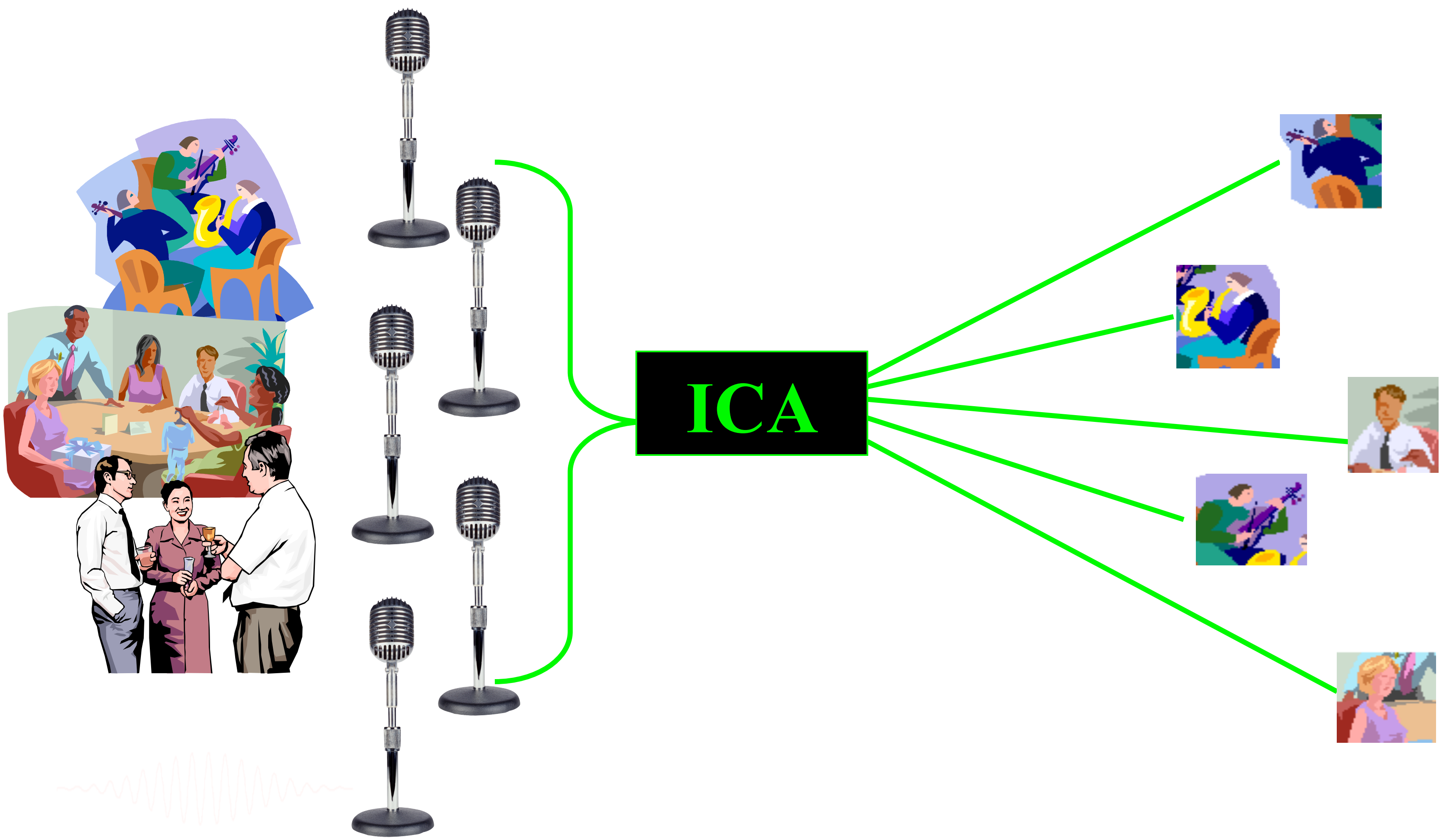
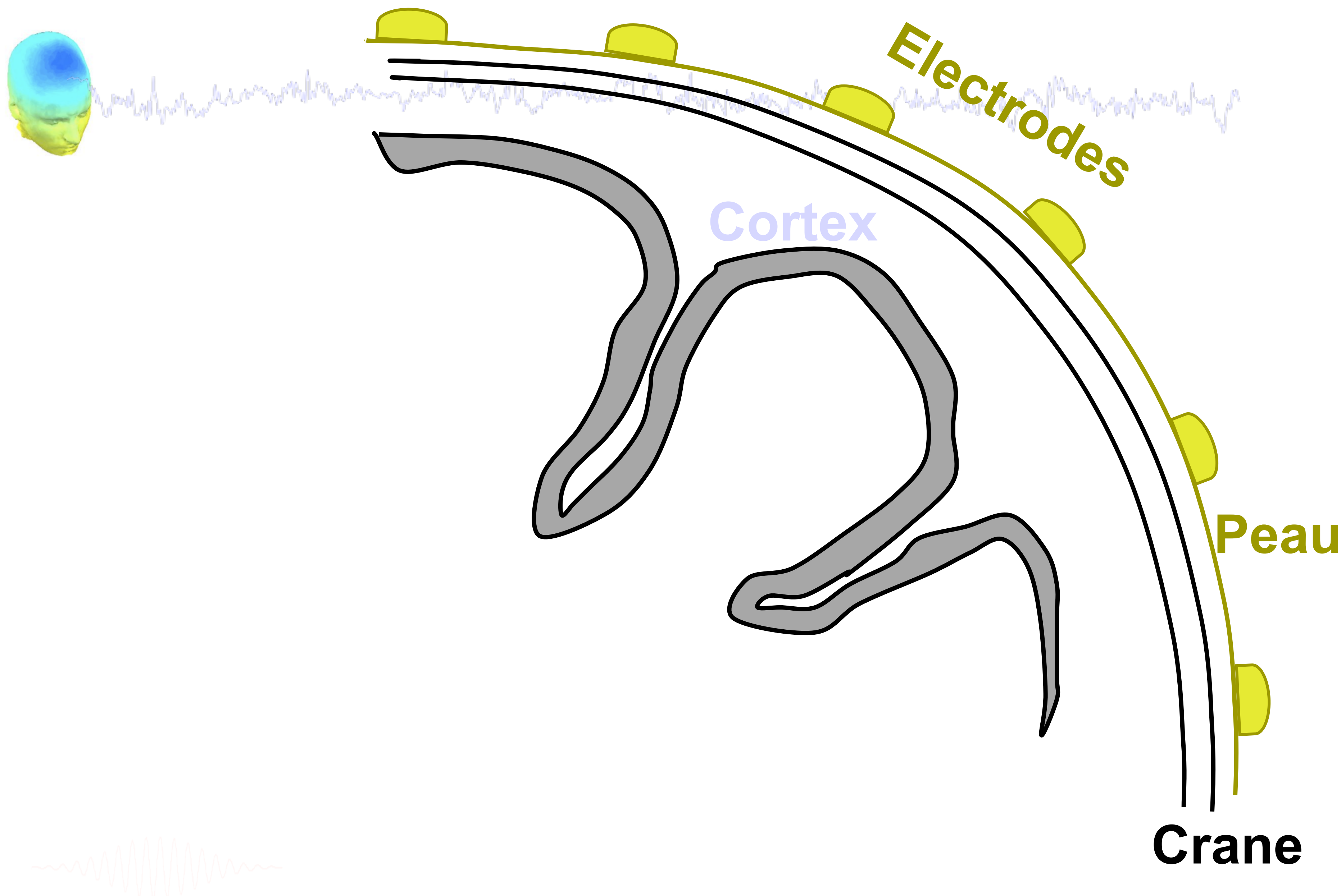
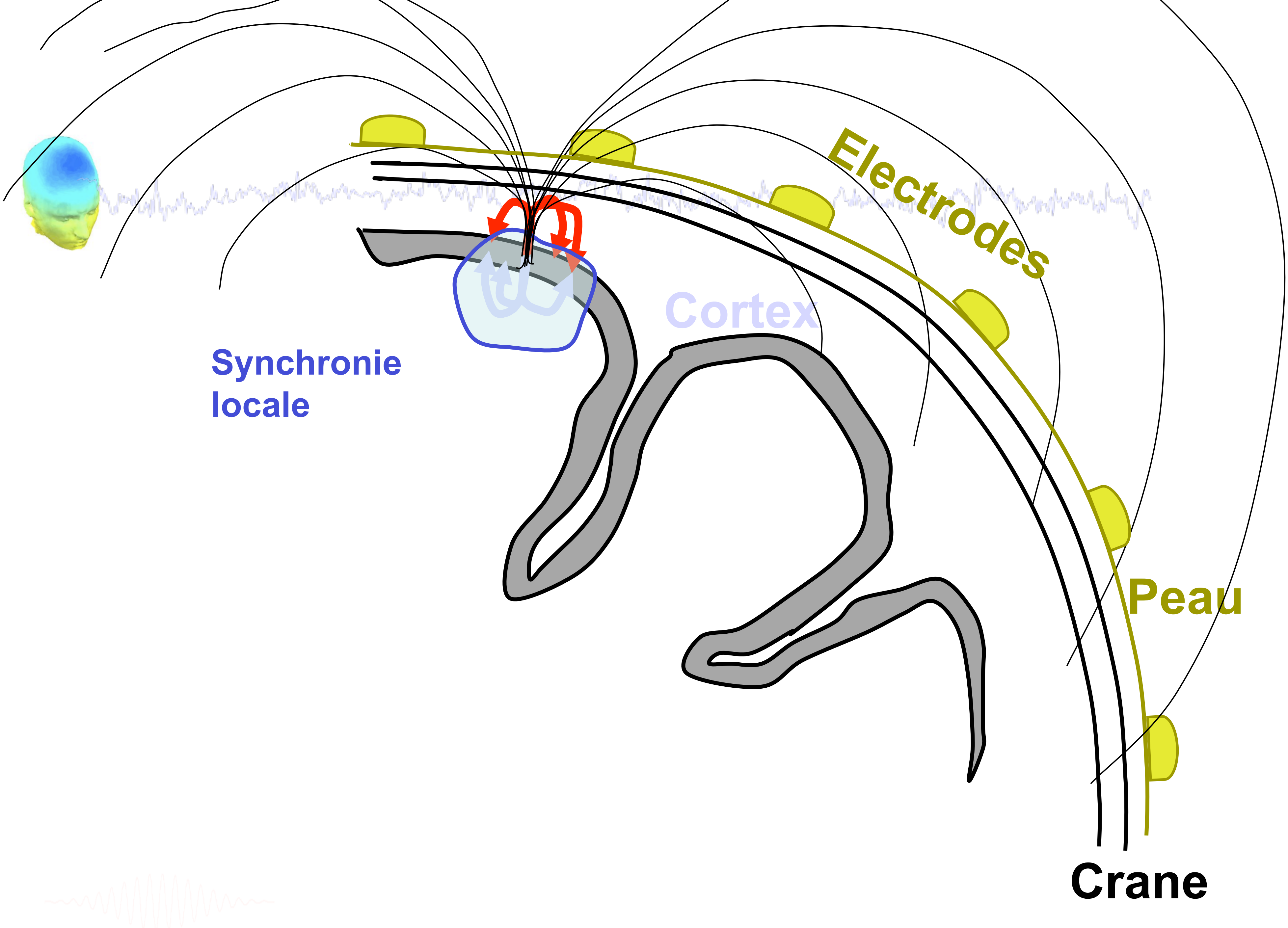
Independent component analysis applied to biophysical time series and EEG

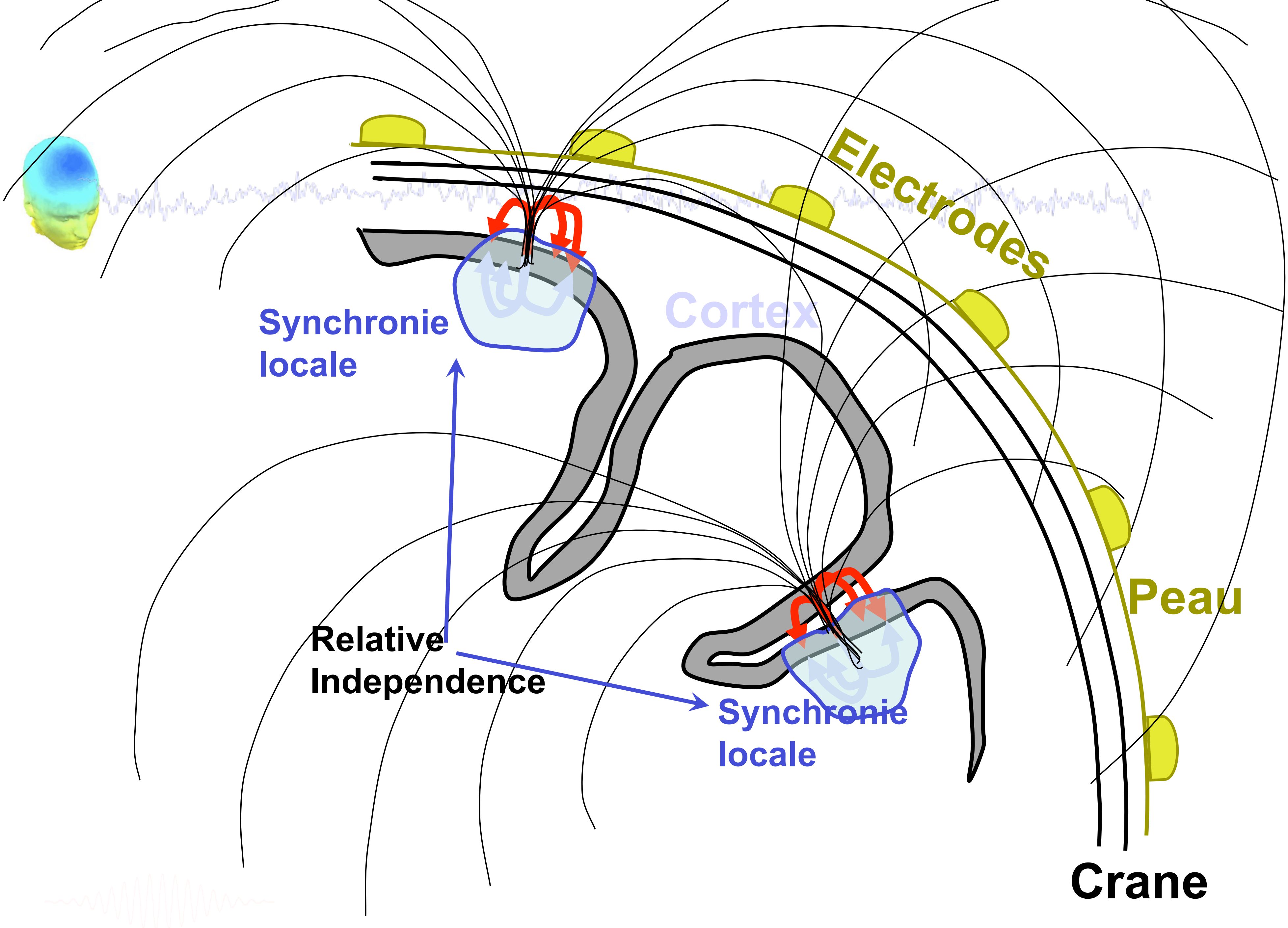
Example: Speech Separation



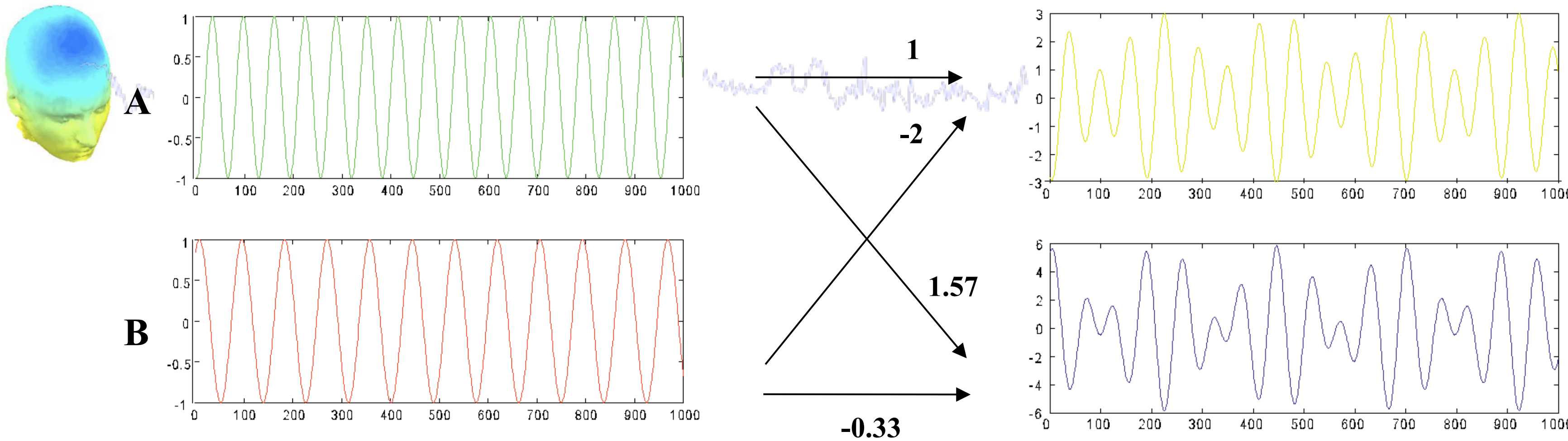

Electrodes
Cortex
Peau
Crane



Electrodes
Cortex
Synchronie locale
Peau
Crane



Electrodes
Cortex
Synchronie locale
Relative Independence
Synchronie locale
Peau
Crane



