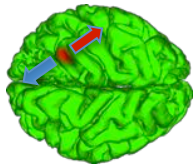


Forward and Inverse EEG Source Modeling



Scott Makeig
Institute for Neural Computation
UCSD, La Jolla CA



Source modeling

forward problem

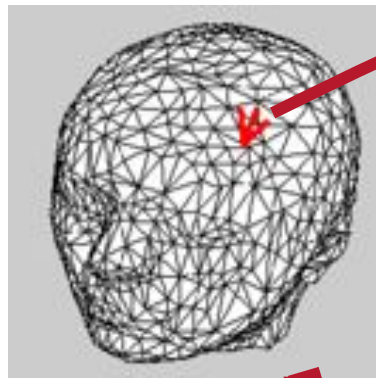


physiological source
electrical current

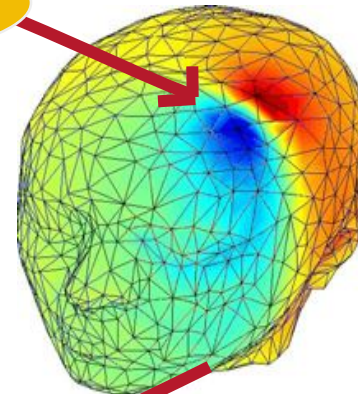
body tissue
volume conductor

observed
potential or field

inverse problem

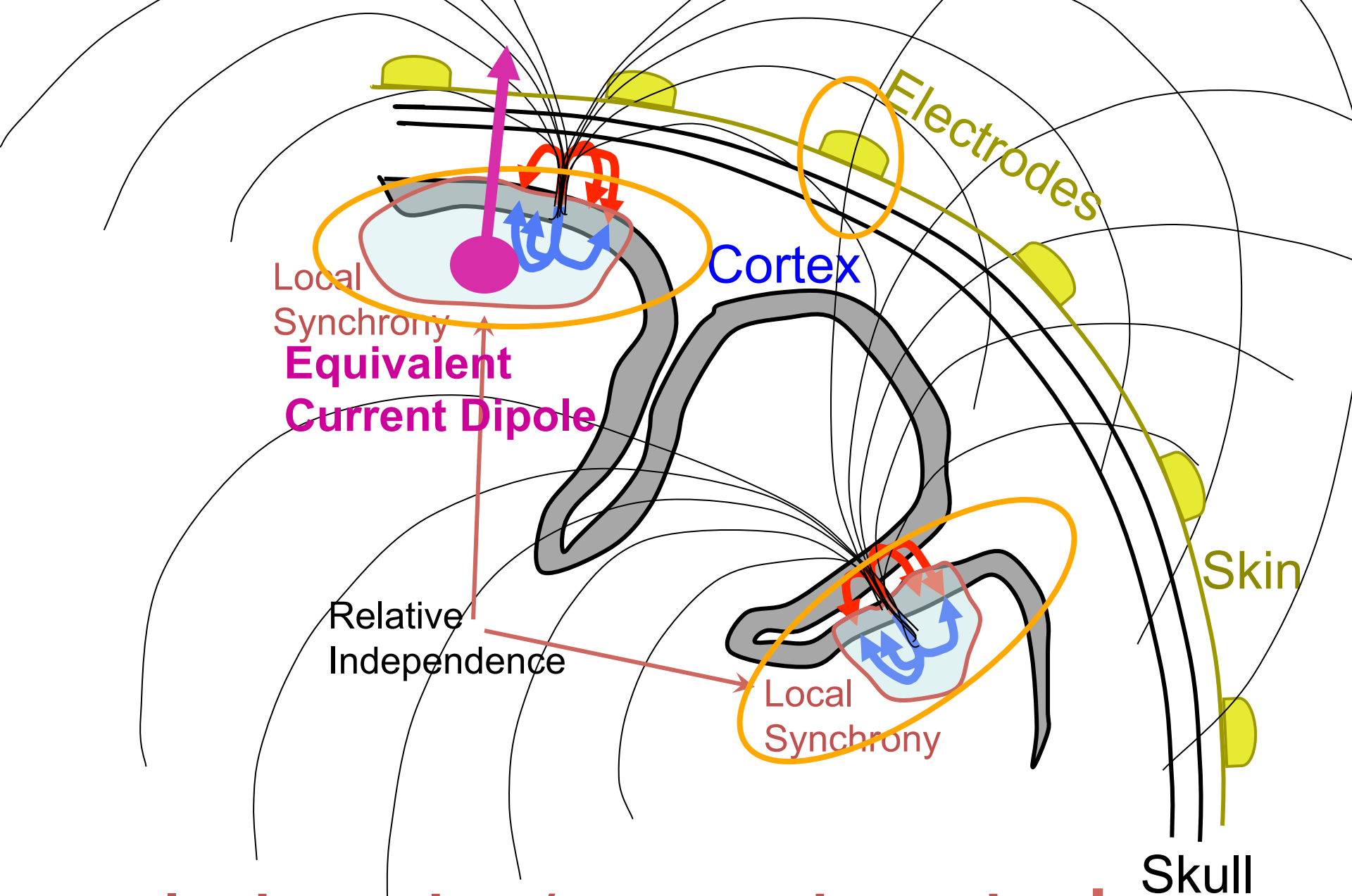


Forward Problem

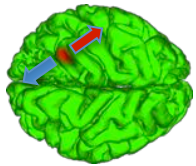


**EEG or
MEG**

Inverse Problem

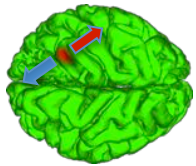


scalp dynamics \neq source dynamics !

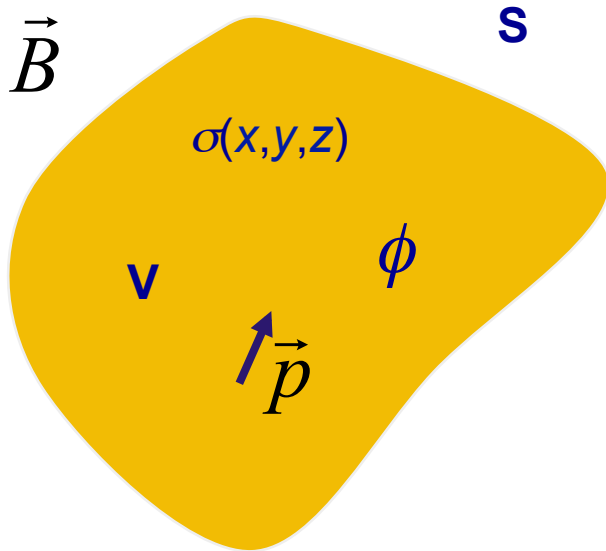


EEG volume conduction

- **Potential difference between electrodes is measured.** This corresponds to current flowing through skin:
 - Only tiny fraction of current passes through skull
 - Therefore the model should describe both skull and skin as accurately as possible.
- **Problems with skull modeling**
 - Poorly visible in anatomical MRI (T2)
 - Thickness varies
 - Conductivity is not homogeneous
 - Complex geometry at front and base of skull



Exact Formulation of the Forward Problem



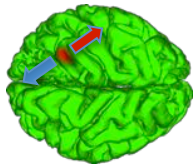
$$\nabla \cdot (\sigma \nabla \Phi) = -\nabla \cdot J^P \quad \text{inside } V$$

$$\sigma \frac{\partial \Phi}{\partial n} = 0 \quad \text{on } S$$

$\sigma(x,y,z)$: conductivity distribution

p : current source

\vec{p}



To Solve the Forward Head Model Problem ...

WE NEED

→ Head model assumptions

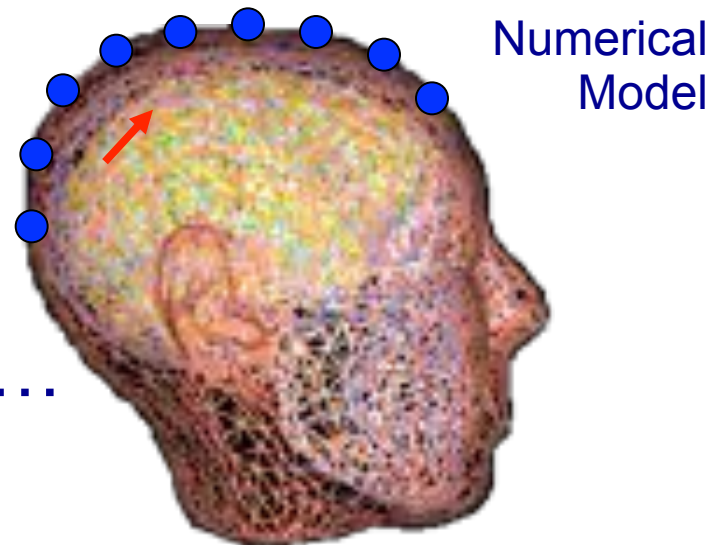
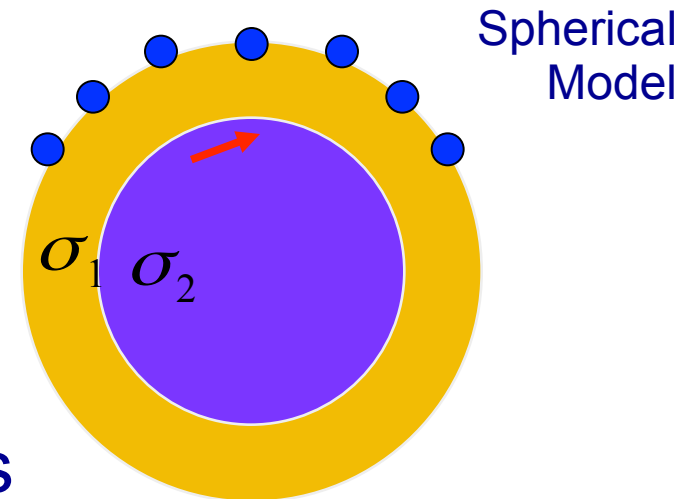
- Conductivity values
- Geometry

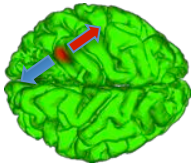
→ Actual sensor locations

→ Source space assumptions

- Magnitudes
- Locations
- Directions

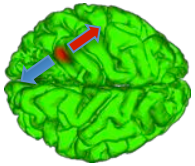
→ Solver builds the model...





