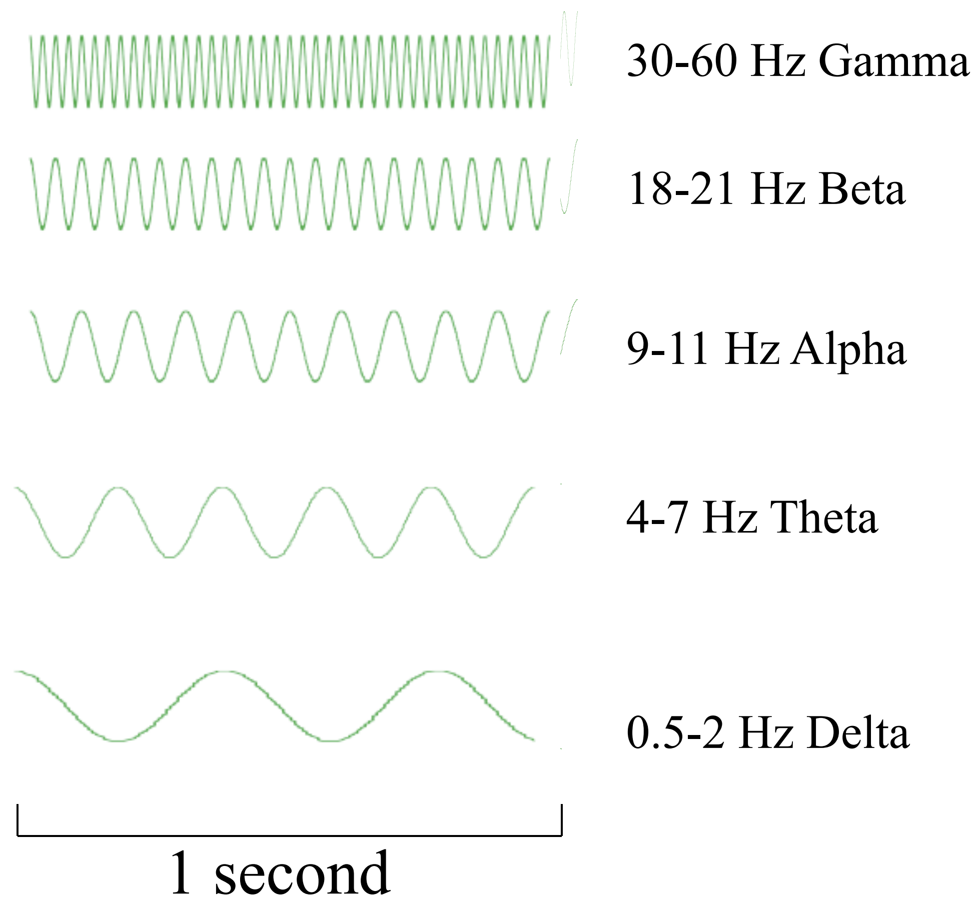
Time-Frequency analysis of biophysical time series

June 14th, Jyväskylä, Finland

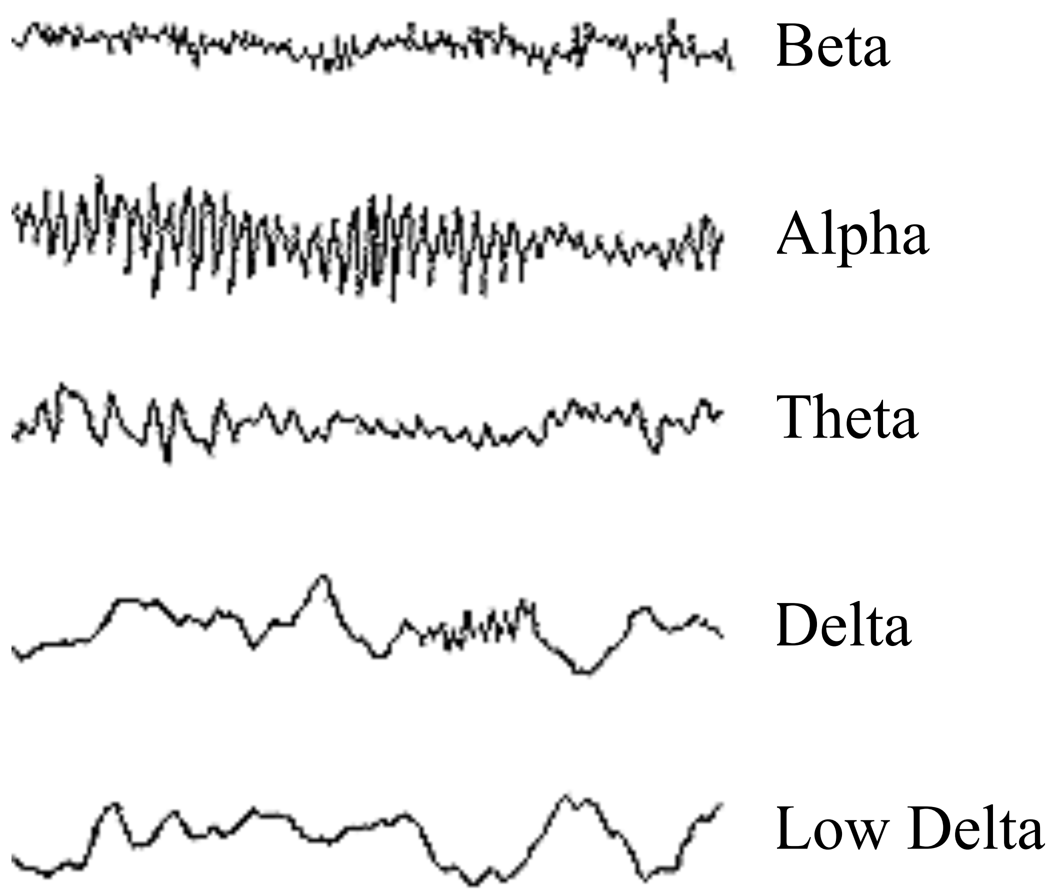
Arnaud Delorme

Frequency analysis

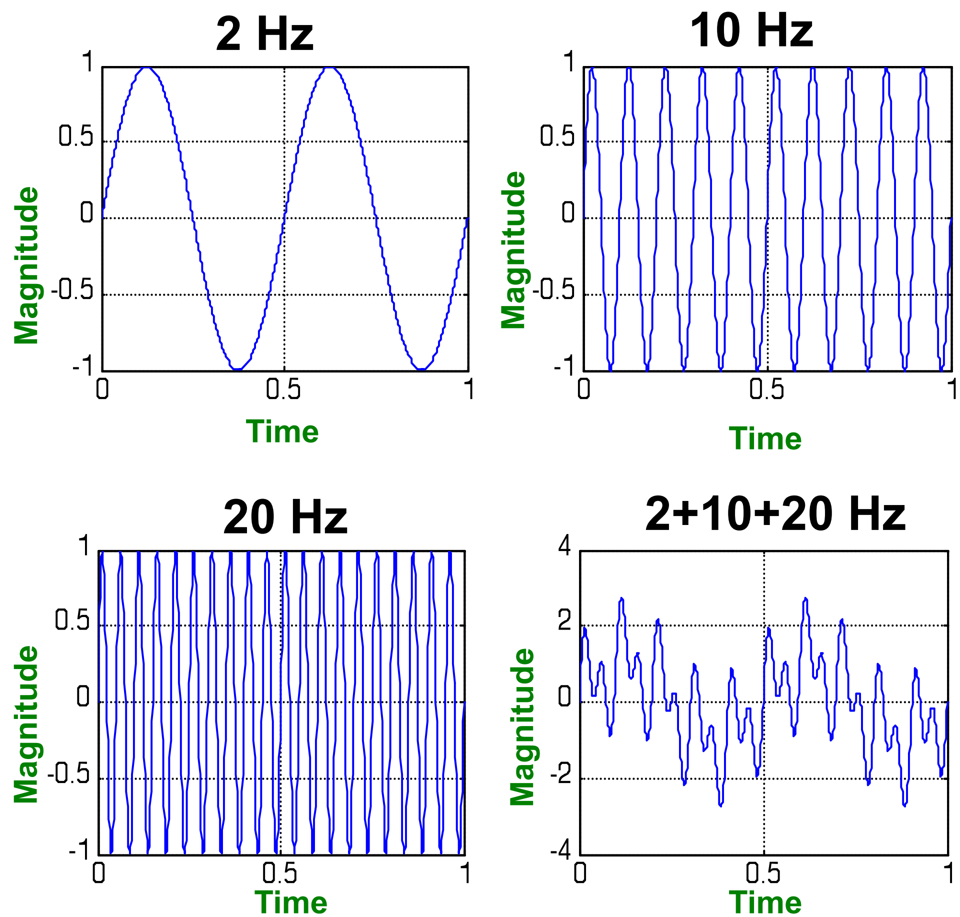
synchronicity of cell excitation determines amplitude and rhythm of the EEG signal



Frequency analysis



Stationary signals

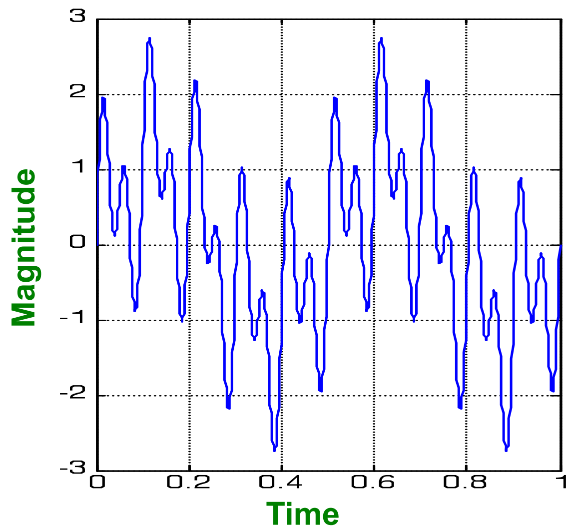


Slide courtesy of Petros Xanthopoulos, Univ. of Florida

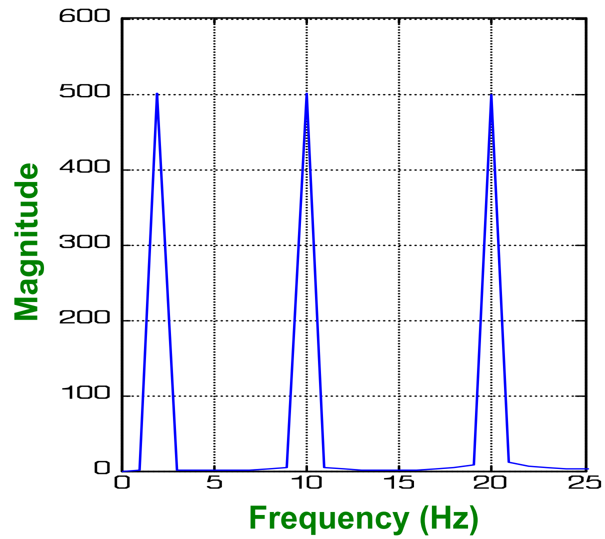
Stationary signal

Stationary

2 Hz + 10 Hz + 20Hz

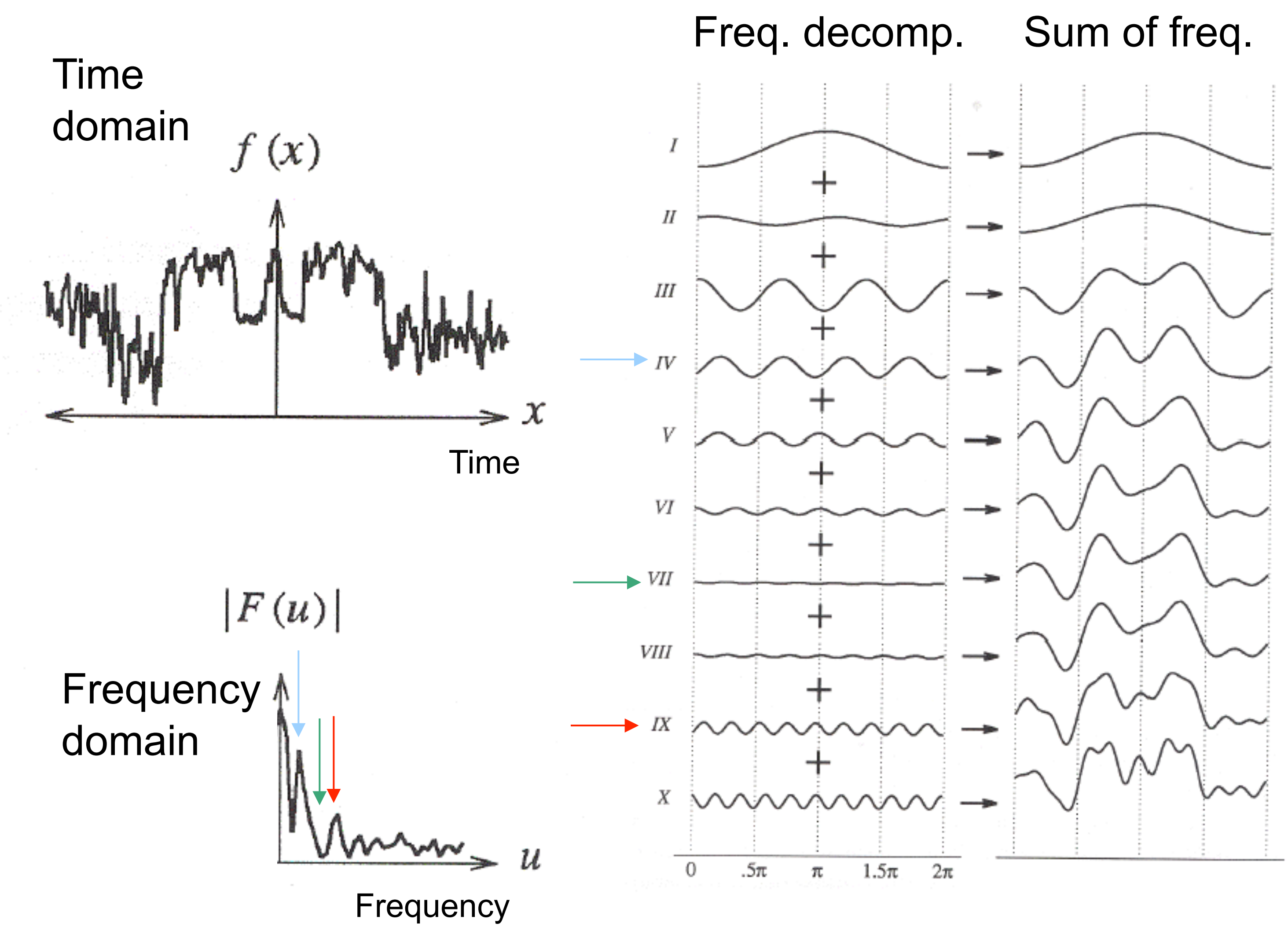


Power spectrum



By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2,10,20Hz respectively).

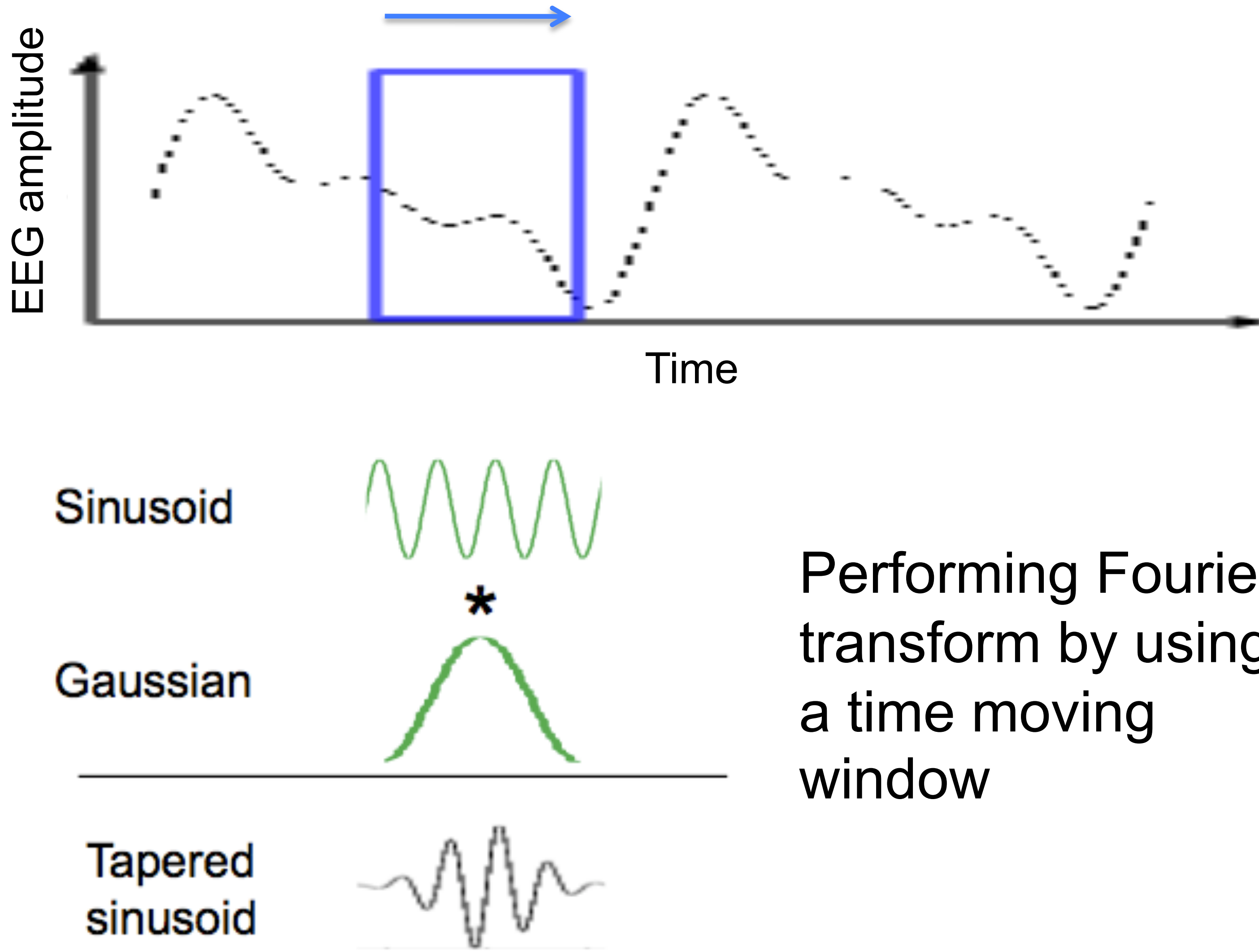
Slide courtesy of Petros Xanthopoulos, Univ. of Florida



