Independent component analysis applied to biophysical time series and EEG

EEGLAB Workshop, Aspet, Arnaud Delorme

Example: Speech Separation











Independent component analysis

Mixture of Brain source activity





EEGLAB Workshop, June 26-29, 2007, Aspet: Arnaud Delorme









ICA activity U

Weight matrix W



Historical Remarks

- Herault & Jutten ("Space or time adaptive signal processing by neural network models", *Neural Nets for Computing Meeting*, Snowbird, Utah, 1986): Seminal paper, neural network
- Bell & Sejnowski (1995): Information Maximization
- Amari et al. (1996): Natural Gradient Learning
- Cardoso (1996): JADE
- Applications of ICA to biomedical signals
 - EEG/ERP analysis (Makeig, Bell, Jung & Sejnowski, 1996).
 - fMRI analysis (McKeown et al. 1998)

ICA Theory – Cost Functions

Family of BSS algorithms

- Information theory (Infomax)
- Bayesian probability theory (Maximum likelihood estimation)
- Negentropy maximization
- Nonlinear PCA
- Statistical signal processing (cumulant maximization, JADE)

A unifying Information-theoretic framework for ICA

- Pearlmutter & Parra showed that InfoMax, ML estimation are equivalent.
- Lee et al. (1999) showed negentropy has the equivalent property to InfoMax.
- Girolami & Fyfe showed nonlinear PCA can be viewed from information-theoretic principle.

Independent Component Analysis

ICA is a method to recover a version, of the original sources by multiplying the data by a unmixing matrix,

U = WX,

While PCA simply decorrelates the outputs (using an orthogonal matrix **W**), ICA attempts to make the outputs **statistically independent**, while placing no constraints on the matrix **W**.



ICA and PCA



Central limit theorem



ICA Training Process



Central limit theorem

- Remove the mean
 x = x <x>
- 'Sphere' the data by diagonalizing its covariance matrix,
 x = <xx^T>^{-1/2}(x-<x>).
- Update W according to

$$\sim\sim\sim\wedge\wedge\Delta\mathbf{W}\proptorac{\partial H(\mathbf{y})}{\partial\mathbf{W}}\mathbf{W}^T\mathbf{W}^T$$



Entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x).$$



Fake dice (make a 6 half of the time): entropy 2.16 (base 2)



$$H = 5\left(-\frac{1}{10}\log_2\left(\frac{1}{10}\right)\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 2.16$$

Entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_b p(x)$$

Joint entropy

$$H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y)\log_b p(x,y),$$

Mutual Information $H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$

Shannon in his landmark 1948 paper ``A Mathematical Theory of Communication.''

From http://planetmath.org/encyclopedia/ShannonsTheoremEntropy.html

Contingency table for stress and emotionality

	STRE						
	1	2	3	4	5	6	Total
EMOT= 1	19	4					23
2	11	63	64	3	1		142
3	2	16	18	20	2	2	60
4	1	4	1	9	6	2	23
5			1	2	4	3	10
6				1	1	1	3
Total	33	87	84	35	13	8	

From http://tecfa.unige.ch/~lemay/thesis/THX-Doctorat/node149.html

Contingency frequencies for stress and emotionality

	STRE					
	1	2	3	4	5	6
EMOT=1	0.07	0.02				
2	0.04	0.24	0.25	0.01		
3	0.01	0.06	0.07	0.08	0.01	0.01
4		0.02		0.03	0.02	0.01
5				0.01	0.02	0.01
6						

Joint entropy 3.46; exercise: compute mutual information $H(X,Y) = -\sum_{(x,y)\in\mathcal{X}\times\mathcal{Y}} p(x,y) \log_b p(x,y)$

ICA learning rule

How to make the outputs statistical independent? Minimize their redundancy or mutual information. Consider the joint entropy of two components, $H(y_1, y_2) = H(y_1) + H(y_2) - I(y_1, y_2).$

Maximizing $H(y_1, y_2) \Longrightarrow$ minimizing $I(y_1, y_2)$.

The learning rule: $\Delta W \propto \frac{\partial H(\mathbf{y})}{\partial \mathbf{W}} \mathbf{W}^T \mathbf{W}$ Entropy extremum
Natural gradient (Amari)



Kurtosis, Super- and Sub-Gaussian

Kurtosis: a measure of how peaked or flat of a probability distribution is.