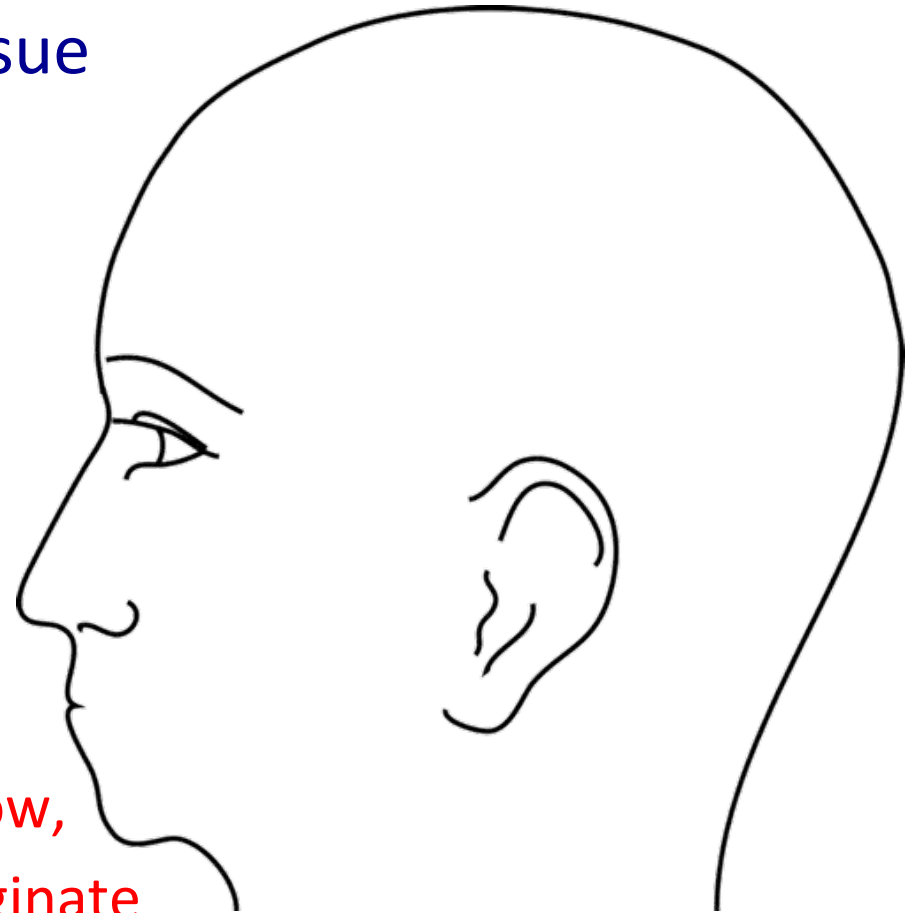
Source Localization Requirements

- ◆ Selected/processed EEG signal
 - **Simple single-source scalp map !**
- ◆ Number/positions of electrodes on the head surface
- ◆ Numerical head model
- ◆ Co-registration of EEG electrodes with head model
- ◆ *A priori* information/guess about the source space
- ◆ Choice of inverse model
- ◆ Choice of numerical method

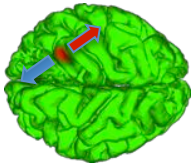


Volume conductor model

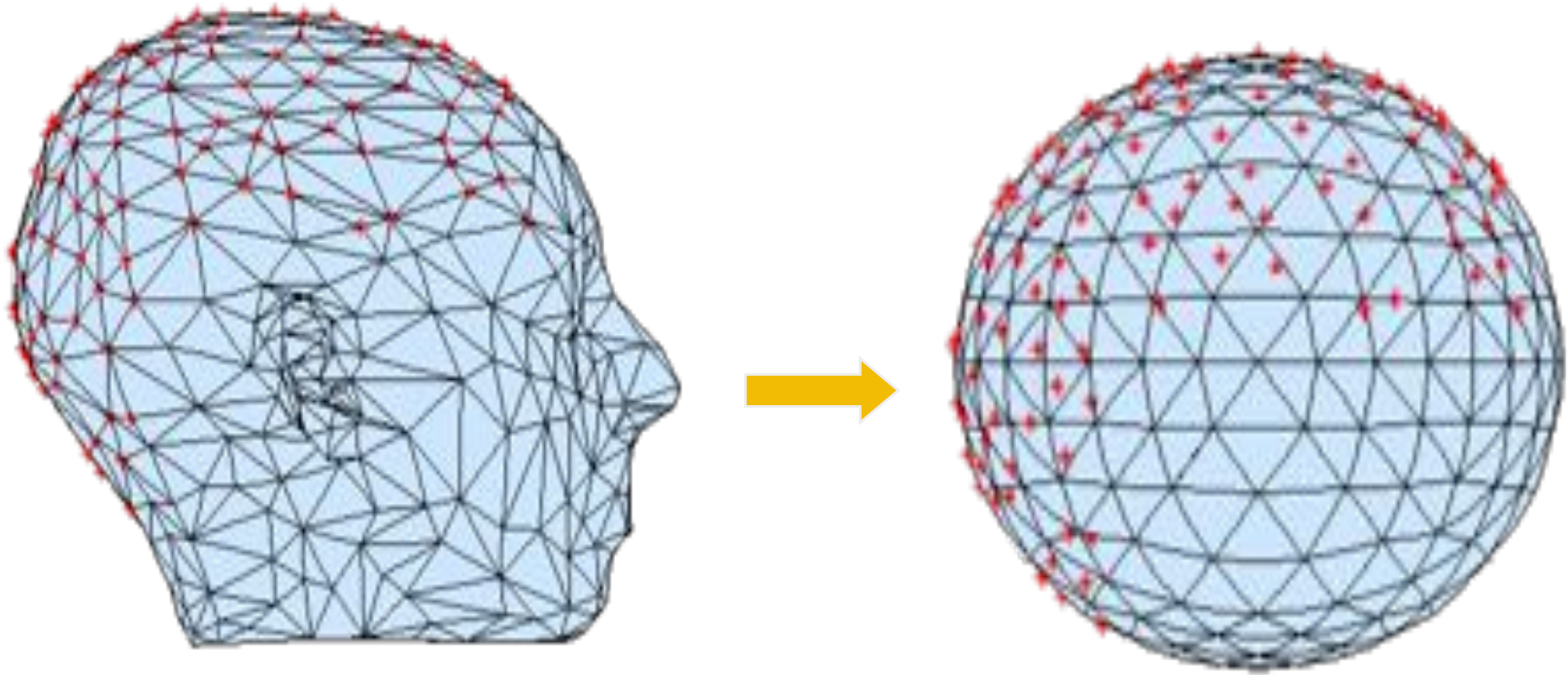
- Electrical properties of tissue
- Geometrical description
 - spherical model
 - realistically shaped model



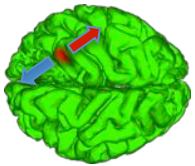
→ Describes how the currents flow,
from where they may originate
to where they are recorded.



Errors in Simple Head Models



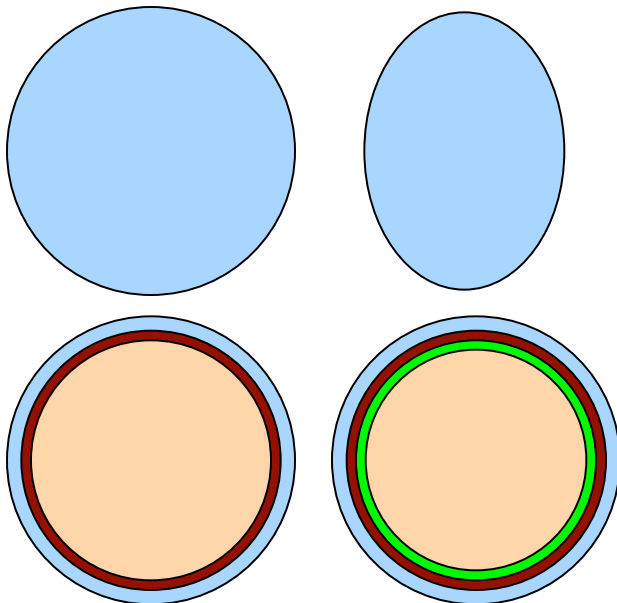
- In the volume conductor model
- In the electrode locations



Head Model Comparison

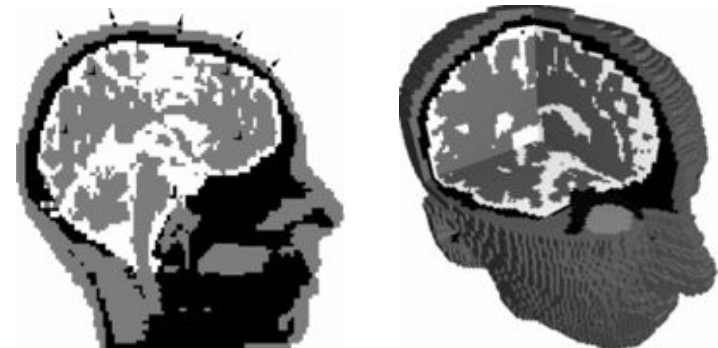
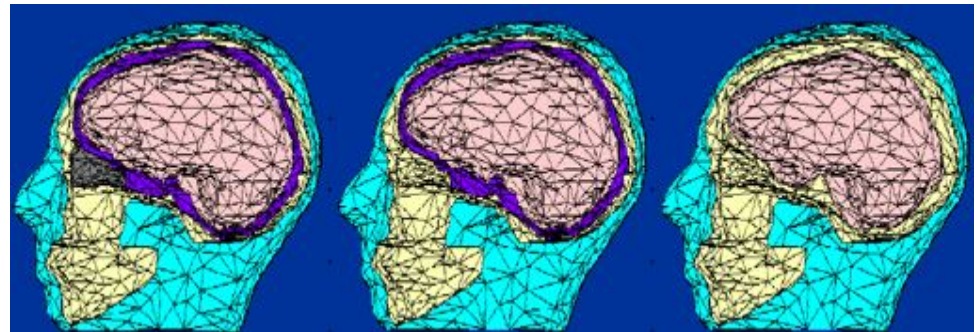
◆ Simple head models