Forward transform $$F(u) = \int_{-\pi}^{\pi} f(x) e^{-2\pi i u x} dx$$

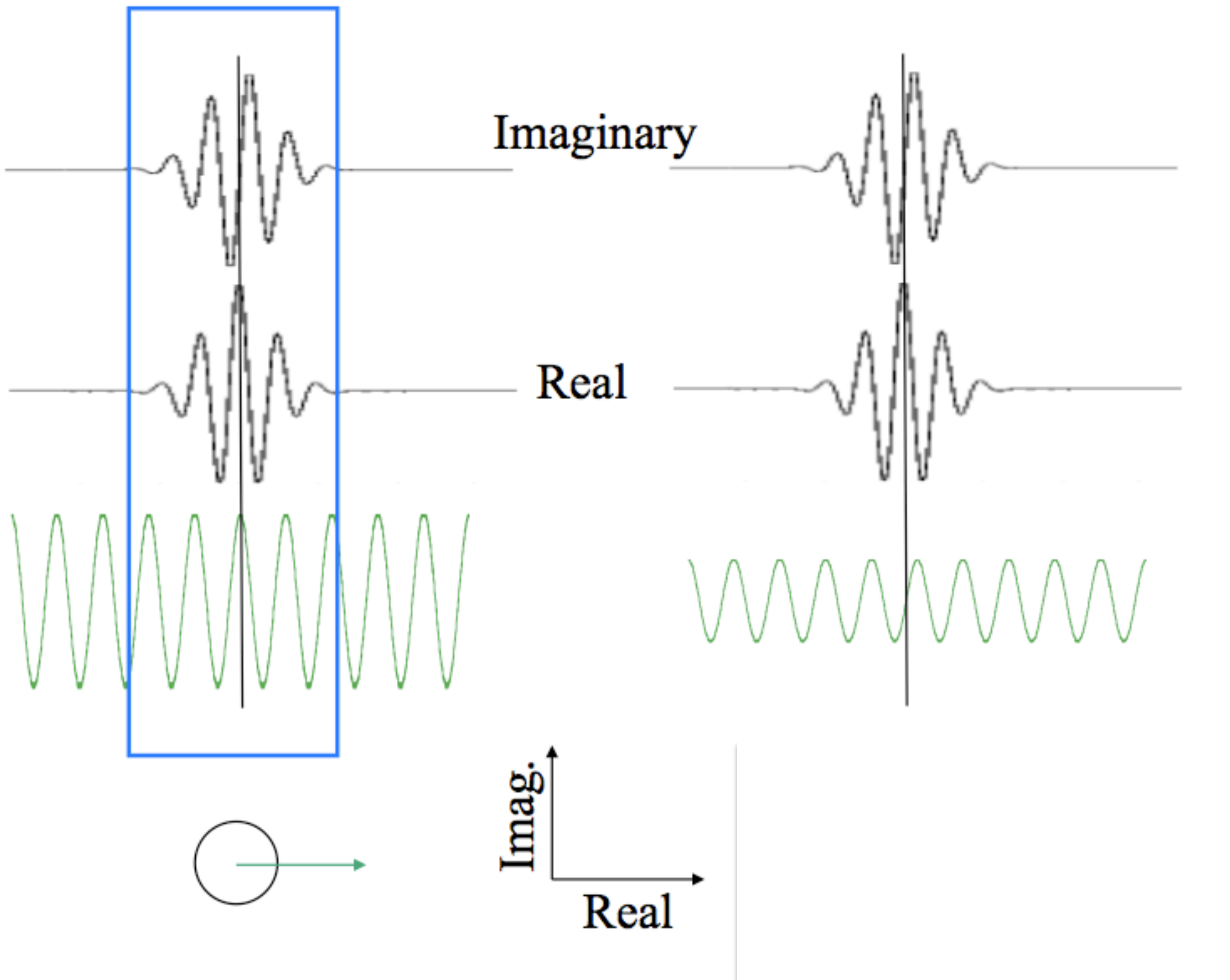
Inverse transform $$f(x) = \sum_{u=-\infty}^{+\infty} F(u) e^{2\pi i u x}$$

Figure, courtesy of Ravi Ramamoorthi & Wolberg

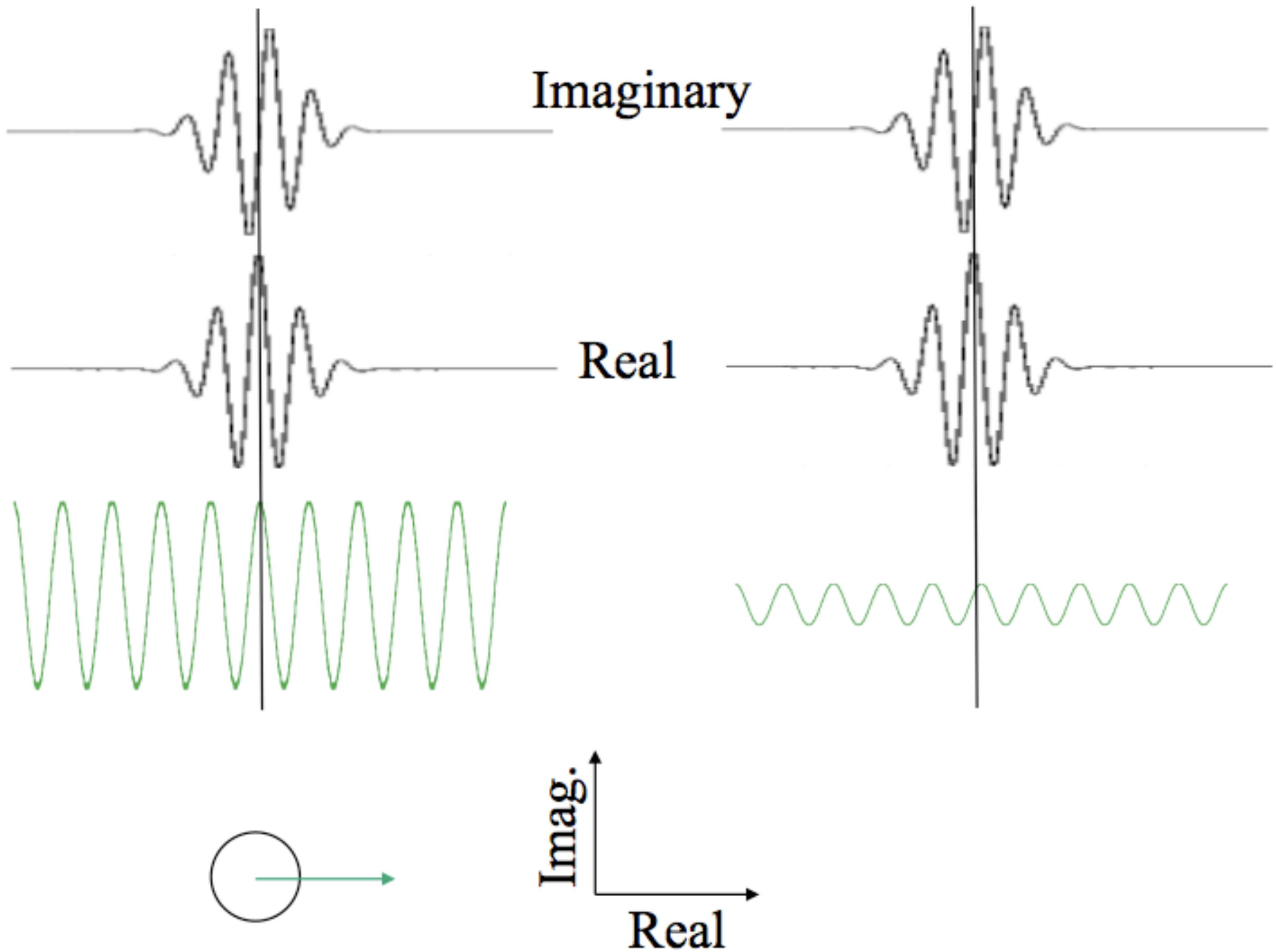


Performing Fourier transform by using a time moving window

Spectral phase and amplitude



Spectral phase and amplitude



```
function [a,b] = dft (y)
%           DFT - The Discrete Fourier Transform
%           [a, b] = DFT (y)
%           a, b are the cosine and sine components

n = length (y);
t = 2*pi*(0:n-1)/n;
f = 2.0 / n;

for j = 0:n2
            cs = cos (j * t);
            ss = sin (j * t);
            a(j+1) = f * (cs * y);
            b(j+1) = f * (ss * y);
end

% boundaries
n2 = floor (n / 2);
a(1) = 0.5 * a(1);
a(n2+1) = 0.5 * a(n2+1);
b(1) = 0.0;
b(n2+1) = 0.0;
```

Loop on frequency

Cosine component

Sine component

Multiply with signal

Discrete Fourrier Transform function

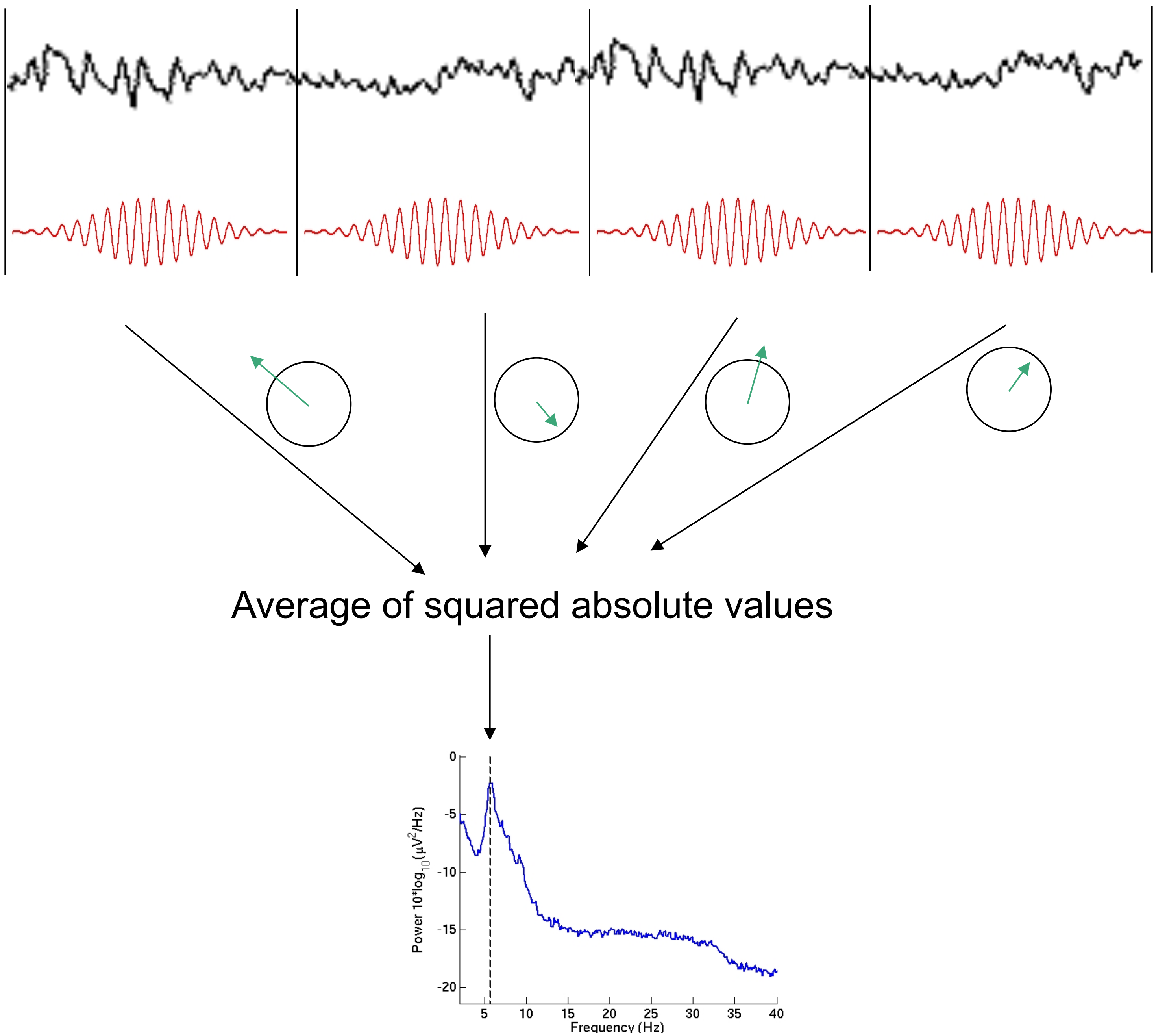
Loop on frequency

```
function X = dft(x)

[N,M] = size(x);
n = 0:N-1;

for k=n
      X(k+1) = exp(-j*2*pi*k*n/N)*x;
end
```