$Y=[A;B]$

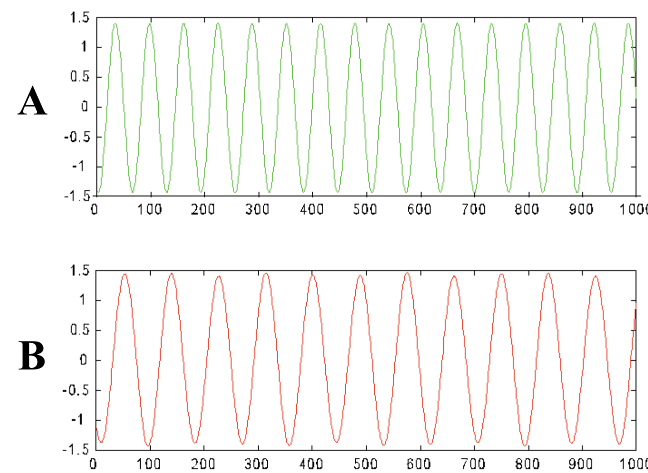
Linear Combination

$X=YW$

ICA

$\tilde{Y}=W^{-1}\tilde{X}$

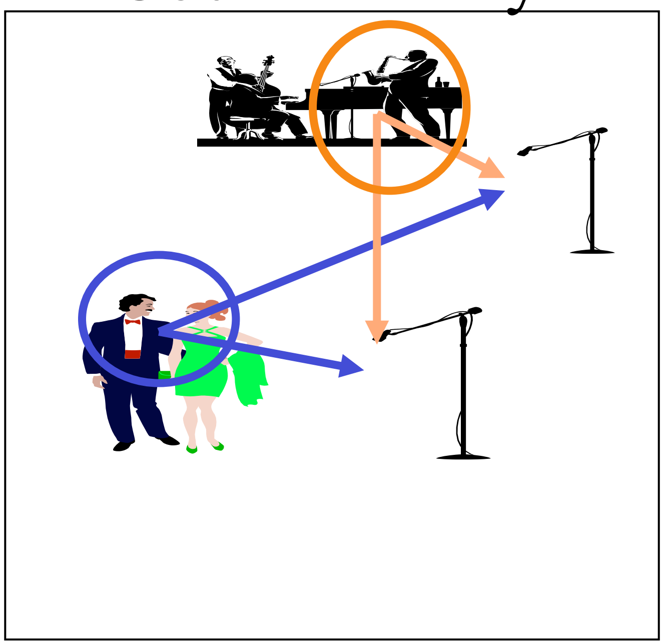
Infomax ICA

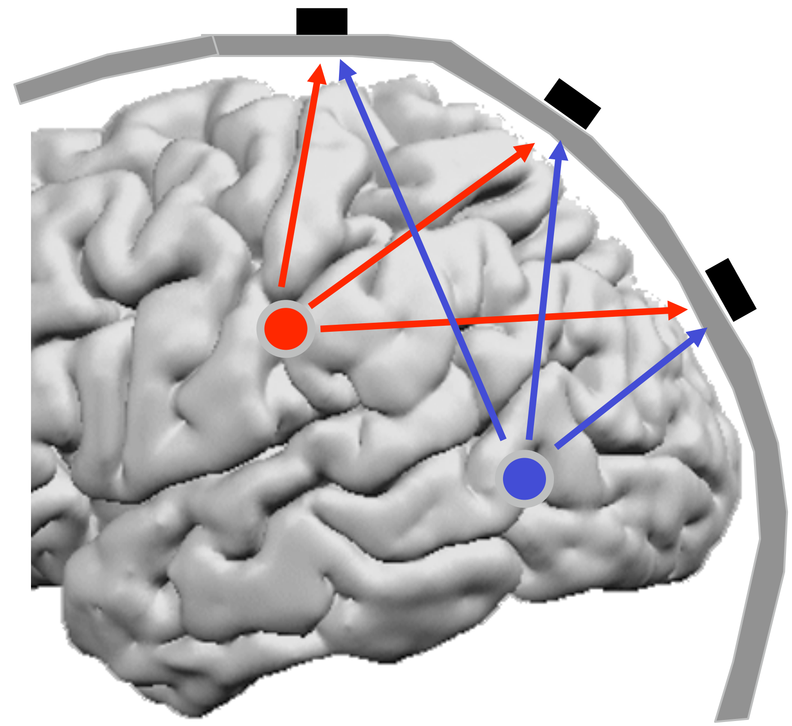


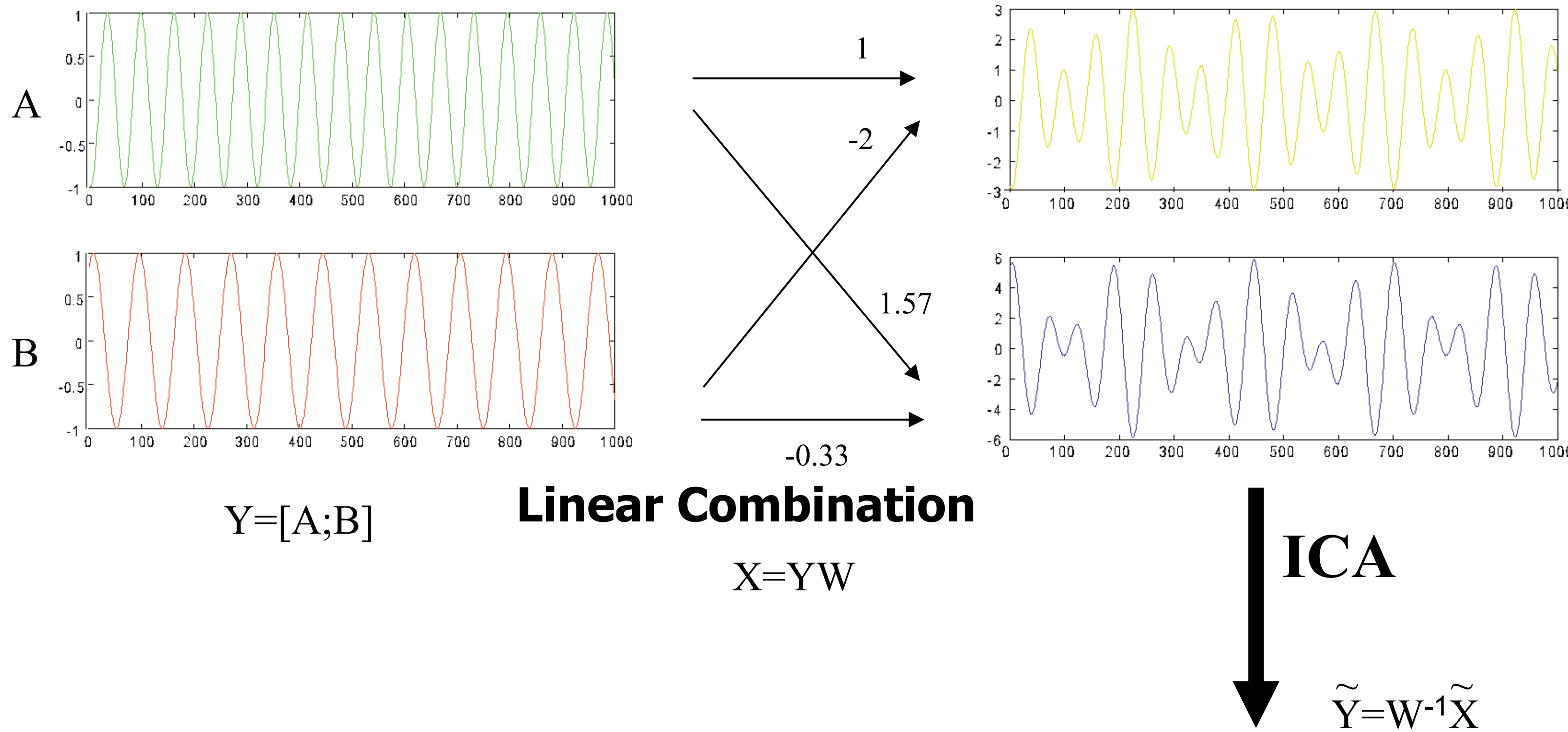
Independent component analysis

Mixture of Brain source activity

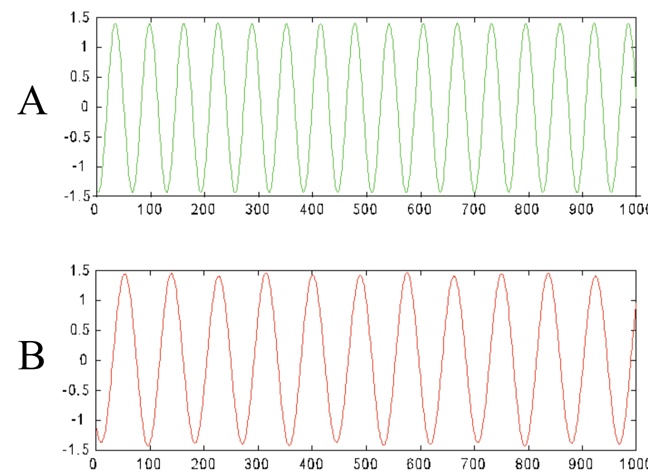
Cocktail Party







ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix



$$\mathbf{U} = \mathbf{W}\mathbf{X}$$

ICA activity

Data

Data X

$$\begin{array}{c c}
\begin{bmatrix} 3 & 2 & 5 & 4 & 3 & 2 & \\ 0 & -2 & -5 & -1 & 1 & -1 & \dots \\ -1 & 2 & 0 & 1 & 0 & -3 & \end{bmatrix} & \begin{array}{l} \leftarrow \text{Channel 1} \\ \leftarrow \text{Channel 2} \\ \leftarrow \text{Channel 3} \end{array}
\end{array}$$

$*$ $*$ $*$

$$\begin{bmatrix} 5 & 3 & -2 \\ 1 & 2 & 4 \\ 0 & -1 & 3 \end{bmatrix} \longrightarrow \begin{array}{c c}
\begin{bmatrix} 3*5+0*3-1*(-2) & 2*5+(-2)*3+2*(-2) & \\ 3*1+0*2-1*4 & 2*1+(-2)*2+2*4 & \dots \\ 5*1-5*2+0*4 & 5*1-5*2+0*4 & \end{bmatrix} & \begin{array}{l} \leftarrow \text{Comp. 1} \\ \leftarrow \text{Comp. 2} \\ \leftarrow \text{Comp. 3} \end{array}
\end{array}$$

Weight matrix W

ICA activity U

$$\mathbf{X}=\mathbf{W}^{-1}\mathbf{U}$$

Data

ICA activity U

$$\begin{array}{cl} \begin{bmatrix} 3 & 2 & 5 & 4 & 3 & 2 & \\ 0 & -2 & -5 & -1 & 1 & -1 & \dots \\ -1 & 2 & 0 & 1 & 0 & -3 & \end{bmatrix} & \begin{array}{l} \leftarrow \text{Comp. } 1 \\ \leftarrow \text{Comp. } 2 \\ \leftarrow \text{Comp. } 3 \end{array} \end{array}$$

$$\begin{bmatrix} 5 & 3 & -2 \\ 1 & 2 & 4 \\ 0 & -1 & 3 \end{bmatrix} \longrightarrow \begin{array}{cl} \begin{bmatrix} 3*5+0*3-1*(-2) & 2*5+(-2)*3+2*(-2) & \\ 3*1+0*2-1*4 & 2*1+(-2)*2+2*4 & \dots \\ 5*1-5*2+0*4 & 5*1-5*2+0*4 & \end{bmatrix} & \begin{array}{l} \leftarrow \text{Chan } 1 \\ \leftarrow \text{Chan } 2 \\ \leftarrow \text{Chan } 3 \end{array} \end{array}$$

Inverse weight matrix W^{-1}

Data X

Historical Remarks

- Herault & Jutten ("Space or time adaptive signal processing by neural network models“, *Neural Nets for Computing Meeting*, Snowbird, Utah, 1986): Seminal paper, neural network
- Bell & Sejnowski (1995): Information Maximization
- Amari et al. (1996): Natural Gradient Learning
- Cardoso (1996): JADE

- **Applications of ICA to biomedical signals**
  - EEG/ERP analysis (Makeig, Bell, Jung & Sejnowski, 1996).
  - fMRI analysis (McKeown et al. 1998)

ICA Theory – Cost Functions

Family of BSS algorithms

- Information theory (Infomax)
- Bayesian probability theory (Maximum likelihood estimation)
- Negentropy maximization
- Nonlinear PCA
- Statistical signal processing (cumulant maximization, JADE)

A unifying Information-theoretic framework for ICA

- Pearlmutter & Parra showed that InfoMax, ML estimation are equivalent.
- Lee et al. (1999) showed negentropy has the equivalent property to InfoMax.
- Girolami & Fyfe showed nonlinear PCA can be viewed from information-theoretic principle.

Independent Component Analysis

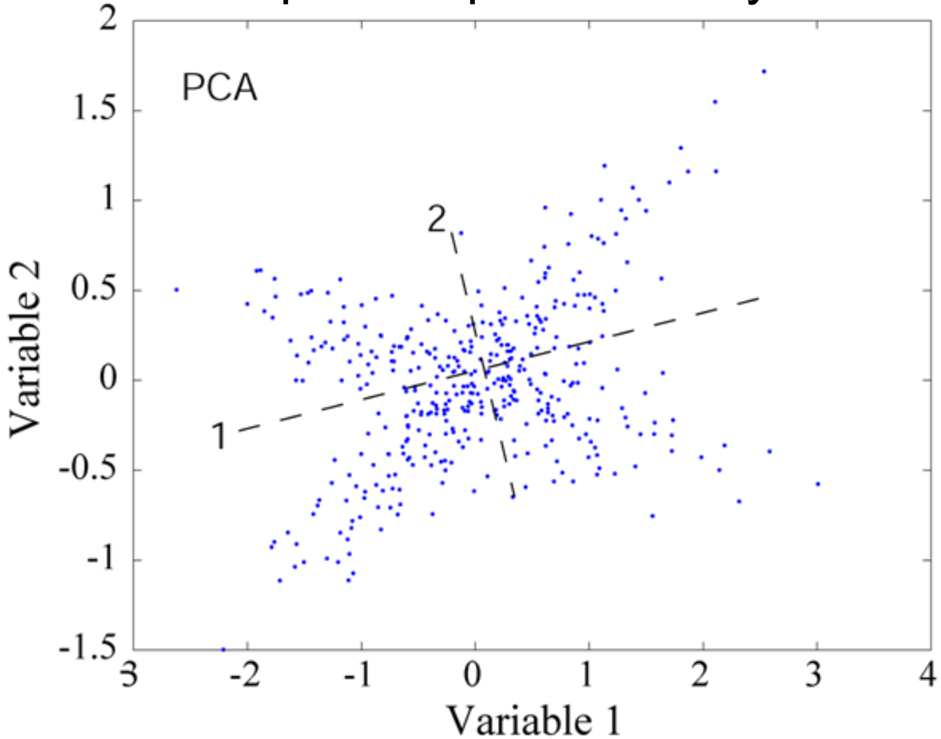
ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix,

$$\mathbf{U} = \mathbf{W}\mathbf{X},$$

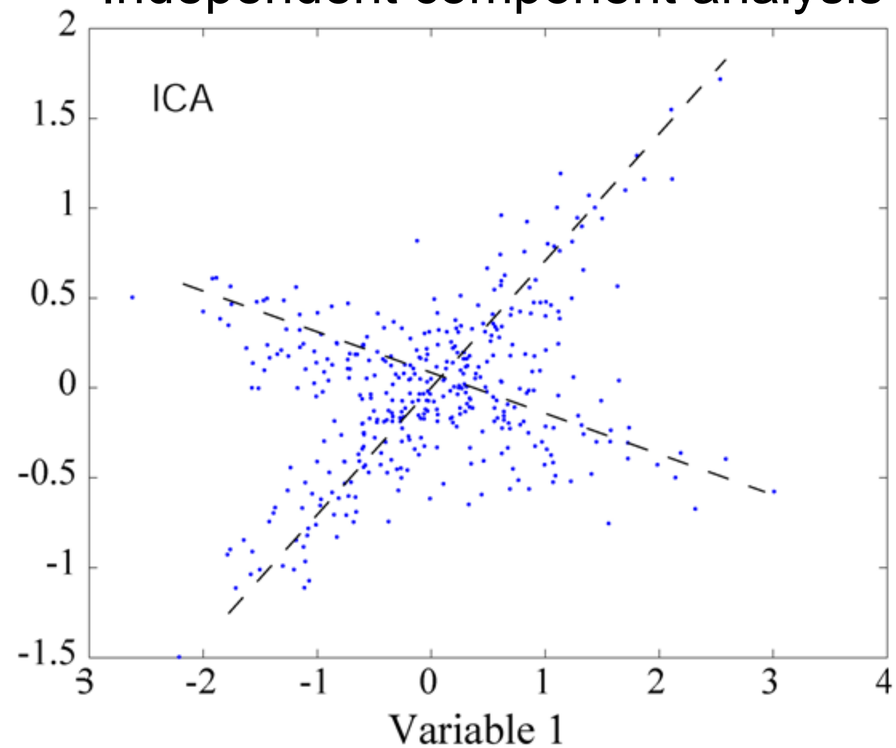
While PCA simply decorrelates the outputs (using an orthogonal matrix **W**), ICA attempts to make the outputs **statistically independent**, while placing no constraints on the matrix **W**.

ICA and PCA

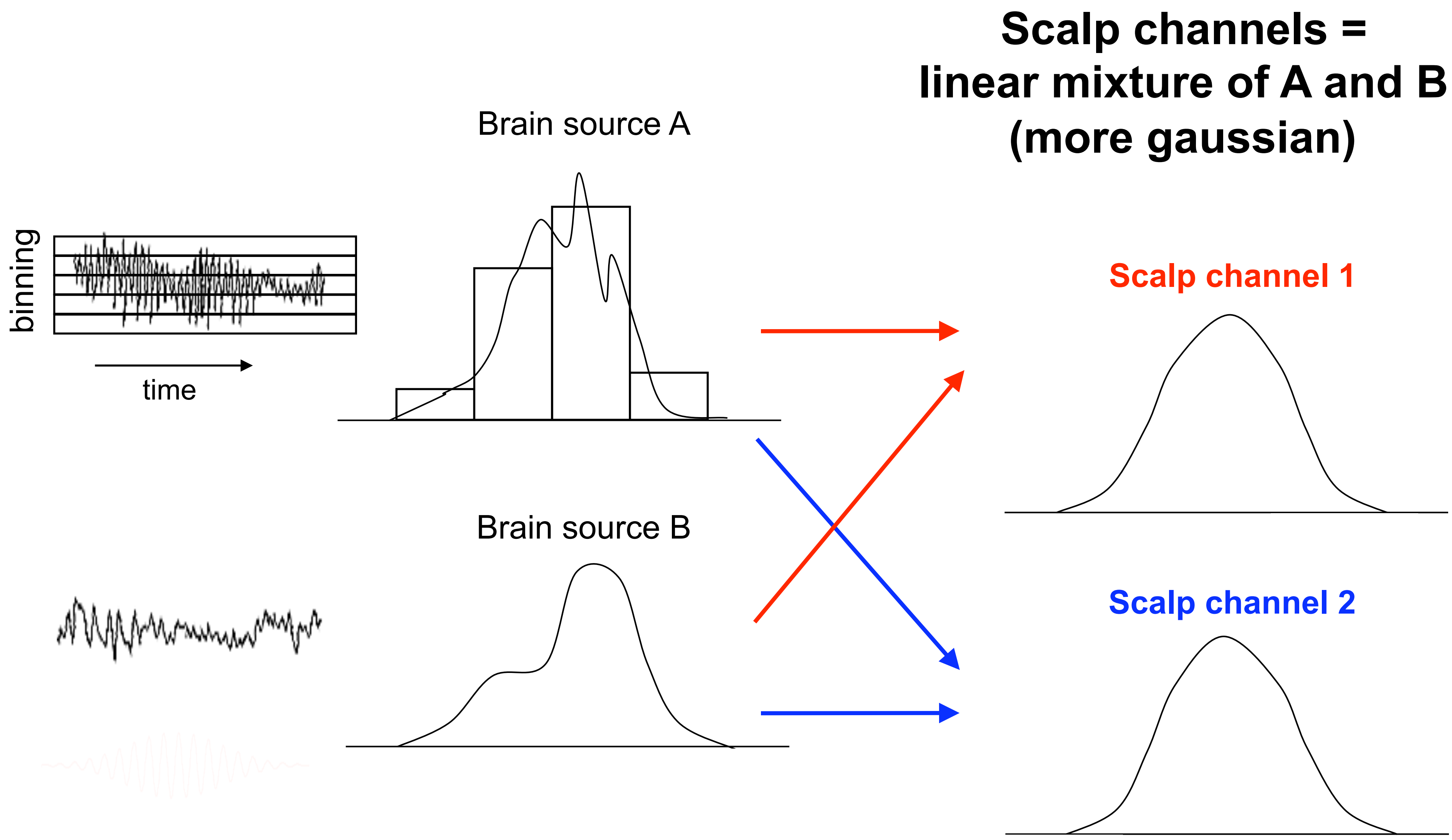
Principal component analysis



Independent component analysis

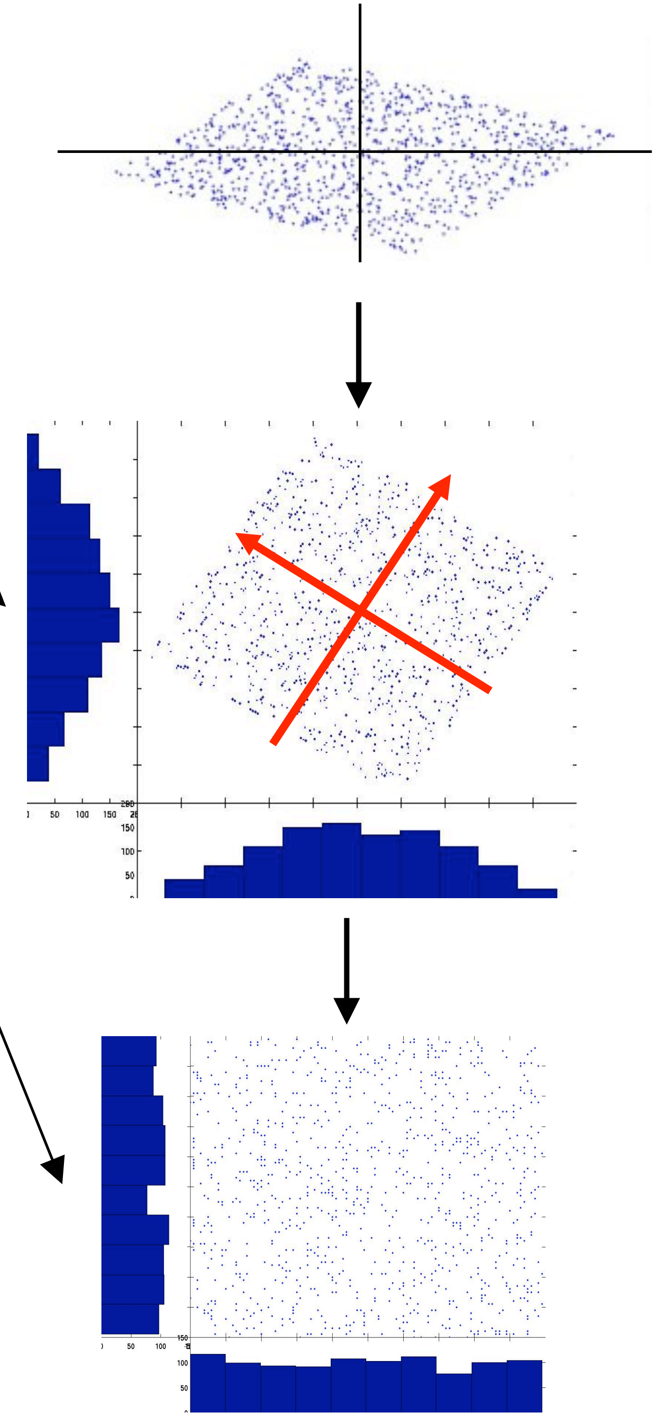


Central limit theorem



ICA Training Process

Central limit theorem



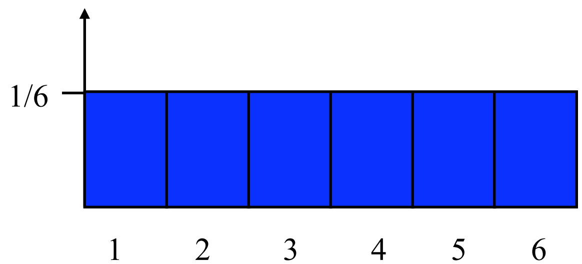
- Remove the mean
  $x = x - \langle x \rangle$
- 'Sphere' the data by diagonalizing its covariance matrix,
  $x = \langle xx^T \rangle^{-1/2}(x - \langle x \rangle)$.
- Update W according to

$$\Delta \mathbf{w} \propto \frac{\partial H(\mathbf{y})}{\partial \mathbf{w}} \mathbf{w}^T \mathbf{w}$$

Entropy

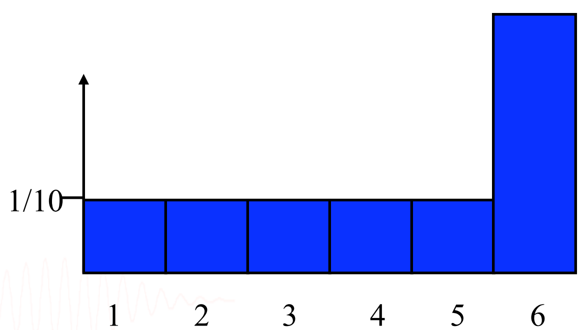
$$H(X) = -\sum_{x\in\mathcal{X}} p(x)\log_b p(x).$$

Dice: 1/6



$$H = 6\left(-\frac{1}{6}\log_2\left(\frac{1}{6}\right)\right) = 2.58$$

Fake dice (make a 6 half of the time): entropy 2.16 (base 2)



$$H = 5\left(-\frac{1}{10}\log_2\left(\frac{1}{10}\right)\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 2.16$$

Entropy

$$H(X) = -\sum_{x\in\mathcal{X}} p(x)\log_b p(x).$$

Joint entropy

$$H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y)\log_b p(x,y).$$

Mutual Information

$$H(y_1,y_2) = H(y_1) + H(y_2) - I(y_1,y_2).$$

Shannon in his landmark 1948 paper ``A Mathematical Theory of Communication."

