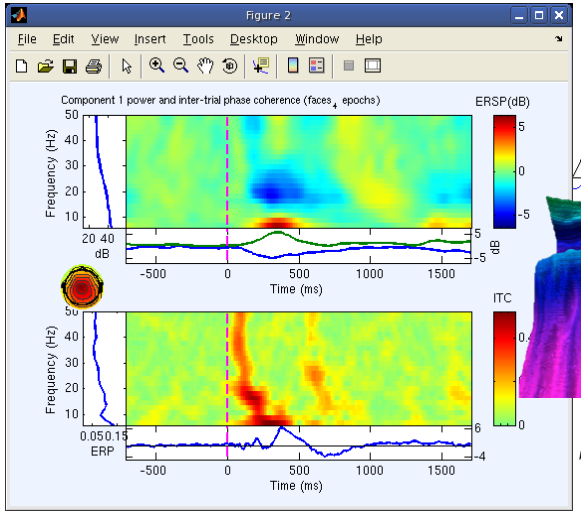
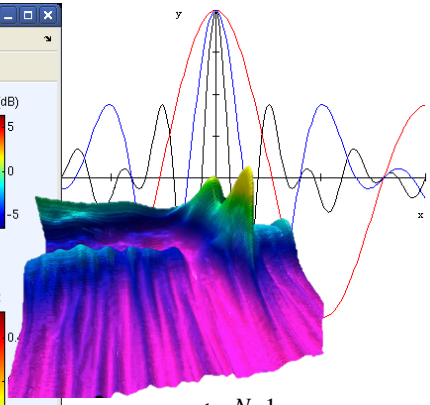
Time-Frequency Analysis of Biophysical Time series

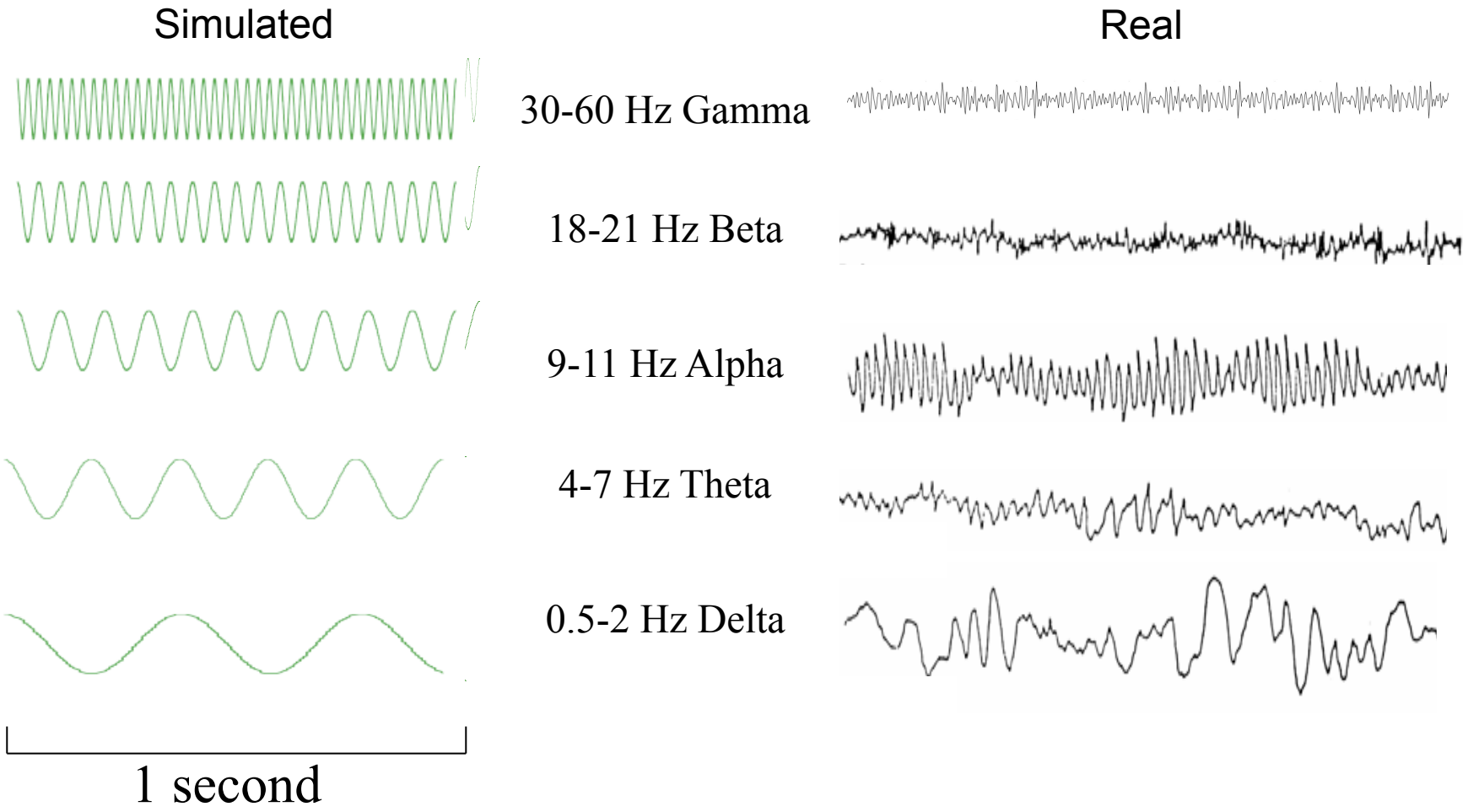
Tim Mullen
(with many contributions from Arnaud Delorme)



