

# Forward and Inverse EEG Source Modeling

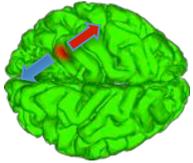


Scott Makeig

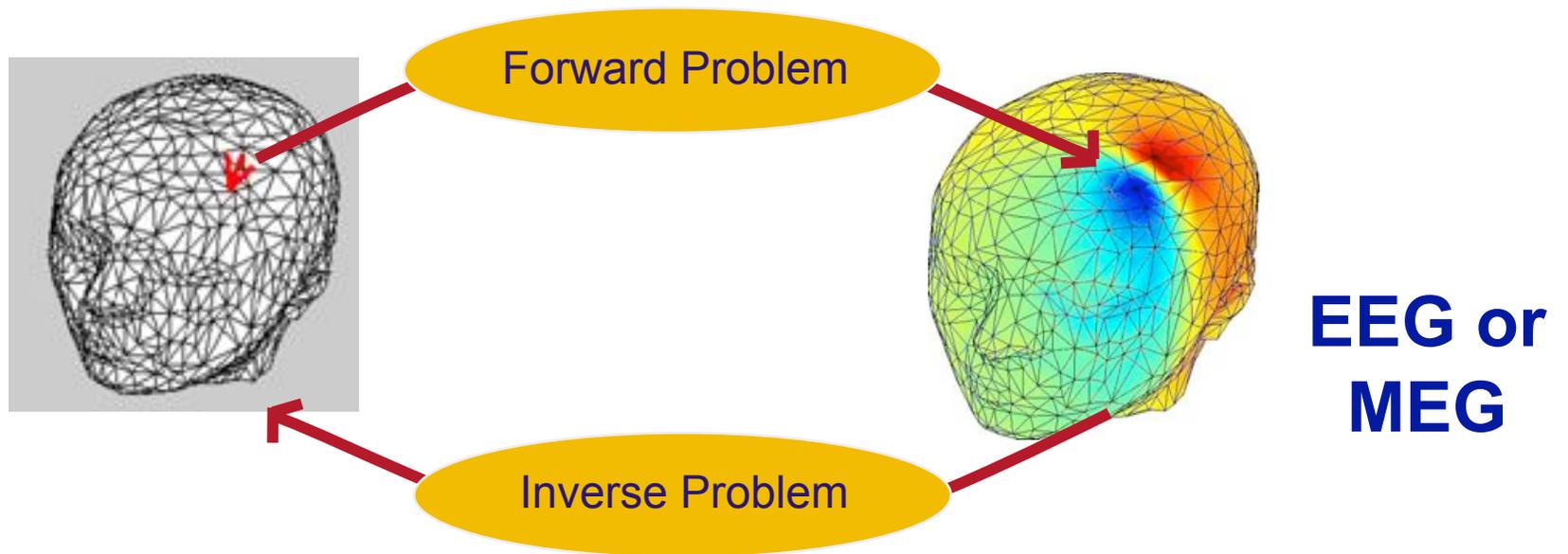
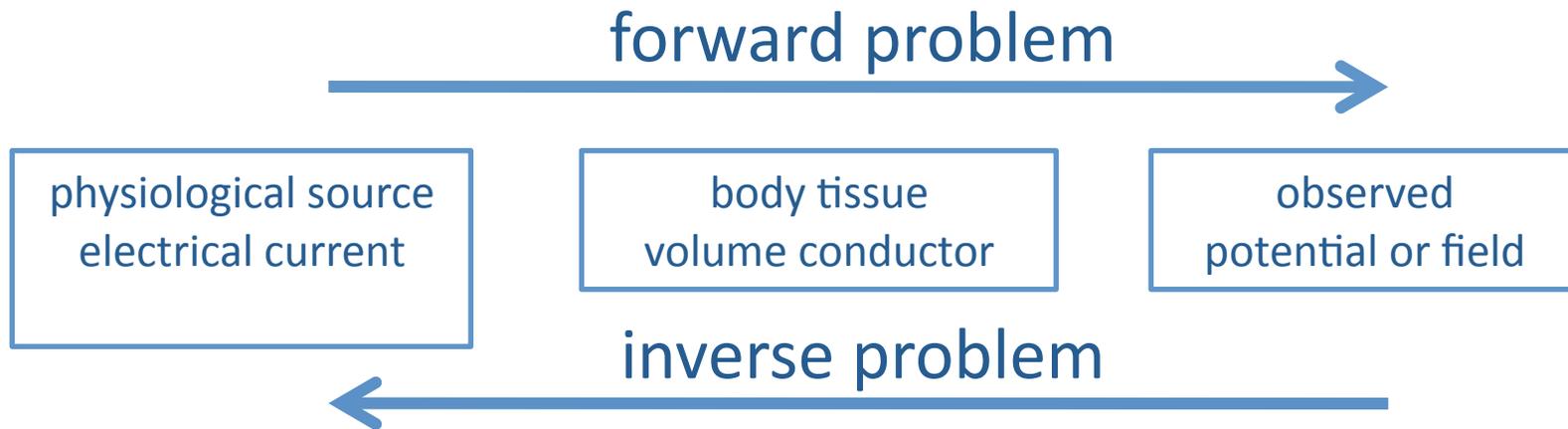
Institute for Neural Computation

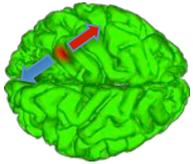
UCSD, La Jolla CA

EEGLAB Workshop, NCTU, Taiwan – Sept. 2010



# Source modeling



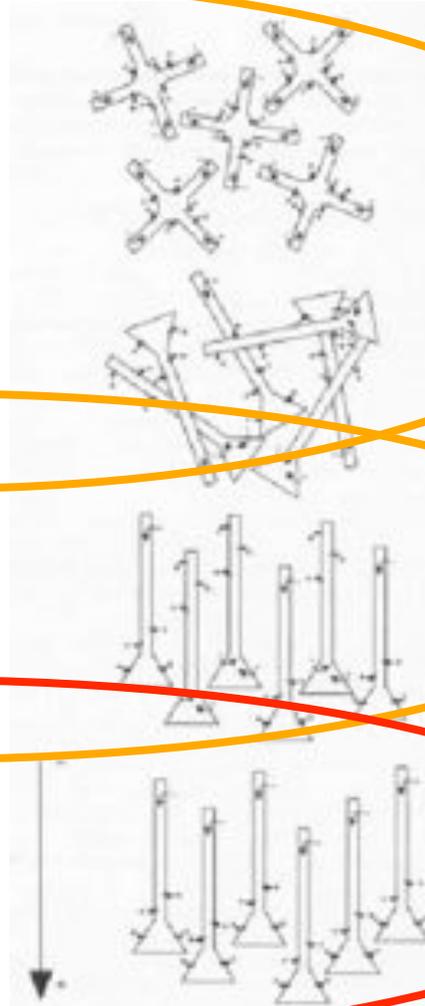


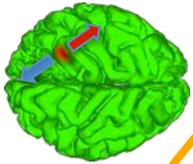
# Symmetry, orientation and activation

radially symmetric, i.e.  
randomly-oriented

asynchronously activated

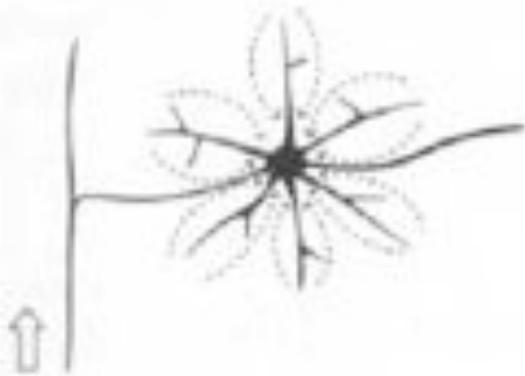
synchronously activated  
parallel-oriented