From http://planetmath.org/encyclopedia/ShannonsTheoremEntropy.html

Contingency table for stress and emotionality

	STRE						
	1	**2**	**3**	**4**	**5**	**6**	**Total**
EMOT= **1**	19	4					23
2	11	63	64	3	1		142
3	2	16	18	20	2	2	60
4	1	4	1	9	6	2	23
5			1	2	4	3	10
6				1	1	1	3
Total	33	87	84	35	13	8	

From http://tecfa.unige.ch/~lemay/thesis/THX-Doctorat/node149.html

Contingency frequencies for stress and emotionality

	STRE					
	1	**2**	**3**	**4**	**5**	**6**
EMOT= **1**	0.07	0.02				
2	0.04	0.24	0.25	0.01		
3	0.01	0.06	0.07	0.08	0.01	0.01
4		0.02		0.03	0.02	0.01
5				0.01	0.02	0.01
6						

Joint entropy 3.46; exercise: compute mutual information

$$H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y)\log_b p(x,y)$$

ICA learning rule

How to make the outputs statistical independent?

Minimize their redundancy or mutual information.

Consider the joint entropy of two components,

$$H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$$

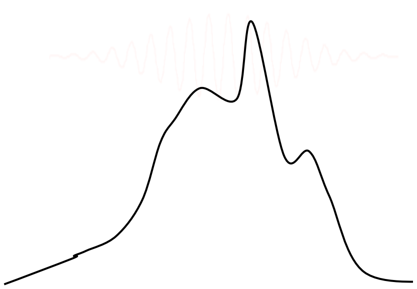
Maximizing $H(y_1, y_2) \Longrightarrow$ minimizing $I(y_1, y_2)$.

The learning rule:

=0 if the two variables are independent

$$\Delta \mathbf{W} \propto \frac{\partial H(\mathbf{y})}{\partial \mathbf{W}} \underbrace{\mathbf{W}^T \mathbf{W}}_{\text{Natural gradient (Amari)}}$$

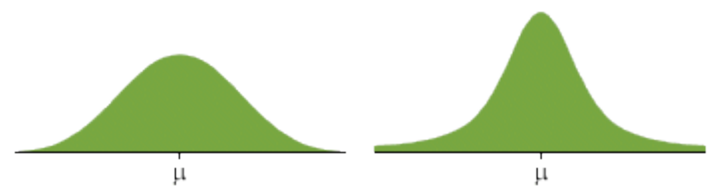
Entropy extremum



Kurtosis, Super- and Sub-Gaussian

Kurtosis: a measure of how peaked or flat of a probability distribution is.

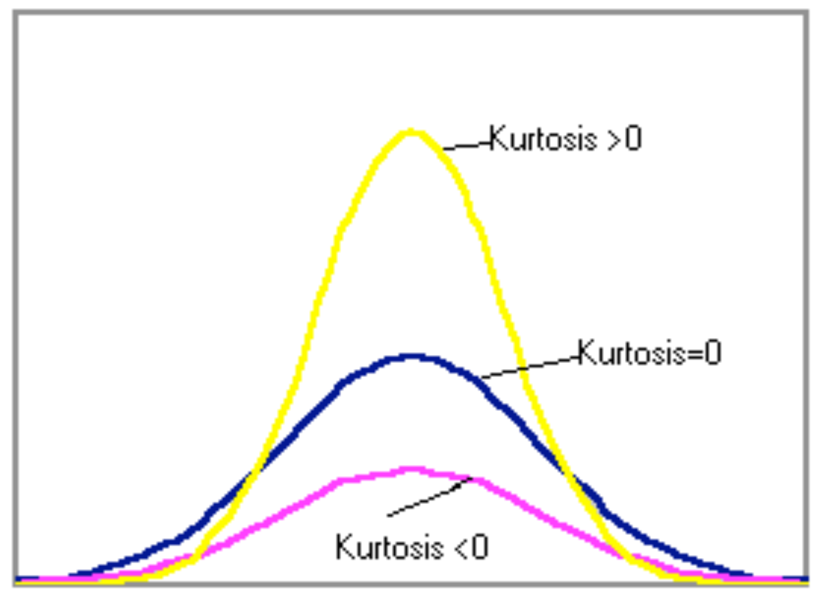
$$kurt(X) = \frac{E\left[(X-\mu)^4\right]}{\sigma^4}$$



Gaussian Dist. Kurtosis = 0

Super-Gaussian: kurtosis > 0

Sub-Gaussian: kurtosis < 0



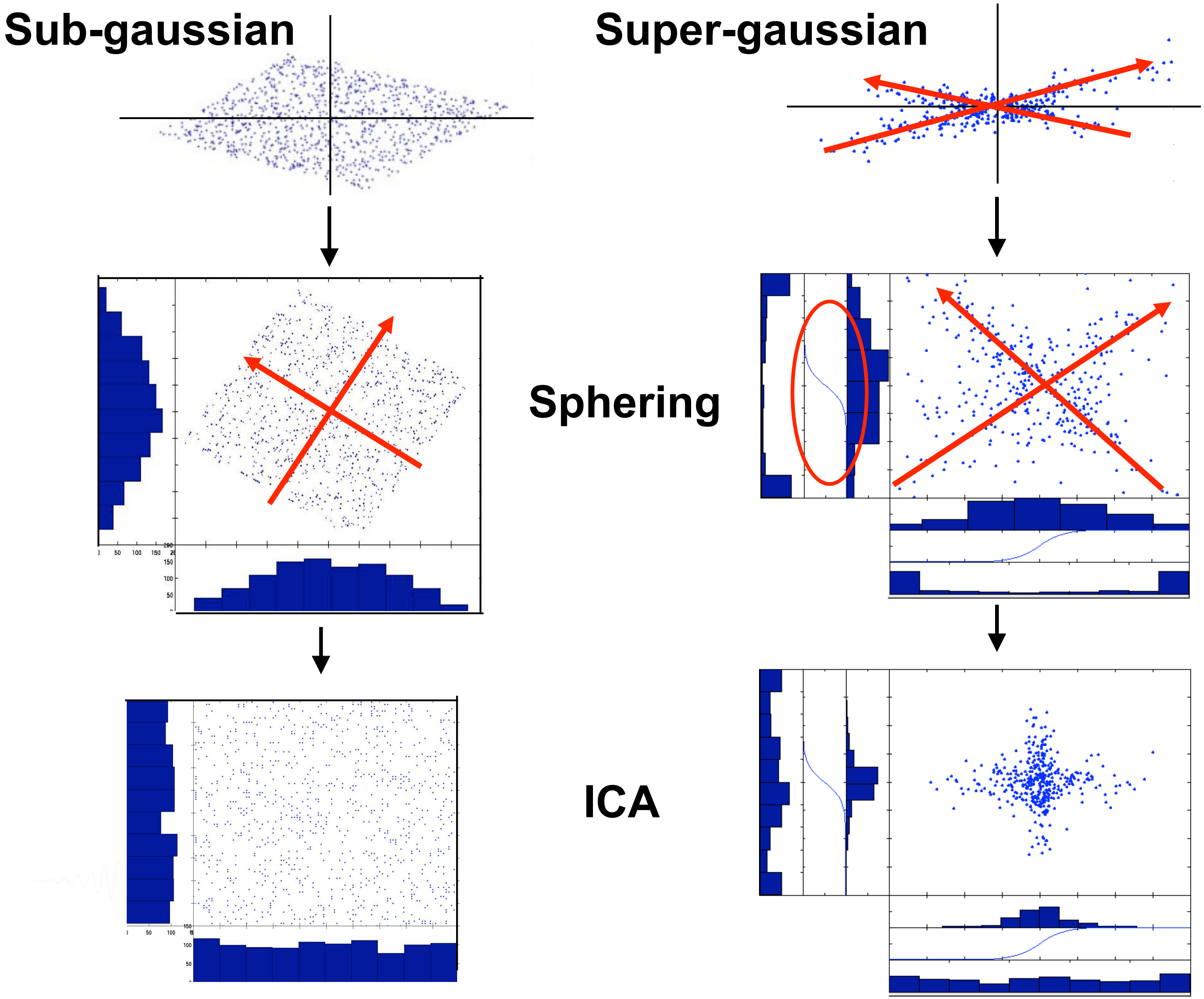
Moments, Cumulants

Moments $\mu_x(n) = E\{x^n\}$

Central moments $m_x(n) = E\{(x - m_x)^n\}$

Cumulants

$$\begin{aligned}
c_1 &= m_1 = \mu && \longleftarrow \text{mean} \\
c_2 &= m_2 = \sigma^2 && \longleftarrow \text{variance} \\
c_3 &= m_3 && \longleftarrow \text{skewness} \\
c_4 &= m_4 - 3m_2^2 && \longleftarrow \text{kurtosis}
\end{aligned}$$

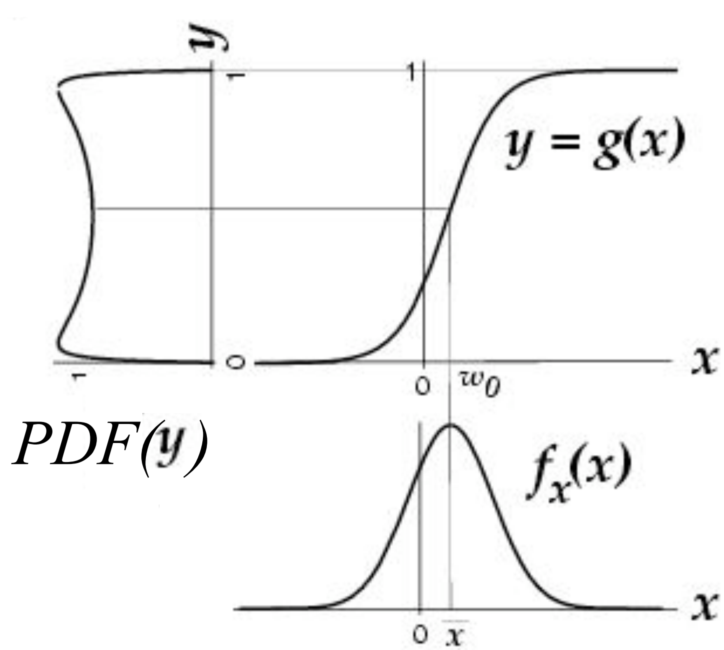

Sub-gaussian
Super-gaussian
Sphering
ICA

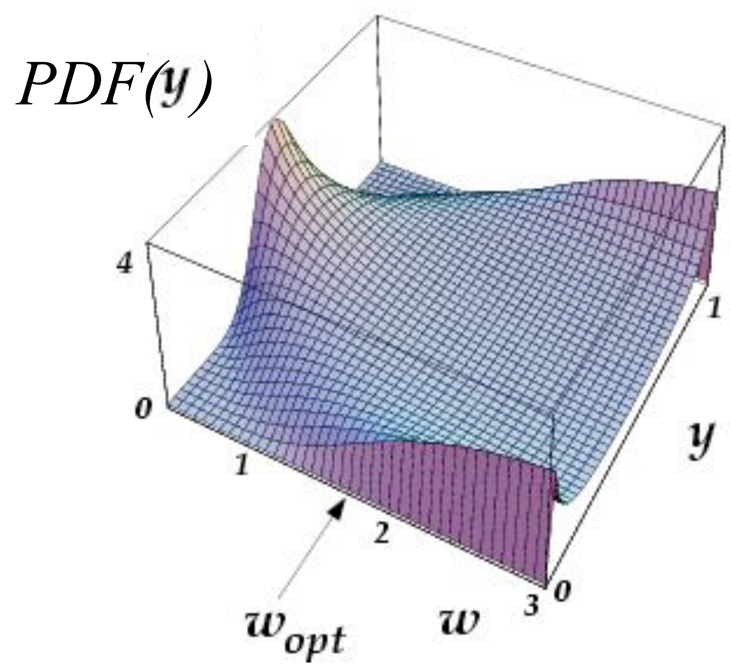
InfoMax (Bell & Sejnowski, 1995)

To make the u_i independent, we need to operate on non-linear transformed output variables, $\mathbf{y} = g(\mathbf{u})$, such as

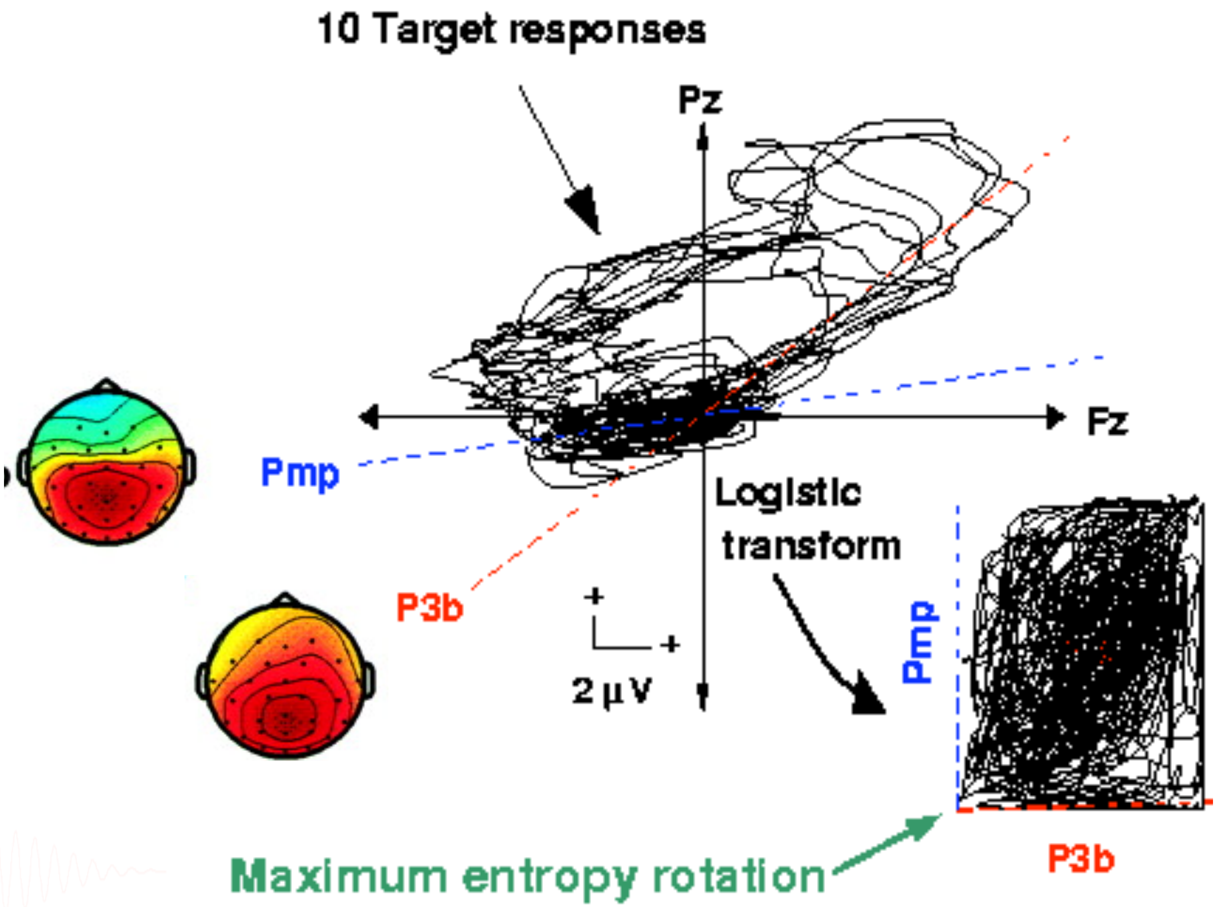
$$\mathbf{y} = \frac{1}{1+e^{-\mathbf{u}}}, \qquad \mathbf{u} = \mathbf{W}\mathbf{x} + \mathbf{w}_0$$

The non-linear function provides all the higher-order statistics necessary to establish independence.



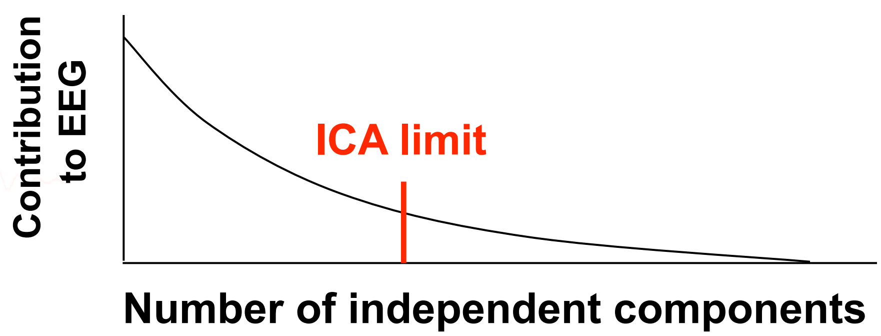


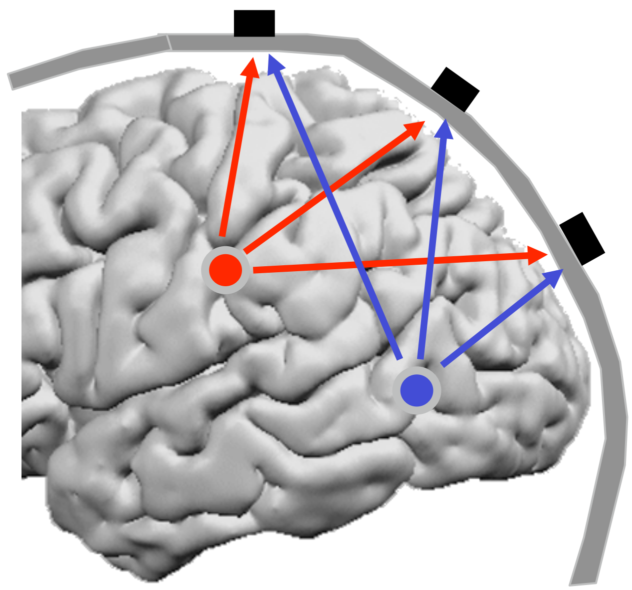
Independent components of EEG/ERP



ICA/EEG Assumptions

- Mixing is linear at electrodes **OK**
- Propagation delays are negligible **OK**
- Component time courses are independent **~**
- Number of components less than the number of channels.