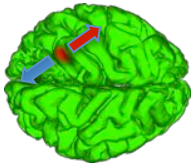
- Single sphere
- 3-4 Layer Spherical
- Spheroid



◆ Realistic head models

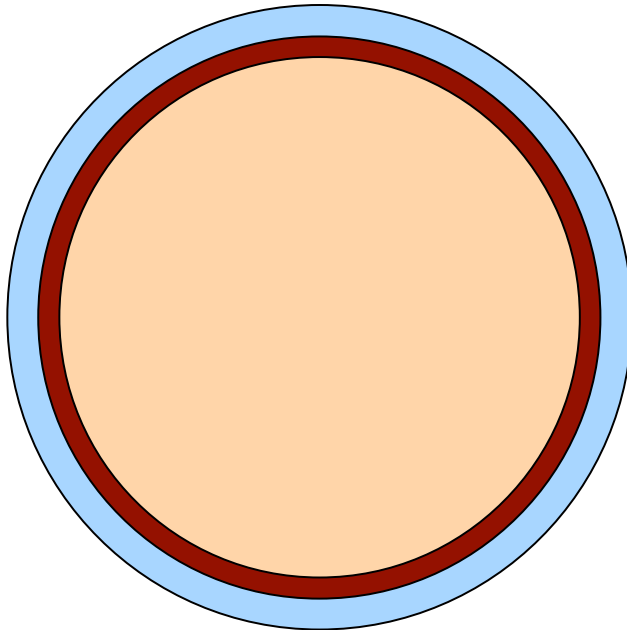
- Boundary Element Method
- Finite Element Method
- Finite Difference Method



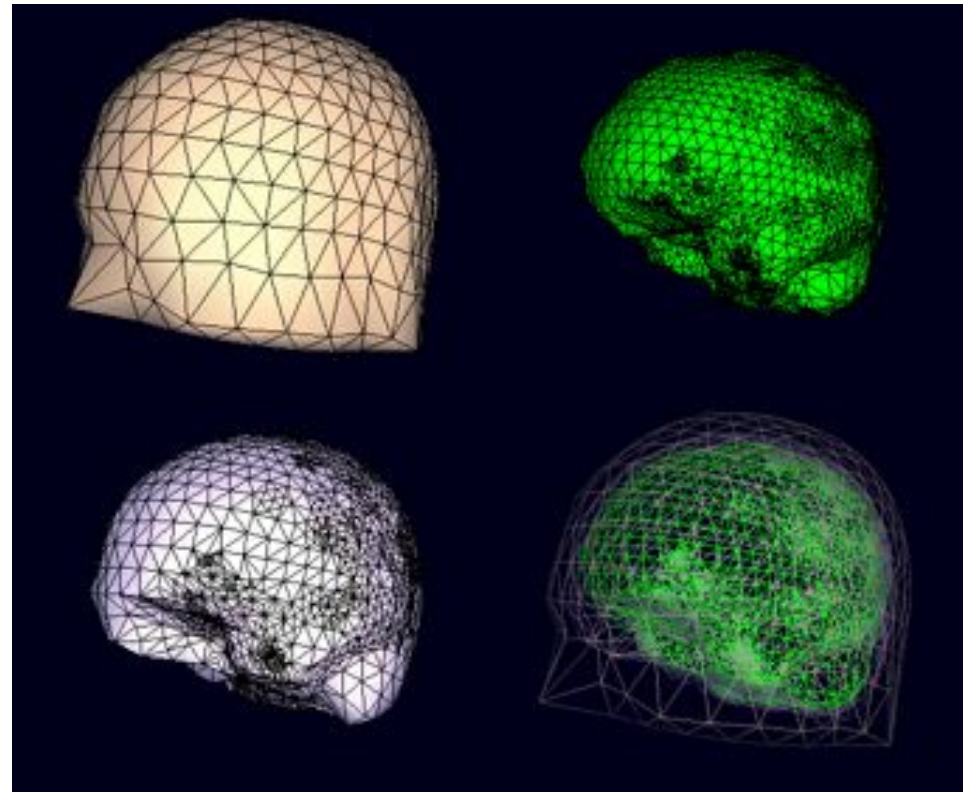


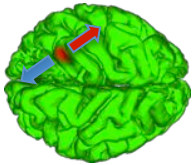
Effects of Head Model

Spherical head model
(3-layer standard)



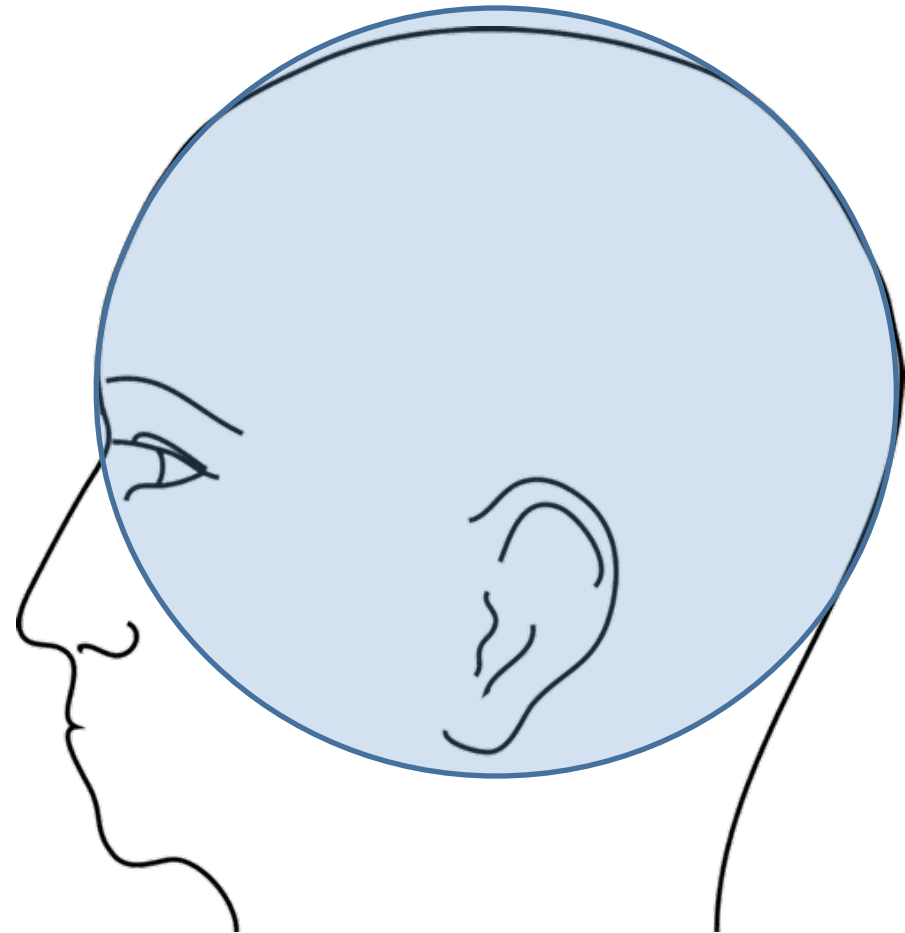
Standard MNI head model
(4-layer mean BEM)

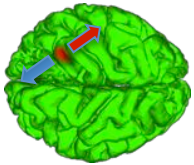




Spherical volume conductor

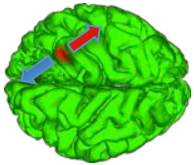
- Advantages of the **spherical** head model
 - mathematically exact
 - fast to compute
 - reasonably accurate
 - easy to use
- Disadvantages of the **spherical** model
 - difficult to align properly
 - inaccurate in some regions





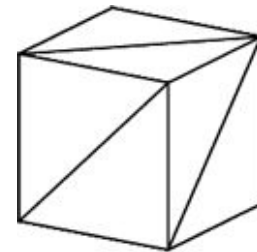
Realistic volume conductor

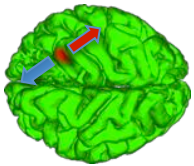
- Advantages of a **realistic** head model
 - a more accurate solution (especially for EEG)
 - Disadvantages of a **realistic** model
 - more work to build from an MR image
 - slower to compute
 - might be numerically unstable
 - harder to make between-subject comparisons
- A pragmatic (easy, cheap) solution is to use a standard (mean) realistic head model (MNI).



