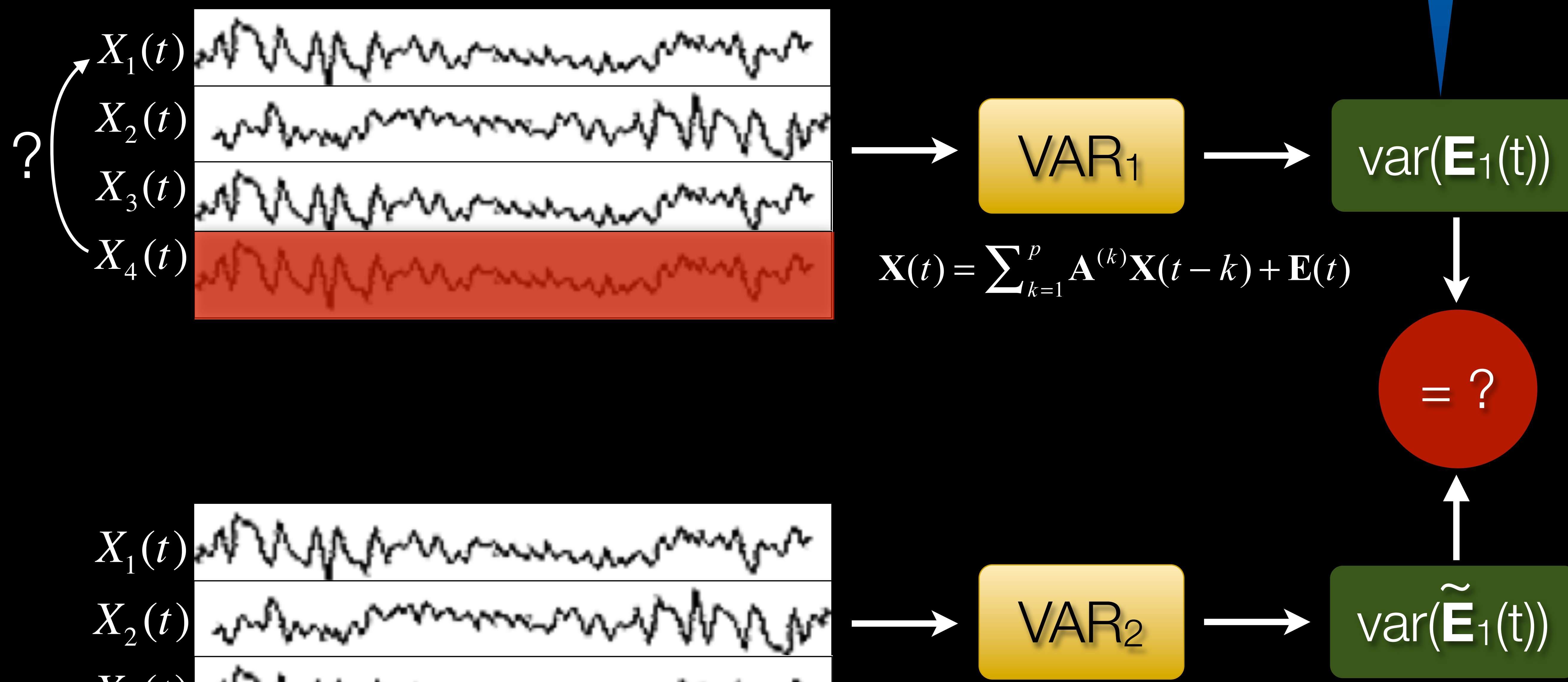


$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{X}_{-4}(t) = \sum_{k=1}^{p} \tilde{\mathbf{A}}^{(k)}\mathbf{X}_{-4}(t-k) + \tilde{\mathbf{E}}(t)$$

Granger Causality

Does **X**$_4$ granger-cause **X**$_1$?
(conditioned on **X**$_2$, **X**$_3$)



$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{X}_{-4}(t) = \sum_{k=1}^{p} \tilde{\mathbf{A}}^{(k)} \mathbf{X}_{-4}(t-k) + \tilde{\mathbf{E}}(t)$$

Granger Causality

Granger Causality

- Granger (1969) quantified this definition for **bivariate** processes in the form of an F-ratio:

reduced model

$$F_{X_1 \leftarrow X_2} = \ln\left(\frac{var(\tilde{E}_1)}{var(E_1)}\right) = \ln\left(\frac{var(X_1(t) \mid X_1(\cdot))}{var(X_1(t) \mid X_1(\cdot), X_2(\cdot))}\right)$$

full model

Granger Causality

- Granger (1969) quantified this definition for **bivariate** processes in the form of an F-ratio:

$$F_{X_1 \leftarrow X_2} = \ln\left(\frac{\overbrace{var(\tilde{E}_1)}^{\text{reduced model}}}{\underbrace{var(E_1)}_{\text{full model}}}\right) = \ln\left(\frac{var(X_1(t) \mid X_1(\cdot))}{var(X_1(t) \mid X_1(\cdot), X_2(\cdot))}\right)$$

- Alternately, for a **multivariate interpretation** we can fit a single MVAR model to all channels and apply the following definition:

Definition 1

X_j granger-causes X_i conditioned on all other variables in $\mathbf{X}$
if and only if $\mathbf{A}_{ij}(k) >> 0$ for some lag $k \in \{1, \ldots, p\}$

Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} -0.5 & 0 \\ 0.7 & 0.2 \end{pmatrix} \begin{pmatrix} X_1(t-1) \\ X_2(t-1) \end{pmatrix} + \begin{pmatrix} E_1(t) \\ E_2(t) \end{pmatrix}$$

$$X_1(t) = -0.5 X_1(t-1) + 0 X_2(t-1) + E_1(t)$$
$$X_2(t) = 0.7 X_1(t-1) + 0.2 X_2(t-1) + E_2(t)$$

Which causal structure does this model correspond to?

a) $1 \rightarrow 2$ b) $1 \leftarrow 2$ c) $1 \leftrightarrow 2$

Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} -0.5 & 0 \\ 0.7 & 0.2 \end{pmatrix} \begin{pmatrix} X_1(t-1) \\ X_2(t-1) \end{pmatrix} + \begin{pmatrix} E_1(t) \\ E_2(t) \end{pmatrix}$$

$$\begin{aligned} X_1(t) &= -0.5 X_1(t-1) + 0 X_2(t-1) + E_1(t) \\ X_2(t) &= 0.7 X_1(t-1) + 0.2 X_2(t-1) + E_2(t) \end{aligned}$$

Which causal structure does this model correspond to?

a) $1 \rightarrow 2$ b) $1 \leftarrow 2$ c) $1 \leftrightarrow 2$

Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} -0.5 & 0 \\ 0.7 & 0.2 \end{pmatrix} \begin{pmatrix} X_1(t-1) \\ X_2(t-1) \end{pmatrix} + \begin{pmatrix} E_1(t) \\ E_2(t) \end{pmatrix}$$

$$X_1(t) = -0.5X_1(t-1) + 0X_2(t-1) + E_1(t)$$

$$X_2(t) = \boxed{0.7X_1(t-1)} + 0.2X_2(t-1) + E_2(t)$$

Which causal structure does this model correspond to?

a) 1 → 2 b) 1 ← 2 c) 1 ↔ 2

Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} -0.5 & 0 \\ 0.7 & 0.2 \end{pmatrix} \begin{pmatrix} X_1(t-1) \\ X_2(t-1) \end{pmatrix} + \begin{pmatrix} E_1(t) \\ E_2(t) \end{pmatrix}$$

$$X_1(t) = -0.5X_1(t-1) + \boxed{0X_2(t-1)} + E_1(t)$$
$$X_2(t) = \boxed{0.7X_1(t-1)} + 0.2X_2(t-1) + E_2(t)$$

Which causal structure does this model correspond to?

a) 1 → 2 b) 1 ← 2 c) 1 ↔ 2

Granger Causality – Frequency Domain

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

Granger Causality – Frequency Domain

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

Fourier-transforming $\mathbf{A}^{(k)}$ we obtain

Likewise, $\mathbf{X}(f)$ and $\mathbf{E}(f)$ correspond to the fourier transforms of the data and residuals, respectively

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

Granger Causality – Frequency Domain

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

Fourier-transforming $\mathbf{A}^{(k)}$ we obtain

Likewise, $\mathbf{X}(f)$ and $\mathbf{E}(f)$ correspond to the fourier transforms of the data and residuals, respectively

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

We can then define the spectral matrix $\mathbf{X}(f)$ as follows:

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

Where $\mathbf{H}(f)$ is the *transfer matrix* of the system.

Granger Causality – Frequency Domain

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

Fourier-transforming $\mathbf{A}^{(k)}$ we obtain

Likewise, **X**(*f*) and **E**(*f*) correspond to the fourier transforms of the data and residuals, respectively

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

We can then define the spectral matrix **X**(*f*) as follows:

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

Where **H**(*f*) is the *transfer matrix* of the system.

Definition 2

X_j granger-causes X_i conditioned on all other variables in **X** <u>if and only if</u> $|\mathbf{A}_{ij}(f)| >> 0$ for some frequency f

Granger Causality – Frequency Domain

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

Fourier-transforming $\mathbf{A}^{(k)}$ we obtain

Likewise, $\mathbf{X}(f)$ and $\mathbf{E}(f)$ correspond to the fourier transforms of the data and residuals, respectively

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

We can then define the spectral matrix $\mathbf{X}(f)$ as follows:

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

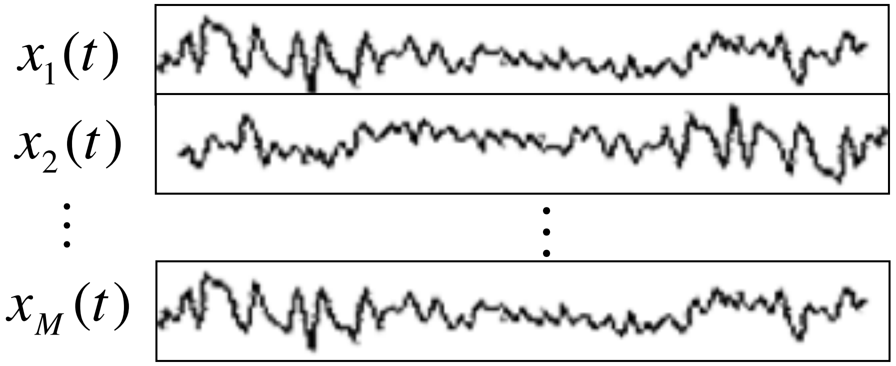
Where $\mathbf{H}(f)$ is the *transfer matrix* of the system.

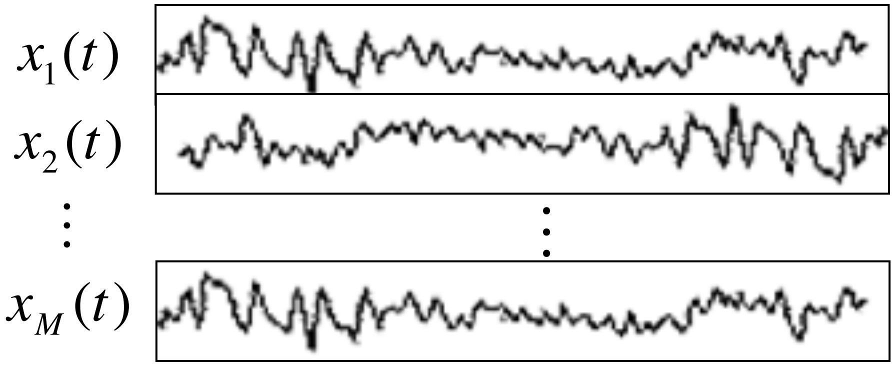
Definition 2

X_j granger-causes X_i conditioned on all other variables in $\mathbf{X}$

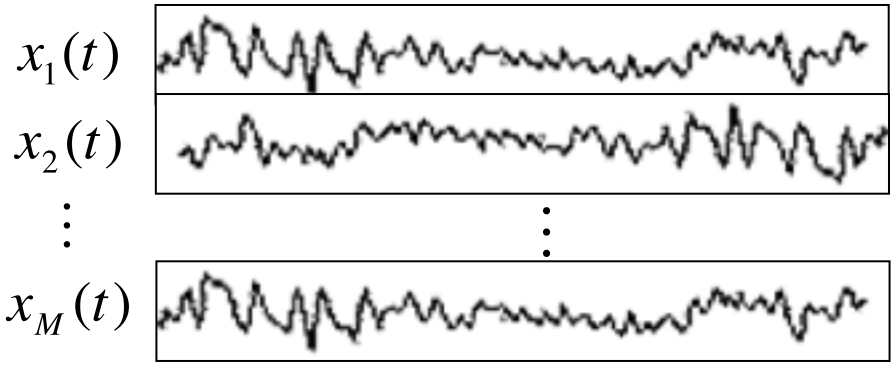
<u>if and only if</u> $|\mathbf{A}_{ij}(f)| >> 0$ for some frequency f

leads to PDC


$x_1(t)$
$x_2(t)$
$x_M(t)$



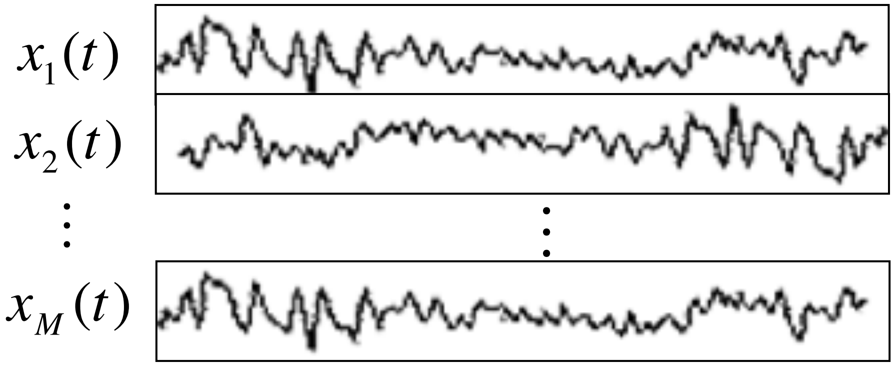
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$



$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

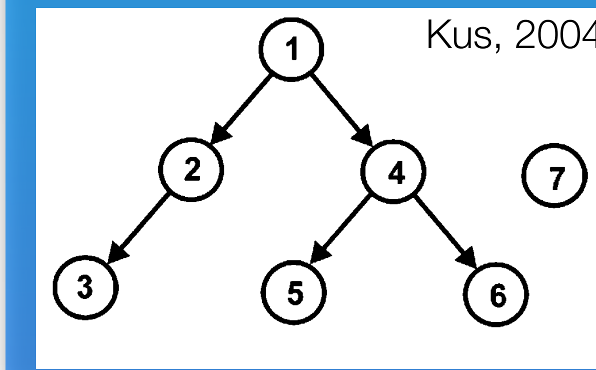


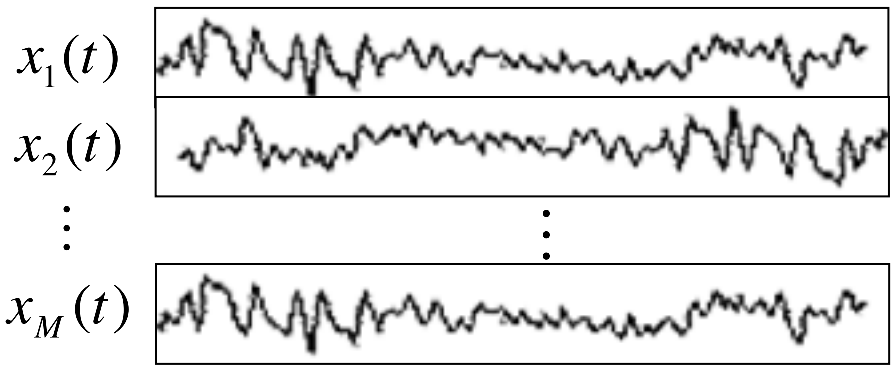
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1} \mathbf{E}(f) = \mathbf{H}(f) \mathbf{E}(f)$$

Ground Truth

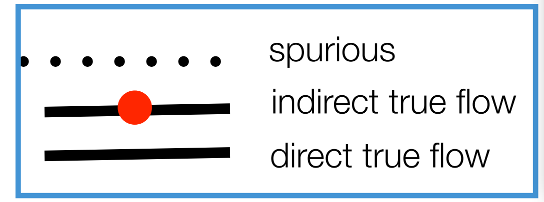




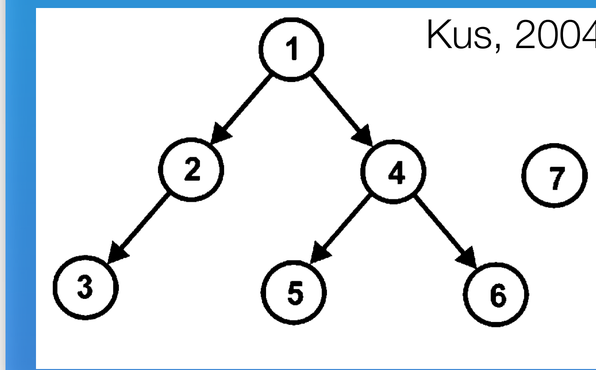
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$



Ground Truth



Functional

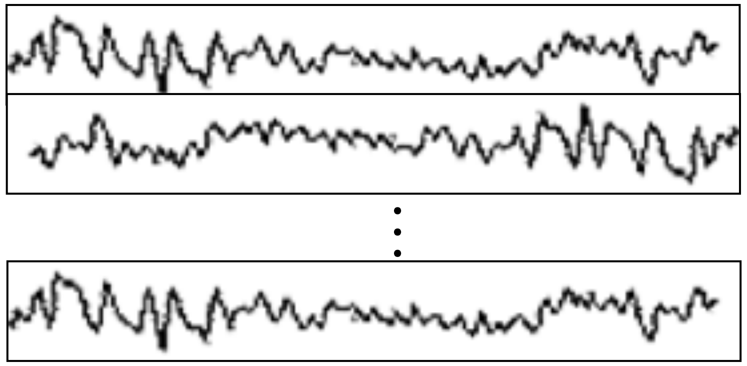
Effective

$x_1(t)$

$x_2(t)$

$\vdots$

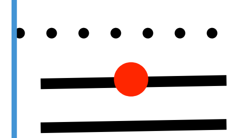
$x_M(t)$



$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$



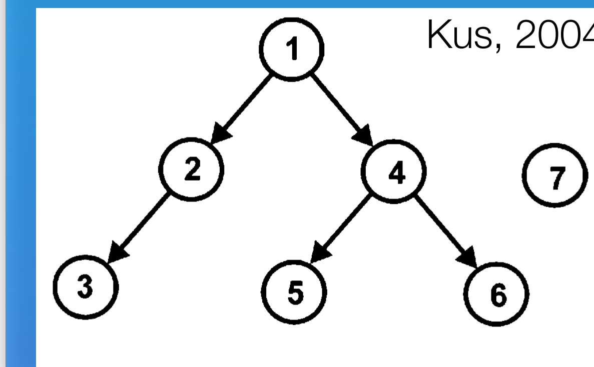
spurious

indirect true flow

direct true flow

Ground Truth

Kus, 2004



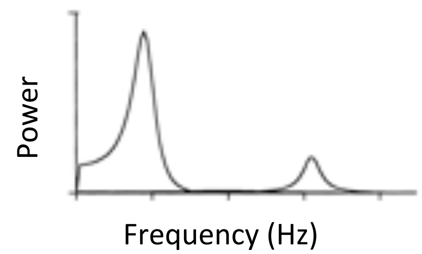
Functional

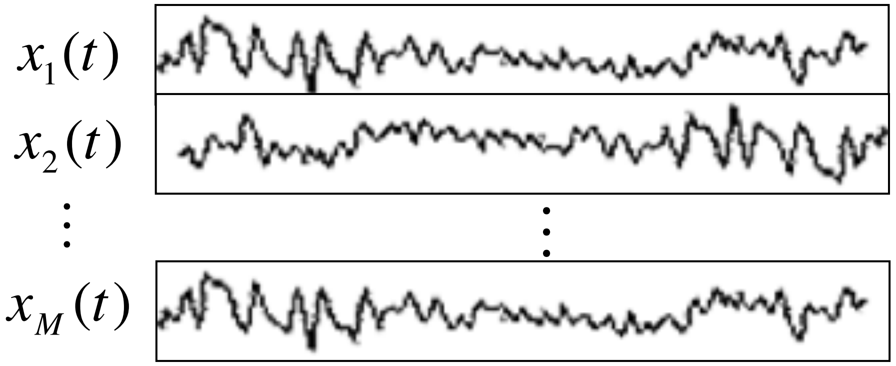
Effective

Spectrum

$$S(f) = \mathbf{X}(f)\mathbf{X}(f)^{*}$$
$$= \mathbf{H}(f)\Sigma\mathbf{H}(f)^{*}$$

(Brillinger, 2001)



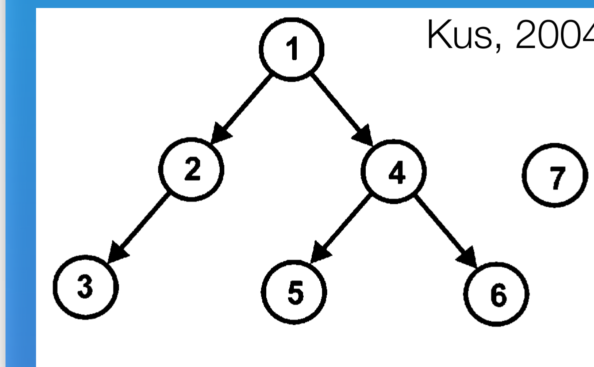


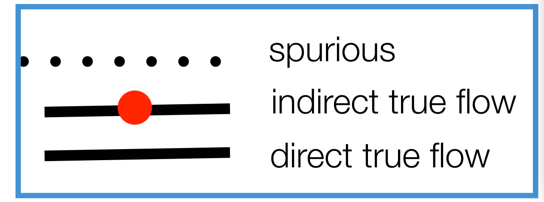
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

Ground Truth





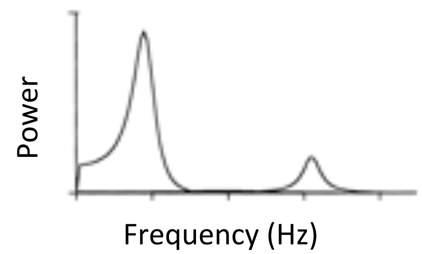
Functional

Effective

Spectrum

$$S(f) = \mathbf{X}(f)\mathbf{X}(f)^* = \mathbf{H}(f)\Sigma\mathbf{H}(f)^*$$

(Brillinger, 2001)

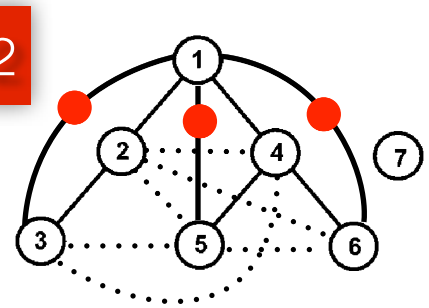


Coherency

$$C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

M=2

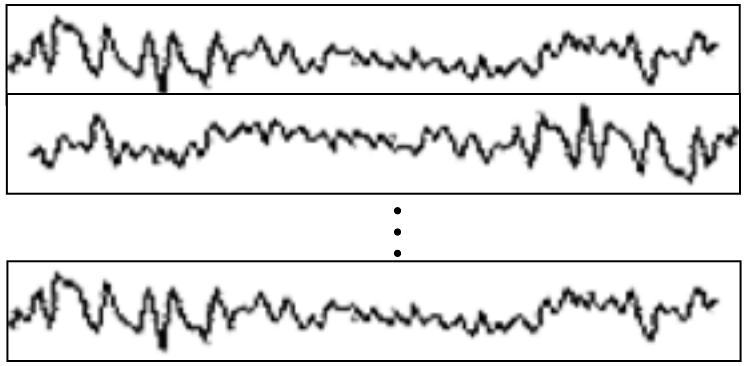


$x_1(t)$

$x_2(t)$

$\vdots$

$x_M(t)$

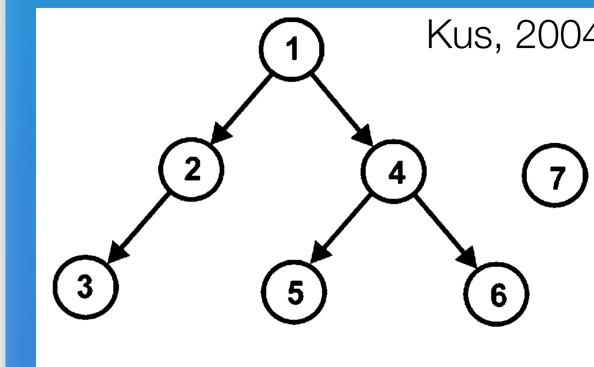


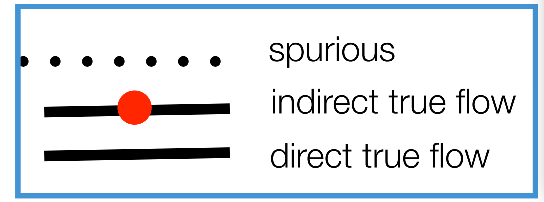
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)} \mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

Ground Truth



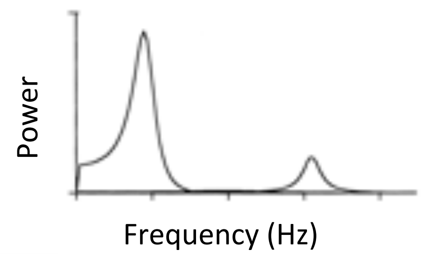


Functional	Effective

Spectrum

$$S(f) = \mathbf{X}(f)\mathbf{X}(f)^* = \mathbf{H}(f)\Sigma\mathbf{H}(f)^*$$

(Brillinger, 2001)

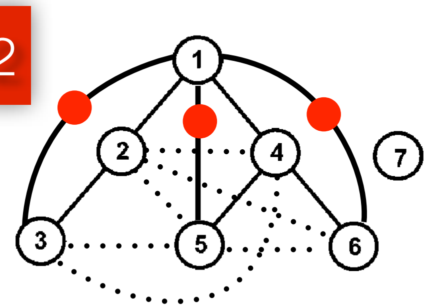


Coherency

M=2

$$C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

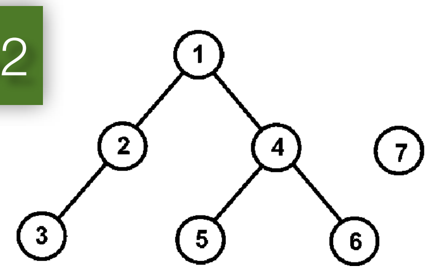


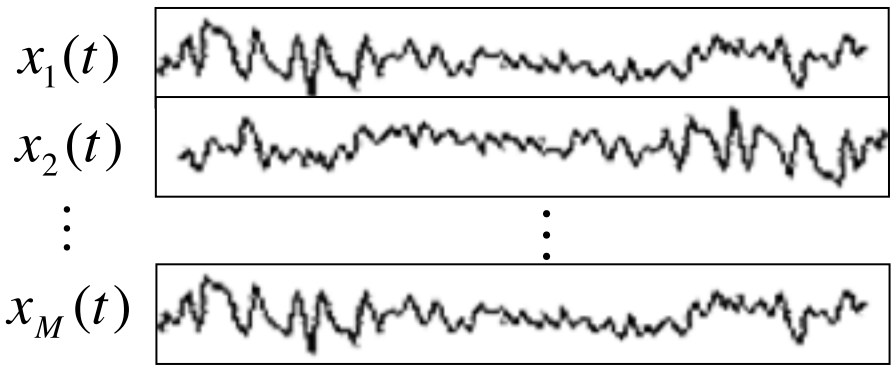
Partial Coherence

M>2

$$P_{ij}(f) = \frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)

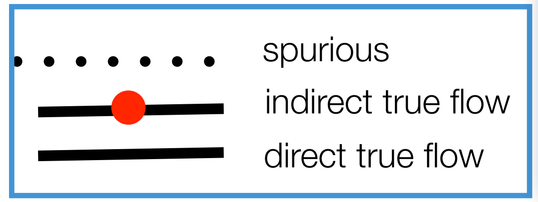




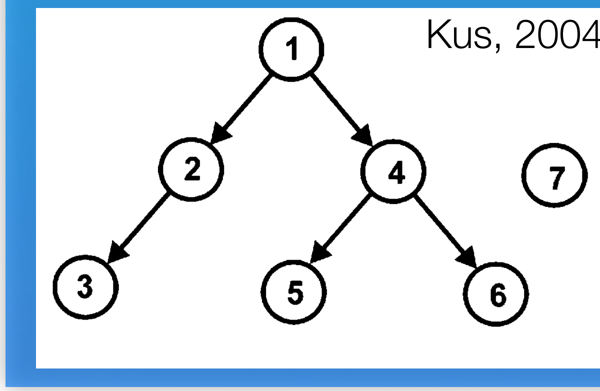
$$\mathbf{X}(t)=\sum_{k=1}^{p}\mathbf{A}^{(k)}\mathbf{X}(t-k)+\mathbf{E}(t)$$

$$\mathbf{A}(f)=-\sum_{k=0}^{p}\mathbf{A}^{(k)}e^{-i2\pi fk};\quad \mathbf{A}^{(0)}=I$$

$$\mathbf{X}(f)=\mathbf{A}(f)^{-1}\mathbf{E}(f)=\mathbf{H}(f)\mathbf{E}(f)$$



Ground Truth



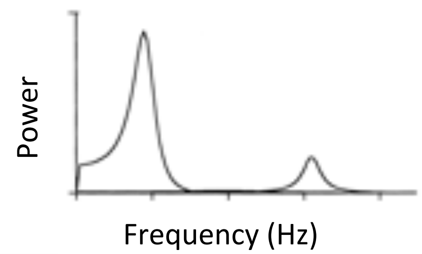
Functional

Effective

Spectrum

$$S(f)=\mathbf{X}(f)\mathbf{X}(f)^{*}$$
$$=\mathbf{H}(f)\Sigma\mathbf{H}(f)^{*}$$

(Brillinger, 2001)

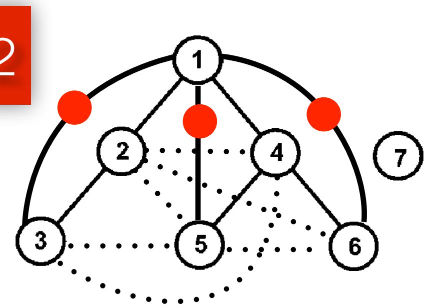


Coherency

M=2

$$C_{ij}(f)=\frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

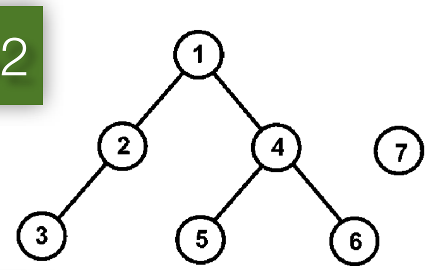


Partial Coherence

M>2

$$P_{ij}(f)=\frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)

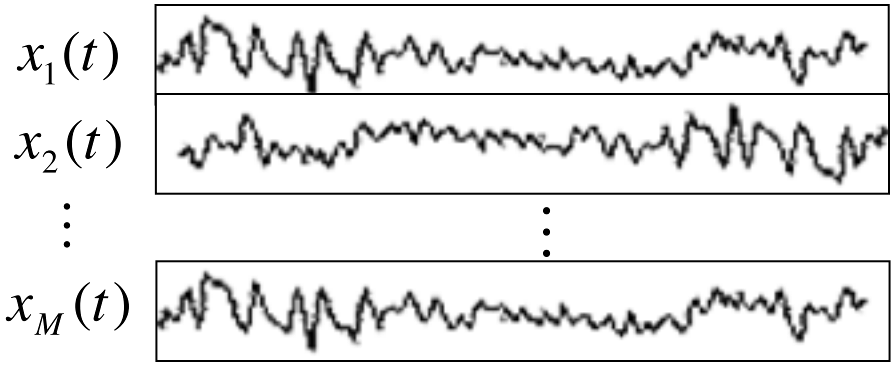


Multiple Coherence

M=1

$$G_{i}(f)=\sqrt{1-\frac{\det(S(f))}{S_{ii}(f)\mathbf{M}_{ii}(f)}}$$

(Brillinger, 2001)

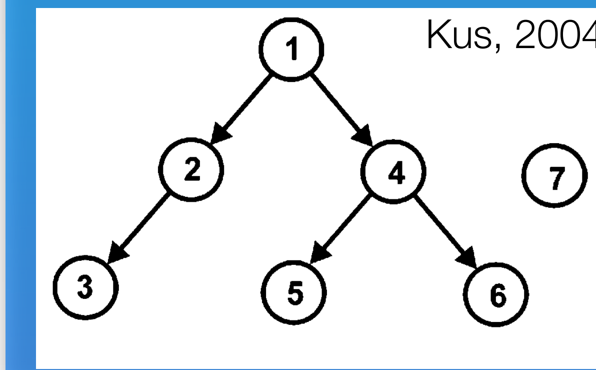


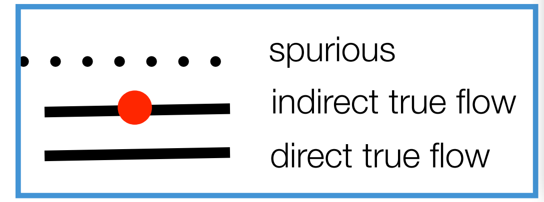
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$

Ground Truth



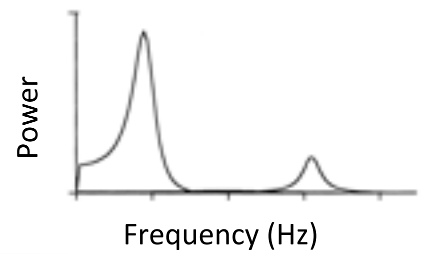


Functional

Spectrum

$$S(f) = \mathbf{X}(f)\mathbf{X}(f)^* = \mathbf{H}(f)\Sigma\mathbf{H}(f)^*$$

(Brillinger, 2001)

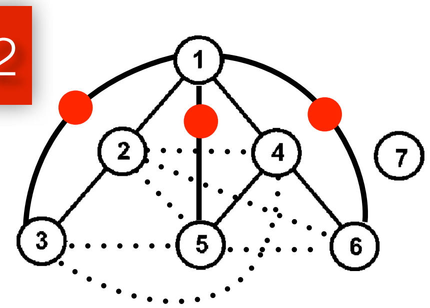


Coherency

M=2

$$C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

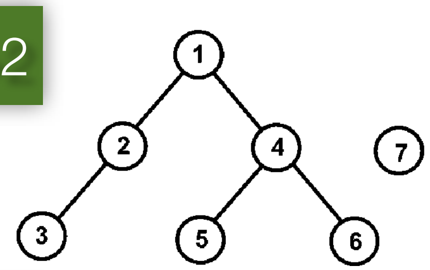


Partial Coherence

M>2

$$P_{ij}(f) = \frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)



Multiple Coherence

M=1

$$G_i(f) = \sqrt{1 - \frac{\det(S(f))}{S_{ii}(f)\mathbf{M}_{ii}(f)}}$$

(Brillinger, 2001)

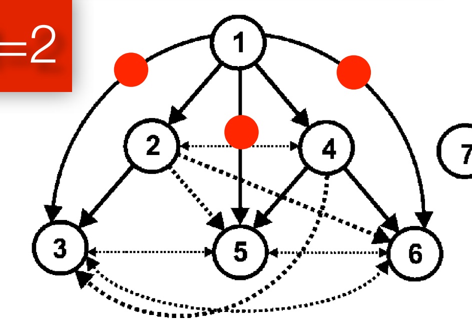
Effective

Granger-Geweke Causality

M=2

$$F_{ij}(f) = \frac{\Sigma_{jj} - (\Sigma_{ij}^2 / \Sigma_{ii}))\,|\,H_{ij}(f)\,|^2}{S_{ii}(f)}$$

(Geweke, 1982; Bressler et al., 2007)

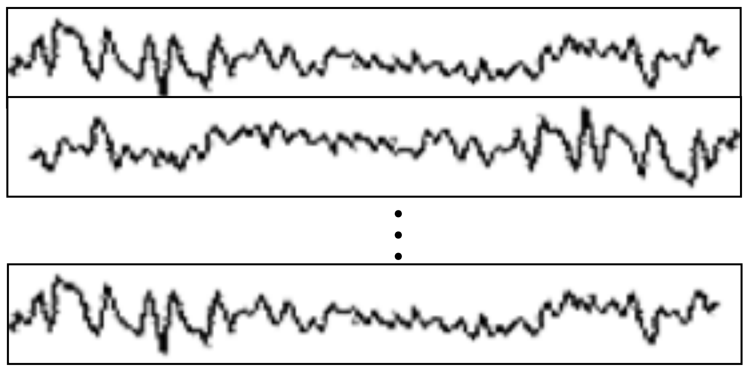


$x_1(t)$

$x_2(t)$

$\vdots$

$x_M(t)$

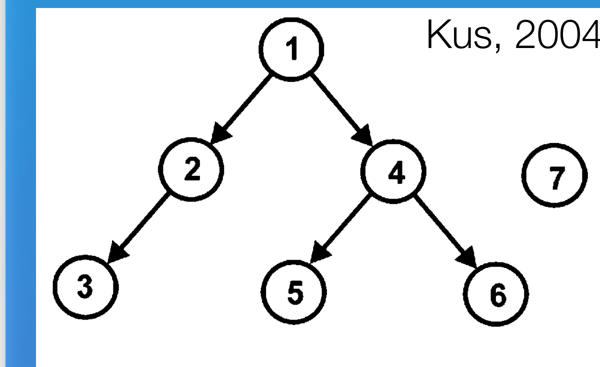


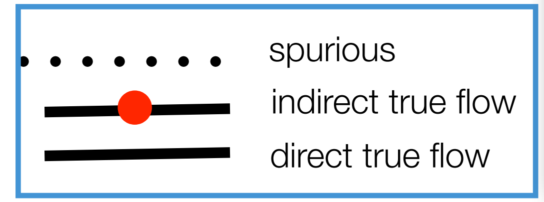
$$\mathbf{X}(t)=\sum_{k=1}^{p}\mathbf{A}^{(k)}\mathbf{X}(t-k)+\mathbf{E}(t)$$

$$\mathbf{A}(f)=-\sum_{k=0}^{p}\mathbf{A}^{(k)}e^{-i2\pi fk};\quad \mathbf{A}^{(0)}=I$$

$$\mathbf{X}(f)=\mathbf{A}(f)^{-1}\mathbf{E}(f)=\mathbf{H}(f)\mathbf{E}(f)$$

Ground Truth



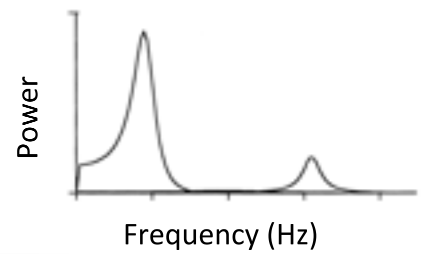


Functional

Spectrum

$$S(f)=\mathbf{X}(f)\mathbf{X}(f)^{*}=\mathbf{H}(f)\Sigma\mathbf{H}(f)^{*}$$

(Brillinger, 2001)

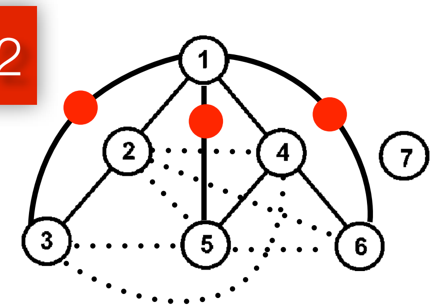


Coherency

M=2

$$C_{ij}(f)=\frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

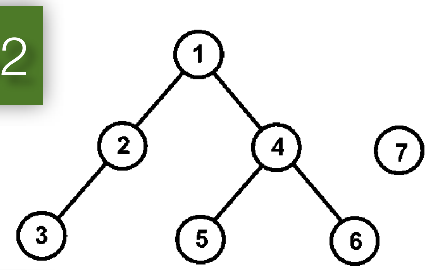


Partial Coherence

M>2

$$P_{ij}(f)=\frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)



Multiple Coherence

M=1

$$G_{i}(f)=\sqrt{1-\frac{\det(S(f))}{S_{ii}(f)\mathbf{M}_{ii}(f)}}$$

(Brillinger, 2001)

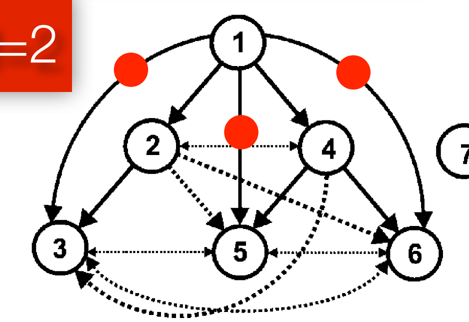
Effective

Granger-Geweke Causality

M=2

$$F_{ij}(f)=\frac{\Sigma_{jj}-(\Sigma_{ij}^{2}/\Sigma_{ii}))\,|H_{ij}(f)|^{2}}{S_{ii}(f)}$$

(Geweke, 1982; Bressler et al., 2007)

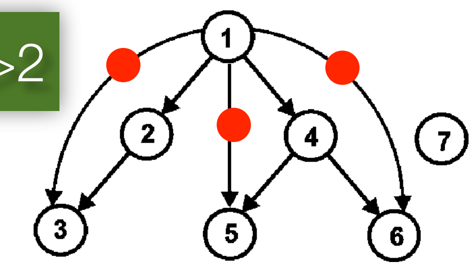


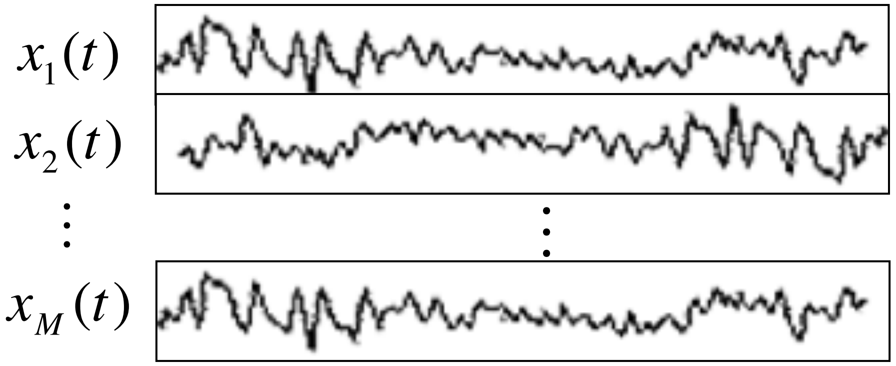
Directed Transfer Function

M>2

$$\eta_{ij}^{2}(f)=\frac{\left|H_{ij}(f)\right|^{2}}{\sum_{f}\sum_{k=1}^{M}\left|H_{ik}(f)\right|^{2}}$$

(Kaminski and Blinowska, 1991)

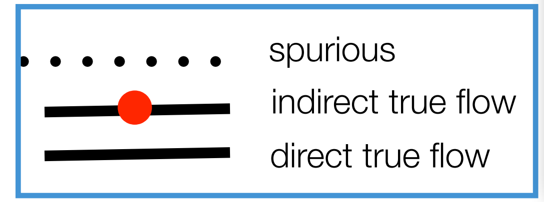




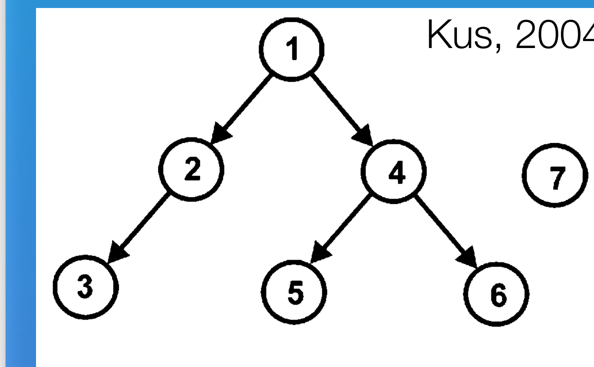
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f) = -\sum_{k=0}^{p} \mathbf{A}^{(k)} e^{-i2\pi fk}; \quad \mathbf{A}^{(0)} = I$$

$$\mathbf{X}(f) = \mathbf{A}(f)^{-1}\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$



Ground Truth

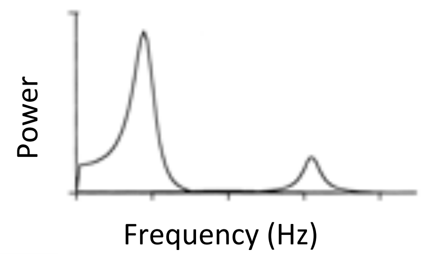


Functional

Spectrum

$$S(f) = \mathbf{X}(f)\mathbf{X}(f)^* = \mathbf{H}(f)\Sigma\mathbf{H}(f)^*$$

(Brillinger, 2001)

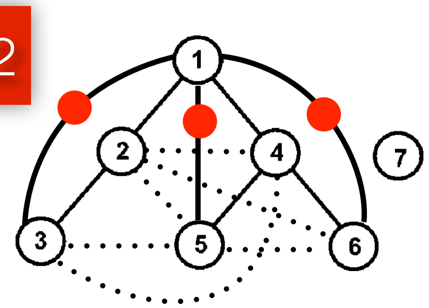


Coherency

M=2

$$C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

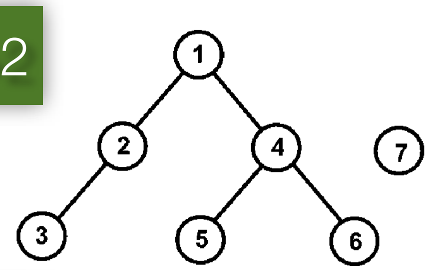


Partial Coherence

M>2

$$P_{ij}(f) = \frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)



Multiple Coherence

M=1

$$G_i(f) = \sqrt{1 - \frac{\det(S(f))}{S_{ii}(f)\mathbf{M}_{ii}(f)}}$$

(Brillinger, 2001)

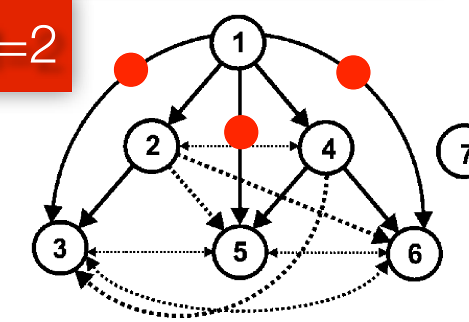
Effective

Granger-Geweke Causality

M=2

$$F_{ij}(f) = \frac{\Sigma_{jj} - (\Sigma_{ij}^2/\Sigma_{ii}))\,|\,H_{ij}(f)\,|^2}{S_{ii}(f)}$$

(Geweke, 1982; Bressler et al., 2007)

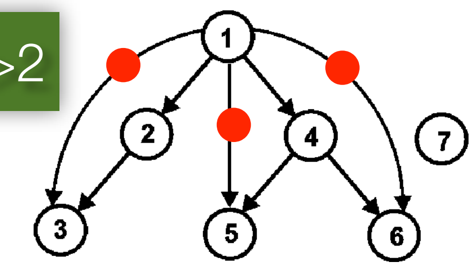


Directed Transfer Function

M>2

$$\eta_{ij}^2(f) = \frac{\left|H_{ij}(f)\right|^2}{\sum_f \sum_{k=1}^{M}\left|H_{ik}(f)\right|^2}$$

(Kaminski and Blinowska, 1991)