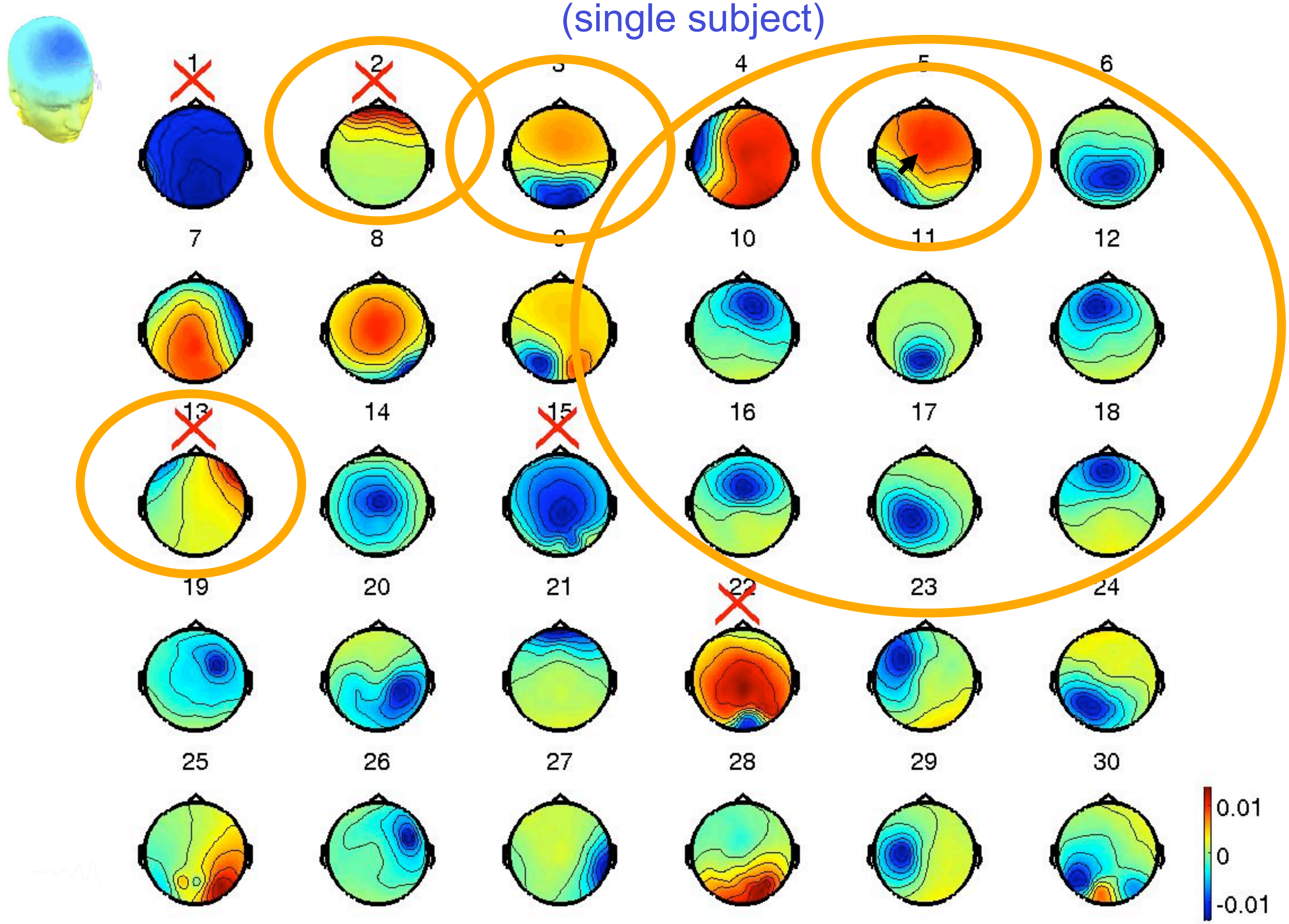
Independent Component Categories

- Artifacts
- Stimulus-locked activity
- Response-locked activity
- Non-phase locked activity
- Event-modulated oscillatory activity

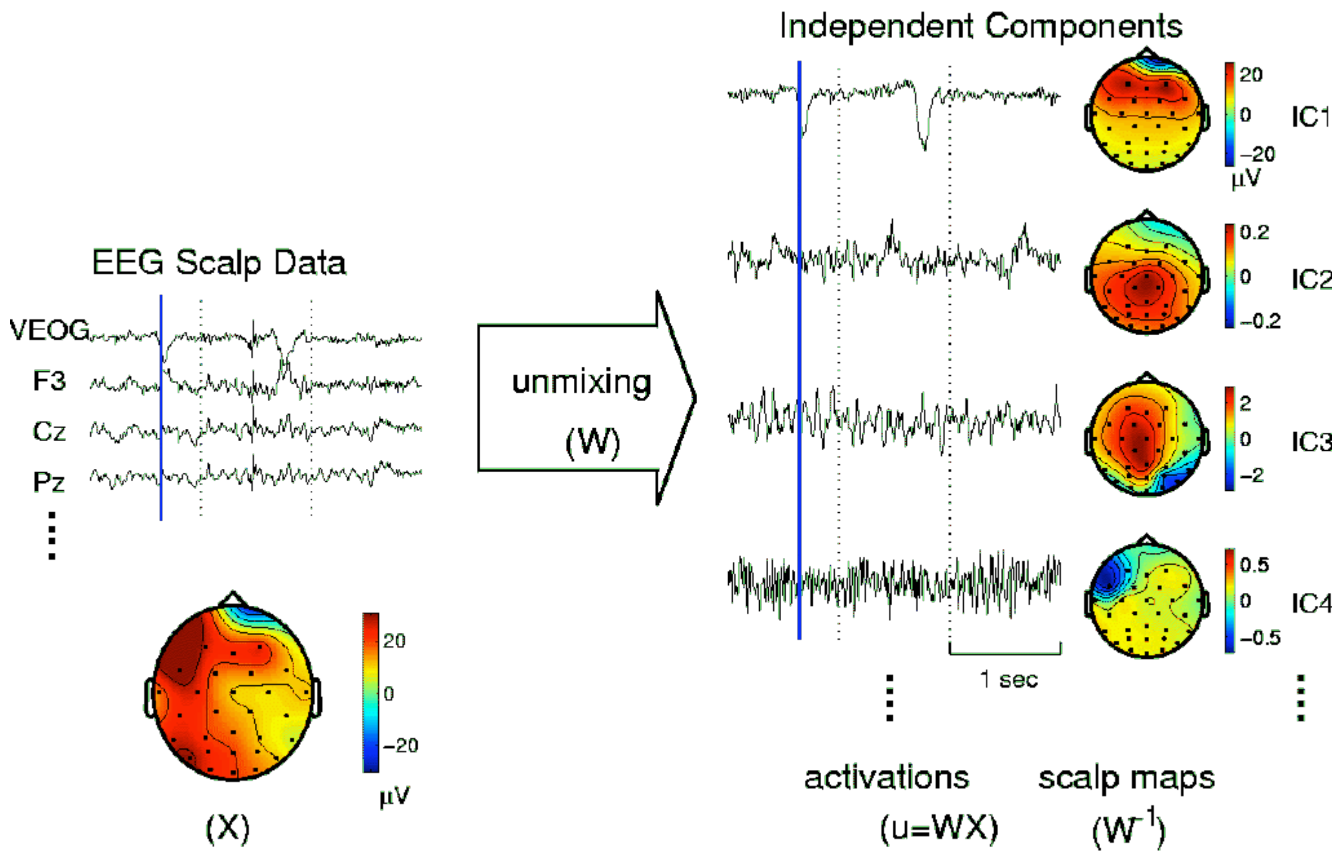
Characteristics of Independent Component of the EEG

- Concurrent Activity
- Maximally Temporally Independent
- Overlapping Maps and Spectra
- Dipolar Scalp Maps
- Functionally Independent
- Between-Subject Regularity

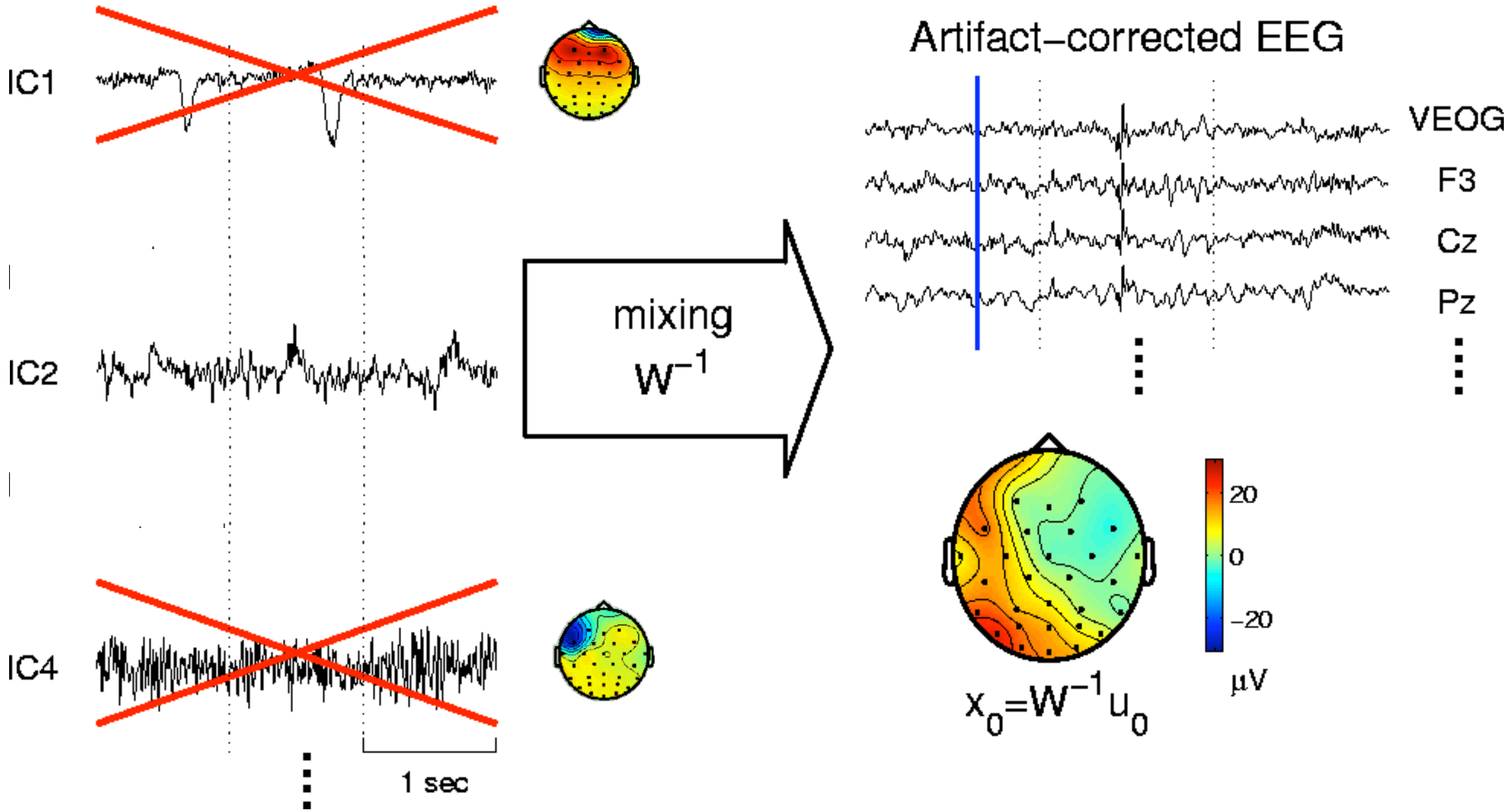
Largest 30 Independent Components
(single subject)



ICA Decomposition into Independent Components



Selective Projection onto Scalp Channels



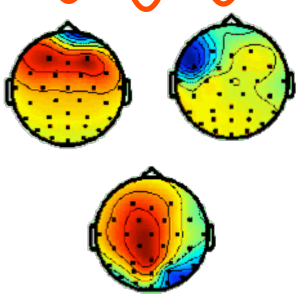
$$X = W^{-1}U$$

Data

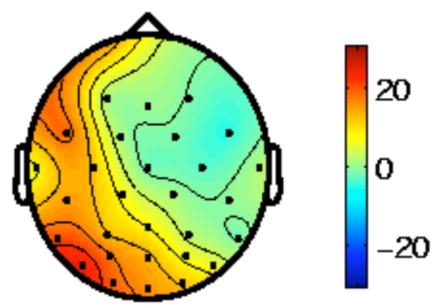
ICA activity U

$$\begin{array}{cl} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & -2 & -5 & -1 & 1 & -1 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \dots \end{bmatrix} & \begin{array}{l} \leftarrow \text{Comp. 1} \\ \leftarrow \text{Comp. 2} \\ \leftarrow \text{Comp. 3} \\ \; \end{array} \end{array}$$

$$\begin{bmatrix} 5 & 3 & -2 & \dots \\ 1 & 2 & 4 & \dots \\ 0 & -1 & 3 & \dots \\ \dots \end{bmatrix} \rightarrow \begin{array}{cl} \begin{bmatrix} 3*5+0*3-1*(-2) & 2*5+(-2)*3+2*(-2) & \dots \\ 3*1+0*2-1*4 & 2*1+(-2)*2+2*4 & \dots \\ 5*1-5*2+0*4 & 5*1-5*2+0*4 & \dots \\ \dots & \dots \end{bmatrix} & \begin{array}{l} \leftarrow \text{Chan 1} \\ \leftarrow \text{Chan 2} \\ \leftarrow \text{Chan 3} \\ \; \end{array} \end{array}$$