Realistic volume conductor

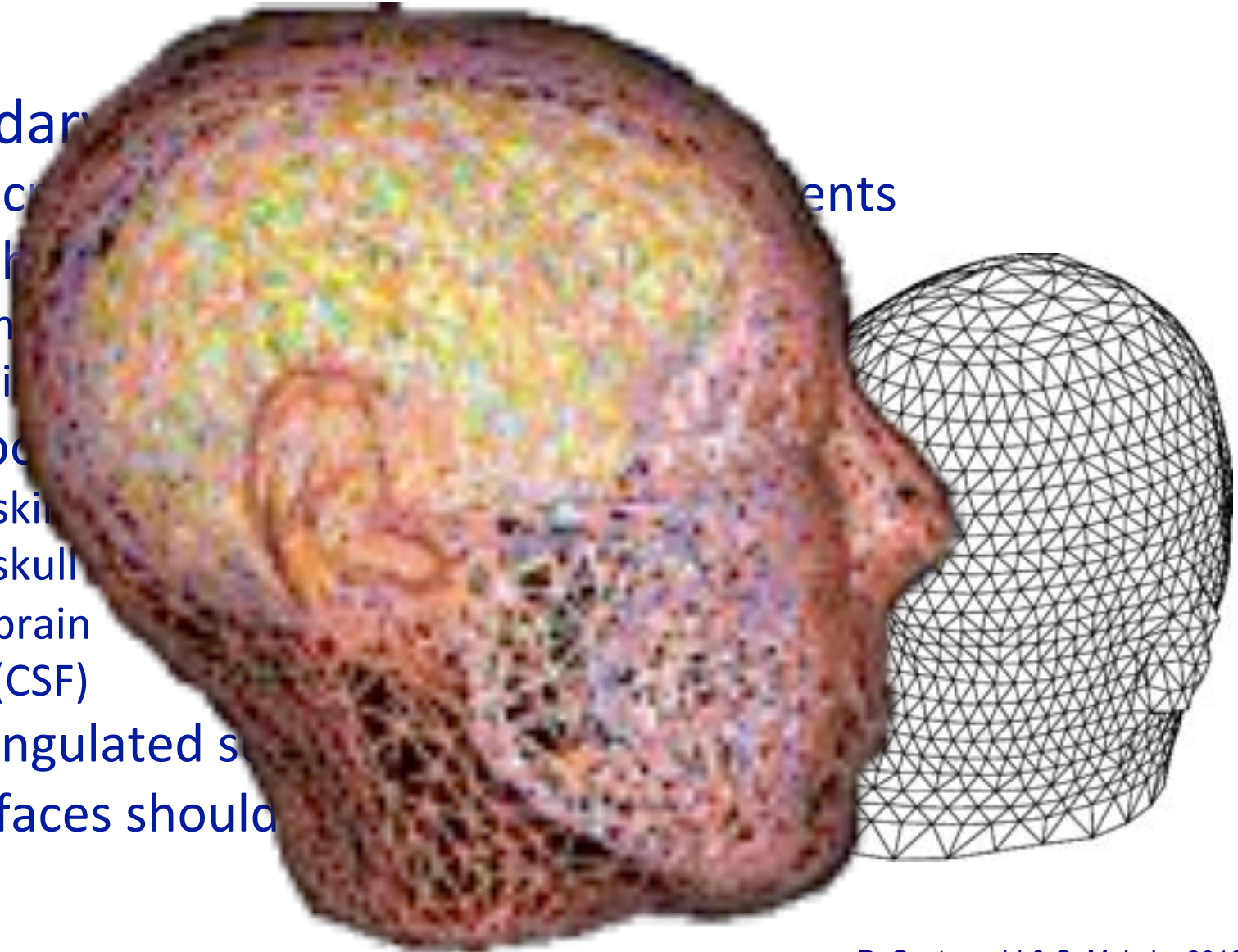
- Computational methods for volume conduction problem that allow realistic geometries:
 - Boundary Element Method (BEM)
 - Finite Element Method (FEM)
- Geometrical description
 - Triangles (planar or quadratic)
 - Tetrahedra (3-D)

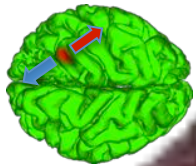




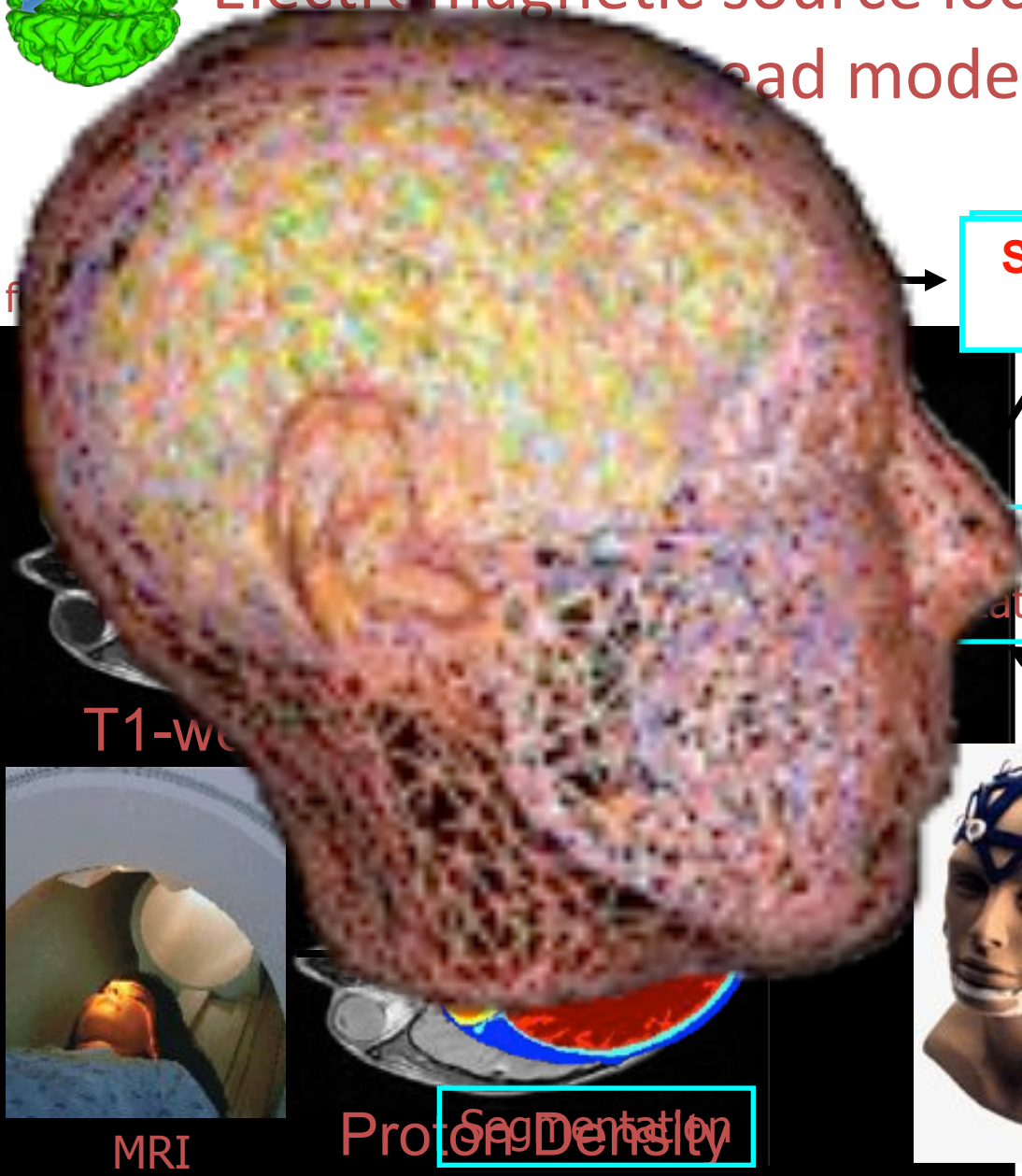
BEM volume conductor

- Boundary
 - describes the geometry of the volume conductor
 - each boundary is defined by a set of points
 - head
 - inner skull
 - important for the calculation of the forward problem
 - skin
 - skull
 - brain
 - (CSF)
 - triangulated surface
 - surfaces should be closed

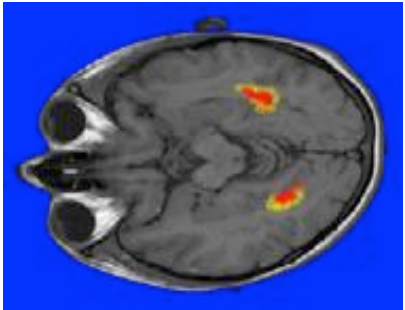




Electromagnetic source localization using realistic head models (NFT)



Simple Map



Source Image

Signal Processing

Coregistration

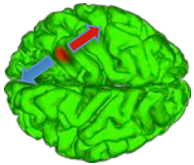


MRI

Proton Density



EEG/MEG



A Four-Layer BEM Head Model

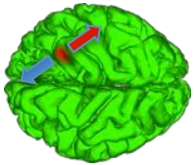


Neuroelectromagnetic
Forward head modeling
Toolbox (**NFT**)

of elements

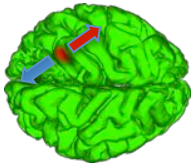
Scalp:	6900
Skull:	6800
CSF:	9000
Brain:	8800

Total	31500
--------------	--------------



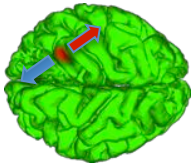
FEM volume conductor

- **Cut the 3-D volume into solid tetrahedra**
 - Large number of elements
 - Each tetrahedron can have its own conductivity
 - Each tetrahedron can have its own anisotropy
- **FEM is most accurate numerical method**
 - Computationally expensive to build
 - Accurate conductivities are not known
 - Accurate anisotropies require DTI & assumptions



Inverse problem methods

- **Single and multiple dipole models**
 - Minimize error between the model and the measured potential/field
- **Distributed dipole models**
 - Perfect fit of model to the measured potential/field
 - Minimize an additional constraint on sources
 - LORETA (assumes a **smooth** distribution)
 - Minimum Norm (L2, **minimum power** at the cortex)
 - Minimum Current (L1, **minimum current** in the cortex)

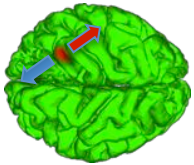


Inverse problem methods

- **Spatial source filtering**

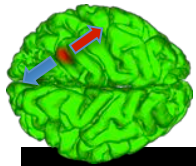
- **Scan whole brain** with single dipole and compute the filter output at every location (second-order covariance matrix)
 - MUSIC algorithm
 - *Beamforming* (e.g., LCMV, SAM, DICS)
- **Perform ICA decomposition** (higher-order statistics)
 - Of the scalp maps at individual moments
 - ICA gives the projections of the sources to the scalp surface, i.e., **‘simple’ maps!**