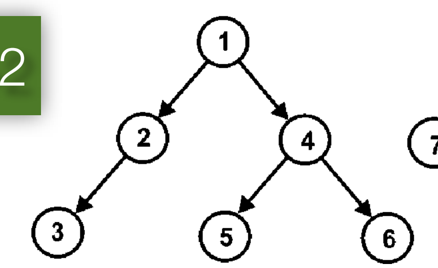


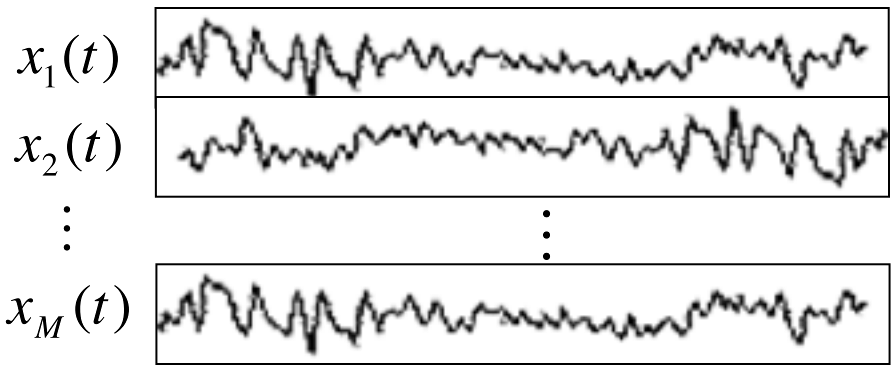
Partial Directed Coherence

M>2

$$\pi_{ij}^2(f) = \frac{|\,A_{ij}(f)\,|^2}{\sum_{k=1}^{M}\left|A_{kj}(f)\right|^2}$$

(Baccalá and Sameshima, 2001)

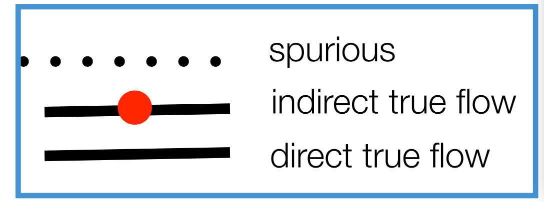




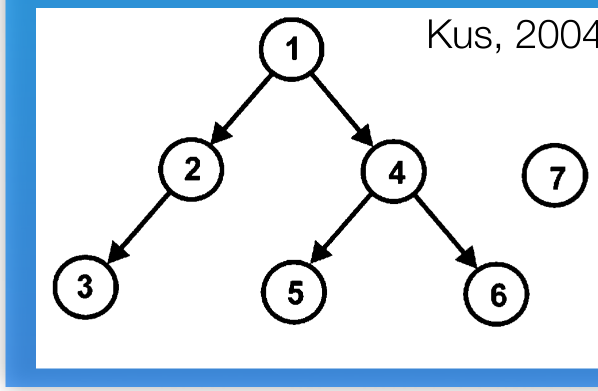
$$\mathbf{X}(t)=\sum_{k=1}^{p}\mathbf{A}^{(k)}\mathbf{X}(t-k)+\mathbf{E}(t)$$

$$\mathbf{A}(f)=-\sum_{k=0}^{p}\mathbf{A}^{(k)}e^{-i2\pi fk};\quad \mathbf{A}^{(0)}=I$$

$$\mathbf{X}(f)=\mathbf{A}(f)^{-1}\mathbf{E}(f)=\mathbf{H}(f)\mathbf{E}(f)$$



Ground Truth

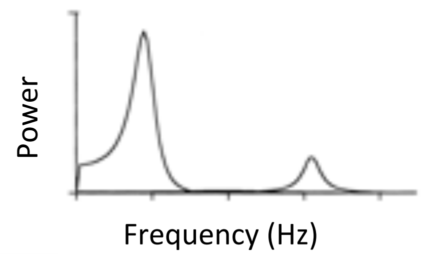


Functional

Spectrum

$$S(f)=\mathbf{X}(f)\mathbf{X}(f)^{*}$$
$$=\mathbf{H}(f)\Sigma\mathbf{H}(f)^{*}$$

(Brillinger, 2001)

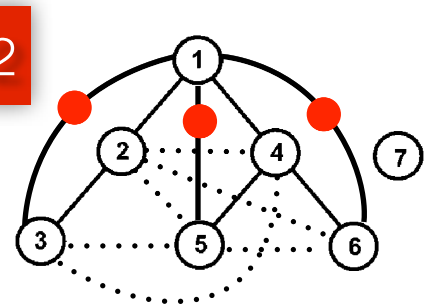


Coherency

M=2

$$C_{ij}(f)=\frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

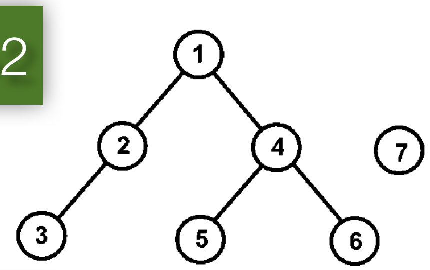


Partial Coherence

M>2

$$P_{ij}(f)=\frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)



Multiple Coherence

M=1

$$G_i(f)=\sqrt{1-\frac{\det(S(f))}{S_{ii}(f)\mathbf{M}_{ii}(f)}}$$

(Brillinger, 2001)

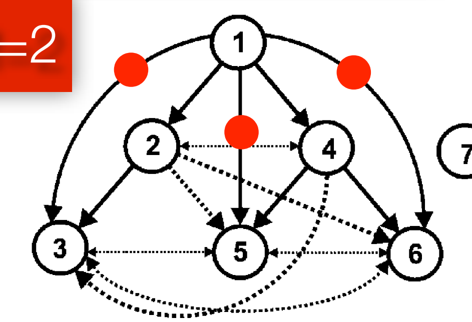
Effective

Granger-Geweke Causality

M=2

$$F_{ij}(f)=\frac{\Sigma_{jj}-(\Sigma_{ij}^{2}/\Sigma_{ii}))\,|H_{ij}(f)|^{2}}{S_{ii}(f)}$$

(Geweke, 1982; Bressler et al., 2007)

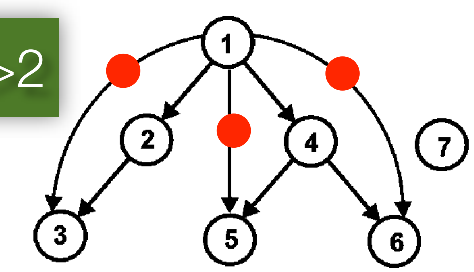


Directed Transfer Function

M>2

$$\eta_{ij}^{2}(f)=\frac{\left|H_{ij}(f)\right|^{2}}{\sum_{f}\sum_{k=1}^{M}\left|H_{ik}(f)\right|^{2}}$$

(Kaminski and Blinowska, 1991)

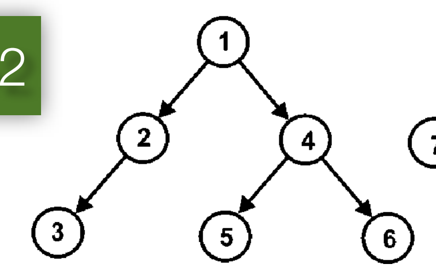


Partial Directed Coherence

M>2

$$\pi_{ij}^{2}(f)=\frac{|A_{ij}(f)|^{2}}{\sum_{k=1}^{M}\left|A_{kj}(f)\right|^{2}}$$

(Baccalá and Sameshima, 2001)

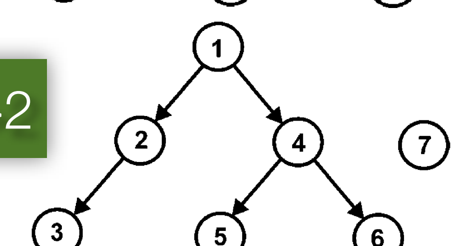


Direct DTF

M>2

$$\delta_{ij}^{2}(f)=\eta_{ij}^{2}(f)P_{ij}^{2}(f)$$

(Korzeniewska, 2003)

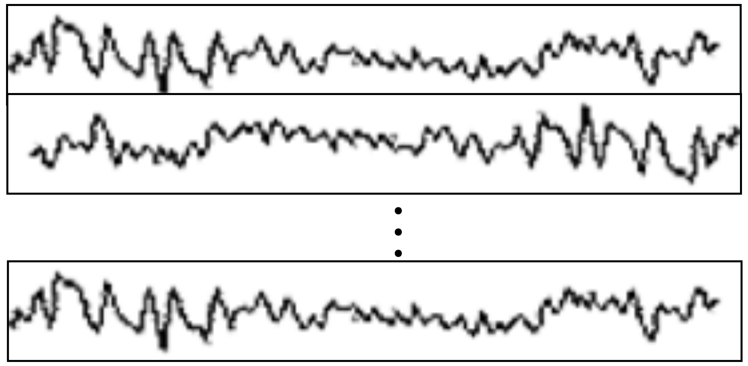


$x_1(t)$

$x_2(t)$

$\vdots$

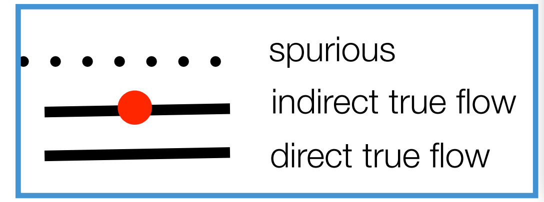
$x_M(t)$



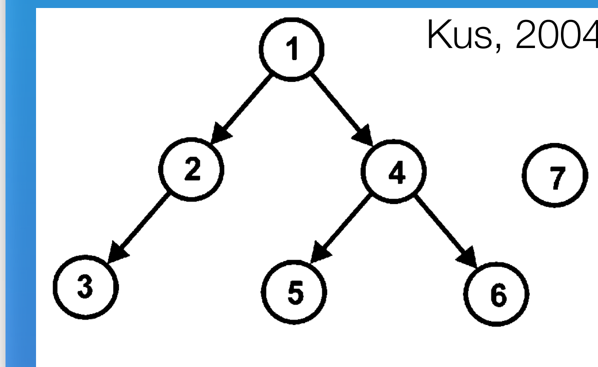
$$\mathbf{X}(t)=\sum_{k=1}^{p}\mathbf{A}^{(k)}\mathbf{X}(t-k)+\mathbf{E}(t)$$

$$\mathbf{A}(f)=-\sum_{k=0}^{p}\mathbf{A}^{(k)}e^{-i2\pi fk};\quad \mathbf{A}^{(0)}=I$$

$$\mathbf{X}(f)=\mathbf{A}(f)^{-1}\mathbf{E}(f)=\mathbf{H}(f)\mathbf{E}(f)$$



Ground Truth

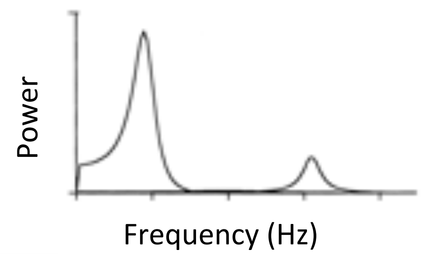


Functional

Spectrum

$$S(f)=\mathbf{X}(f)\mathbf{X}(f)^{*}=\mathbf{H}(f)\Sigma\mathbf{H}(f)^{*}$$

(Brillinger, 2001)

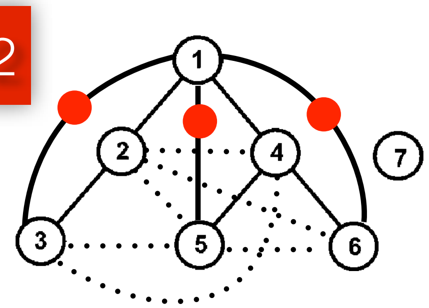


Coherency

M=2

$$C_{ij}(f)=\frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}$$

(Brillinger, 2001)

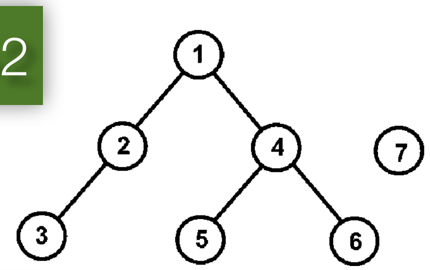


Partial Coherence

M>2

$$P_{ij}(f)=\frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}}$$

(Brillinger, 2001)



Multiple Coherence

M=1

$$G_i(f)=\sqrt{1-\frac{\det(S(f))}{S_{ii}(f)\mathbf{M}_{ii}(f)}}$$

(Brillinger, 2001)

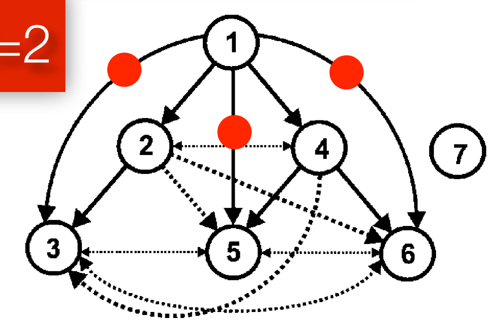
Effective

Granger-Geweke Causality

M=2

$$F_{ij}(f)=\frac{\Sigma_{jj}-(\Sigma_{ij}^2/\Sigma_{ii}))\,|H_{ij}(f)|^2}{S_{ii}(f)}$$

(Geweke, 1982; Bressler et al., 2007)

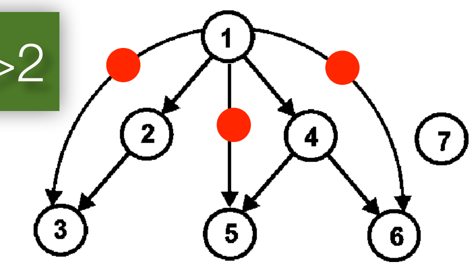


Directed Transfer Function

M>2

$$\eta_{ij}^2(f)=\frac{\left|H_{ij}(f)\right|^2}{\sum_f\sum_{k=1}^{M}\left|H_{ik}(f)\right|^2}$$

(Kaminski and Blinowska, 1991)

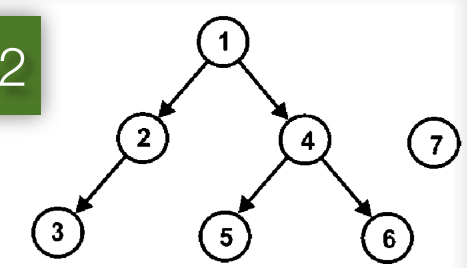


Partial Directed Coherence

M>2

$$\pi_{ij}^2(f)=\frac{|A_{ij}(f)|^2}{\sum_{k=1}^{M}\left|A_{kj}(f)\right|^2}$$

(Baccalá and Sameshima, 2001)

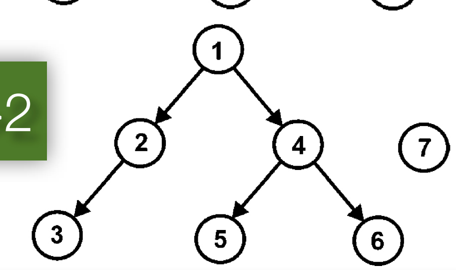


Direct DTF

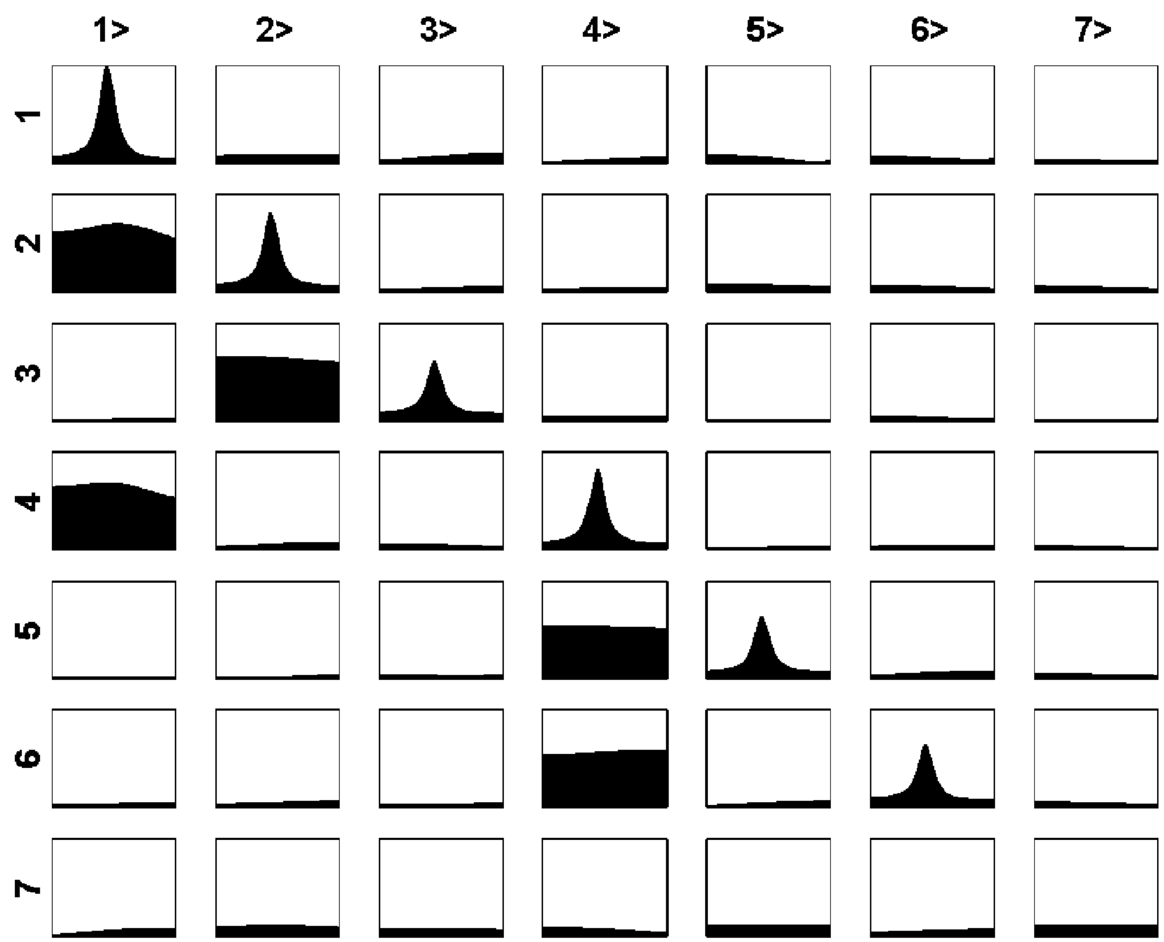
M>2

$$\delta_{ij}^2(f)=\eta_{ij}^2(f)P_{ij}^2(f)$$

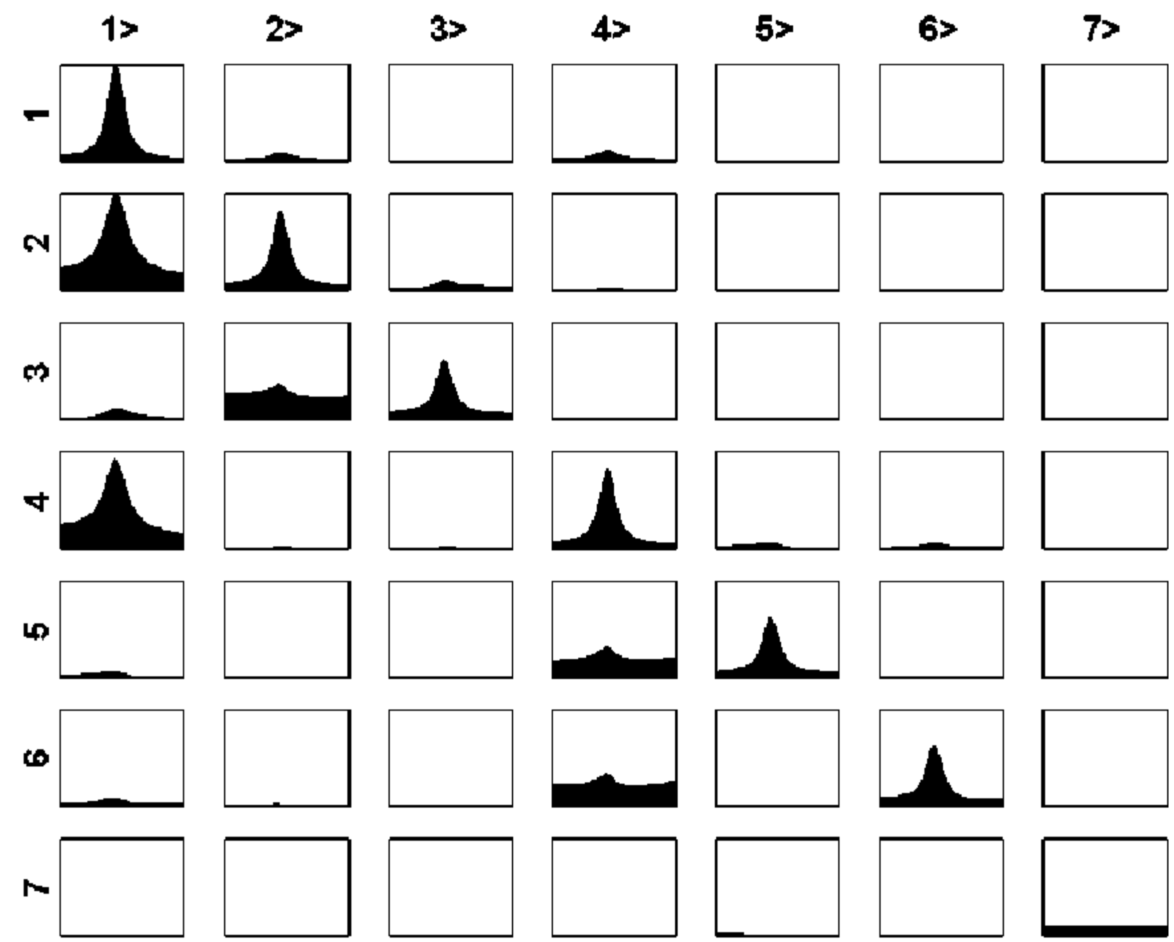
(Korzeniewska, 2003)



PDC versus DTF methods (spectral considerations)



PDC



dDTF

Time-Frequency GC

Time-Frequency GC

- Brain network dynamics often change rapidly with time (non-stationarity)
  - event-related responses
  - transient network changes during information processing

Time-Frequency GC

- Brain network dynamics often change rapidly with time (non-stationarity)
  - event-related responses
  - transient network changes during information processing
- How can we perform time-varying, frequency-domain analysis of network dynamics?

Time-Frequency GC

Time-Frequency GC

- **Many ways to do time-varying MVAR estimation**

Time-Frequency GC

- **Many ways to do time-varying MVAR estimation**
  - Short-Time adaptive multivariate autoregression (AMVAR)

Time-Frequency GC

- **Many ways to do time-varying MVAR estimation**
  - Short-Time adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)

Time-Frequency GC

- **Many ways to do time-varying MVAR estimation**
  - Short-Time adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
  - Kalman Filtering

Time-Frequency GC

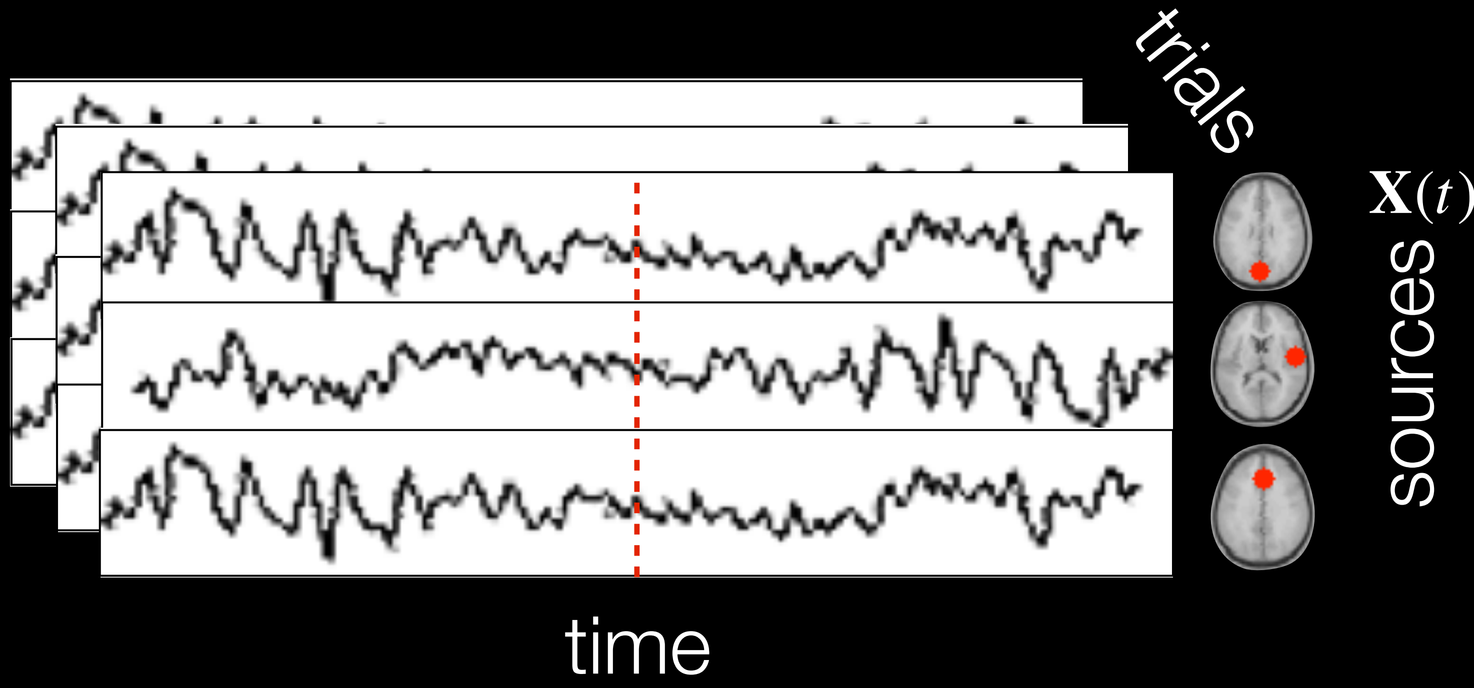
- **Many ways to do time-varying MVAR estimation**
  - Short-Time adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
  - Kalman Filtering
  - ...

Time-Frequency GC

- **Many ways to do time-varying MVAR estimation**
  - Short-Time adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
  - Kalman Filtering
  - ...

Short-Window Time-Frequency GC

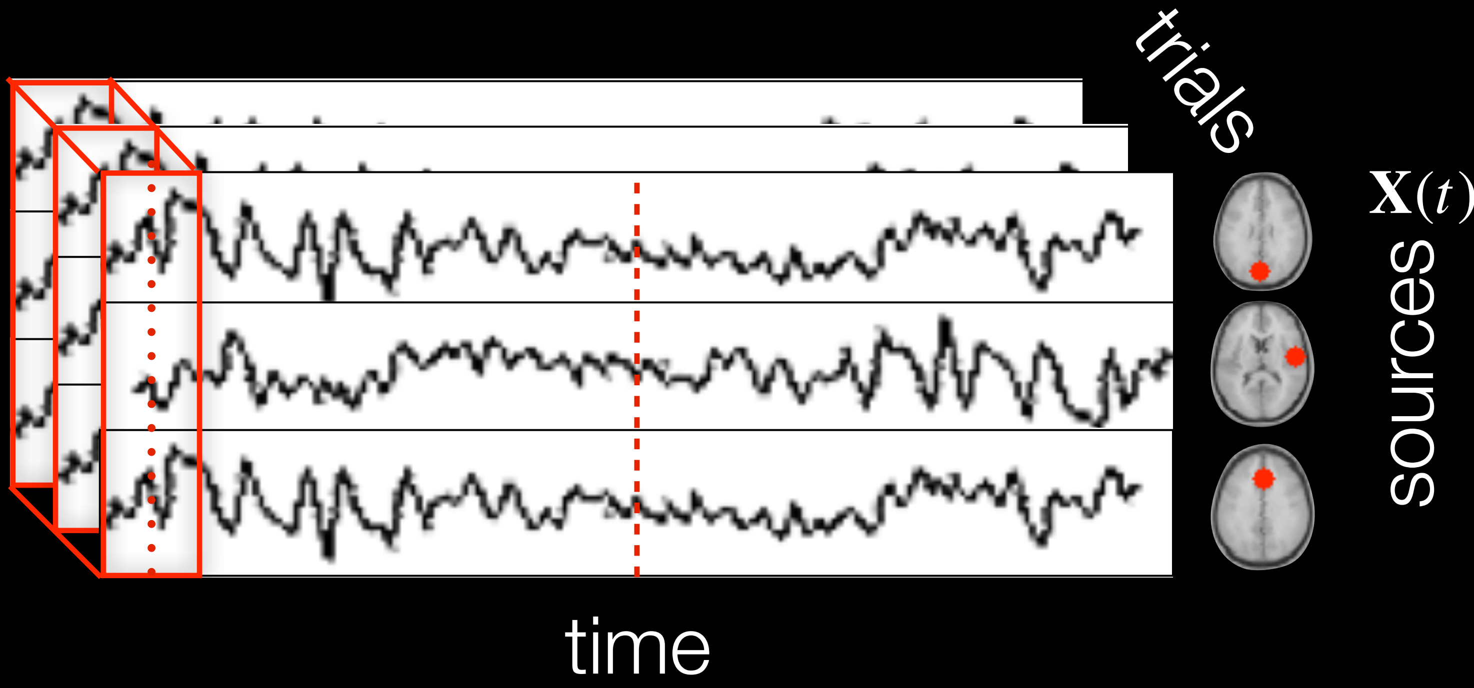
(Ding et al, 2000)



Analogous to short-time Fourier transform

Short-Window Time-Frequency GC

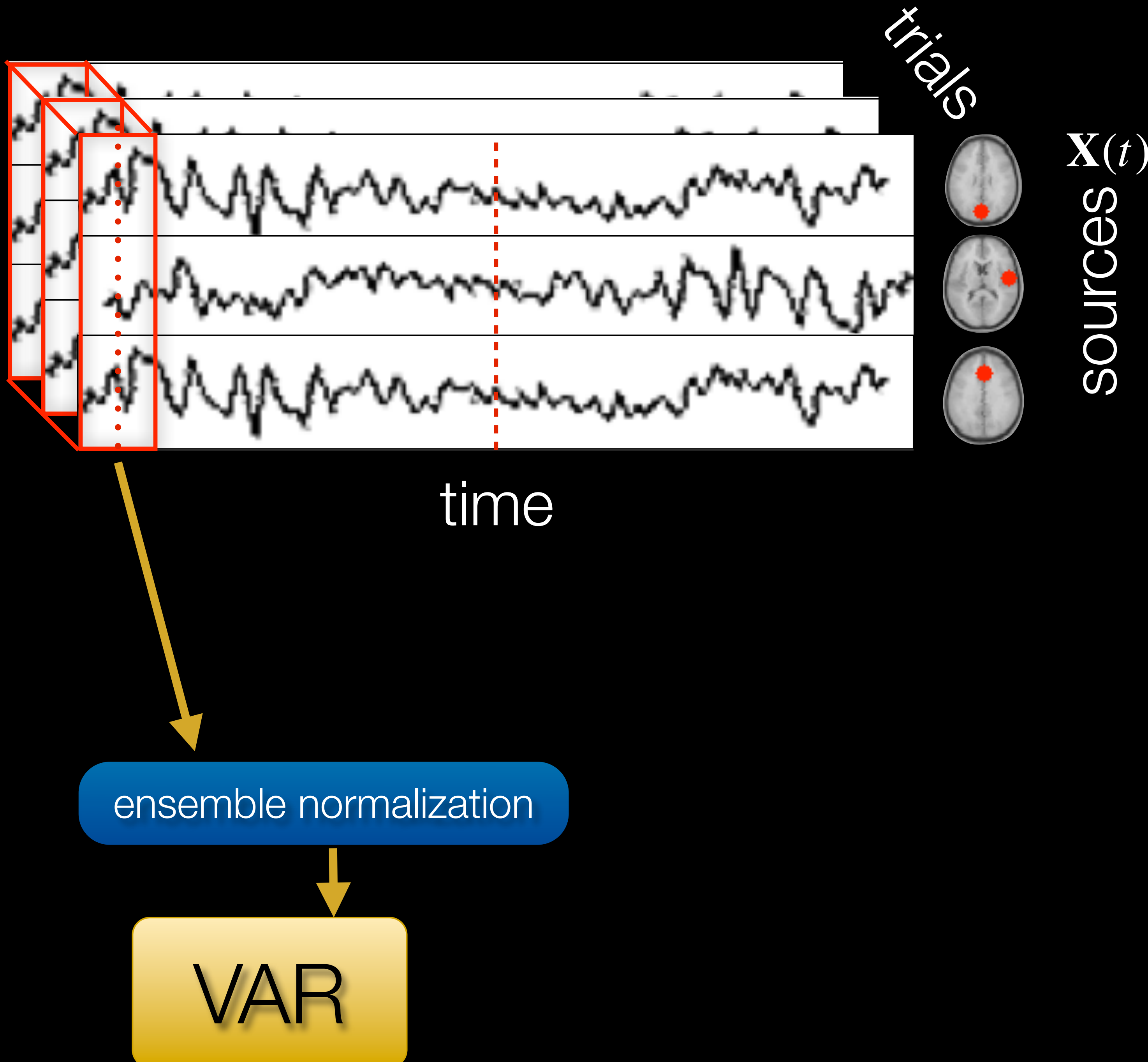
(Ding et al, 2000)



Analogous to short-time Fourier transform

Short-Window Time-Frequency GC

(Ding et al, 2000)



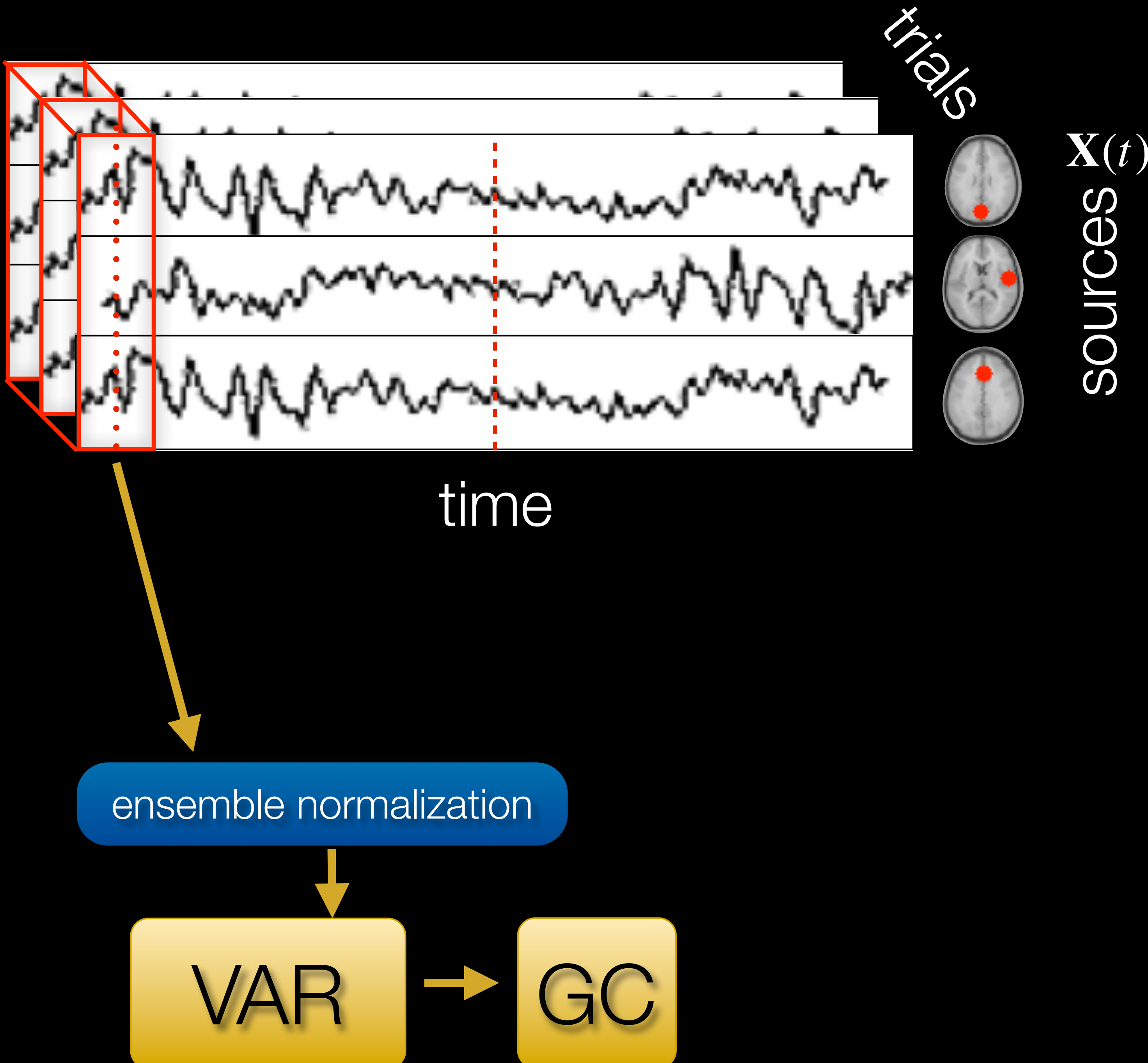
Analogous to short-time Fourier transform

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f,t) = -\sum_{k=0}^{p} \mathbf{A}^{(k)}(t)e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

Short-Window Time-Frequency GC

(Ding et al, 2000)



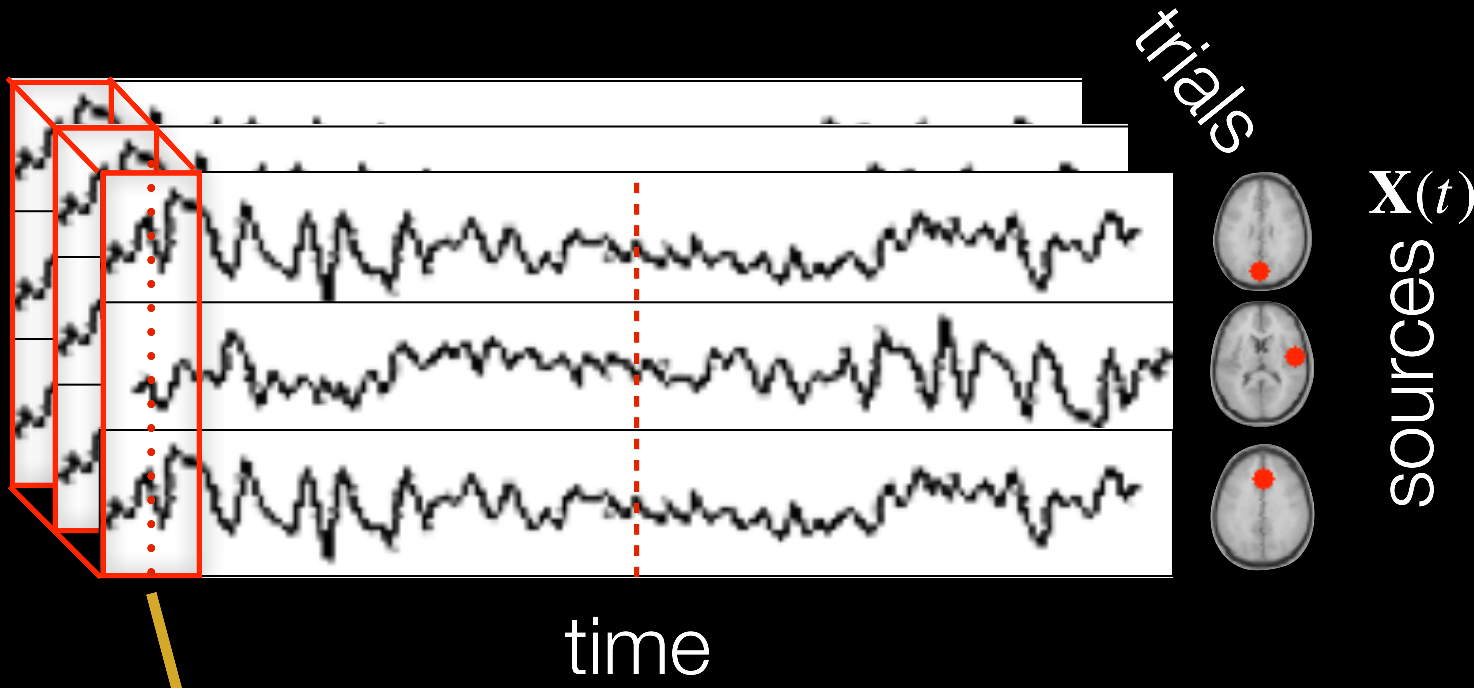
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f,t) = -\sum_{k=0}^{p} \mathbf{A}^{(k)}(t)e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

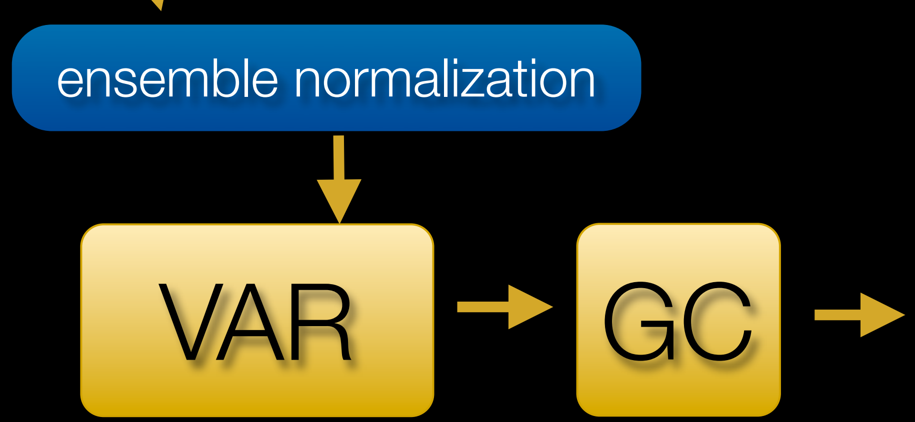
Analogous to short-time Fourier transform

Short-Window Time-Frequency GC

(Ding et al, 2000)

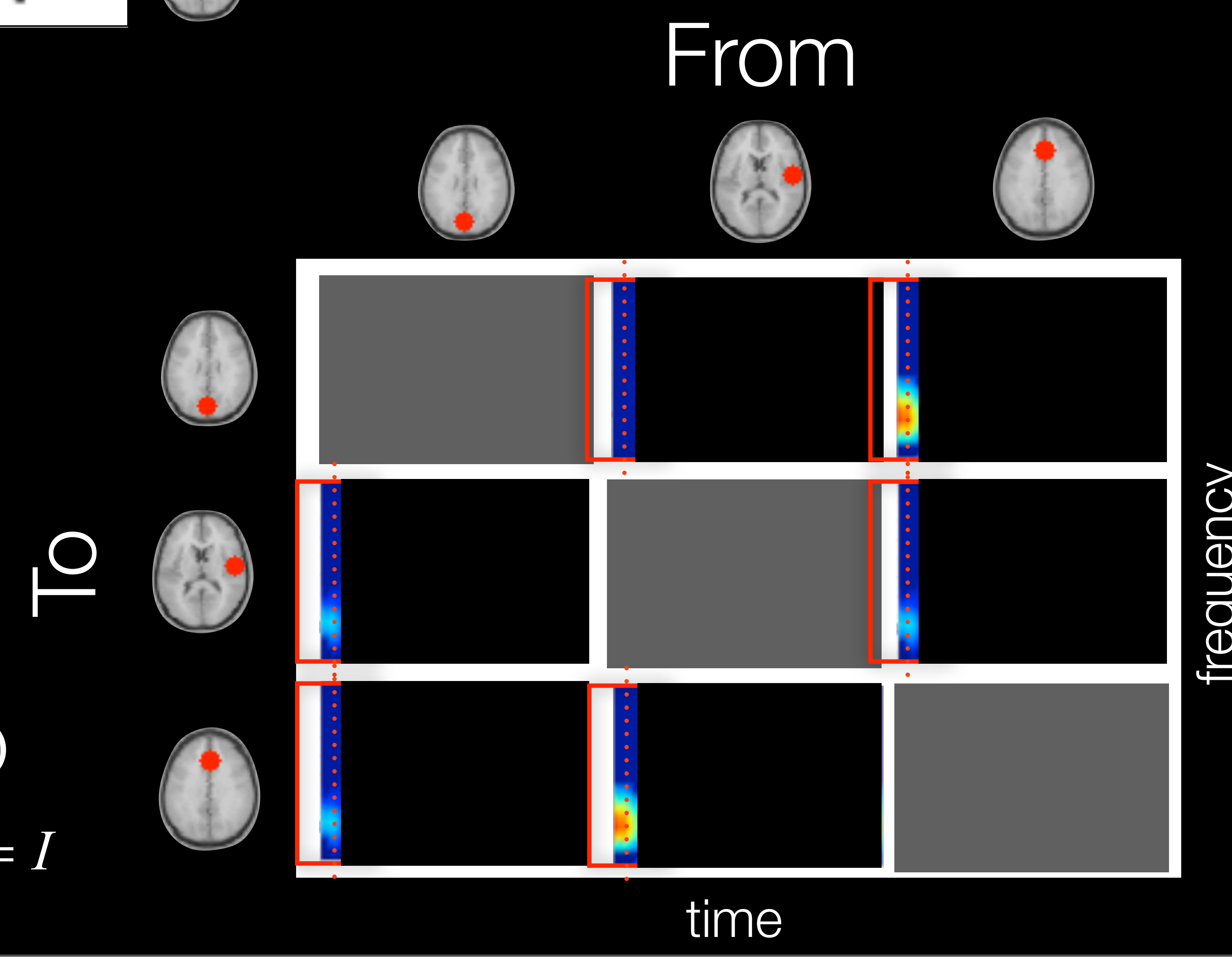


Analogous to short-time Fourier transform



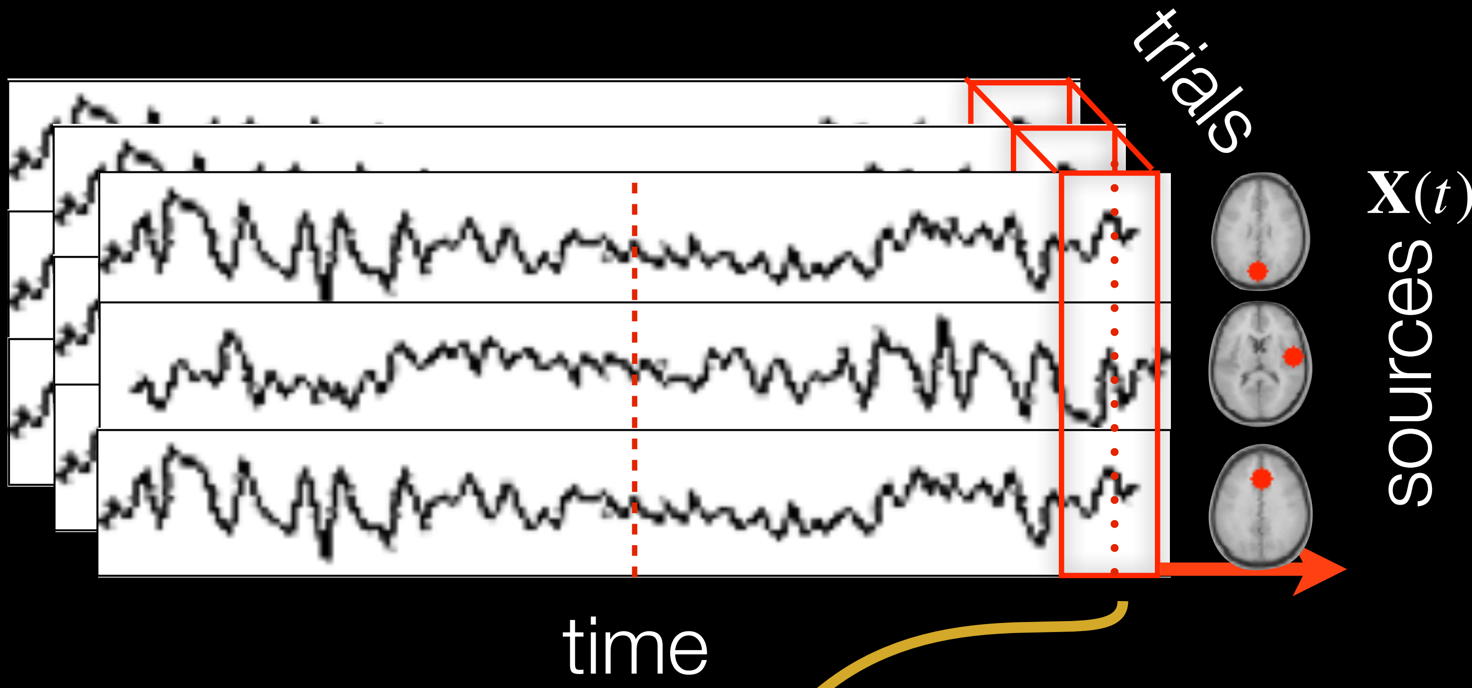
$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f,t) = -\sum_{k=0}^{p} \mathbf{A}^{(k)}(t)e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