Multiply with signal

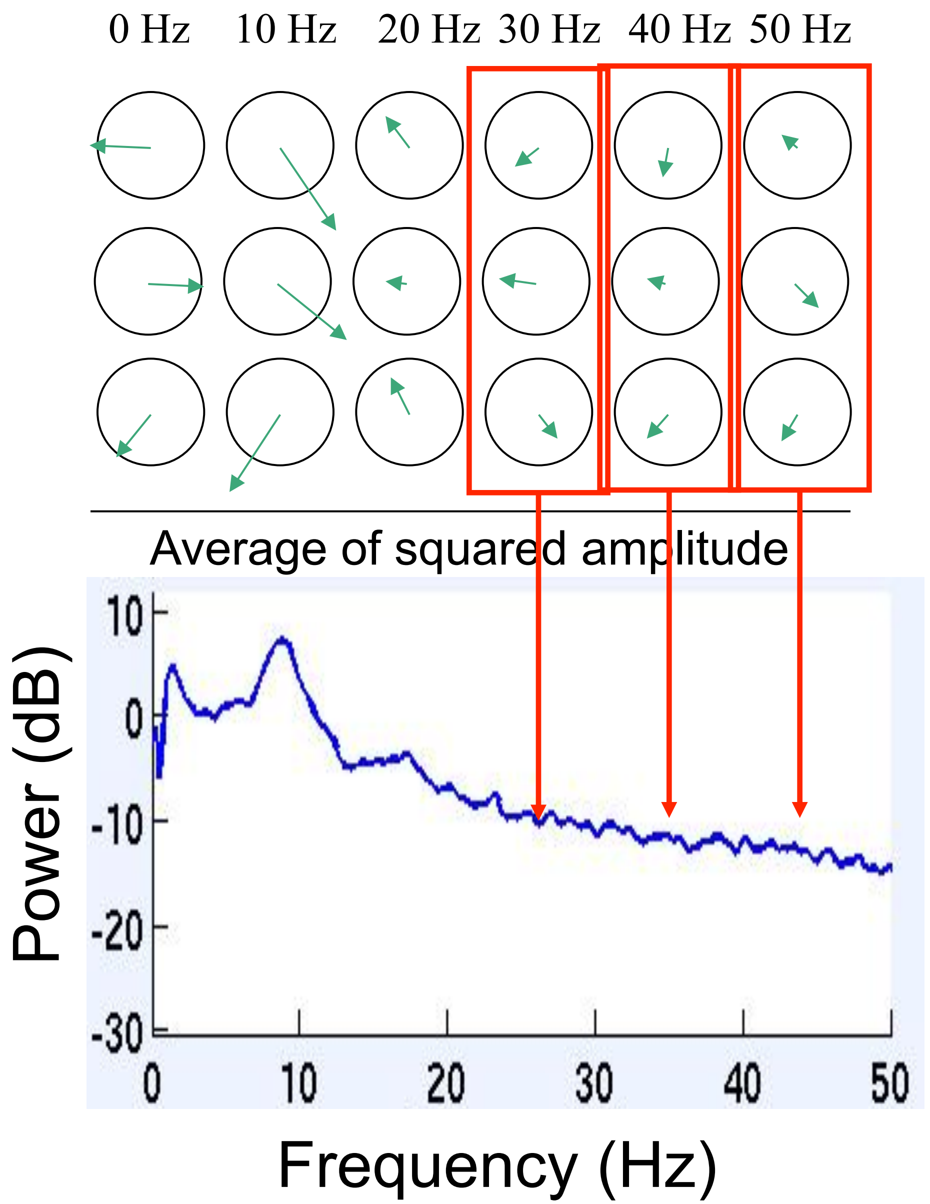
Imaginary part
Sine component

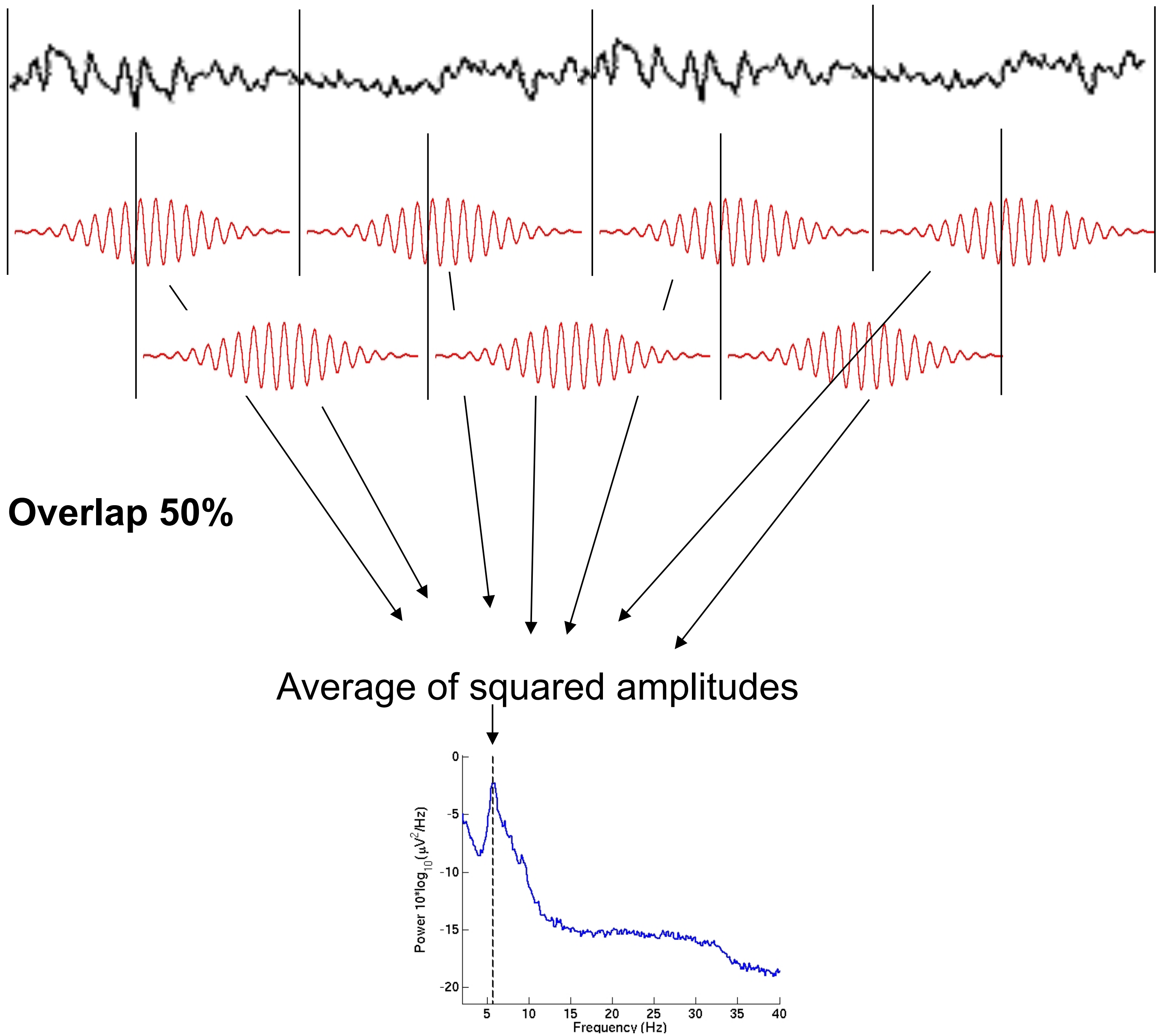
Real part
Cosine component



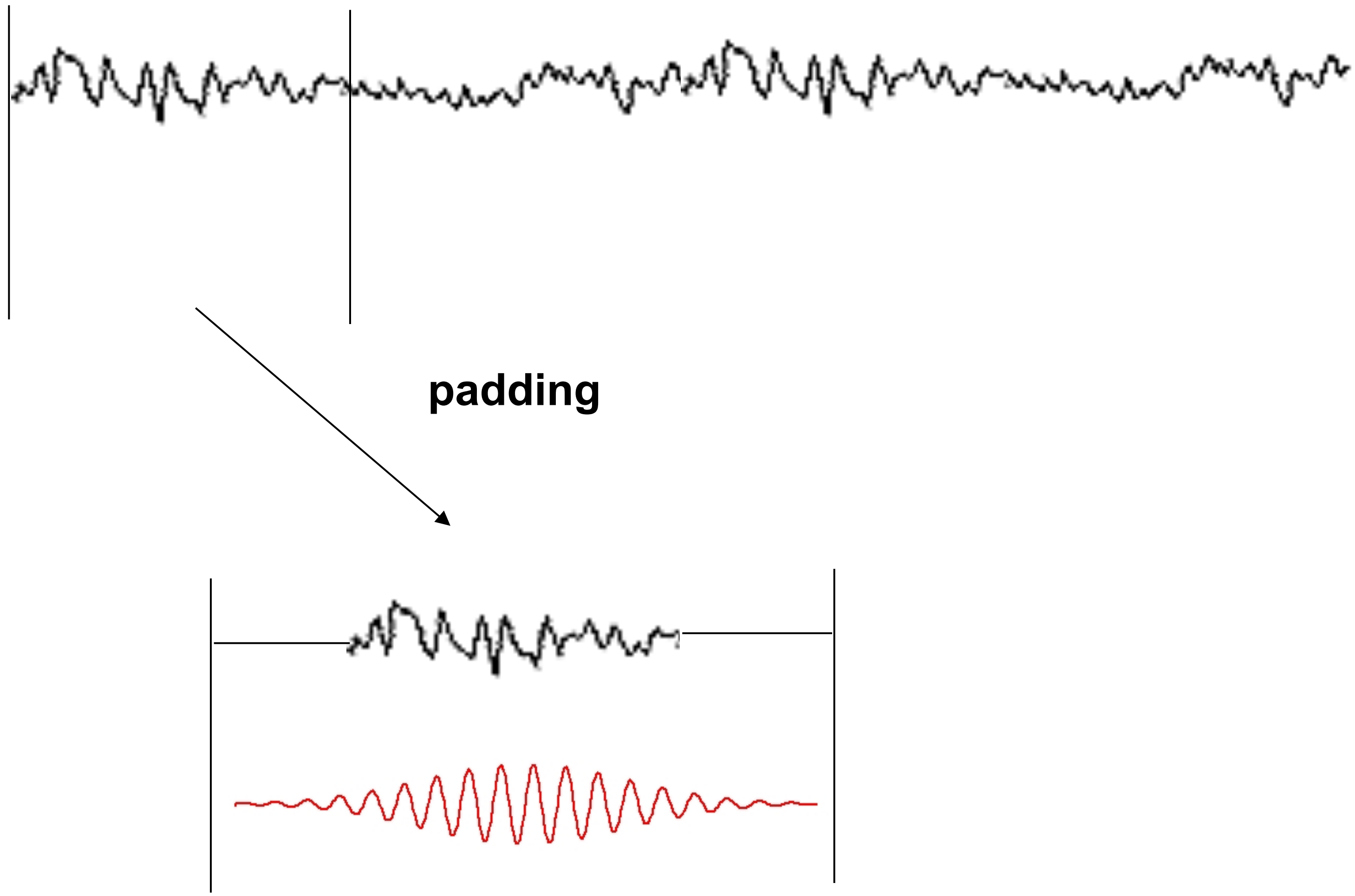
Average of squared absolute values
Power 10*log$_{10}$(μV^2/Hz)
0
-5
-10
-15
-20
5
10
15
20
25
30
35
40
Frequency (Hz)

Spectral power

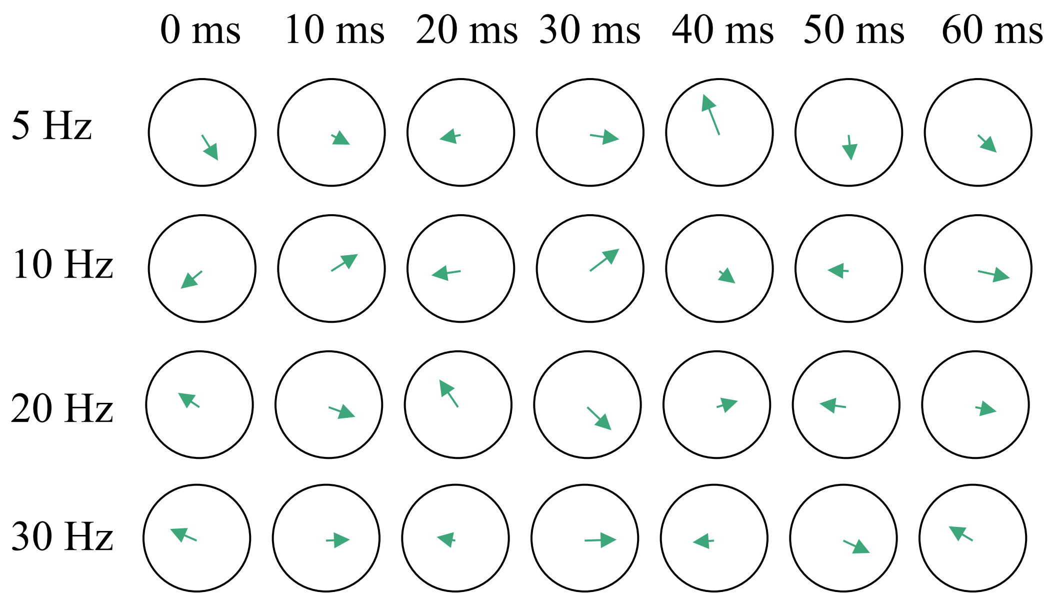




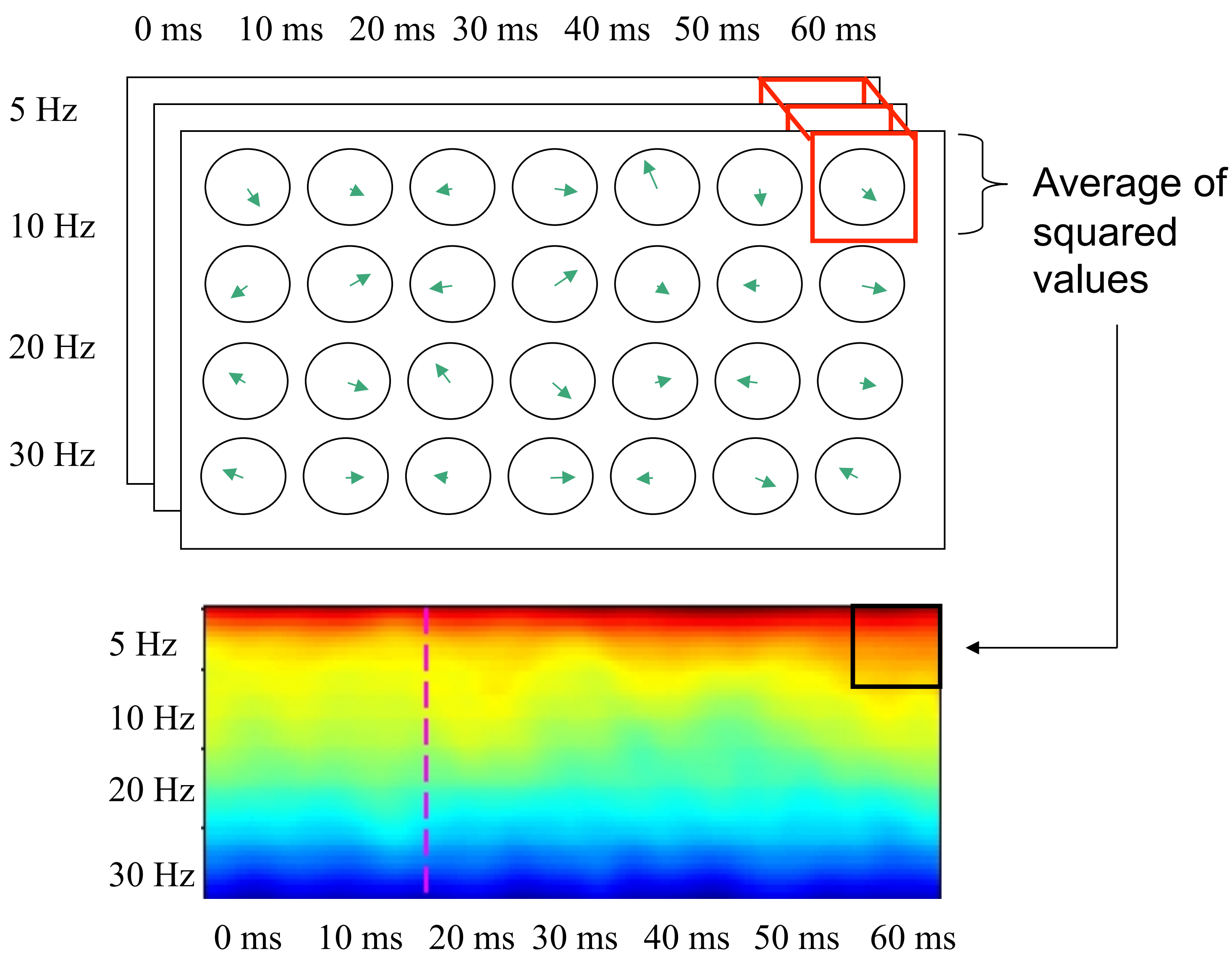
Overlap 50%
Average of squared amplitudes
Power 10*log10(μV²/Hz)
0
-5
-10
-15
-20
5
10
15
20
25
30
35
40
Frequency (Hz)


padding

Spectrogram or ERSP



Spectrogram or ERSP



Power spectrum and event-related spectral perturbation

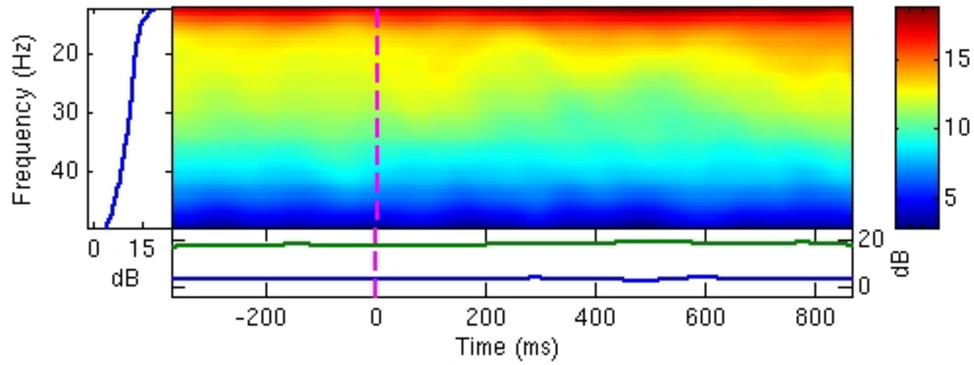
$$ERSP(f,t) = \frac{1}{n}\sum_{k=1}^{n}\left|F_k(f,t)\right|^2$$

Complex number

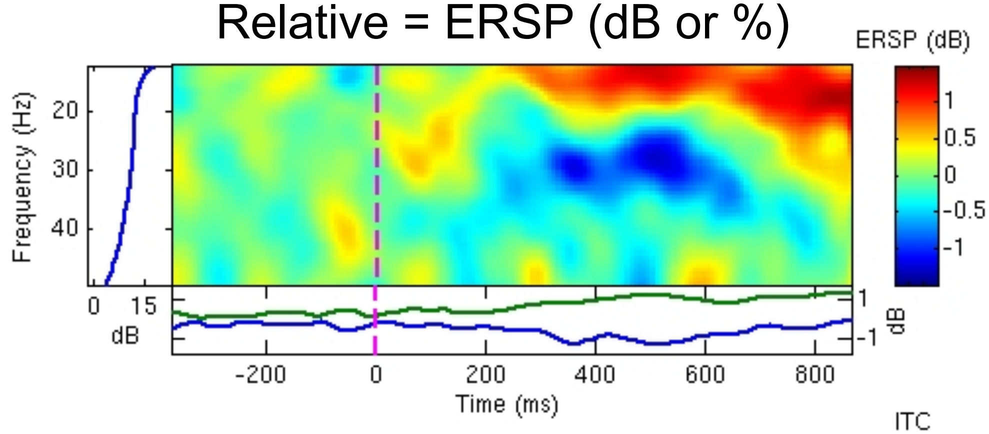
Scaled to dB $10\text{Log}_{10}(\text{ERSP})$