→ ICA solves **‘the first half’ of the inverse problem (‘What?’)**

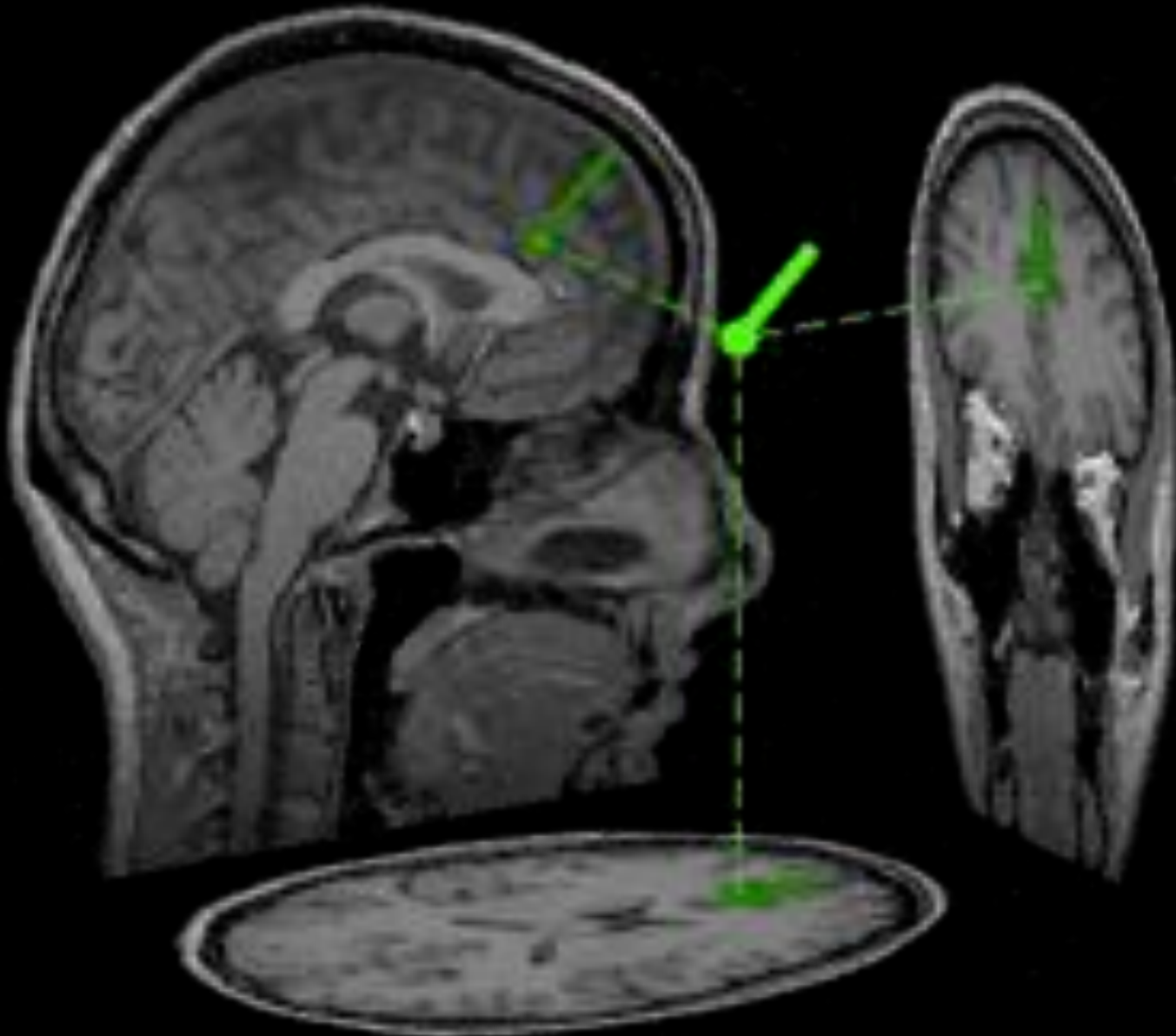


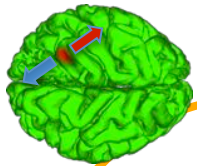
Equivalent current dipoles

- **Physical/mathematical motivation**
 - Any current distribution can be written as a multipole expansion
 - First term: monopole (must be 0)
 - Second term: dipole
 - Higher order terms: quadrupole, ... vanish
 - Synchronous activity across a patch
has an equivalent model dipole!
- **Convenience**
 - **Dipoles** can be used as building blocks
in distributed source models

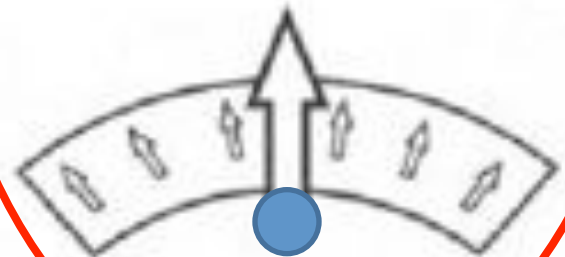
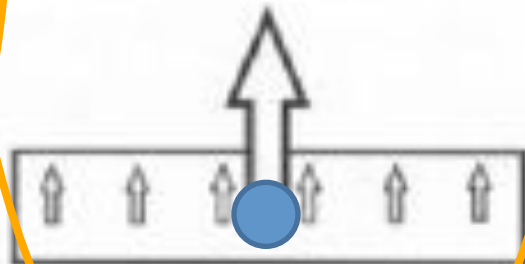
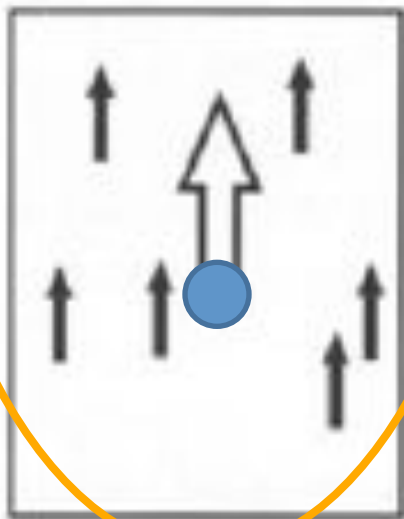
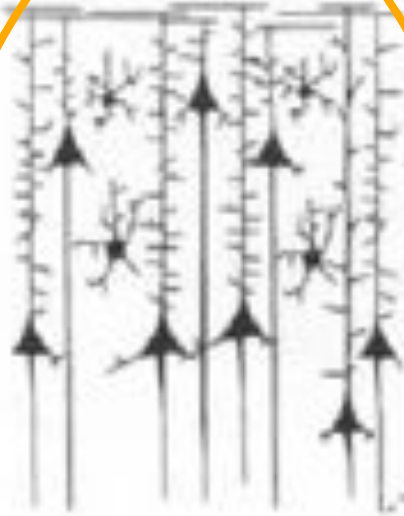


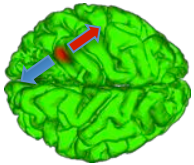
Equivalent current dipoles





Equivalent current dipole





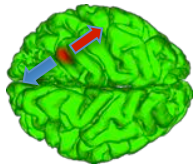
Measured Errors in Dipole Source Localization

◆ Experimental studies

- Phantom → 10 mm loc. error (Henderson & Butler, 1975)
- Human skull → 35 mm (Weinberg et al, 1986)

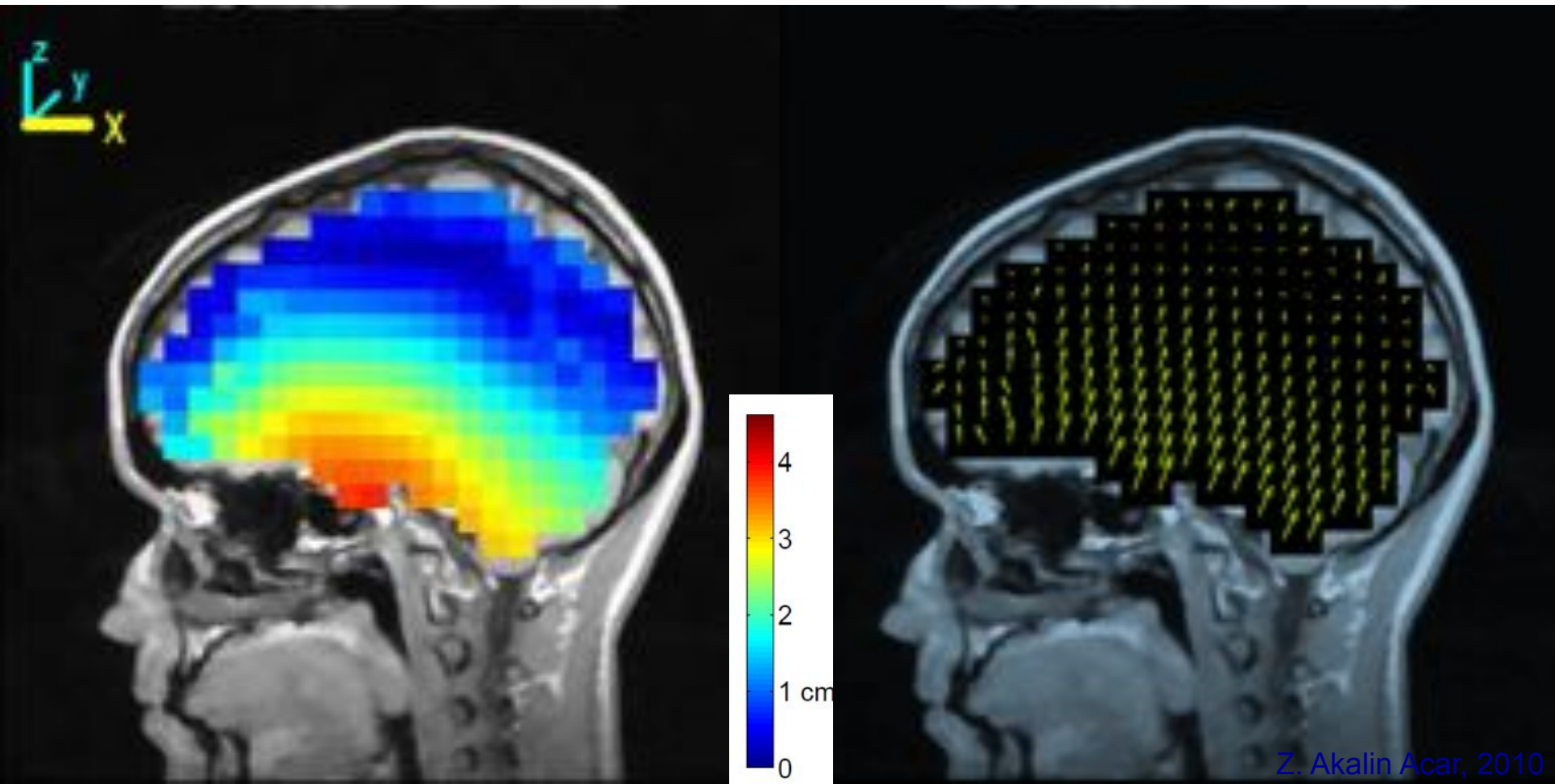
◆ Simulation studies

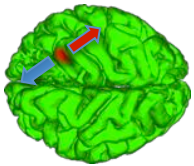
- 3-layer model → 15-25 mm (Roth et al, 1993)
- 3-layer model → 9-14 mm (Vanrumste et al, 2002)
- Human skull → 25 mm (Fletcher et al, 1993)
- 3-layer model → ~8 mm (Akalin Acar, 2005)