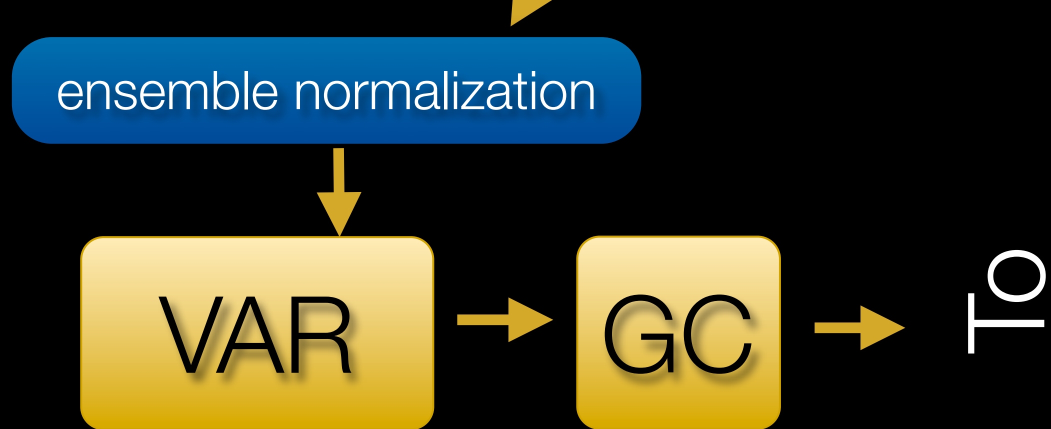


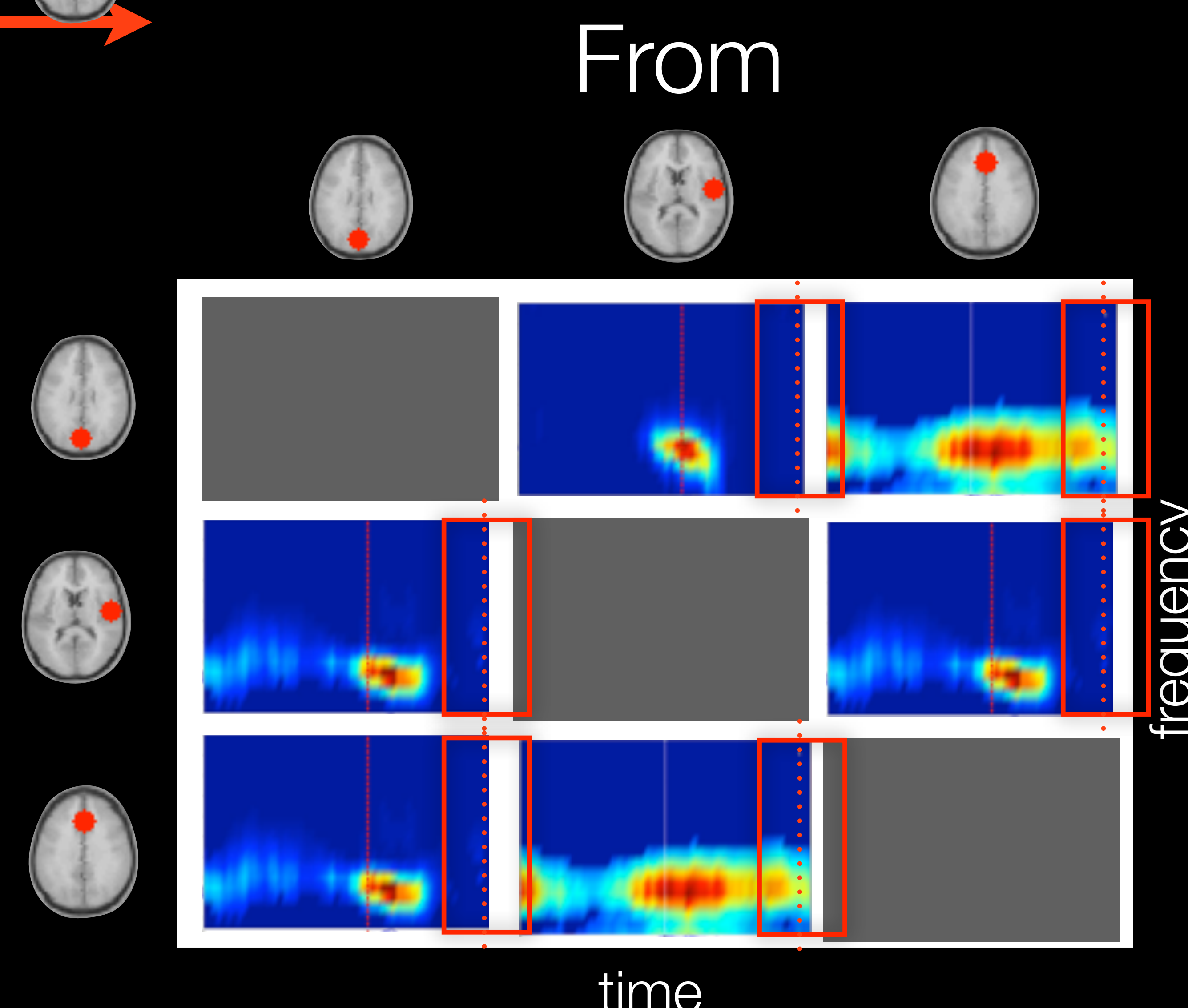
Short-Window Time-Frequency GC

(Ding et al, 2000)



Analogous to short-time Fourier transform





$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}^{(k)}(t)\mathbf{X}(t-k) + \mathbf{E}(t)$$

$$\mathbf{A}(f,t) = -\sum_{k=0}^{p} \mathbf{A}^{(k)}(t)e^{-i2\pi fk}; \mathbf{A}^{(0)} = I$$

Time-Frequency GC

Time-Frequency GC

- **What is a good window length?**

Time-Frequency GC

- **What is a good window length?**
- Considerations:

Time-Frequency GC

- **What is a good window length?**
- Considerations:
  - Temporal smoothing

Time-Frequency GC

- **What is a good window length?**
- Considerations:
  - Temporal smoothing
  - Local stationarity

Time-Frequency GC

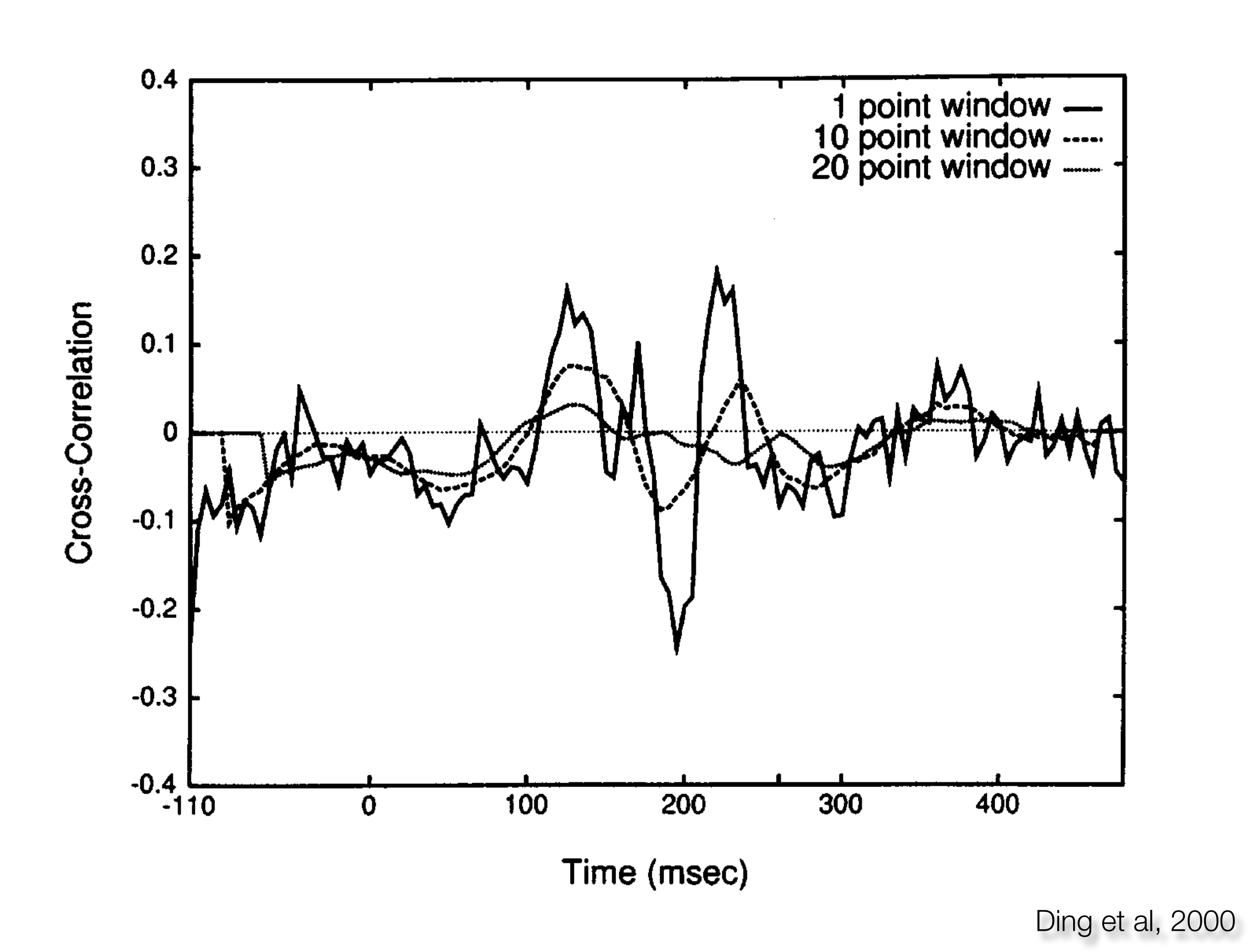
- **What is a good window length?**
- Considerations:
  - Temporal smoothing
  - Local stationarity
  - Sufficient amount of data

Time-Frequency GC

- **What is a good window length?**
- Considerations:
  - Temporal smoothing
  - Local stationarity
  - Sufficient amount of data
  - Process dynamics

Time-Frequency GC

Consideration: Temporal Smoothness

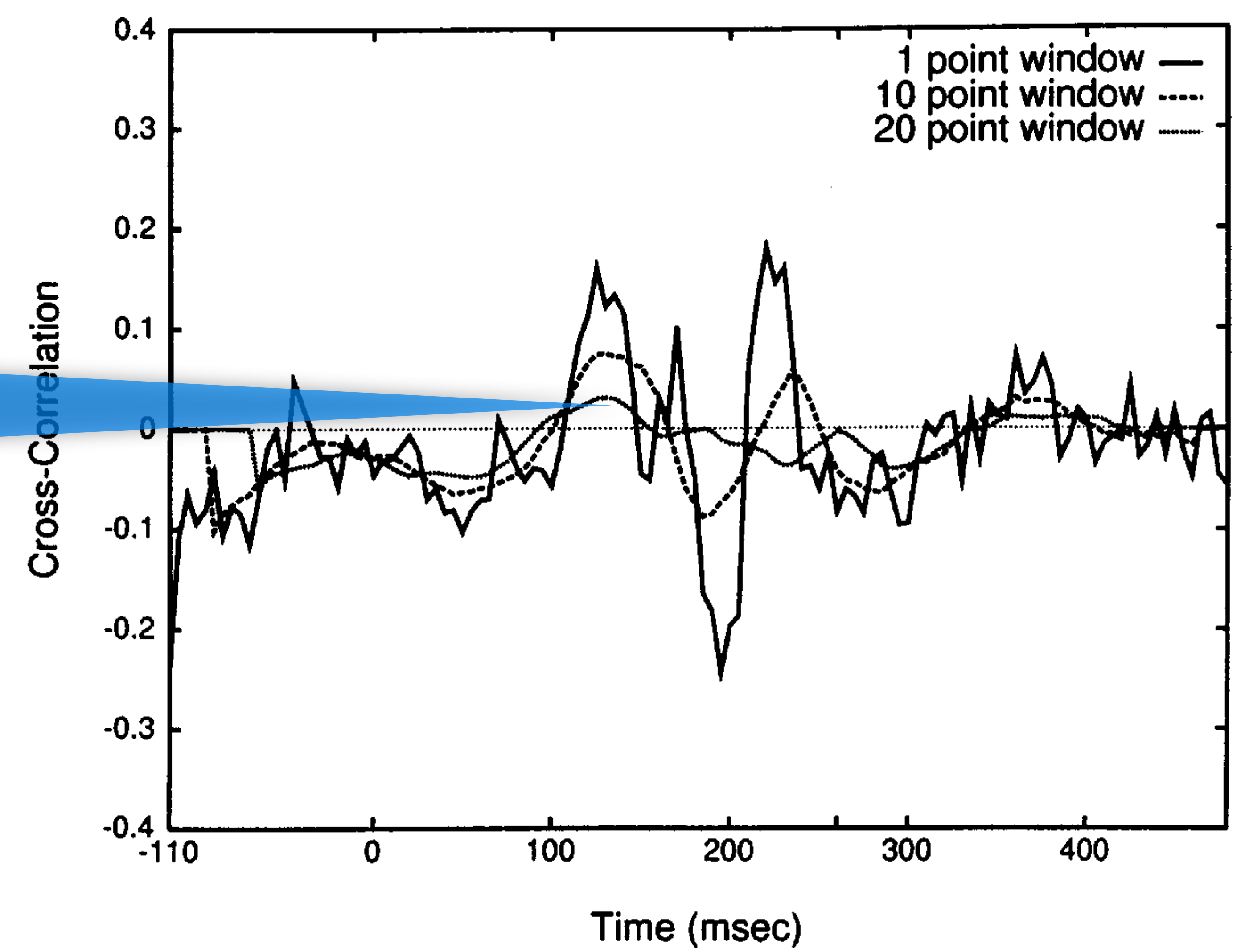


Ding et al, 2000

Time-Frequency GC

Consideration: Temporal Smoothness

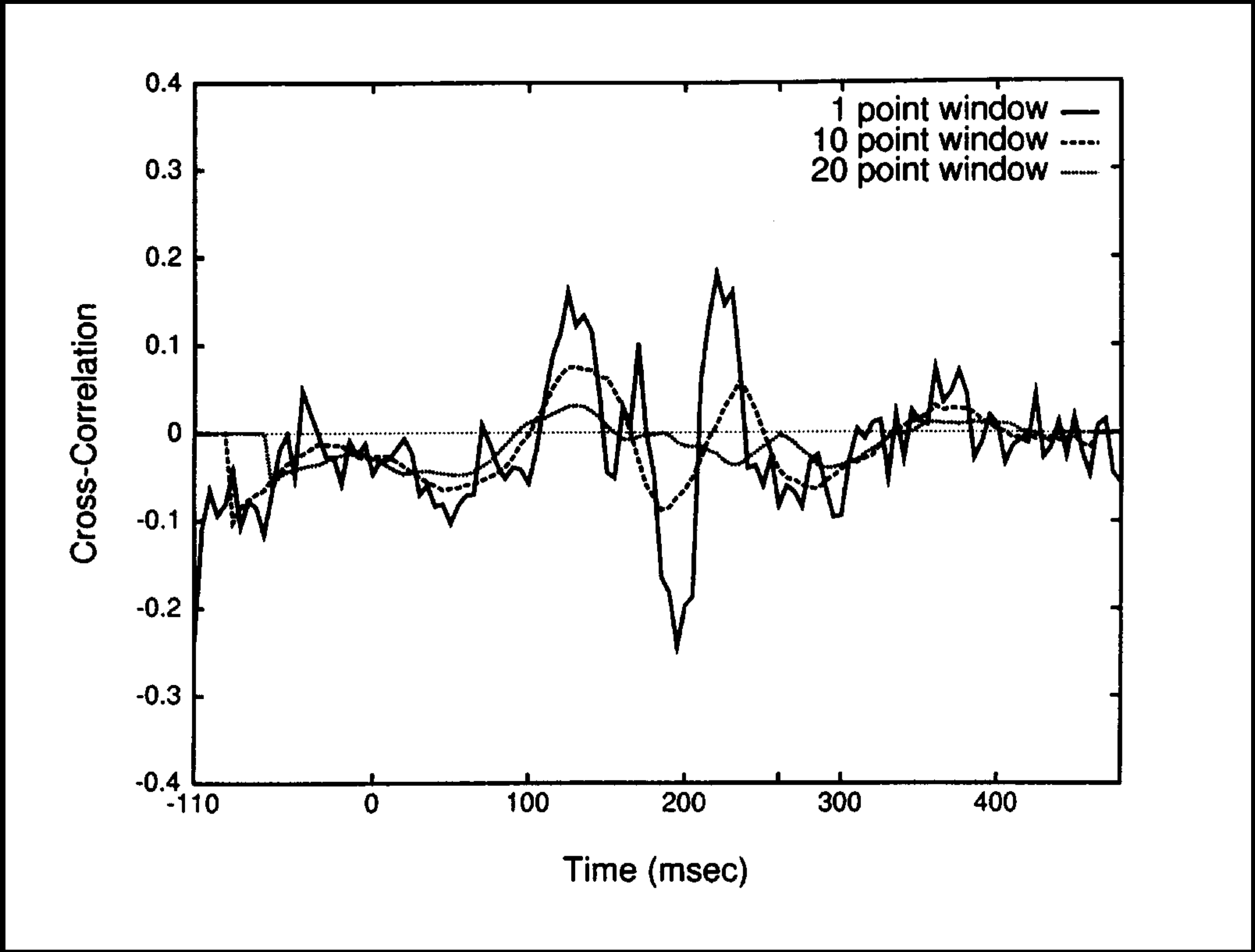
Too-large windows may smooth out interesting transient dynamic features.



Ding et al, 2000

Time-Frequency GC

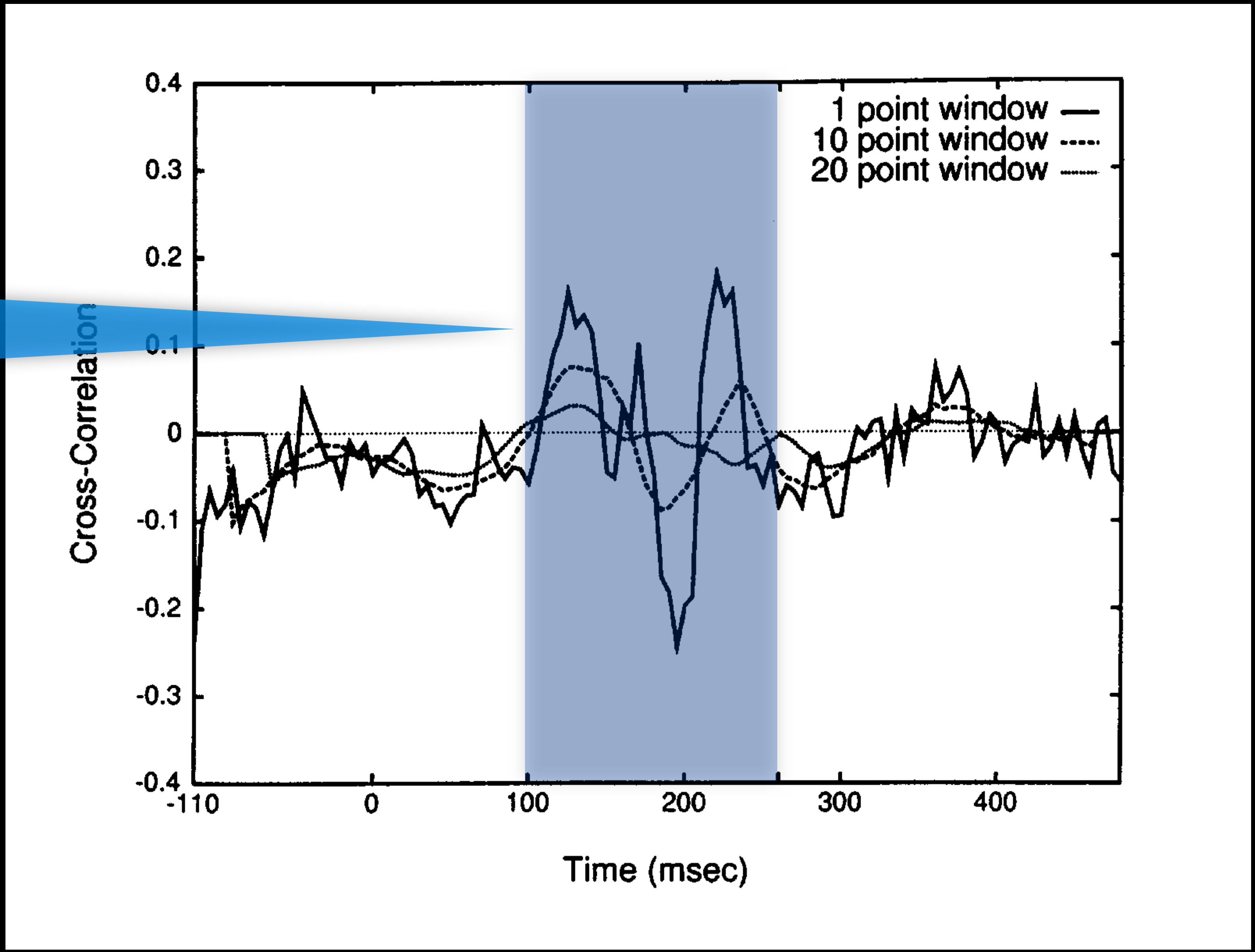
Consideration: Local Stationarity



Time-Frequency GC

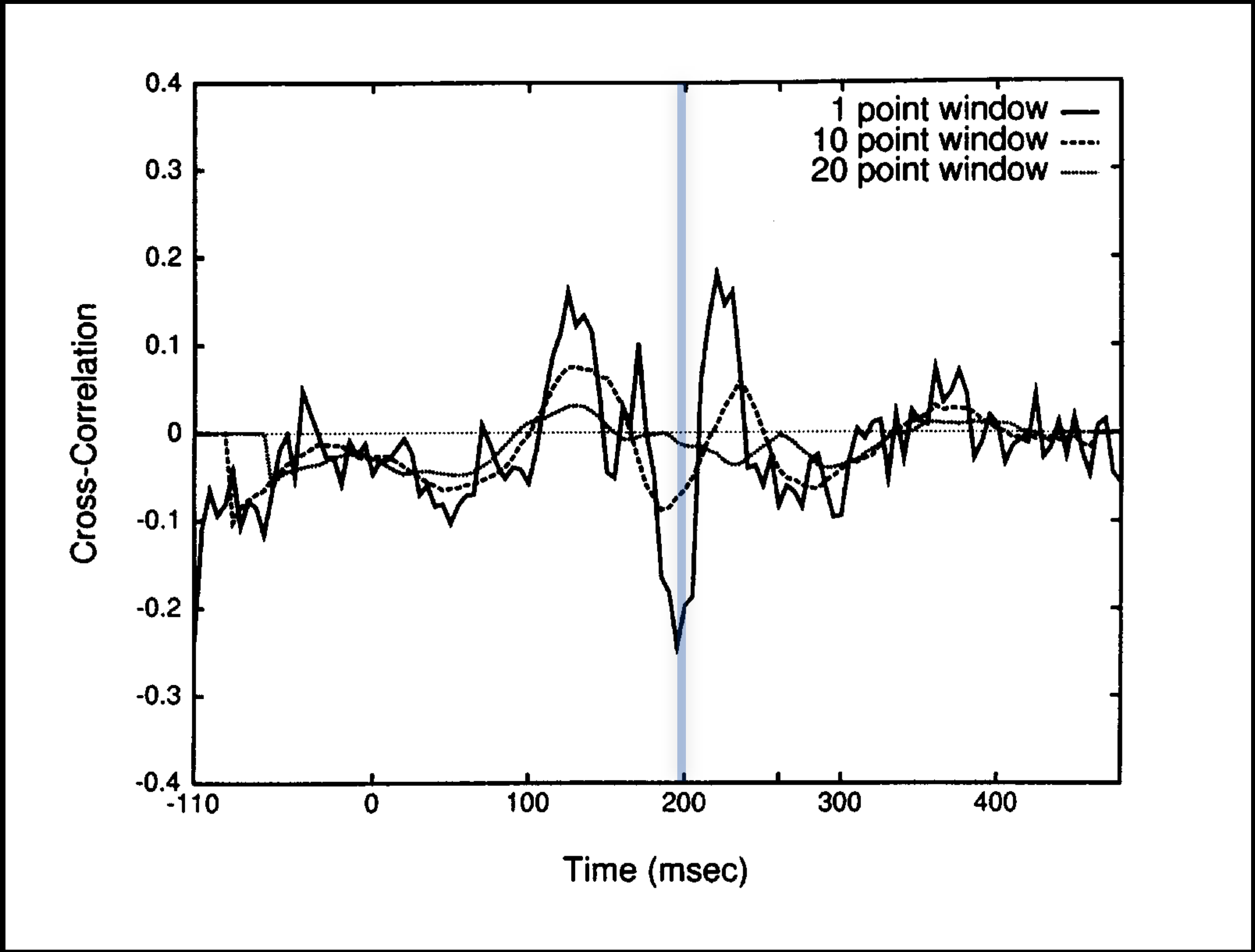
Consideration: Local Stationarity

Too-large windows may not be locally-stationary



Time-Frequency GC

Consideration: Local Stationarity



Time-Frequency GC

Time-Frequency GC

Consideration: Sufficient data

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

We have M^2p model coefficients to estimate. This requires a minimum of M^2p independent samples.

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

We have M^2p model coefficients to estimate. This requires a minimum of M^2p independent samples.
So we have the constraint $M^2p <= N_{tr} W$.

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

We have M^2p model coefficients to estimate. This requires a minimum of M^2p independent samples.
So we have the constraint $M^2p <= N_{tr} W$.
In practice, however, a better heuristic is $M^2p <= (1/10) N_{tr} W$.

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

We have M^2p model coefficients to estimate. This requires a minimum of M^2p independent samples.
So we have the constraint $M^2p \le N_{tr} W$.
In practice, however, a better heuristic is $M^2p \le (1/10) N_{tr} W$.

Or: $\mathbf{W \ge 10(M^2p/N_{tr})}$

10x more data points than parameters to estimate

Time-Frequency GC

Consideration: Sufficient data

M = number of variables

p = model order

N_{tr} = number of trials

W = length of each window (sample points)

We have M^2p model coefficients to estimate. This requires a minimum of M^2p independent samples.
So we have the constraint $M^2p <= N_{tr} W$.
In practice, however, a better heuristic is $M^2p <= (1/10)N_{tr} W$.

Or: **$W >= 10(M^2p/N_{tr})$**

10x more data points than parameters to estimate

SIFT will let you know if your window length is not optimal

Time-Frequency GC

Time-Frequency GC

Consideration: Process dynamics

Time-Frequency GC

Consideration: Process dynamics

- Your window must be larger than the maximum expected interaction time lag between any two processes.

Time-Frequency GC

Consideration: Process dynamics

- Your window must be larger than the maximum expected interaction time lag between any two processes.
- Your window should be large enough to span ~1 cycle of the lowest frequency of interest (remember the Heisenberg uncertainty principle)

Time-Frequency GC

Consideration: Process dynamics

- Your window must be larger than the maximum expected interaction time lag between any two processes.
- Your window should be large enough to span ~1 cycle of the lowest frequency of interest (remember the Heisenberg uncertainty principle)

Time-Frequency GC

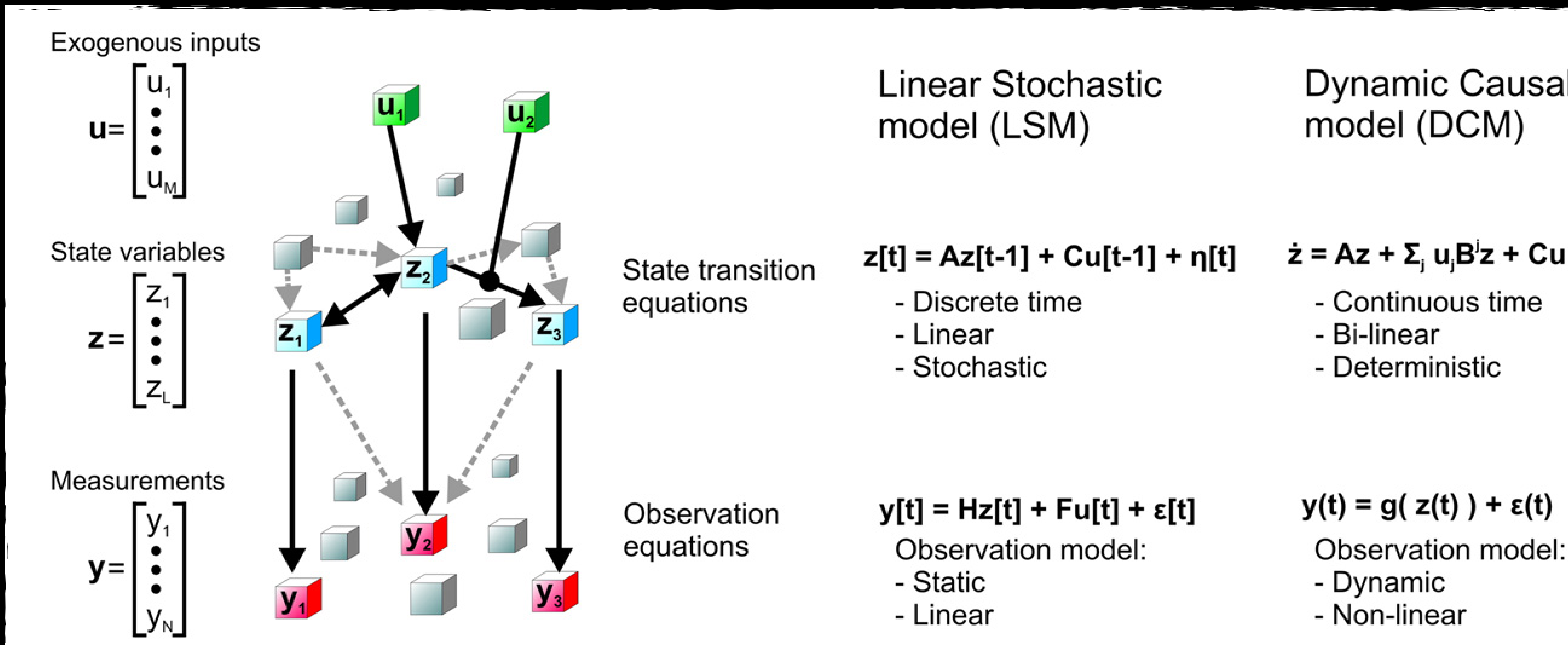
Consideration: Process dynamics

- Your window must be larger than the maximum expected interaction time lag between any two processes.
- Your window should be large enough to span ~1 cycle of the lowest frequency of interest (remember the Heisenberg uncertainty principle)

Time-Frequency GC

- **Many ways to do time-varying MVAR estimation**
  - Short-Time adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
  - Kalman Filtering
  - ...

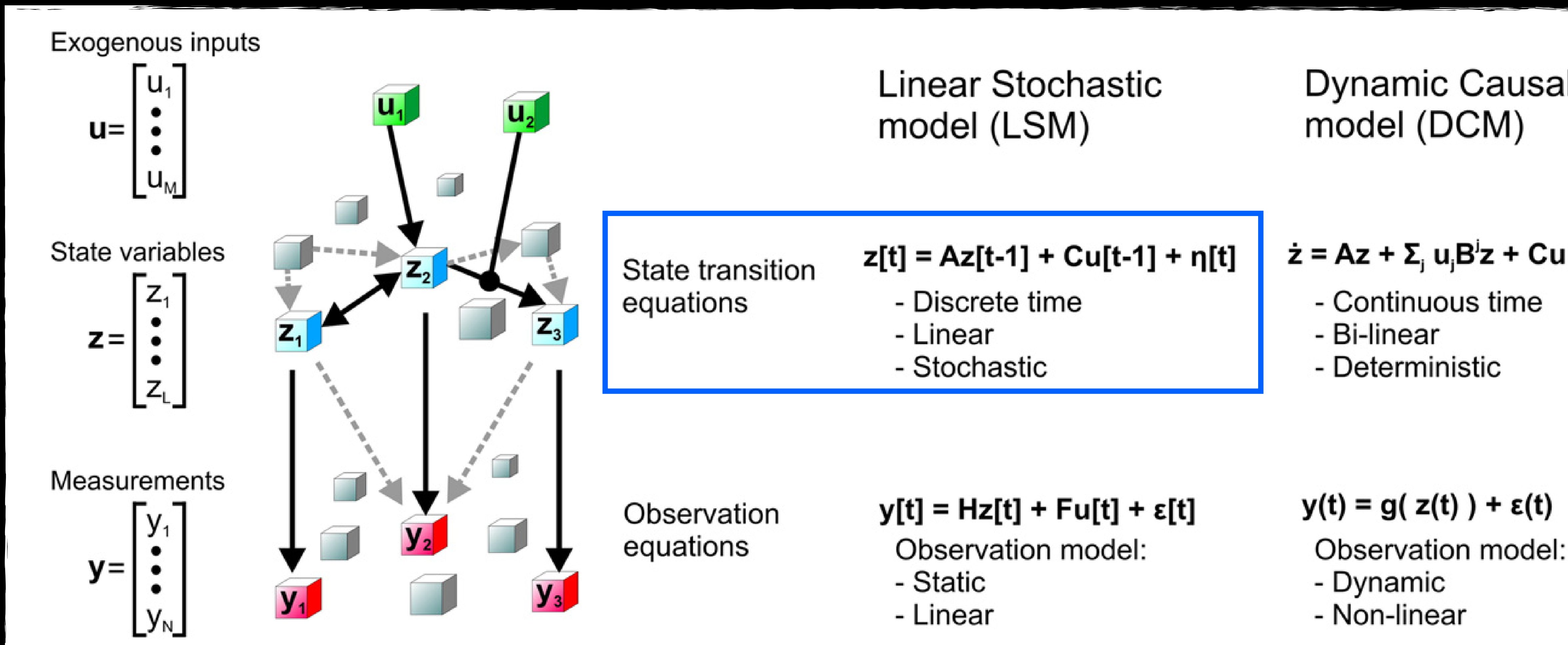
The State-Space Model



Roebroeck, et al NeuroImage, 2009

- Based on rich dynamical systems theory.
- Well-established state-space algorithms for tracking in non-stationary, high-dimensional, partially-observed, noisy systems
- Easily extendable to nonlinear systems
- Allows for the additional modeling of (known or inferred) exogenous inputs
- Allows for estimation of additional unknown sources (as additional states)

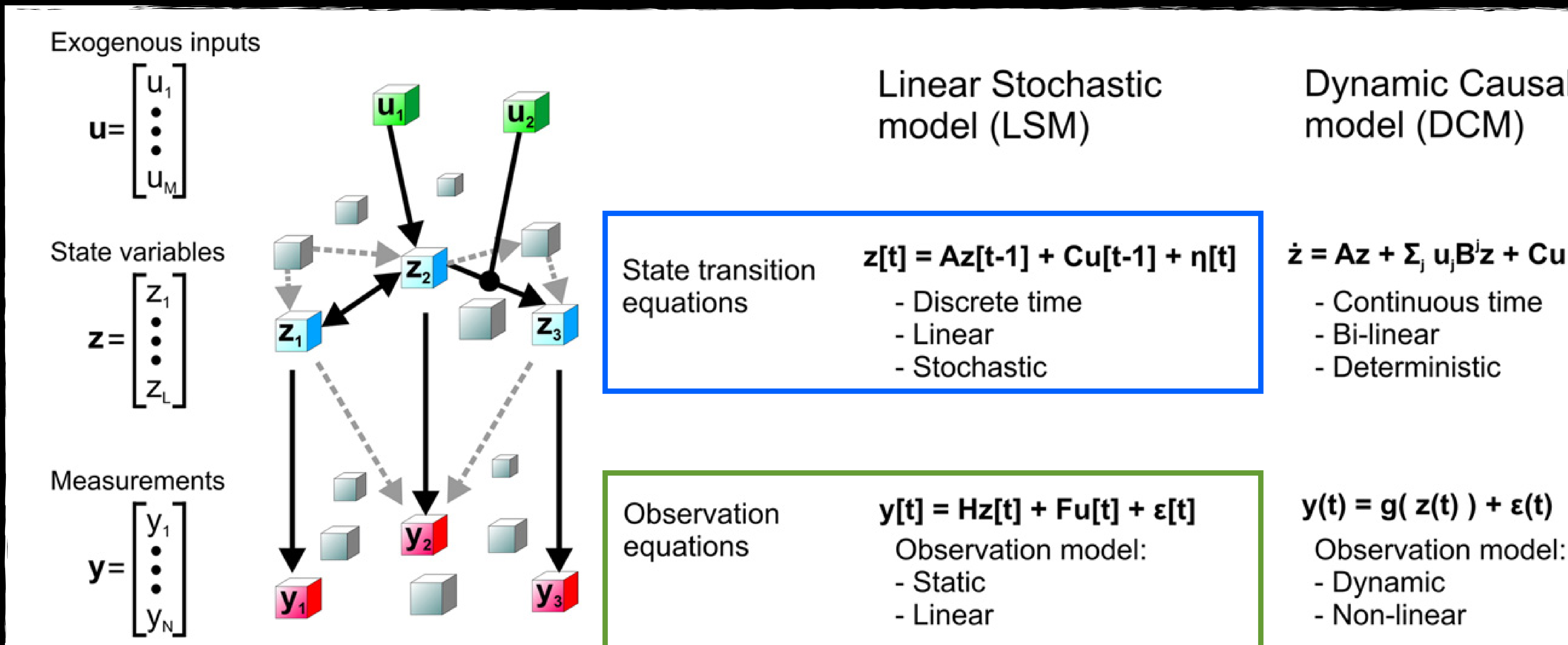
The State-Space Model



Roebroeck, et al NeuroImage, 2009

- Based on rich dynamical systems theory.
- Well-established state-space algorithms for tracking in non-stationary, high-dimensional, partially-observed, noisy systems
- Easily extendable to nonlinear systems
- Allows for the additional modeling of (known or inferred) exogenous inputs
- Allows for estimation of additional unknown sources (as additional states)

The State-Space Model



Roebroeck, et al NeuroImage, 2009

- Based on rich dynamical systems theory.
- Well-established state-space algorithms for tracking in non-stationary, high-dimensional, partially-observed, noisy systems
- Easily extendable to nonlinear systems
- Allows for the additional modeling of (known or inferred) exogenous inputs
- Allows for estimation of additional unknown sources (as additional states)

The State-Space Model

The State-Space Model

$$z(t) = vec\left([A^{(1)}(t),\ldots,A^{(p)}(t)]^T\right)_{[M^2 p\times 1]}$$

unknown VAR parameters

$$y(t) = X(t)$$

$$H(t) = I_M \otimes vec\left(\begin{bmatrix} X(t-1) & \ldots & X(t-p) \end{bmatrix}^T\right)^T_{[1\times Mp]}$$

The State-Space Model

$$z(t) = vec\left([A^{(1)}(t),\ldots,A^{(p)}(t)]^T\right)_{[M^2p\times 1]}$$

unknown VAR parameters

$$y(t) = X(t)$$

$$H(t) = I_M \otimes vec\left(\begin{bmatrix} X(t-1) & \ldots & X(t-p) \end{bmatrix}^T\right)^T_{[1\times Mp]}$$

State-Space Model

The State-Space Model

$$z(t) = vec\left([A^{(1)}(t),\ldots,A^{(p)}(t)]^T\right)_{[M^2p\times 1]}$$

unknown VAR parameters

$$y(t) = X(t)$$

$$H(t) = I_M \otimes vec\left(\begin{bmatrix} X(t-1) & \ldots & X(t-p) \end{bmatrix}^T\right)^T_{[1\times Mp]}$$

$$\boxed{z(t) = z(t-1) + v(t)}$$

state transition equation (random walk)

State-Space Model

The State-Space Model

$$z(t) = vec\left([A^{(1)}(t),\ldots,A^{(p)}(t)]^T\right)_{[M^2p\times 1]}$$

unknown VAR parameters

$$y(t) = X(t)$$

$$H(t) = I_M \otimes vec\left(\begin{bmatrix} X(t-1) & \ldots & X(t-p) \end{bmatrix}^T\right)^T_{[1\times Mp]}$$

$$\boxed{\begin{aligned} z(t) &= z(t-1) + v(t) \\ y(t) &= H(t)z(t) + \epsilon(t) \end{aligned}}$$

state transition equation (random walk)

observation equation (VAR model)

State-Space Model

The State-Space Model

$$z(t) = vec\left([A^{(1)}(t),\ldots,A^{(p)}(t)]^T\right)_{[M^2 p\times 1]}$$

unknown VAR parameters

$$y(t) = X(t)$$

$$H(t) = I_M \otimes vec\left(\begin{bmatrix} X(t-1) & \ldots & X(t-p) \end{bmatrix}^T\right)^T_{[1\times Mp]}$$

$$\boxed{\begin{aligned} z(t) &= z(t-1) + v(t) \\ y(t) &= H(t)z(t) + \epsilon(t) \end{aligned}}$$

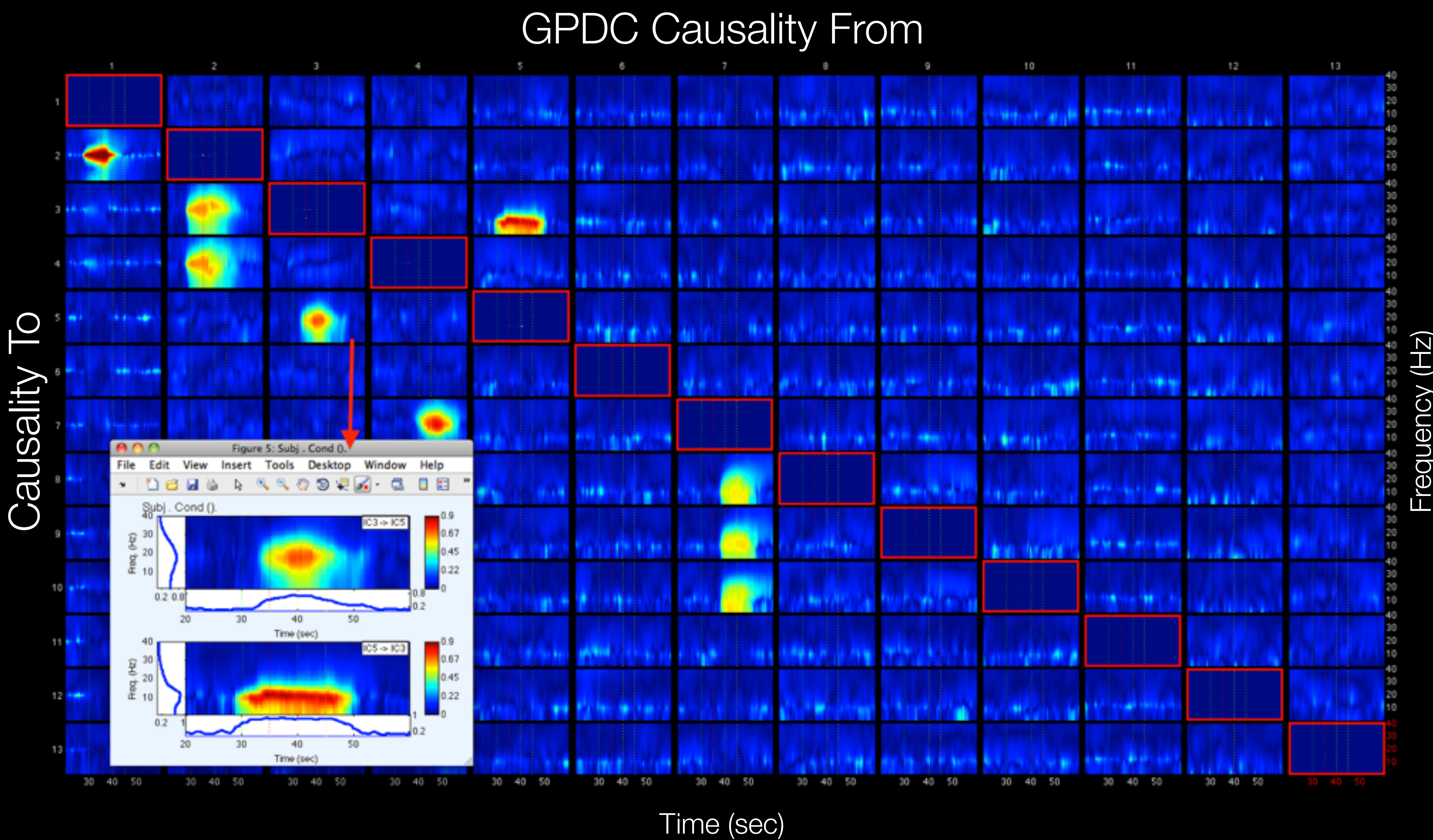
state transition equation (random walk)

observation equation (VAR model)

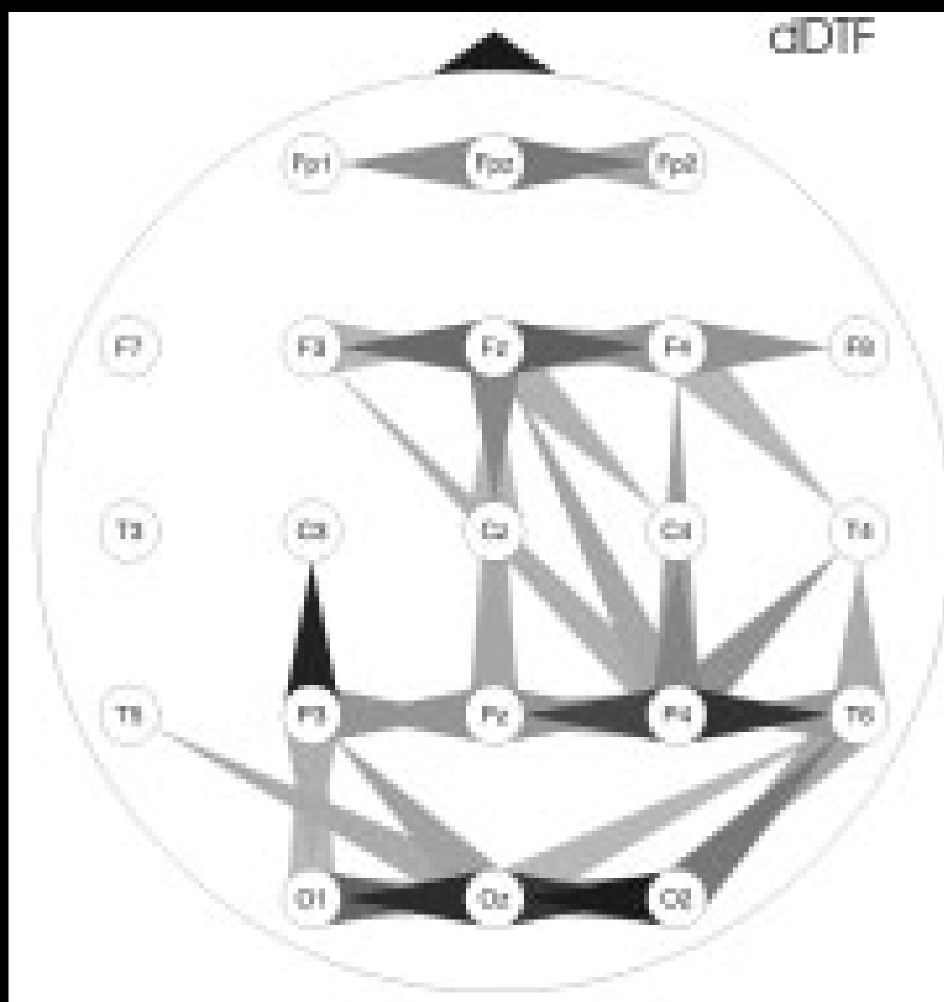
State-Space Model

- How do we solve for the time-varying unknown states?
- Kalman Filtering (and extensions)