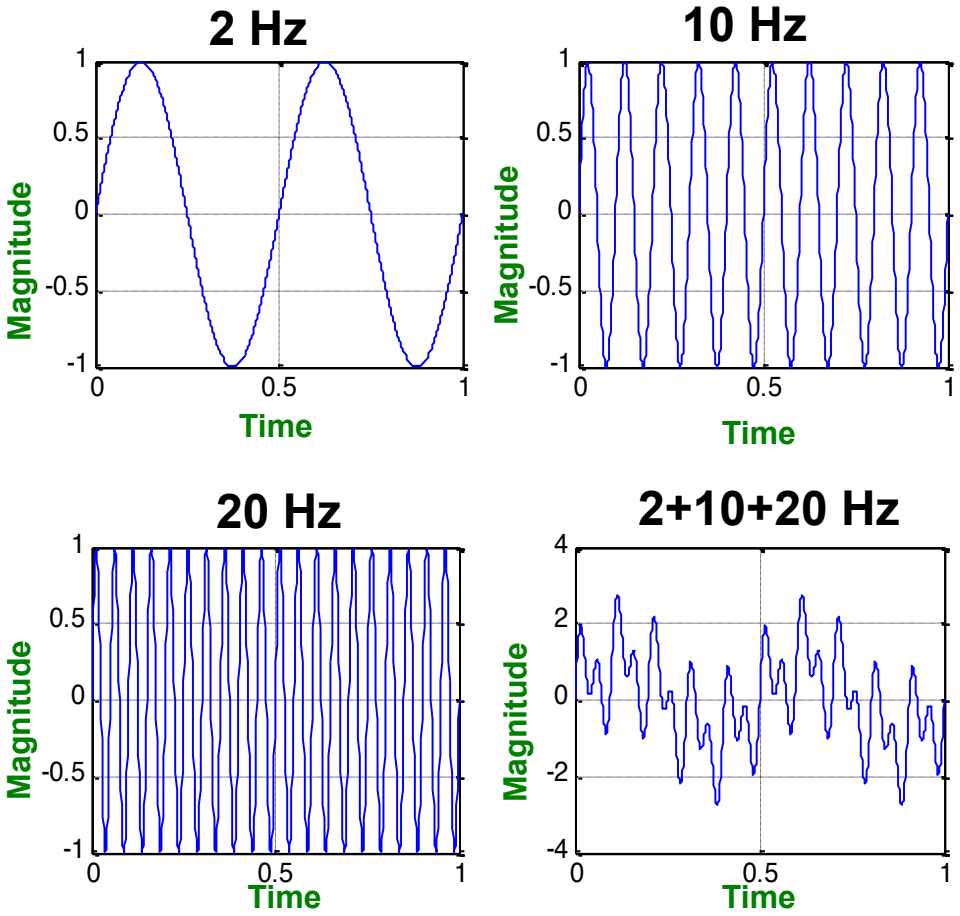


$$S(f) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-2\pi ift/N}$$

Common Oscillatory Modes in EEG



Wide-sense stationary signals

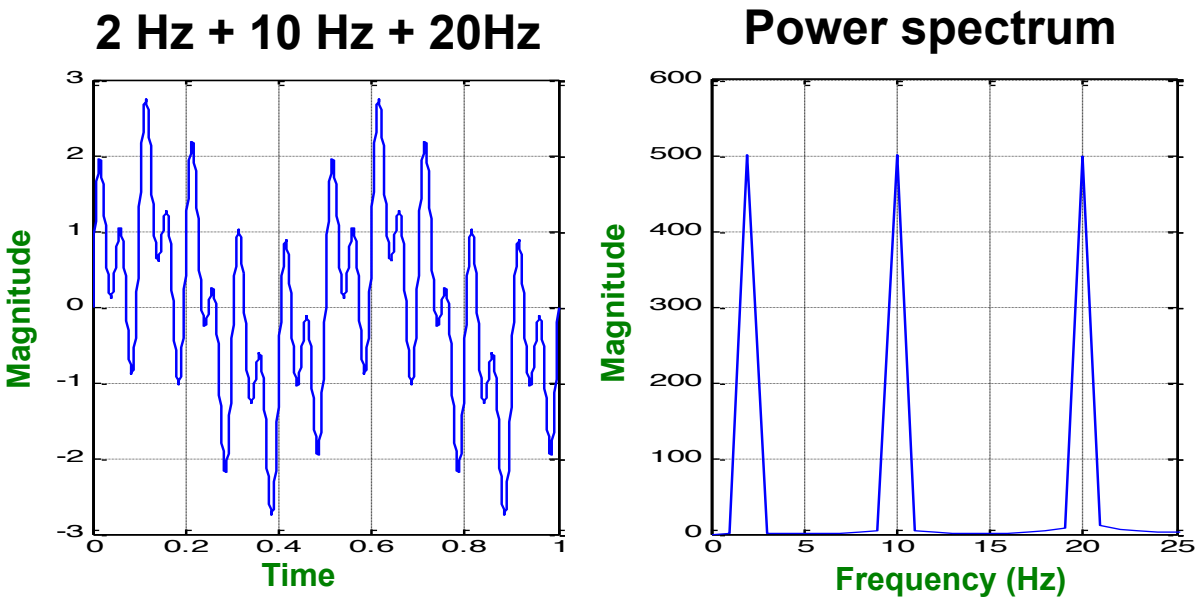


Wide-sense Stationary

Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Wide-sense stationary signals

Wide-sense Stationary



By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2, 10, 20Hz respectively).