# Neuronal currents

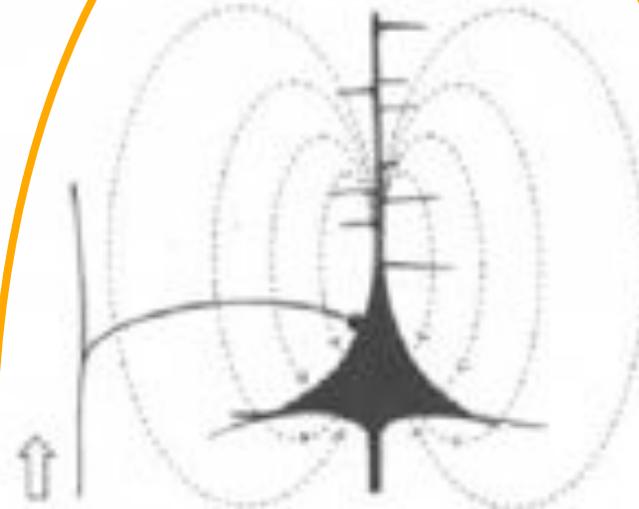
Stellate cell



Closed field

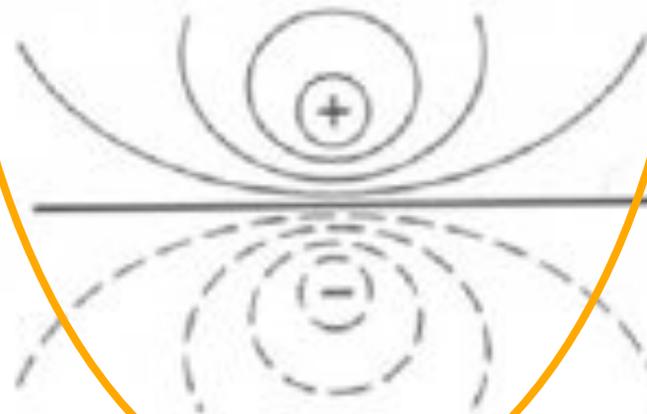


Pyramidal cell

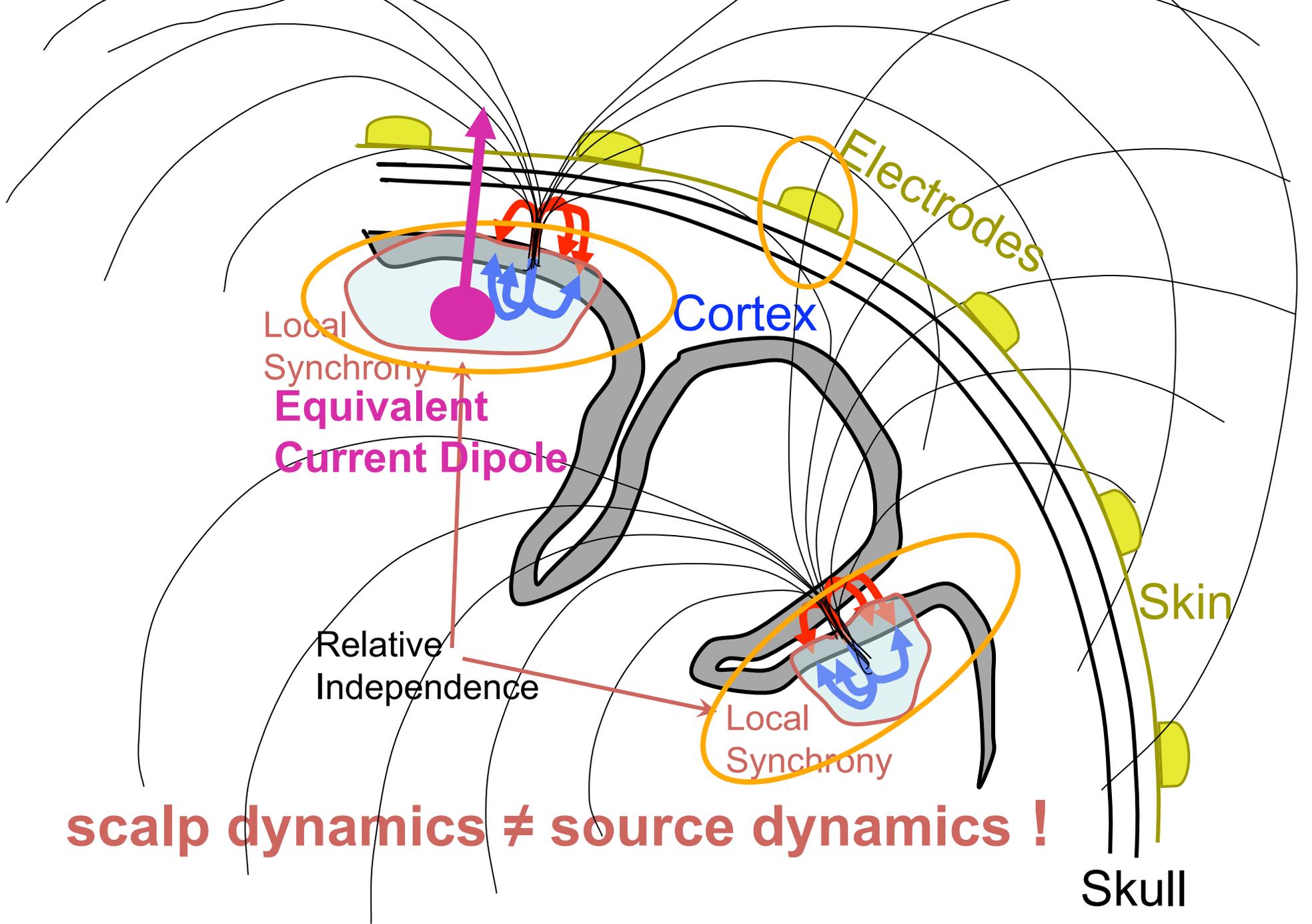


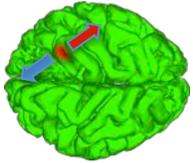
EEG

Open field



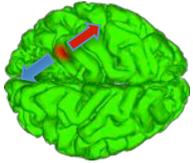
MEG



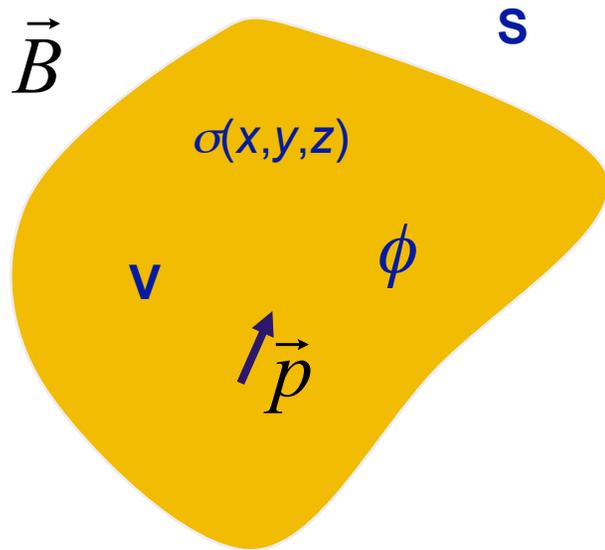


# EEG volume conduction

- **Potential difference between electrodes is measured.** This corresponds to current flowing through skin:
  - Only tiny fraction of current passes through skull
  - Therefore the model should describe both skull and skin as accurately as possible.
- **Problems with skull modeling**
  - Poorly visible in anatomical MRI (T2)
  - Thickness varies
  - Conductivity is not homogeneous
  - Complex geometry at front and base of skull



# Exact Formulation of the Forward Problem

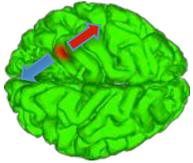


$$\begin{aligned}\nabla \cdot (\sigma \nabla \Phi) &= -\nabla \cdot J^P \quad \text{inside } V \\ \sigma \frac{\partial \Phi}{\partial n} &= 0 \quad \text{on } S\end{aligned}$$

$\sigma(x,y,z)$  : conductivity distribution

$p$  : current source

$\vec{p}$



# To Solve the Forward Head Model Problem ...

## WE NEED

### → Head Model

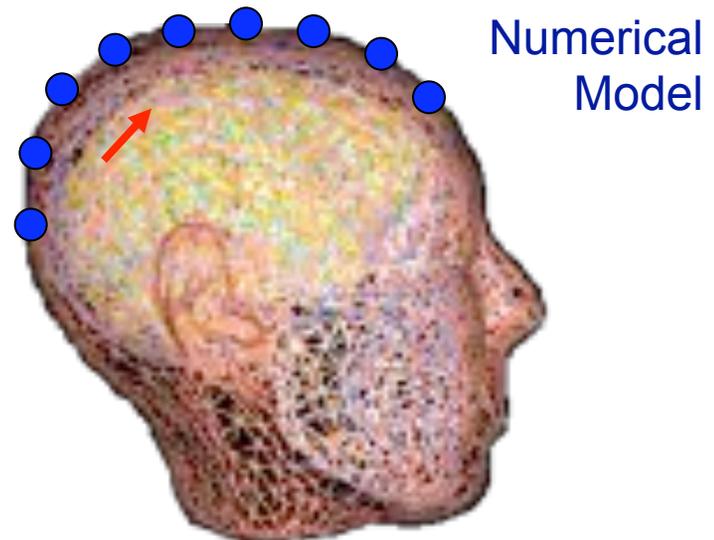
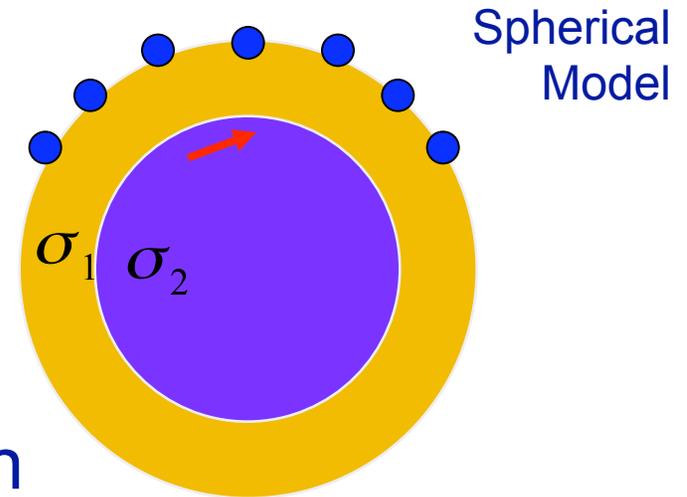
- Conductivity values
- Geometry

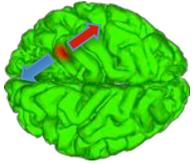
### → Sensor Locations

### → Possible source distribution

- Magnitudes
- Locations
- Directions

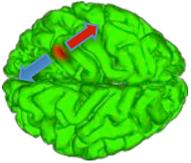
### → Solver





## Source Localization Requirements

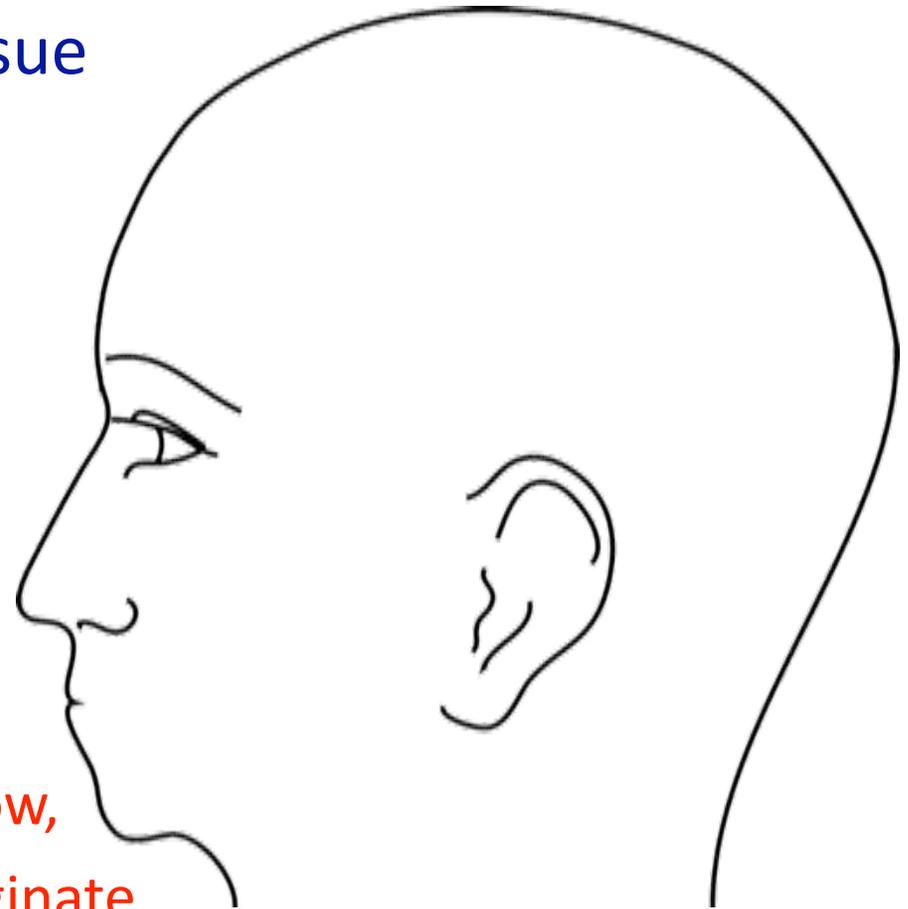
- ◆ Selected/processed EEG signal
  - **Simple single-source scalp map !**
- ◆ Number/positions of electrodes on the head surface
- ◆ Numerical head model
- ◆ Co-registration of EEG electrodes with head model
- ◆ *A priori* information/guess about the source space
- ◆ Choice of inverse model
- ◆ Choice of numerical method