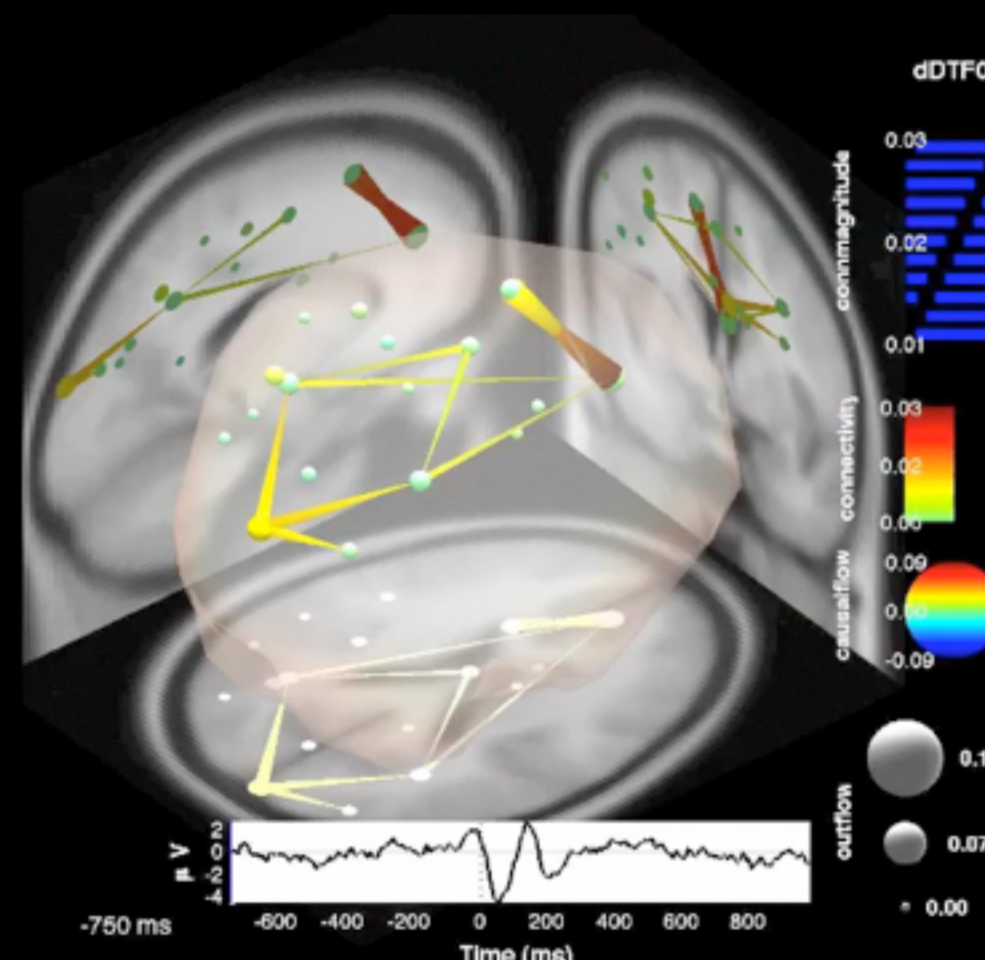
Kalman Filtering



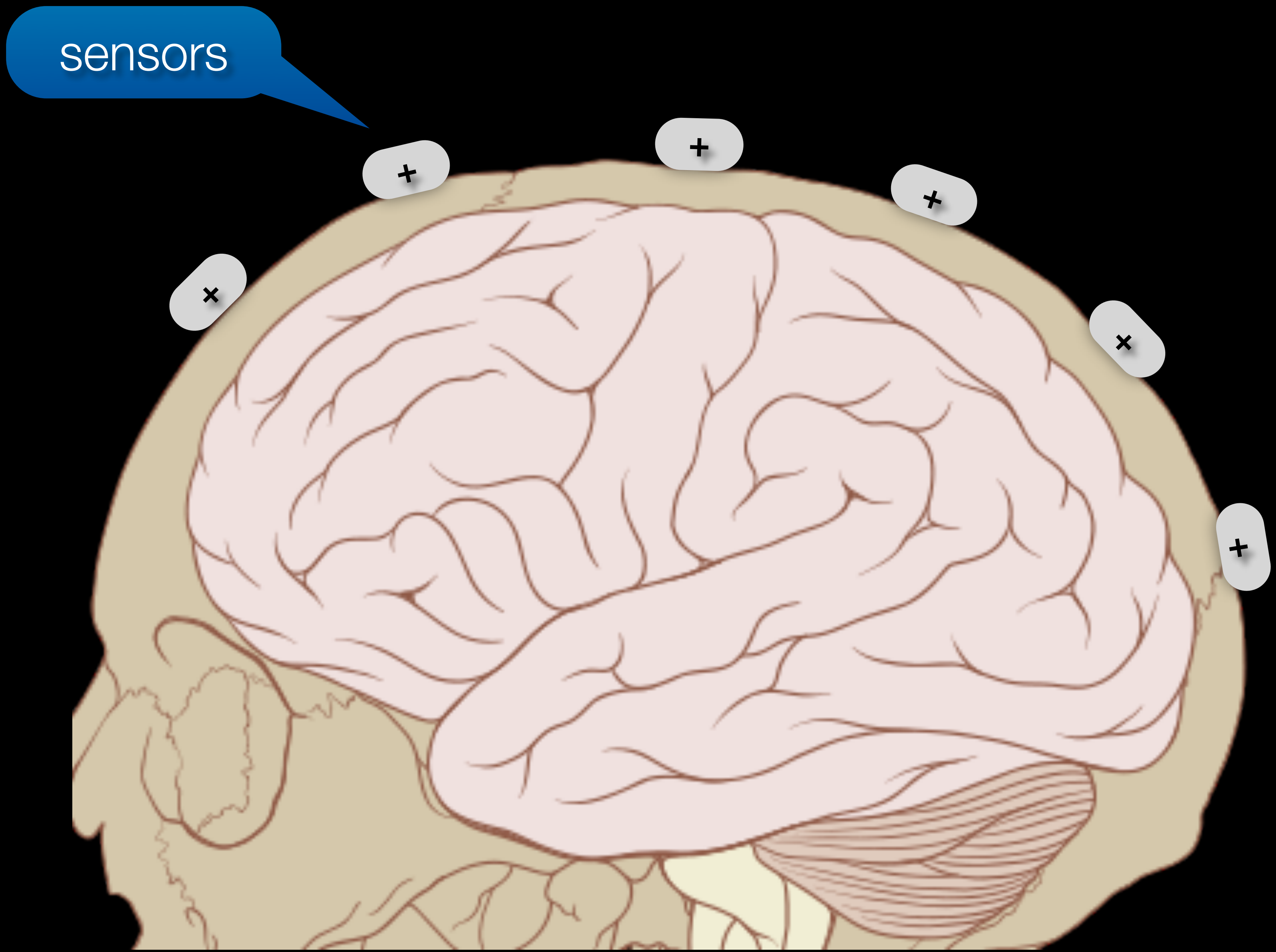
Scalp or Source?



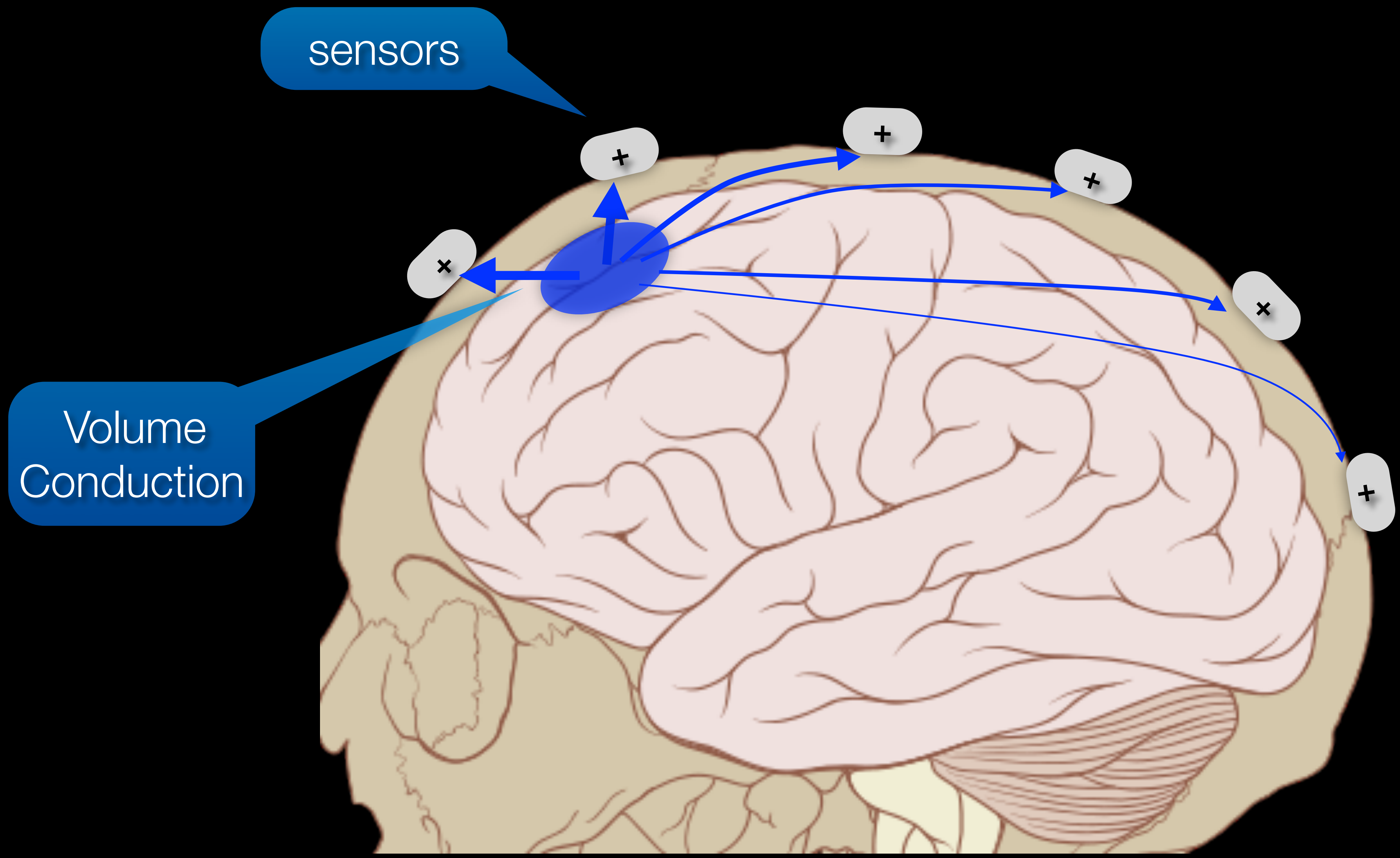
or



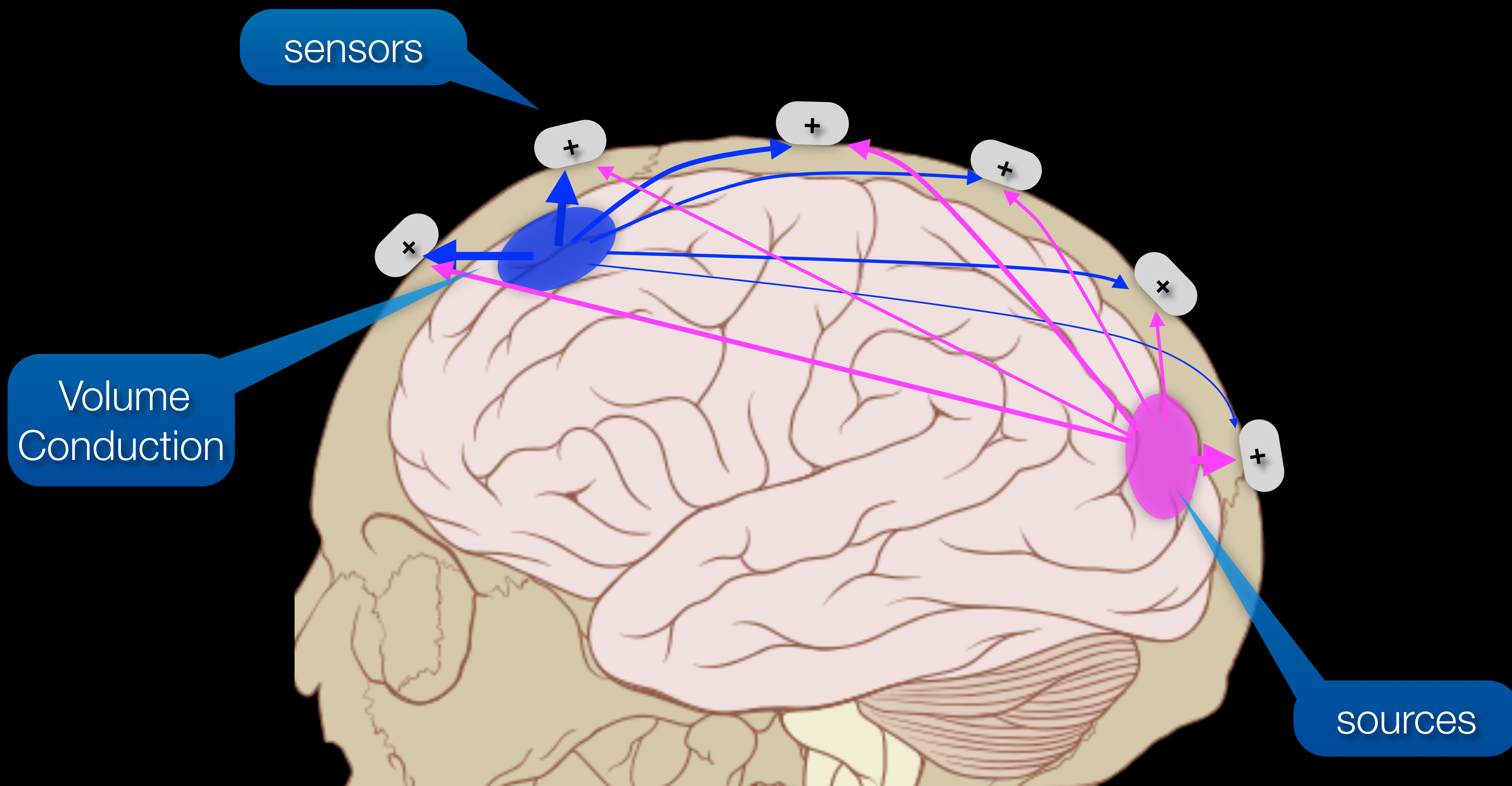
Channel or Source?



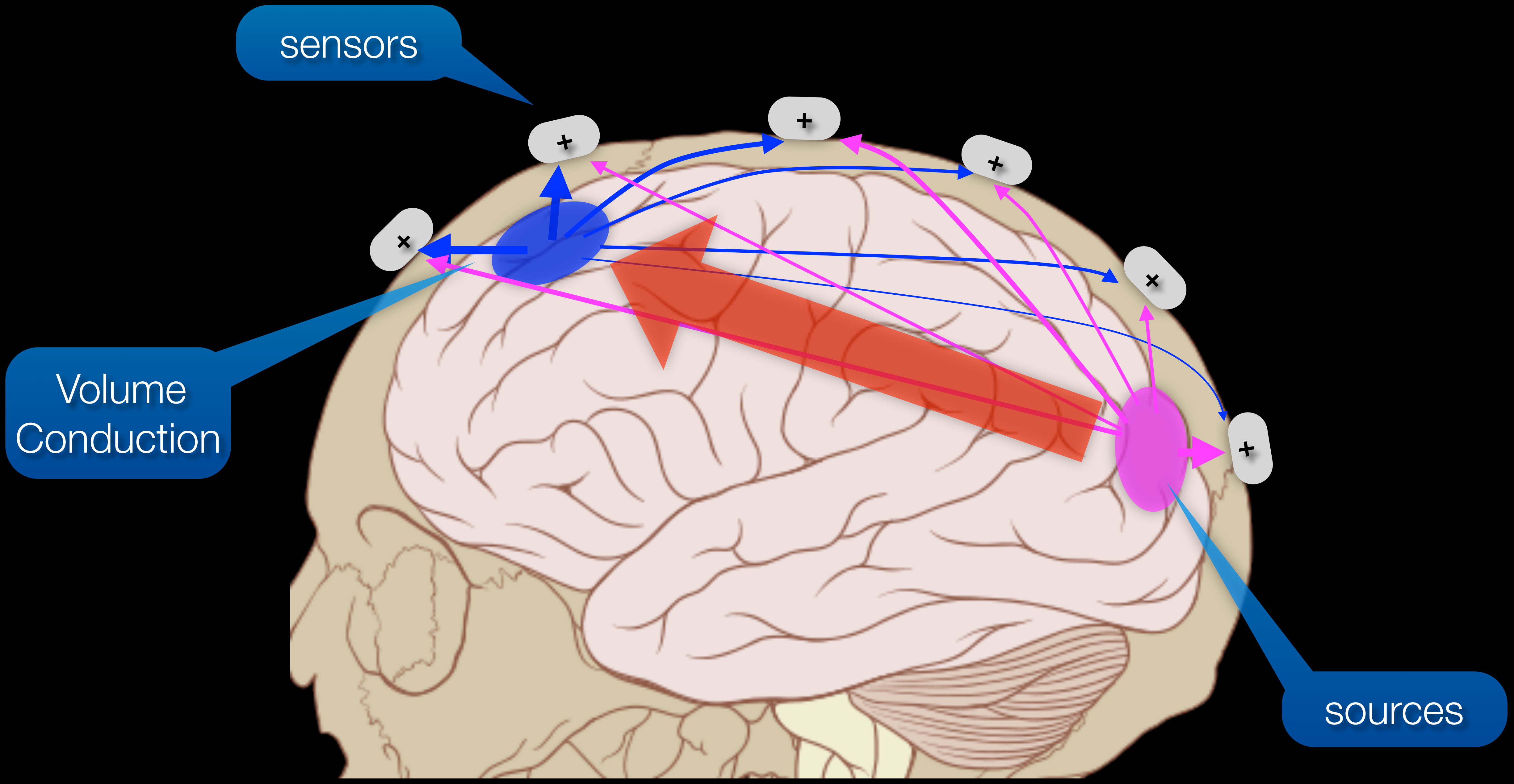
Channel or Source?



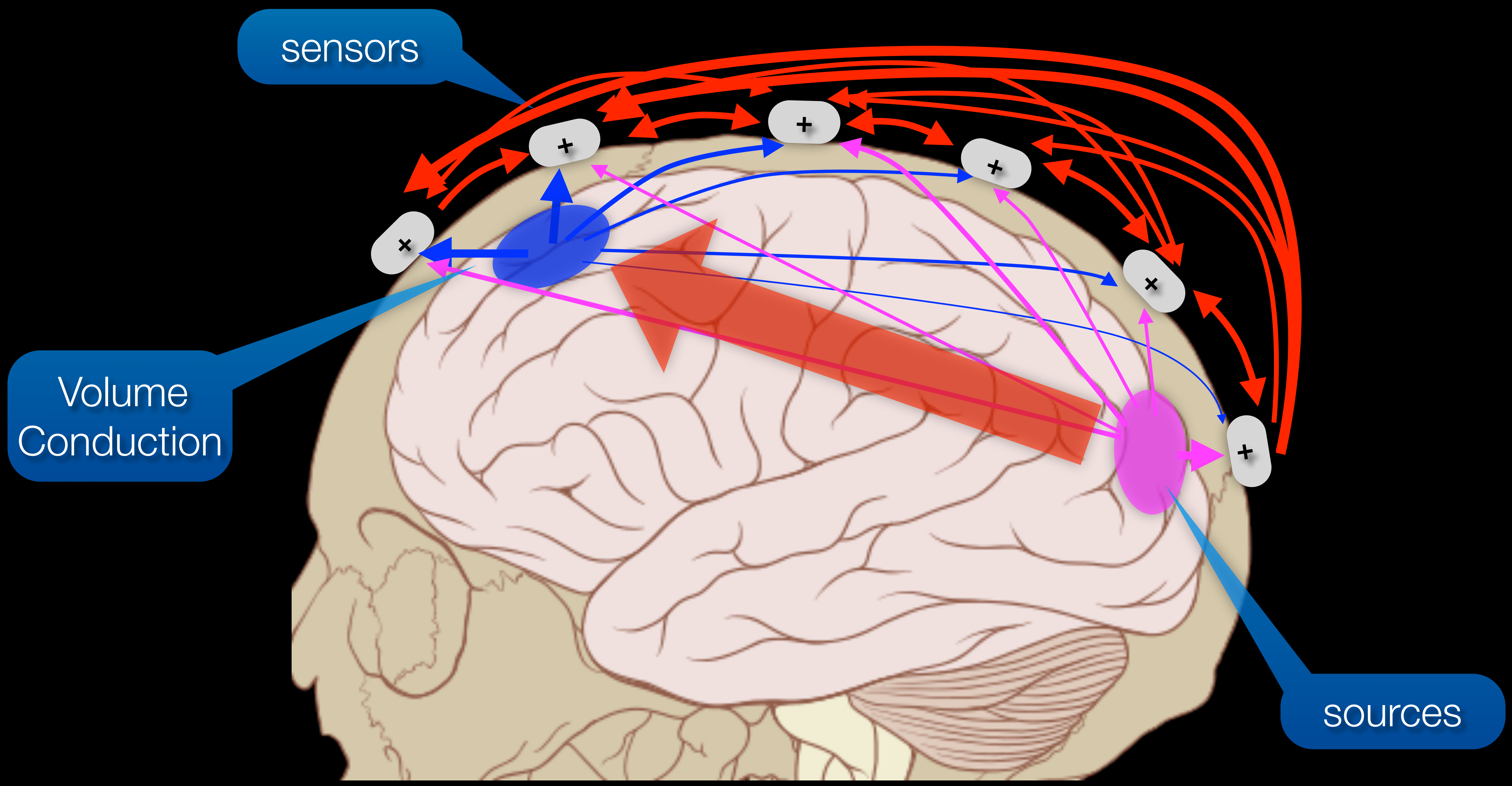
Channel or Source?



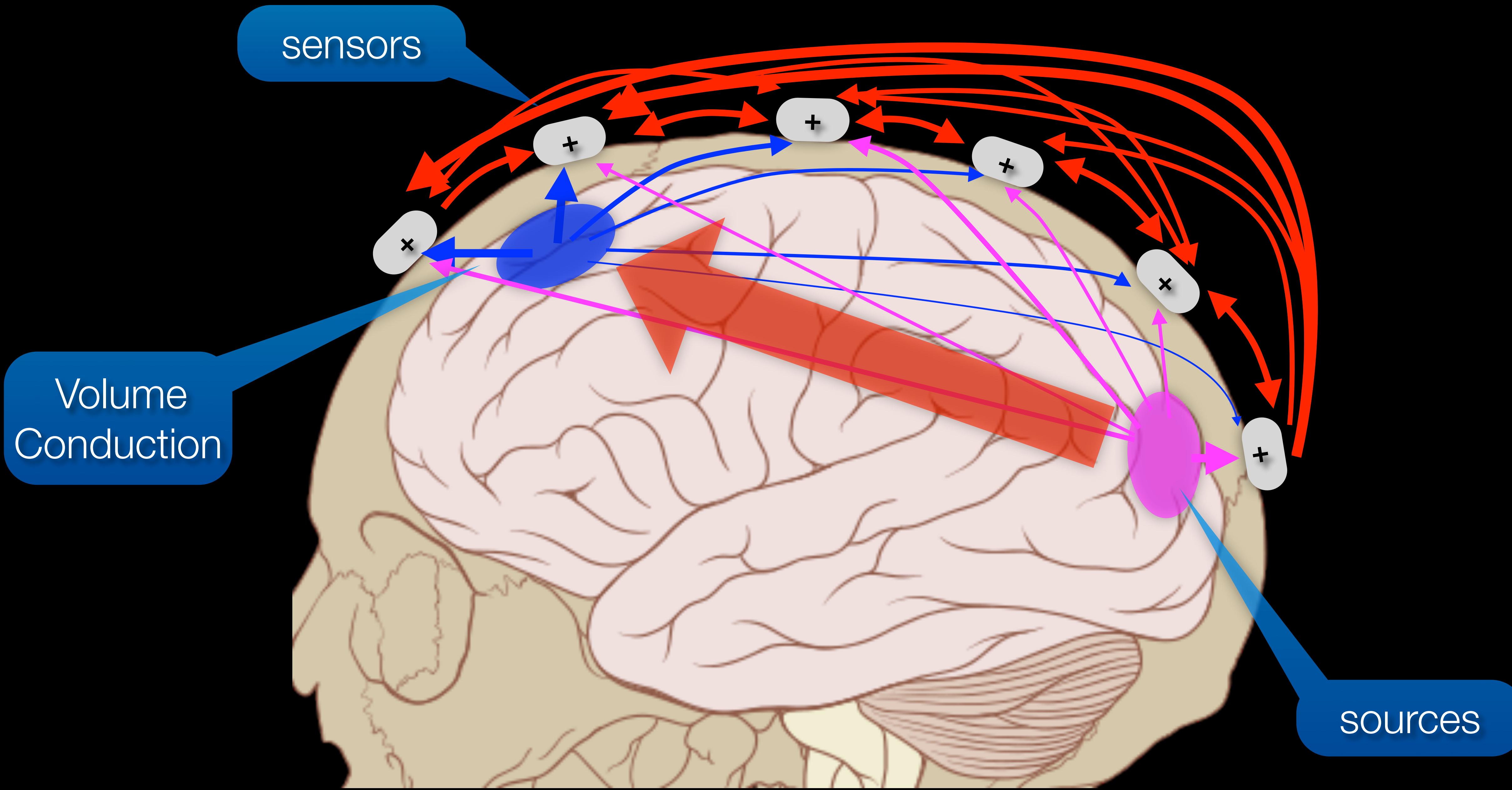
Channel or Source?



Channel or Source?

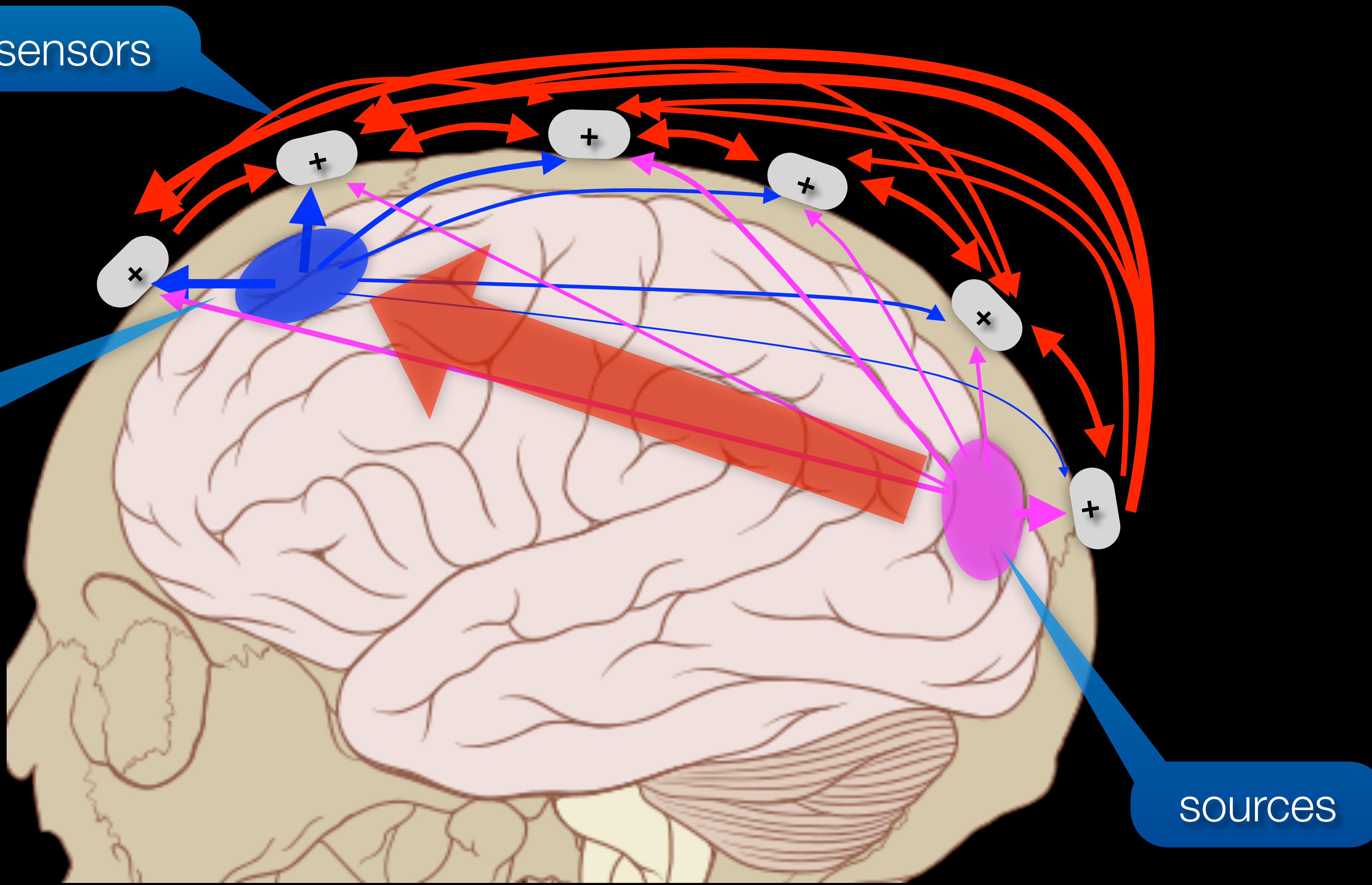


Channel or Source?



$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Channel or Source?

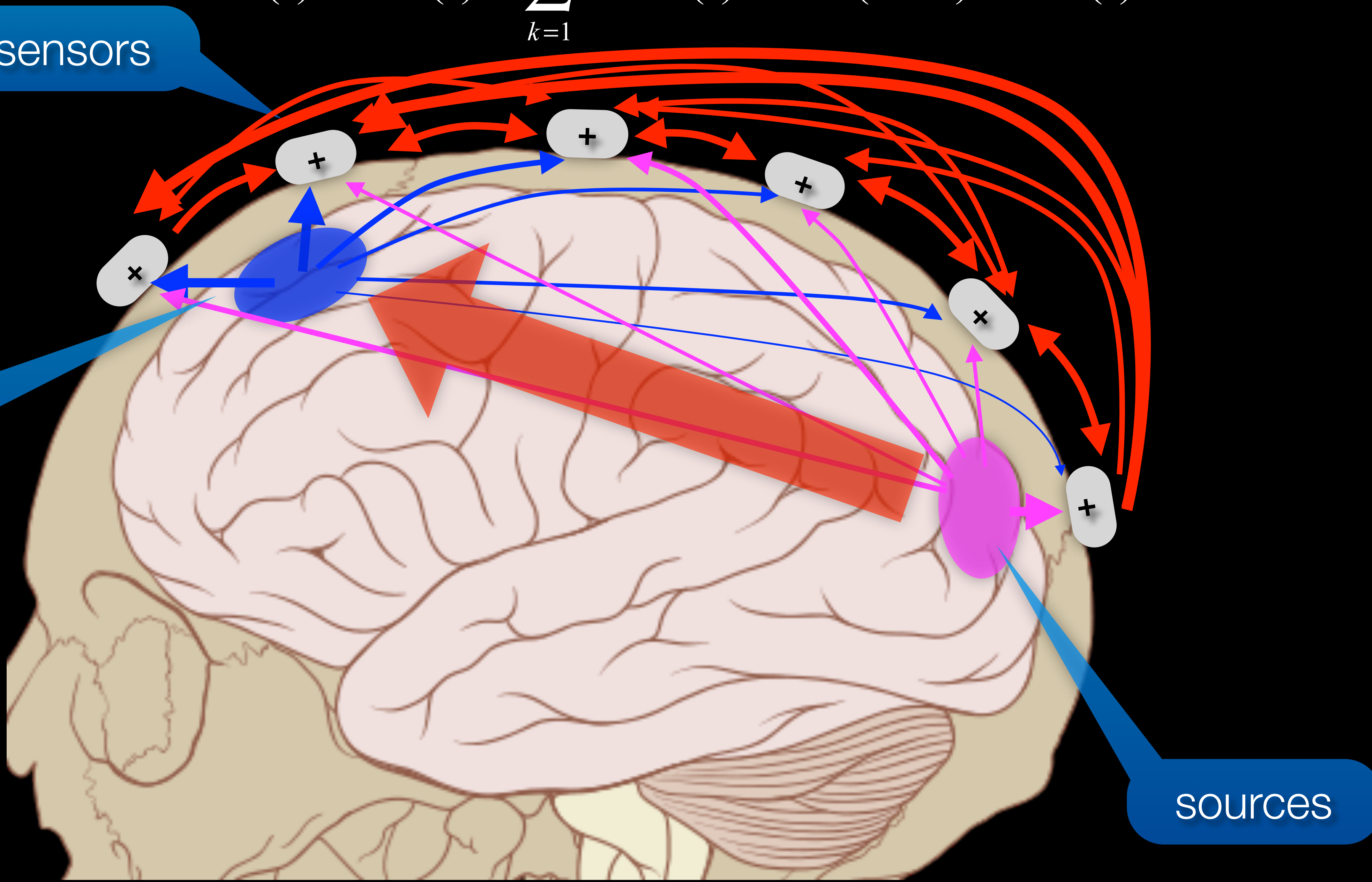


$$X(t) = MS(t)$$

$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Channel or Source?

$$X(t) = MS(t) = \sum_{k=1}^{p} MA^{(k)}(t) M^{-1} X(t-k) + ME(t)$$

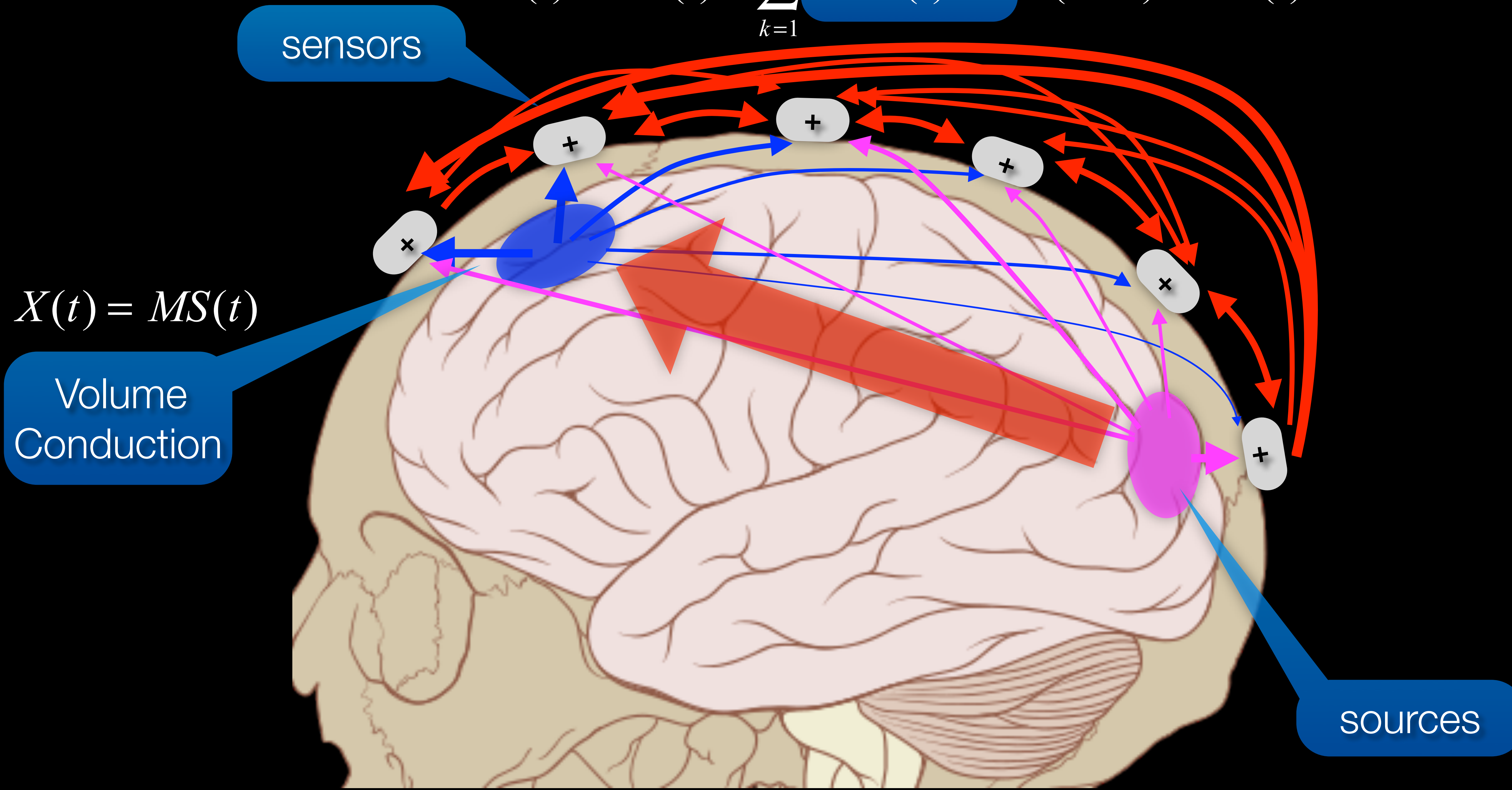


$$X(t) = MS(t)$$

$$S(t) = \sum_{k=1}^{p} A^{(k)}(t) S(t-k) + E(t)$$

Channel or Source?

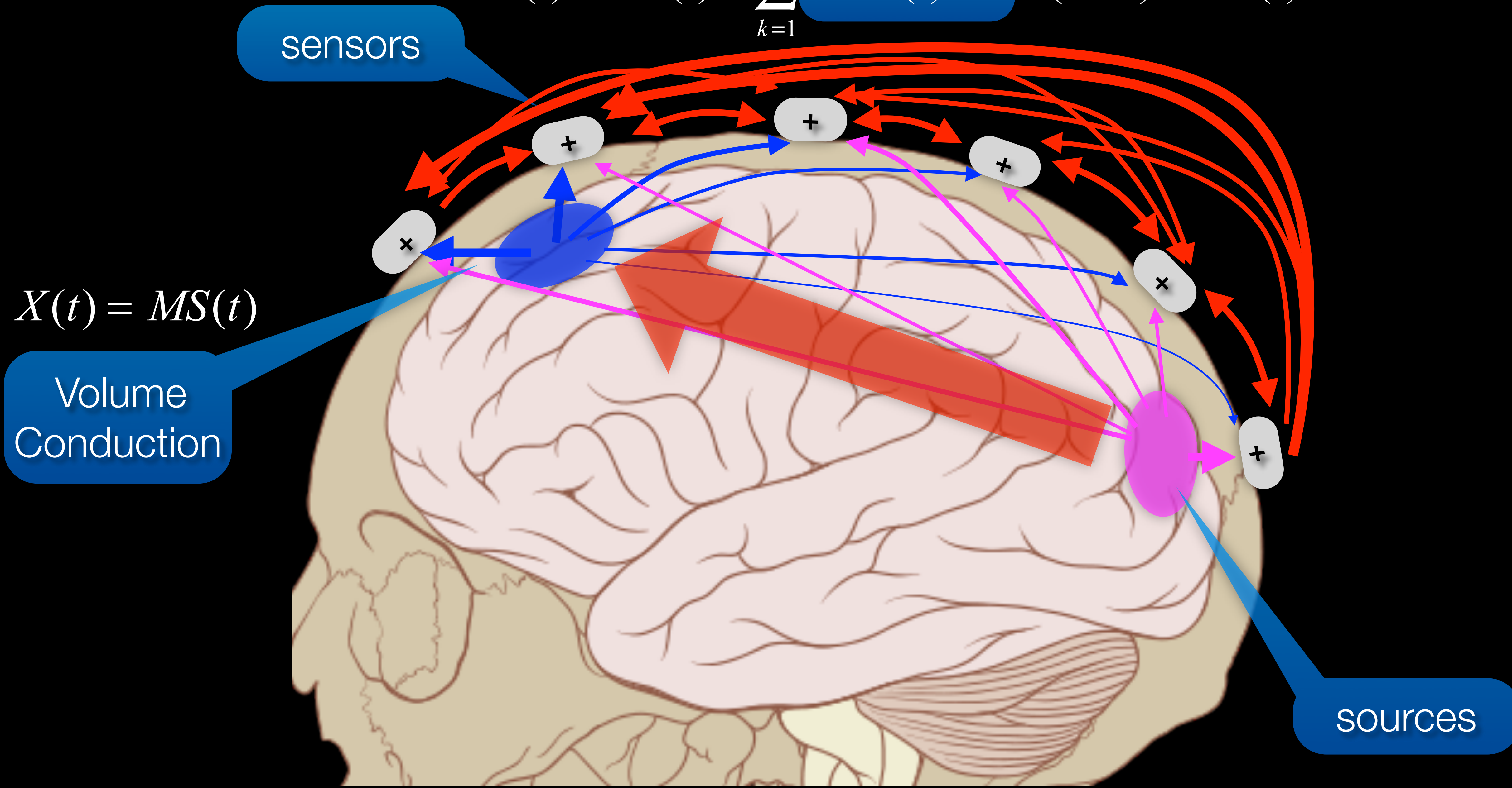
$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$



$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Channel or Source?

$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$

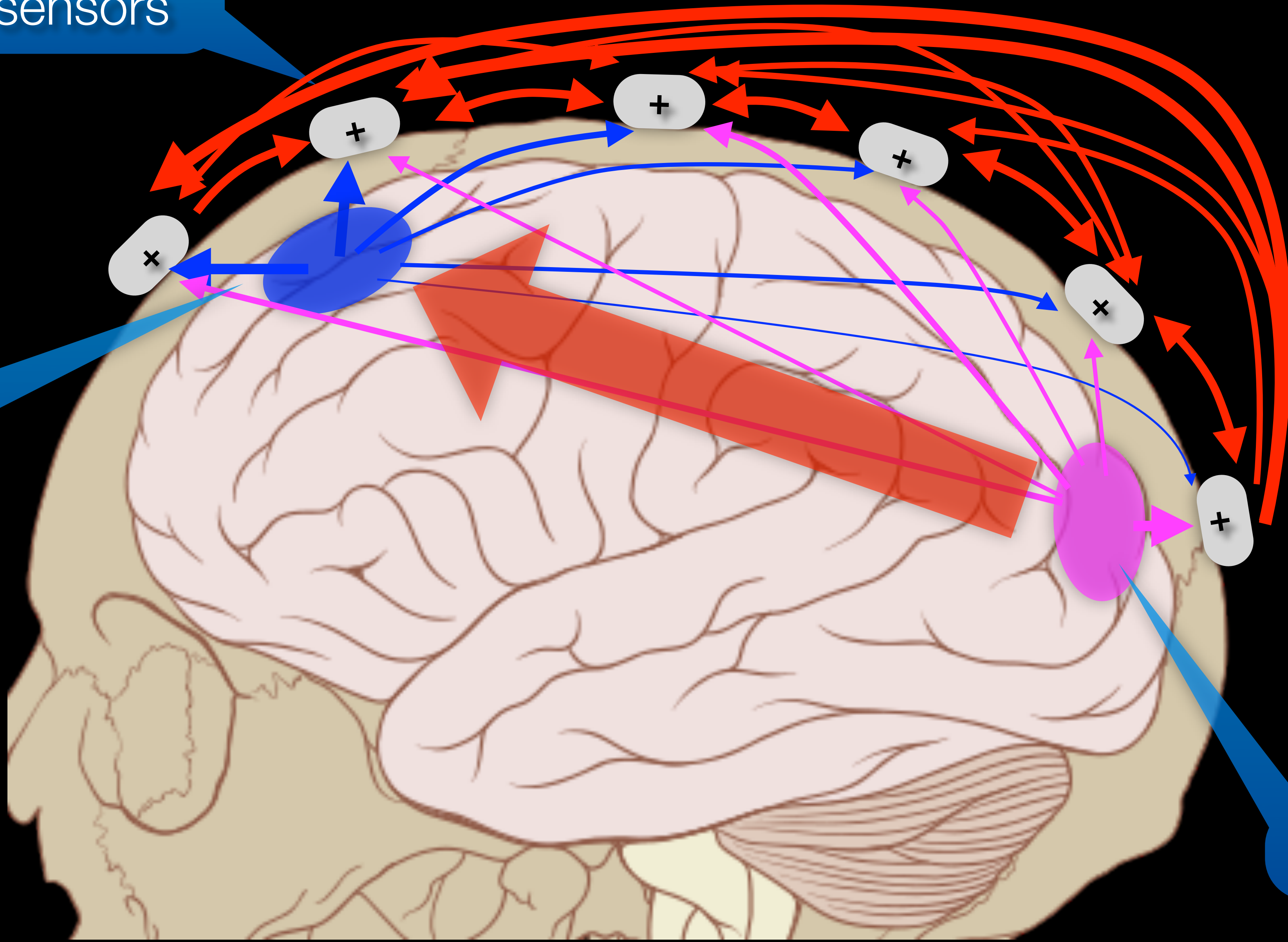


Solution? Source Separation

$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Channel or Source?