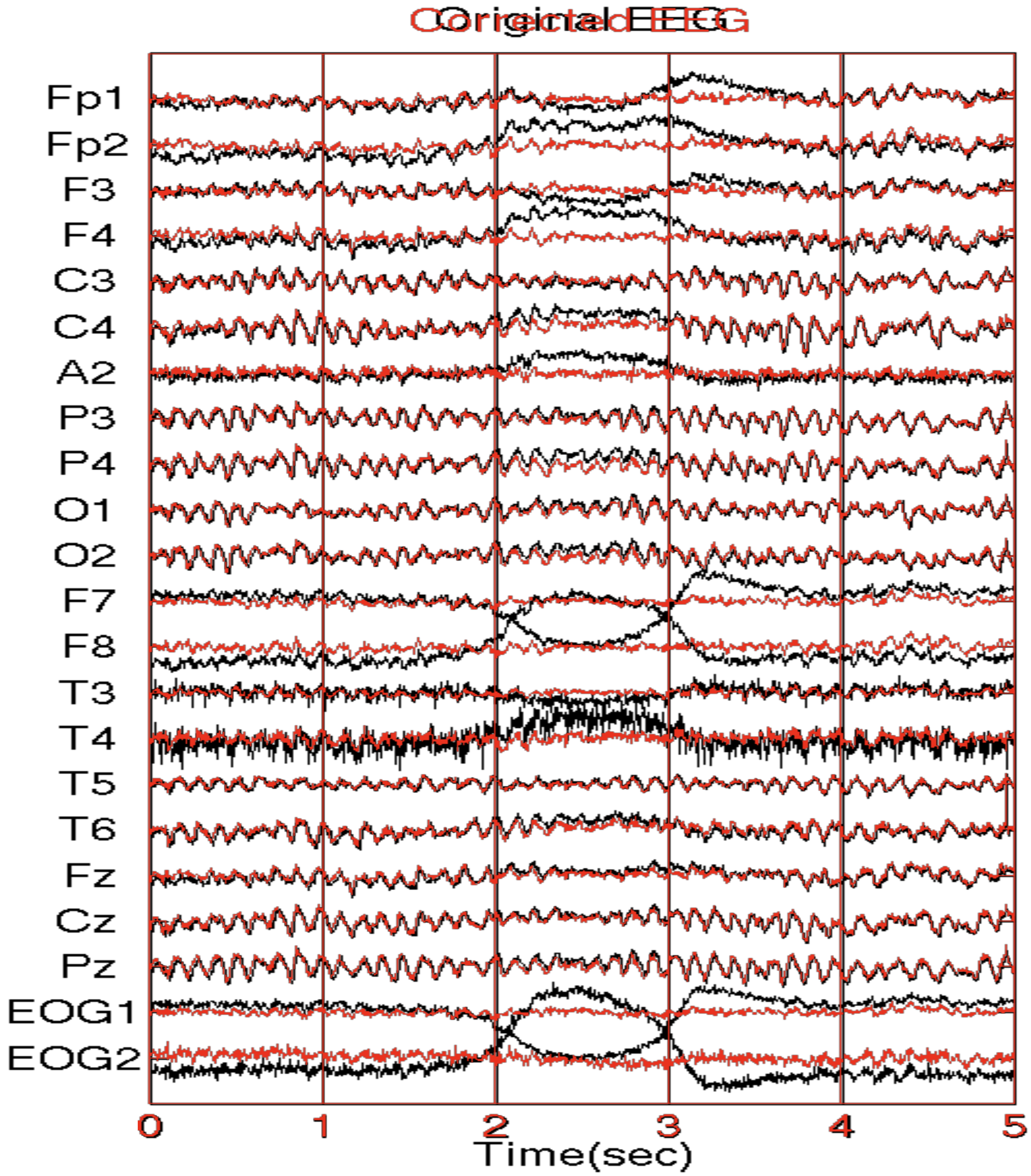


Data X

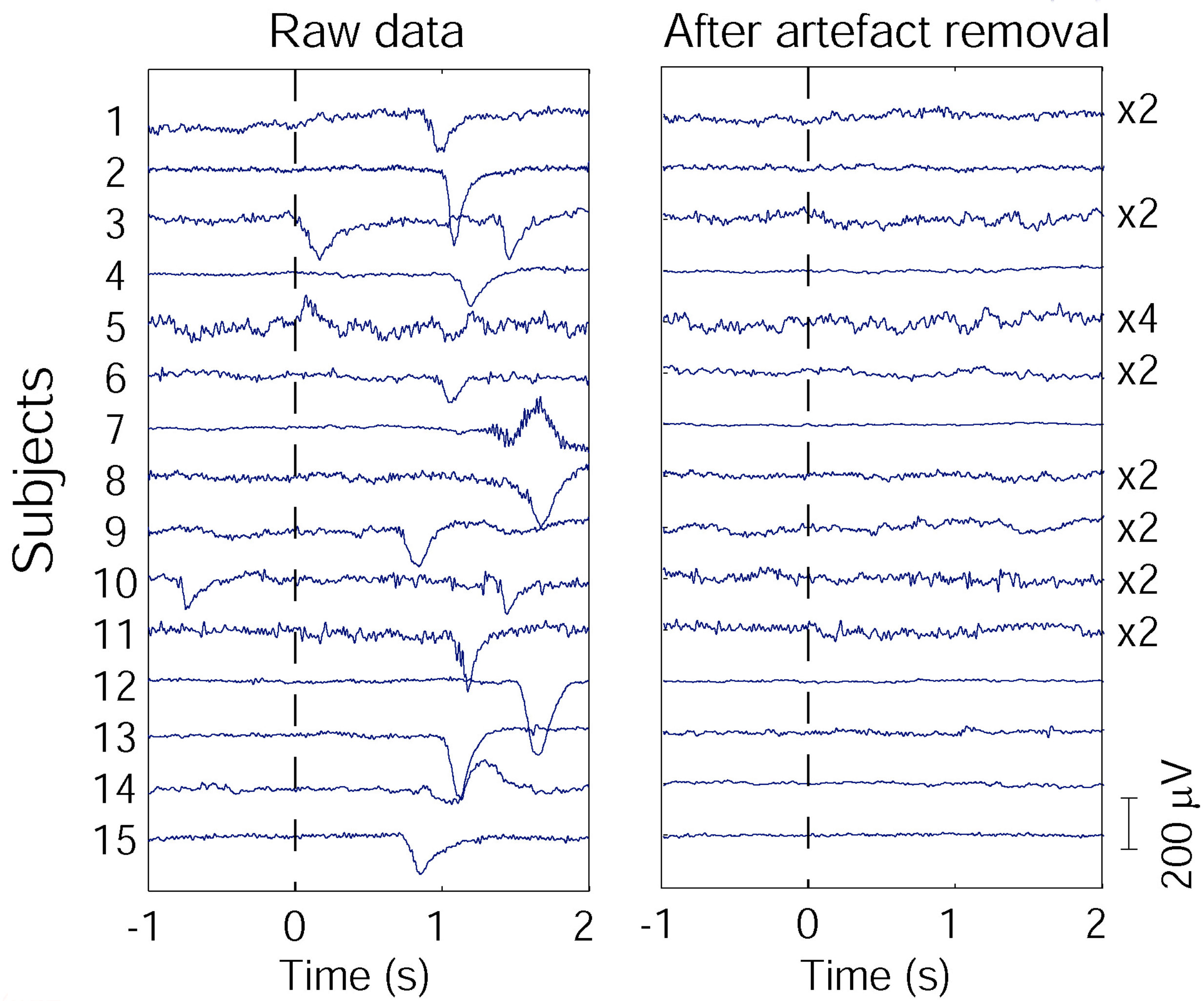


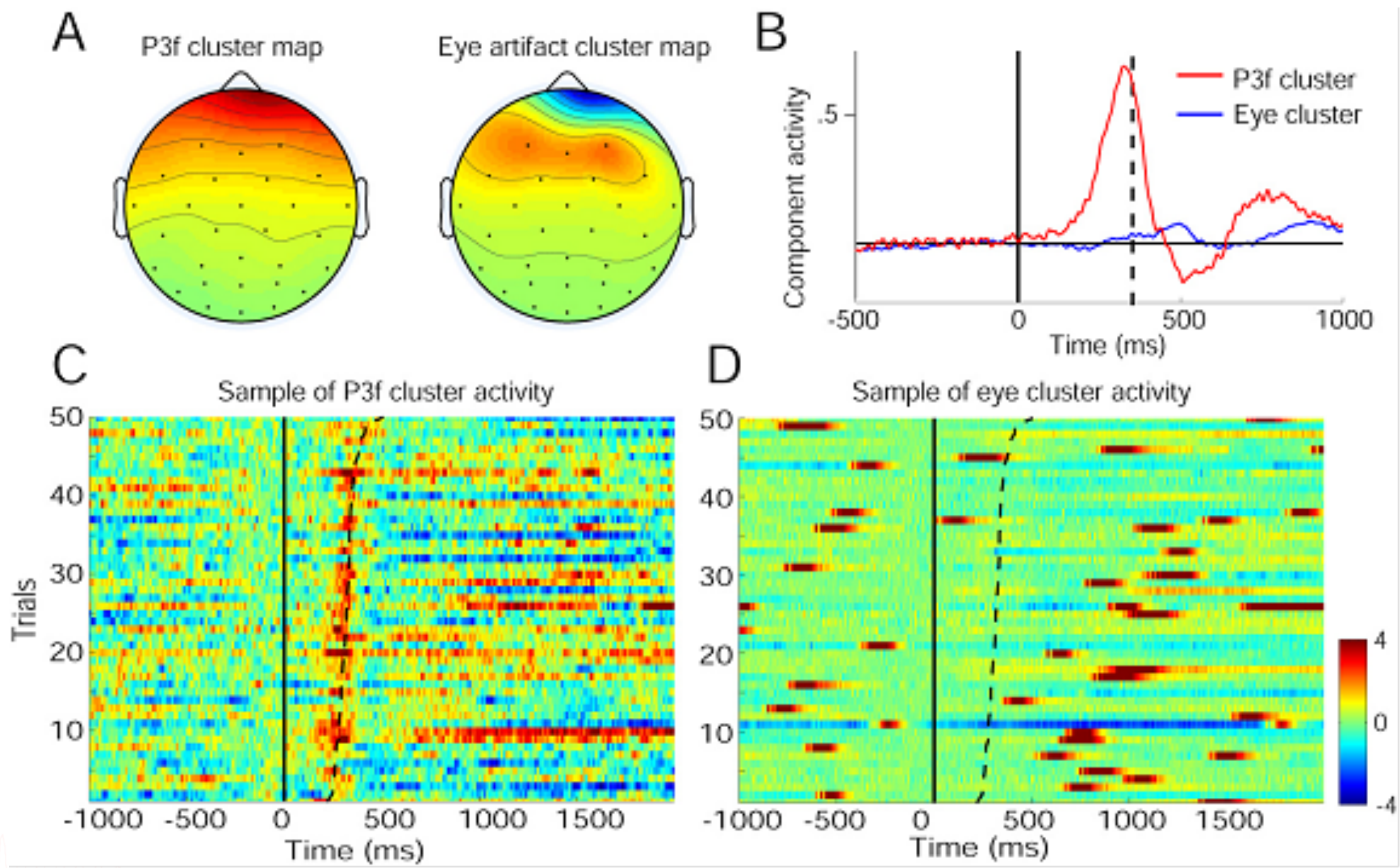
Inverse weight matrix W^{-1}

ICA-based Artifact Removal



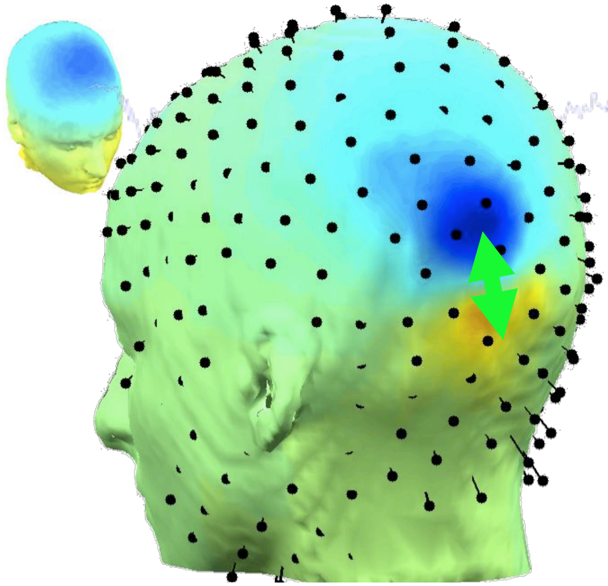
Artifact removal using ICA



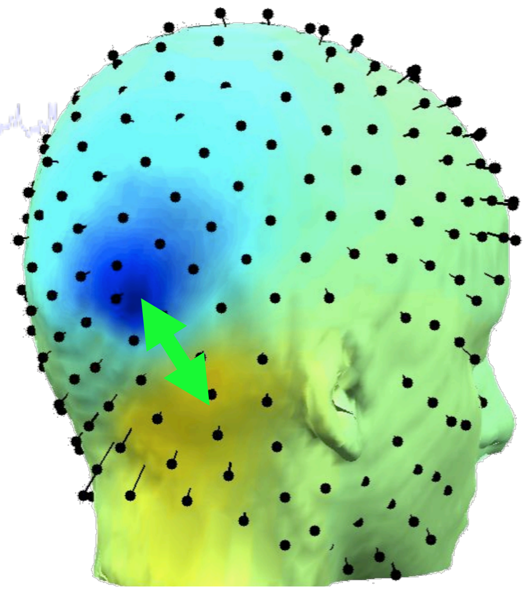

A
P3f cluster map
Eye artifact cluster map
B
Component activity
.5
P3f cluster
Eye cluster
-500
0
500
1000
Time (ms)
C
Sample of P3f cluster activity
Trials
50
40
30
20
10
-1000
-500
0
500
1000
1500
Time (ms)
D
Sample of eye cluster activity
50
40
30
20
10
-1000
-500
0
500
1000
1500
Time (ms)
4
0
-4