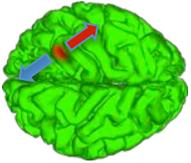


# Volume conductor model

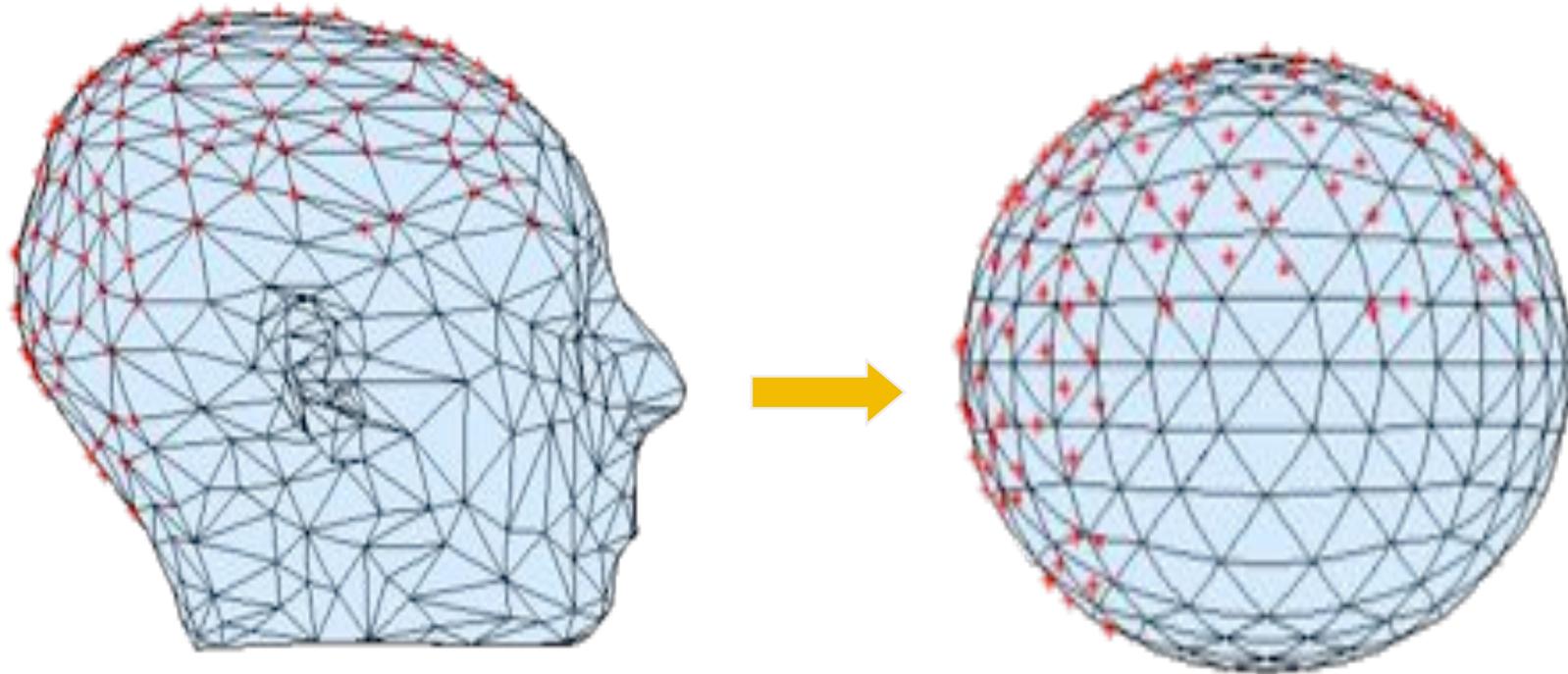
- Electrical properties of tissue
- Geometrical description
  - spherical model
  - realistically shaped model

→ Describes how the currents flow,  
from where they may originate

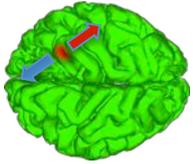




## Errors in Simple Head Models



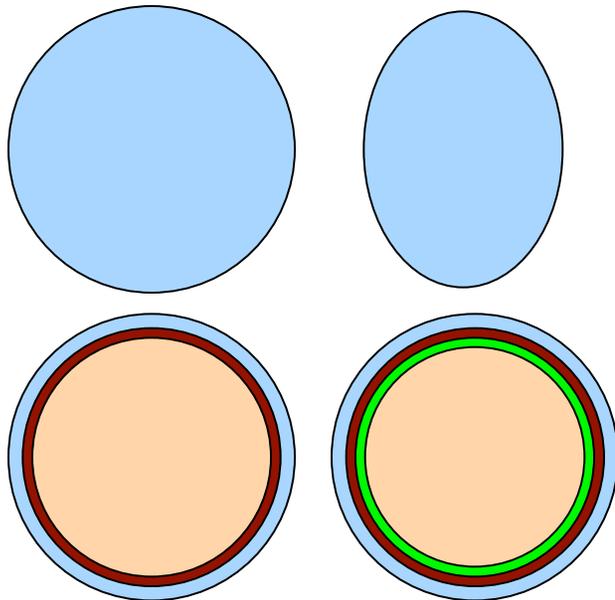
- In the volume conductor model
- In the electrode locations



# Head Model Comparison

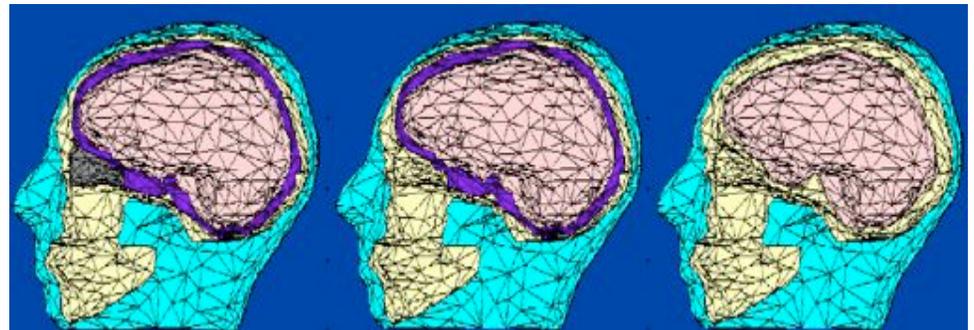
## ◆ Simple head models

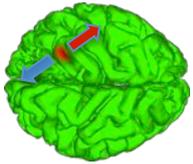
- Single sphere
- 3-4 Layer Spherical
- Spheroid



## ◆ Realistic head models

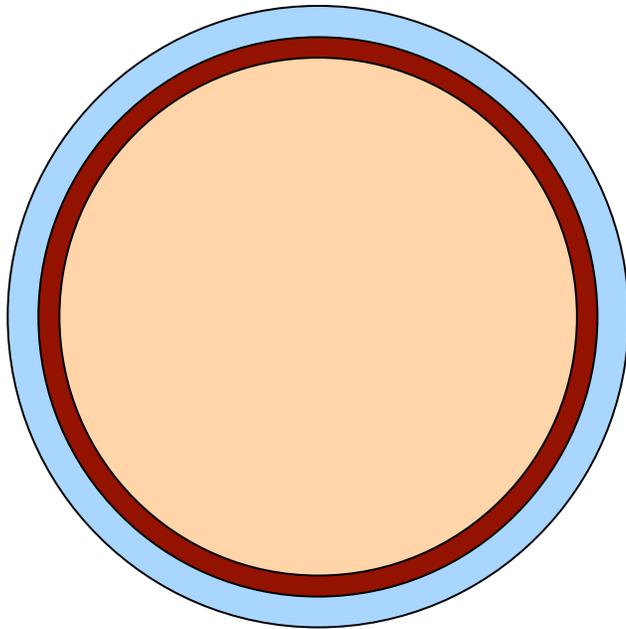
- Boundary Element Method
- Finite Element Method
- Finite Difference Method



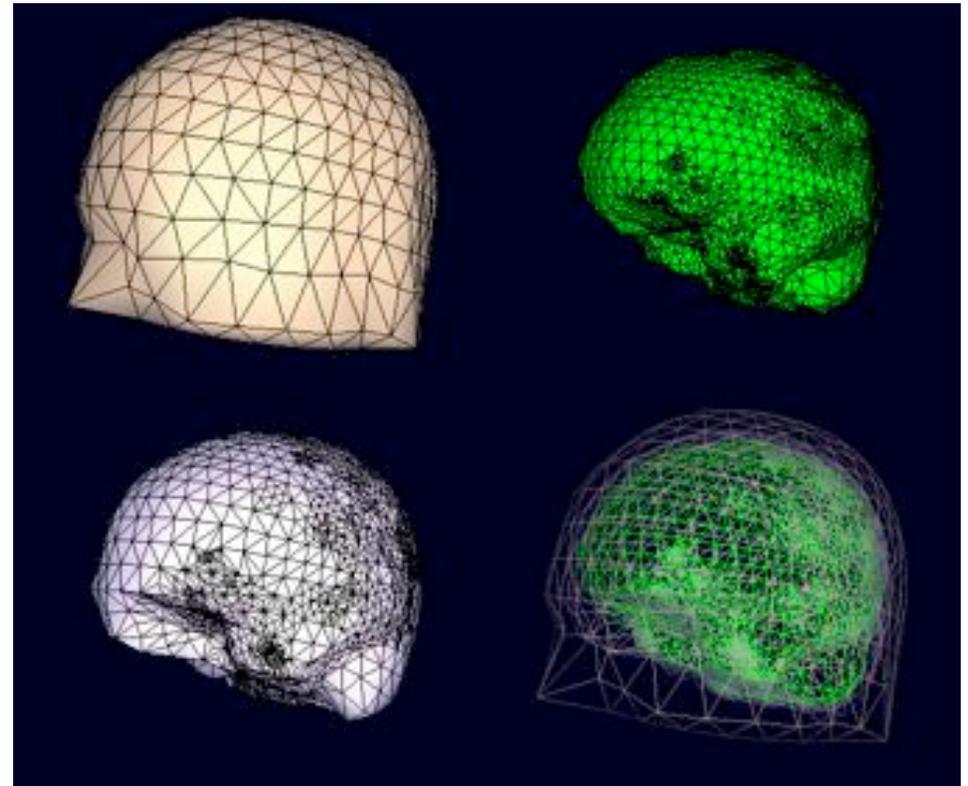


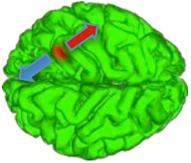
# Effects of Head Model

Spherical head model  
(3-layer standard)