Absolute versus relative power

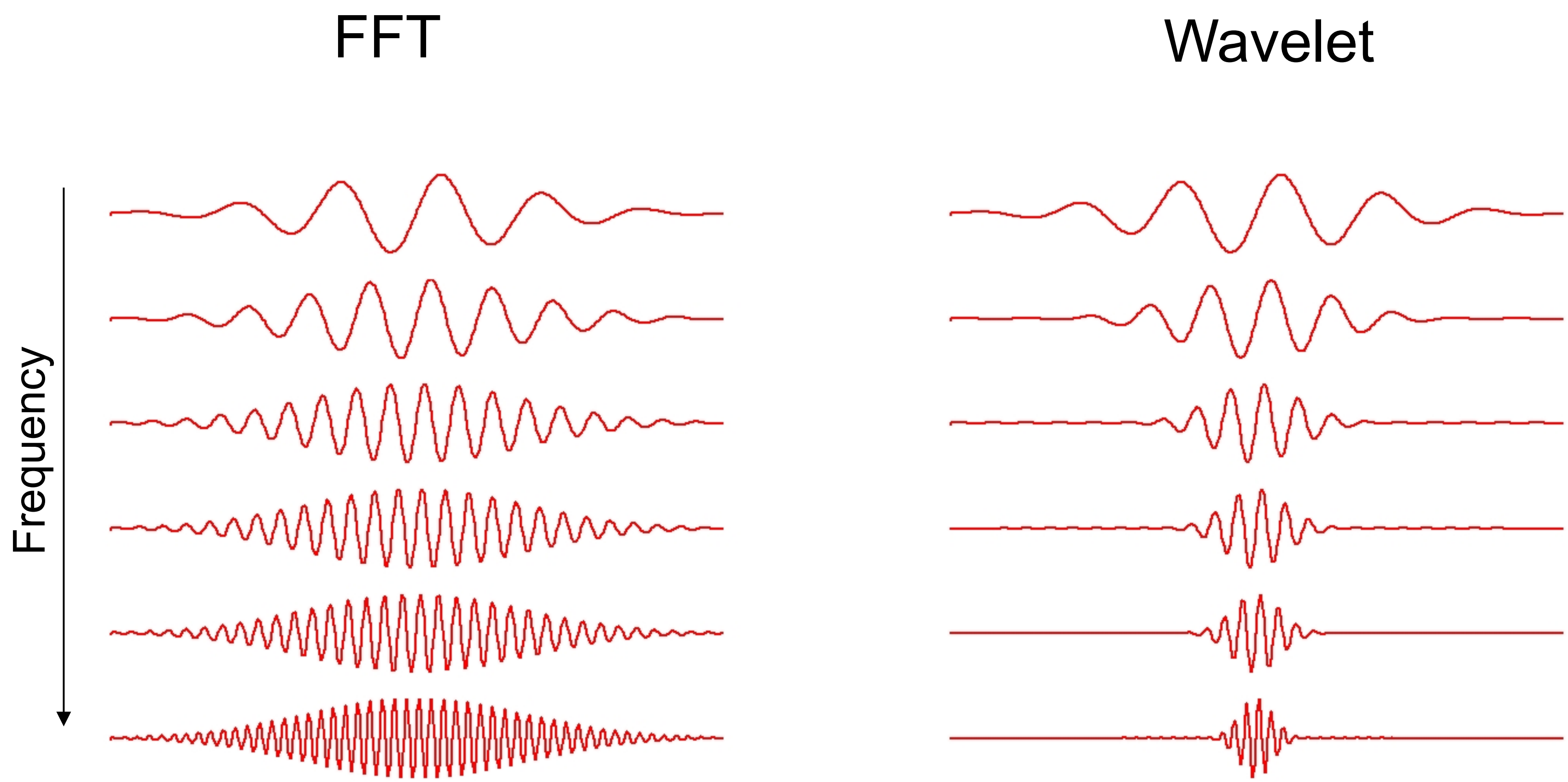
Absolute = ERS



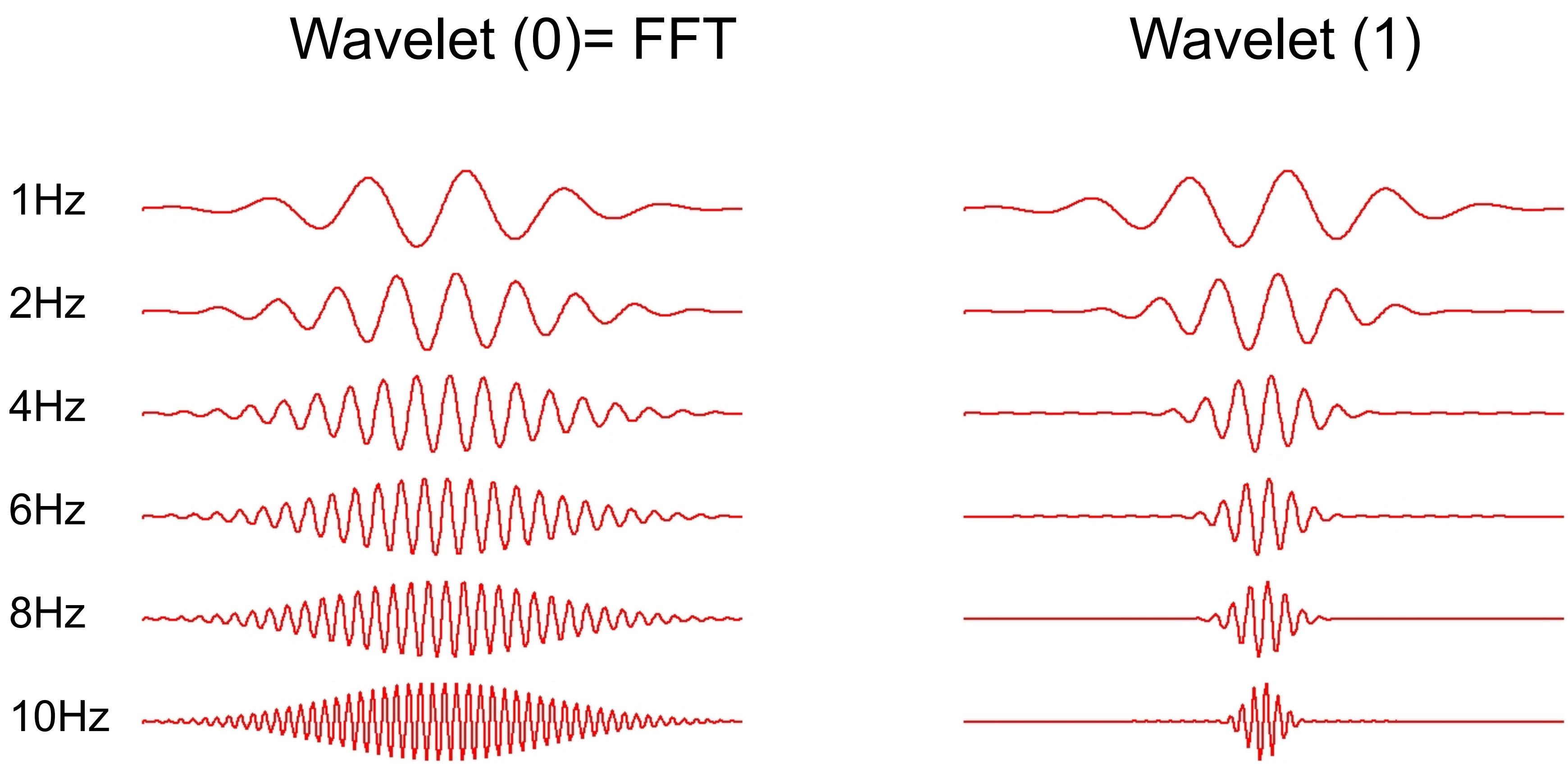
Relative = ERSP (dB or %)



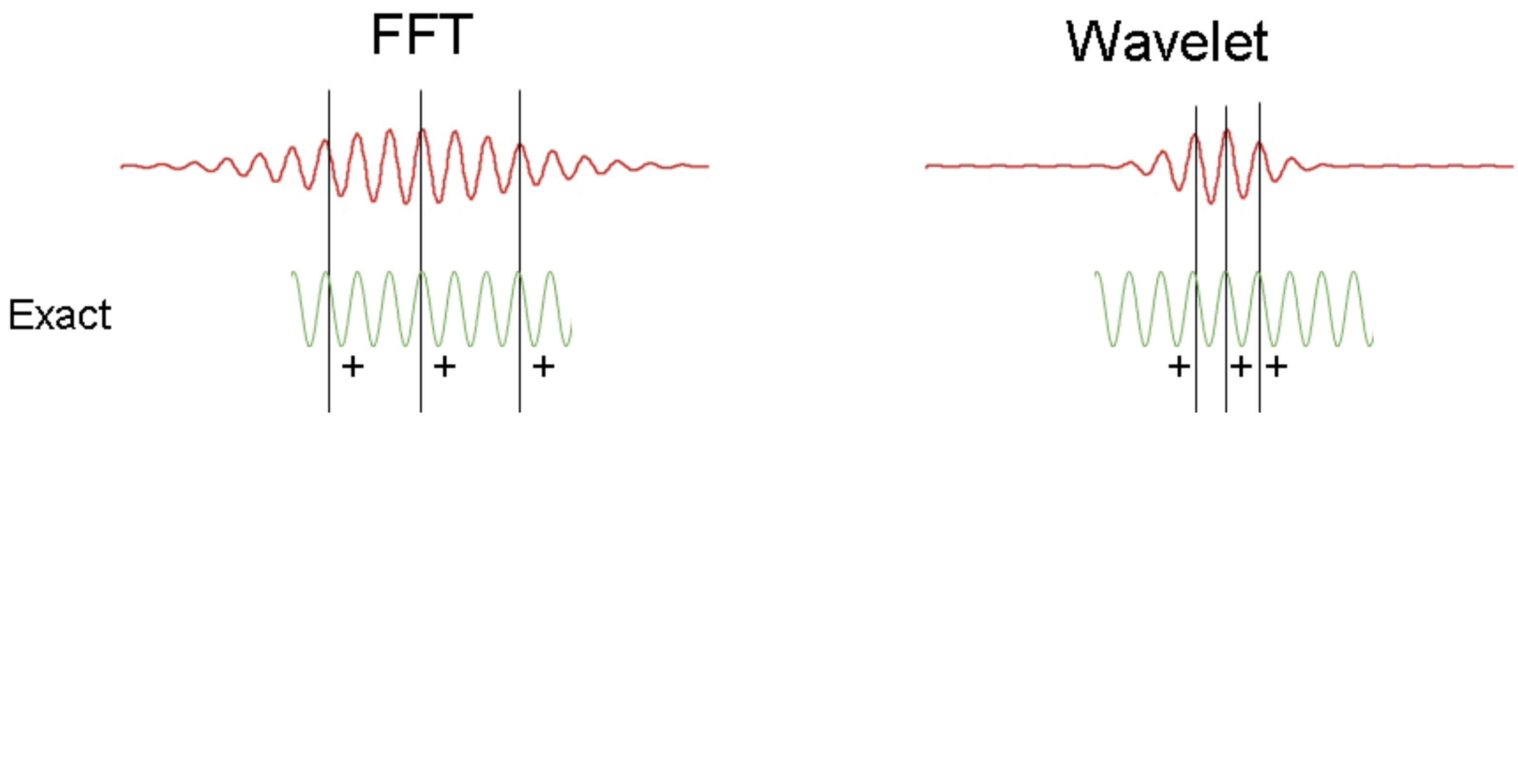
Difference between FFT and wavelets



Wavelets factor



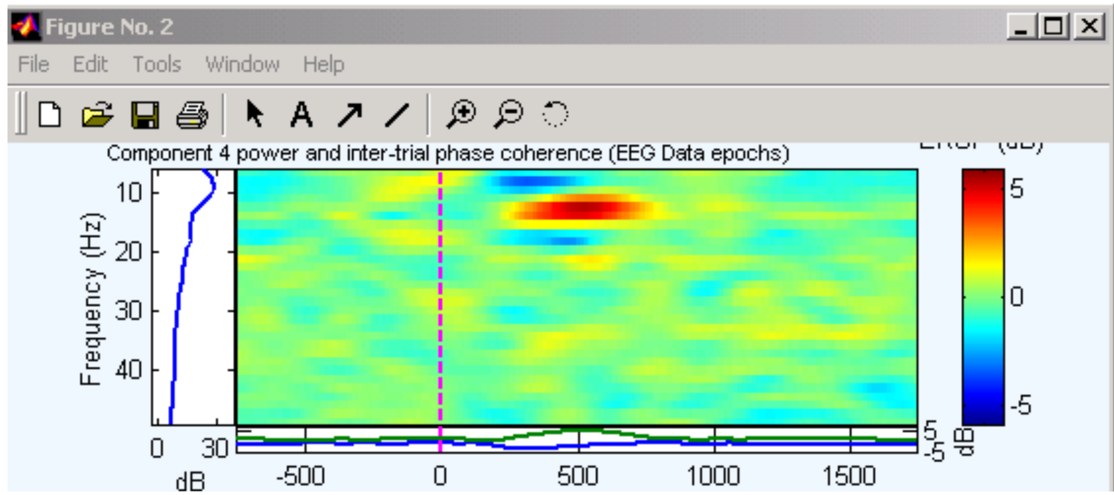
Time-frequency resolution trade off



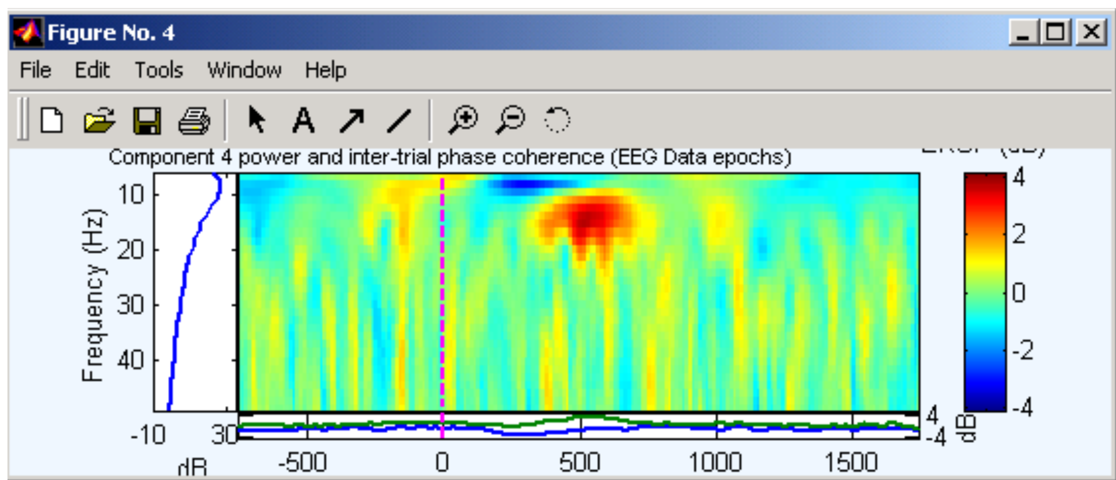
High freq. resolution
low time-resolution

Low freq. resolution
high time-resolution

FFT



Pure wavelet



The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/position and in frequency/momentum.

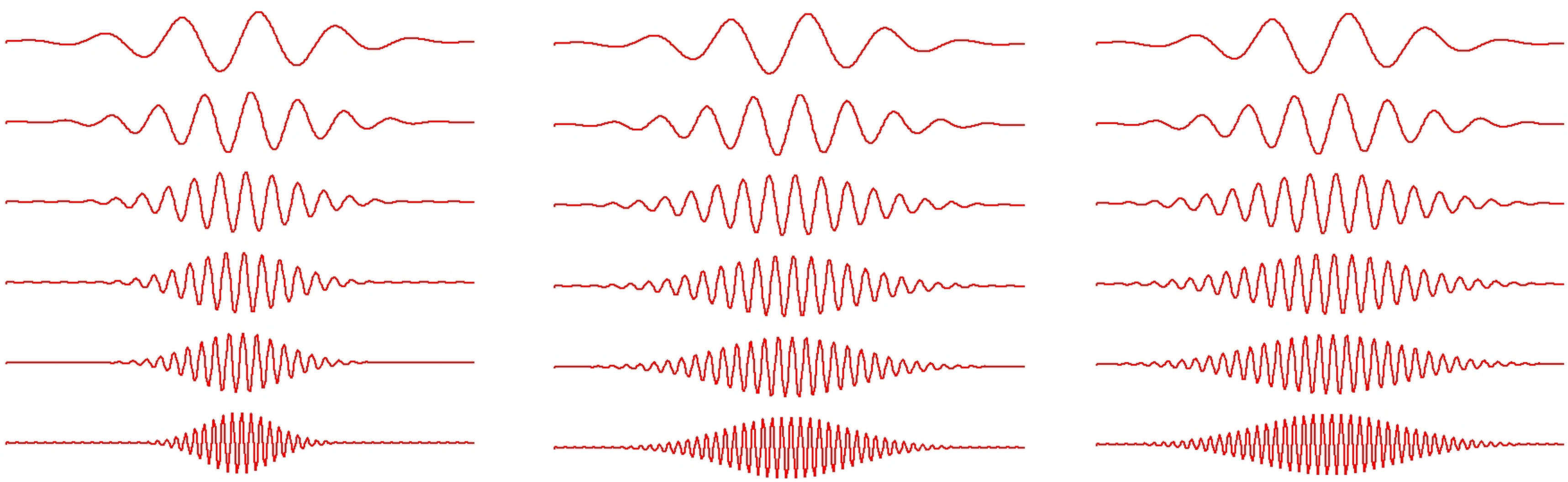
There exists a lower bound to the *Heisenberg's product*:

$$\Delta t \, \Delta f \geq 1/(4\pi)$$

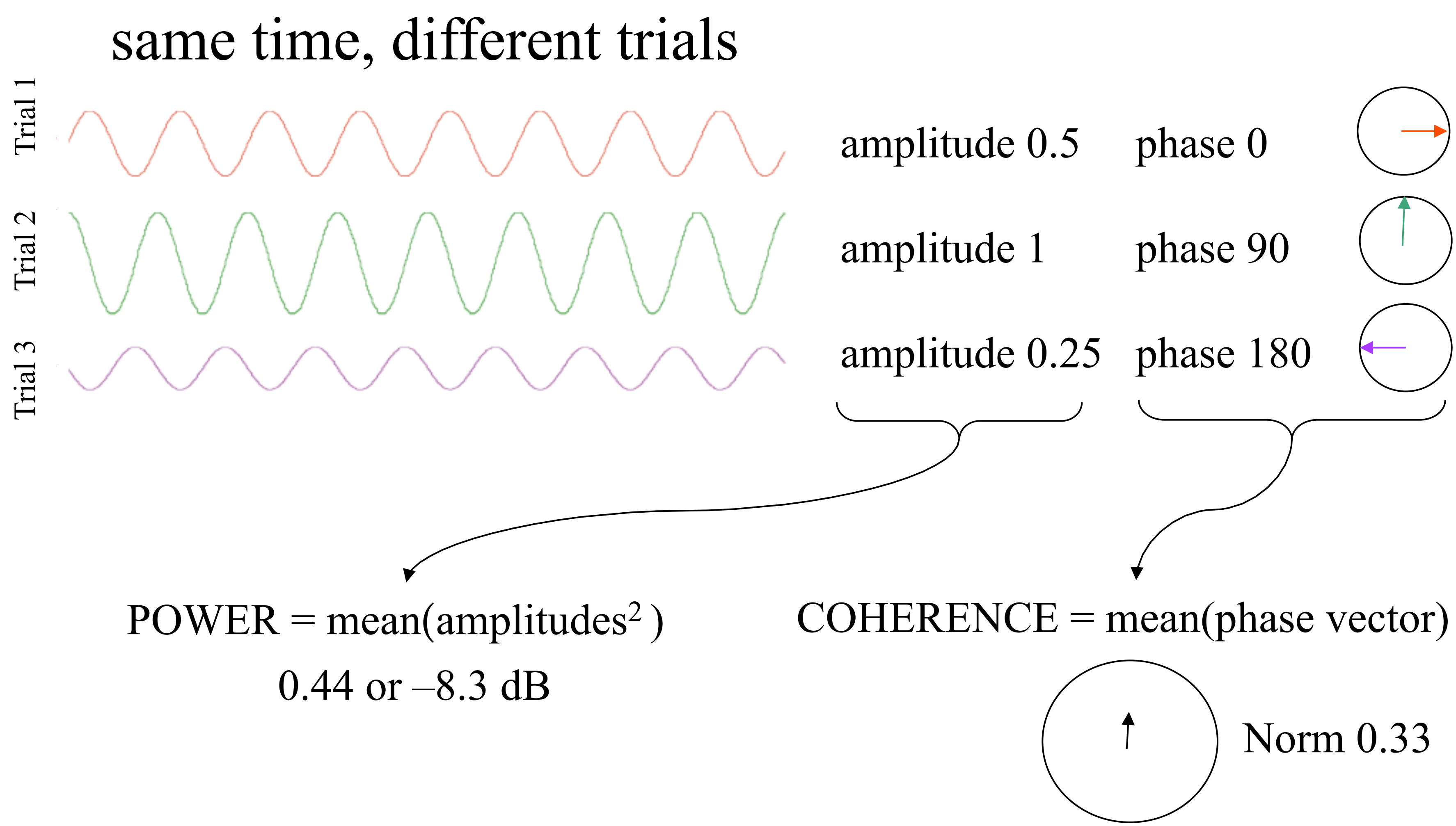
$\Delta f = 1Hz$, $\Delta t = 80$ msec or $\Delta f = 2Hz$, $\Delta t = 40$ msec