 $kurt(X) = \frac{E[(X-\mu)^4]}{-4}$



Gaussian Dist. Kurtosis = 0 Super-Gaussian: kurtosis > 0 Sub-Gaussian: kurtosis < 0

Moments, Cumulants

Moments
$$\mu_x(n) = E\{x^n\}$$

Central moments $m_x(n) = E\{(x - m_x)^n\}$

Cumulants	c_1	=	$m_1 = \mu$	◀	mean
	c_2	=	$m_2 = \sigma^2$	•	variance
	c_3	=	m_3	←───	skewness
	c_4	=	$m_4 - 3m_2^2$	←	kurtosis



To make the u_i independent, we need to operate on nonlinear transformed output variables, y = g(u), such as

$$\mathbf{y} = \frac{1}{1 + e^{-\mathbf{u}}}, \quad \mathbf{u} = \mathbf{W}\mathbf{x} + \mathbf{w}_0$$

The non-linear function provides all the higher-order statistics necessary to establish independence.



Independent components of EEG/ERP



ICA/EEG Assumptions

- Mixing is linear at electrodes **OK**
- Propagation delays are negligible
- Component time courses are independent
- Number of components less than the number of channels.



Number of independent components

OK

Independent Component Categories

- Artifacts
- Stimulus-locked activity
- Response-locked activity
- Non-phase locked activity
- Event-modulated oscillatory activity



Characteristics of Independent Component of the EEG

- Concurrent Activity
- Maximally Temporally Independent
- Overlapping Maps and Spectra
- Dipolar Scalp Maps
- Functionally Independent
- Between-Subject Regularity





ICA Decomposition into Independent Components



Selective Projection onto Scalp Channels





ICA-based Artifact Removal

Corigicteld



Artifact removal using ICA








Two Neck Muscle Processes

IC31











Credit: J. Onto

Why analyze source activity instead of channels?





Credit: J. Onto













Localization



Patch of Cortex Acting as a Dipole



Separating EEG source activities





Dipolar Scalp Projections

ICA creates a spatial filter for each temporally independent source







Localization of activity



Localization of activity









ICA component comparison

Data

- 13 subjects performing a memory task
- 71 electrodes including EOGs
- more than 300,000 data points/subject

Decomposition

• 23 ICA algorithms plus PCA and Promax

Analysis

• Localization of all components with a single dipole (4-shell spherical model)



Algorithm (Matlab func.)	D%	MIR	Origin
Extended Infomax (runica).	29.9	178	EEGLAB 4.515
Pearson	29.1	169	ICAcentral (6).
Infomax (runica).	28.2	160	EEGLAB 4.515
ERICA	26.9	184	ICALAB 1.5.2
SONS	25.4	183	ICALAB 1.5.2
SHIBBS	23.7	169	ICAcentral (5).
FastICA*	23.5	169	ICAcentral (2).
JADE (jader).	23.4	169	EEGLAB 4.515
TICA	23.4	169	ICALAB 1.5.2
JADE optimized (jade_op).	21.4	169	ICALAB 1.5.2
JADE w/ time delay (jade_td).	20.2	169	ICALAB 1.5.2
eeA	19.0	305	ICAcentral (8).
Infomax (icaML) †	18.8	212	ICA DTU Tbox
FOBI	18.6	169	ICALAB 1.5.2
SOBIRO (acsobiro).	17.9	167	EEGLAB 4.515
EVD 24	17.7	169	ICALAB 1.5.2
EVD	17.0	169	ICALAB 1.5.2
SOBI	16.1	583	EEGLAB 4.515
icaMS†	10.6	169	ICA DTU Tbox
AMUSE	8.5	169	ICALAB 1.5.2
PCA	3.1	583	EEGLAB 4.515
Promax	33.7	467	EEGLAB 4.515
Whitening/Sphering	57.6	164	EEGLAB 4.515



 $\sim\sim\sim$



Correlations entre les décompositions



AMICA









dip/mm³ 0.25 0.19 0.13 0.06 0

Nombre de composants inférieurs à chaque seuil de variance résiduelle



Plus d'indépendence implique plus de composants biologiquement plausibles





FASTICA Ext. Infomax

AMICA





Frontal midline



For example: frontal midline theta cluster



FM0 Cluster





Onton, Delorme and Makeig, NeuroImage 27 (2005) 341 - 356

Goal: to cluster matching ICs across subjects



Credit J. Ontor

What makes an ICA component?



Subject differences?





0.5

0

-0.5

Subject differences?



Subject differences?






McKeown et al., Human Brain Map., 1998

Independent fMRI Components









Rejection methods

- Detection of peaks of activity (thresholding)
- Detection of linear trends (R²)
- Detection of improbable events (Joint Probability)
- Detection of peaky distributions of activity (Kurtosis)
- Detection of frequency peaks (frequency thresholding)