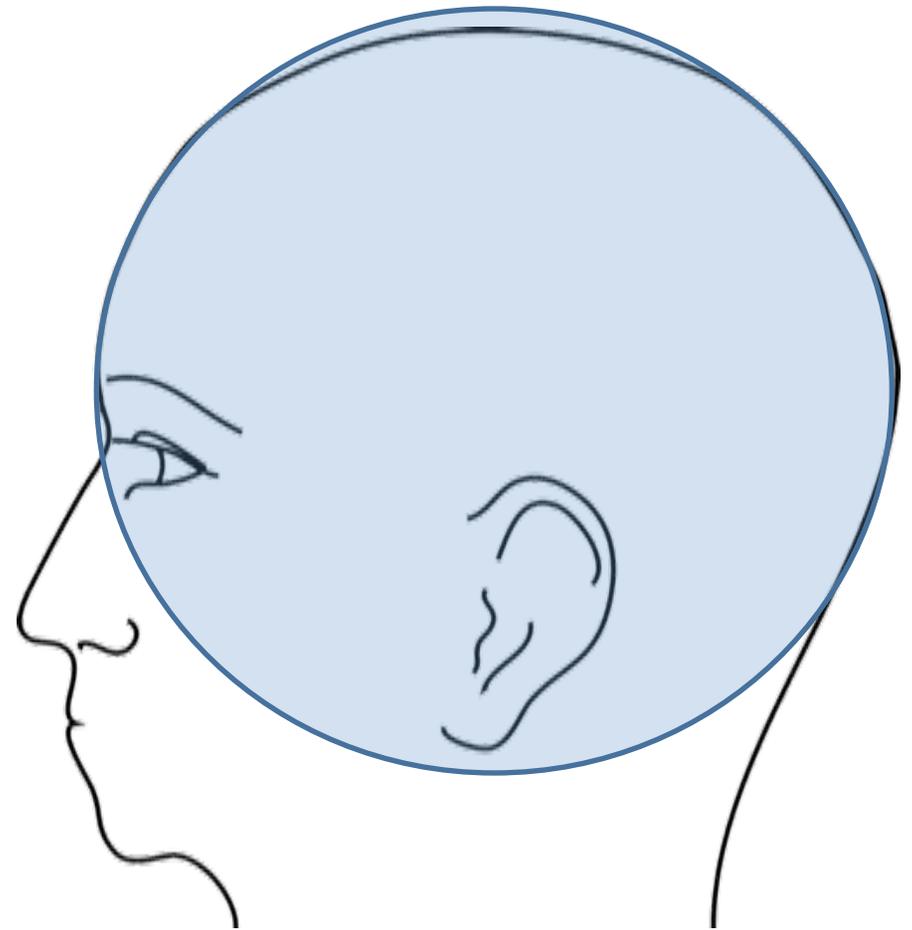
Standard MNI head model  
(4-layer mean BEM)

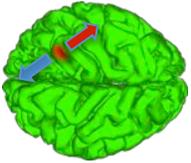




# Spherical volume conductor

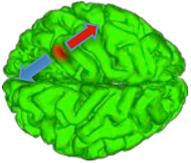
- Advantages of the **spherical** head model
  - mathematically exact
  - fast to compute
  - reasonably accurate
  - easy to use
- Disadvantages of the **spherical** model
  - difficult to align properly
  - inaccurate in some regions





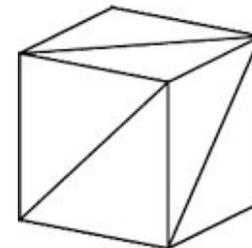
## Realistic volume conductor

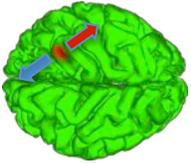
- Advantages of a **realistic** head model
    - a more accurate solution (especially for EEG)
  - Disadvantages of a **realistic** model
    - more work to build from an MR image
    - slower to compute
    - might be numerically unstable
    - harder to make between-subject comparisons
- A pragmatic (easy, cheap) solution is to use a standard (mean) realistic head model (MNI).



## Realistic volume conductor

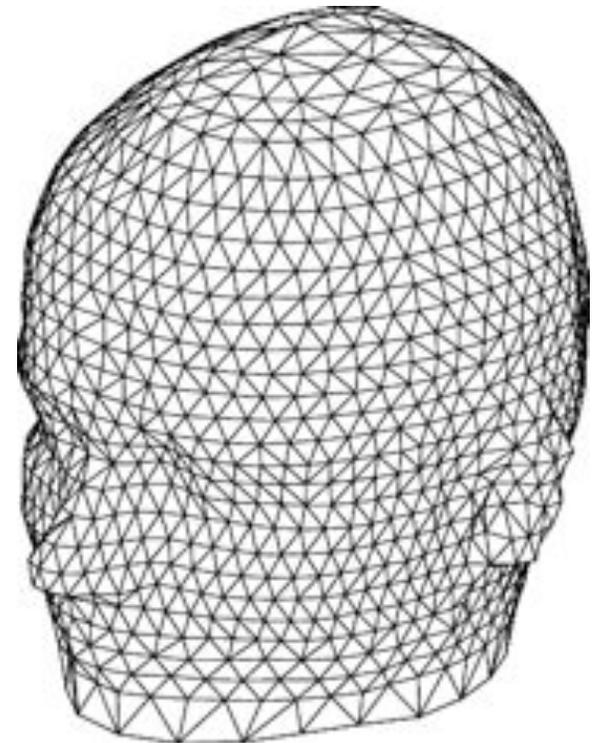
- Computational methods for volume conduction problem that allow realistic geometries:
  - Boundary Element Method (BEM)
  - Finite Element Method (FEM)
- Geometrical description
  - Triangles (planar or quadratic)
  - Tetrahedra (3-D)

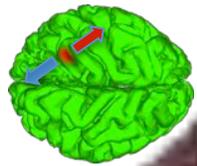




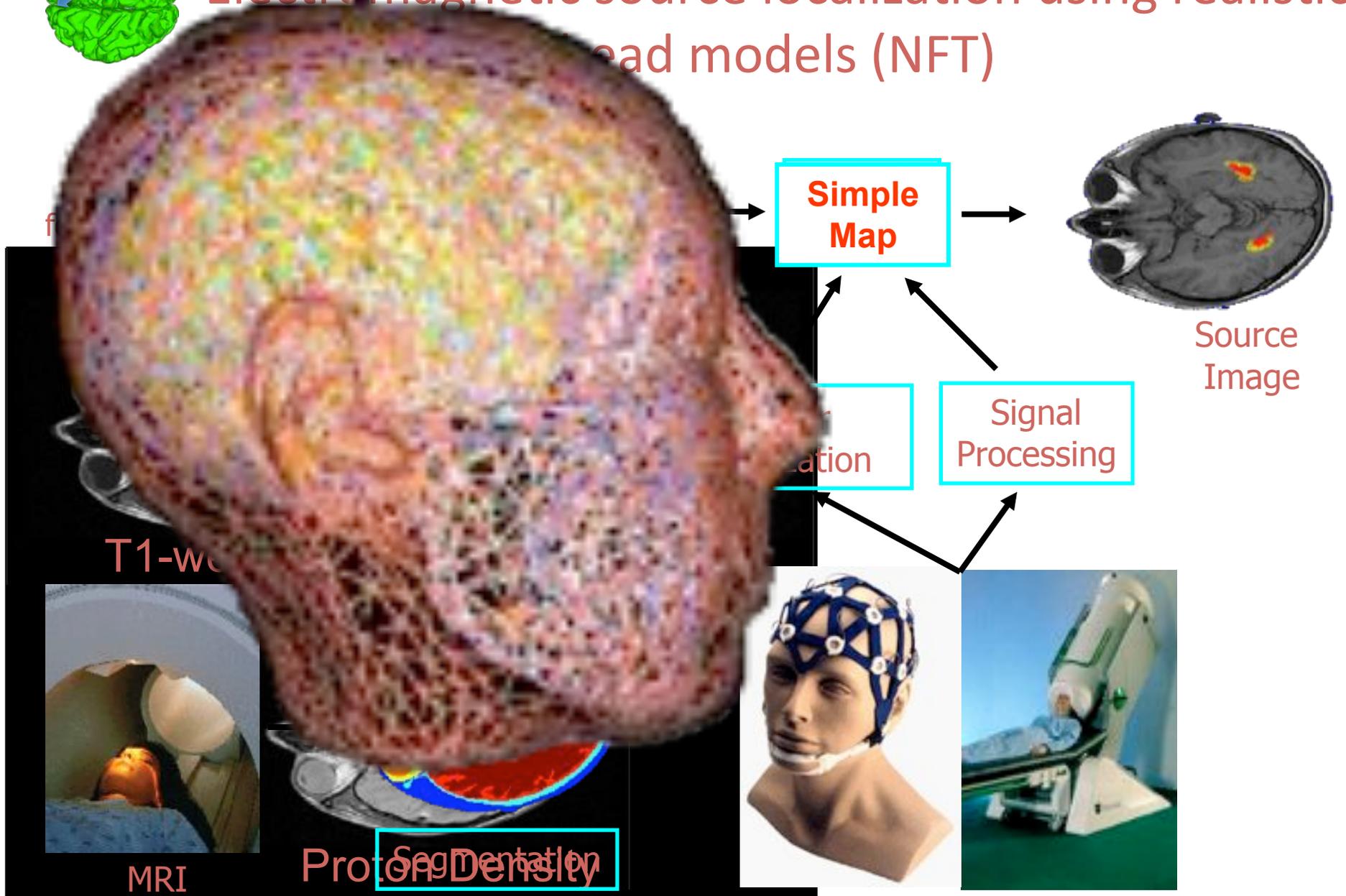
# BEM volume conductor

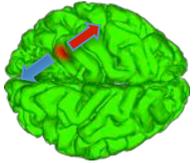
- Boundary Element Method
  - description of geometry by compartments
  - each compartment is
    - homogenous
    - isotropic
  - important tissues
    - skin
    - skull
    - brain
    - (CSF)
  - triangulated surfaces as boundaries
  - surfaces should be closed