Modified wavelets

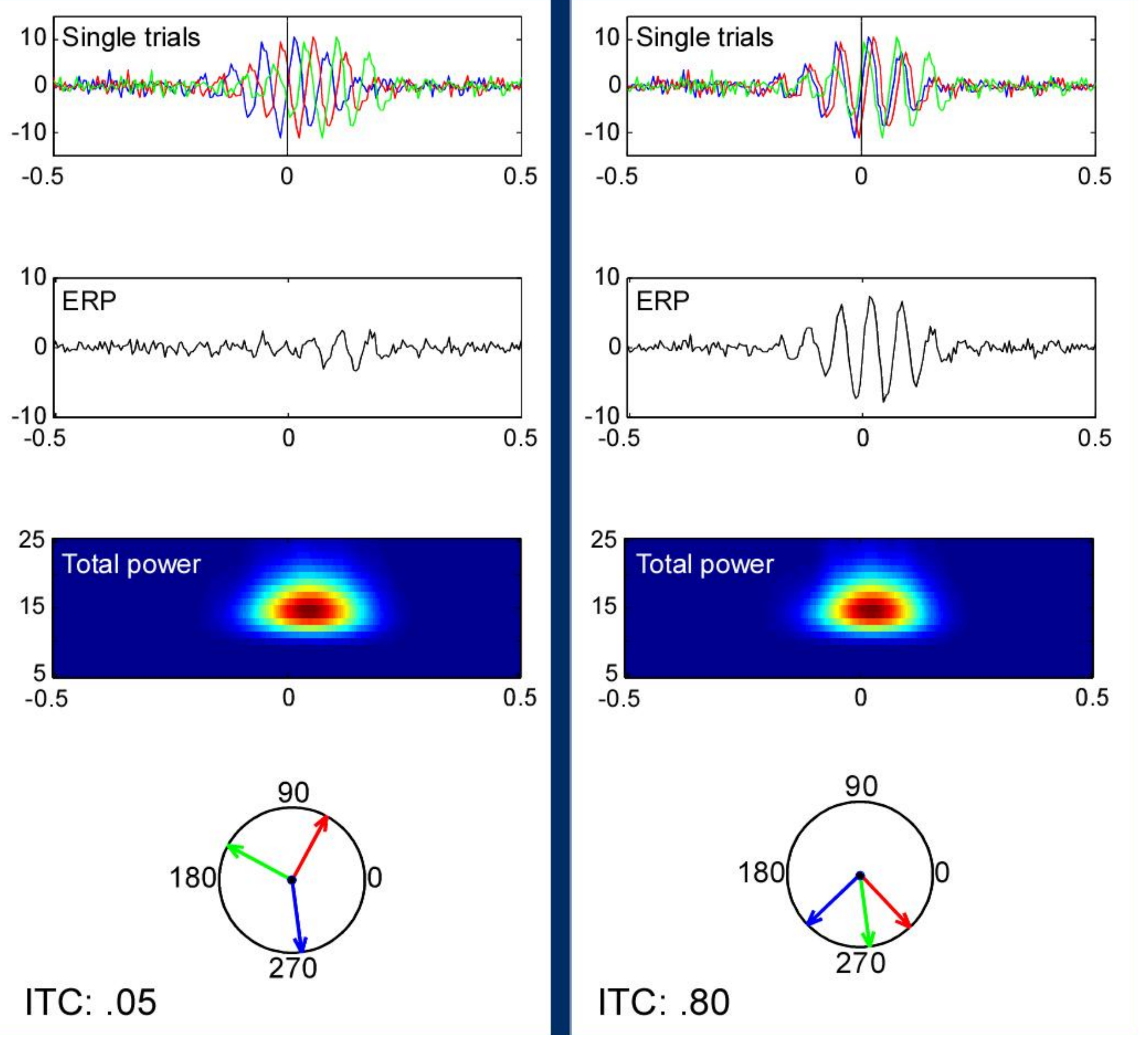
Wavelet (0.8) Wavelet (0.5) Wavelet (0.2)



Inter trial coherence



Intertrial Coherence (ITC)



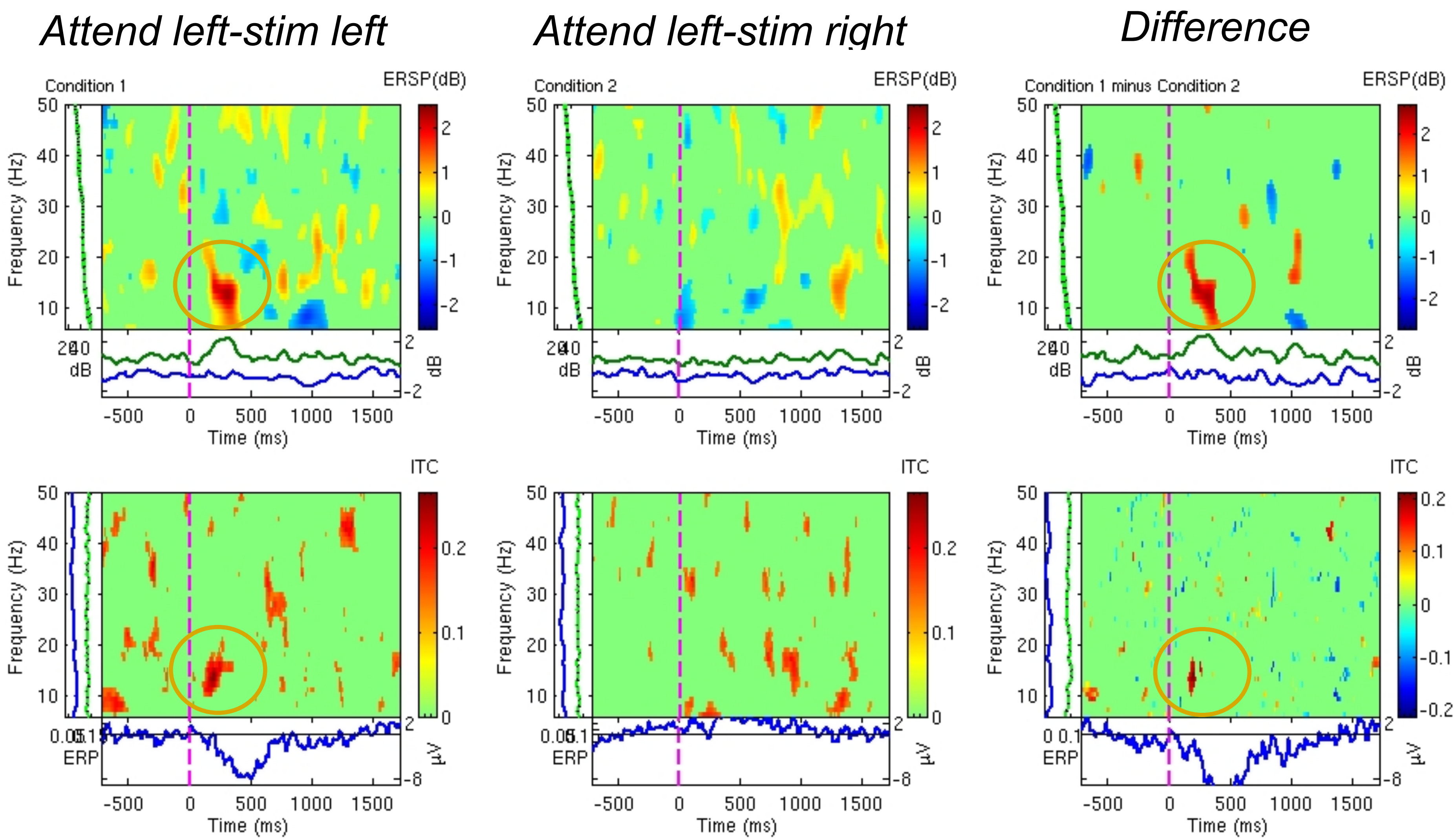
Slide courtesy of Stefan Debener

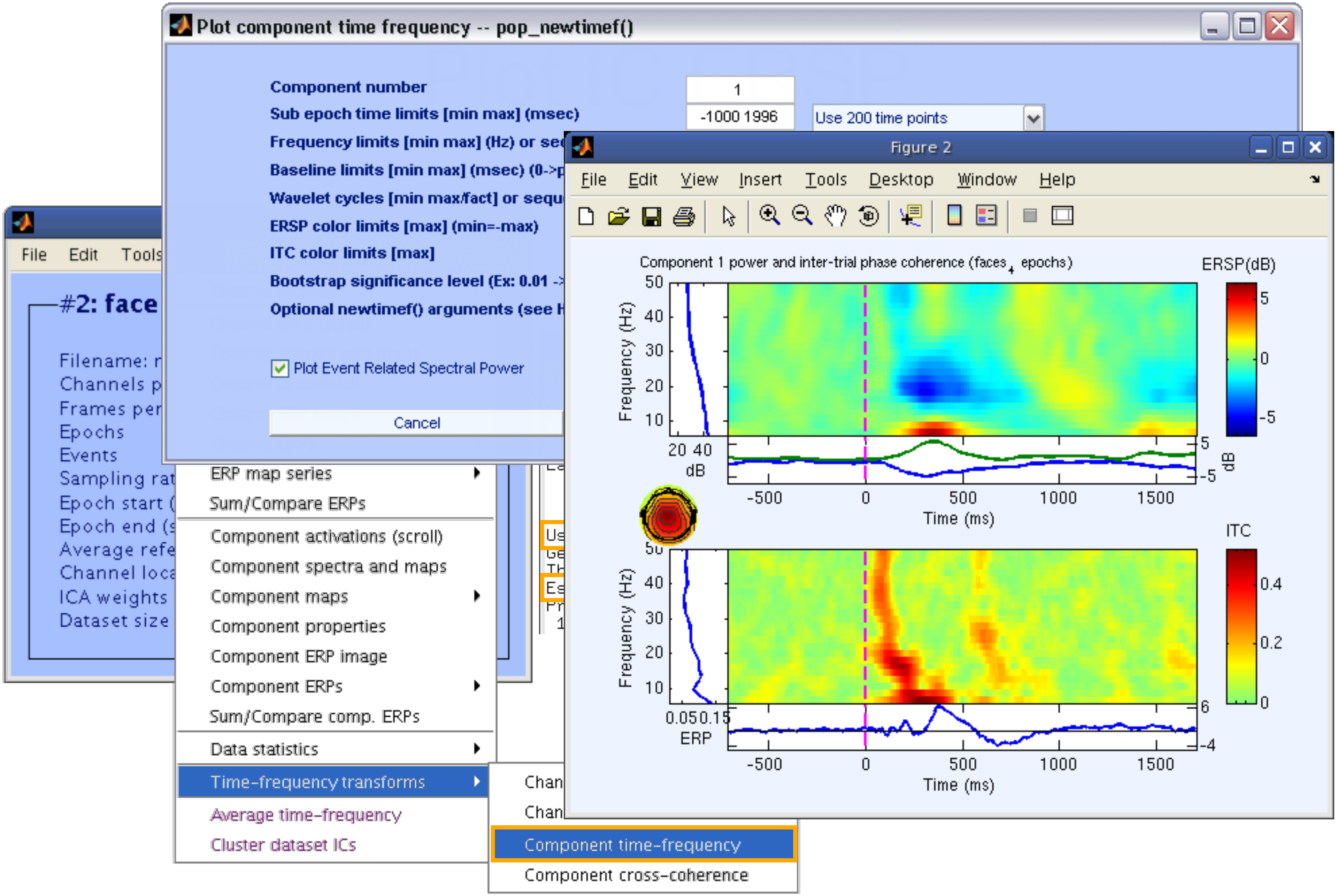
Phase ITC

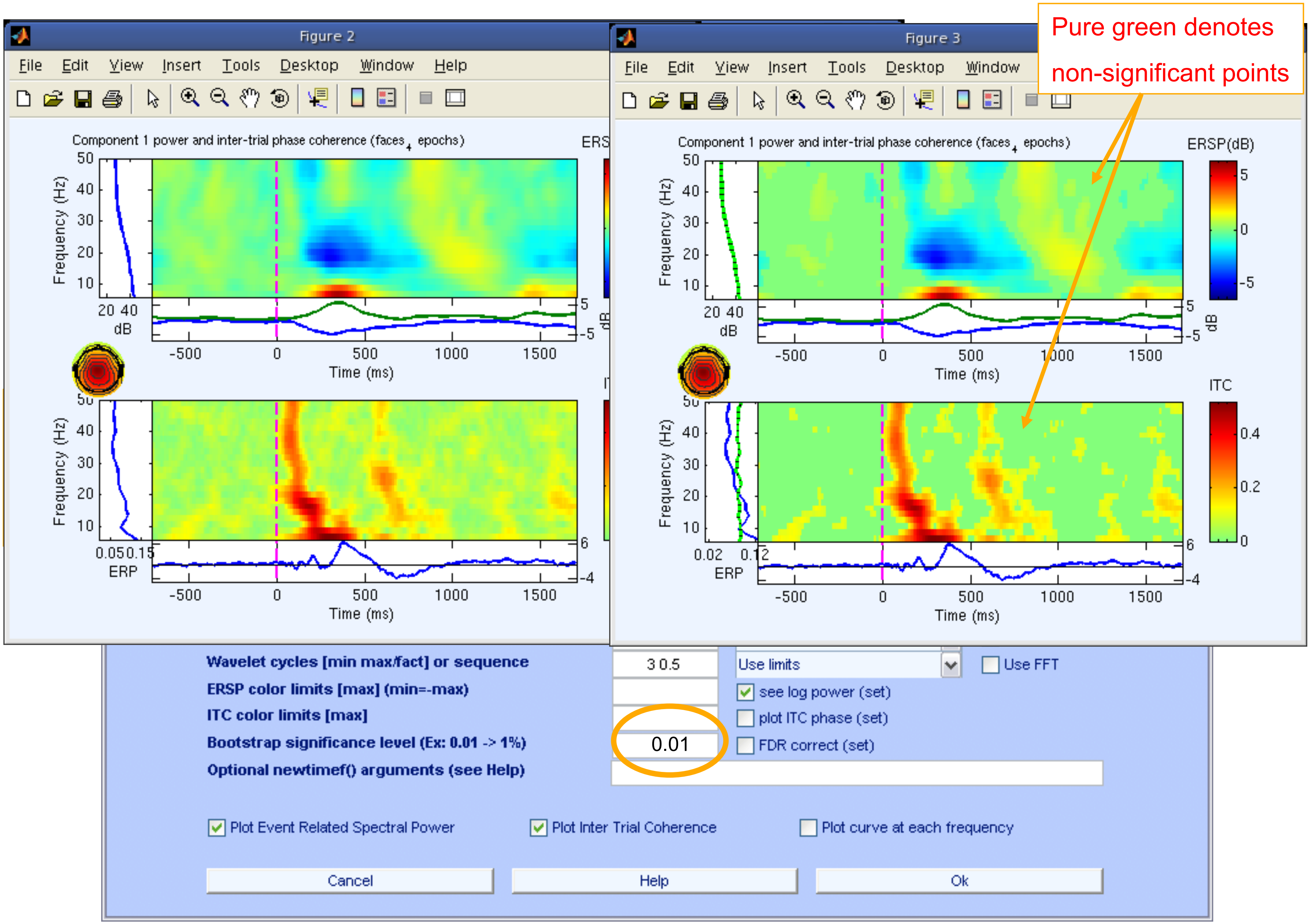
$$ITPC(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k(f,t)}{\left|F_k(f,t)\right|}$$

Normalized
(no amplitude information)

Power and inter trial coherence




Plot component time frequency -- pop_newtimef()
Component number
1
Sub epoch time limits [min max] (msec)
-1000 1996
Use 200 time points
Plot Event Related Spectral Power
Cancel
Figure 2
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
ERSP(dB)
Frequency (Hz)
Time (ms)
ITC
ERP
ERP map series
Sum/Compare ERPs
Component activations (scroll)
Component spectra and maps
Component maps
Component properties
Component ERP image
Component ERPs
Sum/Compare comp. ERPs
Data statistics
Time-frequency transforms
Average time-frequency
Cluster dataset ICs
Component time-frequency
Component cross-coherence