$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$



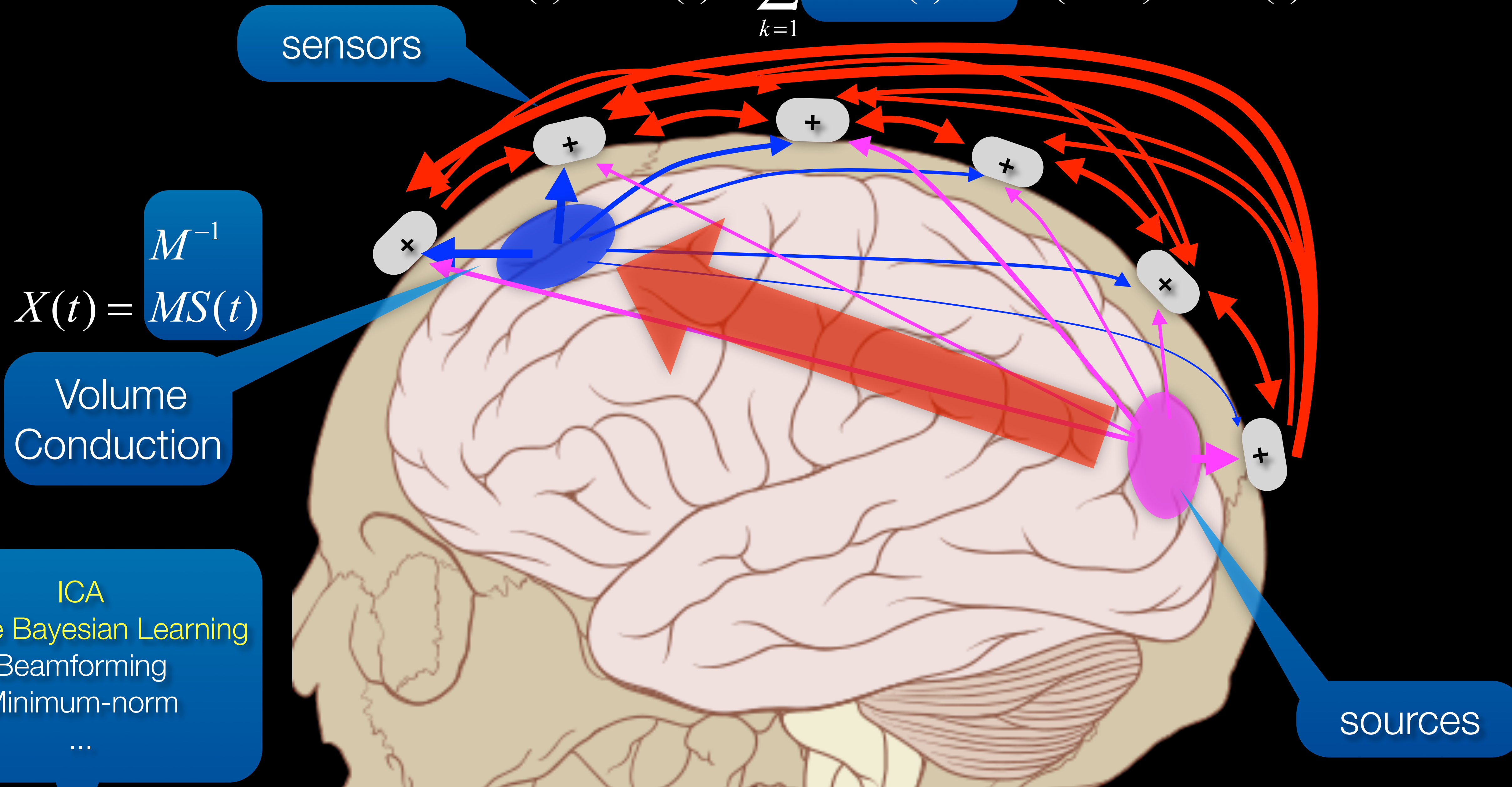
ICA
Sparse Bayesian Learning
Beamforming
Minimum-norm
...

Solution? Source Separation

$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Channel or Source?

$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$



$$X(t) = \boxed{M^{-1} MS(t)}$$

ICA
Sparse Bayesian Learning
Beamforming
Minimum-norm
...

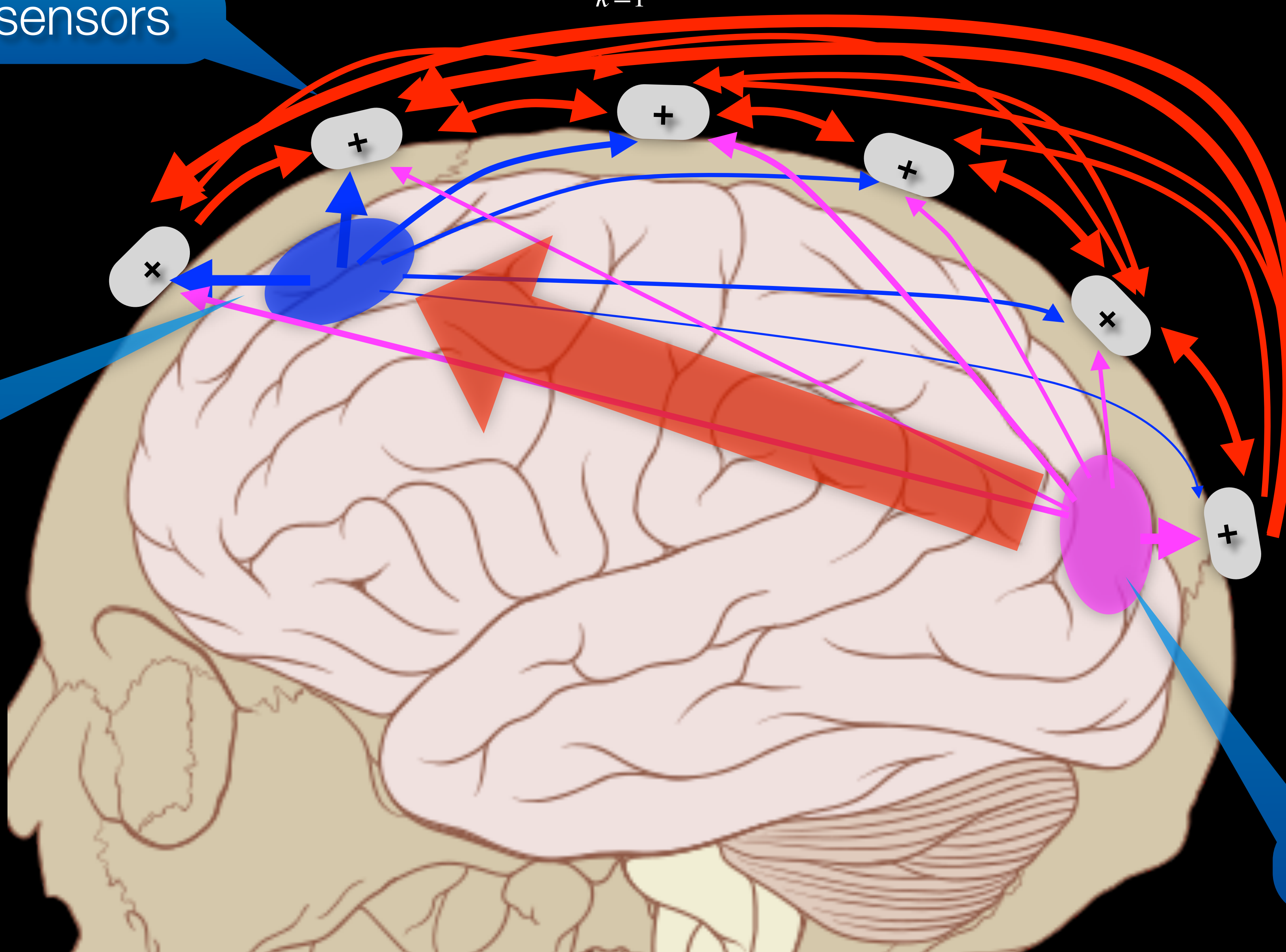
Solution? Source Separation

$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Channel or Source?

$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$

$$X(t) = M^{-1}MS(t)$$



ICA
Sparse Bayesian Learning
Beamforming
Minimum-norm
...

Solution? Source Separation

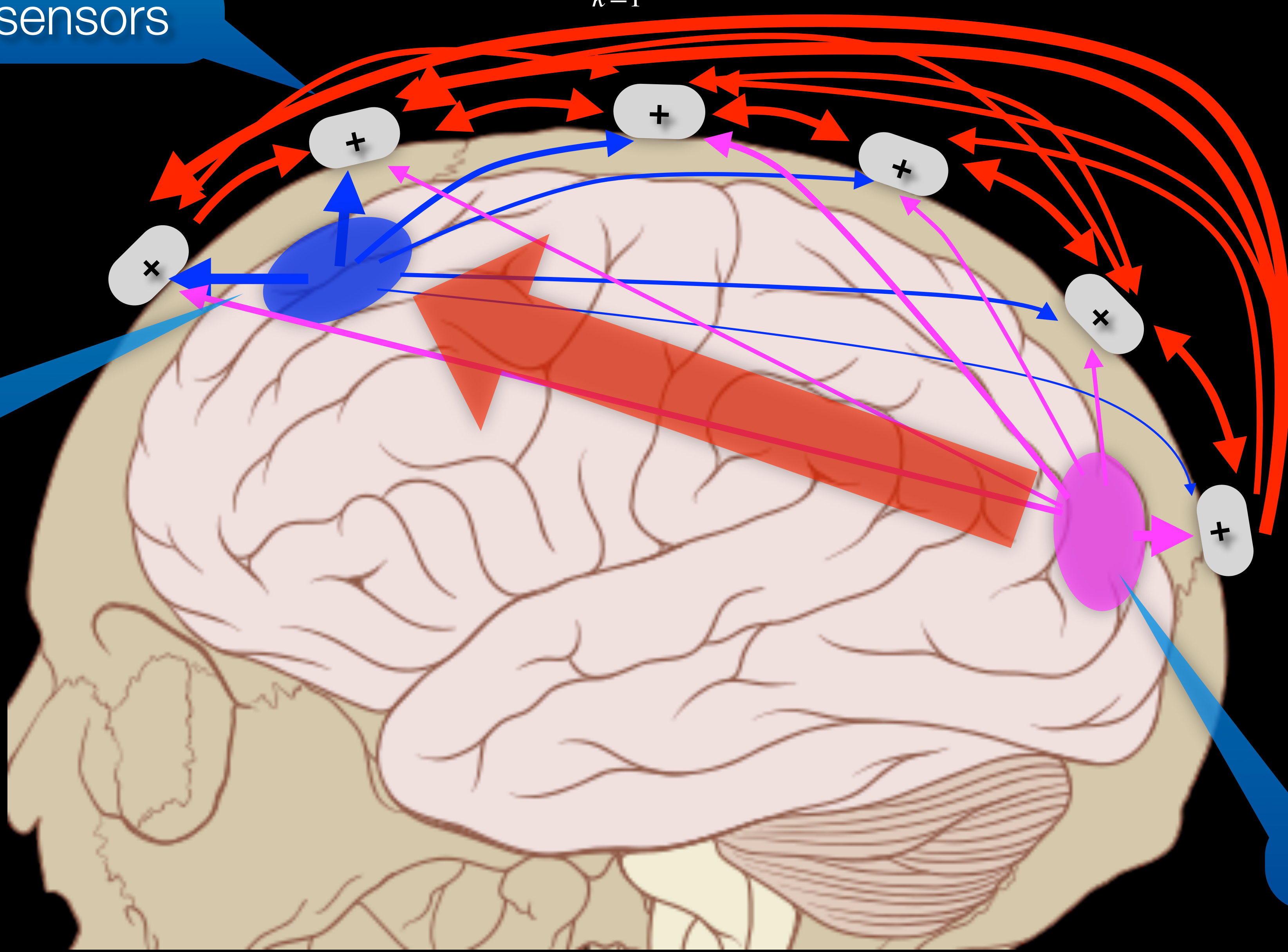
$$S(t) = \sum_{k=1}^{p} A^{(k)}(t)S(t-k) + E(t)$$

Saturday, September 24, 2011

Channel or Source?

$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$

$$X(t) = \boxed{M^{-1}} \; MS(t)$$



ICA
Sparse Bayesian Learning
Beamforming
Minimum-norm
...

Solution? Source Separation

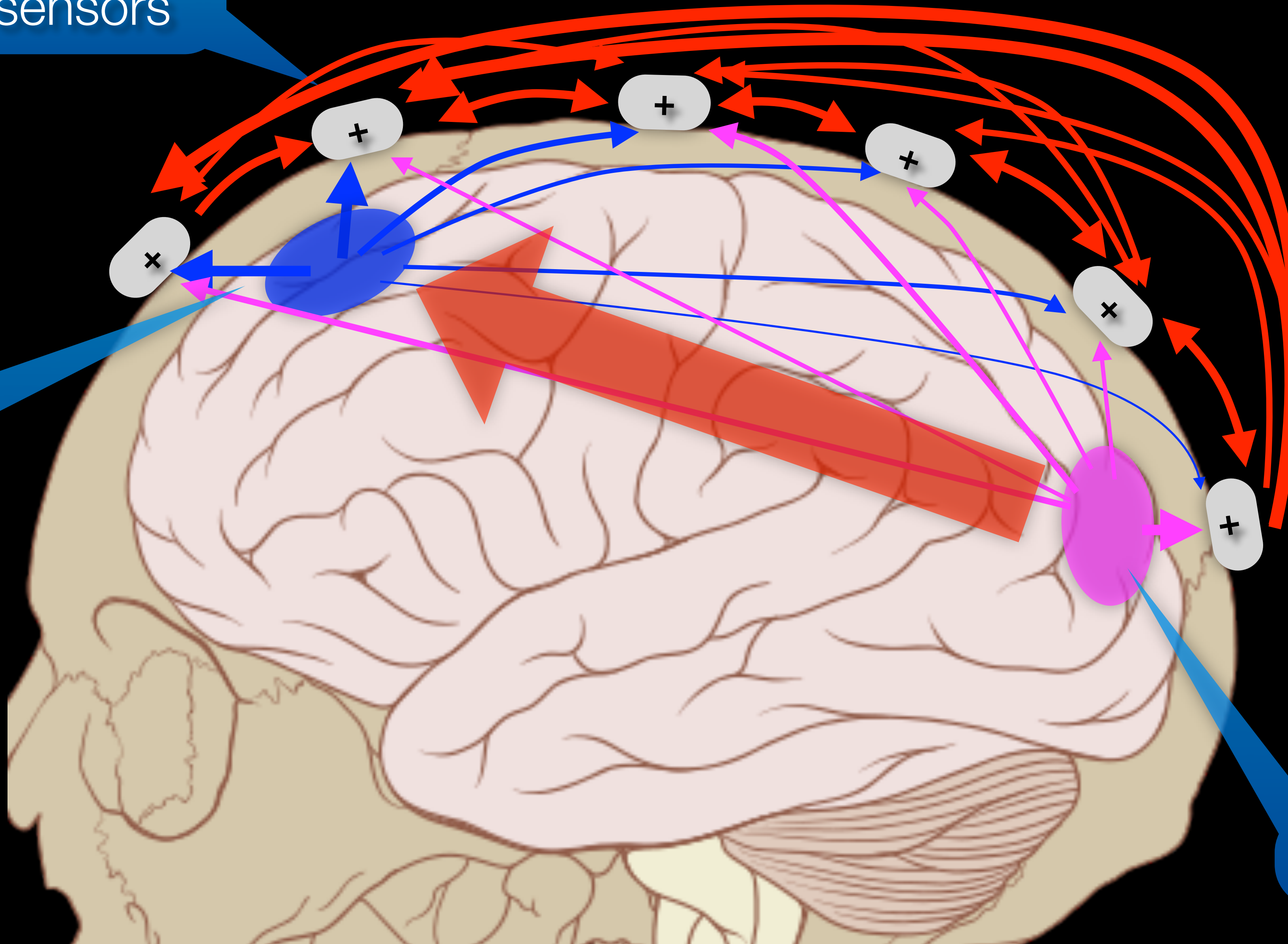
$$S(t) = \sum_{k=1}^{p} \boxed{A^{(k)}(t)} S(t-k) + E(t)$$

Channel or Source?

Volume conduction exists for ECoG too!

$$X(t) = MS(t) = \sum_{k=1}^{p} \boxed{MA^{(k)}(t)M^{-1}} X(t-k) + ME(t)$$

sensors



$$X(t) = \boxed{M^{-1}} \\ \boxed{MS(t)}$$

Volume Conduction

sources

ICA
Sparse Bayesian Learning
Beamforming
Minimum-norm
...

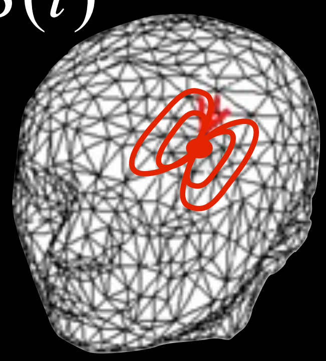
Solution? Source Separation

$$S(t) = \sum_{k=1}^{p} \boxed{A^{(k)}(t)} S(t-k) + E(t)$$

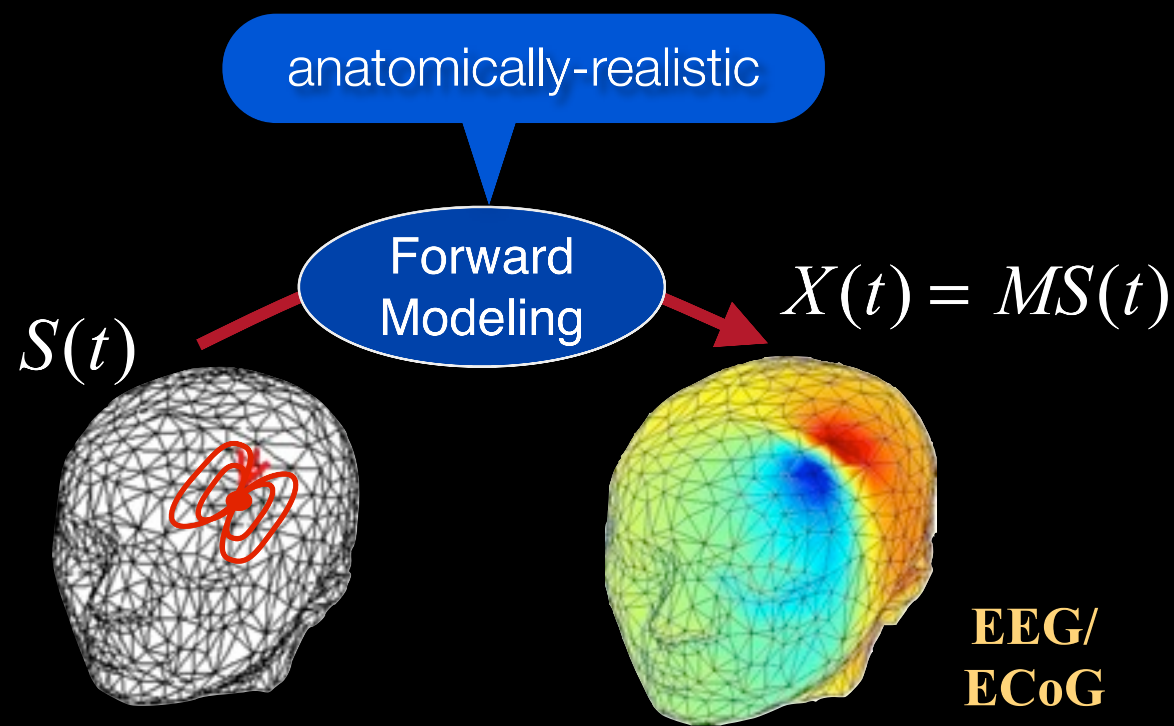
Forward/Inverse Modeling

$S(t)$

Akalin Acar

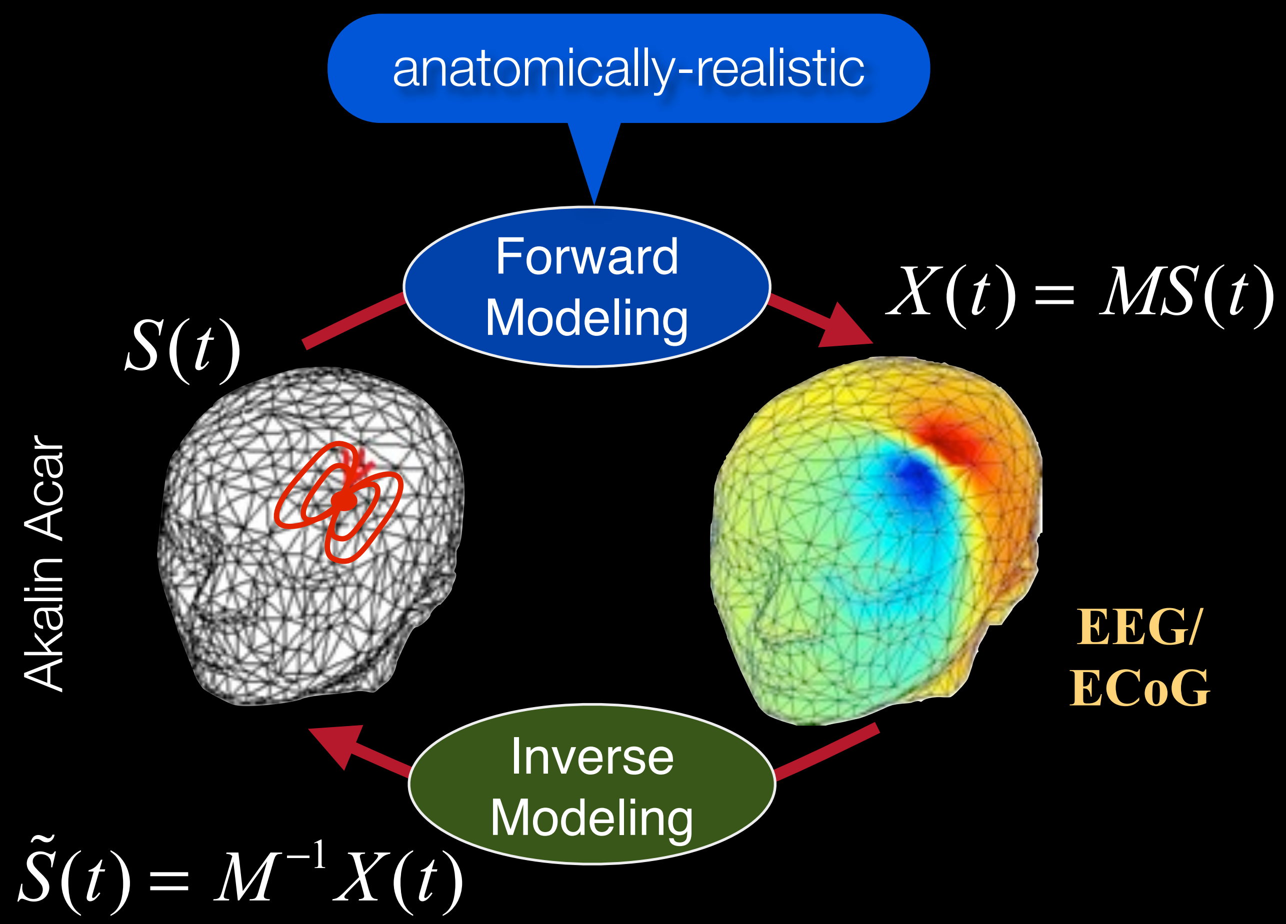


Forward/Inverse Modeling

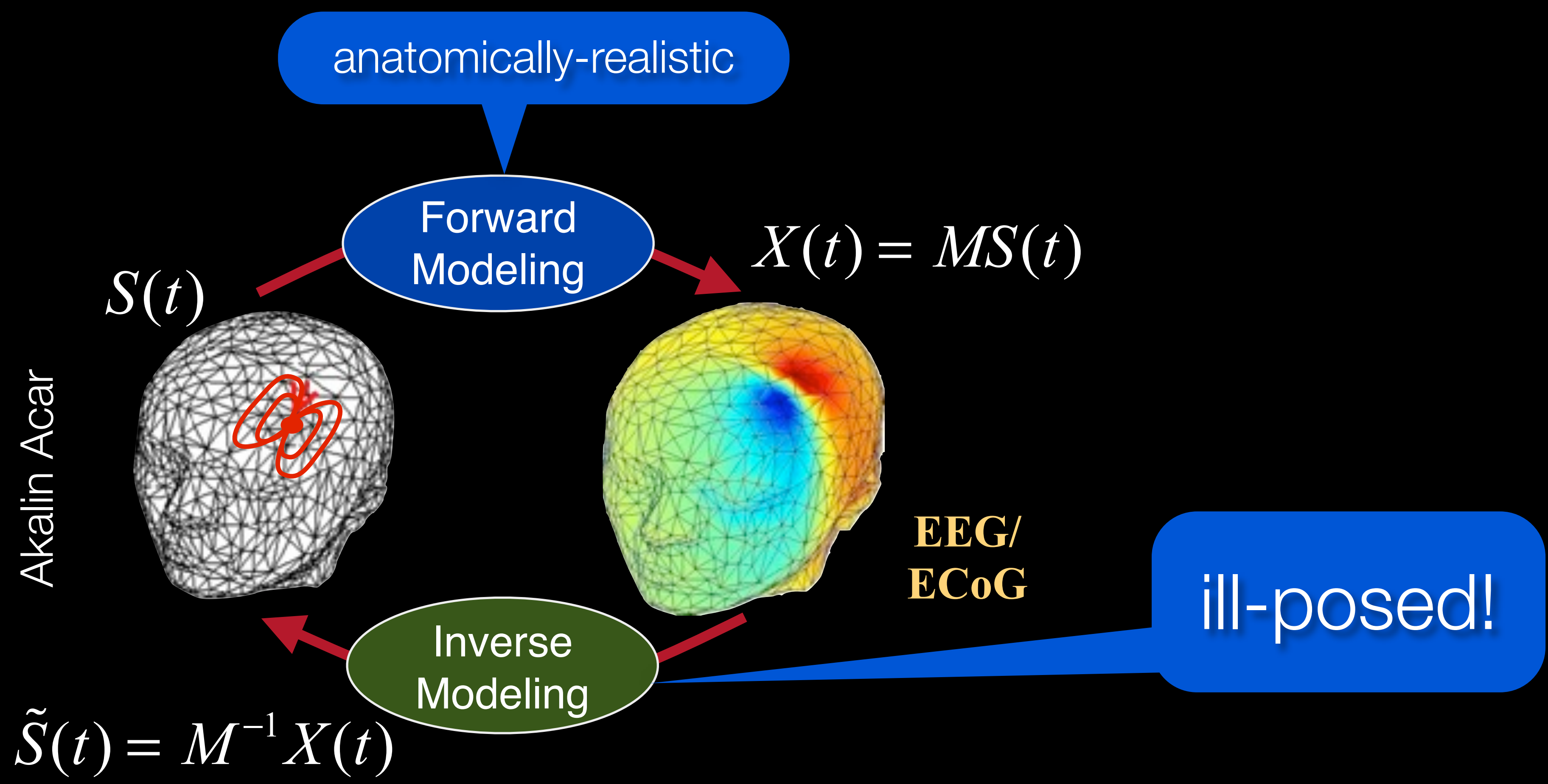


Akalin Acar

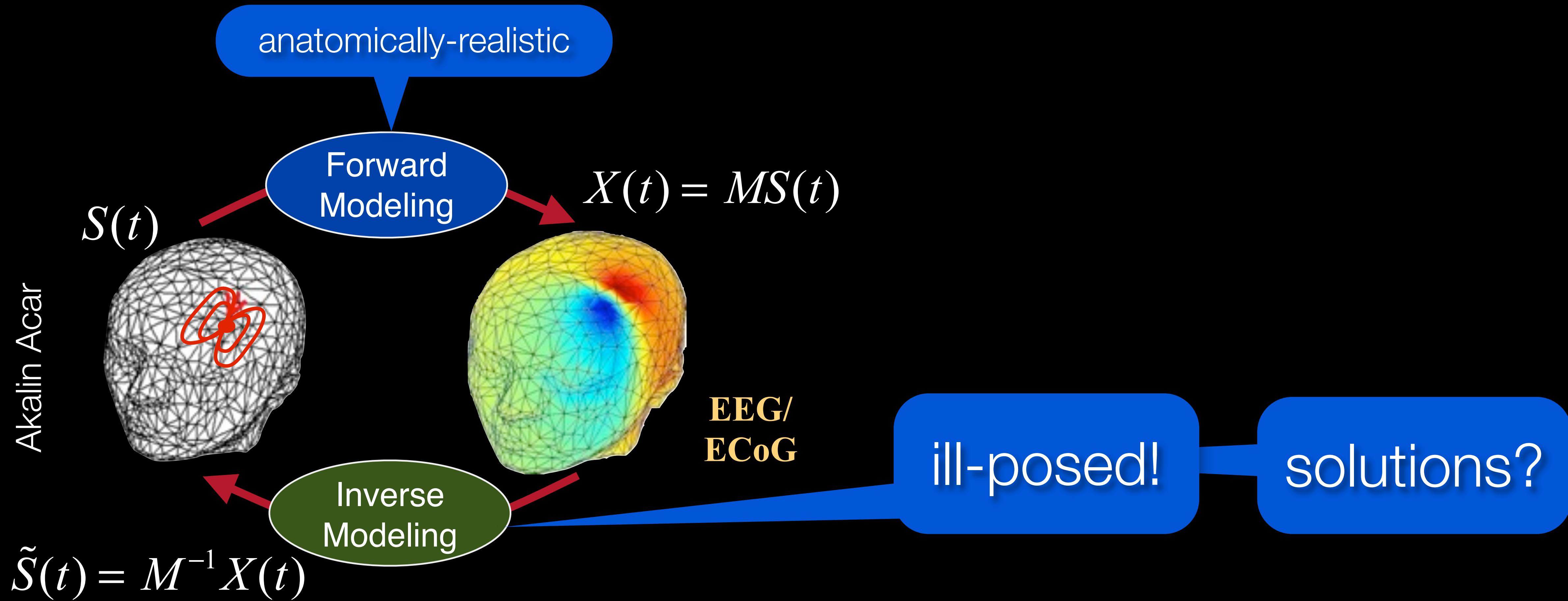
Forward/Inverse Modeling



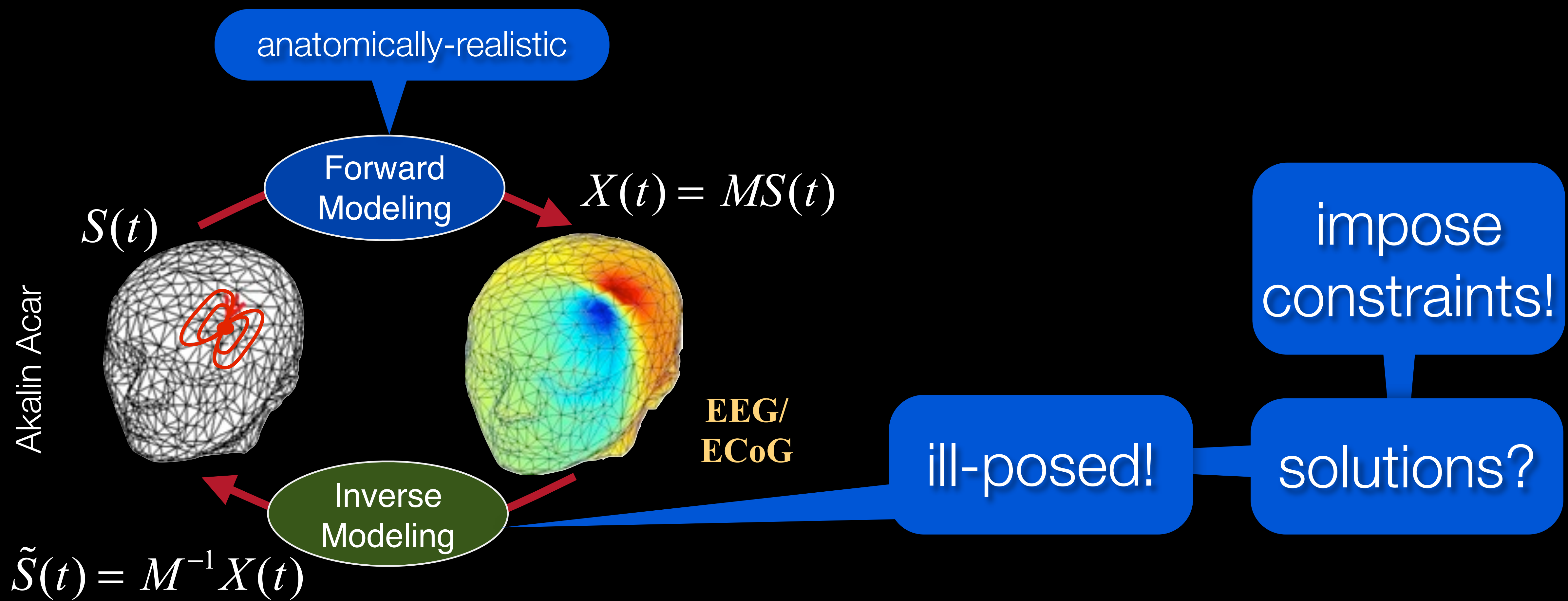
Forward/Inverse Modeling



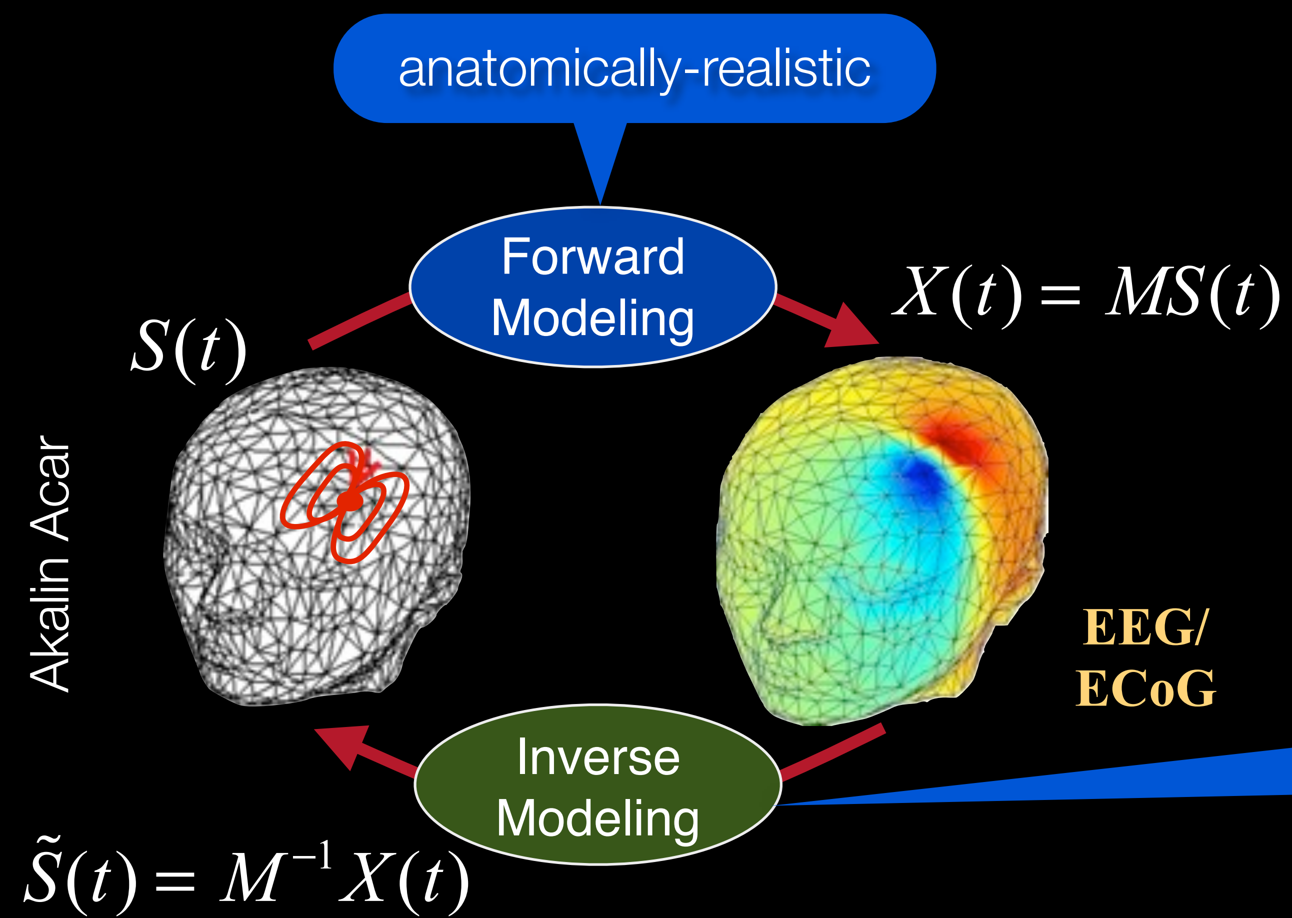
Forward/Inverse Modeling



Forward/Inverse Modeling



Forward/Inverse Modeling



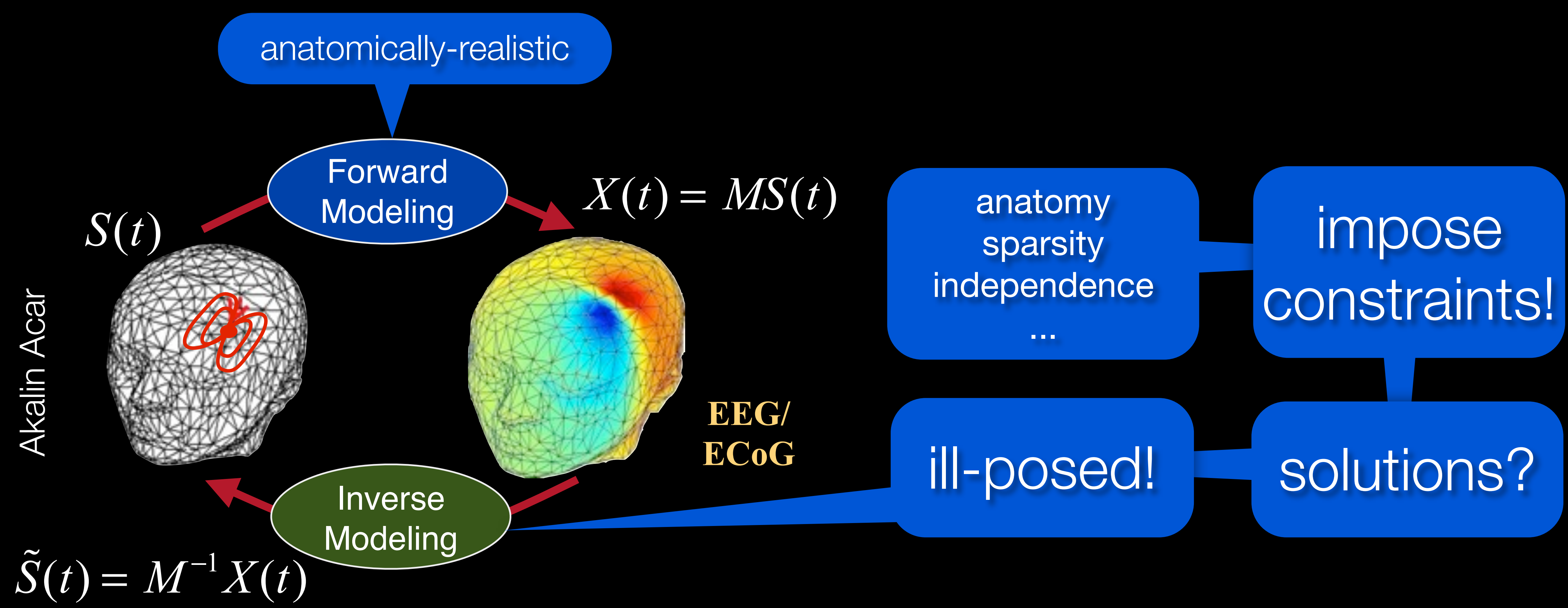
anatomy
sparsity
independence
...

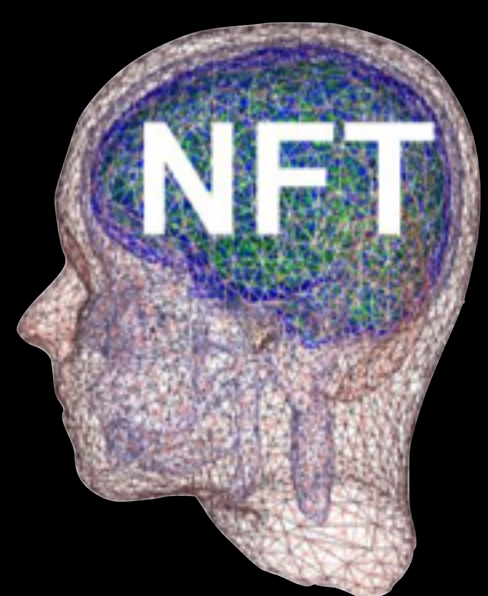
impose constraints!

ill-posed!

solutions?

Forward/Inverse Modeling





A Recipe for Reducing Errors:

- Anatomically Realistic Forward Model
- Appropriately Constrained Inverse Model

Akalin Acar and Makeig, 2010

Estimating Dependency of Independent Components ?

Estimating Dependency of Independent Components ?

- Isn’t it a contradiction to examine dependence between Independent Components?

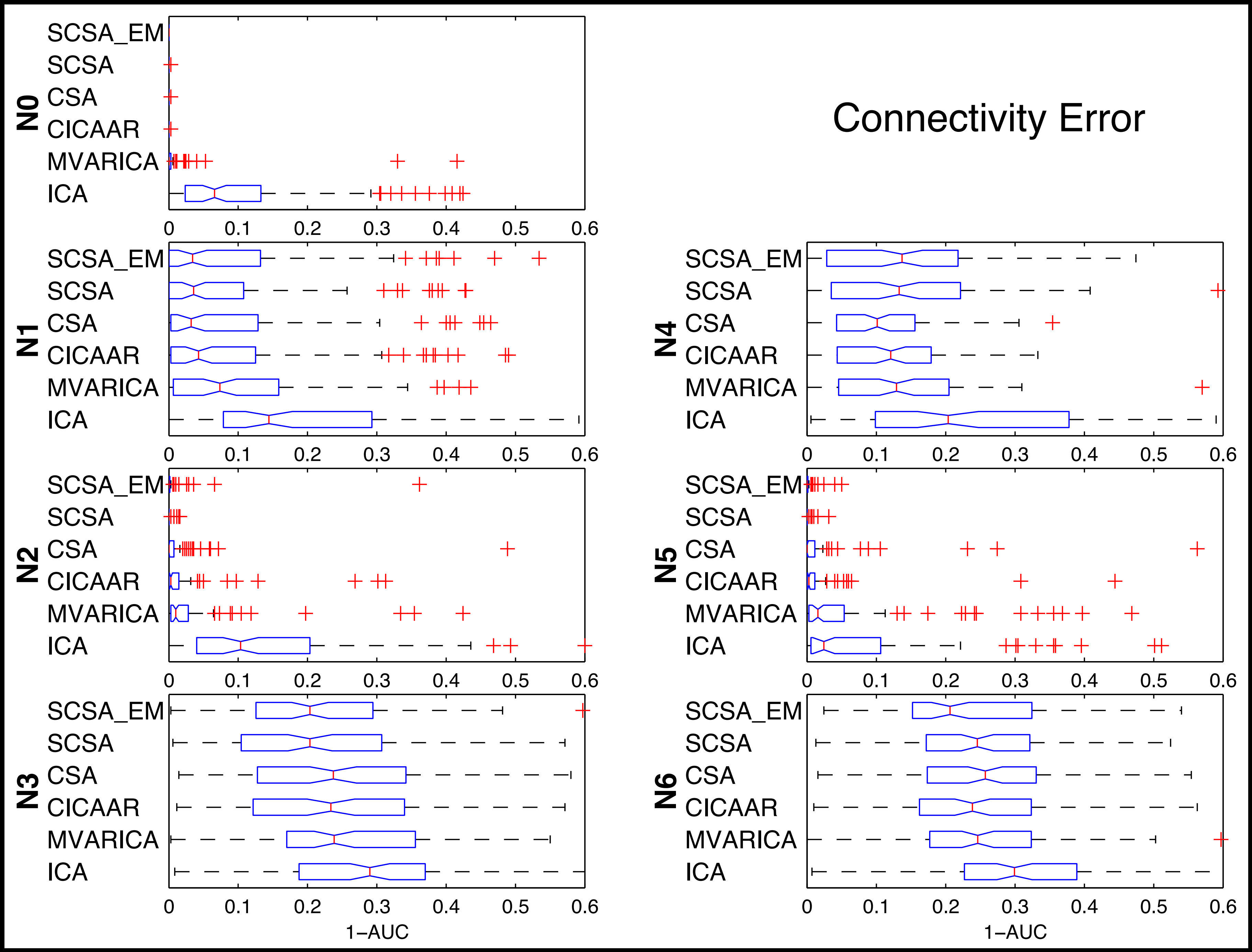
Estimating Dependency of Independent Components ?

- Isn’t it a contradiction to examine dependence between Independent Components?
- Instantaneous (e.g., Infomax) ICA only explicitly enforces *instantaneous* independence. Time-delayed dependencies may be preserved

Estimating Dependency of Independent Components ?

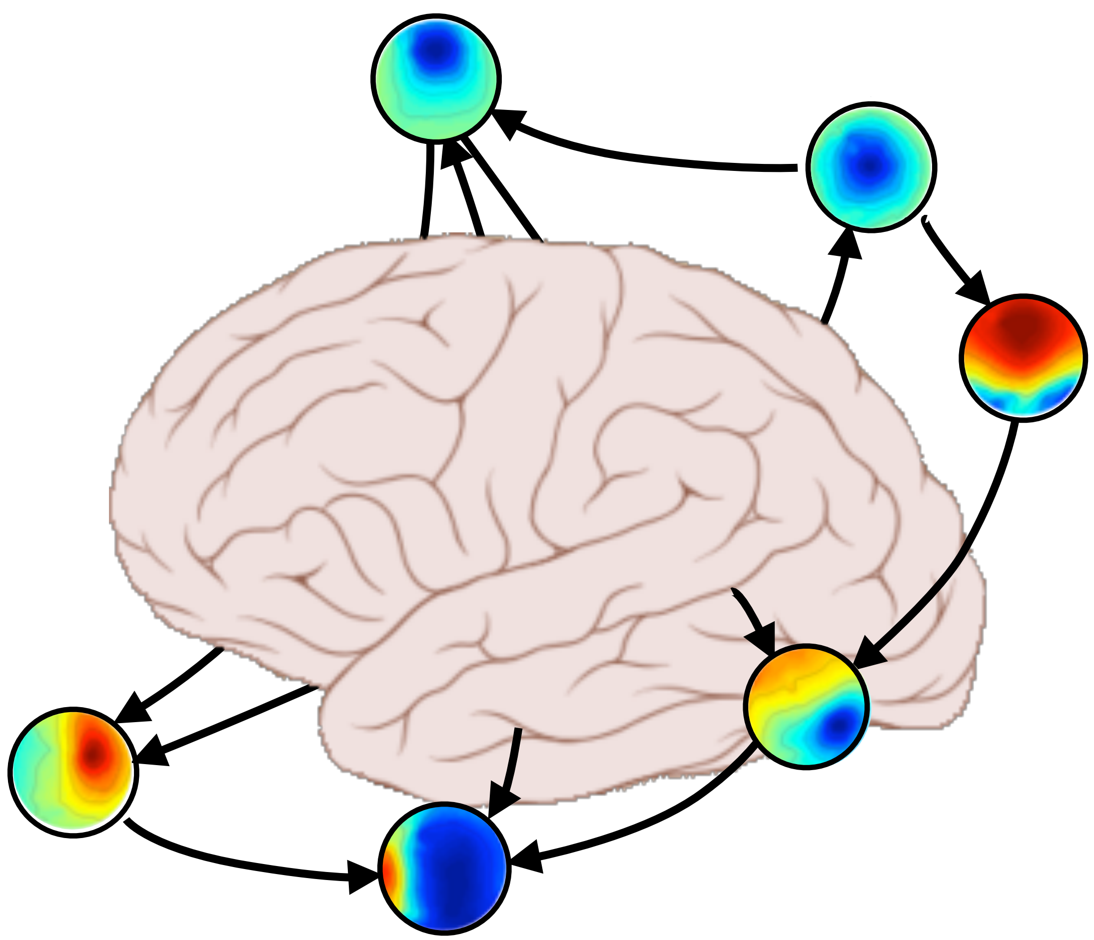
- Isn't it a contradiction to examine dependence between Independent Components?
- Instantaneous (e.g., Infomax) ICA only explicitly enforces *instantaneous* independence. Time-delayed dependencies may be preserved
- ICA seeks to maximize *global* independence (over entire recording session), transient dependencies are often preserved

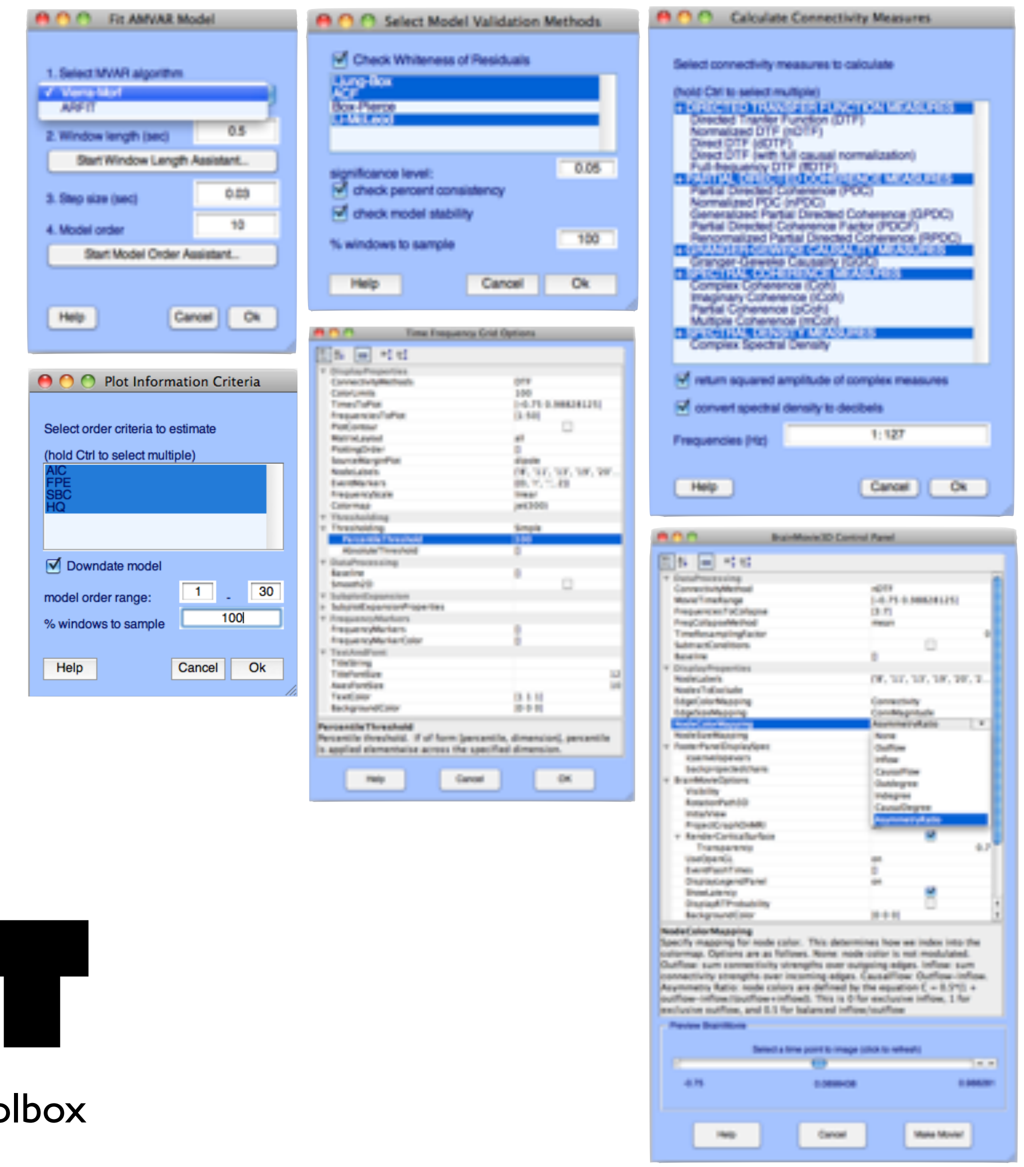
Estimating Dependency of Independent Components ?