Source Localization Errors

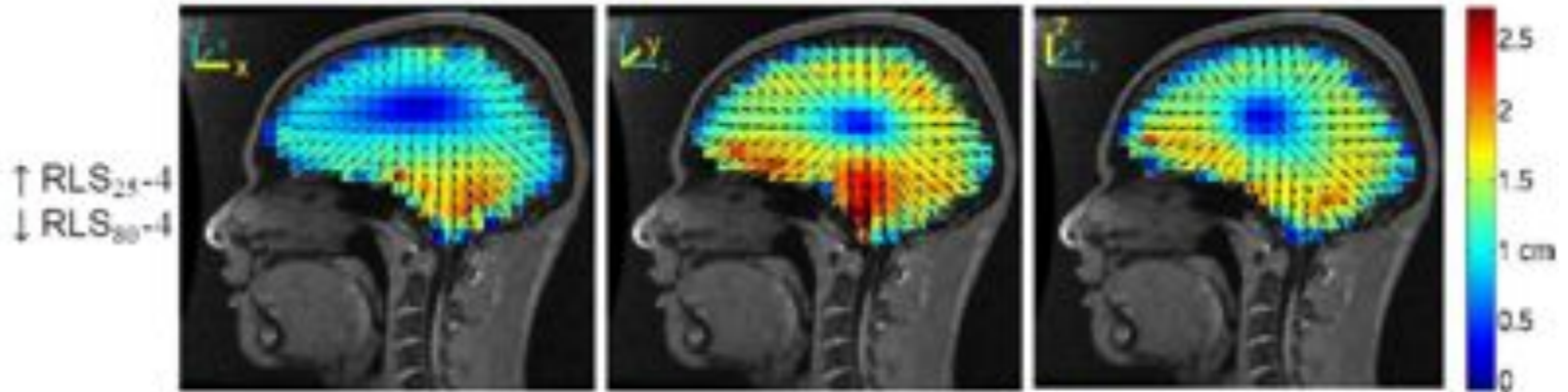
- For a **3-layer spherical** head model
- Relative to 4-layer realistic BEM head model



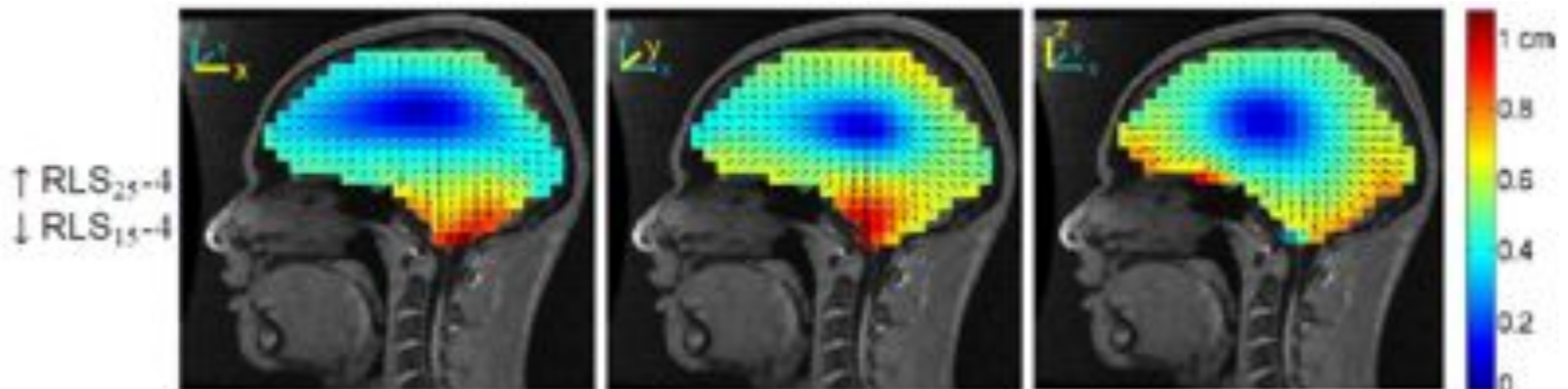


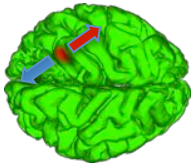
Source Localization Errors

- Forward model (individual BEM) – brain/skull cond. 25
- Inverse model (individual BEM) – brain/skull cond. 80



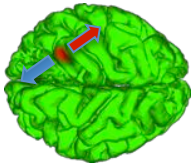
- Forward model (individual BEM) – brain/skull cond. 25
- Inverse model (individual BEM) – brain/skull cond. 15





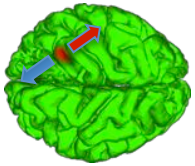
Single vs. multiple dipole models

- Manipulate source parameters to **minimize error** between measured and model data
 - **Position** of each source
 - **Orientation** of each source
 - **Strength** of each source
- **Orientation** and **strength** together correspond to the “dipole moment” and can be estimated *linearly*
 - **Position** is estimated *non-linearly* by iterative source parameter estimation



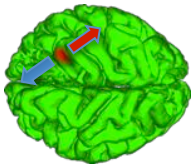
Dipole scanning: grid search

- Define grid with allowed dipole locations
- Compute optimal dipole moment for each location
- Compute value of goal-function
- Plot value of goal-function on grid
- Number of evaluations:
 - single dipole, 1 cm grid: $\sim 4,000$
 - single dipole, $\frac{1}{2}$ cm grid: $\sim 32,000$
 - BUT two dipoles, 1 cm grid: $\sim 16,000,000$



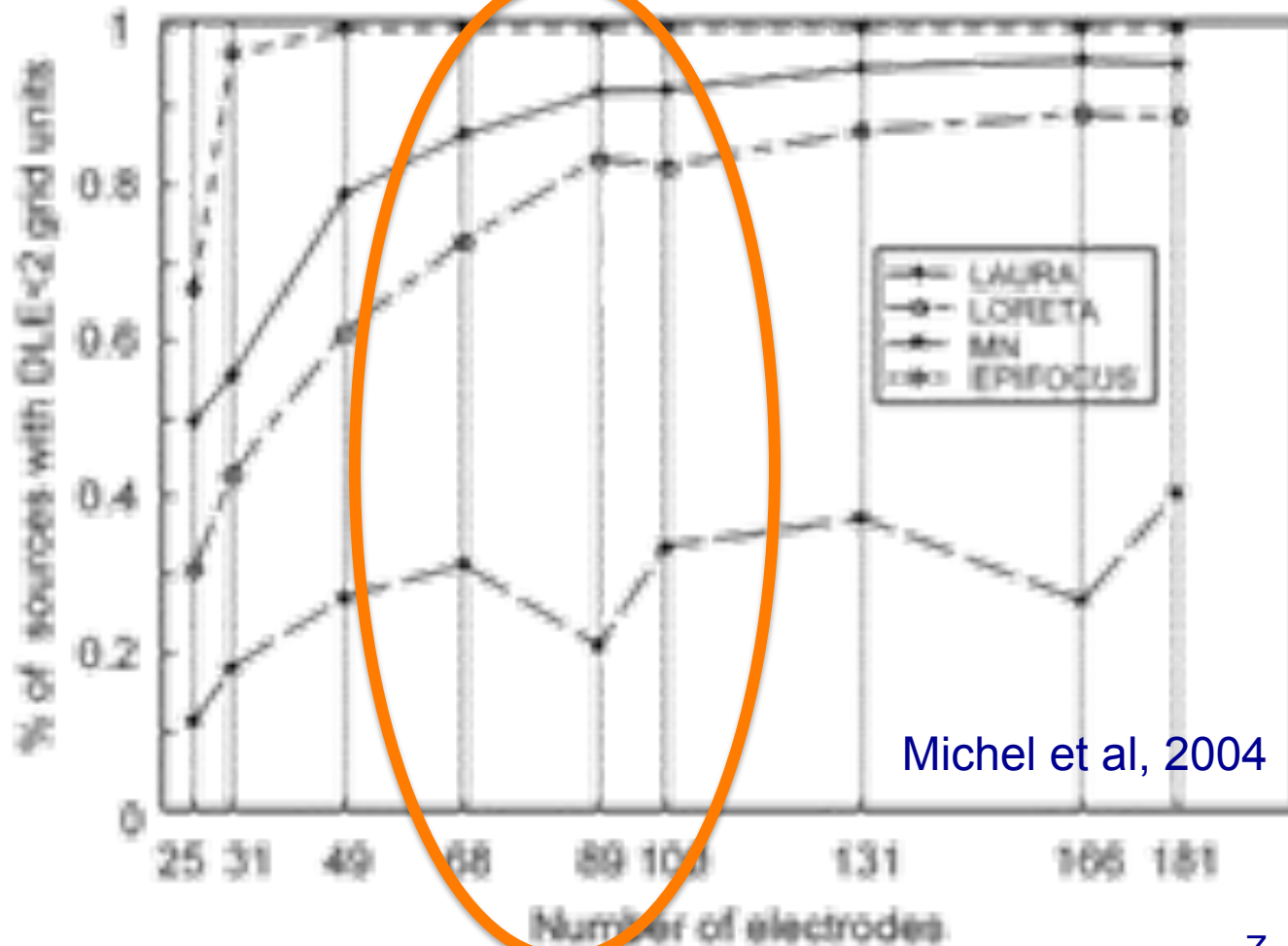
Dipole fitting: nonlinear search

- **Start with an initial guess from coarse fitting**
 - Evaluate the local derivative of goal-function
 - “Walk down hill” to the most optimal solution
 - Number of evaluations needed ~ 100