Pure green denotes
non-significant points
Figure 2
Figure 3
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
ERSP(dB)
ITC
Frequency (Hz)
Time (ms)
dB
ERP
Wavelet cycles [min max/fact] or sequence
3 0.5
Use limits
Use FFT
ERSP color limits [max] (min=-max)
see log power (set)
ITC color limits [max]
plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)
0.01
FDR correct (set)
Optional newtimef() arguments (see Help)
Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency
Cancel
Help
Ok

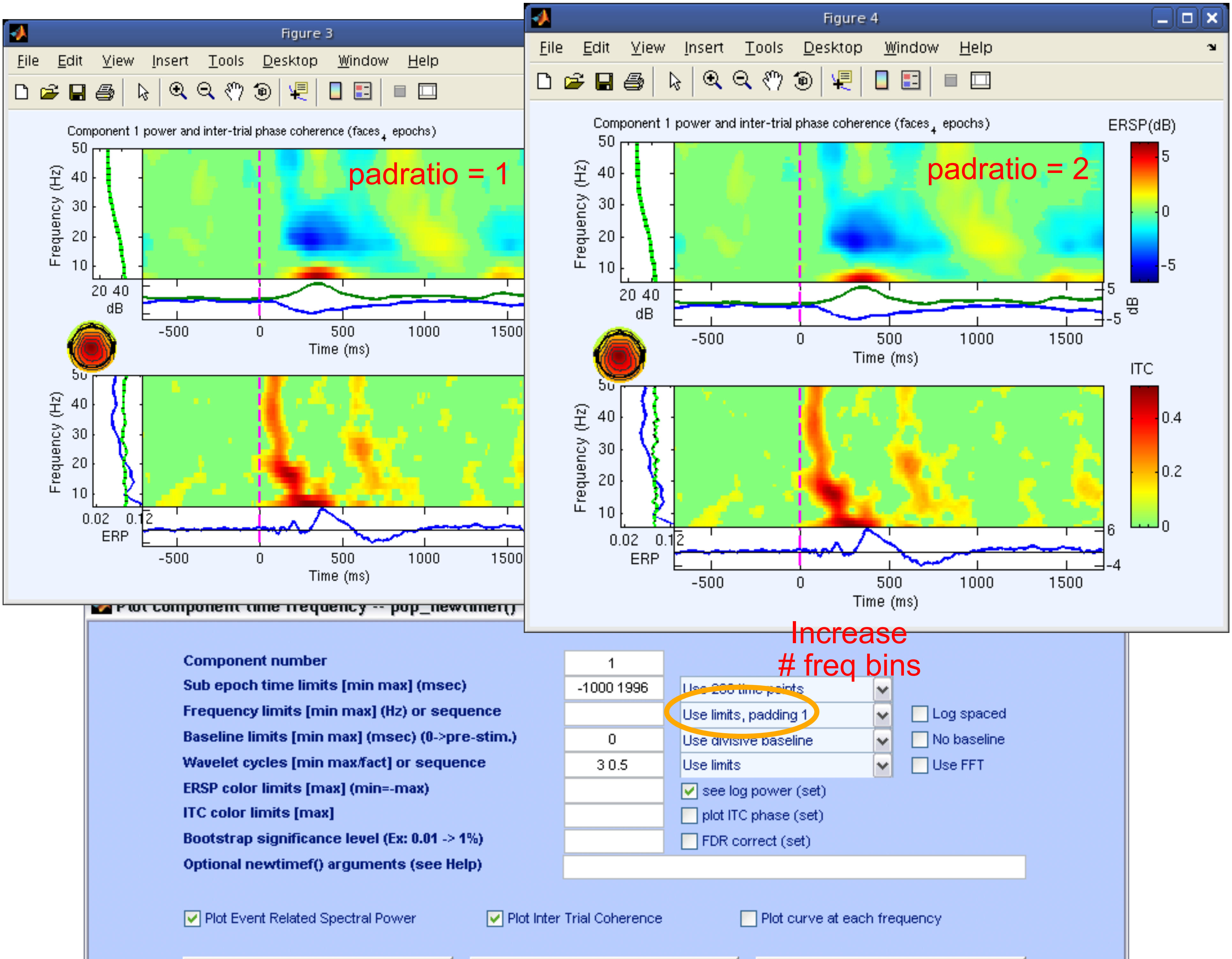
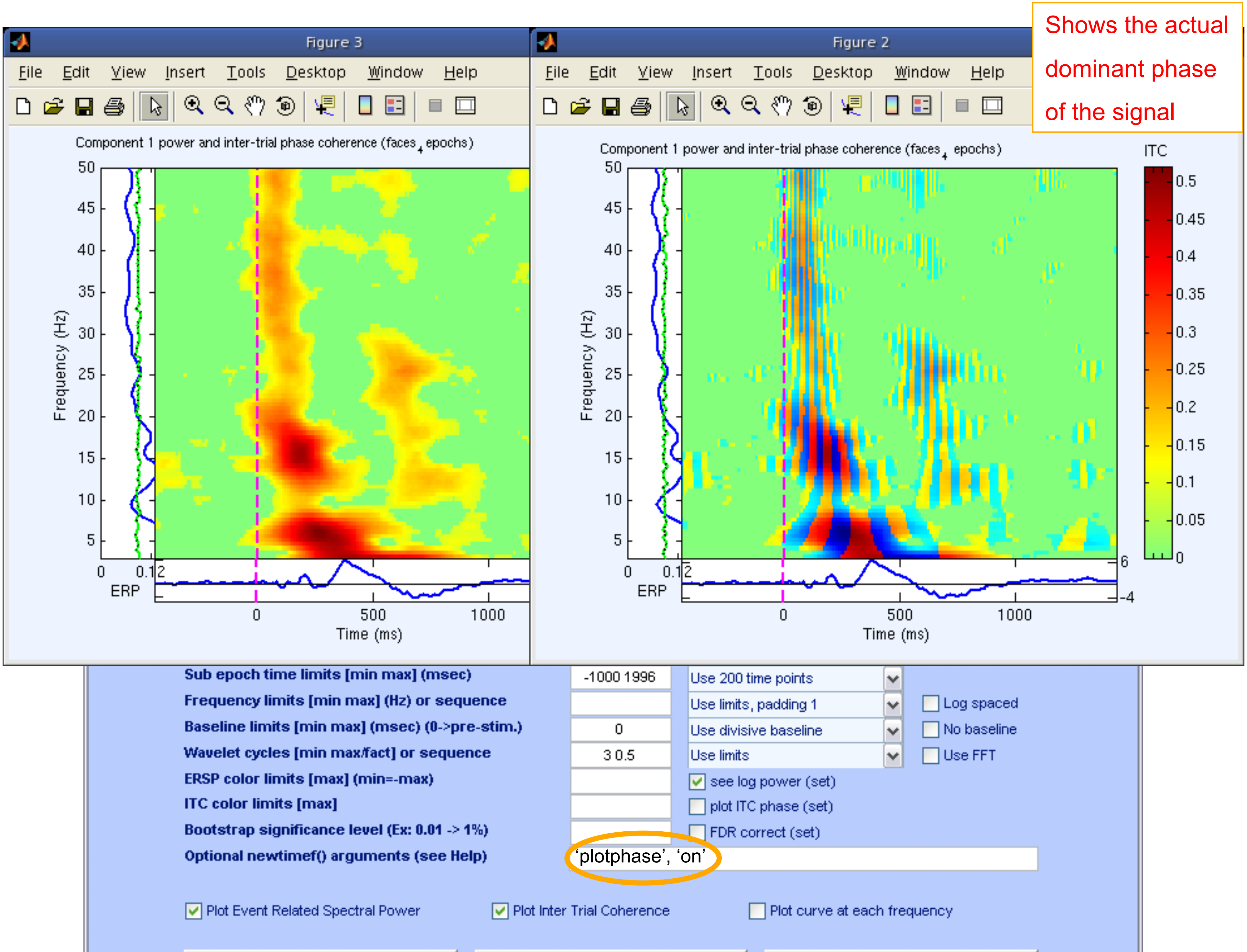
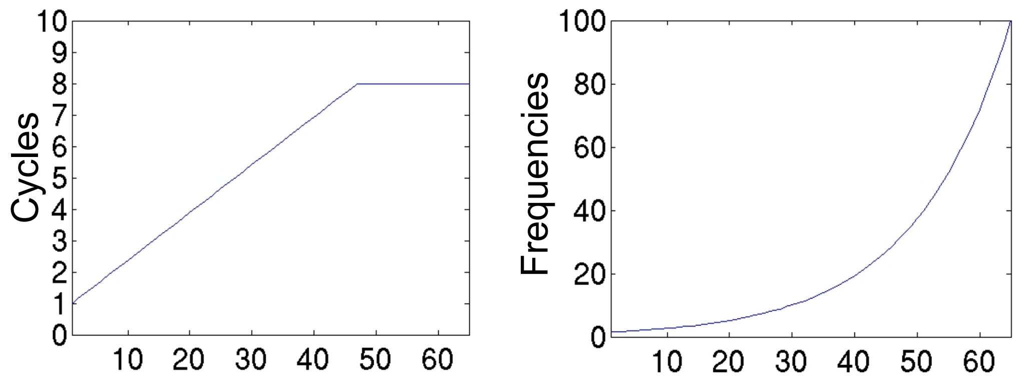


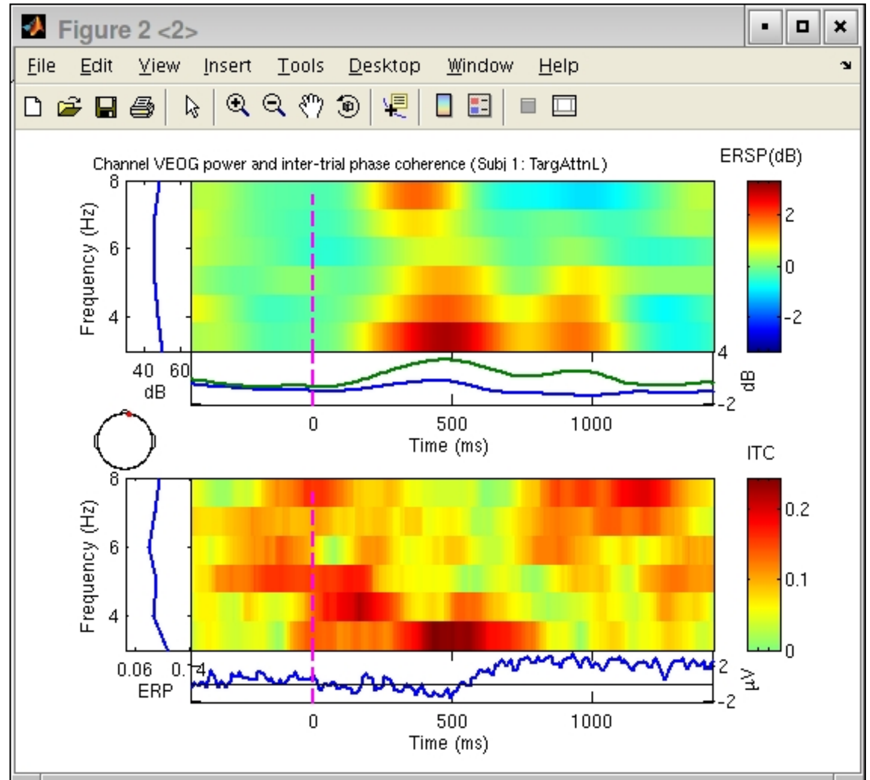
Figure 3
Figure 4
Component 1 power and inter-trial phase coherence (faces, epochs)
padratio = 1
padratio = 2
Frequency (Hz)
Time (ms)
ERSP(dB)
ITC
ERP
dB
Increase
freq bins
Plot component time frequency -- pop_newtimef()
Component number
Sub epoch time limits [min max] (msec)
Frequency limits [min max] (Hz) or sequence
Baseline limits [min max] (msec) (0->pre-stim.)
Wavelet cycles [min max/fact] or sequence
ERSP color limits [max] (min=-max)
ITC color limits [max]
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional newtimef() arguments (see Help)
1
-1000 1996
0
3 0.5
Use limits, padding 1
Use limits
Log spaced
No baseline
Use FFT
see log power (set)
plot ITC phase (set)
FDR correct (set)
Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency

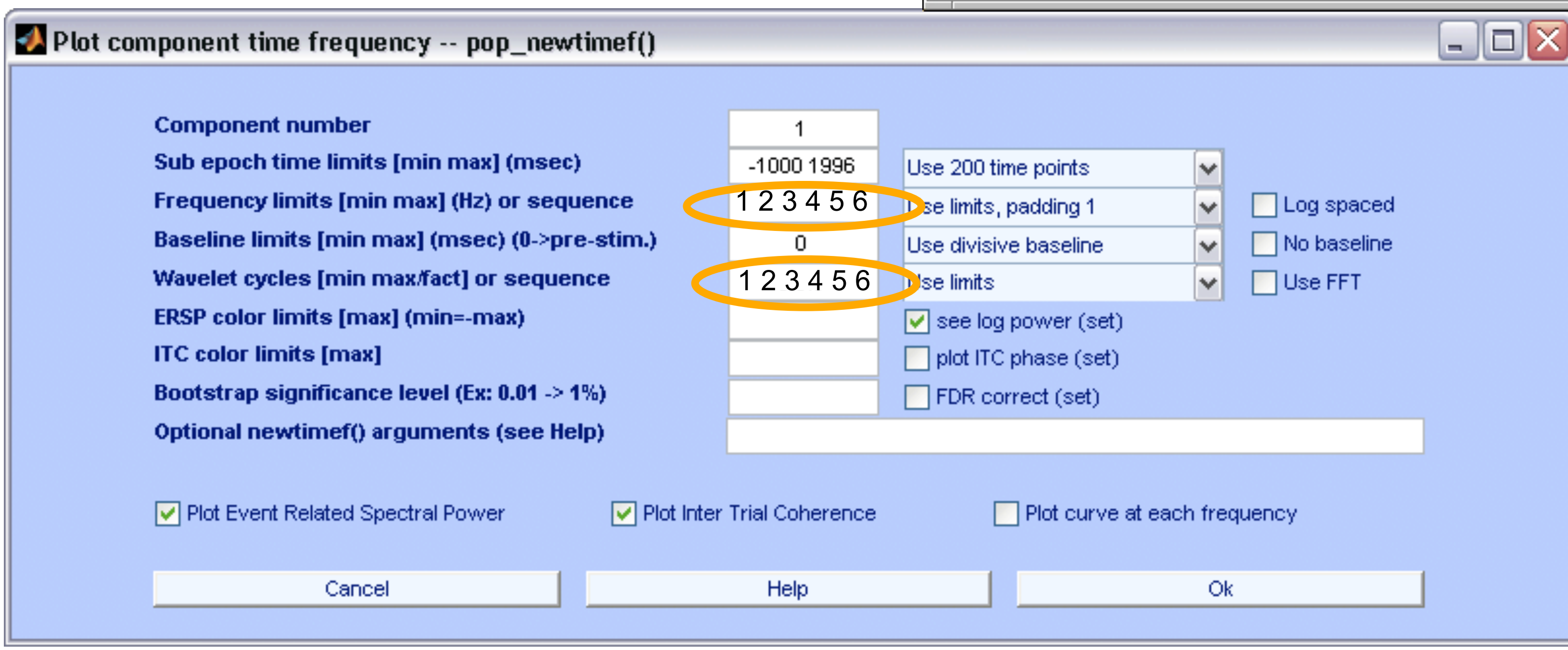

Shows the actual dominant phase of the signal
Figure 3
Figure 2
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces + epochs)
Frequency (Hz)
ERP
Time (ms)
ITC
Sub epoch time limits [min max] (msec)
-1000 1996
Use 200 time points
Frequency limits [min max] (Hz) or sequence
Use limits, padding 1
Log spaced
Baseline limits [min max] (msec) (0->pre-stim.)
0
Use divisive baseline
No baseline
Wavelet cycles [min max/fact] or sequence
3 0.5
Use limits
Use FFT
ERSP color limits [max] (min=-max)
see log power (set)
ITC color limits [max]
plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)
FDR correct (set)
Optional newtimef() arguments (see Help)
'plotphase', 'on'
Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency

To visualize both low and high frequencies

```
freqs  = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
```





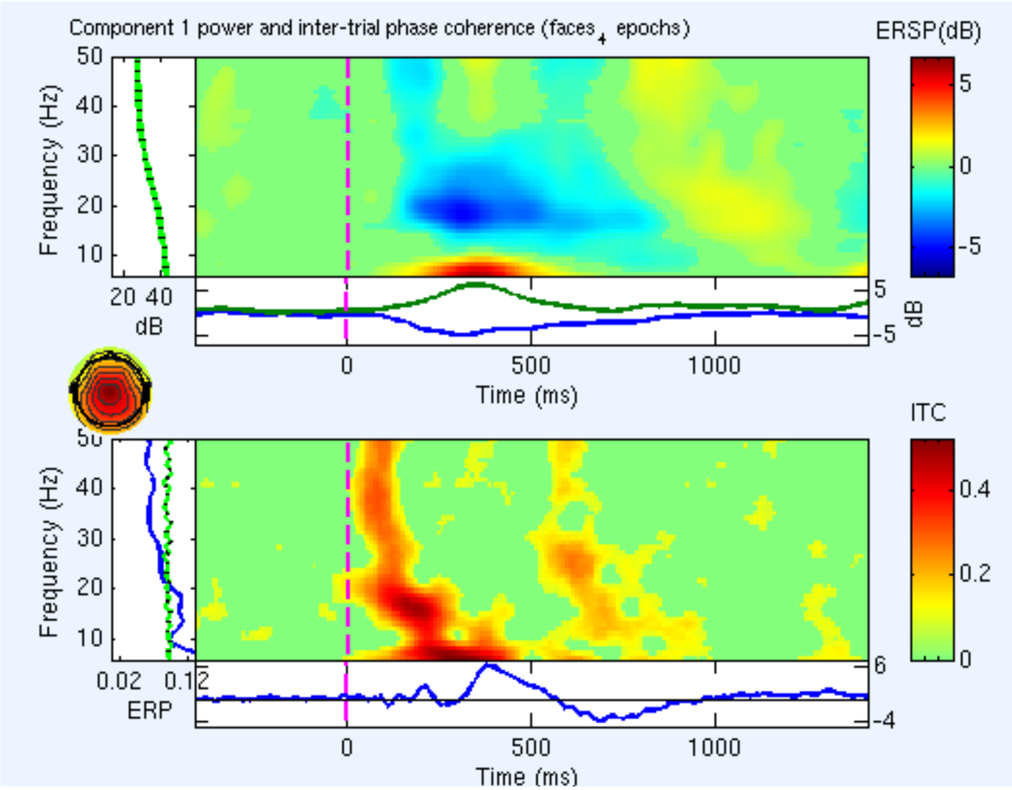


Evoked versus induced

- Evoked = ERSP of the average ERP
- Induced = usually standard ERSP
- Real induced