Haufe et al, 2008

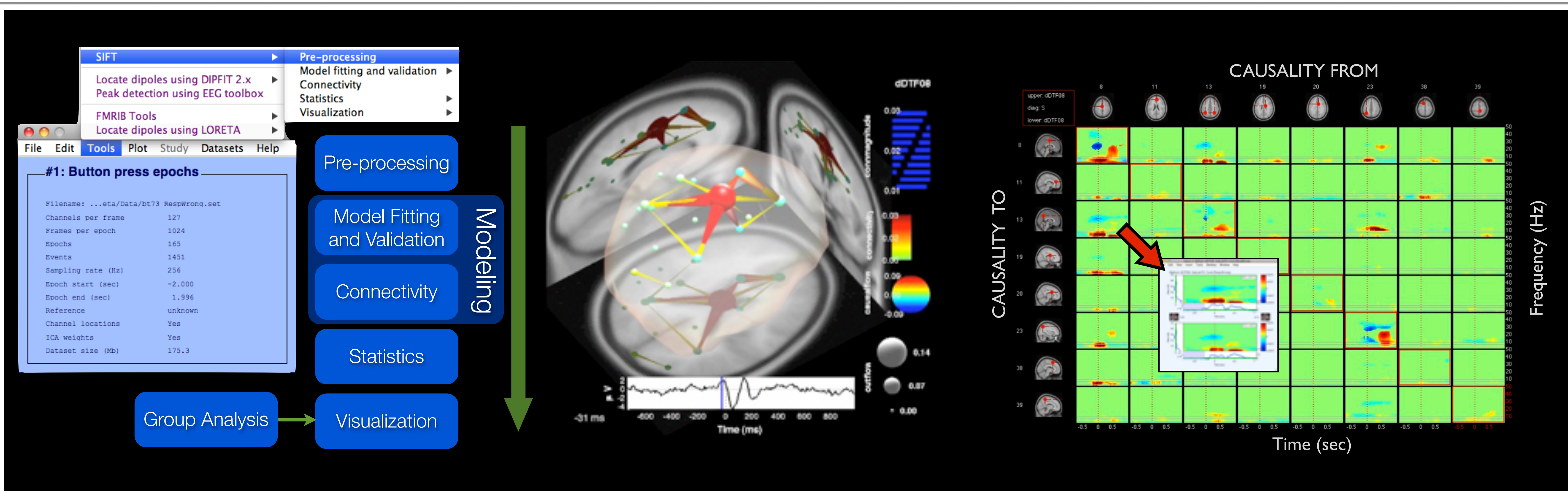
http://sccn.ucsd.edu/wiki/SIFT
Mullen, et al, *Journal of Neuroscience Methods* (in prep, 2011)
Mullen, Delorme, Kothe, Makeig, *Society for Neuroscience*, 2010
Delorme, Mullen, Kothe et al, *Computational Intelligence and Neuroscience*, vol 12, 2011



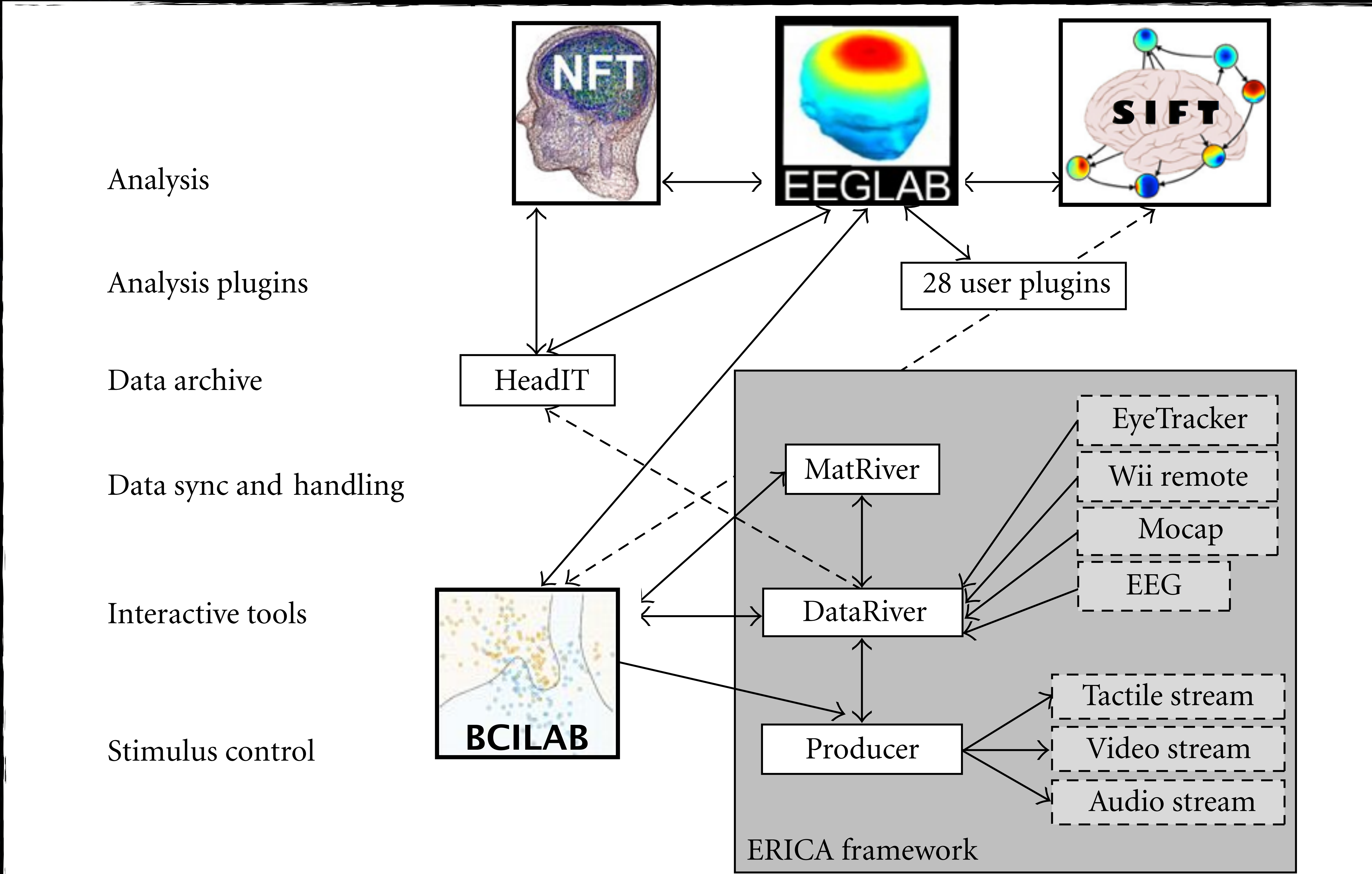


SIFT

Source Information Flow Toolbox



EEGLAB Software framework



Delorme, Mullen, Kothe, Akalin Acar, Bigdely-Shamlo, Vankov, Makeig, *Computational Intelligence and Neuroscience*, vol 12, 2011

Source Information Flow Toolbox (SIFT)

Source Information Flow Toolbox (SIFT)

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment

Source Information Flow Toolbox (SIFT)

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches

Source Information Flow Toolbox (SIFT)

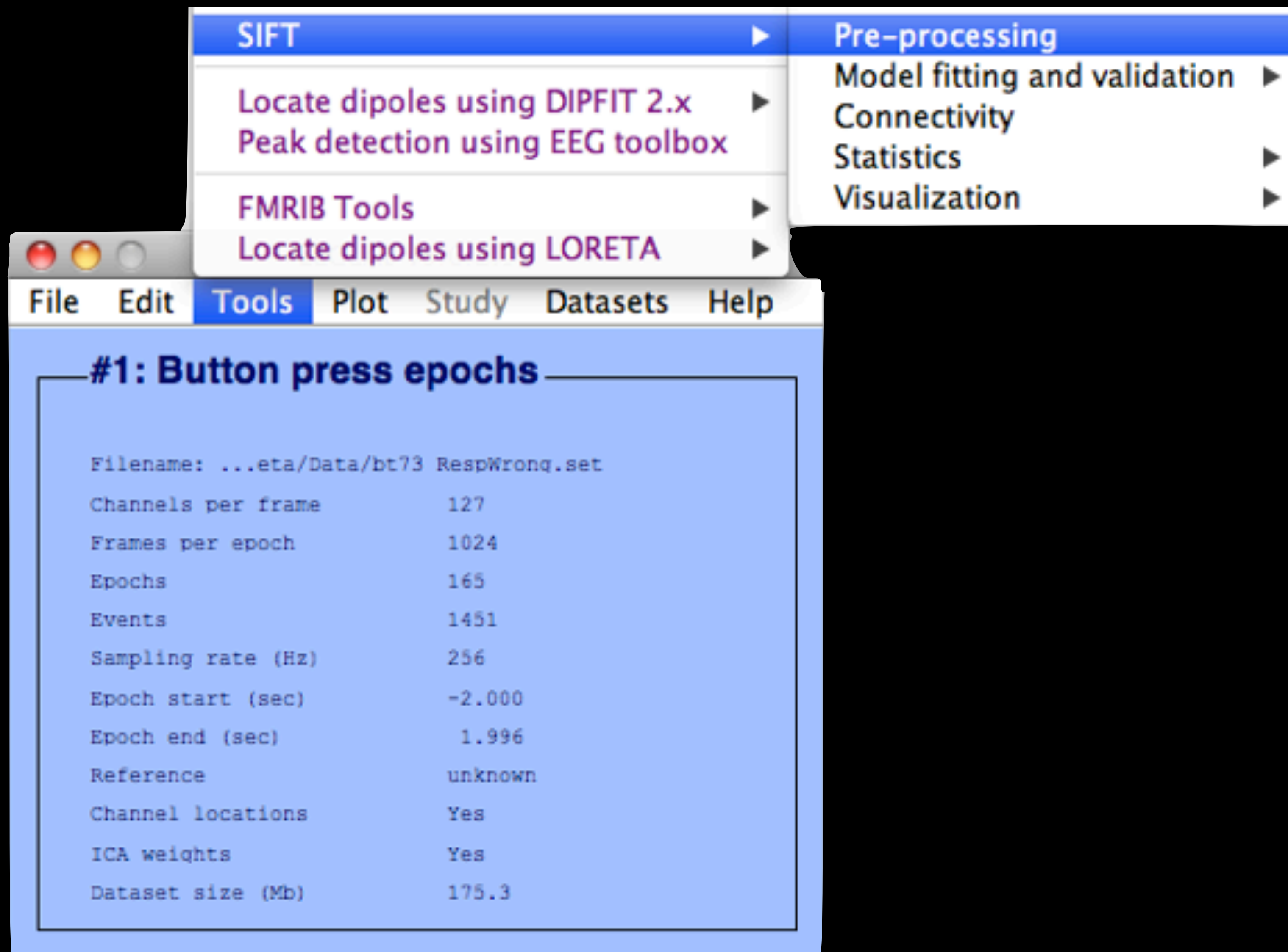
- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
- Emphasis on vector autoregression and time-frequency domain approaches

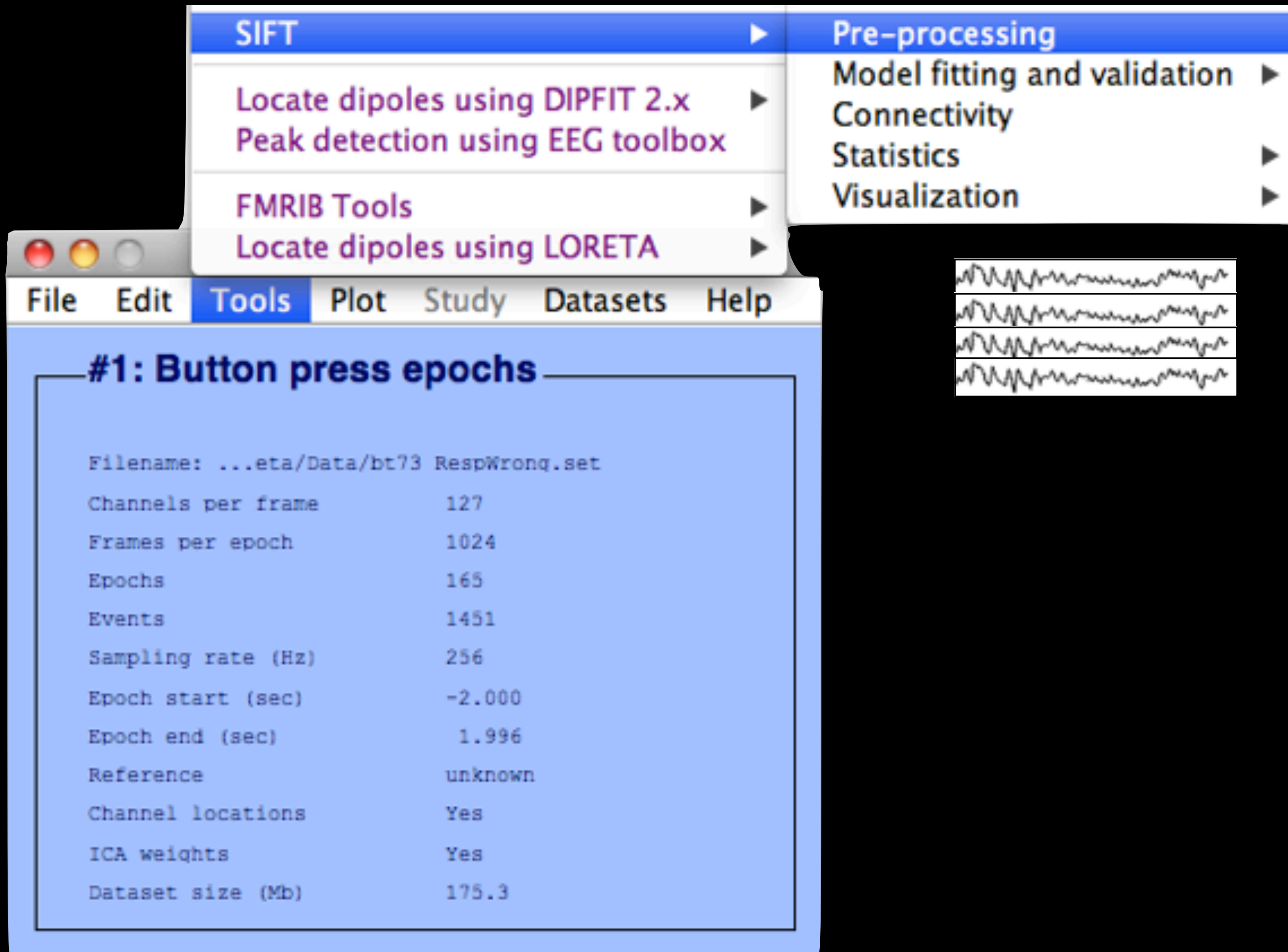
Source Information Flow Toolbox (SIFT)

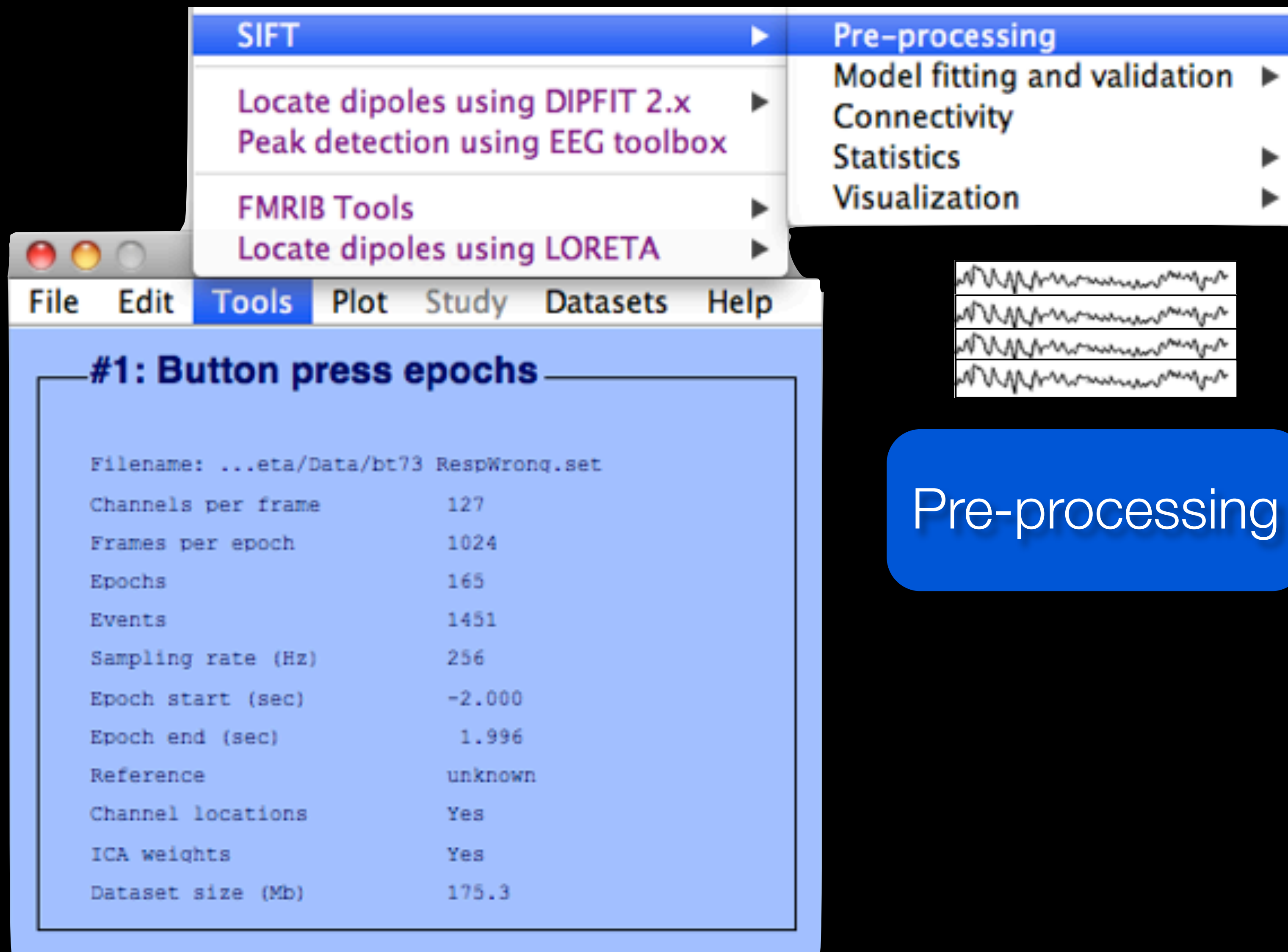
- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
- Emphasis on vector autoregression and time-frequency domain approaches
- Standard and novel interactive visualization methods for exploratory analysis of connectivity across time, frequency, and spatial location

Source Information Flow Toolbox (SIFT)

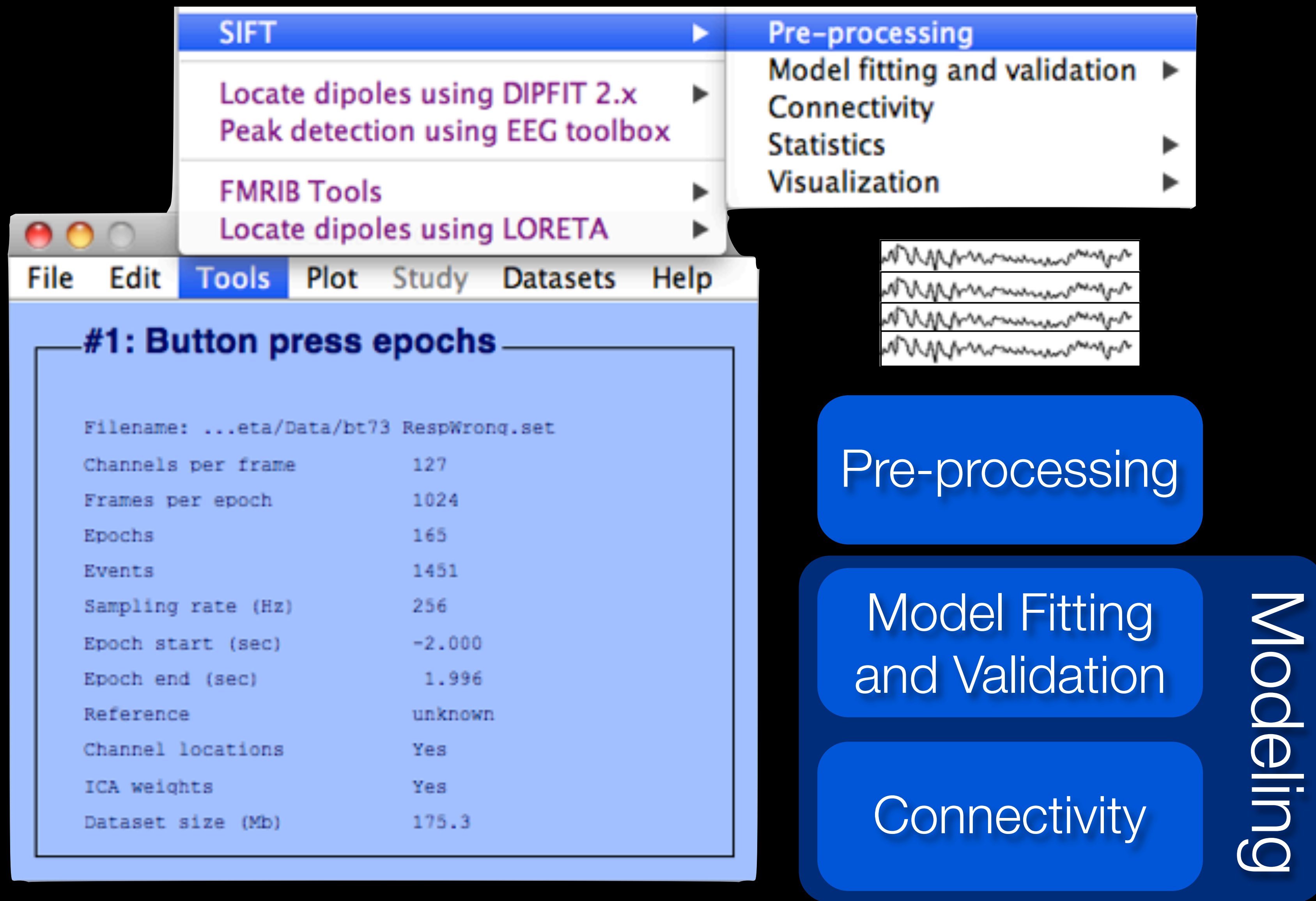
- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
- Emphasis on vector autoregression and time-frequency domain approaches
- Standard and novel interactive visualization methods for exploratory analysis of connectivity across time, frequency, and spatial location
- **Requirements**: EEGLAB, MATLAB™ 2008b, Signal Processing Toolbox, Statistics Toolbox (for some functions -- may be removed in the future)

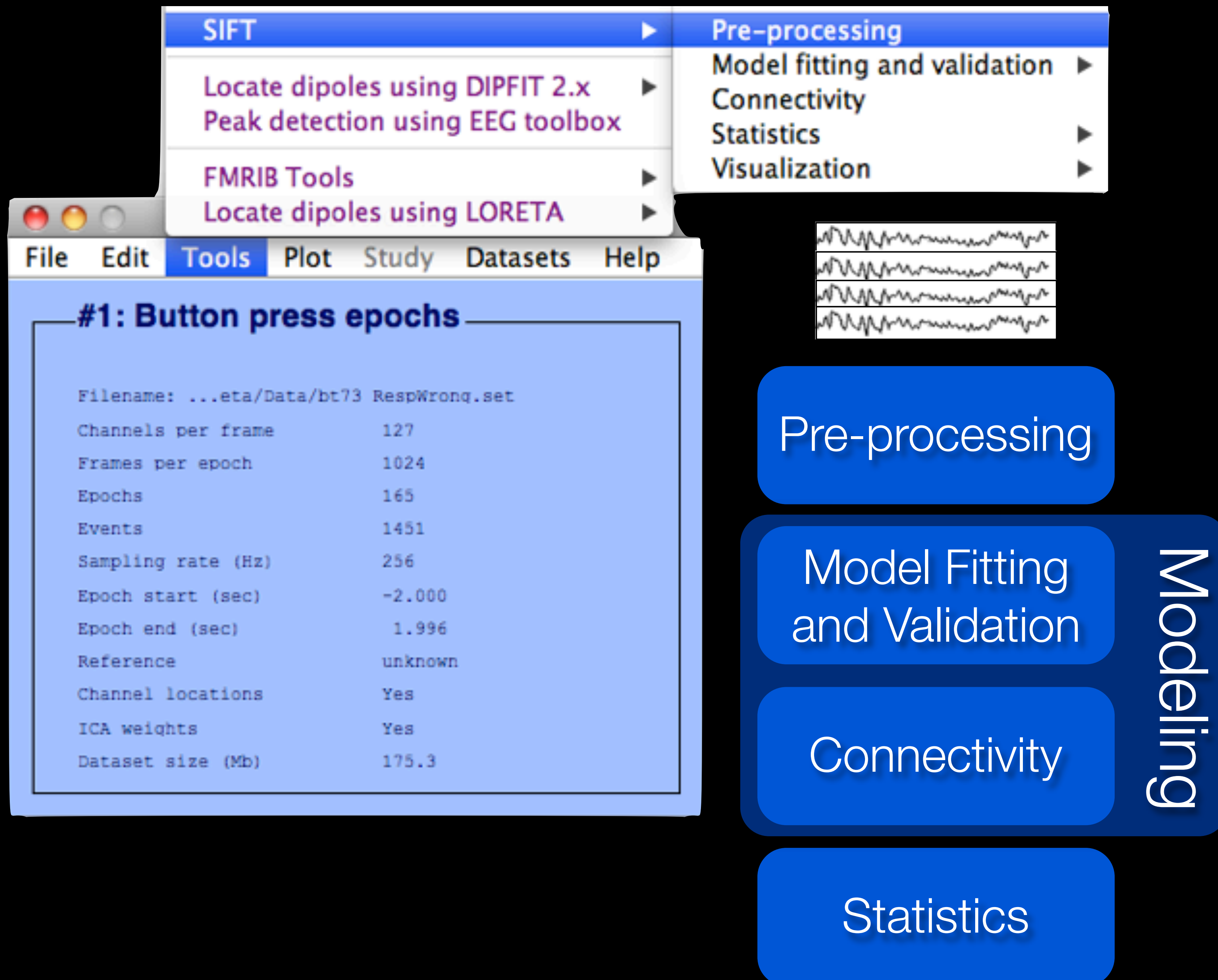

SIFT
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
File Edit Tools Plot Study Datasets Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3


SIFT
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
File
Edit
Tools
Plot
Study
Datasets
Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3

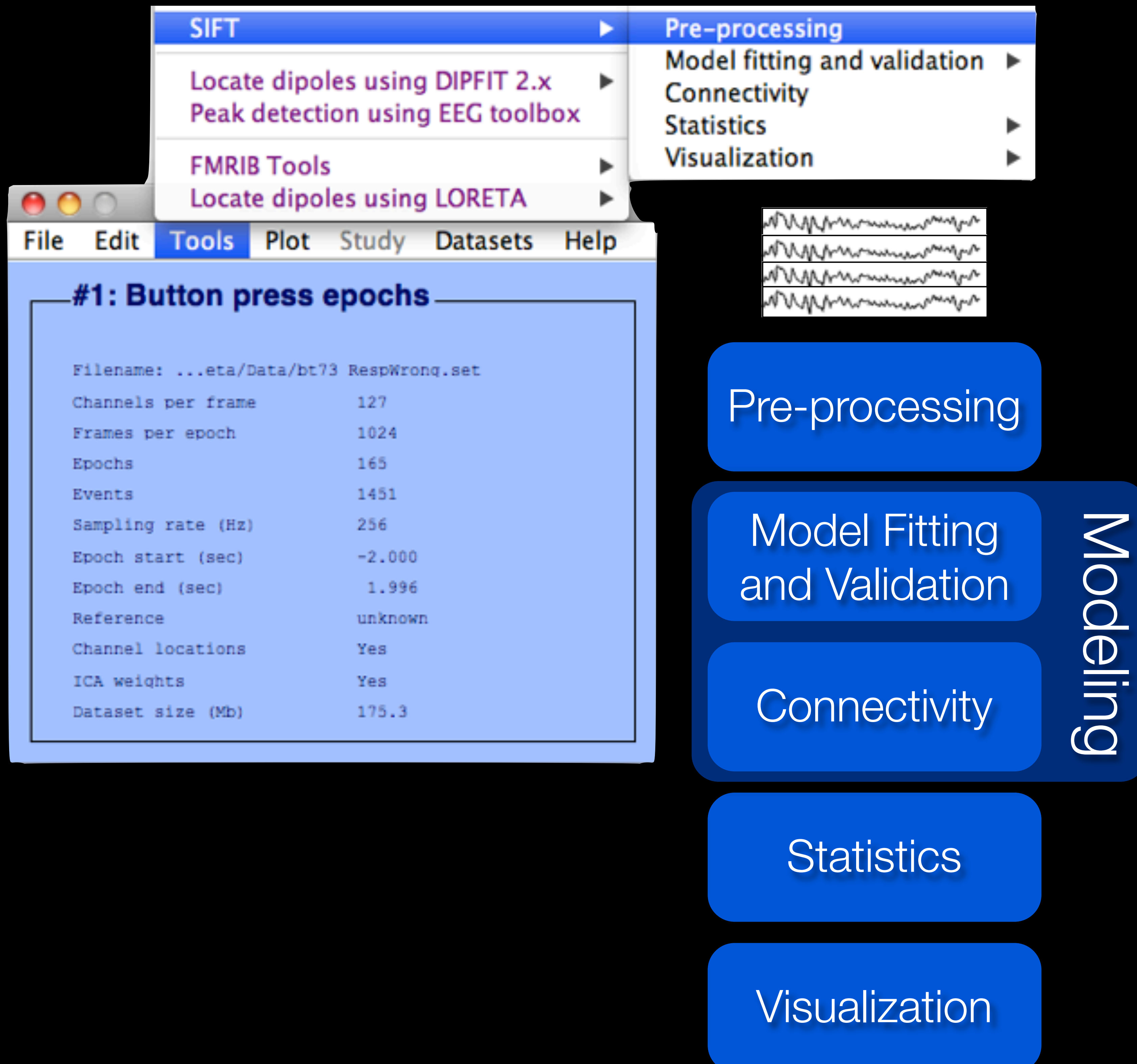


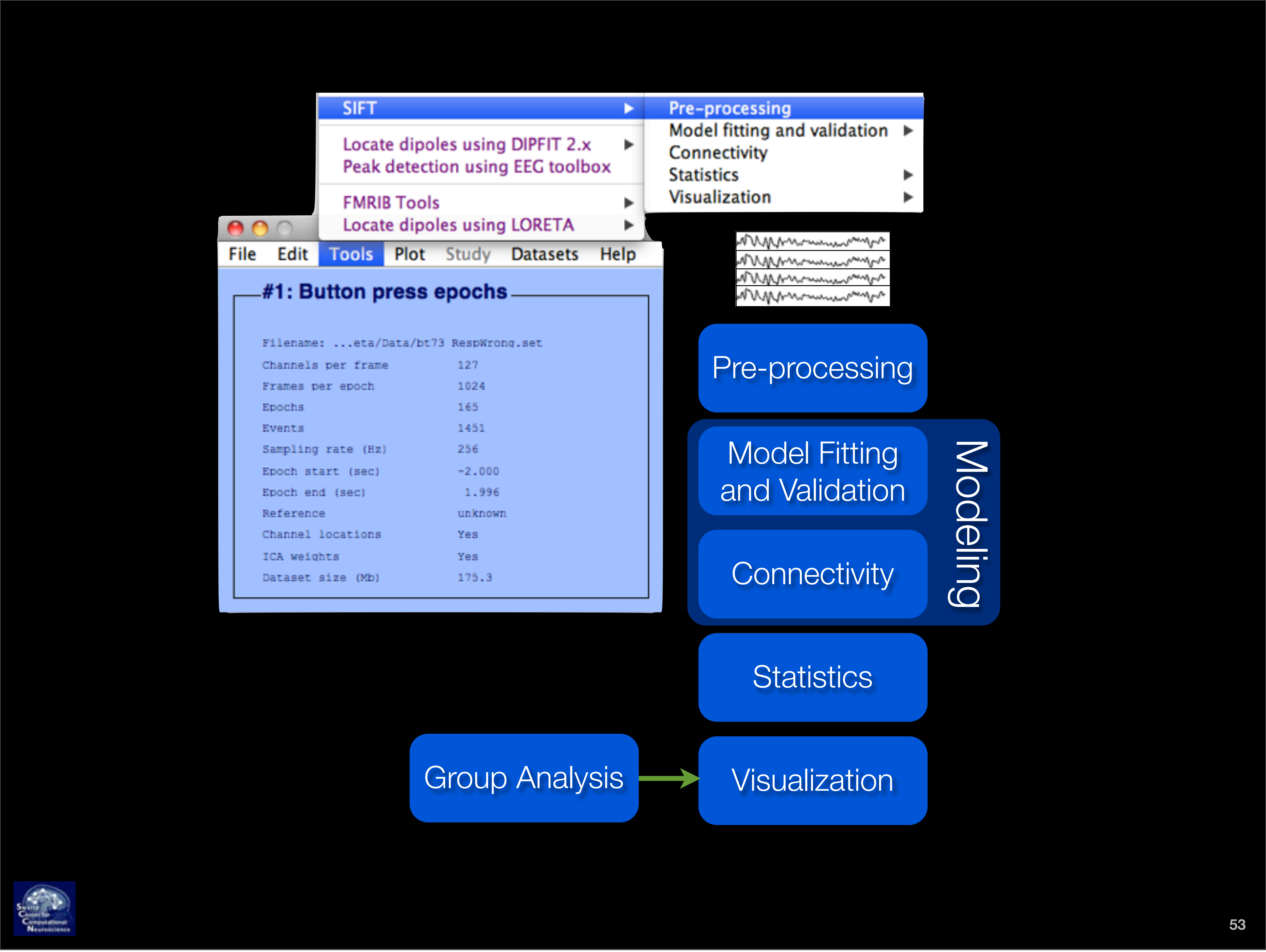
Pre-processing

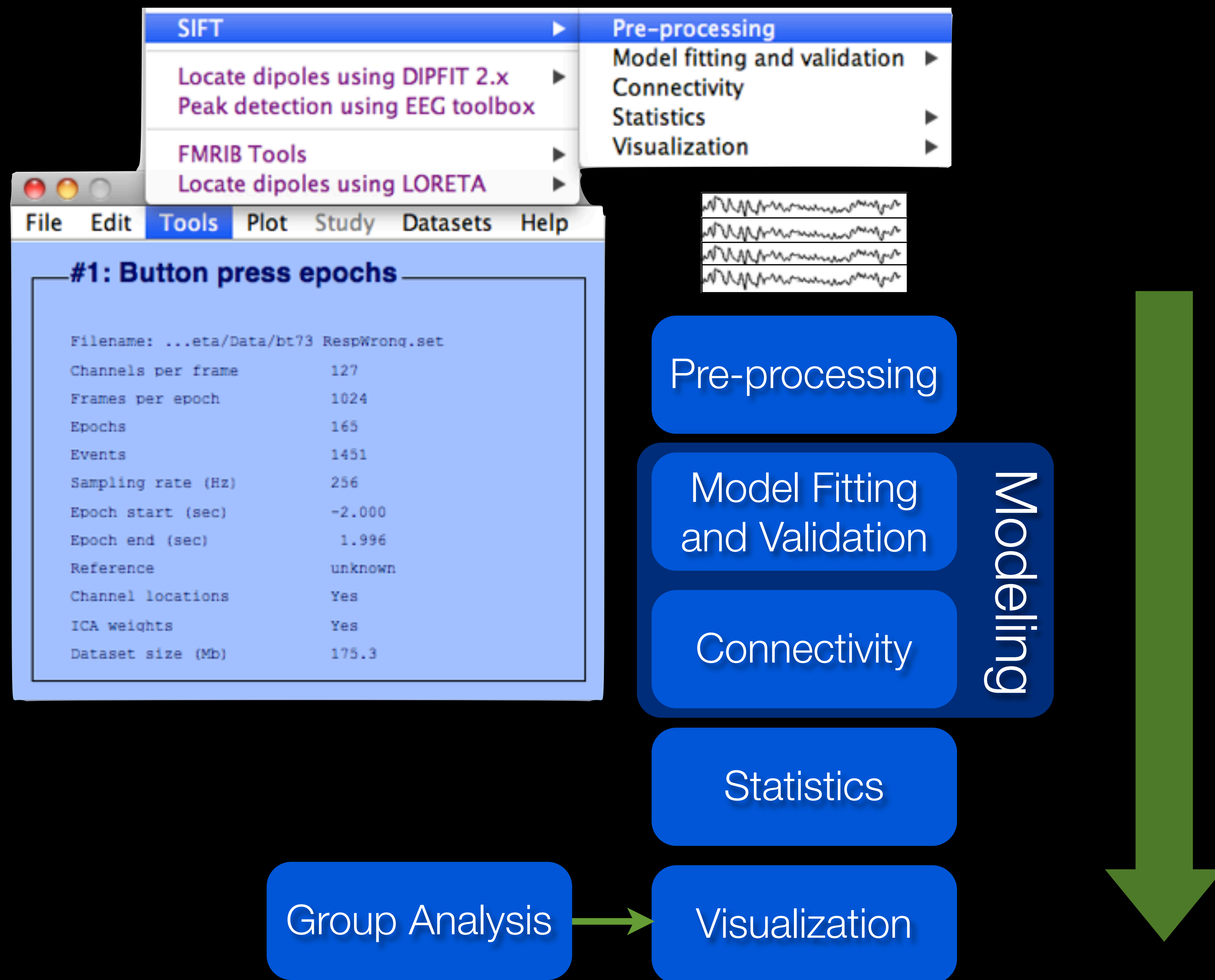

SIFT
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
File Edit Tools Plot Study Datasets Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3
Pre-processing
Model Fitting and Validation
Connectivity
Modeling



SIFT
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
File Edit Tools Plot Study Datasets Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3
Pre-processing
Model Fitting and Validation
Connectivity
Modeling
Statistics


SIFT
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
File Edit Tools Plot Study Datasets Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3
Pre-processing
Model Fitting and Validation
Connectivity
Modeling
Statistics
Visualization


SIFT
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
File Edit Tools Plot Study Datasets Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3
Pre-processing
Model Fitting and Validation
Connectivity
Modeling
Statistics
Group Analysis
Visualization


SIFT
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
Locate dipoles using DIPFIT 2.x
Peak detection using EEG toolbox
FMRIB Tools
Locate dipoles using LORETA
File Edit Tools Plot Study Datasets Help
#1: Button press epochs
Filename: ...eta/Data/bt73 RespWrong.set
Channels per frame 127
Frames per epoch 1024
Epochs 165
Events 1451
Sampling rate (Hz) 256
Epoch start (sec) -2.000
Epoch end (sec) 1.996
Reference unknown
Channel locations Yes
ICA weights Yes
Dataset size (Mb) 175.3
Pre-processing
Model Fitting and Validation
Connectivity
Modeling
Statistics
Group Analysis
Visualization

Preprocessing | Modeling | Statistics | Visualization

- **Source-separation and localization**
  (performed externally using EEGLAB or other toolboxes)
- Filtering/Detrending
- Downsampling
- Differencing
- Normalization (temporal or ensemble)
- Trial balancing
- Tests for stationarity of the data (linear methods)

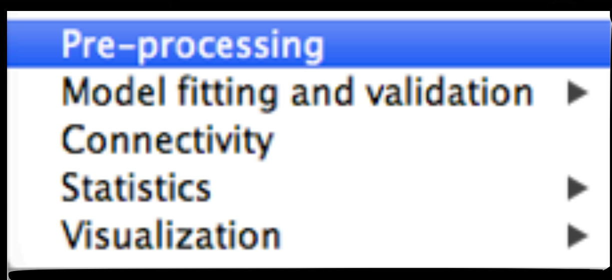
...