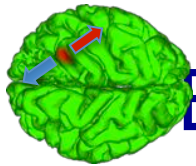


Effect of Number of Electrodes

- Single dipole source
- 3-layer spherical head model
- 1152 solution points



Michel et al, 2004



Effects of the Skull Conductivity Estimate

Measurements of skull conductivity:

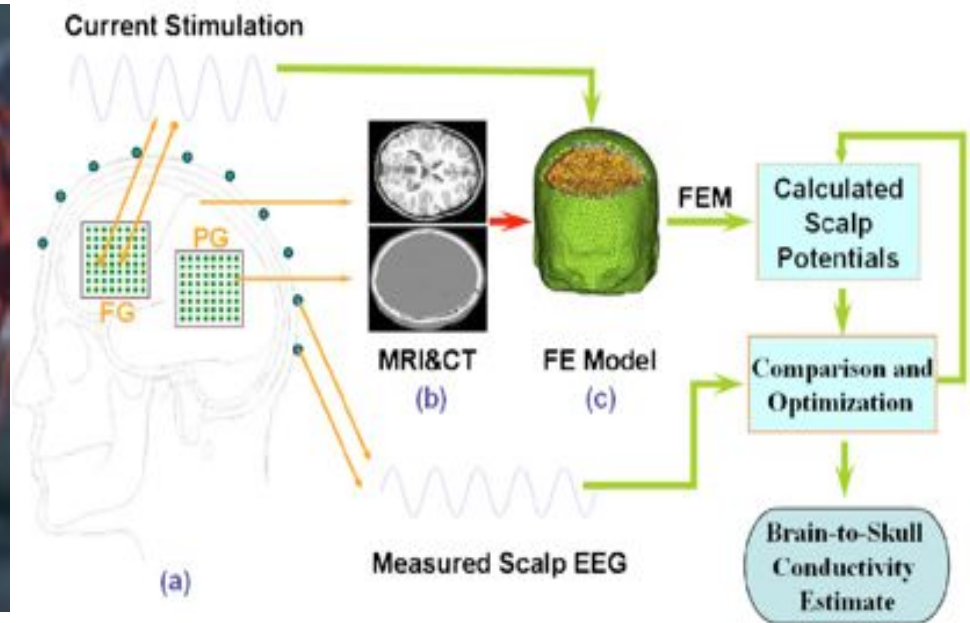
- MR-EIT
- Magnetic stimulation
- Current injection

In vivo

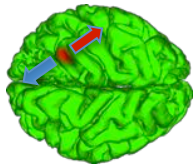


Hoekama et al, 2003

In vitro



He et al, 2005



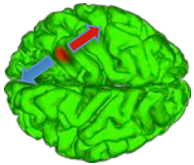
Skull Conductivity Measurements

Brain to skull ratio		
Rush and Driscoll	1968	80
Cohen and Cuffin	1983	80
Oostendorp et al	2000	15
Lai et al	2005	25

Measurement	Age	σ (mS/m)	Sd (mS/m)
Agar-agar phantom	—	43.6	3.1
Patient 1	11	80.1	5.5
Patient 2	25	71.2	8.3
Patient 3	36	53.7	4.3
Patient 4	46	34.4	2.3
Patient 5	50	32.0	4.5
Post mortem skull	68	21.4	1.3

Skull conductivity
by age

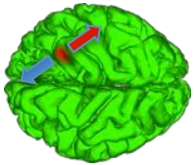
Hoekama et al, 2003



Effect of reference electrode

“The choice of a particular reference electrode ... does not change in any way the biophysical information contained in the potential distribution. It does not in any way change the relation between source and potential, except for an additive constant of no physical significance.”

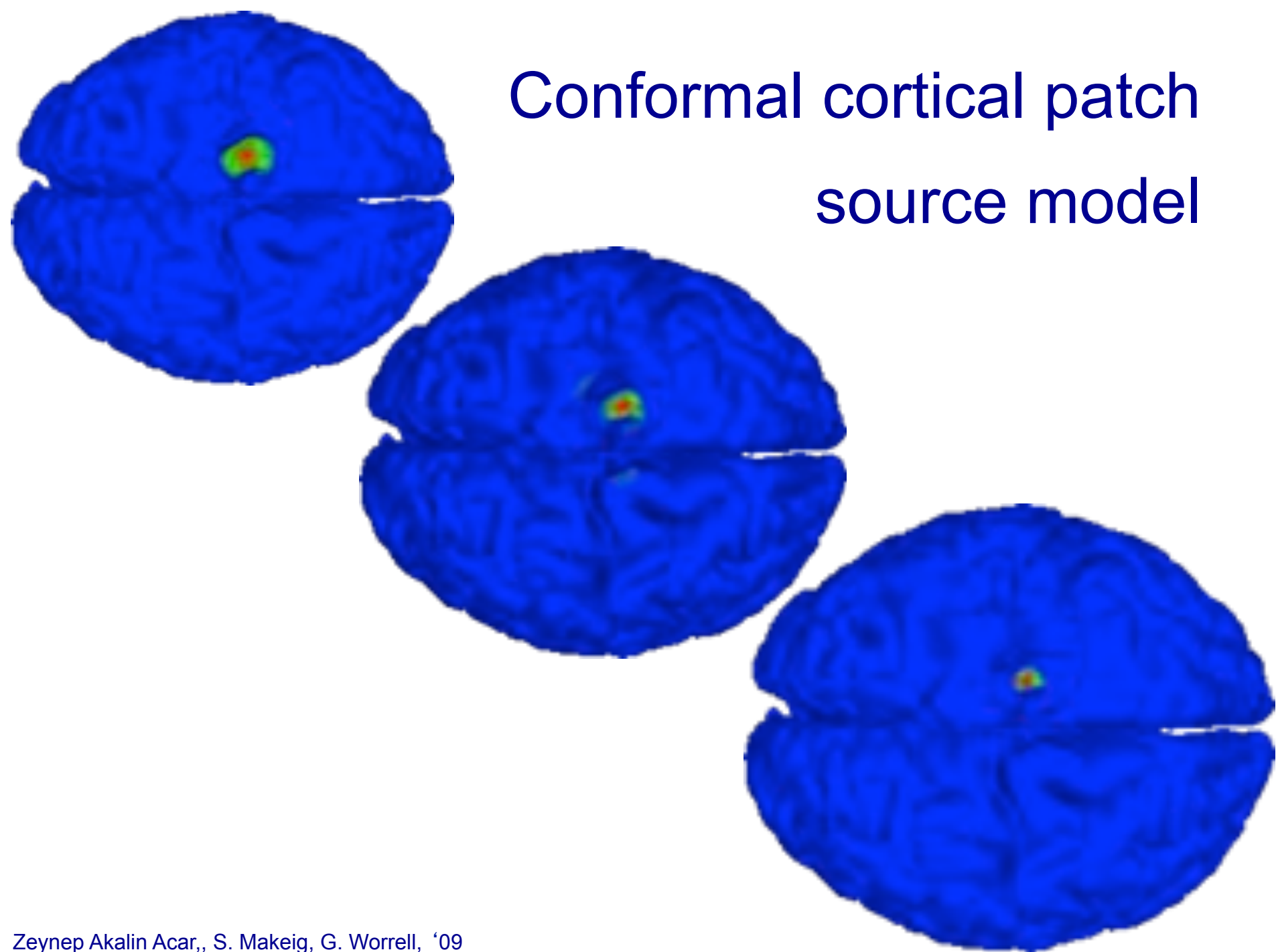
- Geselowitz, 1998

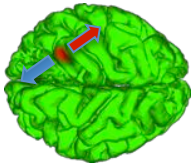


Distributed source models

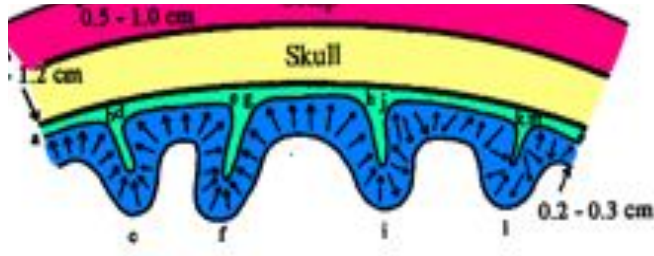
- **Position of the source is not estimated as such**
 - Pre-defined grid (3-D volume or cortical sheet)
 - Strength is estimated at each grid element
 - In principle, a linear problem, easy to solve, BUT...
 - More “unknowns” (parameters) than “knowns”
(channels, measurements)
 - An infinite number of solutions can explain the data perfectly
(not necessarily physiologically plausible!)
 - **So → Additional constraints are required**