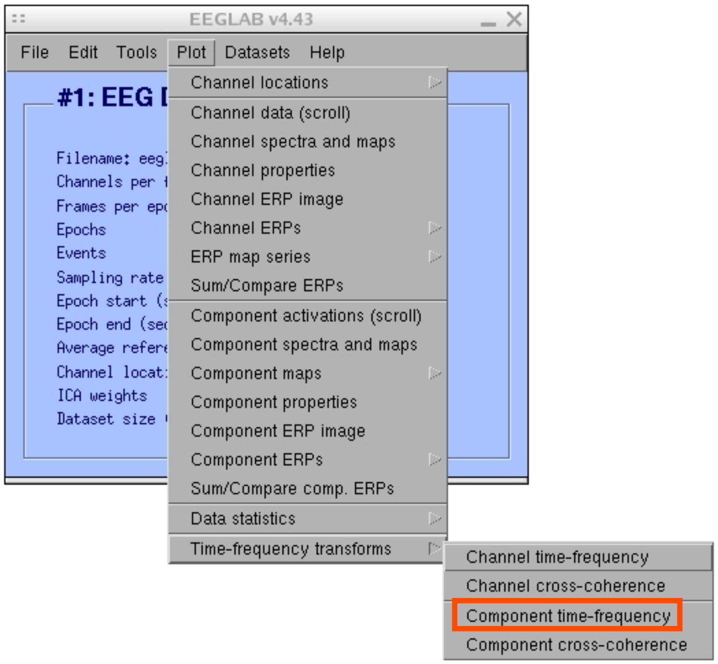
(1) standard ERSP with ERP regressed out of every trial

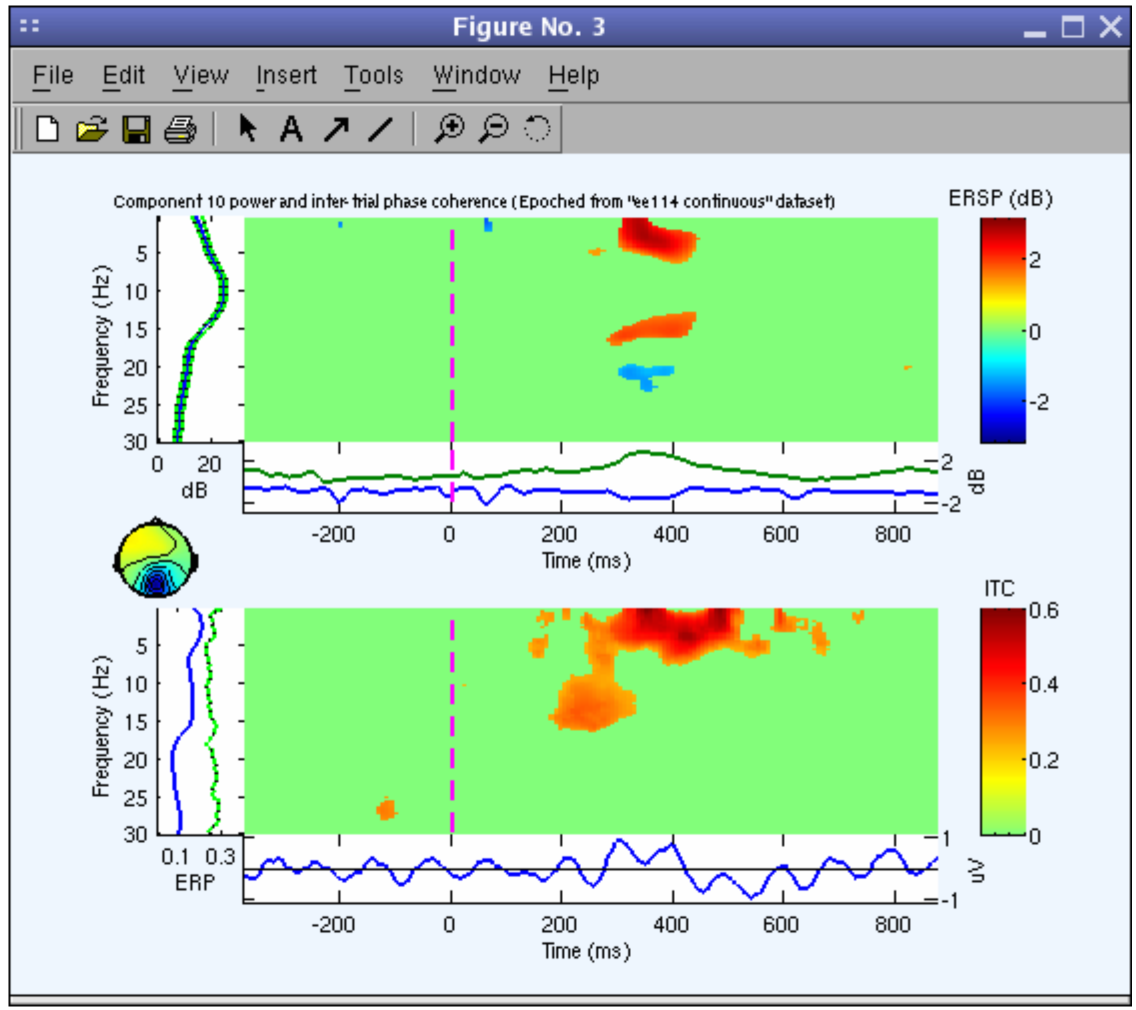
(2) standard ERSP minus ERSP of the average ERP scaled for averaging effect



In any case, looking at the ITC provides the amount of synchronization in the time-frequency decomposition that account for ERPs

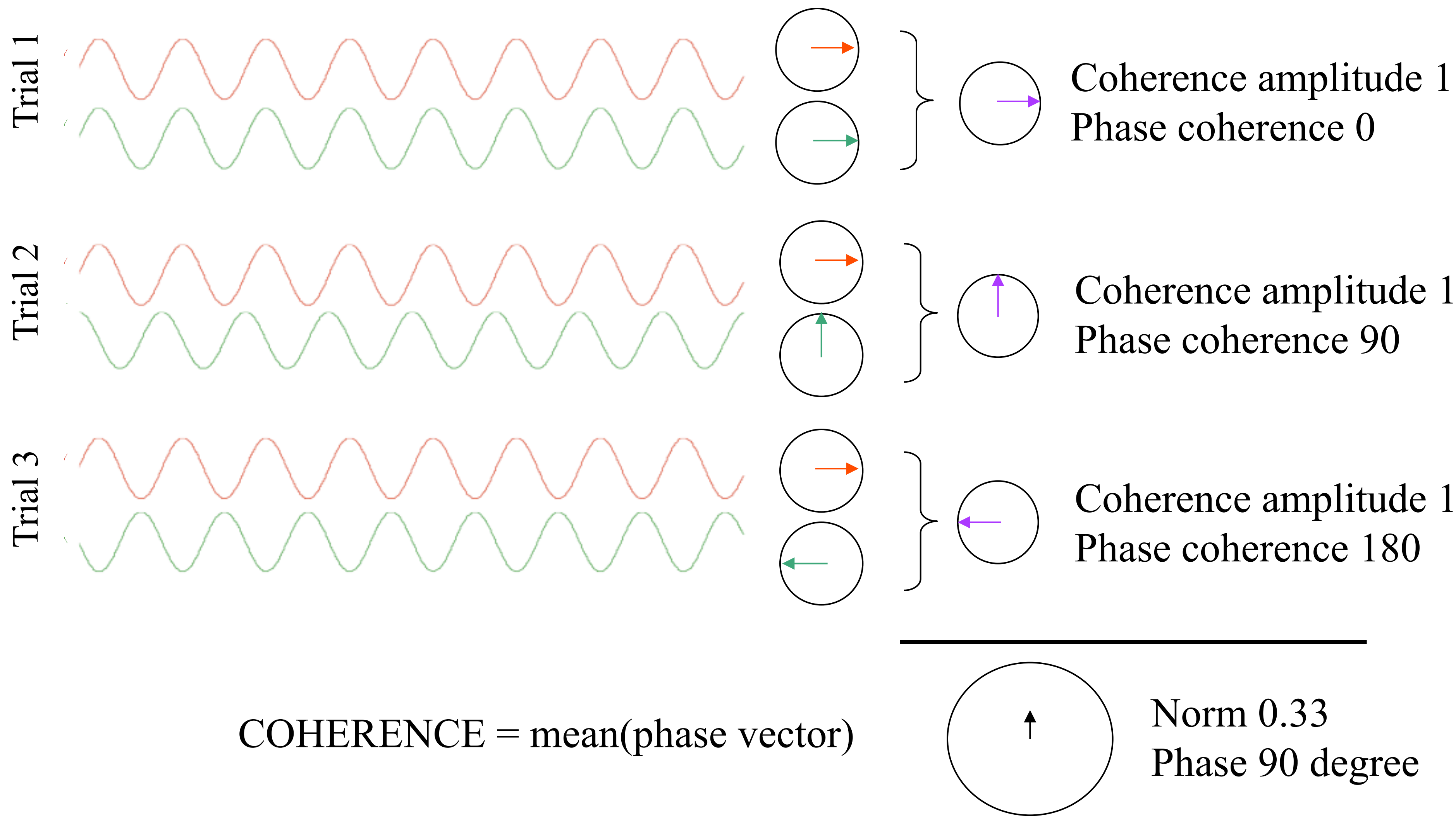
Component time-frequency





Cross-coherence amplitude and phase

2 components, comparison on the same trials



Phase coherence (default)

$$ERPCOH^{a,b}(f,t) = \frac{1}{n}\sum_{k=1}^{n}\frac{F_k^a(f,t)F_k^b(f,t)^*}{\left|F_k^a(f,t)\right|\left|F_k^b(f,t)\right|}$$

Only phase information component b

Only phase information component a

Other spectral measures

Amplitude correlation

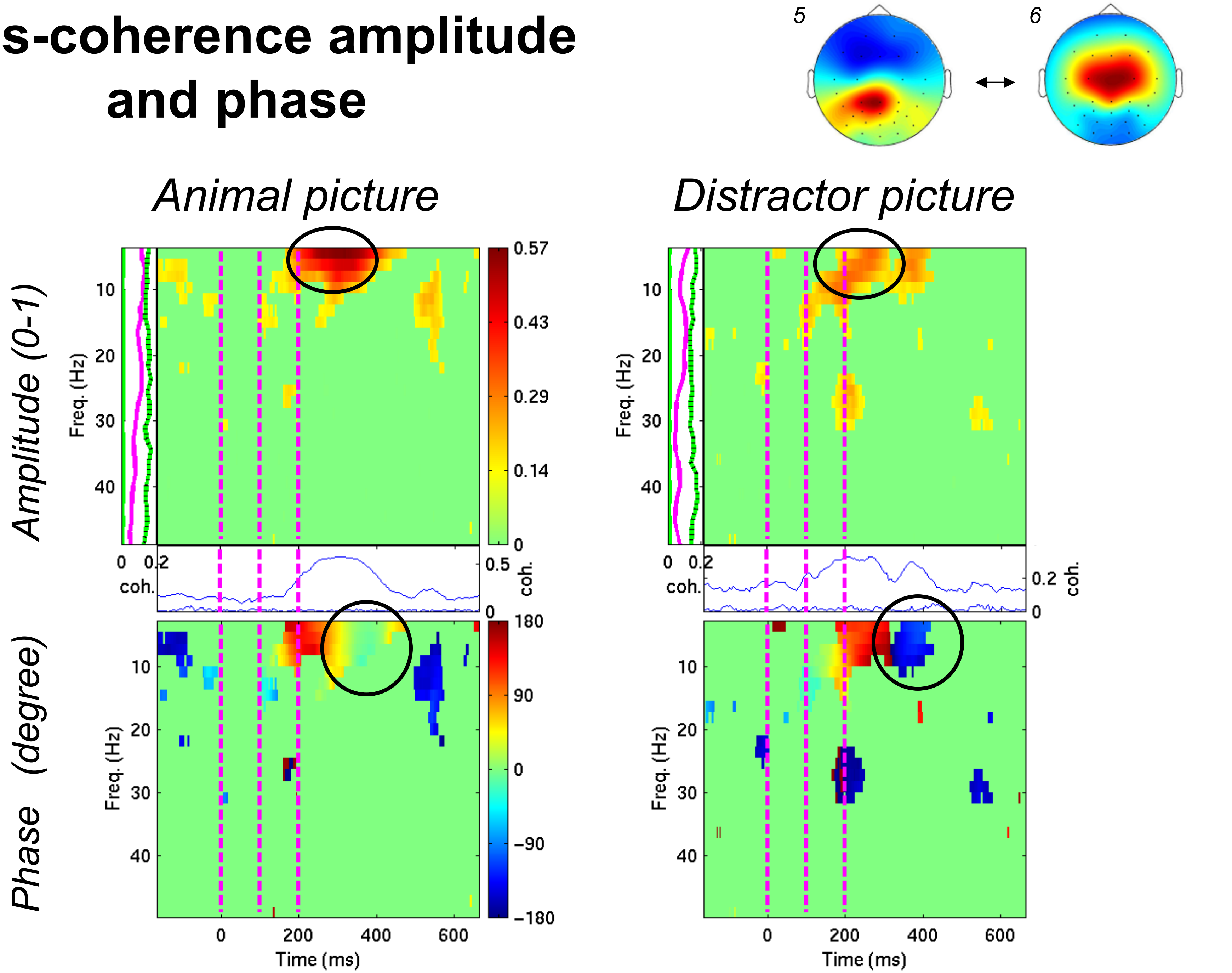
Covariance

$$corr^{a,b}(f,t) = \frac{\sum_{k=1}^{n} \left(\left| F_k^a(f,t) \right| - \overline{\left| F^a(f,t) \right|} \right) \left(\left| F_k^b(f,t) \right| - \overline{\left| F^b(f,t) \right|} \right)}{n \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left(\left| F_k^a(f,t) \right| - \overline{\left| F^a(f,t) \right|} \right)^2} \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left(\left| F_k^b(f,t) \right| - \overline{\left| F^b(f,t) \right|} \right)^2}}$$