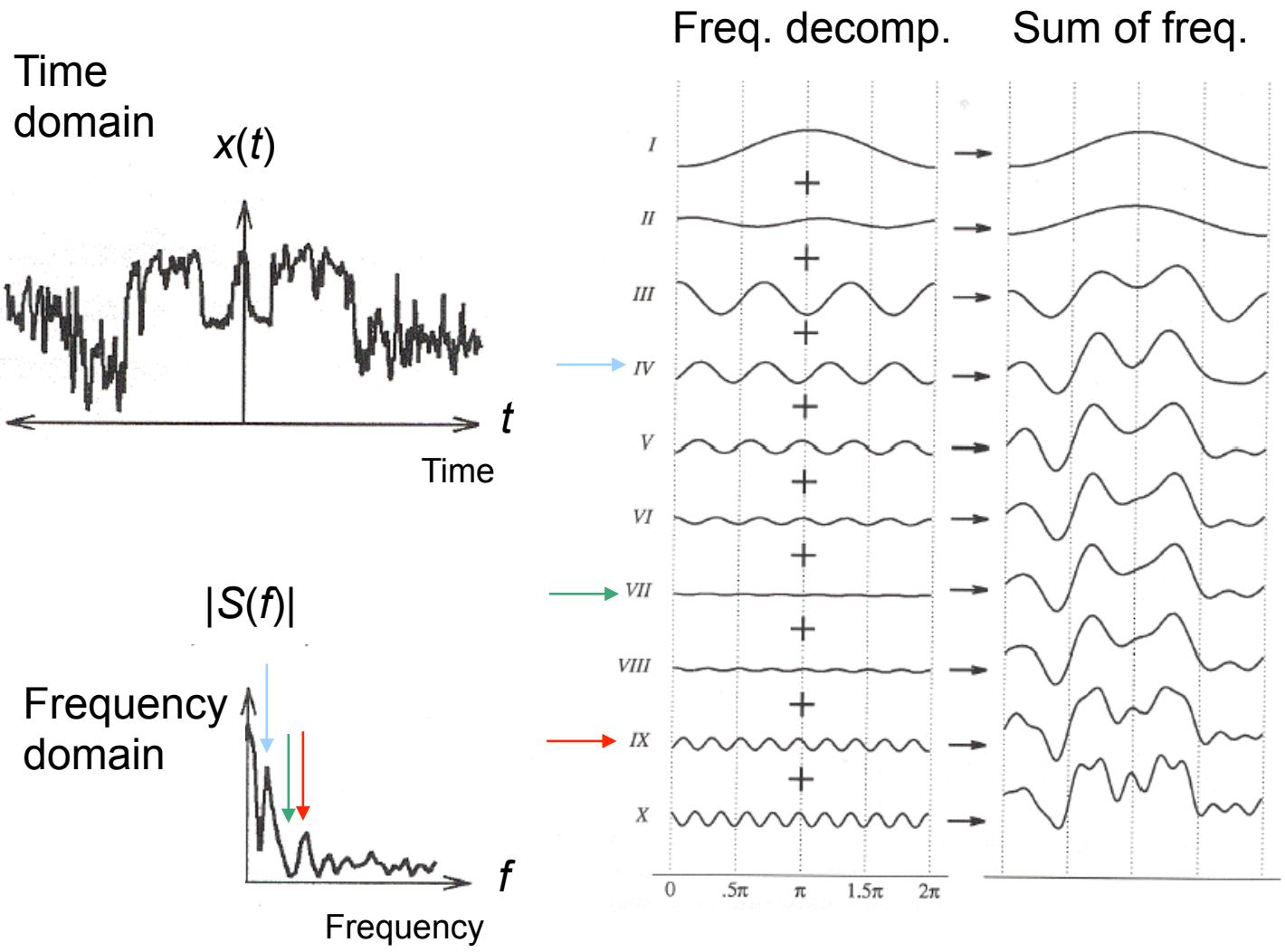
Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Fourier's Theorem

Any stationary, continuous process can be exactly