# Electromagnetic source localization using realistic head models (NFT)





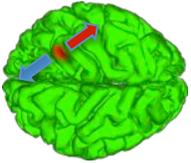
# A Four-Layer BEM Head Model



**N**euroelectromagnetic  
**F**orward head modeling  
**T**oolbox (**NFT**)

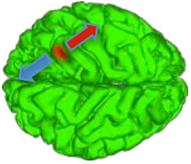
## # of elements

Scalp:	6900
Skull:	6800
CSF:	9000
Brain:	8800
<b>Total</b>	<b>31500</b>



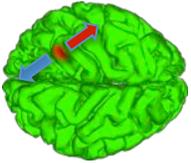
## FEM volume conductor

- Tessellate the 3-D volume into solid tetrahedra
  - Large number of elements
  - Each tetrahedron can have its own conductivity
  - Each tetrahedron can have its own anisotropy
- FEM is most accurate numerical method
  - Computationally expensive
  - Accurate conductivities are not known



# Inverse problem methods

- **Single and multiple dipole models**
  - Minimize error between the model and the measured potential/field
- **Distributed dipole models**
  - Perfect fit of model to the measured potential/field
  - Minimize an additional constraint on sources
    - LORETA (assume a smooth distribution)
    - Minimum Norm (L2, minimum power at the cortex)
    - Minimum Current (L1, minimum current in the cortex)

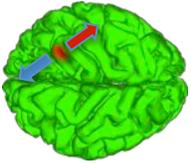


# Inverse problem methods

- **Spatial source filtering**

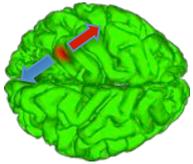
- **Scan whole brain** with single dipole and compute the filter output at every location (second-order covariance matrix)
  - MUSIC algorithm
  - *Beamforming* (e.g., LCMV, SAM, DICS)
- **Perform ICA decomposition** (higher-order statistics)
  - Of the scalp maps at individual moments
  - ICA gives the projections of the sources to the scalp surface, i.e., **‘simple’ maps!**

→ ICA solves **‘the first half’** of the inverse problem (**‘What?’**)



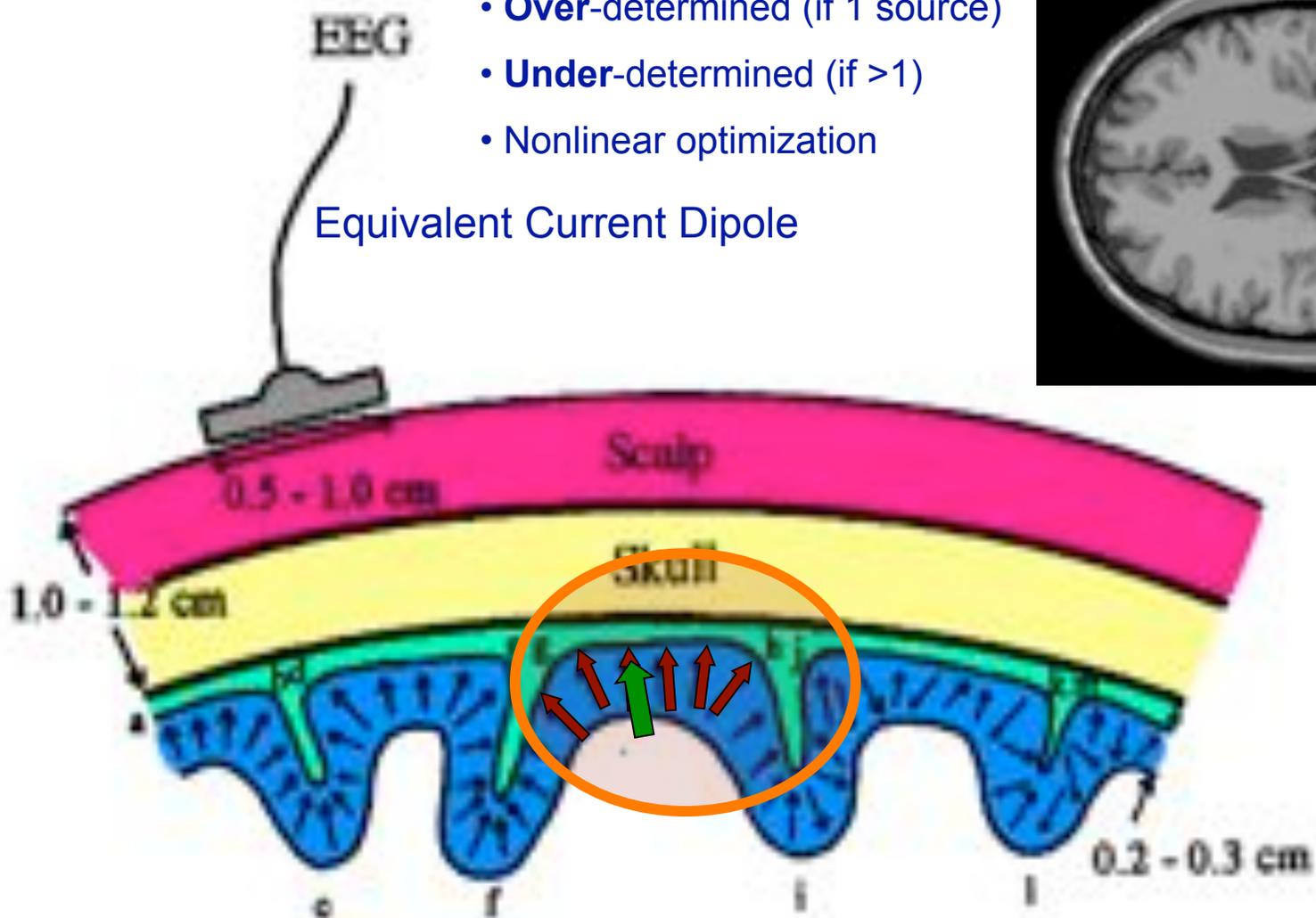
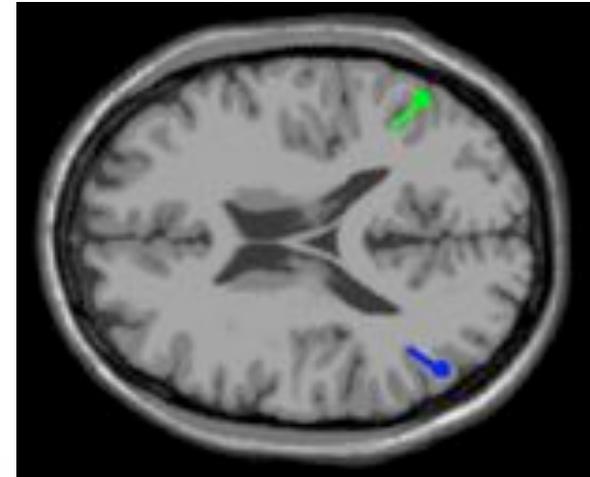
# Equivalent current dipoles

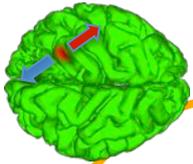
- Physical/mathematical motivation
  - Any current distribution can be written as a multipole expansion
  - First term: monopole (must be 0)
  - Second term: dipole
  - Higher order terms: quadrupole, ...
- Convenience
  - **Dipoles** can be used as building blocks in distributed source models



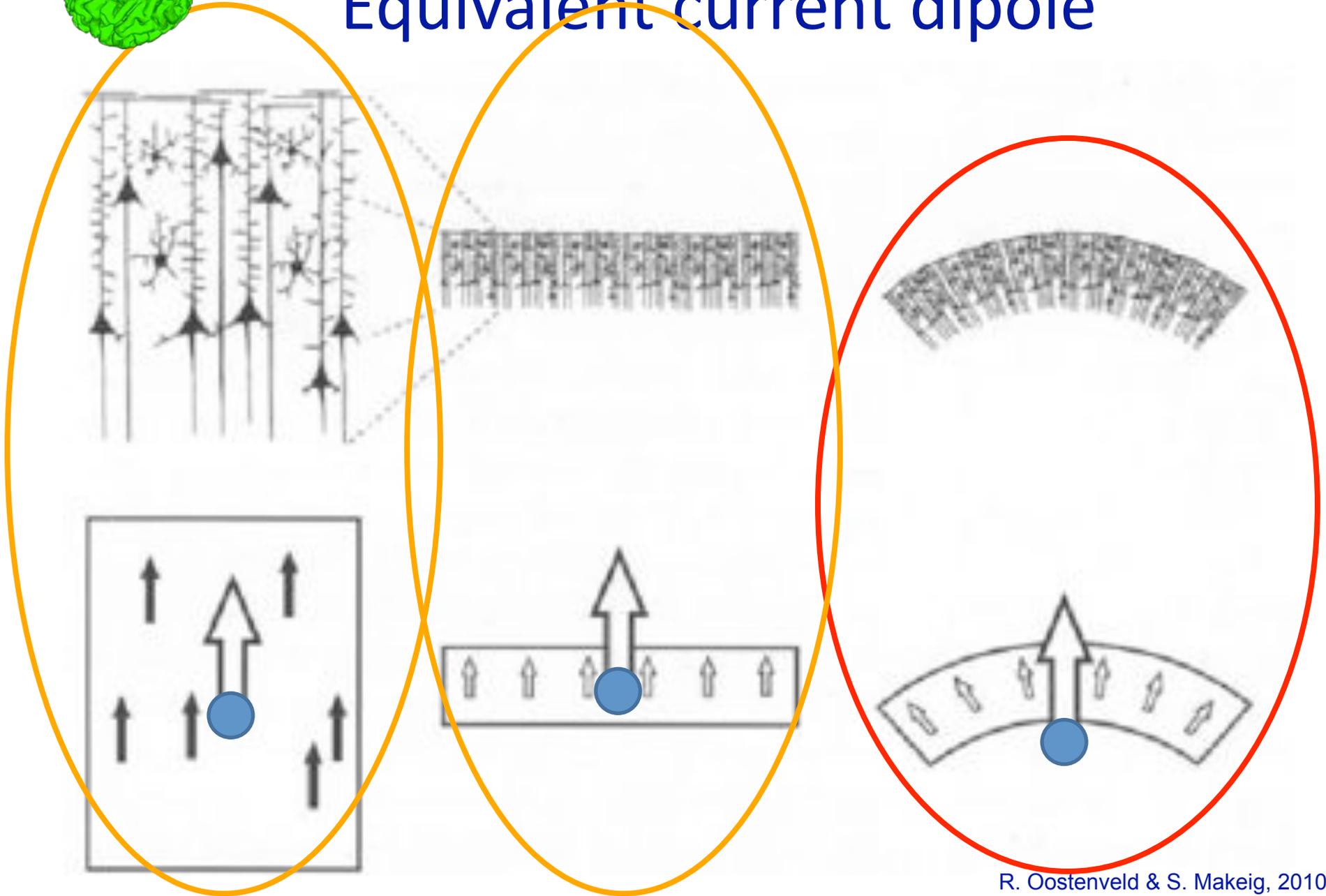
# Equivalent current dipoles

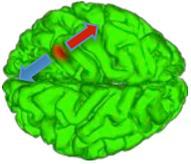
- **Over-determined** (if 1 source)
  - **Under-determined** (if  $>1$ )
  - Nonlinear optimization
- Equivalent Current Dipole