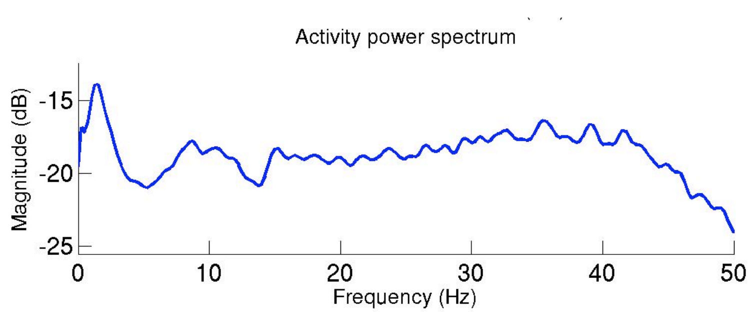
Two Neck Muscle Processes

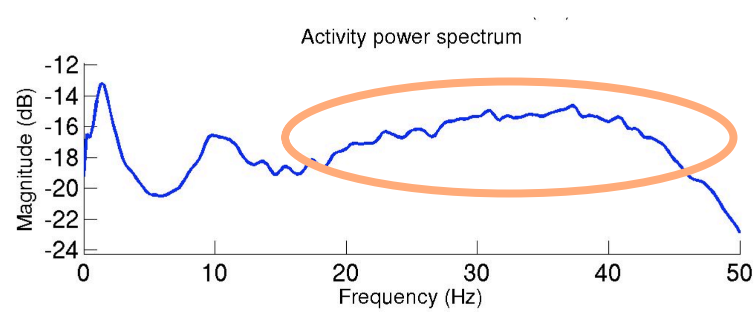
IC39



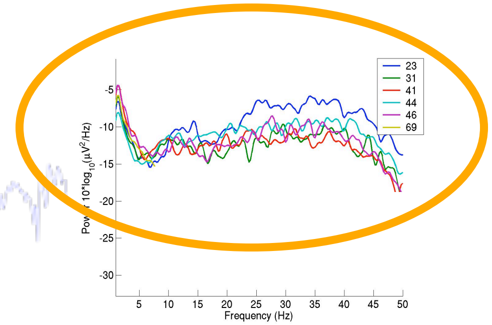
IC31

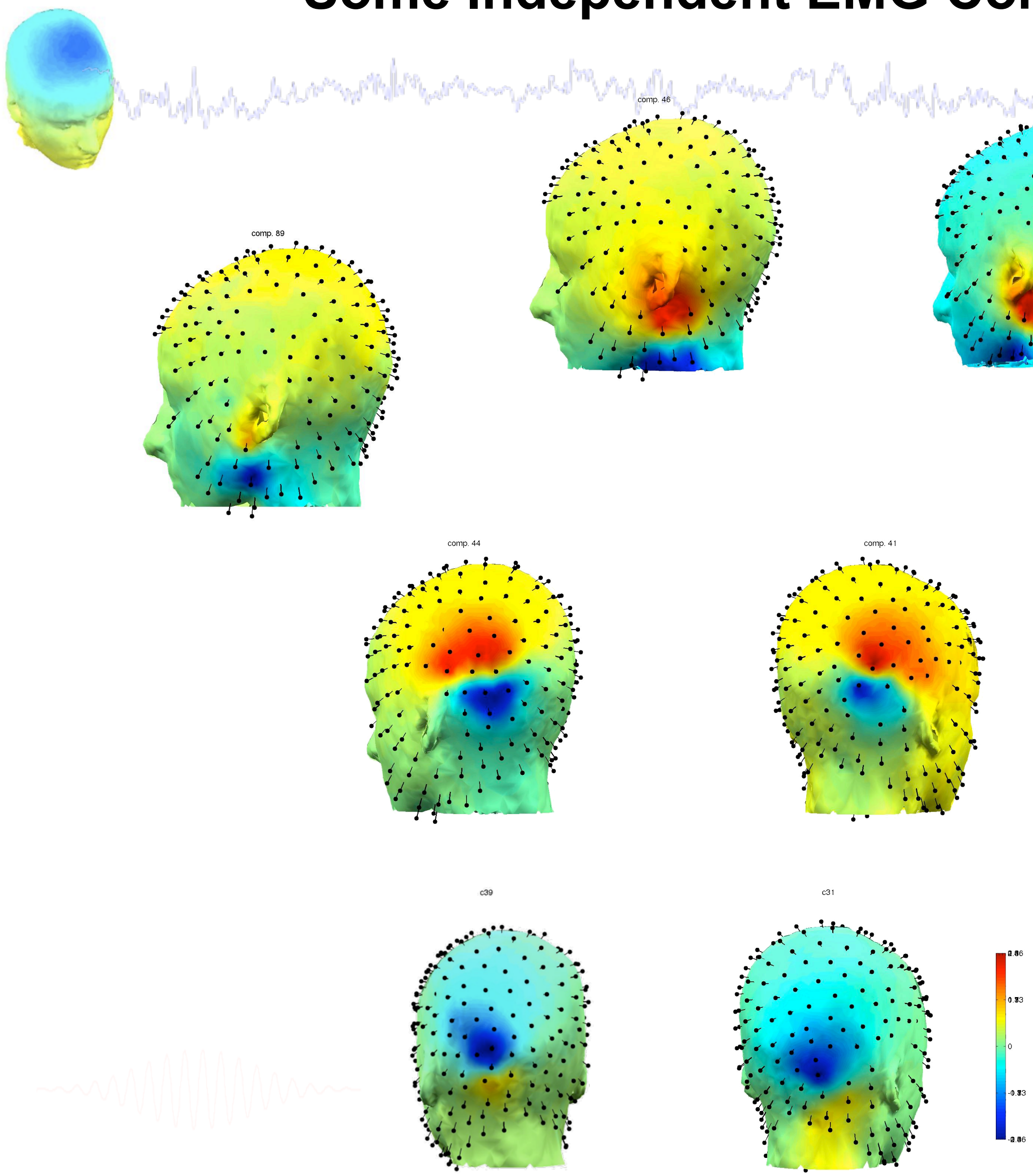


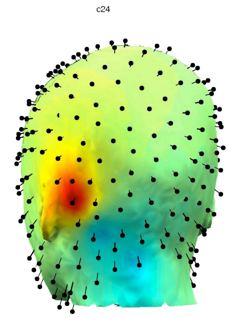


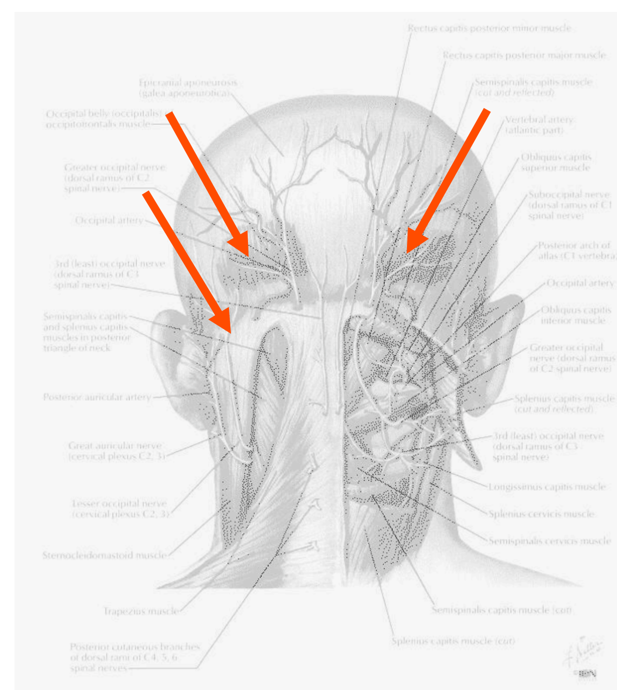


Some Independent EMG Components

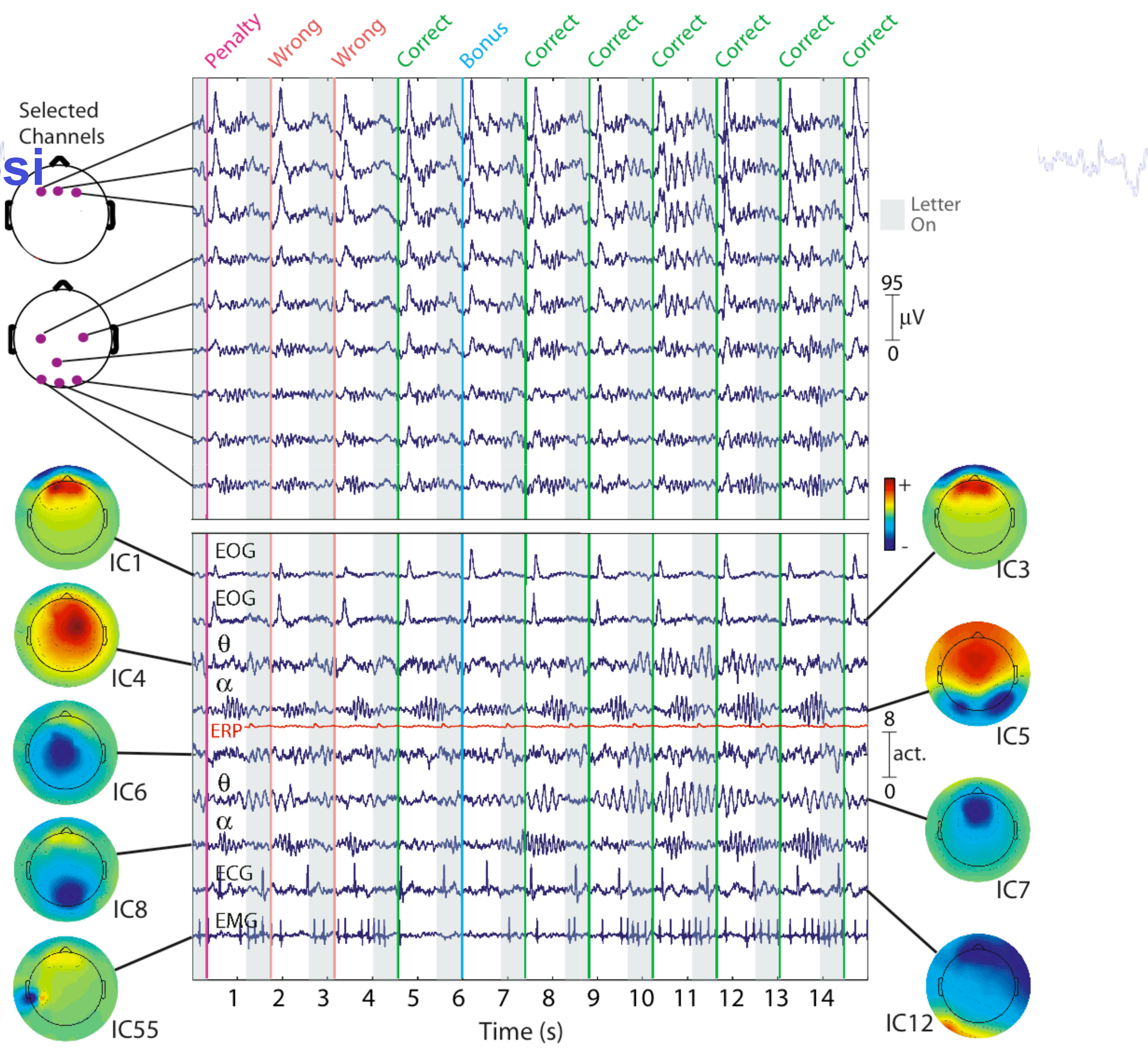








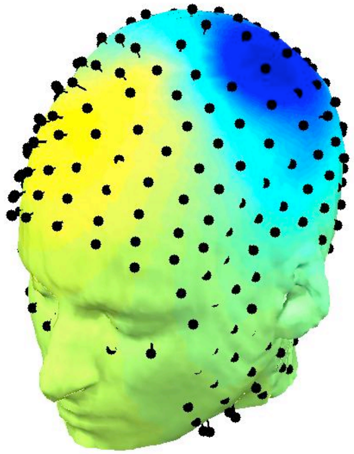
Sample EEG Decomposition

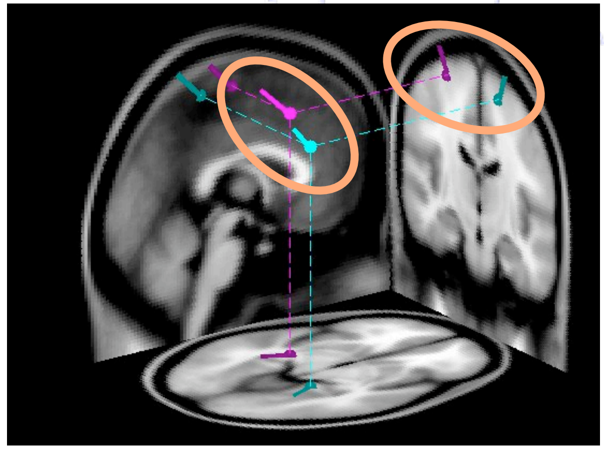


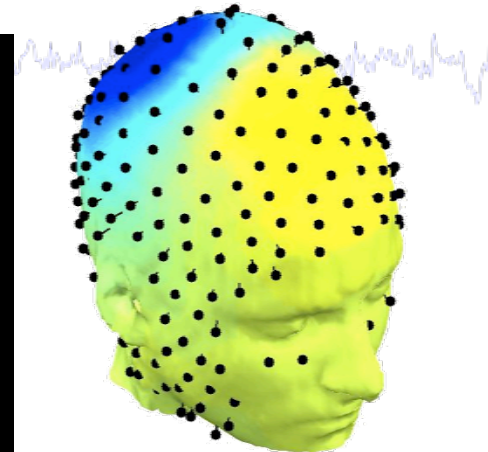
IC14

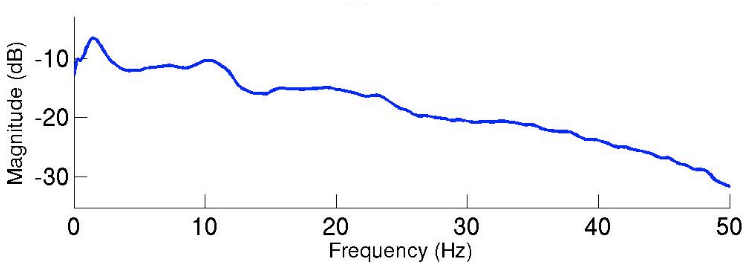
IC18

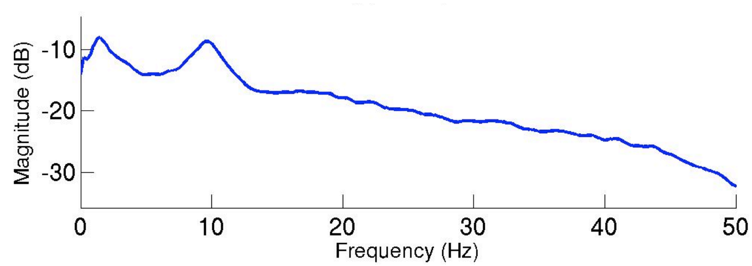
Two Lateral Alpha Processes





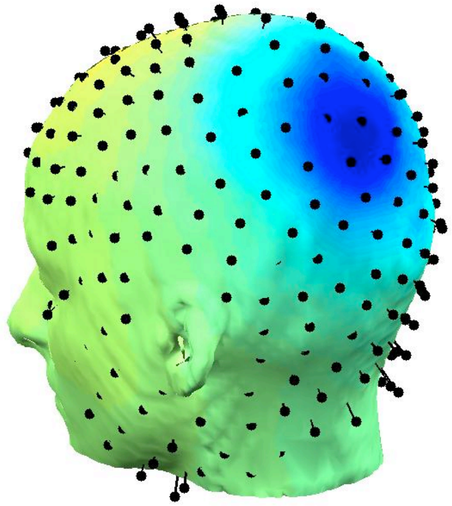


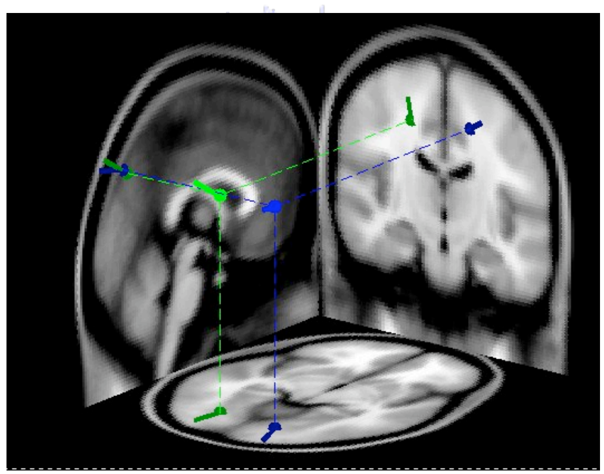




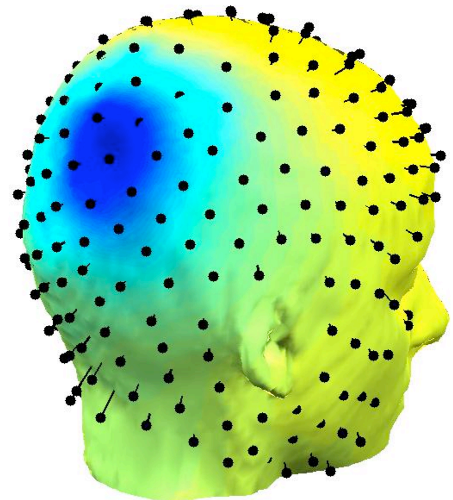
Two Central Alpha Processes

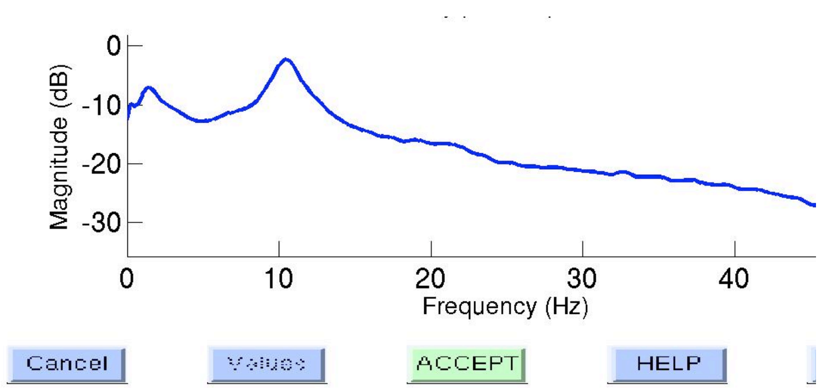
IC9

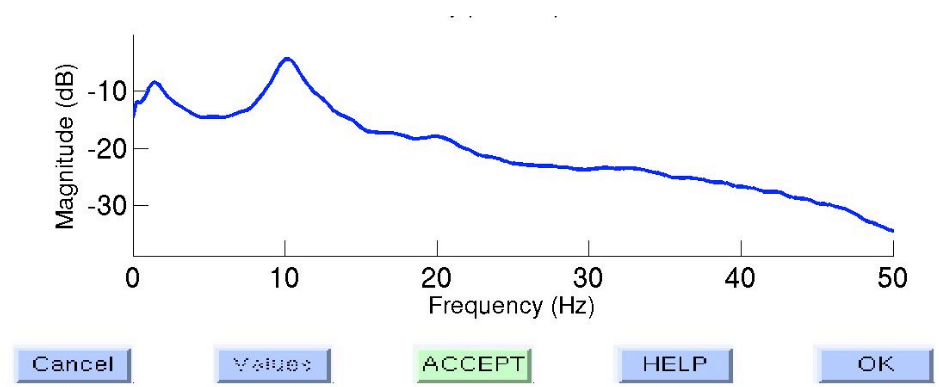


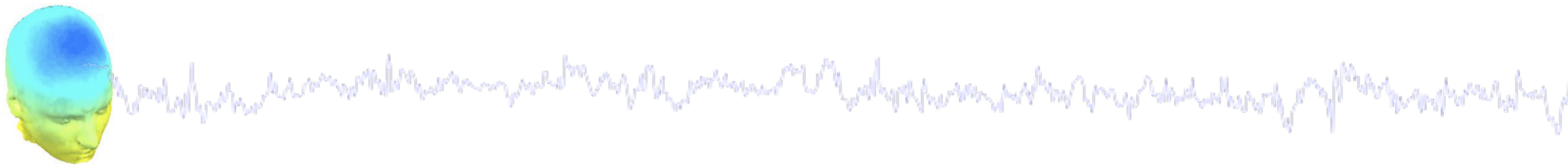


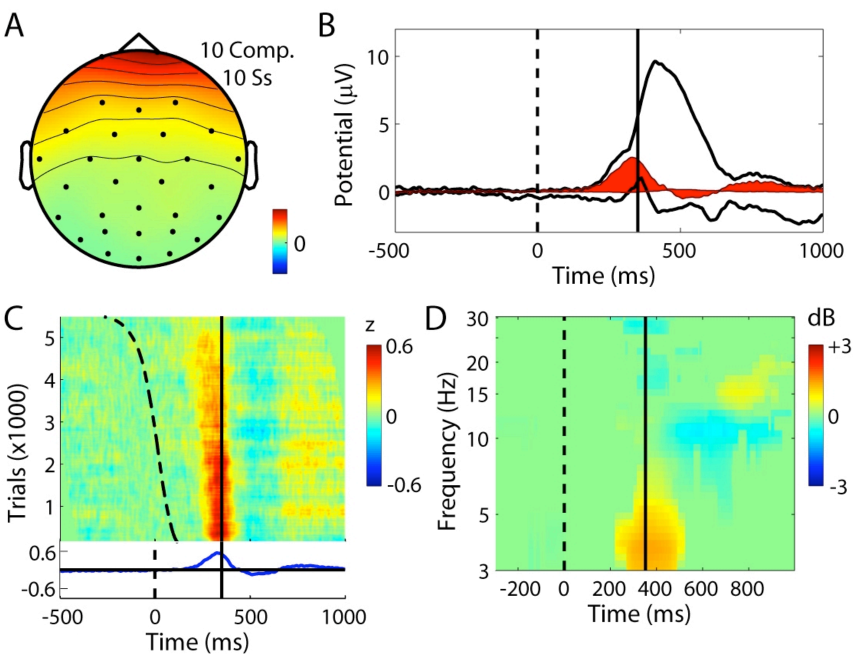
IC11



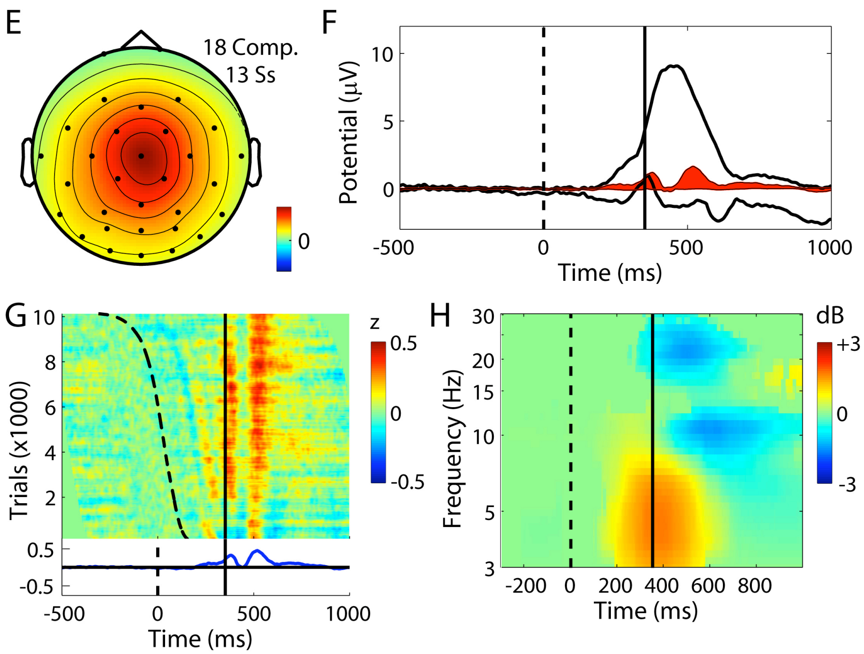


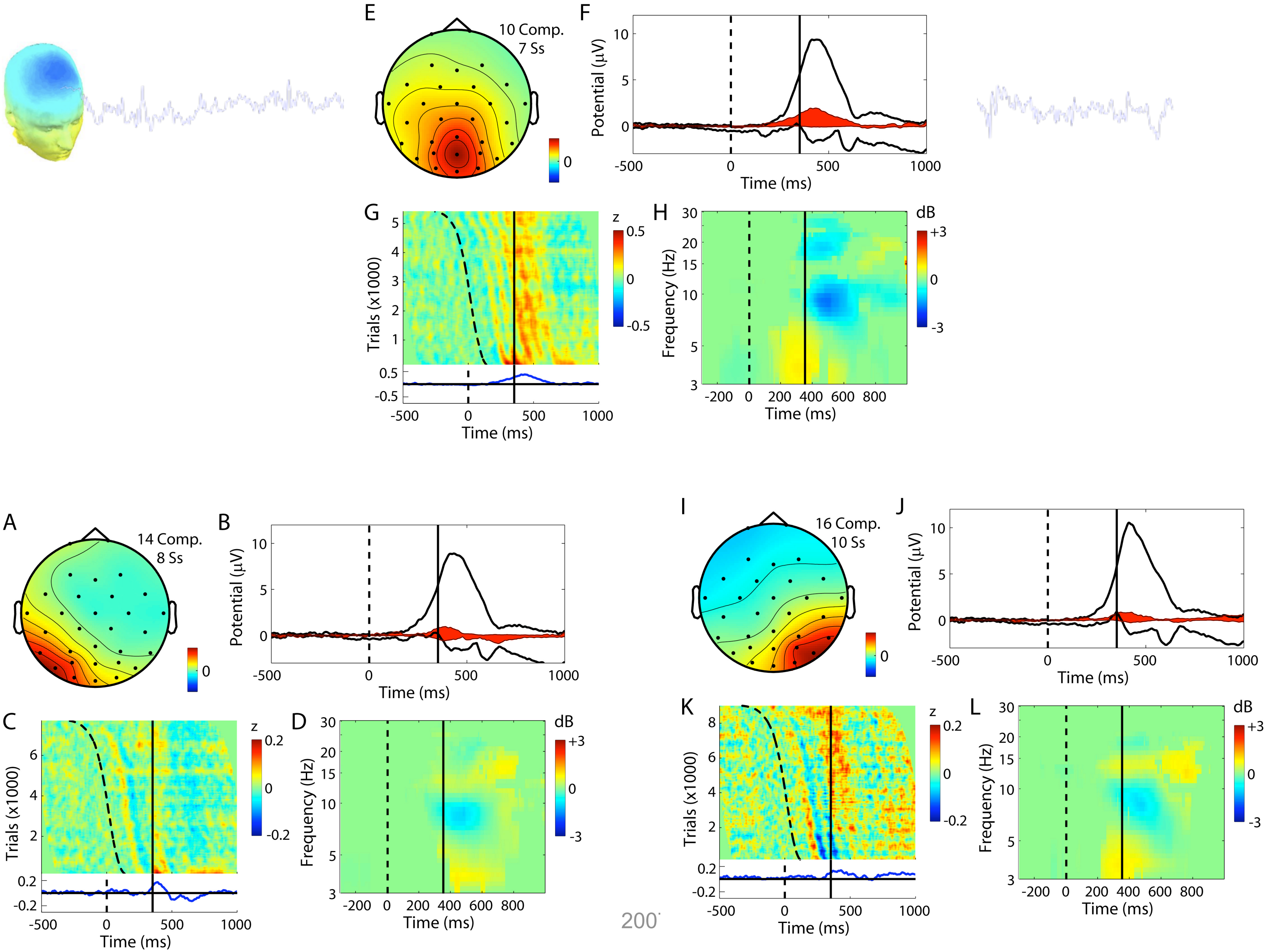




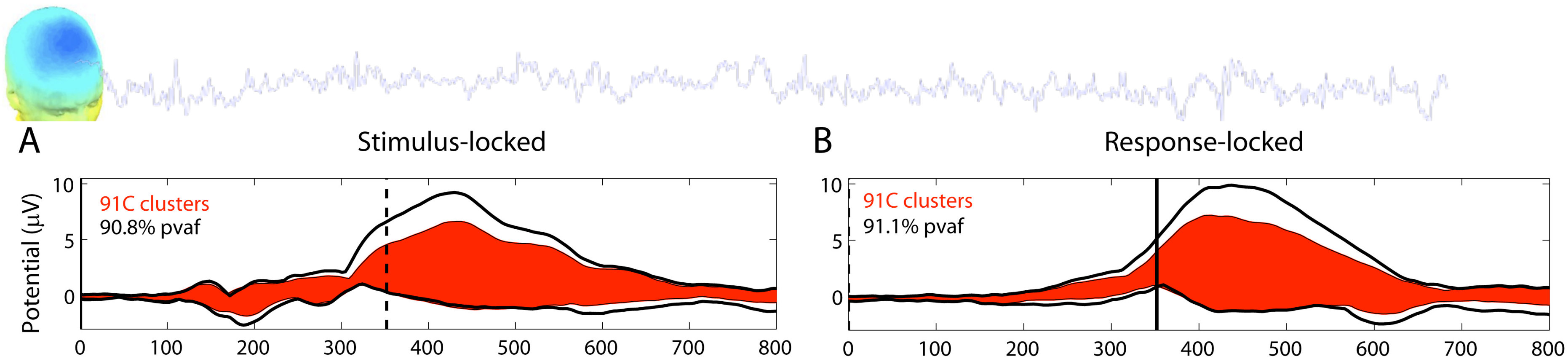


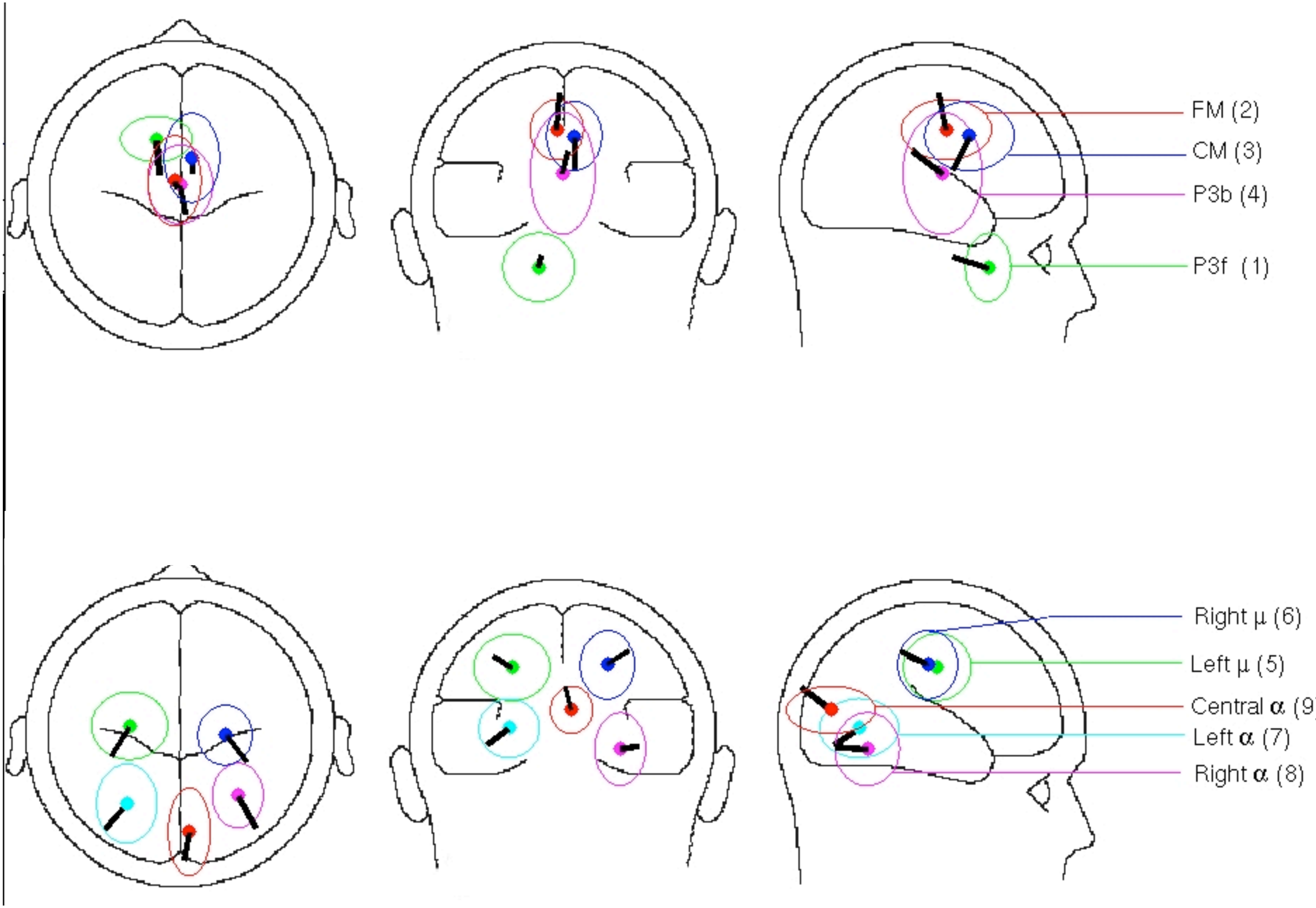
A
10 Comp.
10 Ss
0
B
Potential (µV)
Time (ms)
C
Trials (x1000)
z
0.6
0
-0.6
Time (ms)
D
Frequency (Hz)
dB
+3
0
-3
Time (ms)


E
18 Comp.
13 Ss
0
F
Potential (μV)
10
5
0
-500
0
500
1000
Time (ms)
G
Trials (x1000)
10
8
6
4
2
0.5
-0.5
-500
0
500
1000
Time (ms)
z
0.5
0
-0.5
H
Frequency (Hz)
30
20
15
10
5
3
-200
0
200
400
600
800
Time (ms)
dB
+3
0
-3



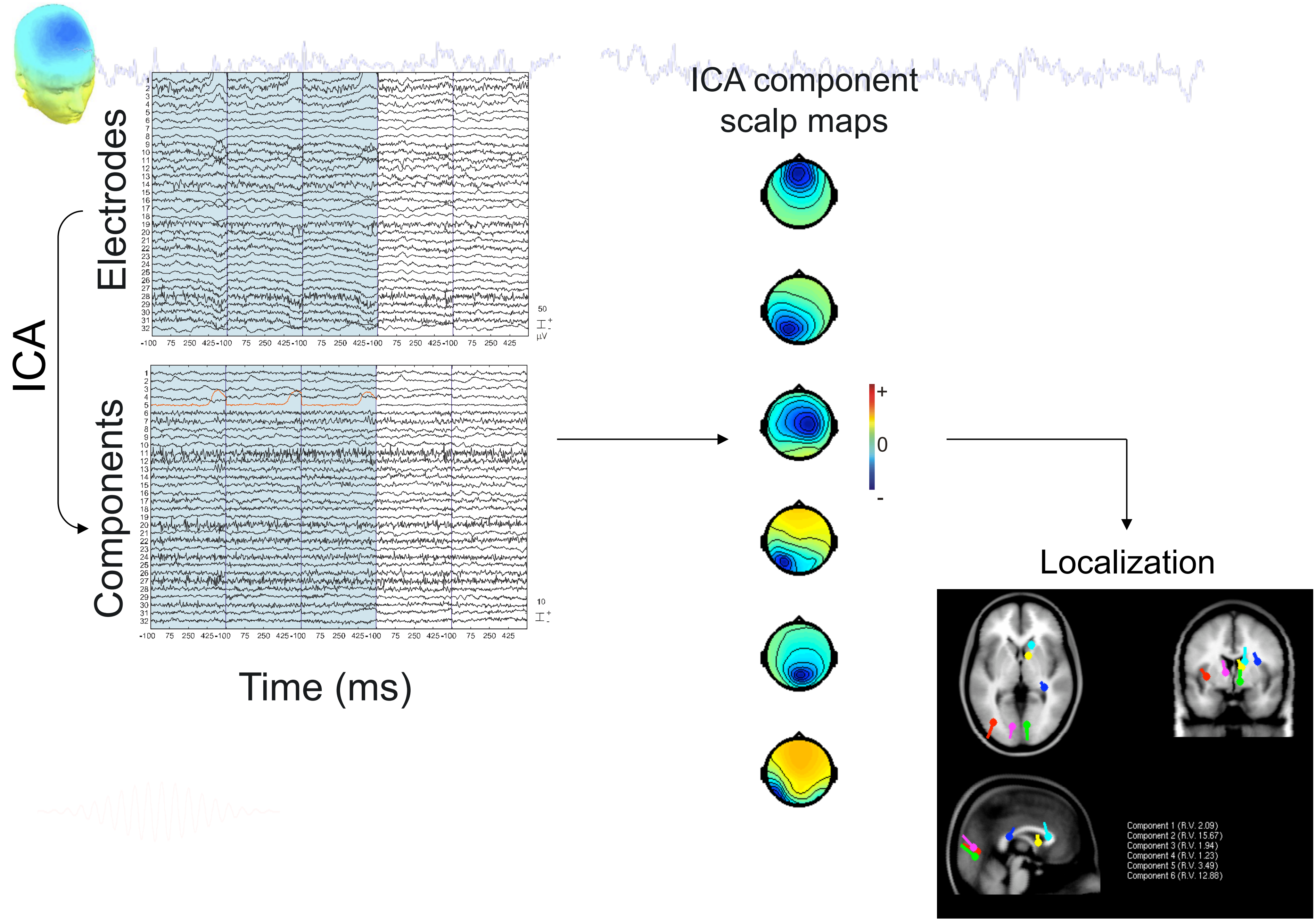
E
10 Comp.
7 Ss
F
Potential (μV)
Time (ms)
G
Trials (x1000)
z
Time (ms)
H
Frequency (Hz)
dB
Time (ms)
A
14 Comp.
8 Ss
B
Potential (μV)
Time (ms)
C
Trials (x1000)
z
Time (ms)
D
Frequency (Hz)
dB
Time (ms)
I
16 Comp.
10 Ss
J
Potential (μV)
Time (ms)
K
Trials (x1000)
z
Time (ms)
L
Frequency (Hz)
dB
Time (ms)