described by an infinite sum of sinusoids of different amplitudes and phases.

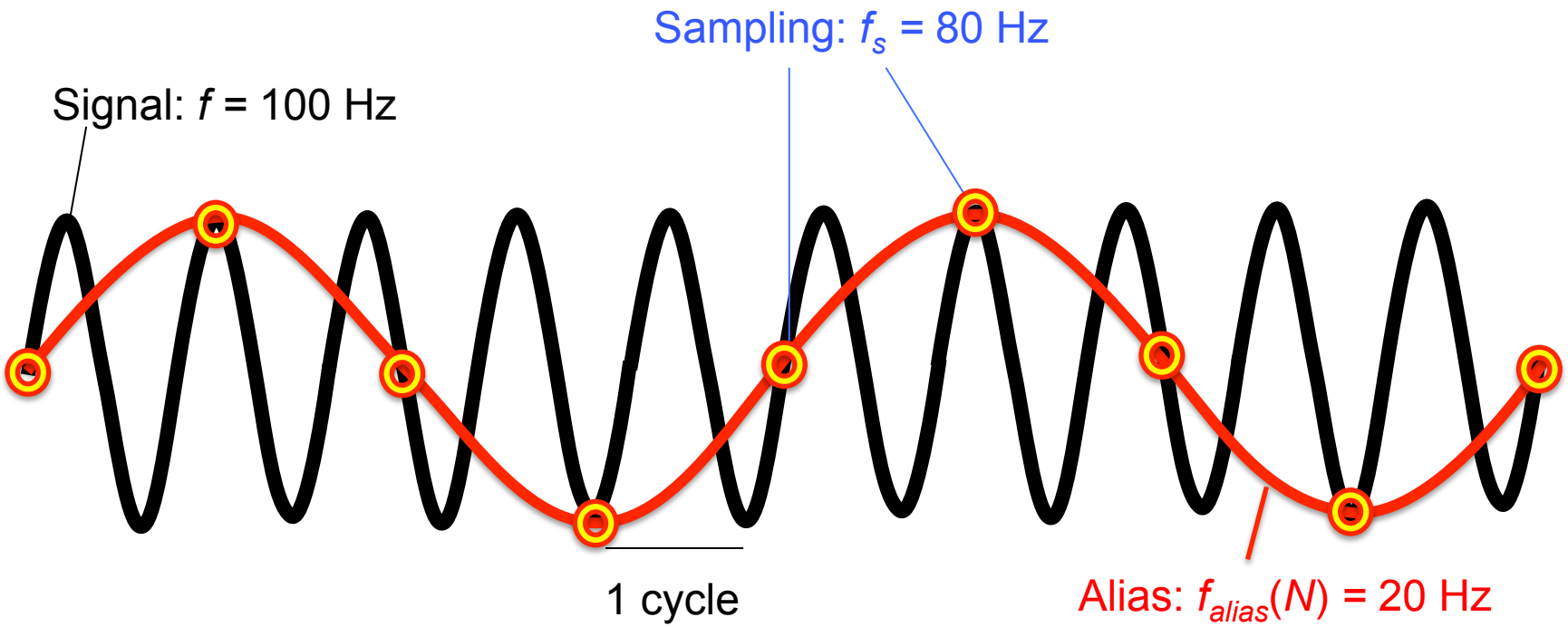
Jean Baptiste Joseph Fourier
(1768 –1830)