# Equivalent current dipole





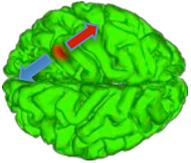
# Measured Errors in Dipole Source Localization

## ◆ Experimental studies

- Phantom → 10 mm loc. error (Henderson & Butler, 1975)
- Human skull → 35 mm (Weinberg et al, 1986)

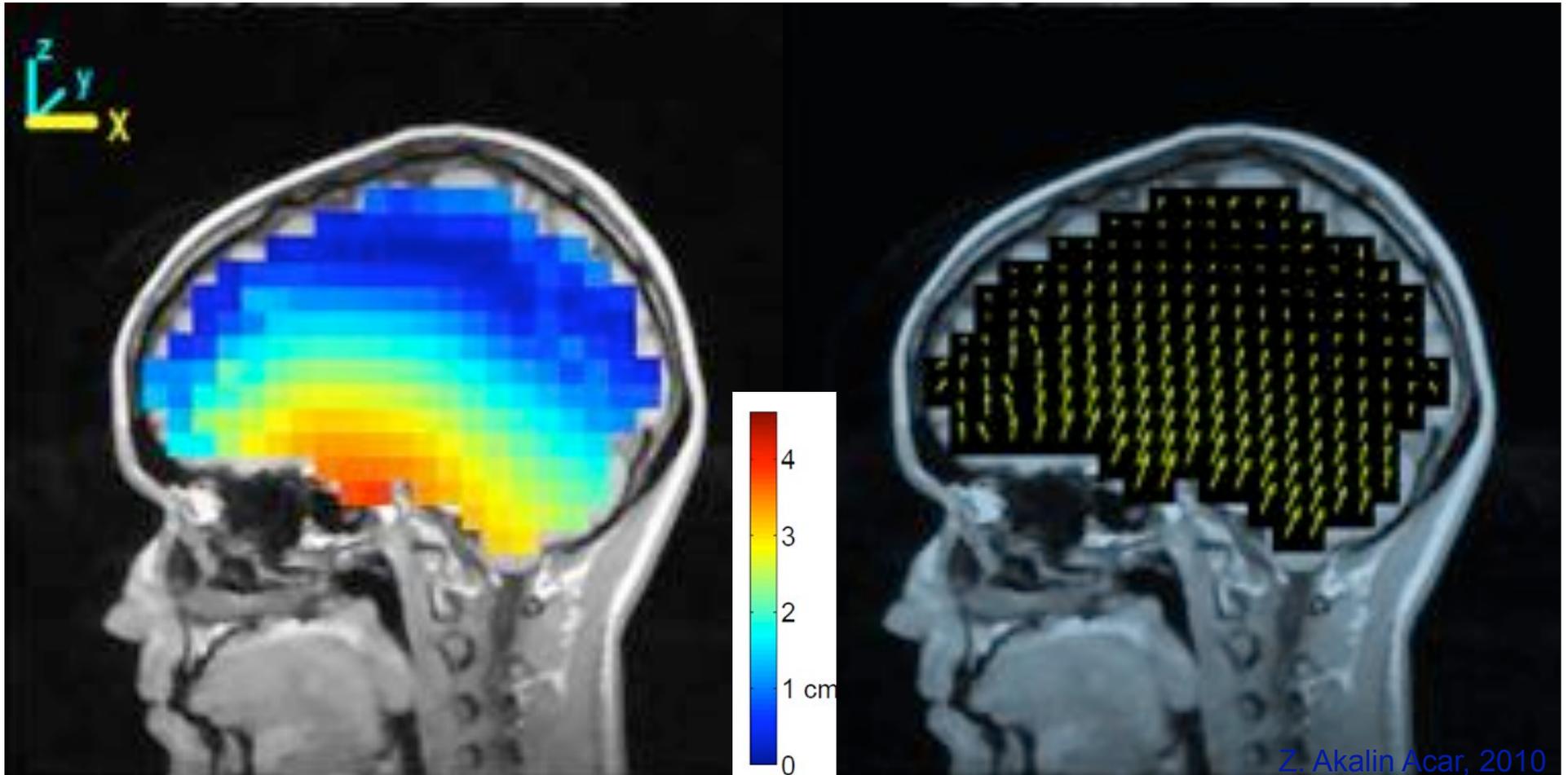
## ◆ Simulation studies

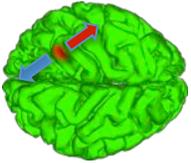
- 3-layer model → 15-25 mm (Roth et al, 1993)
- 3-layer model → 9-14 mm (Vanrumste et al, 2002)
- Human skull → 25 mm (Fletcher et al, 1993)
- 3-layer model → ~8 mm (Akalin Acar, 2005)



# Source Localization Errors

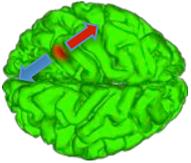
- For a **3-layer spherical** head model
- Relative to 4-layer realistic BEM head model





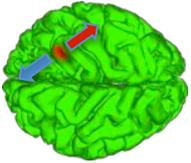
## Single vs. multiple dipole models

- Manipulate source parameters to **minimize error** between measured and model data
  - **Position** of each source
  - **Orientation** of each source
  - **Strength** of each source
- **Orientation** and **strength** together correspond to the “dipole moment” and can be estimated *linearly*
  - **Position** is estimated *non-linearly* by iterative source parameter estimation



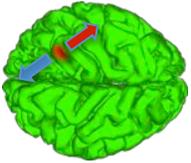
## Dipole scanning: grid search

- Define grid with allowed dipole locations
- Compute optimal dipole moment for each location
- Compute value of goal-function
- Plot value of goal-function on grid
- Number of evaluations:
  - single dipole, 1 cm grid:  $\sim 4,000$
  - single dipole,  $\frac{1}{2}$  cm grid:  $\sim 32,000$
  - BUT two dipoles, 1 cm grid:  $\sim 16,000,000$



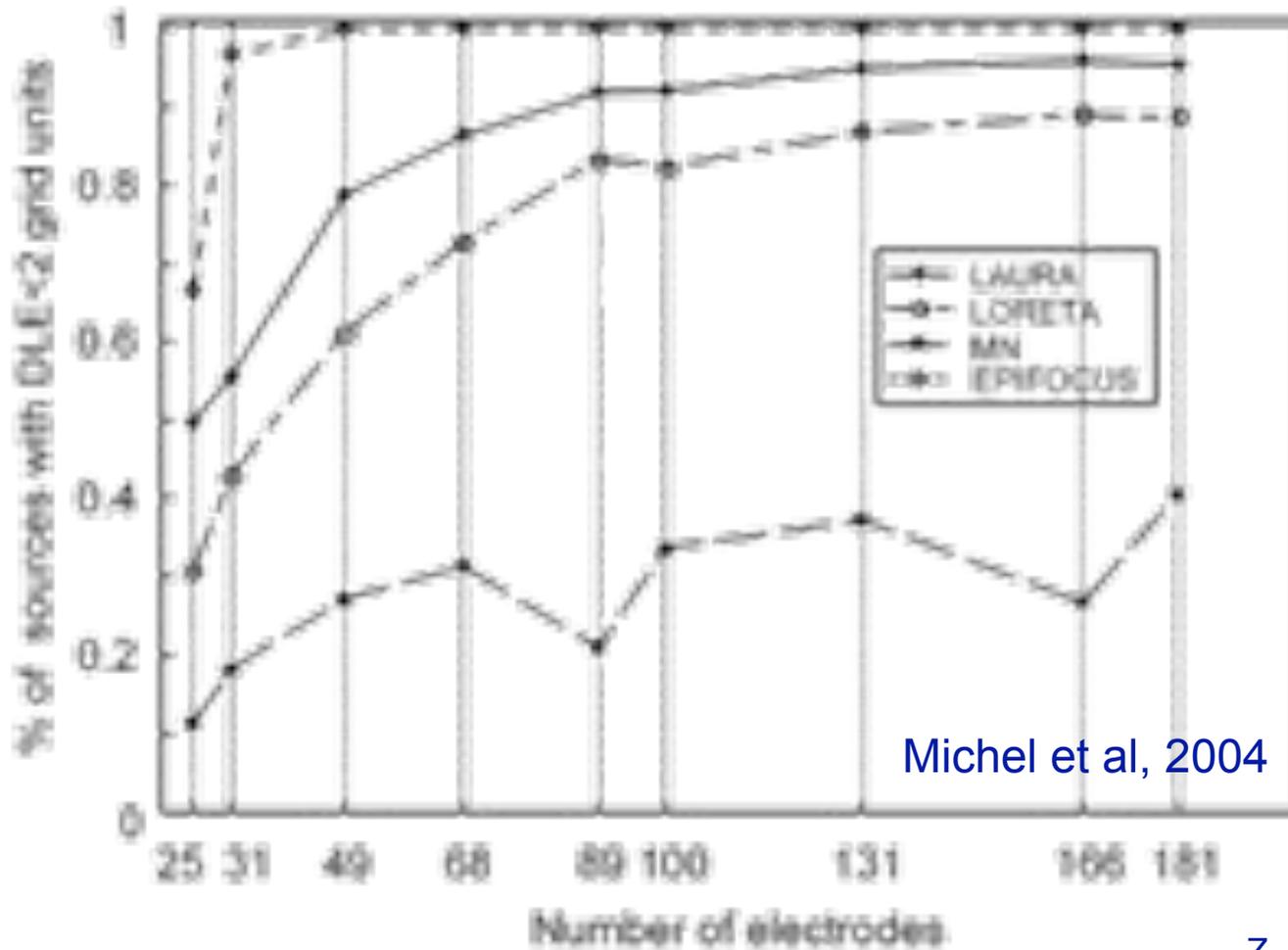
## Dipole fitting: nonlinear search

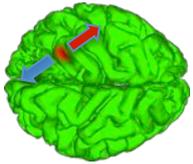
- Start with an initial guess from coarse fitting
  - evaluate the local derivative of goal-function
  - “walk down hill” to the most optimal solution
- Number of evaluations needed  $\sim 100$