A
Stimulus-locked
B
Response-locked
Potential (µV)
91C clusters
90.8% pvaf
91C clusters
91.1% pvaf
10
5
0
0
100
200
300
400
500
600
700
800

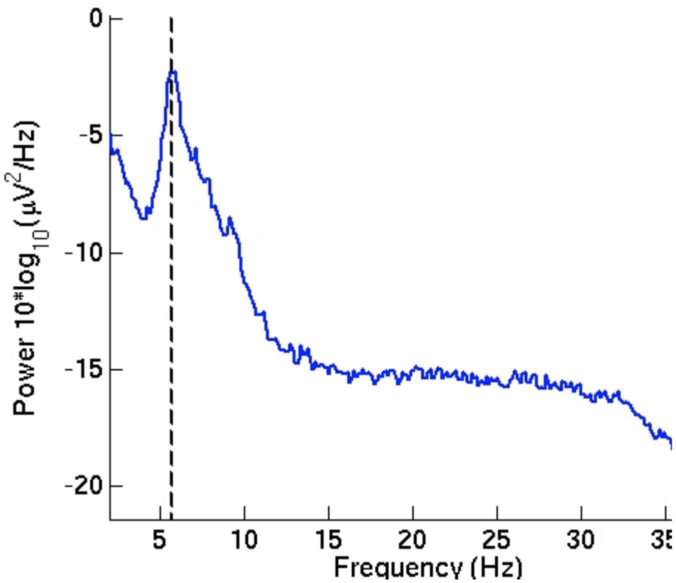


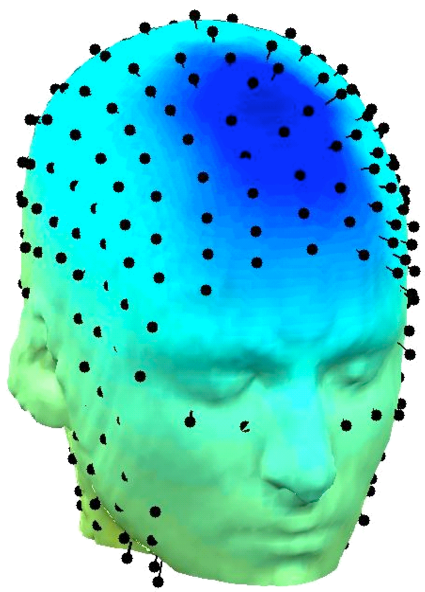
FM (2)
CM (3)
P3b (4)
P3f (1)
Right μ (6)
Left μ (5)
Central α (9)
Left α (7)
Right α (8)

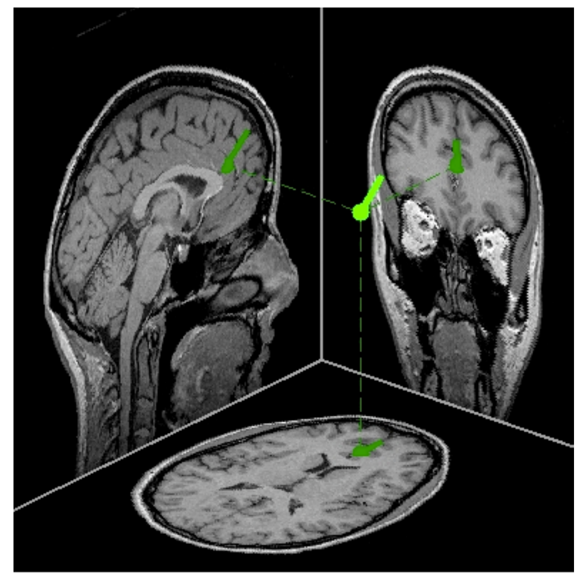
Localization

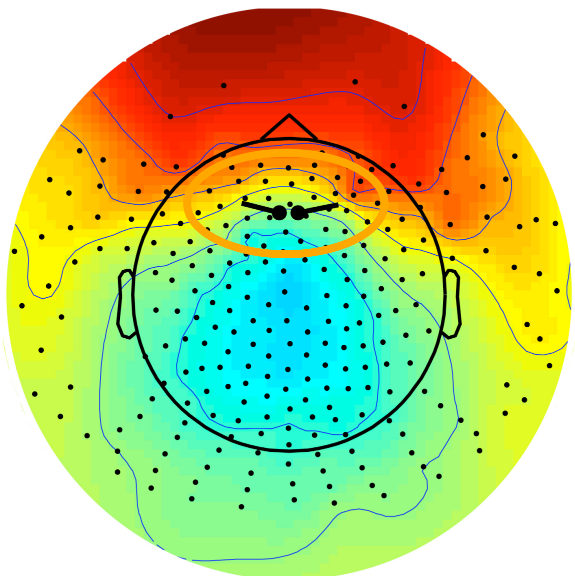
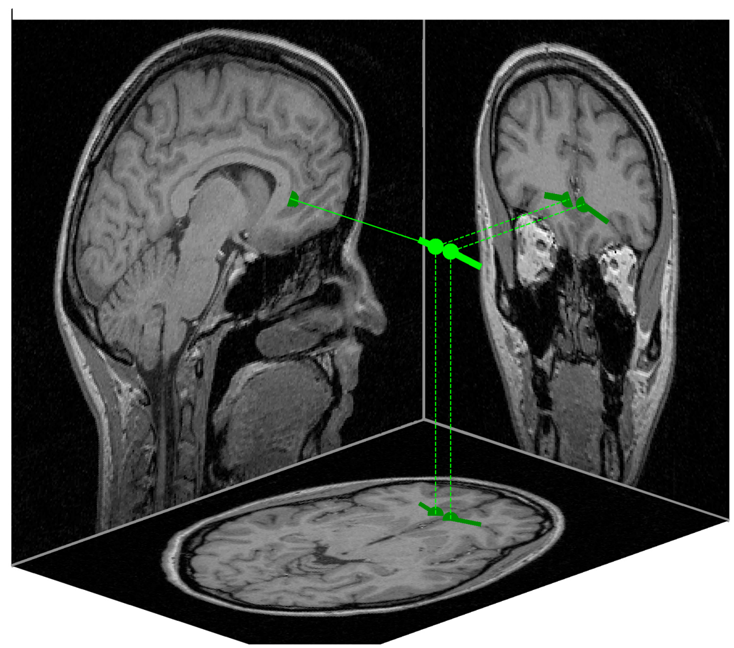