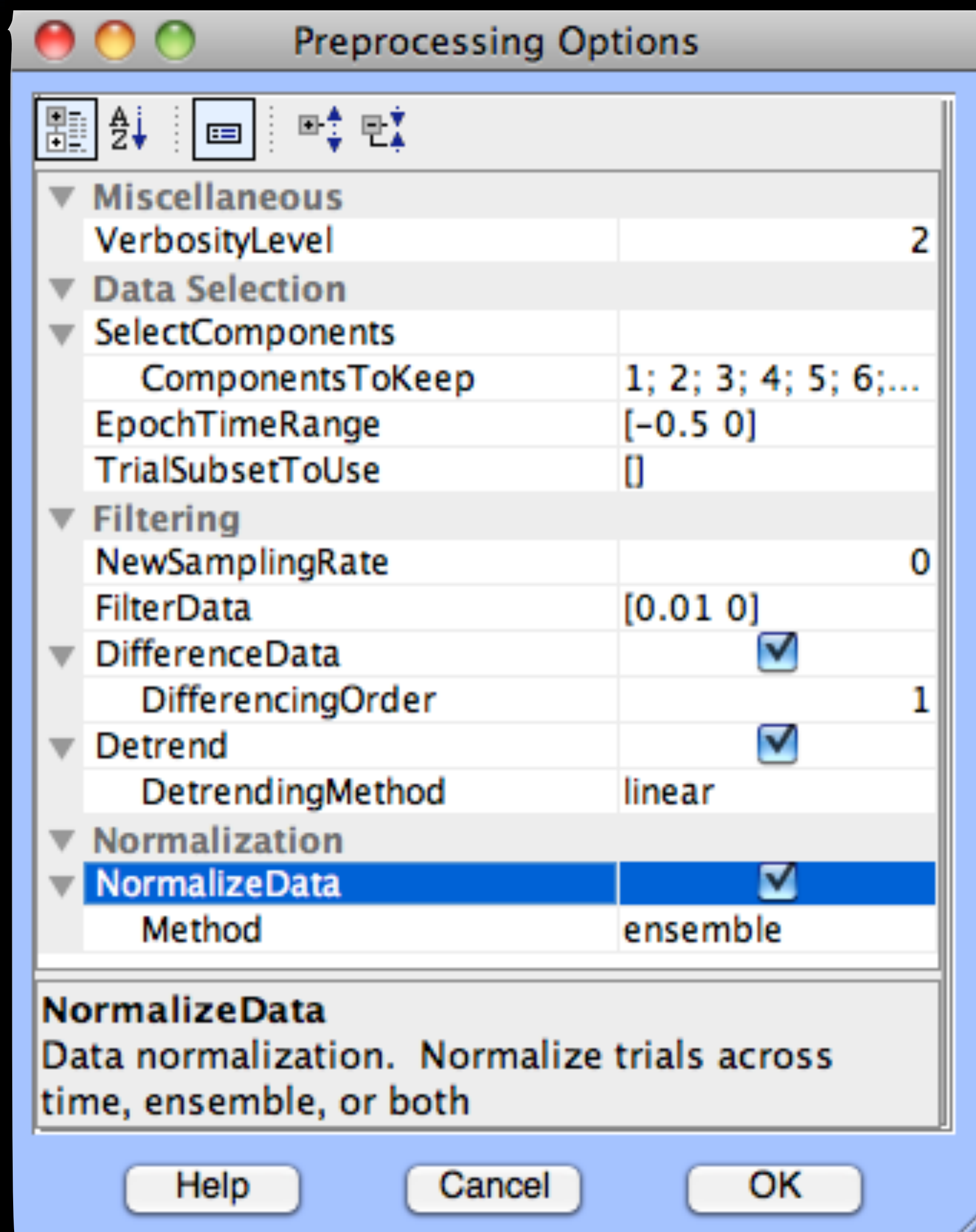
Preprocessing

Modeling

Statistics

Visualization





Preprocessing

Modeling

Statistics

Visualization

Model Fitting

Validation

Connectivity

Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | Validation | Connectivity

Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | Validation | Connectivity

	Linear	Nonlinear
Parametric	MVAR Modeling Sparse MVAR Linear Kalman Filtering	Extended/Cubature Kalman Filtering
Nonparametric	Nonparametric MVAR (minimum-phase spectral factorization) Multivariate phase distribution	Transfer Entropy

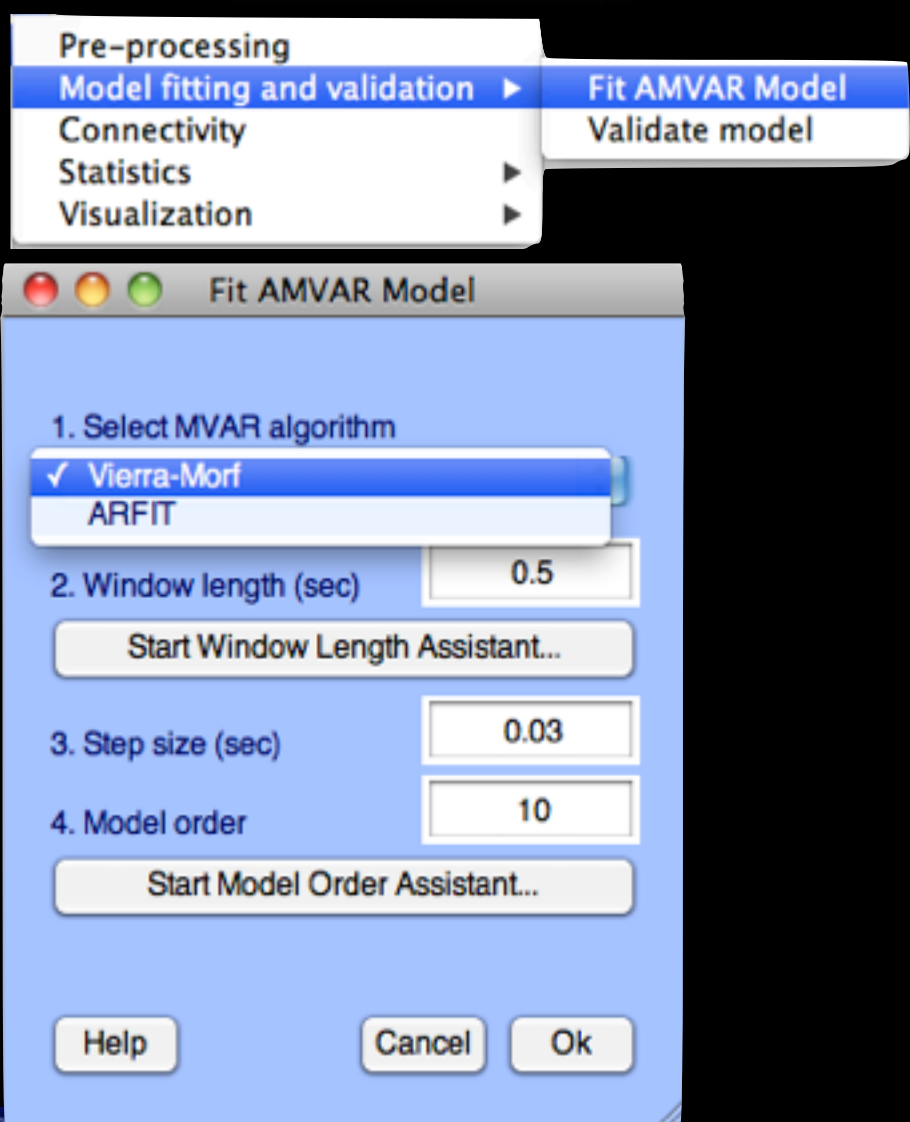
fully implemented — partially-developed — coming soon

Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | Validation | Connectivity

Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | Validation | Connectivity



Preprocessing

Modeling

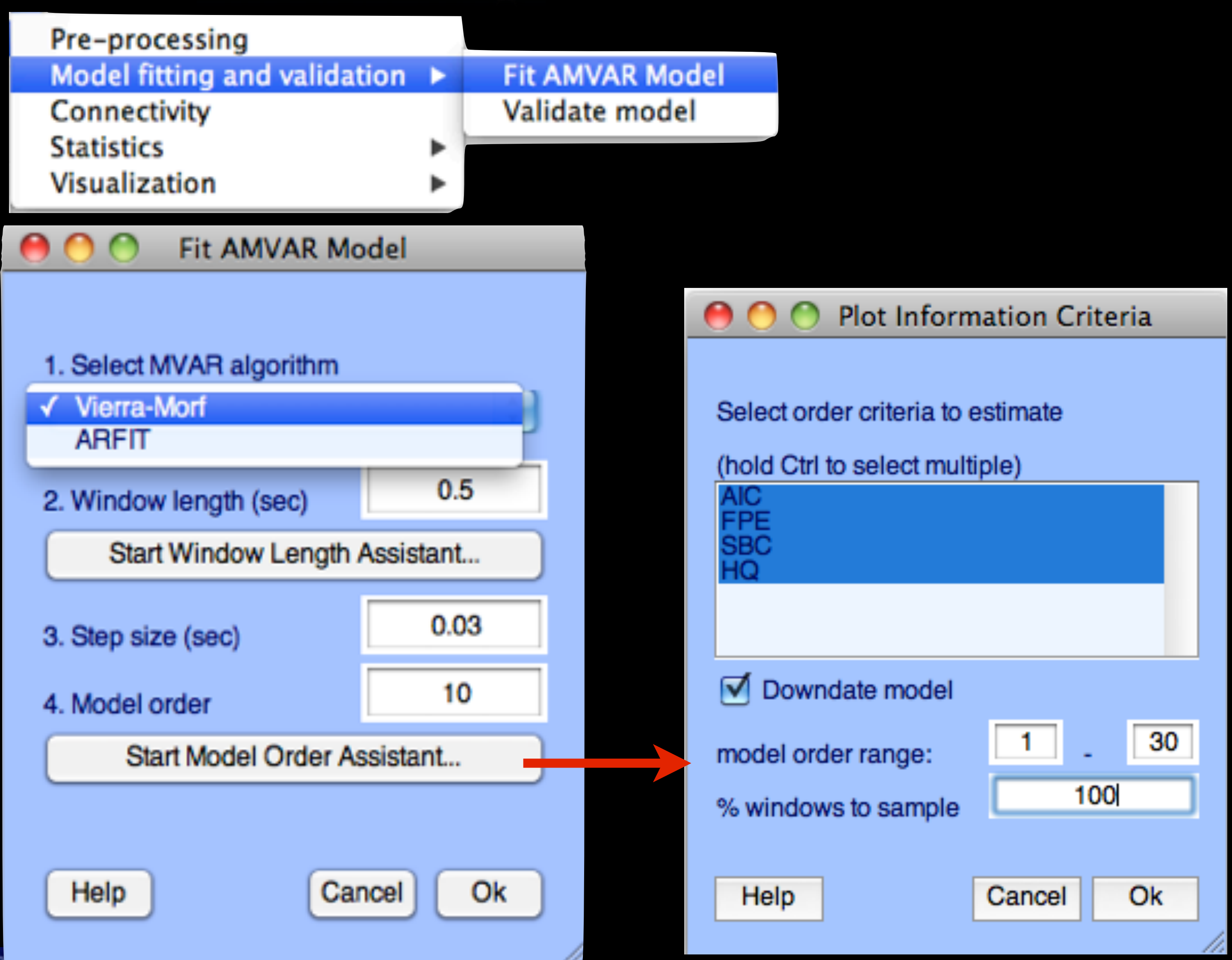
Statistics

Visualization

Model Fitting

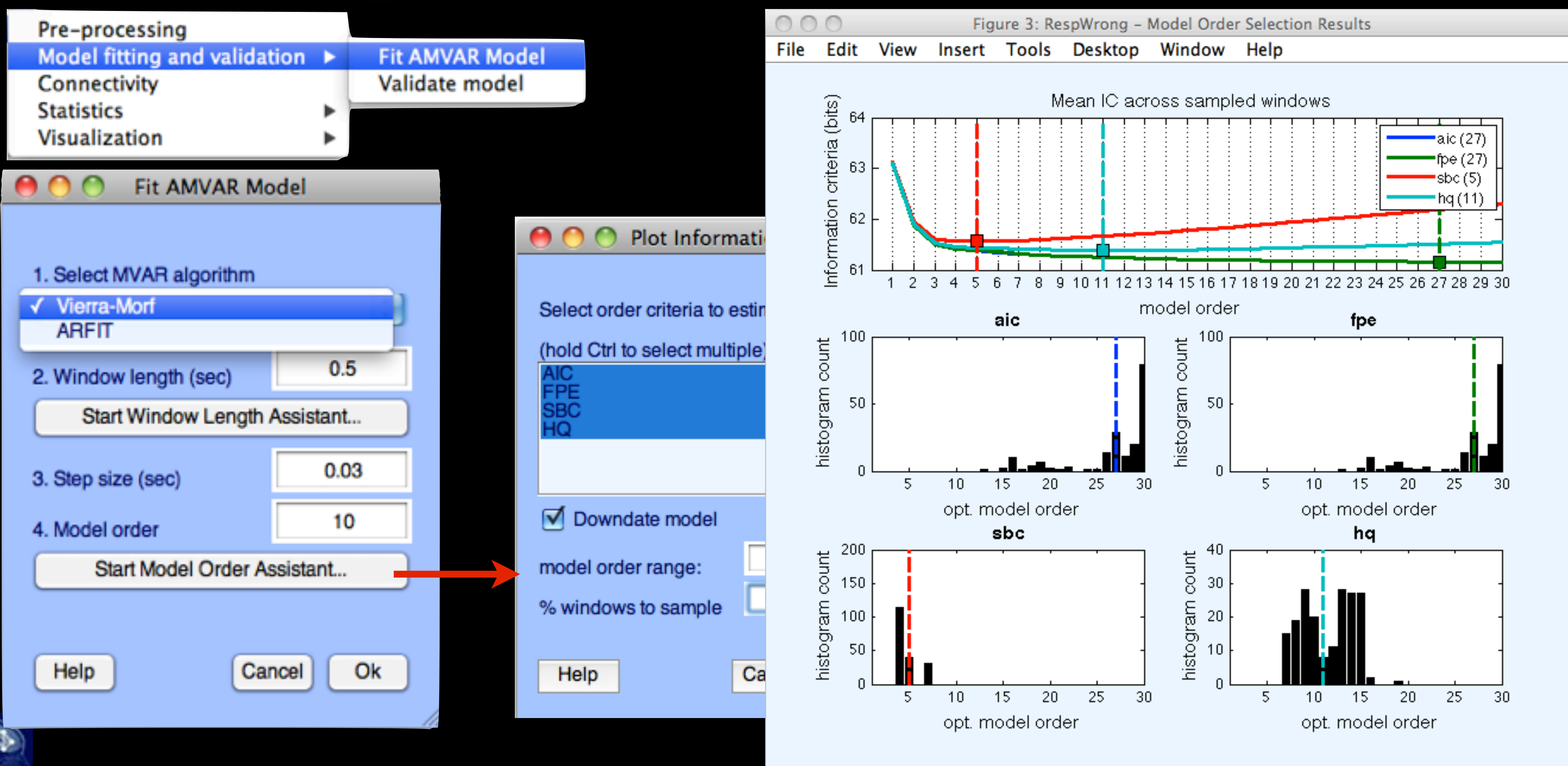
Validation

Connectivity



Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | Validation | Connectivity



Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | **Validation** | Connectivity

- Whiteness of Residuals
  - Portmanteau tests
  - Autocorrelation function
- Model Consistency
- Model Stability

fully implemented | partially-developed | coming soon

Preprocessing

Modeling

Statistics

Visualization

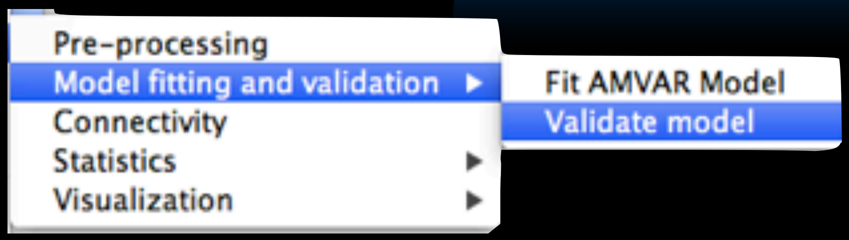
Model Fitting

Validation

Connectivity

Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | **Validation** | Connectivity



Preprocessing

Modeling

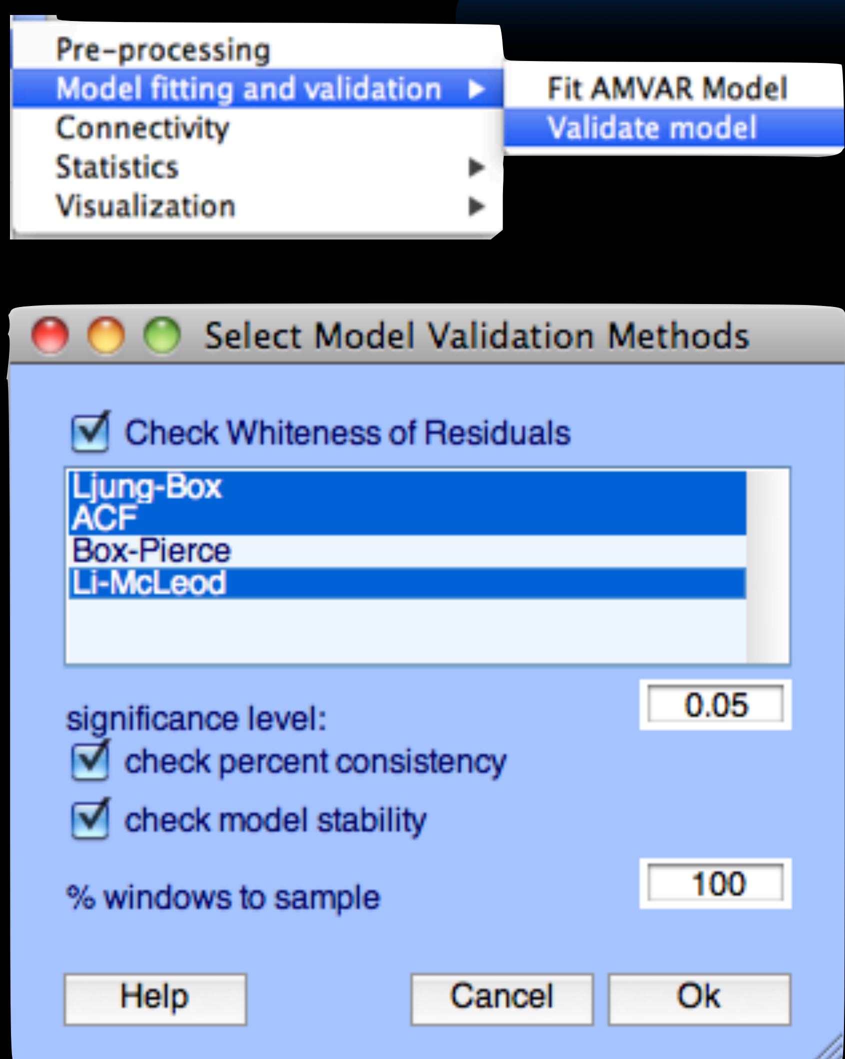
Statistics

Visualization

Model Fitting

Validation

Connectivity



Preprocessing

Modeling

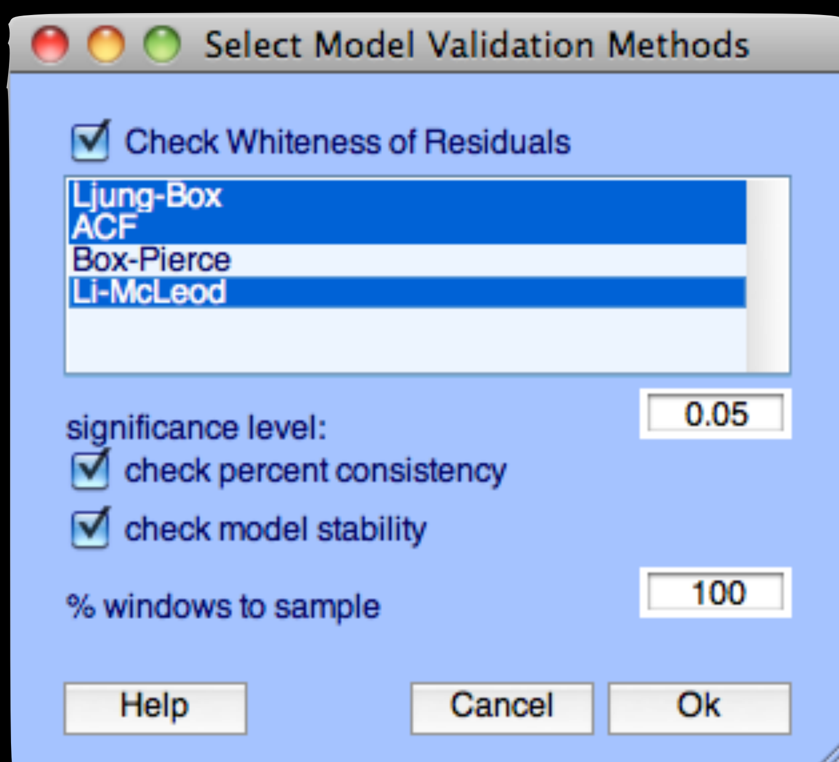
Statistics

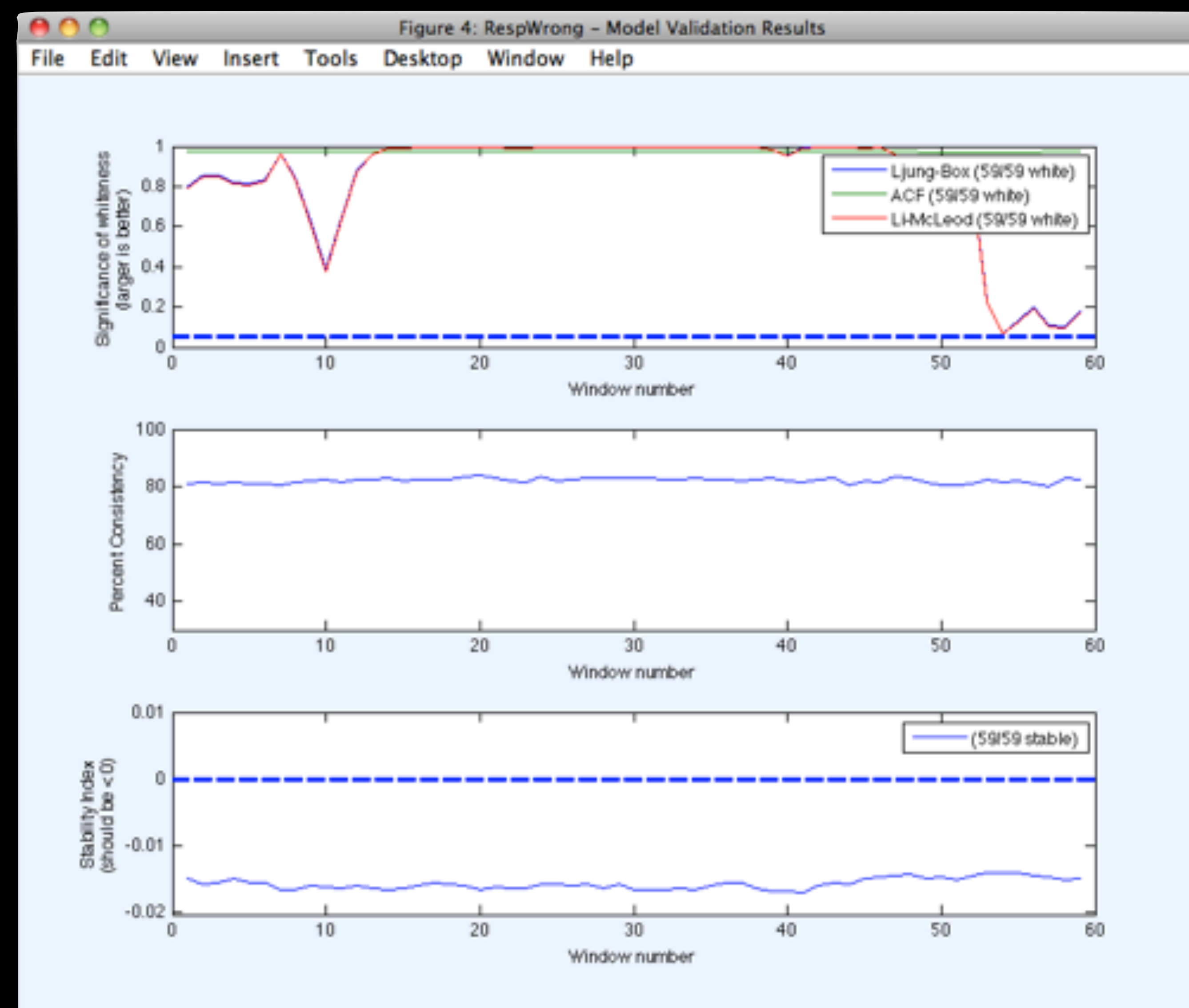
Visualization

Model Fitting

Validation

Connectivity



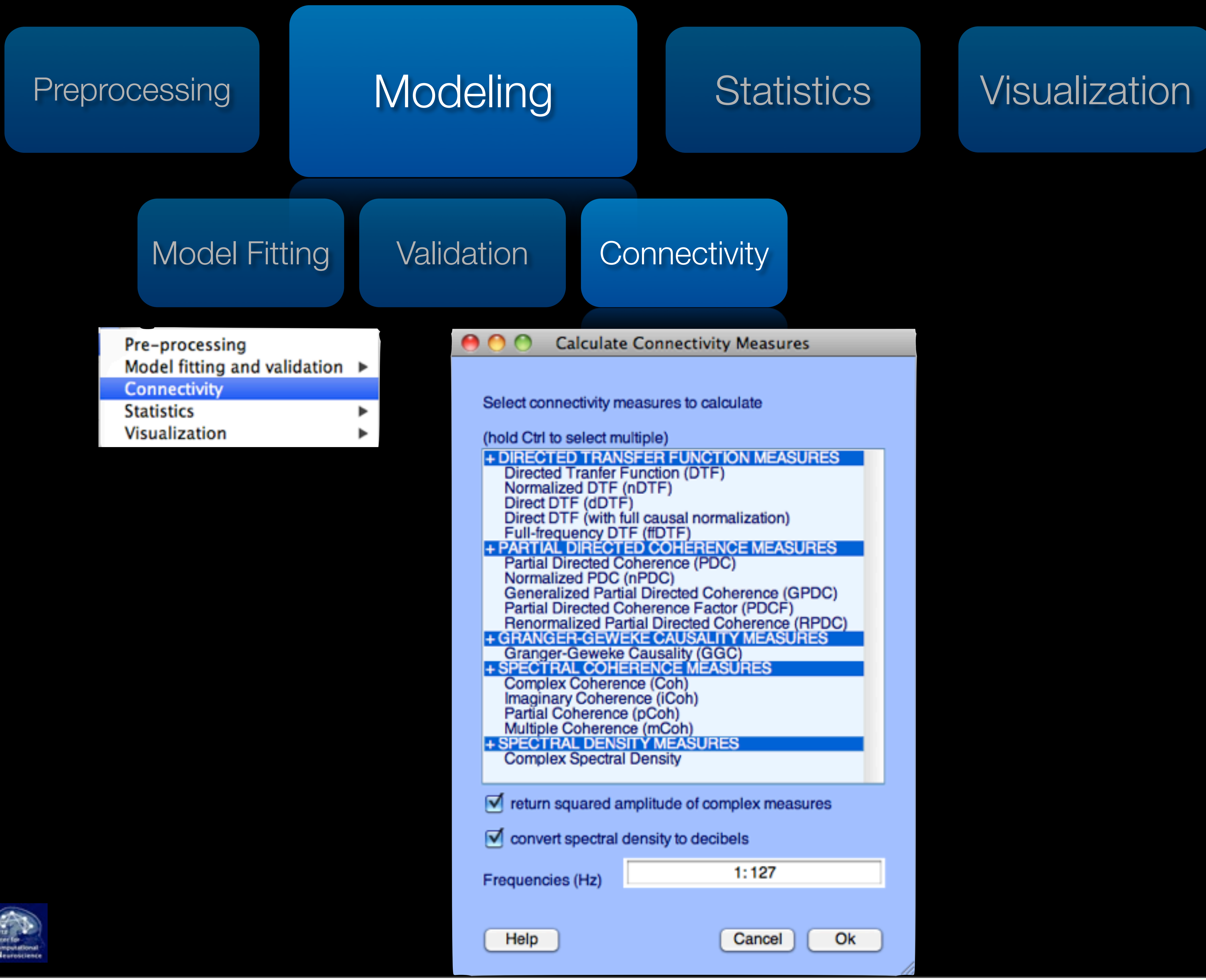


Preprocessing | **Modeling** | Statistics | Visualization

Model Fitting | Validation | **Connectivity**

VAR	Other
-Power spectrum (ERSP) -Coherence (Coh), Partial Coherence (pCoh), Multiple Coherence (mCoh) -Partial Directed Coherence (PDC) -Generalized PDC (GPDC) -Partial Directed Coherence Factor (PDCF) -Renormalized PDC (rPDC) * -Directed Transfer Function (DTF) -Direct Directed Transfer Function (dDTF) -Granger-Geweke Causality (GGC) -Conditional GGC -Blockwise GGC *	-Transfer Entropy * -Multivariate phase-locking value (mPLV) *

fully implemented | partially-developed | coming soon



Preprocessing
Modeling
Statistics
Visualization
Model Fitting
Validation
Connectivity
Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization
Calculate Connectivity Measures
Select connectivity measures to calculate
(hold Ctrl to select multiple)
+DIRECTED TRANSFER FUNCTION MEASURES
Directed Tranfer Function (DTF)
Normalized DTF (nDTF)
Direct DTF (dDTF)
Direct DTF (with full causal normalization)
Full-frequency DTF (ffDTF)
+PARTIAL DIRECTED COHERENCE MEASURES
Partial Directed Coherence (PDC)
Normalized PDC (nPDC)
Generalized Partial Directed Coherence (GPDC)
Partial Directed Coherence Factor (PDCF)
Renormalized Partial Directed Coherence (RPDC)
+GRANGER-GEWEKE CAUSALITY MEASURES
Granger-Geweke Causality (GGC)
+SPECTRAL COHERENCE MEASURES
Complex Coherence (Coh)
Imaginary Coherence (iCoh)
Partial Coherence (pCoh)
Multiple Coherence (mCoh)
+SPECTRAL DENSITY MEASURES
Complex Spectral Density
return squared amplitude of complex measures
convert spectral density to decibels
Frequencies (Hz)
1:127
Help
Cancel
Ok

Preprocessing

Modeling

Statistics

Visualization

Beta Release

Preprocessing

Modeling

Statistics

Visualization

Parametric

Asymptotic analytic estimates of confidence intervals

Applies to: PDC, nPDC, DTF, nDTF, rPDC

Tests: H_{null}, H_{base}, H_{AB}

Confidence intervals using thin-plate smoothing splines

Applies to: dDTF

Tests: H_{base}, H_{AB}

Beta Release

$H_{null} : \mathbf{C}_{ij} \leq \mathbf{C}_{null}$ $H_{base}: \mathbf{C}_{ij} \leq \mathbf{C}_{baseline}$ $H_{AB}: \mathbf{C}^{A}_{ij} = \mathbf{C}^{B}_{ij}$

fully implemented partially-developed coming soon

Preprocessing | Modeling | **Statistics** | Visualization

Parametric

Asymptotic analytic estimates of confidence intervals
- Applies to: PDC, nPDC, DTF, nDTF, rPDC
- Tests: H_{null}, H_{base}, H_{AB}

Confidence intervals using thin-plate smoothing splines
- Applies to: dDTF
- Tests: H_{base}, H_{AB}

Non-parametric

Phase-randomization
- Applies to: all
- Tests: H_{null}

Permutation Tests
- Applies to: all
- Tests: H_{AB}, H_{base}

Bootstrap and Jacknife
- Applies to: all
- Tests: H_{AB}, H_{base}

Beta Release

$H_{null} : \mathbf{C}_{ij} \leq \mathbf{C}_{null}$ $H_{base}: \mathbf{C}_{ij} \leq \mathbf{C}_{baseline}$ $H_{AB}: \mathbf{C}^{A}_{ij} = \mathbf{C}^{B}_{ij}$

fully implemented | partially-developed | coming soon

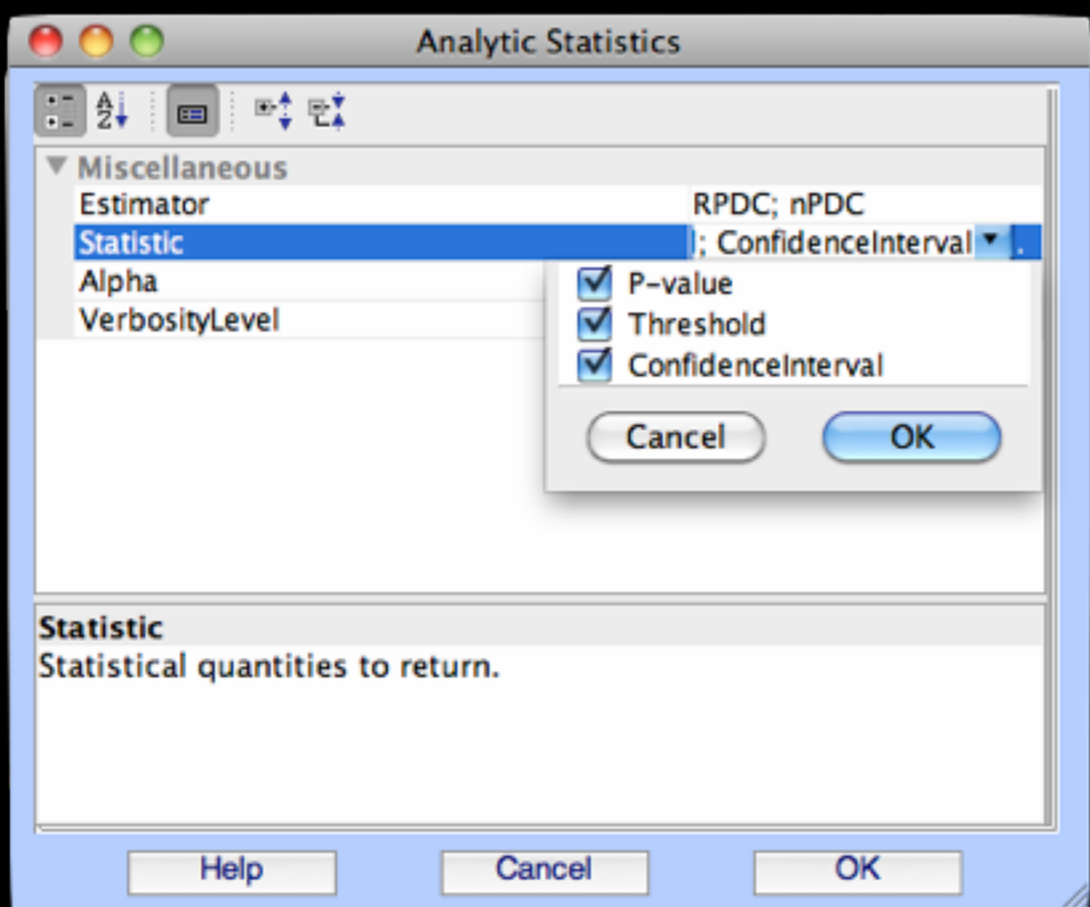
Preprocessing

Modeling

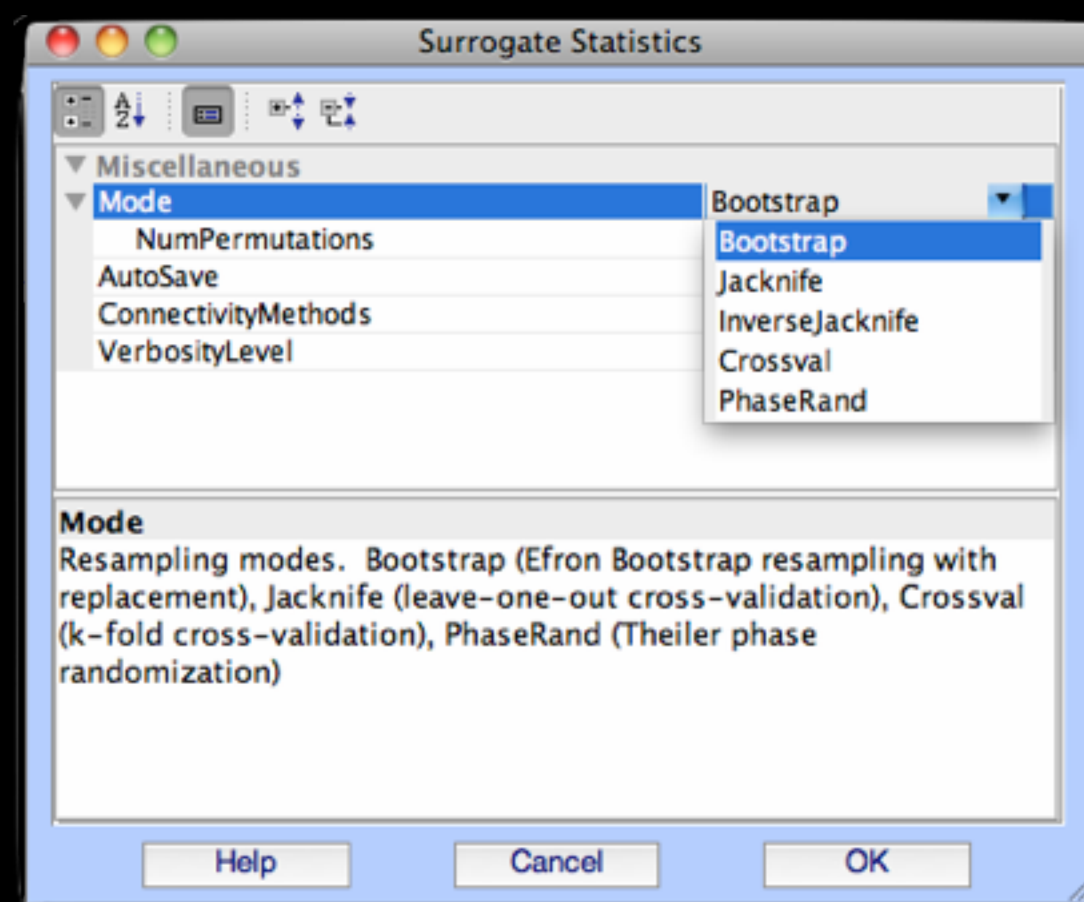
Statistics

Visualization

Parametric



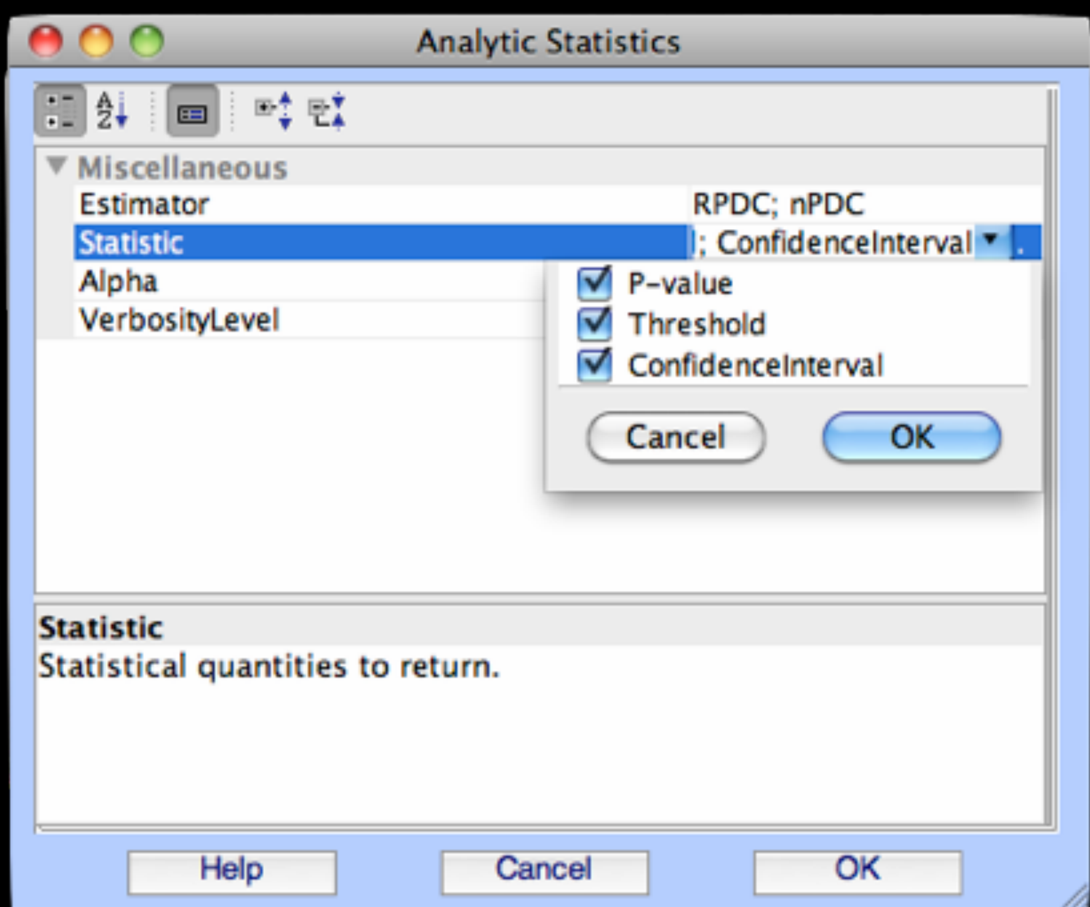
Non-parametric

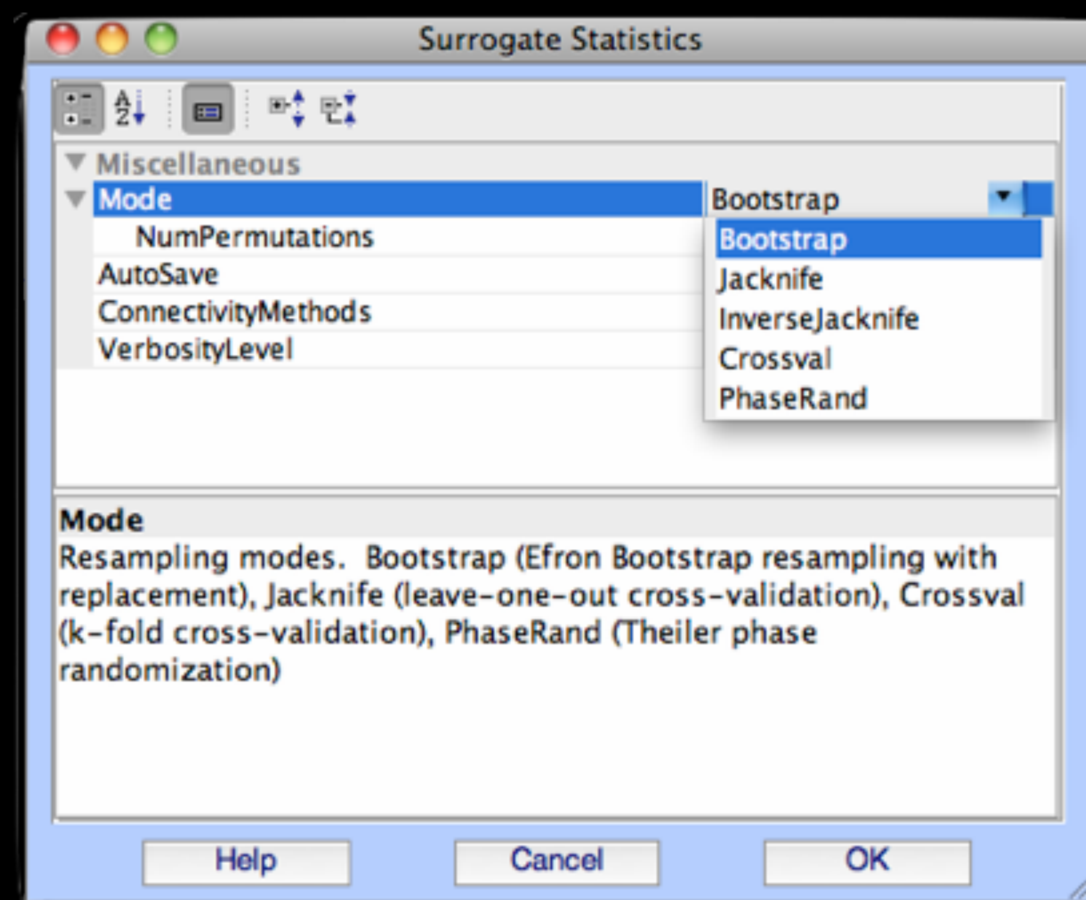


Preprocessing Modeling **Statistics** Visualization

Parametric

Non-parametric





H_{null} : $\mathbf{C}_{ij} \leq \mathbf{C}_{null}$ H_{base}: $\mathbf{C}_{ij} \leq \mathbf{C}_{baseline}$ H_{AB}: $\mathbf{C}^{A}_{ij} = \mathbf{C}^{B}_{ij}$

fully implemented partially-developed coming soon

Preprocessing

Modeling

Statistics

Visualization

fully implemented

partially-developed

coming soon

Preprocessing

Modeling

Statistics

Visualization

Interactive Time-Frequency Grid

fully implemented

partially-developed

coming soon

Preprocessing | Modeling | Statistics | **Visualization**

Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

fully implemented — partially-developed — coming soon

Preprocessing | Modeling | Statistics | **Visualization**

Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

fully implemented | partially-developed | coming soon

Preprocessing | Modeling | Statistics | **Visualization**

Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

Directed Graphs on anatomicals (ECoG)

fully implemented | partially-developed | coming soon

Preprocessing | Modeling | Statistics | **Visualization**

Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

Directed Graphs on anatomicals (ECoG)

and more...

fully implemented | partially-developed | coming soon

Preprocessing | Modeling | Statistics | **Visualization**

Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

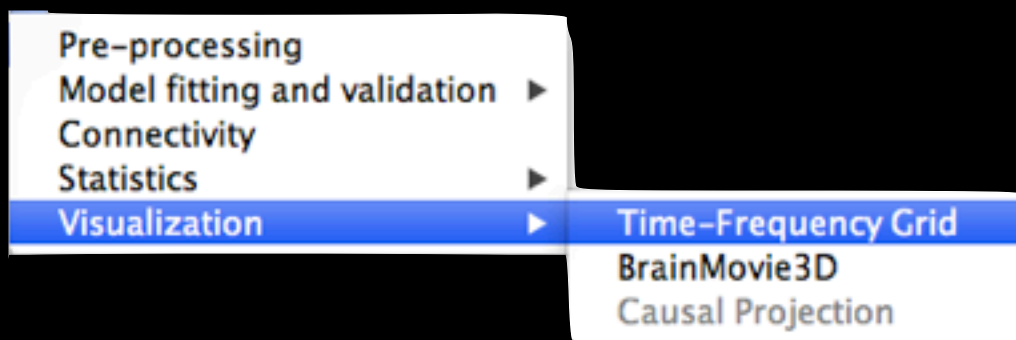
Directed Graphs on anatomicals (ECoG)

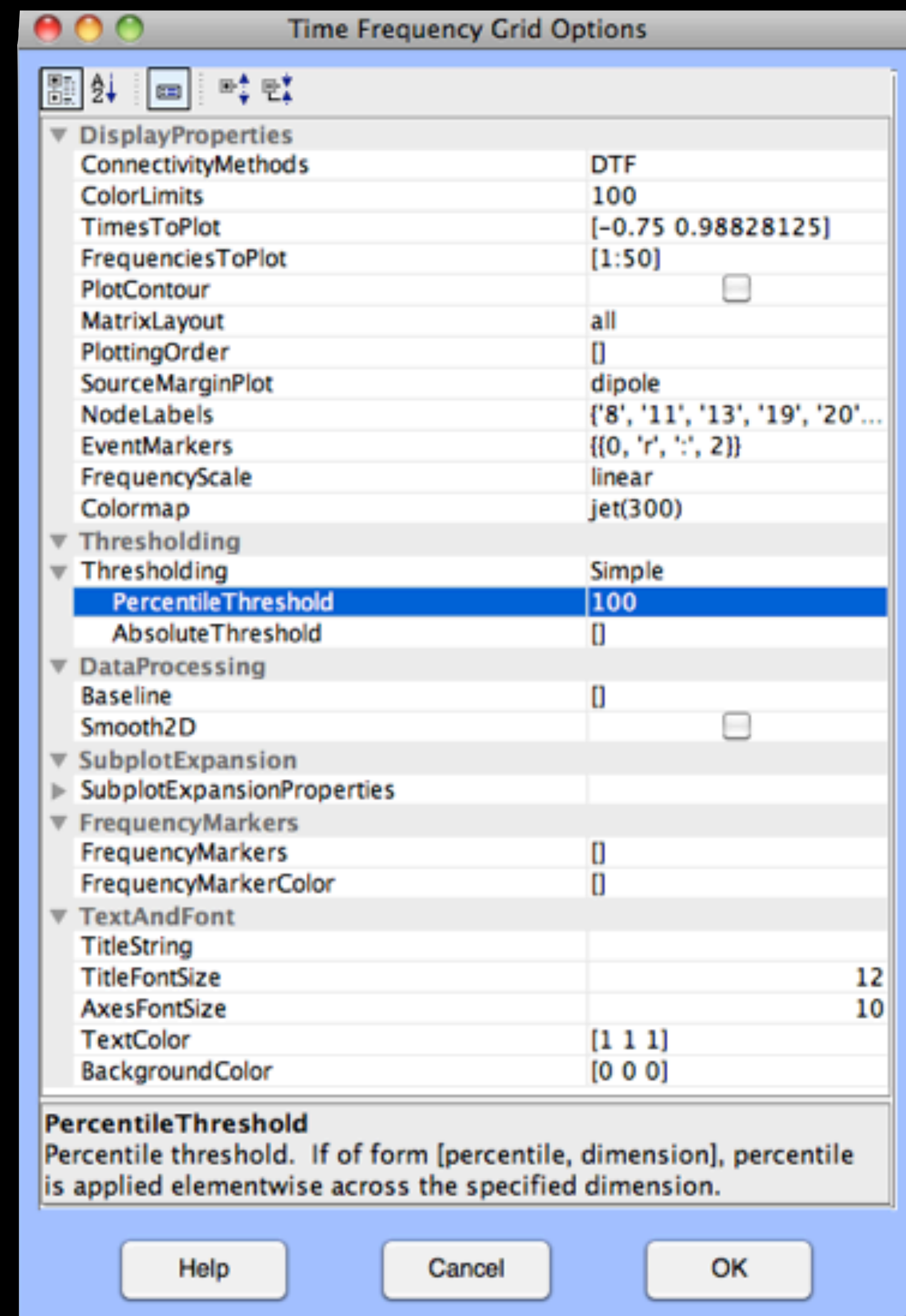
and more...