## Effect of Number of Electrodes

- Single dipole source
- 3-layer spherical head model
- 1152 solution points



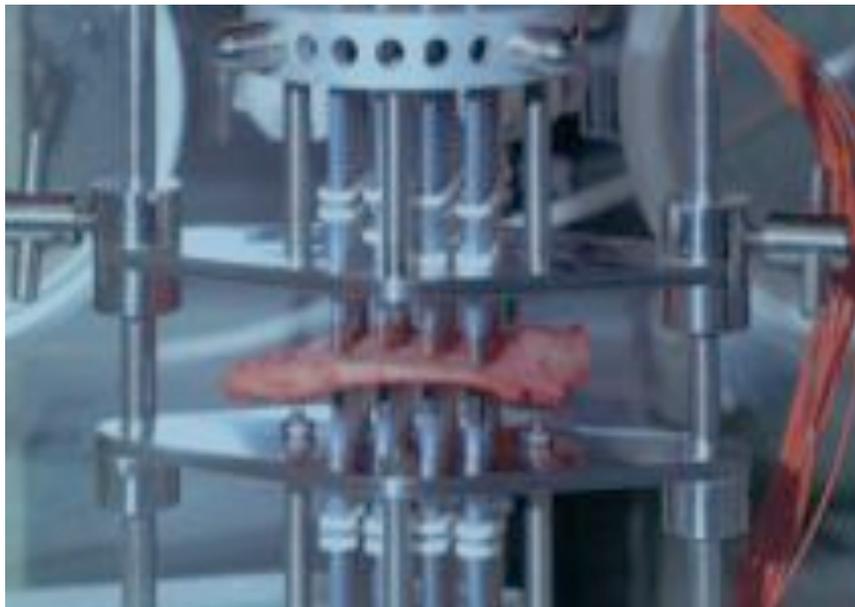


# Effects of Skull Conductivity Estimate

Measurements of skull conductivity:

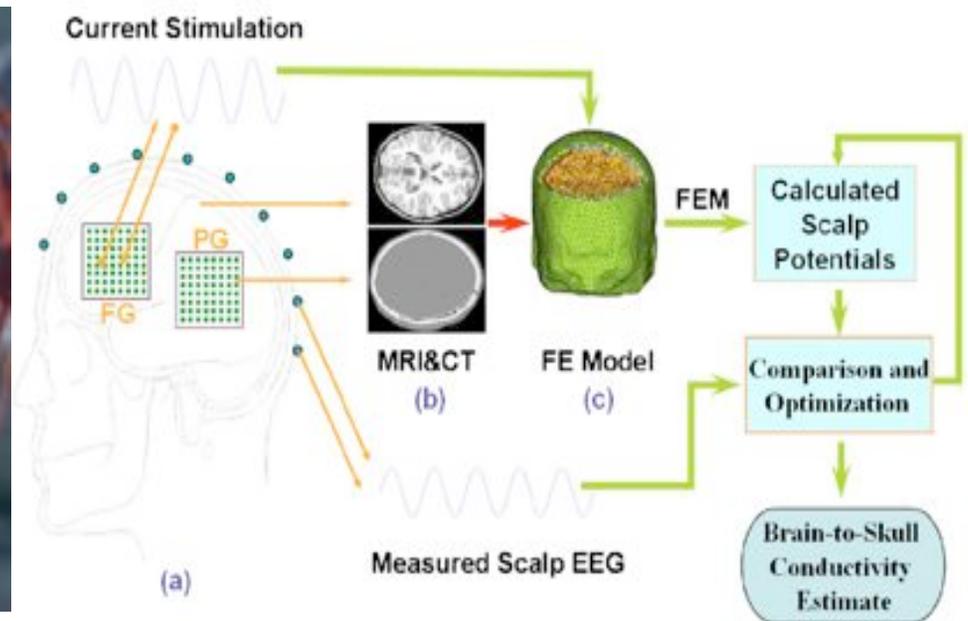
- MR-EIT
- Magnetic stimulation
- Current injection

In vivo



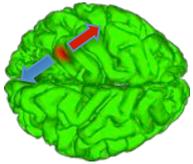
Hoekama et al, 2003

In vitro



He et al, 2005

Z. Akalin Acar, 2010



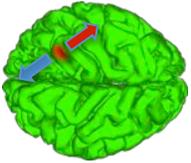
# Effects of Skull Conductivity Estimate

Brain to skull ratio		
Rush and Driscoll	1968	80
Cohen and Cuffin	1983	80
Oostendorp et al	2000	15
Lai et al	2005	25

Measurement	Age	$\sigma$ (mS/m)	Sd (mS/m)
Agar-agar phantom	–	43.6	3.1
Patient 1	11	80.1	5.5
Patient 2	25	71.2	8.3
Patient 3	36	53.7	4.3
Patient 4	46	34.4	2.3
Patient 5	50	32.0	4.5
Post mortem skull	68	21.4	1.3

Skull conductivity  
by age

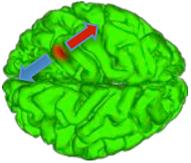
Hoekama et al, 2003



## Effect of reference electrode

“The choice of a particular reference electrode ... does not change in any way the biophysical information contained in the potential distribution. It does not in any way change the relation between source and potential, except for an additive constant of no physical significance.”

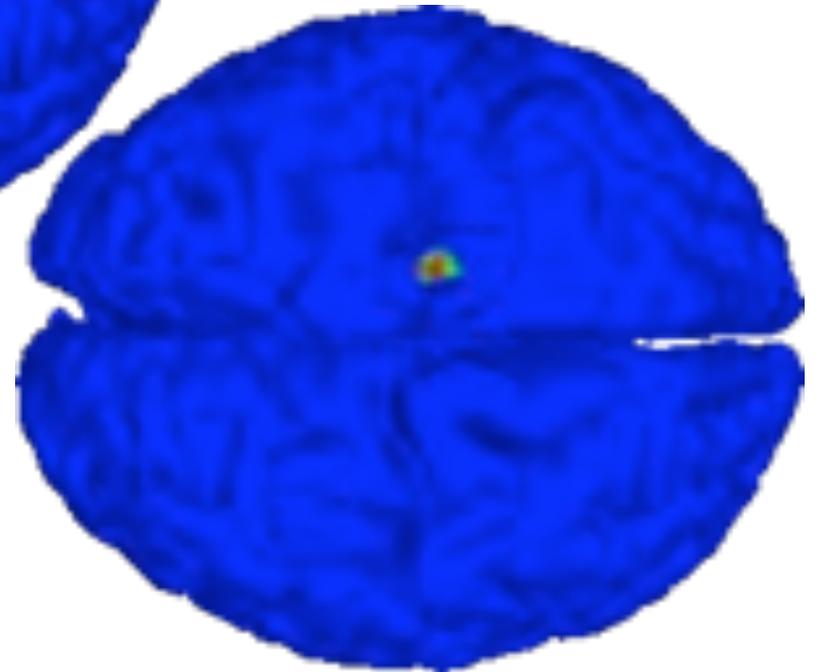
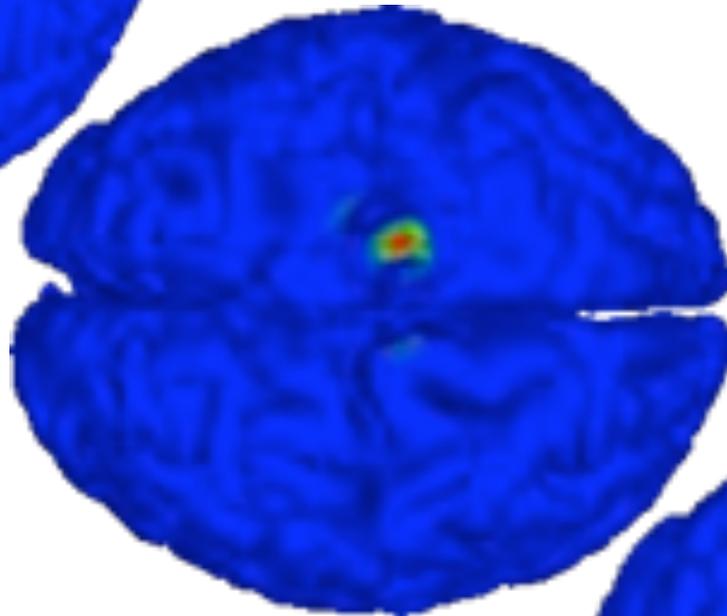
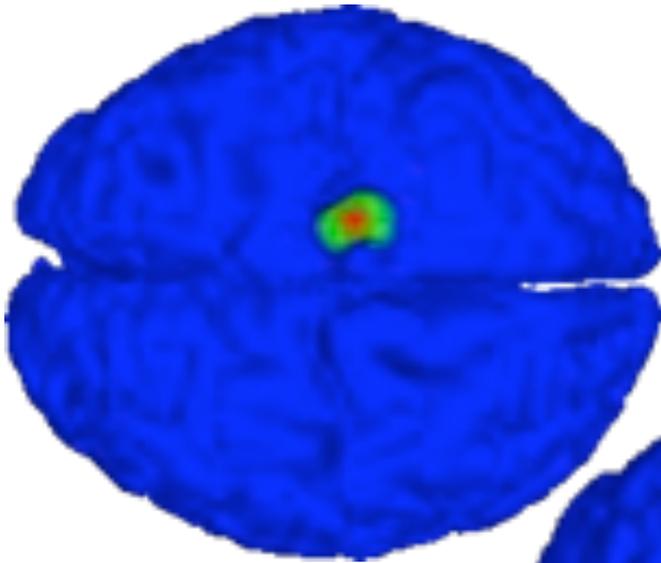
- Geselowitz, 1998

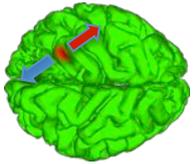


# Distributed source models

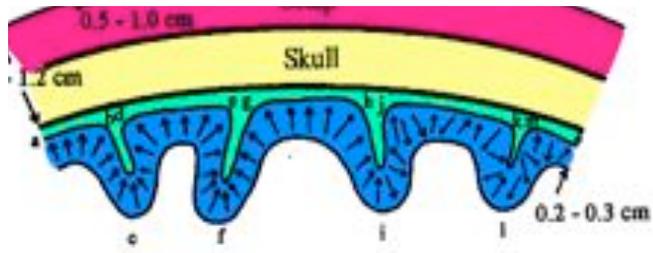
- Position of the source is not estimated as such
  - Pre-defined grid (3-D volume or cortical sheet)
  - Strength is estimated at each grid element
  - In principle, a linear problem, easy to solve, BUT...
    - More “unknowns” (parameters) than “knowns”  
(channels, measurements)
    - An infinite number of solutions can explain the data perfectly  
(not necessarily physiologically plausible!)
  - **So**, additional constraints are required ...