Frontal midline

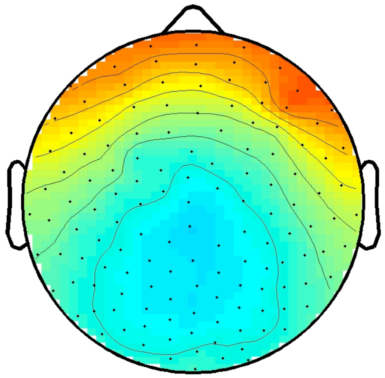


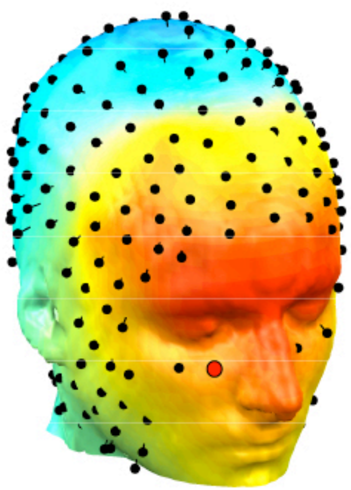


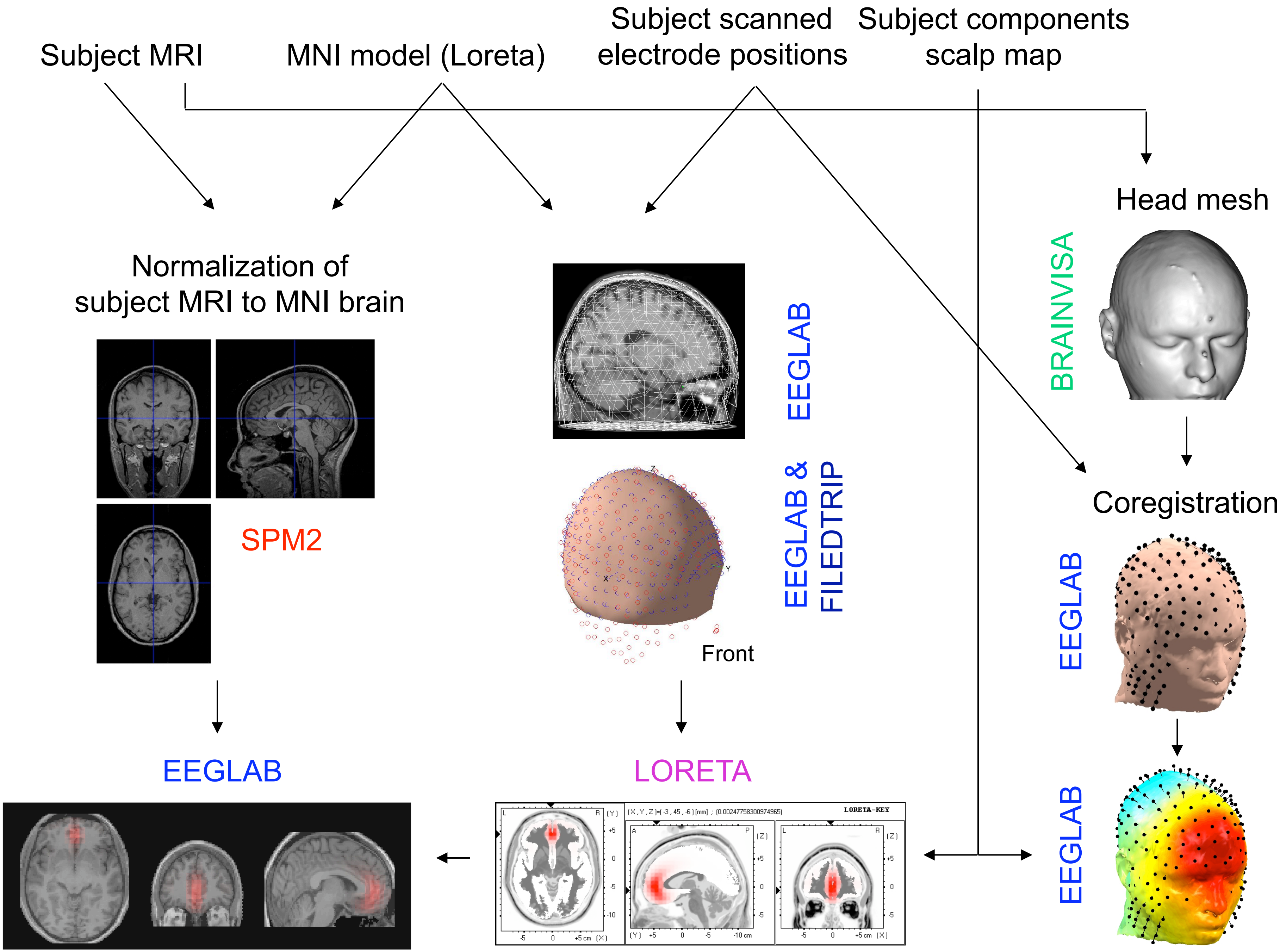


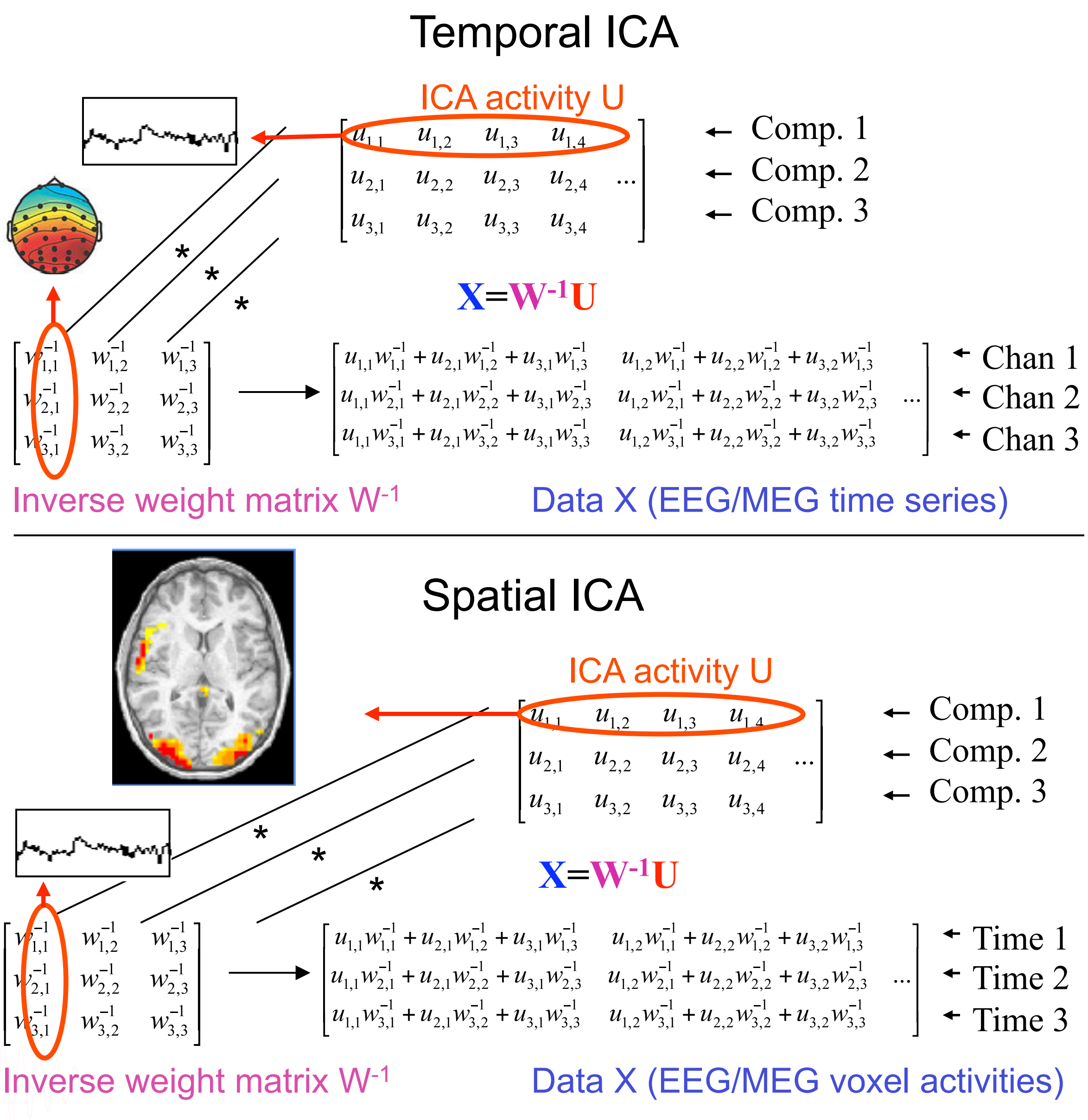




Localization of activity

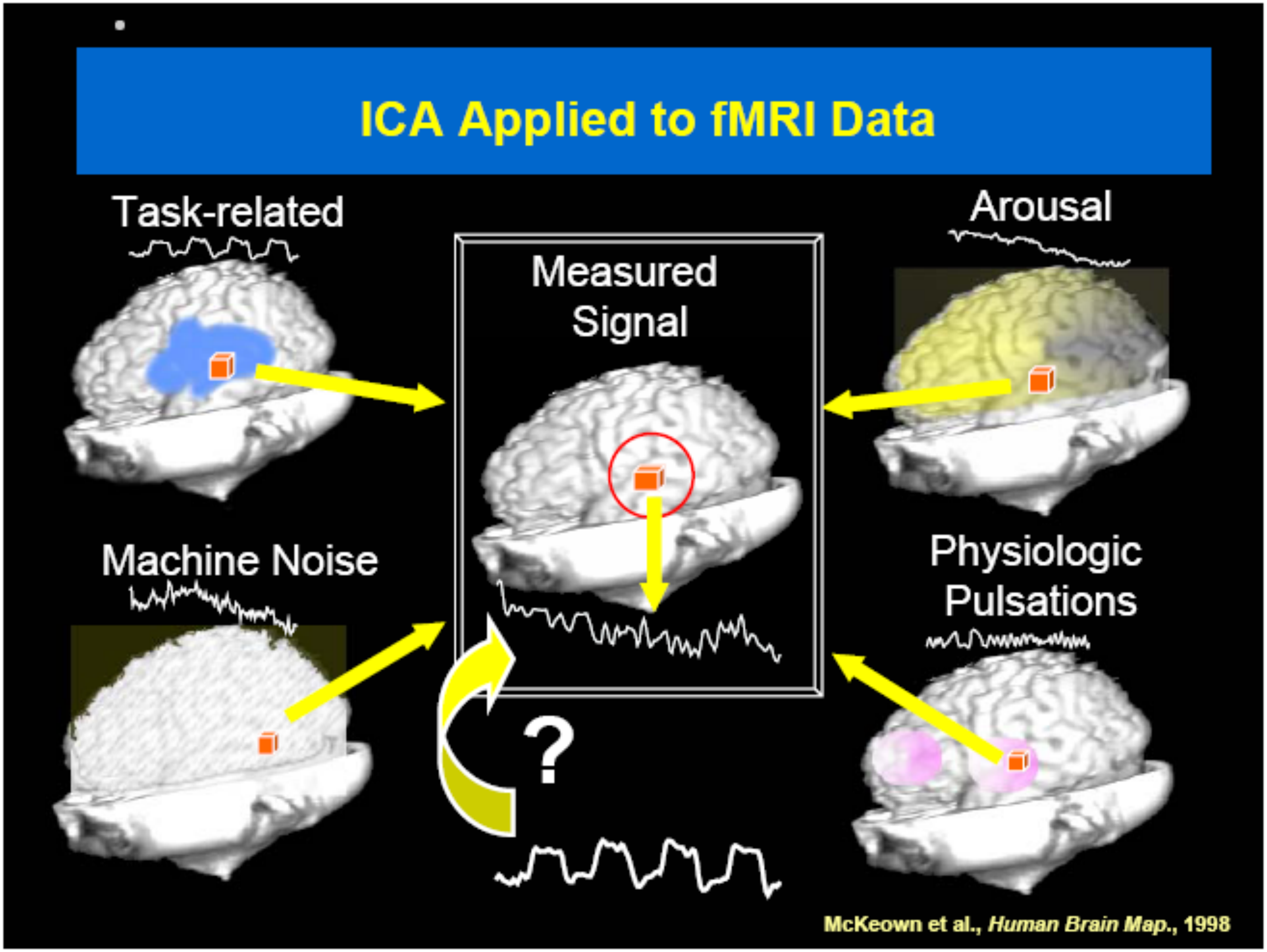


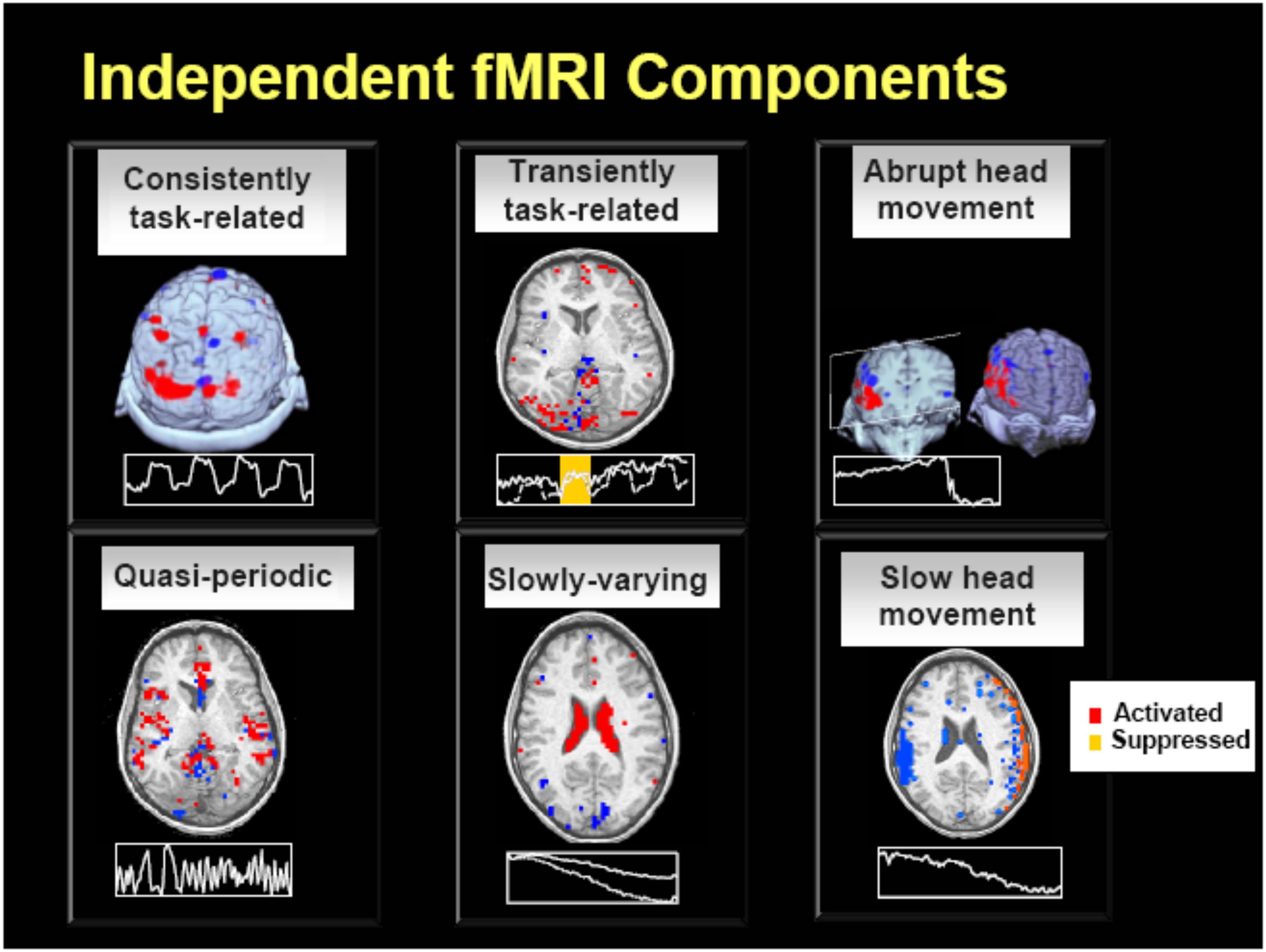



Subject MRI
MNI model (Loreta)
Subject scanned electrode positions
Subject components scalp map
Head mesh
BRAINVISA
Normalization of subject MRI to MNI brain
SPM2
EEGLAB
EEGLAB & FILEDTRIP
Front
Coregistration
EEGLAB
EEGLAB
EEGLAB
LORETA
LORETA-KEY



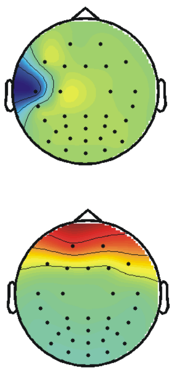
Temporal ICA
ICA activity U
Comp. 1
Comp. 2
Comp. 3
X=W-1U
Chan 1
Chan 2
Chan 3
Inverse weight matrix W-1
Data X (EEG/MEG time series)
Spatial ICA
ICA activity U
Comp. 1
Comp. 2
Comp. 3
X=W-1U
Time 1
Time 2
Time 3
Inverse weight matrix W-1
Data X (EEG/MEG voxel activities)


ICA Applied to fMRI Data
Task-related
Measured
Signal
Arousal
Machine Noise
Physiologic
Pulsations
?
McKeown et al., *Human Brain Map.*, 1998


Independent fMRI Components
Consistently task-related
Transiently task-related
Abrupt head movement
Quasi-periodic
Slowly-varying
Slow head movement
Activated
Suppressed

Rejection: Raw Data vs. ICA

Artificial artifacts



1 - Transient high-frequency events (20-60 Hz) modeling temporal muscle artifacts

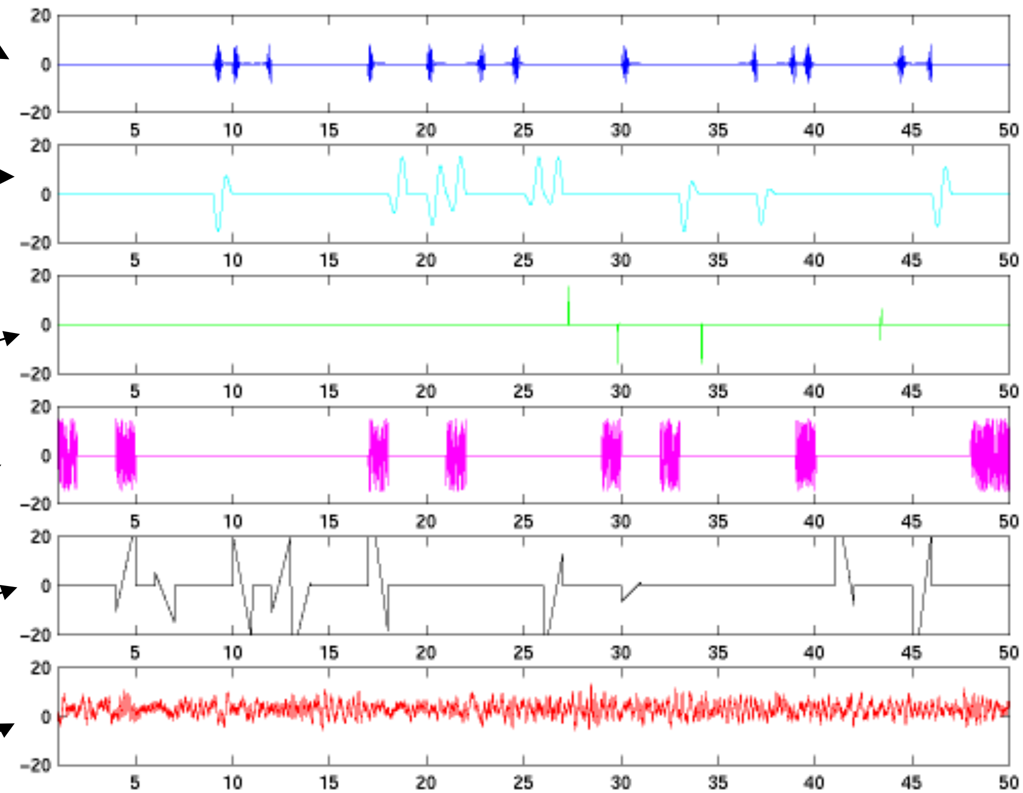
2 - Low-frequency events (1-3 Hz) modeling eye movement artifacts

3 - Signal discontinuities from electrical artifacts

4 - High noise EEG artifacts

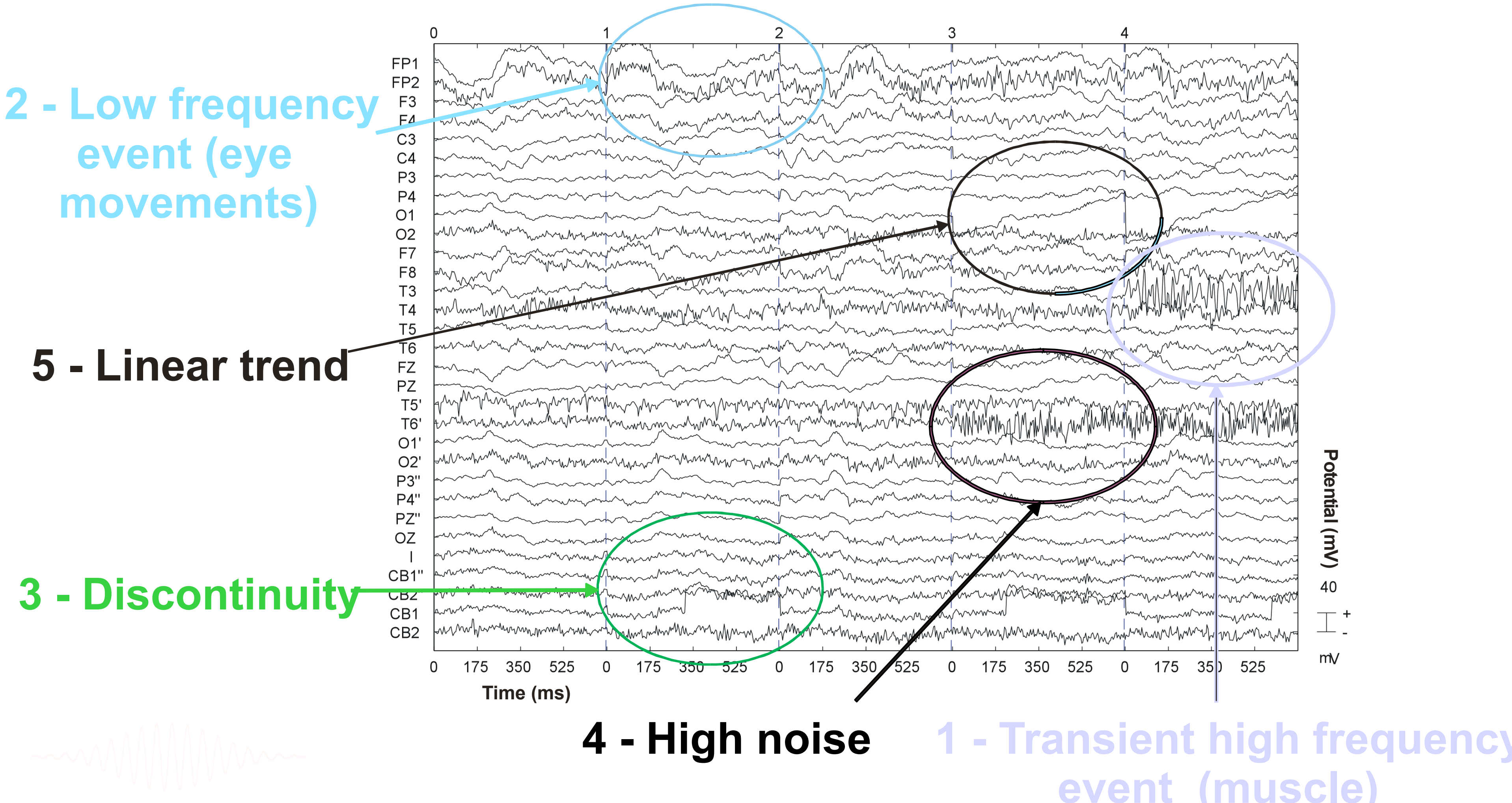
5 - Linear trends from electrical artifacts

EEG activity (31 channels x 100 trials of artifact-free EEG)



Rejection: Raw Data vs. ICA

Simulated data



Rejection: Raw Data vs. ICA

Rejection methods

- Detection of peaks of activity (thresholding)
- Detection of linear trends (R^2)
- Detection of improbable events (Joint Probability)
- Detection of peaky distributions of activity (Kurtosis)
- Detection of frequency peaks (frequency thresholding)

Vary artifacts' amplitude

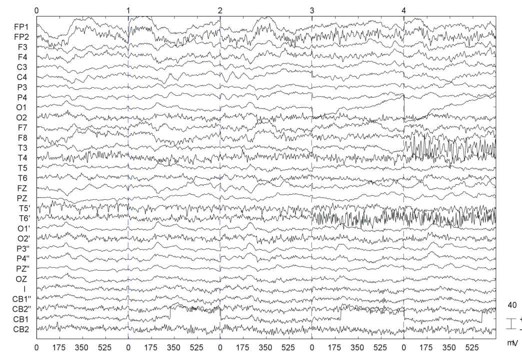
Known artifacts

Optimize the free parameter for each method

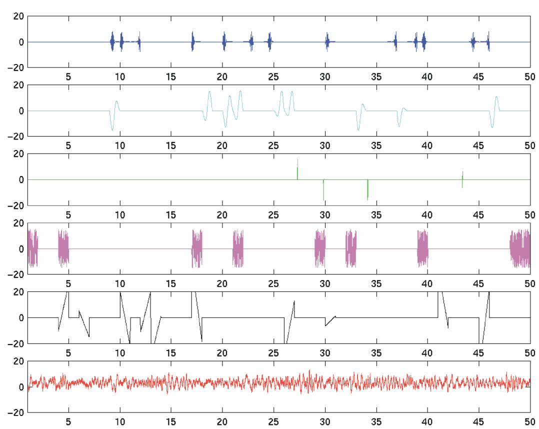
Performance of each method

Rejection: Raw Data vs. ICA

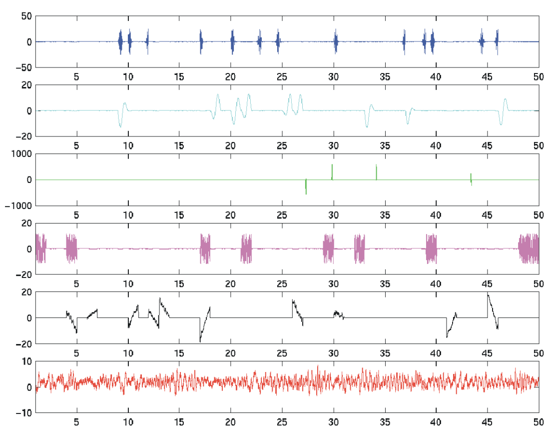
Mixture

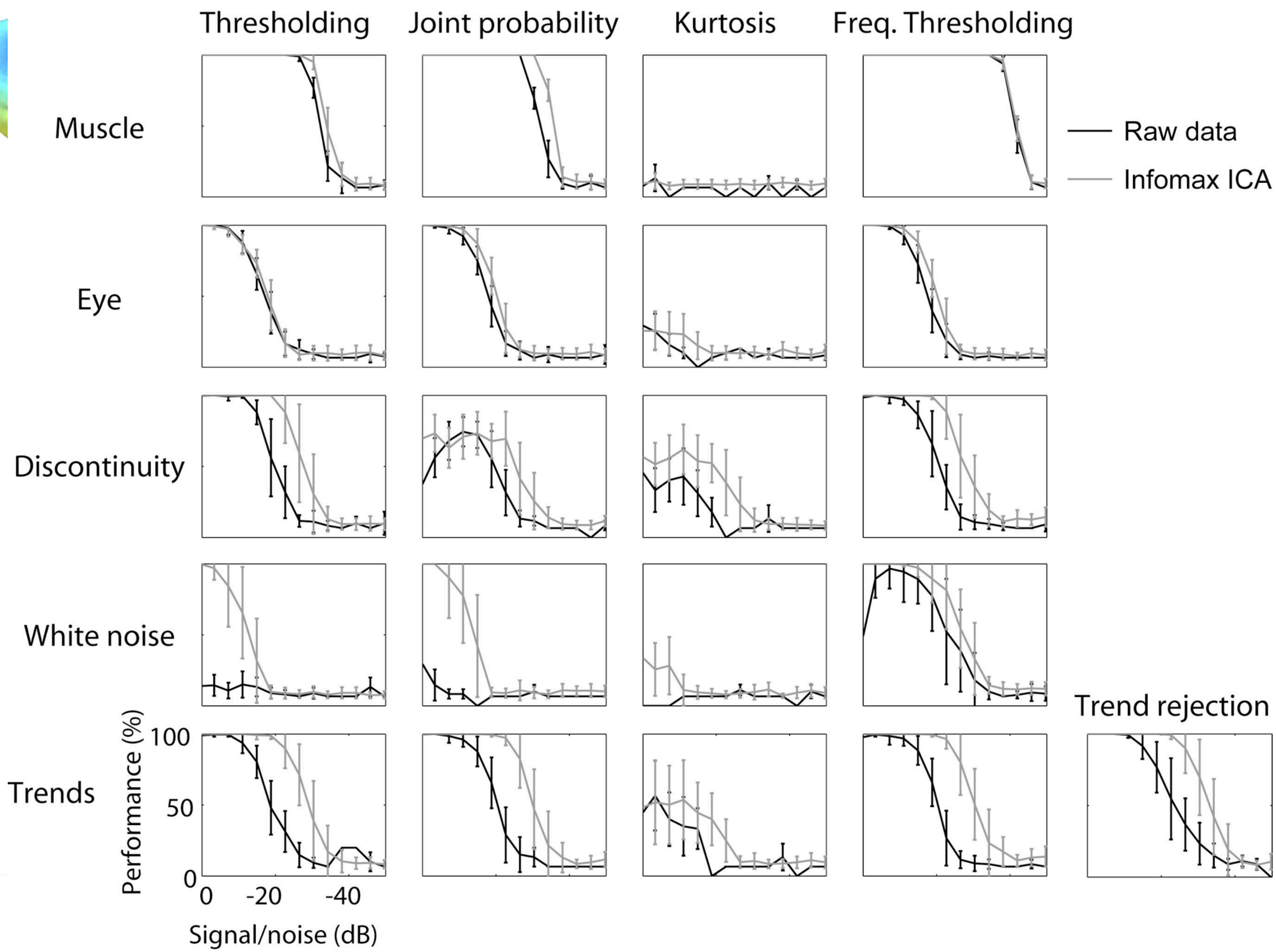


Original data

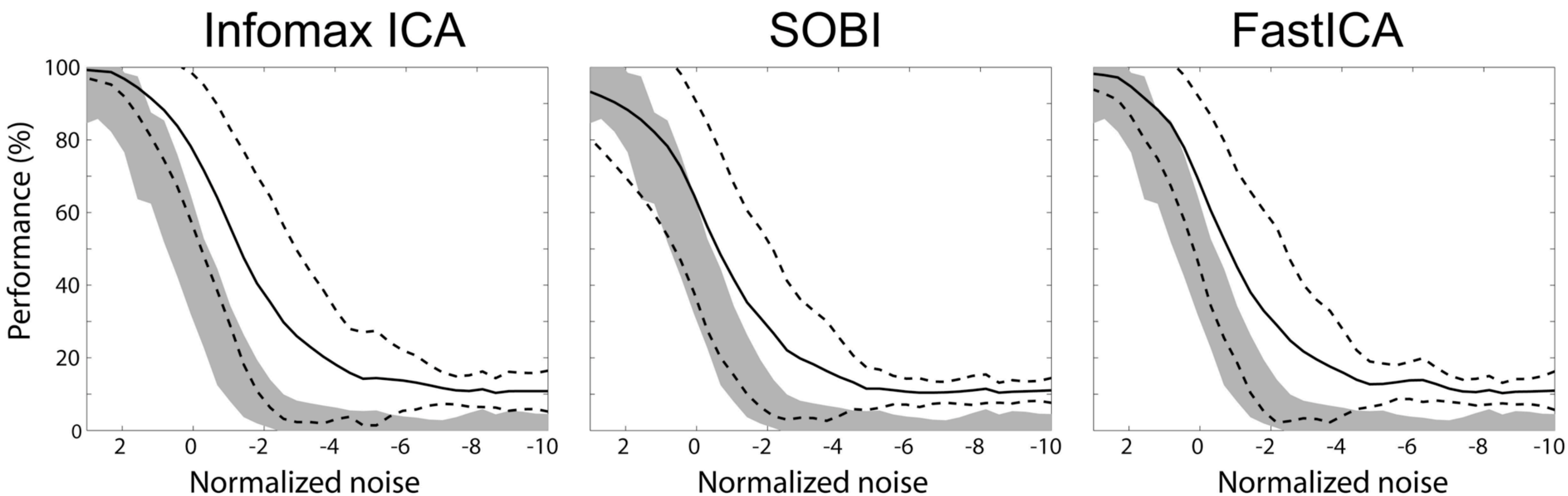


Infomax ICA





Thresholding
Joint probability
Kurtosis
Freq. Thresholding
Muscle
Eye
Discontinuity
White noise
Trends
Trend rejection
Raw data
Infomax ICA
Performance (%)
100
50
0
0
-20
-40
Signal/noise (dB)


Infomax ICA
SOBI
FastICA
Performance (%)
100
80
60
40
20
0
2
0
-2
-4
-6
-8
-10
Normalized noise