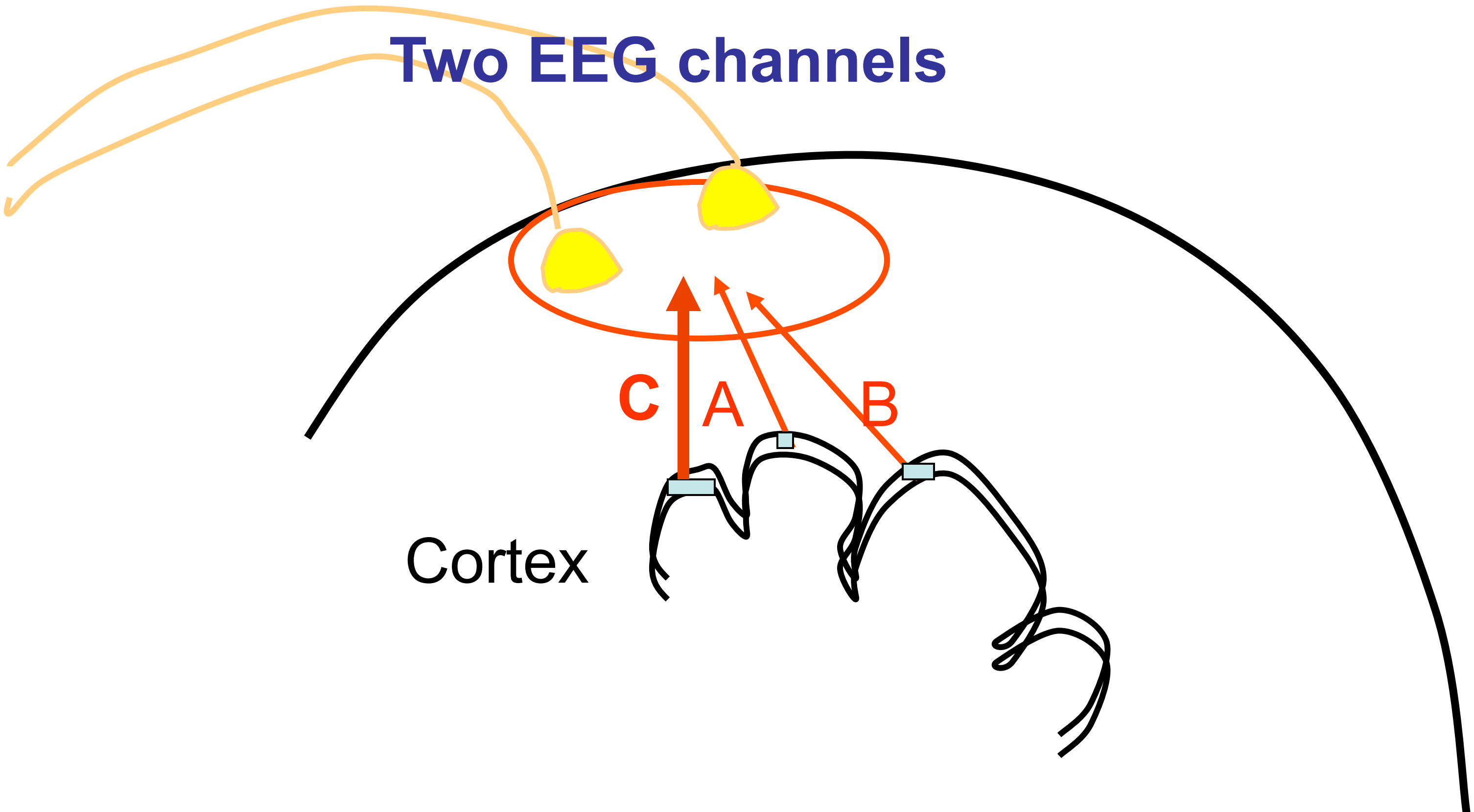
Standard deviation for a

Standard deviation for b

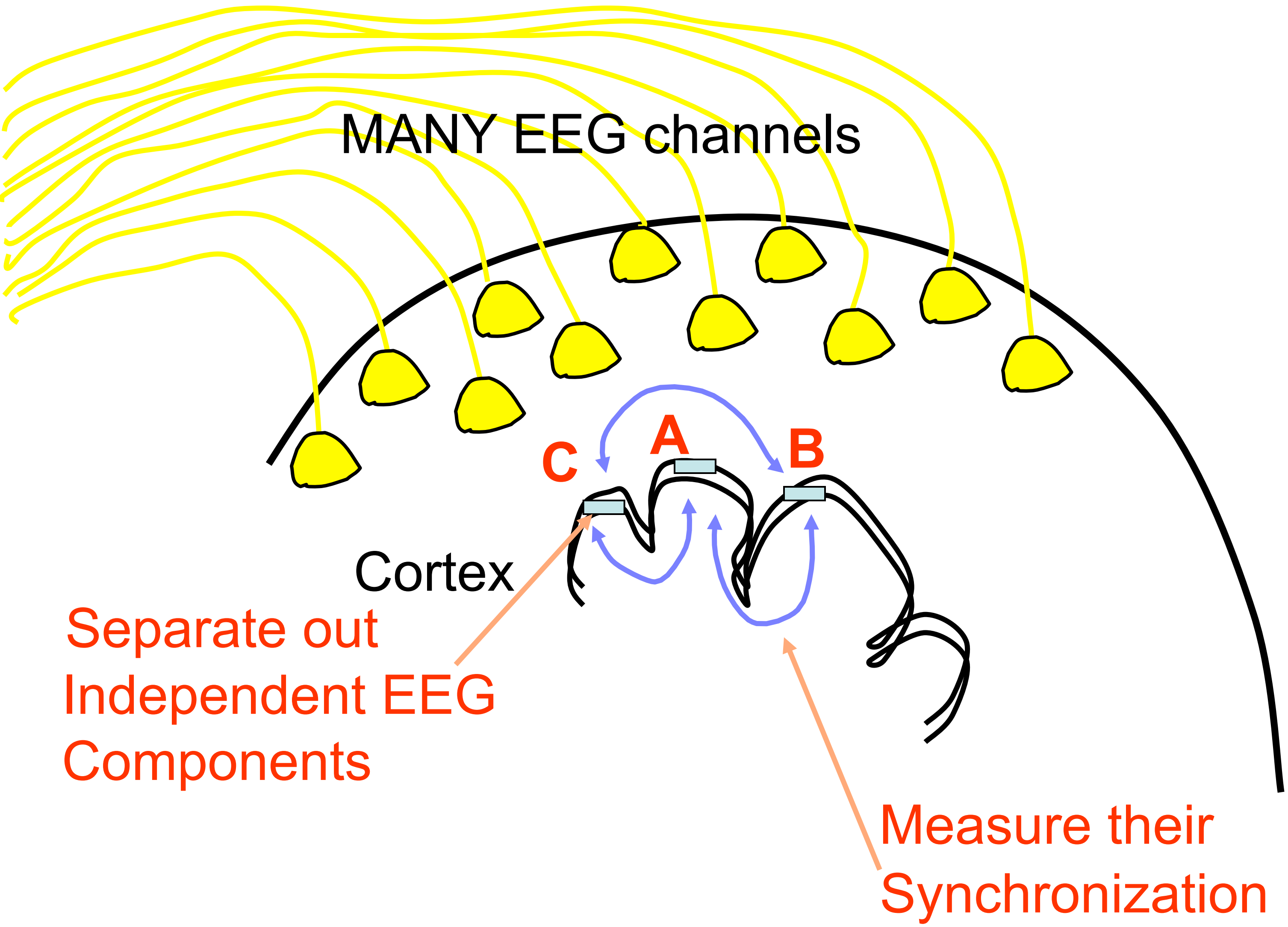
Cross-coherence amplitude and phase



Two EEG channels

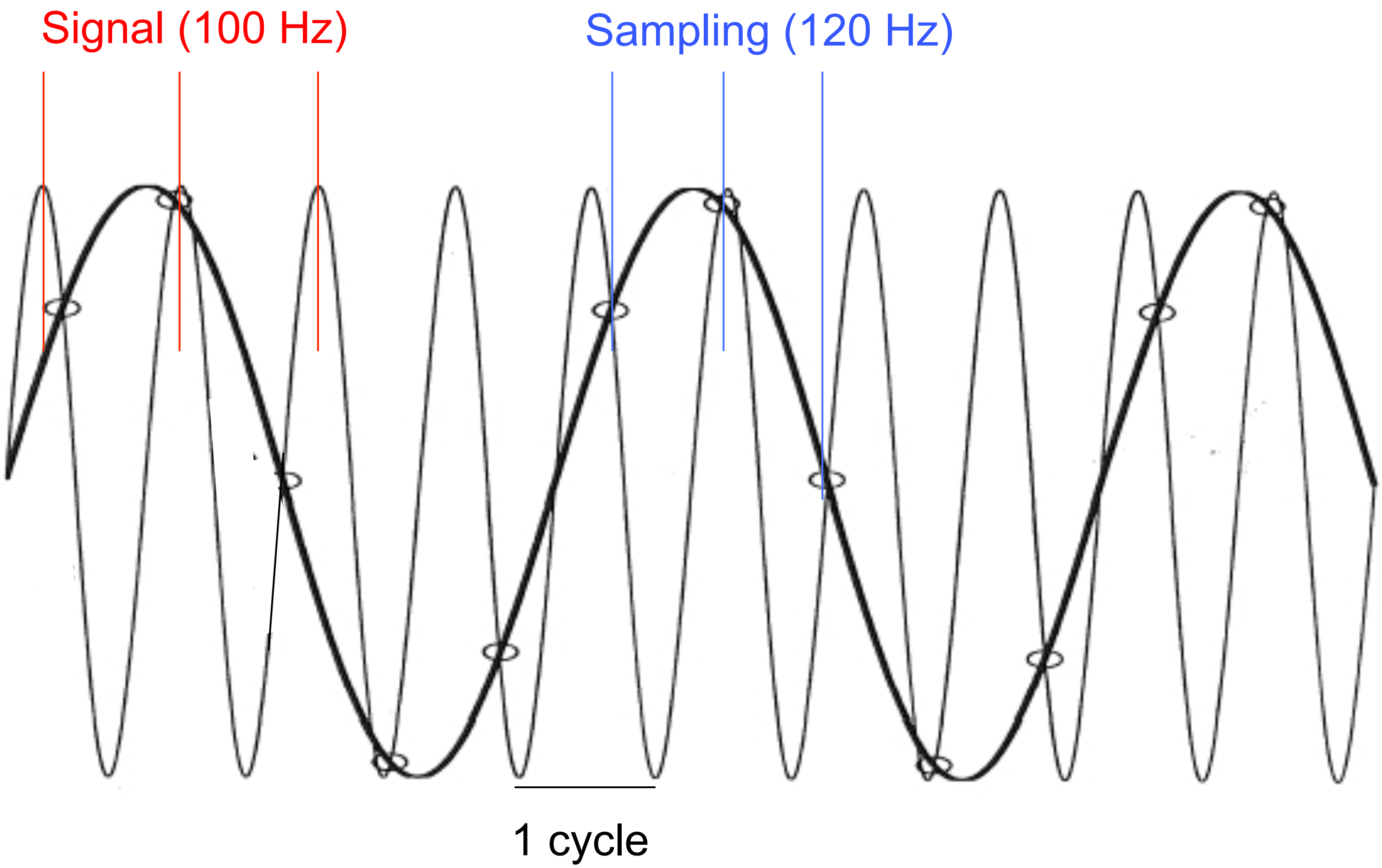


Scalp channel coherence → source confounds!



source dynamics!

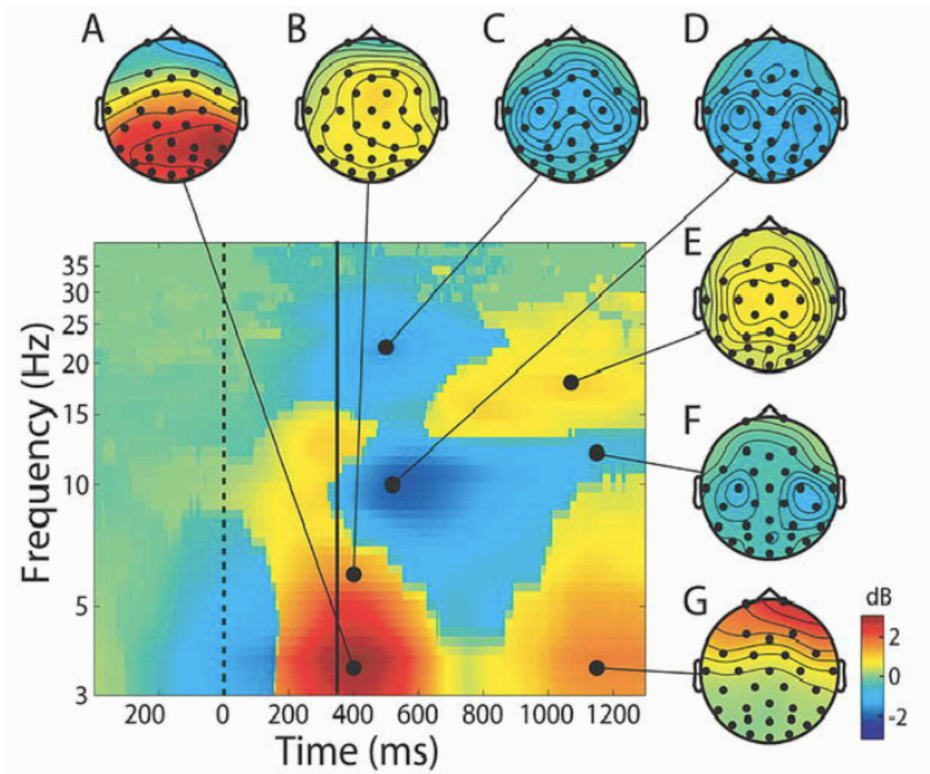
Niquist frequency: Aliasing



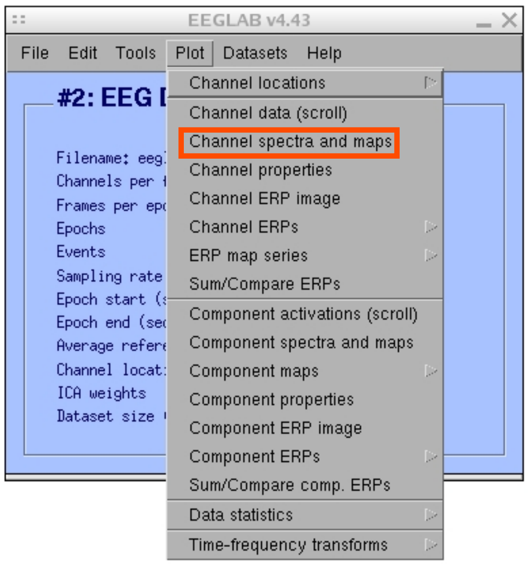
e.g. 100 Hz sampled at 120 Hz

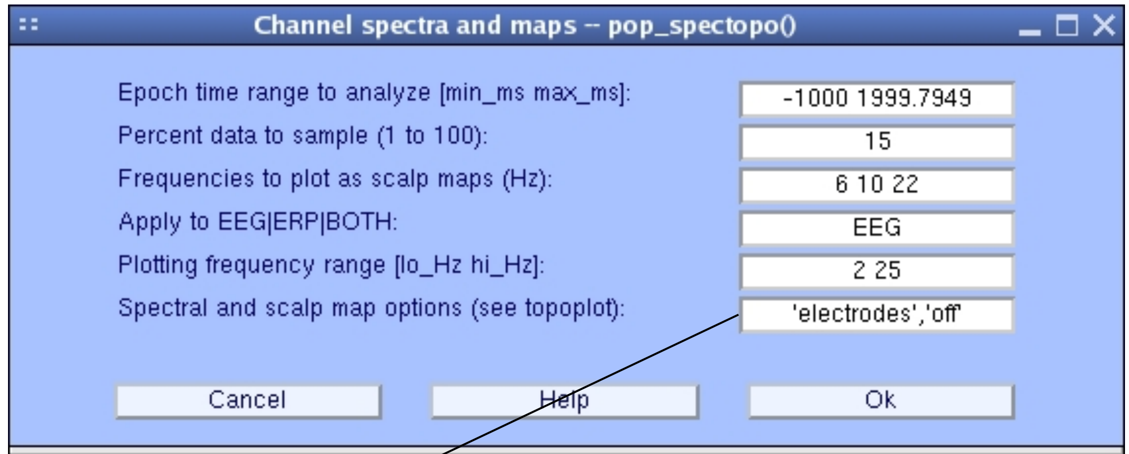
Advanced time-frequency functions

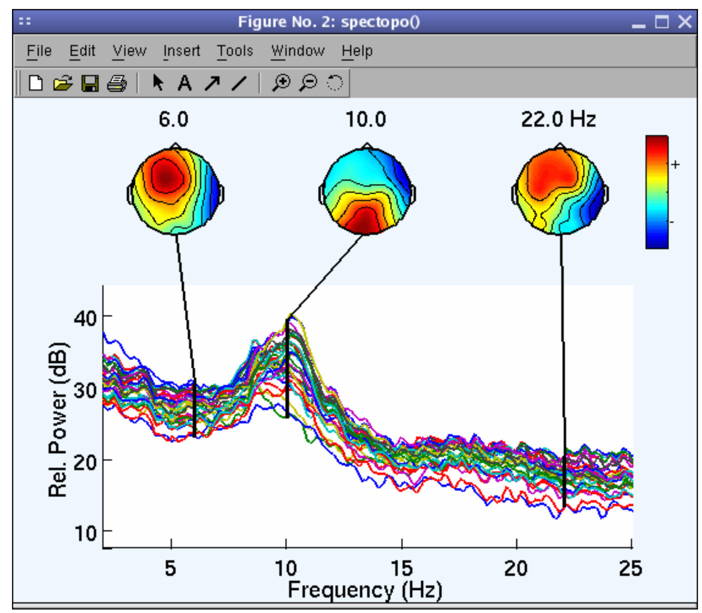
- Tftopo(): allow visualizing time-frequency power distribution over the scalp



Plot data spectrum using EEGLAB







'winsize', 256 (change FFT window length)
'nfft', 256 (change FFT padding)
'overlap', 128 (change window overlap)

Exercise

- **ALL**

  Start EEGLAB, from the menu load *sample_data/eeglab_data_epochs_ica.set* or your own data (epoch, reject noise if not done already)

- **Novice**

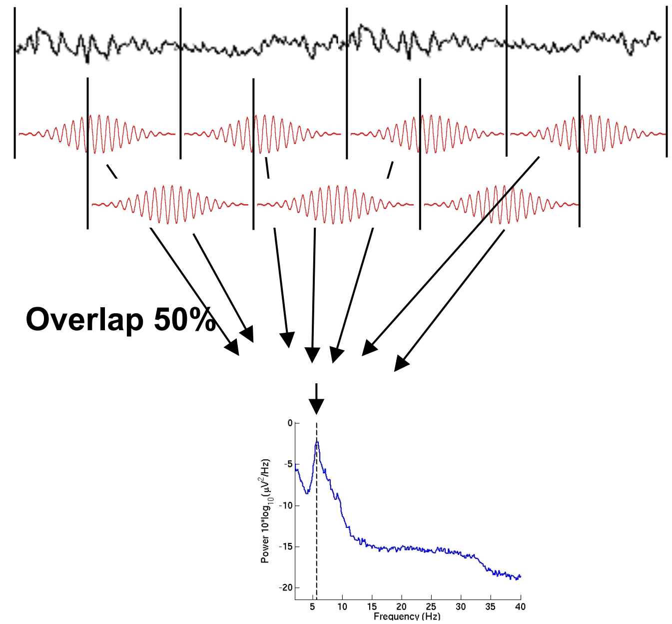
  From the GUI, Plot spectral decomposition with 100% data and 50% overlap (‘overlap’). Try reducing window length (‘winsize’) and FFT length (‘nfft’)

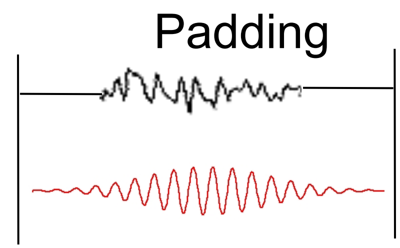
- **Intermediate**

  Same as novice but using a command line call to the *pop_spectopo()* function. Use GUI then history to see a standard call (“eegh”).

- **Advanced**

  Same as novice but using a command line call to the *spectopo()* function.





Exercise - newtimef

- **Novice**

  From the GUI, pick an interesting IC and plot component ERSP. Try changing parameters window size, number of wavelet cycles, padratio,

- **Intermediate**

  From the command line, use newtimef() to tailor your time/frequency output to your liking. Look up the help to try not to remove the baseline, change baseline length and plot in log scale. Enter custom frequencies and cycles (2 slides back).

- **Advanced**

  Compare FFT, the different wavelet methods (see help), and multi-taper methods (use timef function not newtimef). Enter custom frequencies and cycles. Look up newtimef help to compare conditions. Vizualise single-trial timef-frequency power using erpimage.