# Conformal cortical patch source model

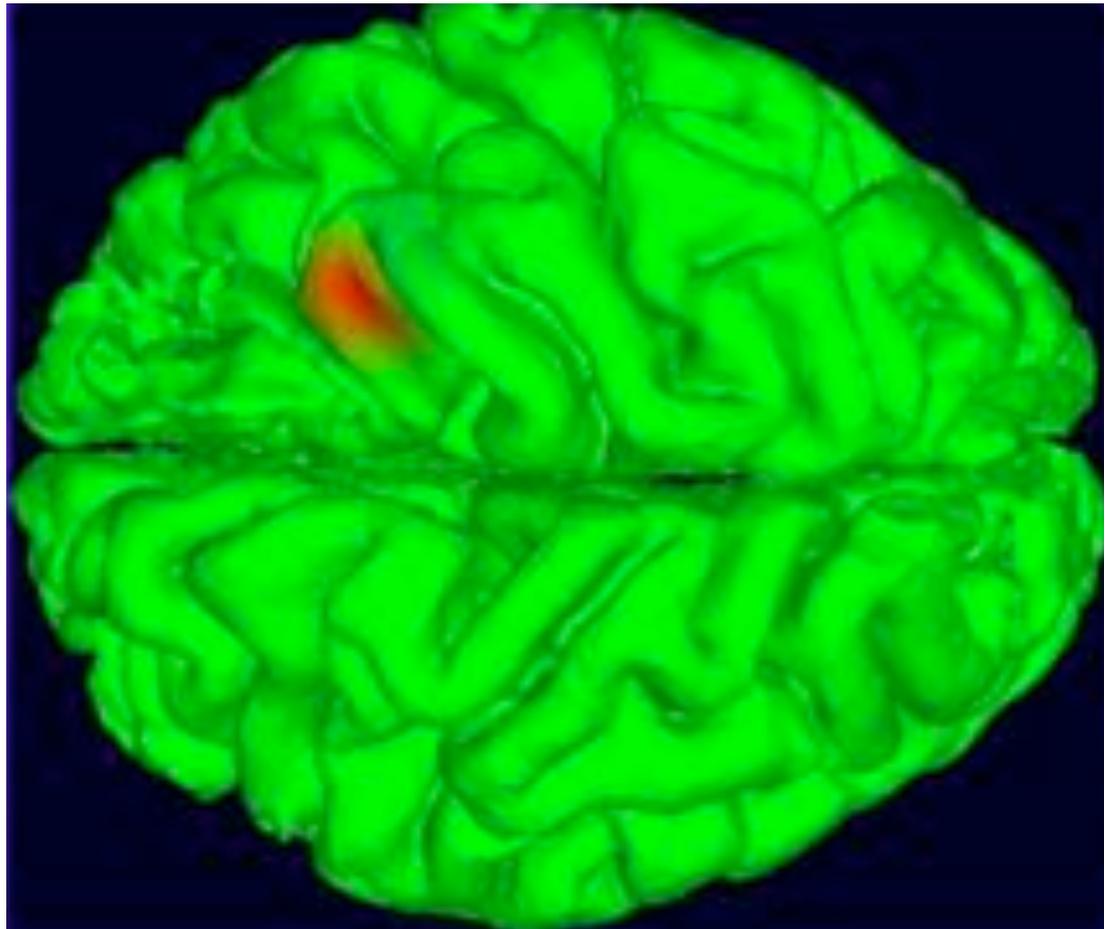


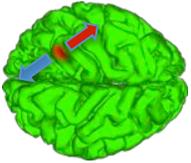


# Conformal cortical patch source model



Model a source estimate as a sum of overlapping patches

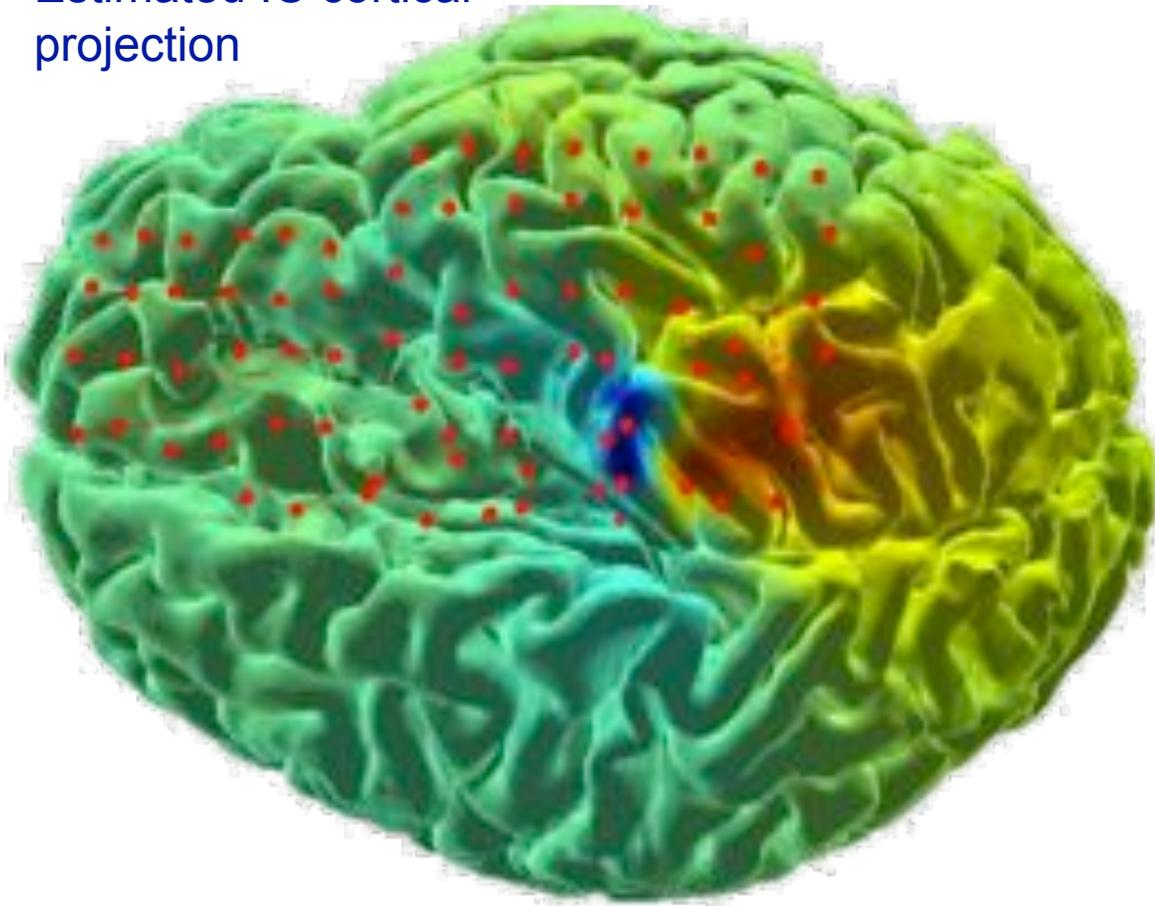




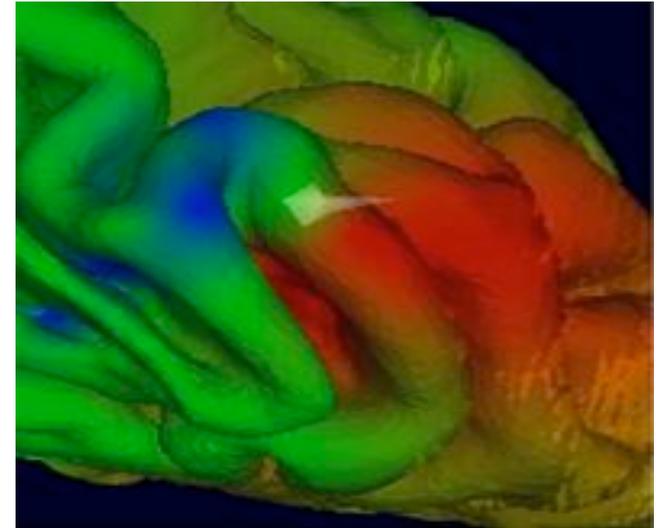
# Comparing source models

for an IC of an intracranial data set

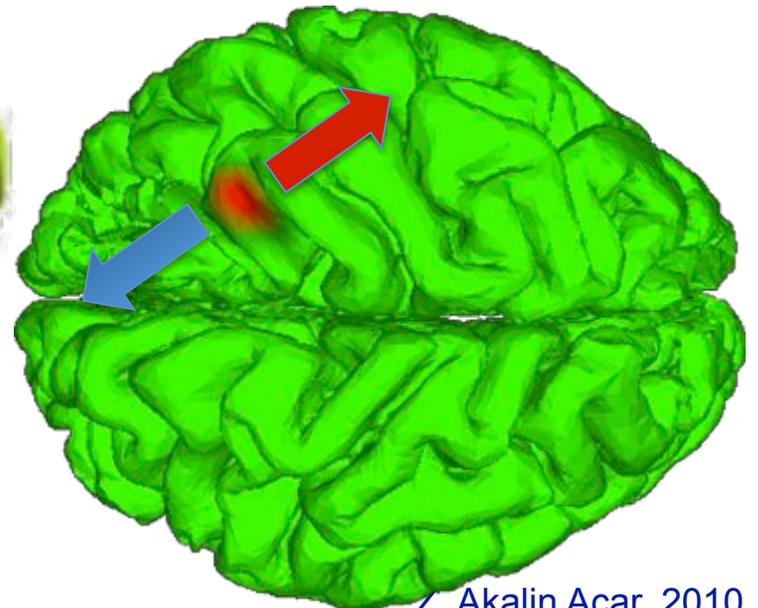
Estimated IC cortical projection

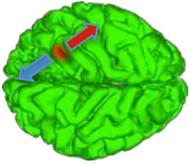


Equivalent Current Dipole Model



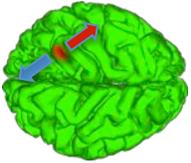
Sparse Patch Basis Model





# Summary I

- **Forward modeling**  
is required for the interpretation of scalp topographies
- Interpretation of scalp topographies  
is **inverse modelling** “source estimation”
- Mathematical techniques are available  
to aid in interpreting scalp topographies  
→ These are **inverse source models**



# Summary II

- **Inverse modeling**
  - Model assumption for volume conductor
  - Model assumption for source (i.e., dipole)
  - Additional assumptions on source
- **Single point-like sources**
- **Multiple point-like sources**
- **Distributed sources**
  - Different mathematical solutions
    - Dipole fitting (linear and nonlinear)
    - Linear estimation (regularized)