All of these currently support single-subject or (in beta version) group analysis
ROI connectivity analysis can currently be performed using dipole clustering

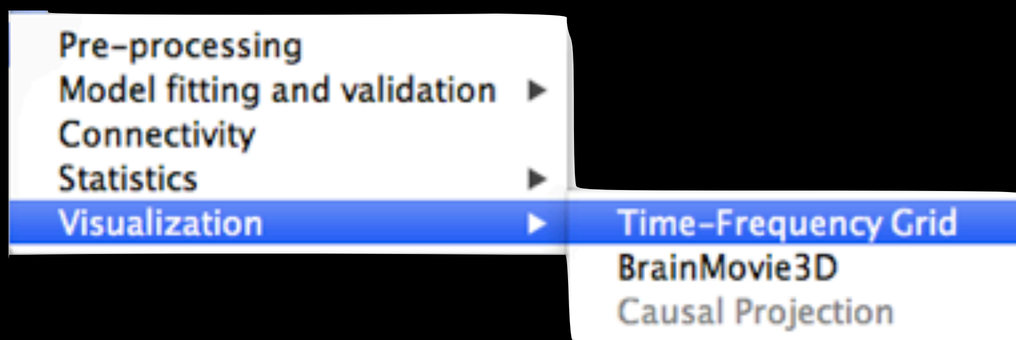
fully implemented | partially-developed | coming soon

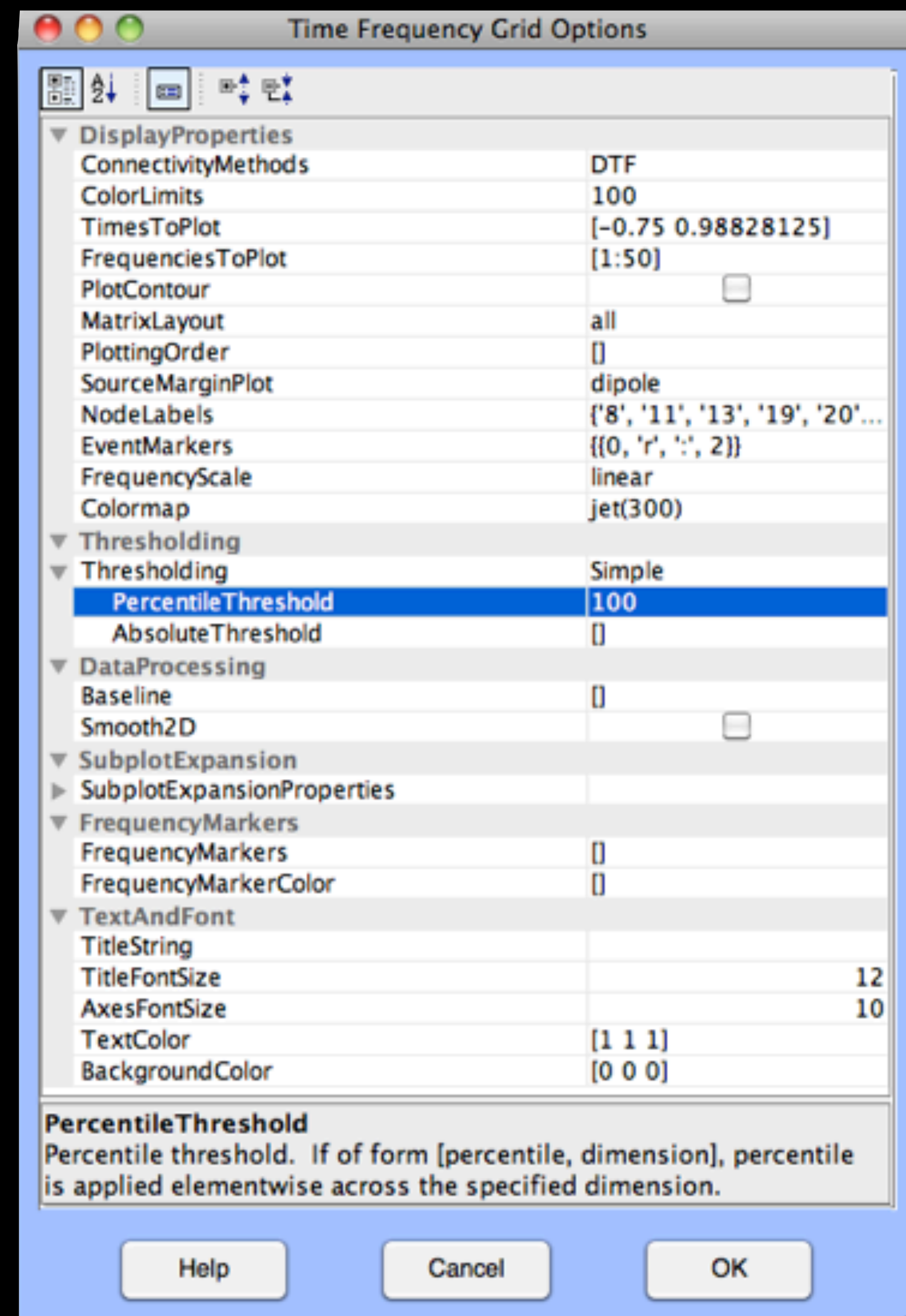
Interactive Time-Frequency Grid



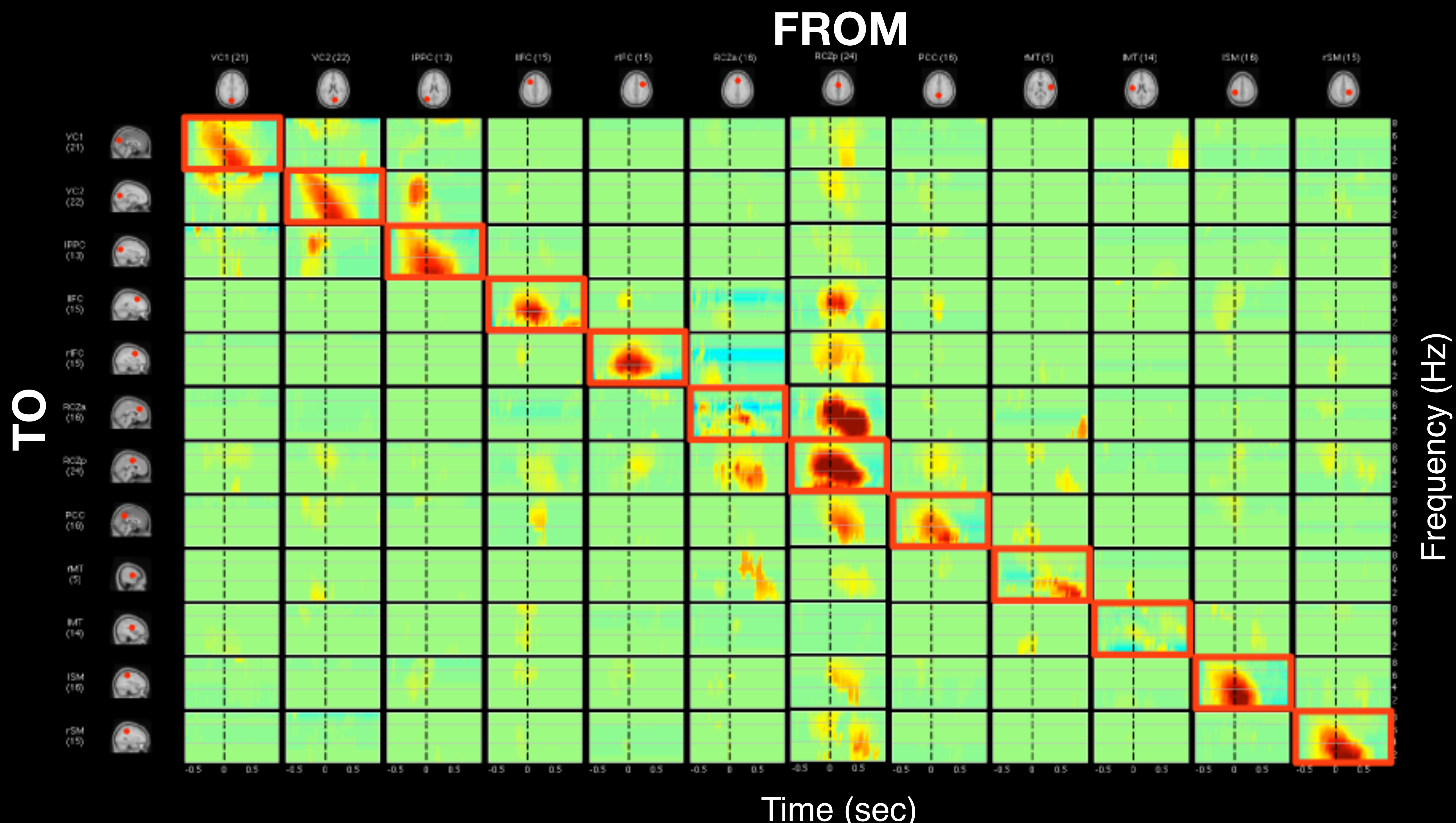


Interactive Time-Frequency Grid

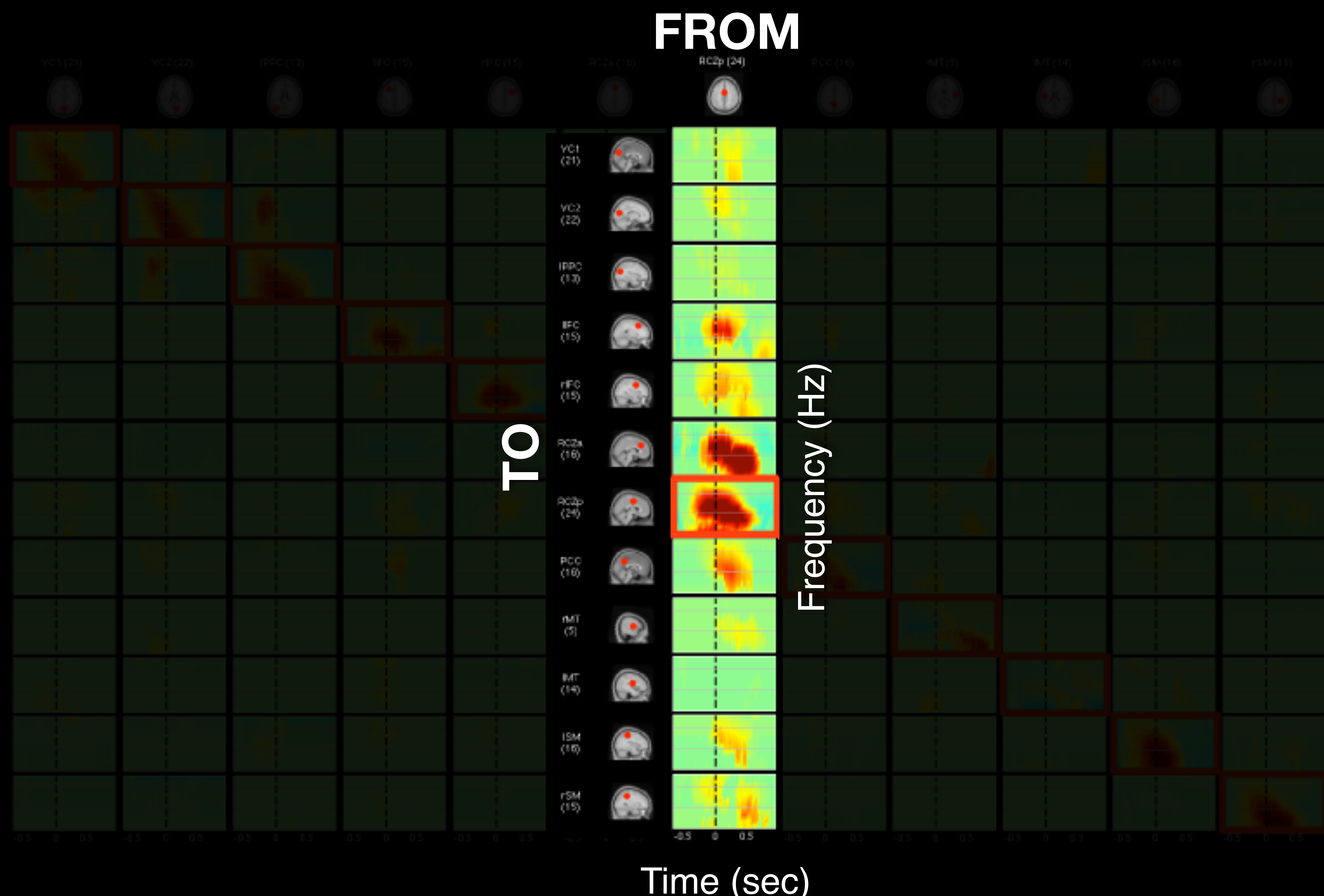




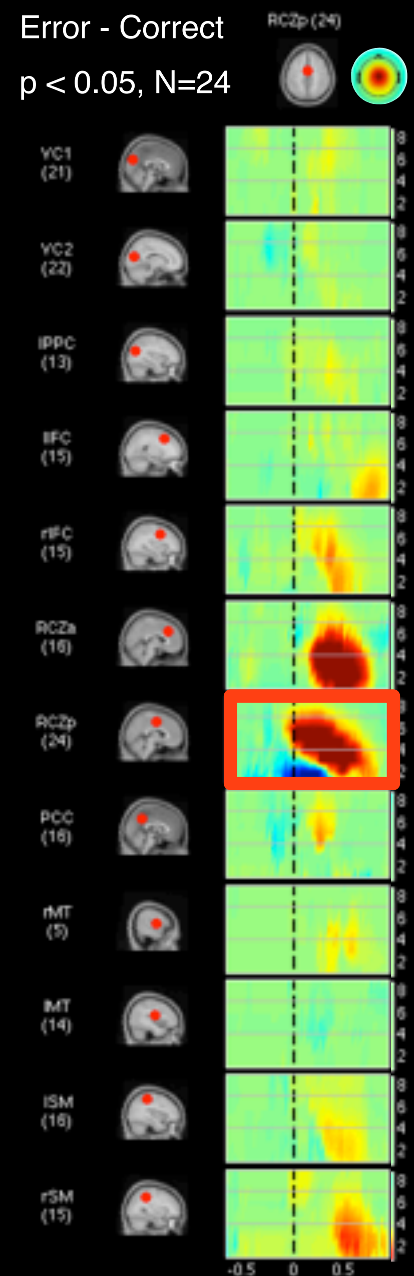
Causal Time-Frequency Grid



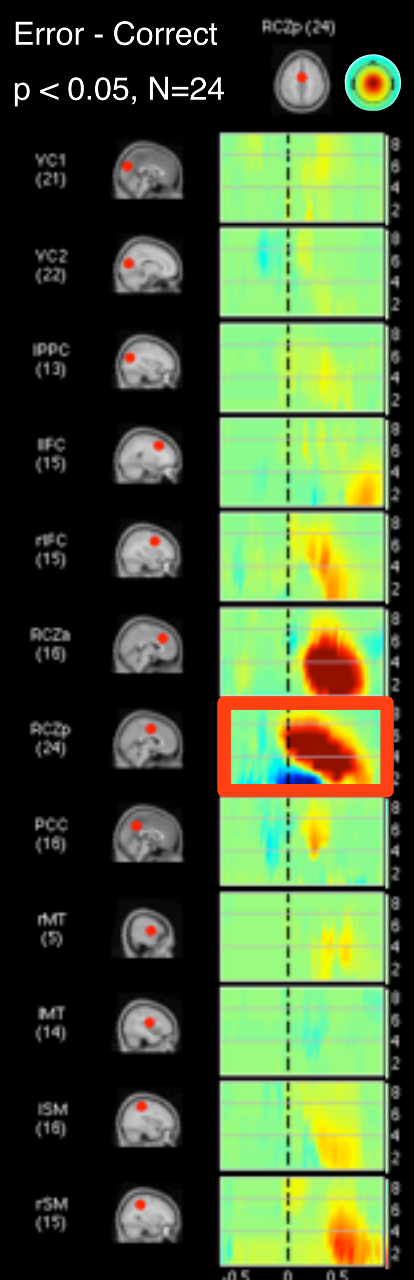
Causal Time-Frequency Grid



Causal Time-Frequency Grid



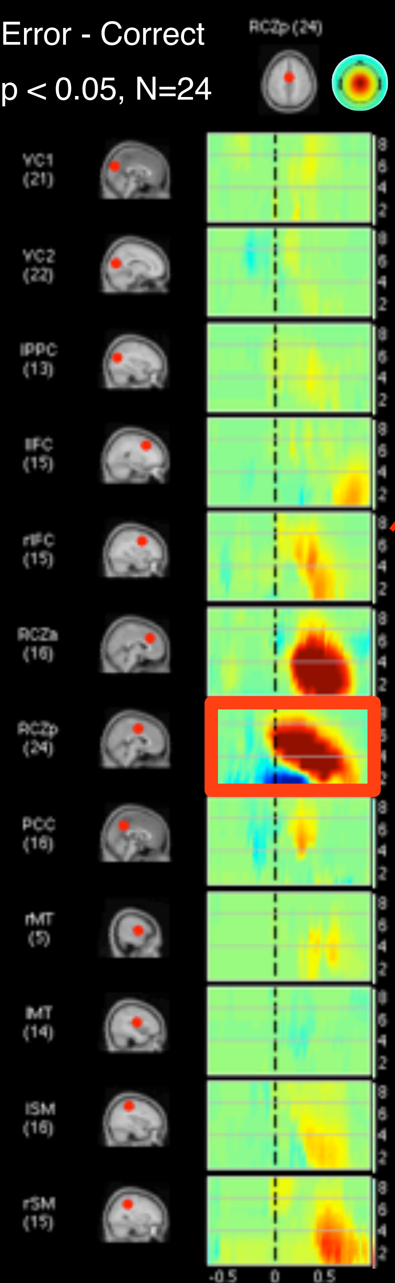
Causal Time-Frequency Grid

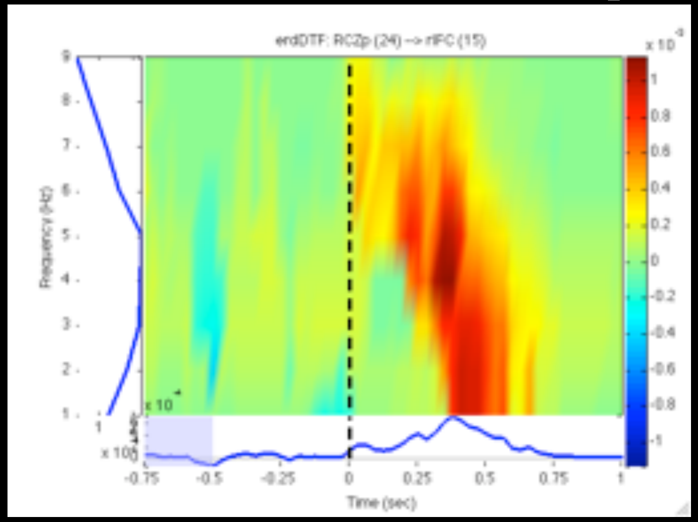


Causal Time-Frequency Grid

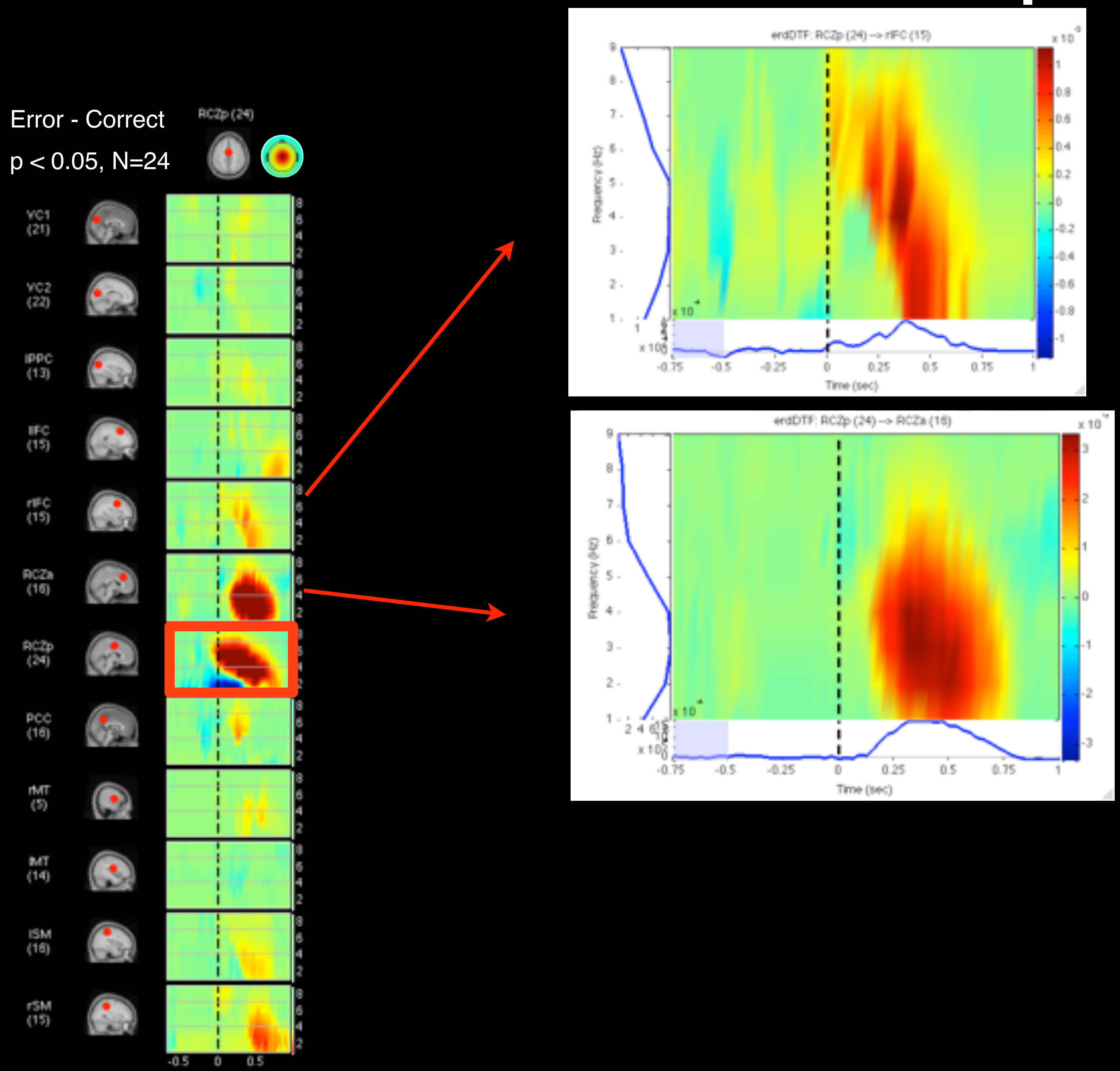
Error - Correct

p < 0.05, N=24

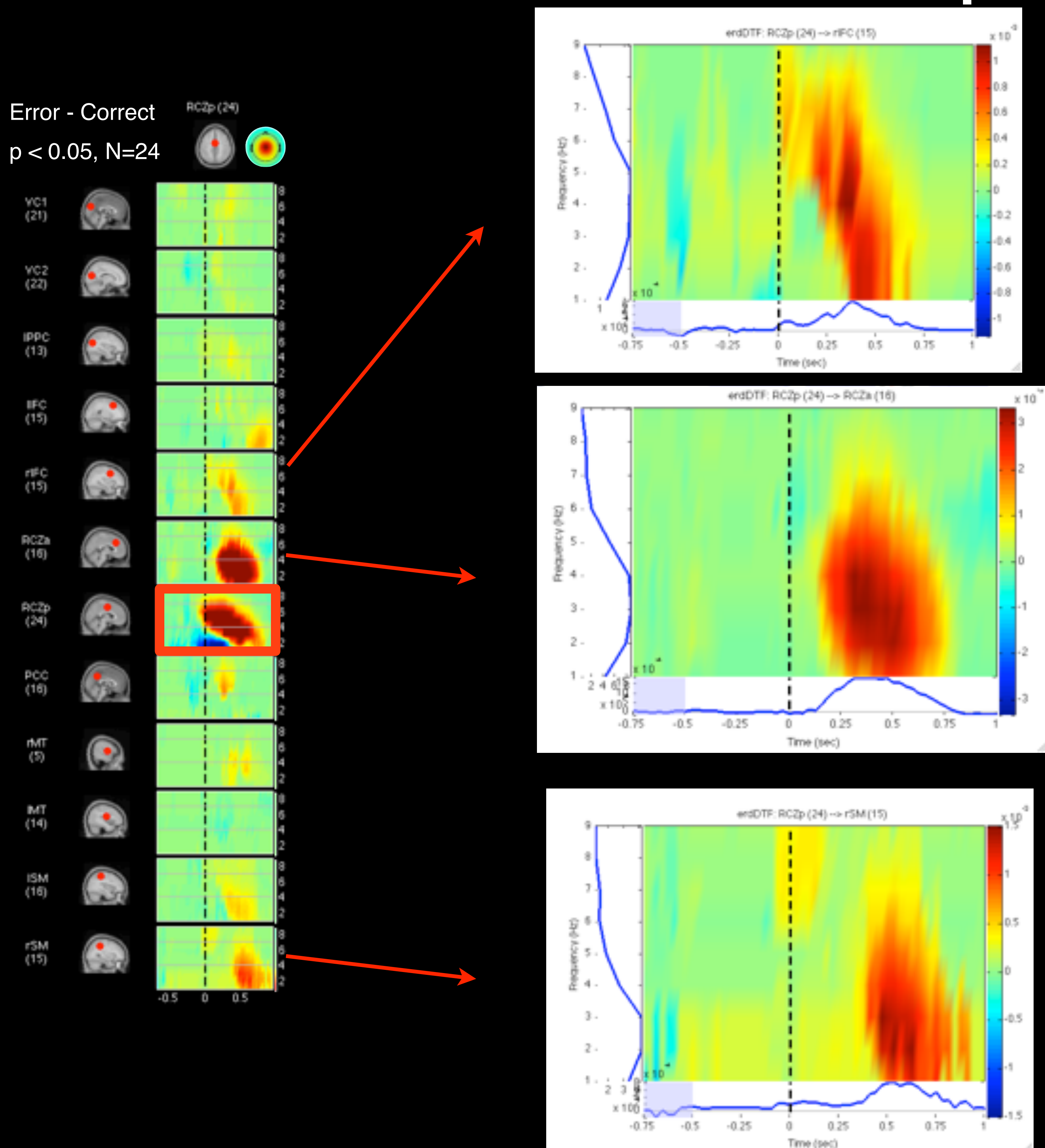




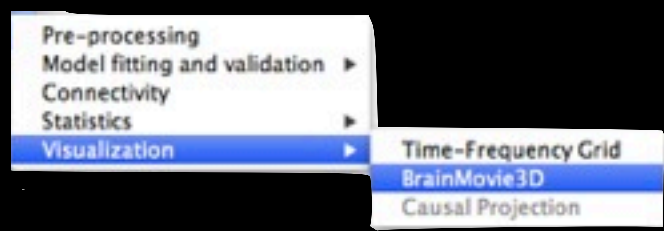
Causal Time-Frequency Grid



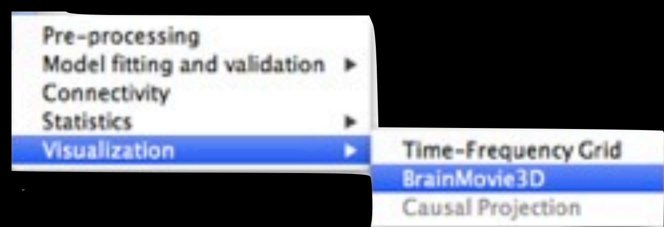
Causal Time-Frequency Grid

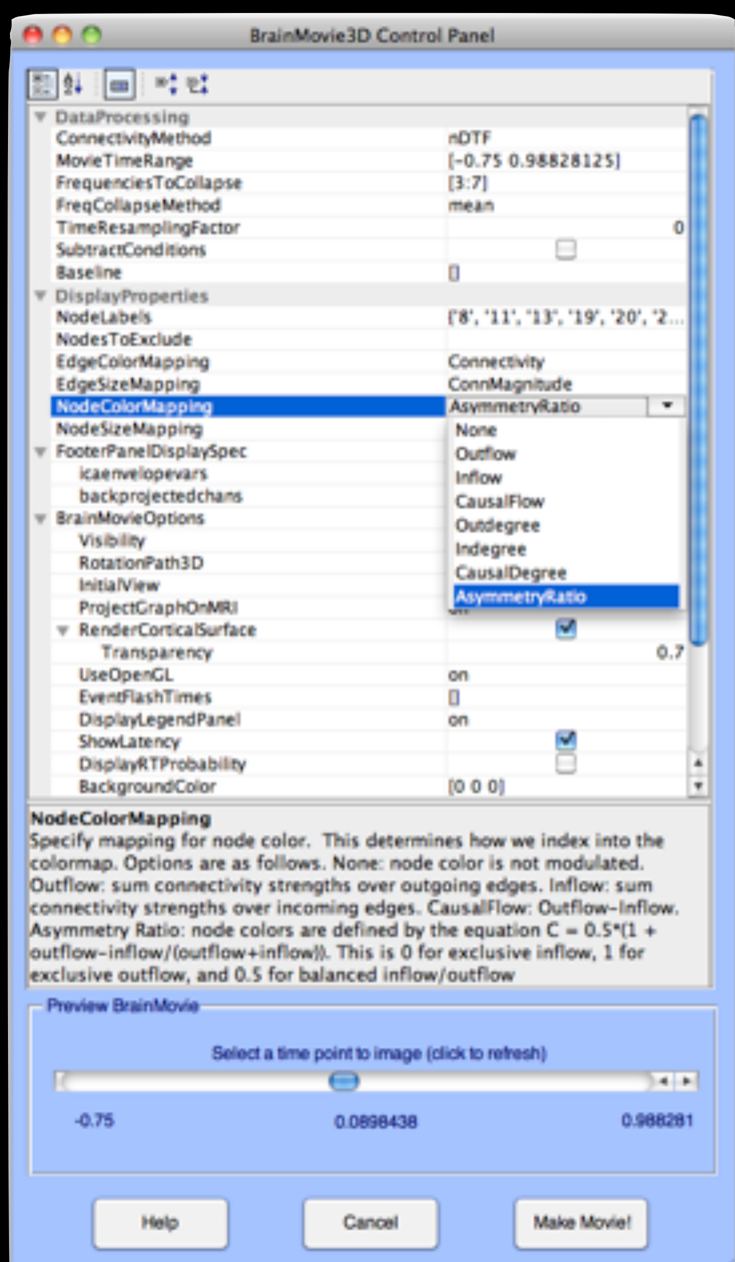


Interactive BrainMovie3D

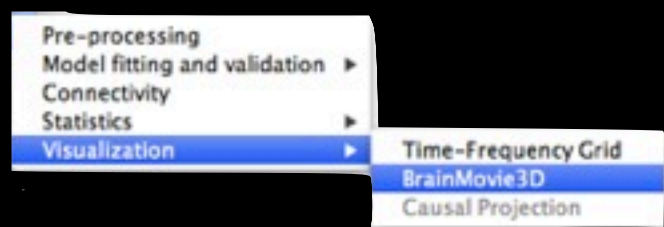


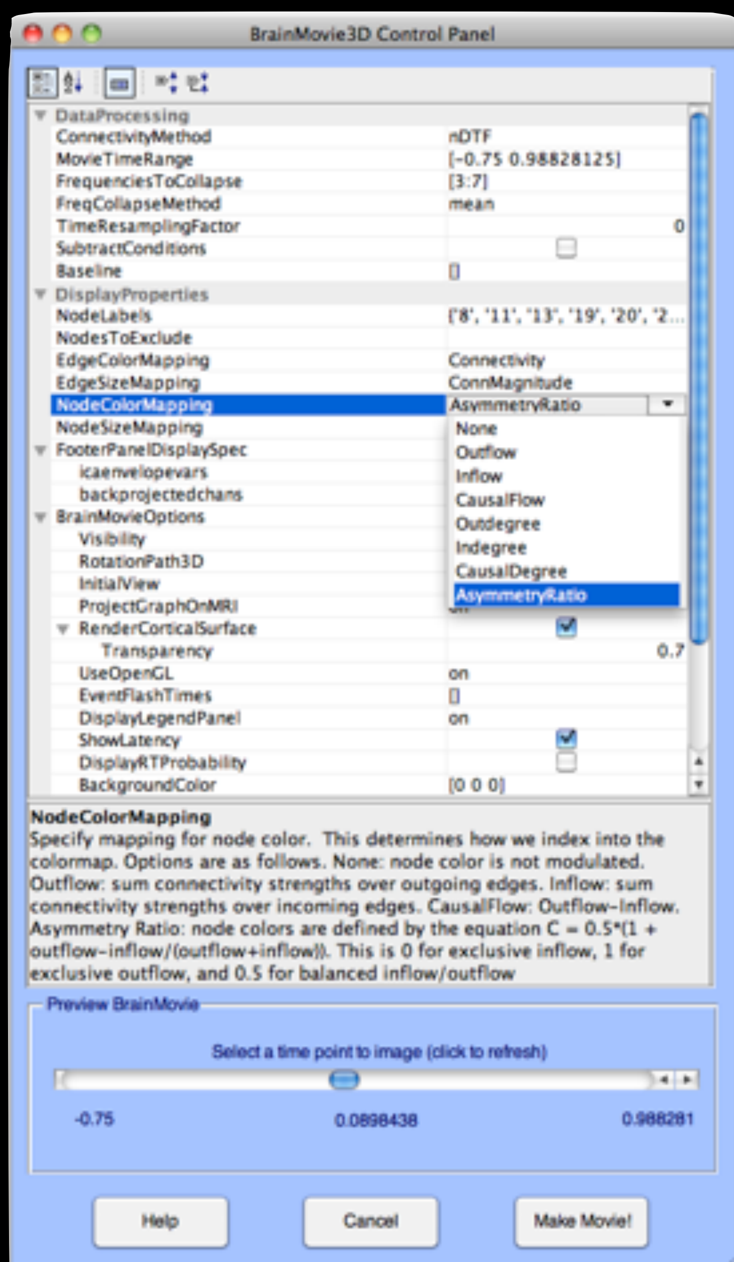
Interactive BrainMovie3D

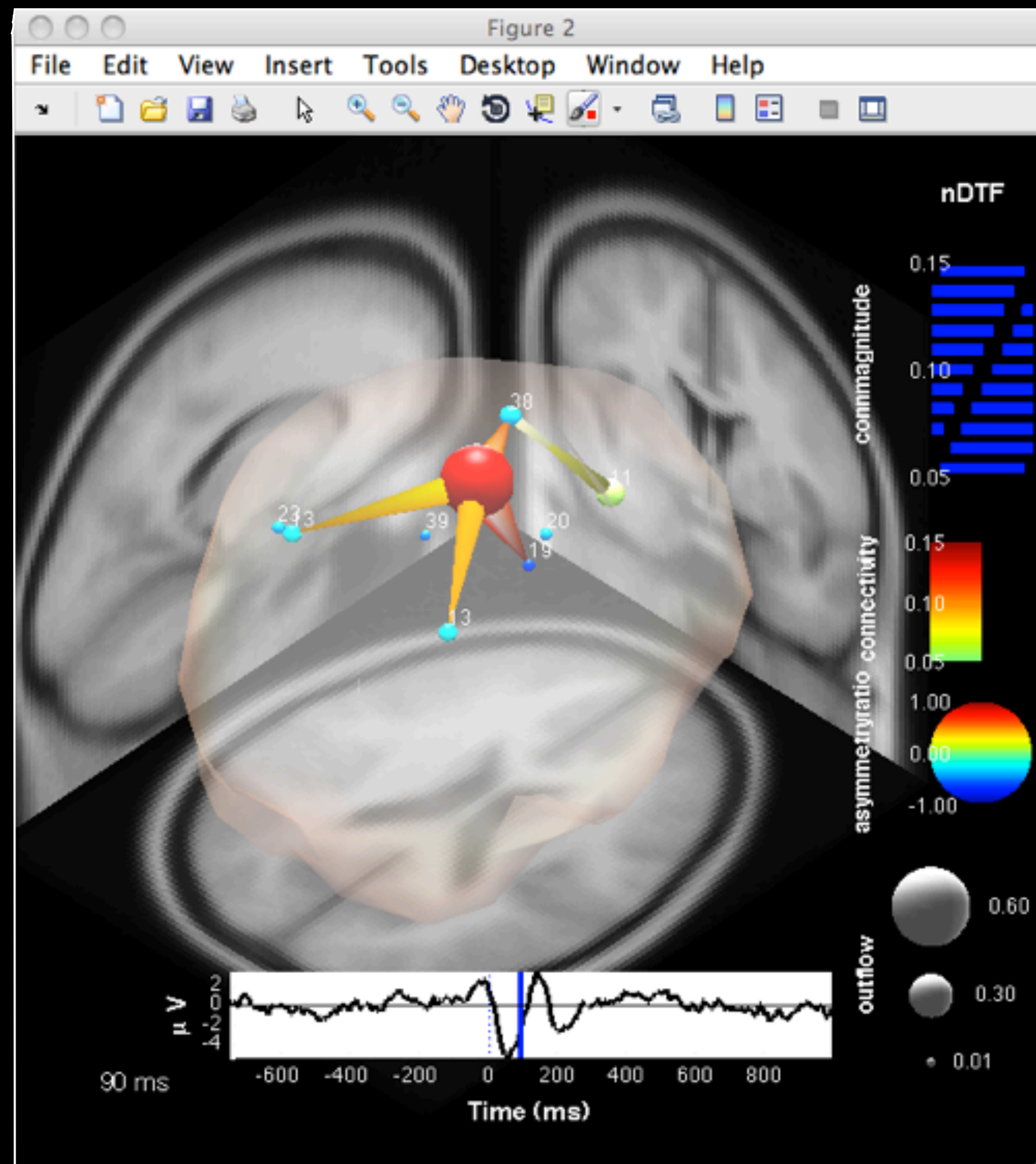




Interactive BrainMovie3D





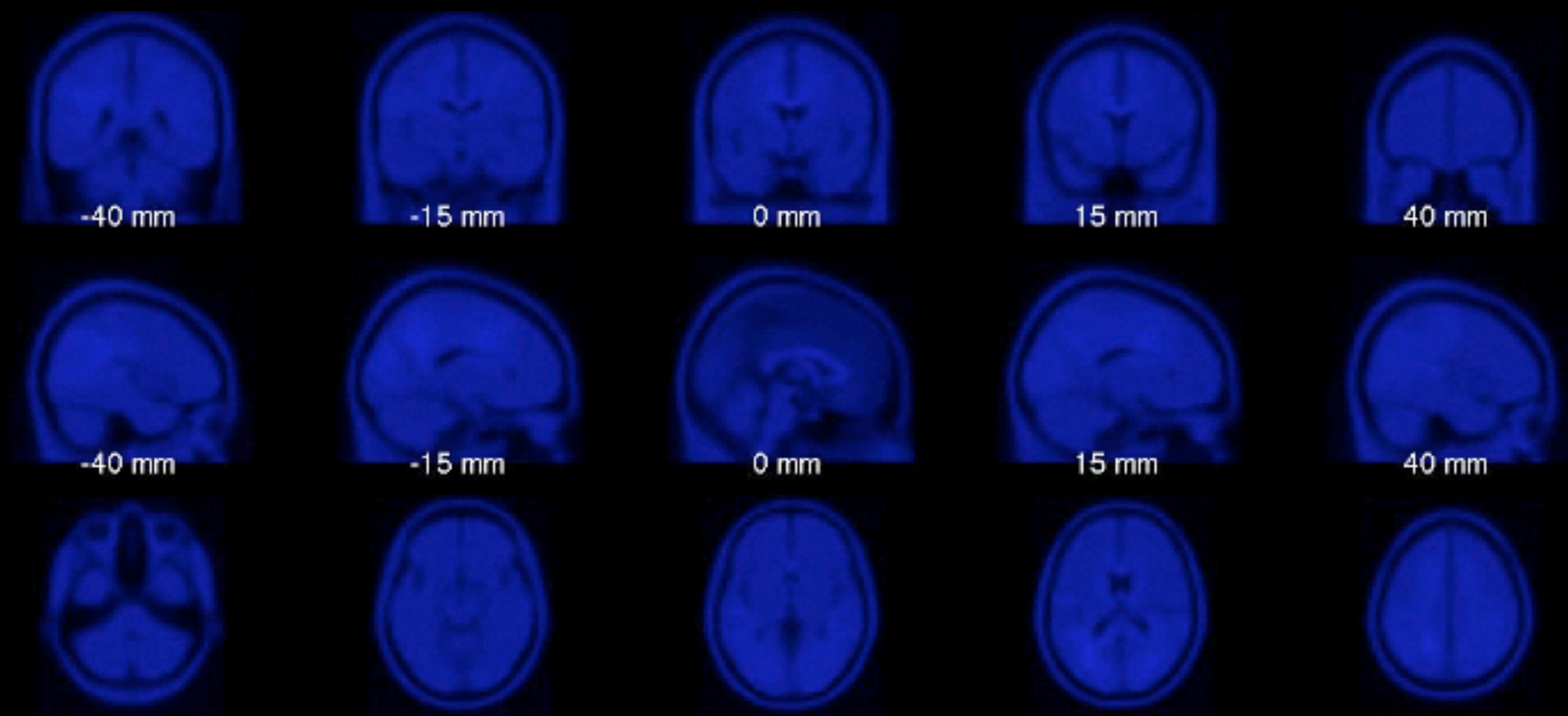


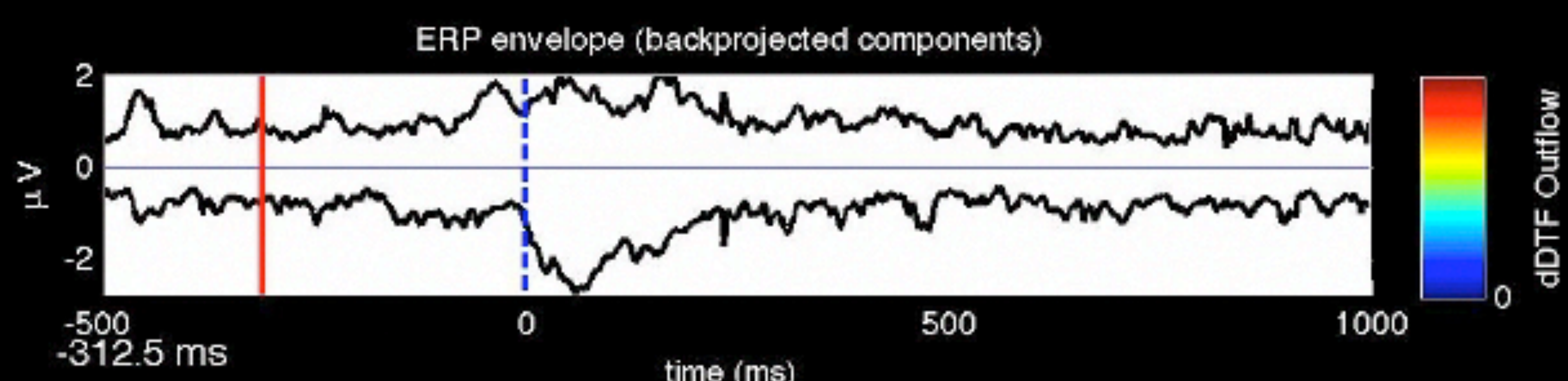
Causal Projection

Error > Correct ($p < 0.05$, N=24)

dDTF

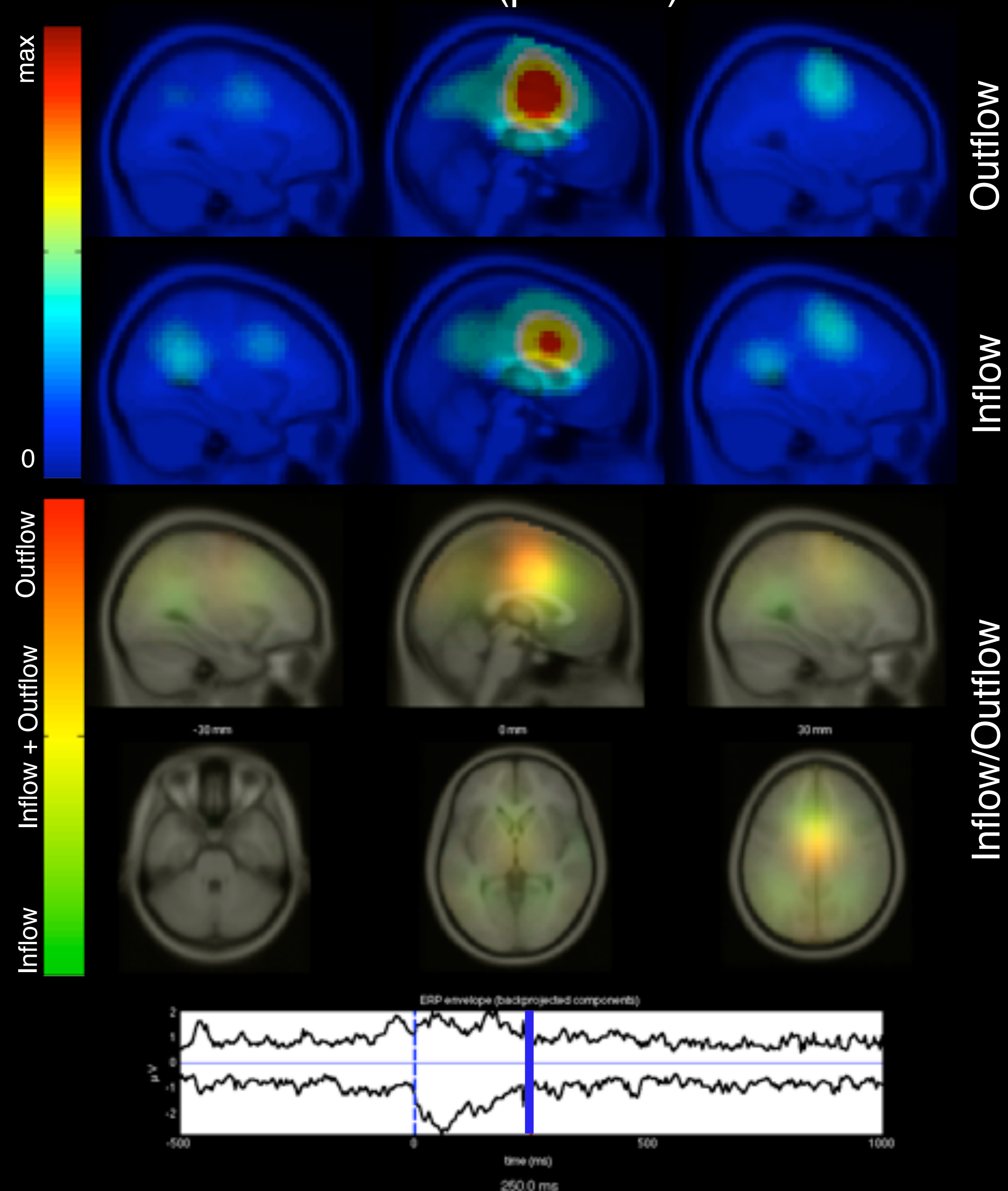
3-7 Hz





Causal Projection

Error > Correct ($p < 0.05$) 3-7 Hz



Group Analysis

partially-developed

Group Analysis

Disjoint Clustering

This approach adopts a 3-stage process:
1. Identify K ROI's (clusters) by affinity clustering of sources across subject population using EEGLAB's Measure-Product clustering.
2. Average all incoming and outgoing statistically significant connections between each pair of ROIs to create a [K X K [x freq x time]] group connectivity matrix.
3. Visualize the results using any of SIFTs visualization routines. This method suffers from low statistical power when subjects do not have high agreement in terms of source locations (missing variable problem).

partially-developed

Group Analysis

Disjoint Clustering

This approach adopts a 3-stage process:
1. Identify K ROI's (clusters) by affinity clustering of sources across subject population using EEGLAB's Measure-Product clustering.
2. Average all incoming and outgoing statistically significant connections between each pair of ROIs to create a [K X K [x freq x time]] group connectivity matrix.
3. Visualize the results using any of SIFTs visualization routines. This method suffers from low statistical power when subjects do not have high agreement in terms of source locations (missing variable problem).

Bayesian Mixture Model

A more robust approach (in development with Wes Thompson and to be released in SIFT 1.0b) uses smoothing splines and Monte-Carlo methods for joint estimation of posterior probability (with confidence intervals) of cluster centroid location and between-cluster connectivity. This method takes into account the "missing variable" problem inherent to the disjoint clustering approach and provides robust group connectivity statistics.

See Thompson and Mullen et al (2011), *ICON XI*

partially-developed

Bayesian Group Inference

Error > Baseline ($p < 0.01$, N=24)

dDTF

3-7 Hz