Conformal cortical patch source model

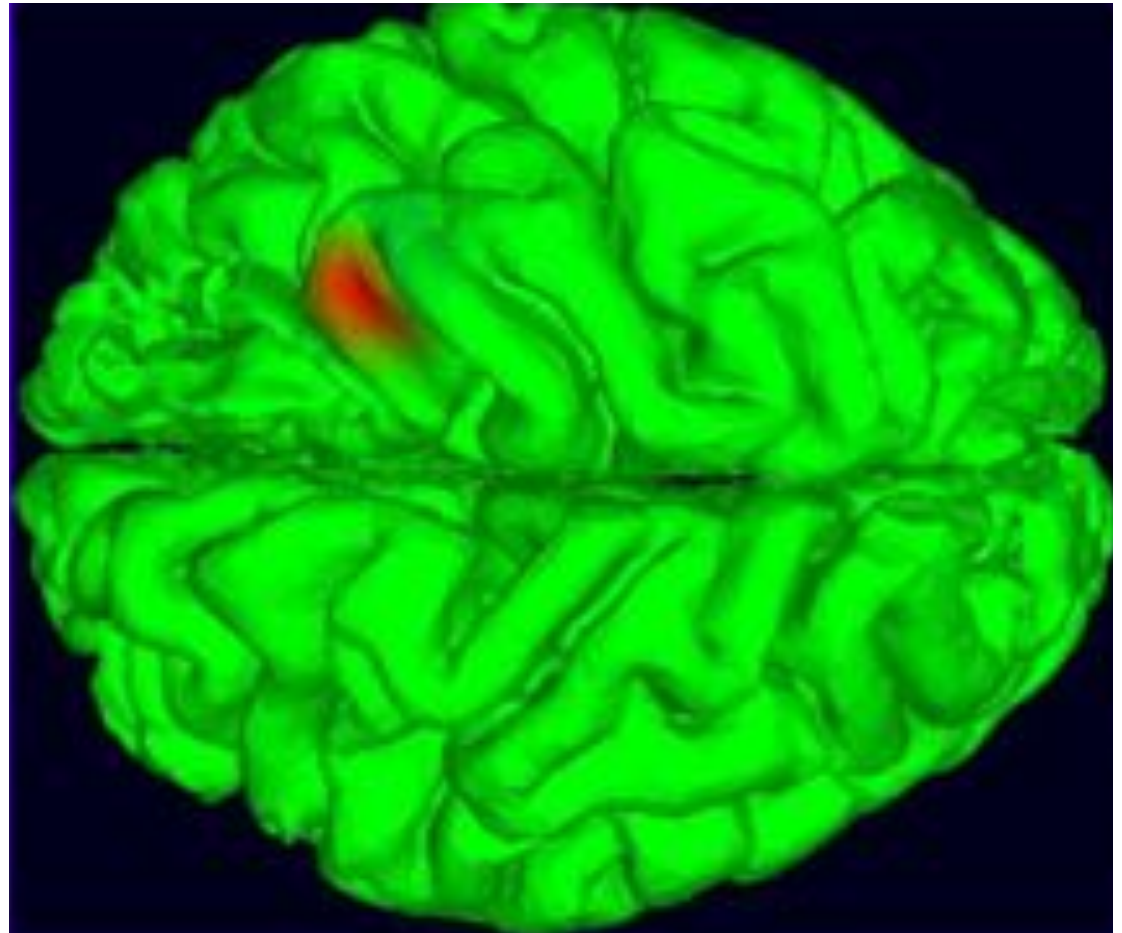


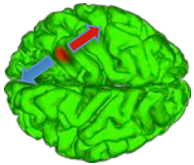


Conformal cortical patch source model



Model a source estimate as a sum of overlapping patches

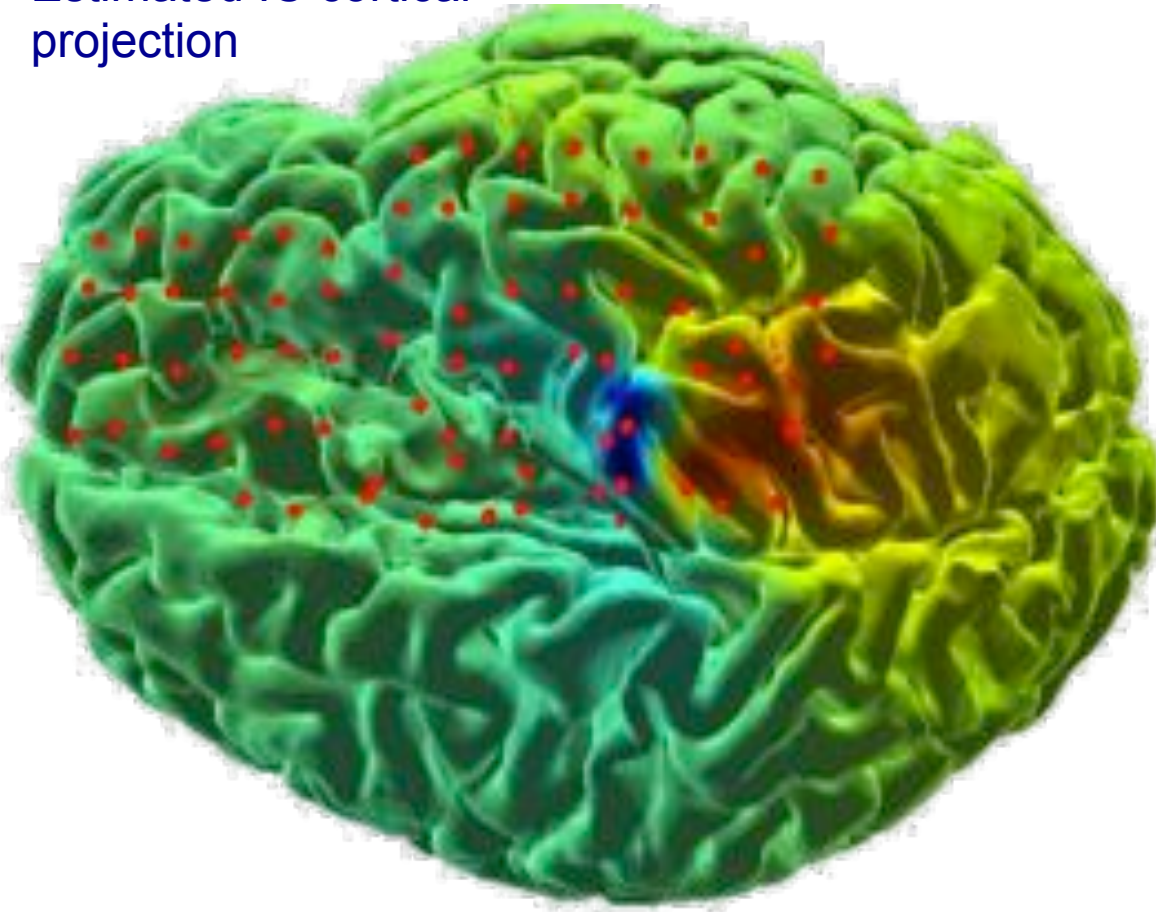




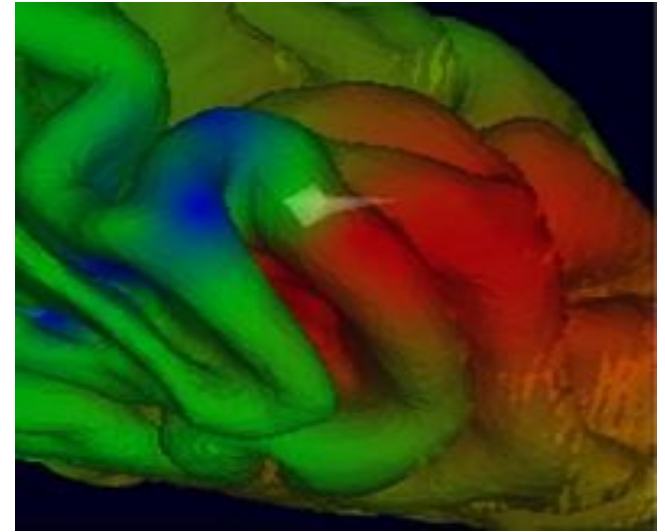
Comparing source models

for an IC of an intracranial data set

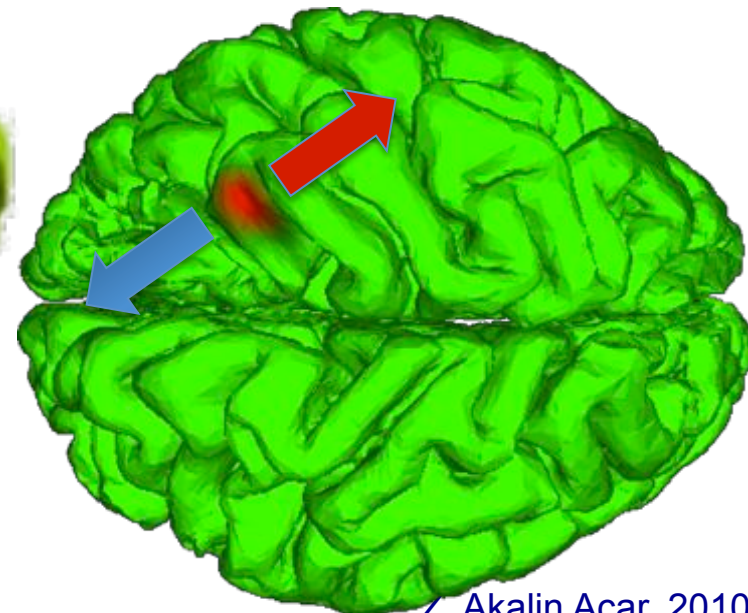
Estimated IC cortical
projection

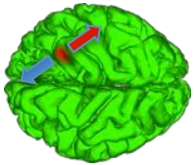


Equivalent Current Dipole Model



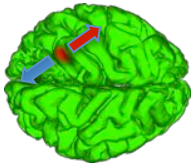
Sparse Patch Basis Model





Summary I

- **Forward modeling**
is required to interpret the scalp topographies
- Interpreting scalp topographies means
inverse modelling or “**source estimation**”
- Mathematical techniques are available
to aid in interpreting scalp topographies
→ These are **inverse source models**



Summary II

- **Inverse modeling**
 - Model assumptions for the (volume conductor) head
 - Model assumptions for source (equiv. dipole source)
 - Additional assumptions on source location/orientation
- **Single point-like sources**
- **Multiple point-like sources**
- **Distributed sources**
 - Different mathematical solutions
 - Dipole fitting (linear and nonlinear)
 - Linear estimation (regularized)
- **For EEG inverse modeling, conductivity is key!**

Next ... NFT!