Fourier's Theorem



Figure, courtesy of Ravi Ramamoorthi & Wolberg

Aliasing and the Nyquist Frequency



Nyquist Frequency:

The maximum frequency that can be uniquely recovered at a sampling rate of f_s

$$f_N = f_s / 2$$

$$f_{alias}(N) = \left| f + N f_s \right|$$

$$f_s = \text{sampling rate}$$

Quiz: What is N in the example above?

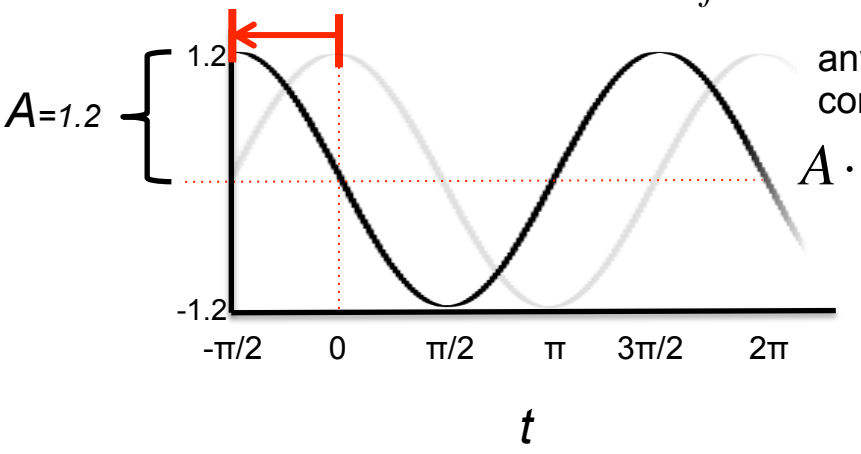
Euler's Formula

phase shift

$\theta = \pi / 2$

angular frequency

$\omega = 2\pi f = 2\pi$ rad/sec



any sinusoid can be expressed as the sum of two complex numbers...

instantaneous complex amplitude and phase

$$A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t+\theta)} + \frac{A}{2} e^{-i(\omega t+\theta)}$$

$$= \mathrm{Re}\{A e^{i(\omega t+\theta)}\} = \mathrm{Re}\{S(\omega, t)\}$$

Another version:

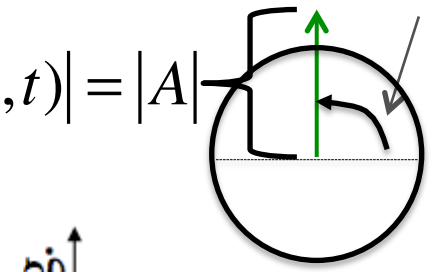
$$e^{i(\omega t+\theta)} = \cos(\omega t + \theta) + i \sin(\omega t + \theta)$$

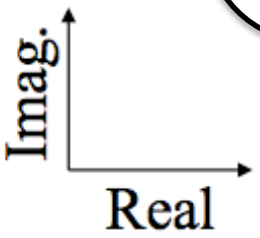
Real part
Cosine component

Imaginary part
Sine component

$$|S(\omega, t)| = |A|$$

$$\theta = \angle S(\omega, t) = \pi / 2$$

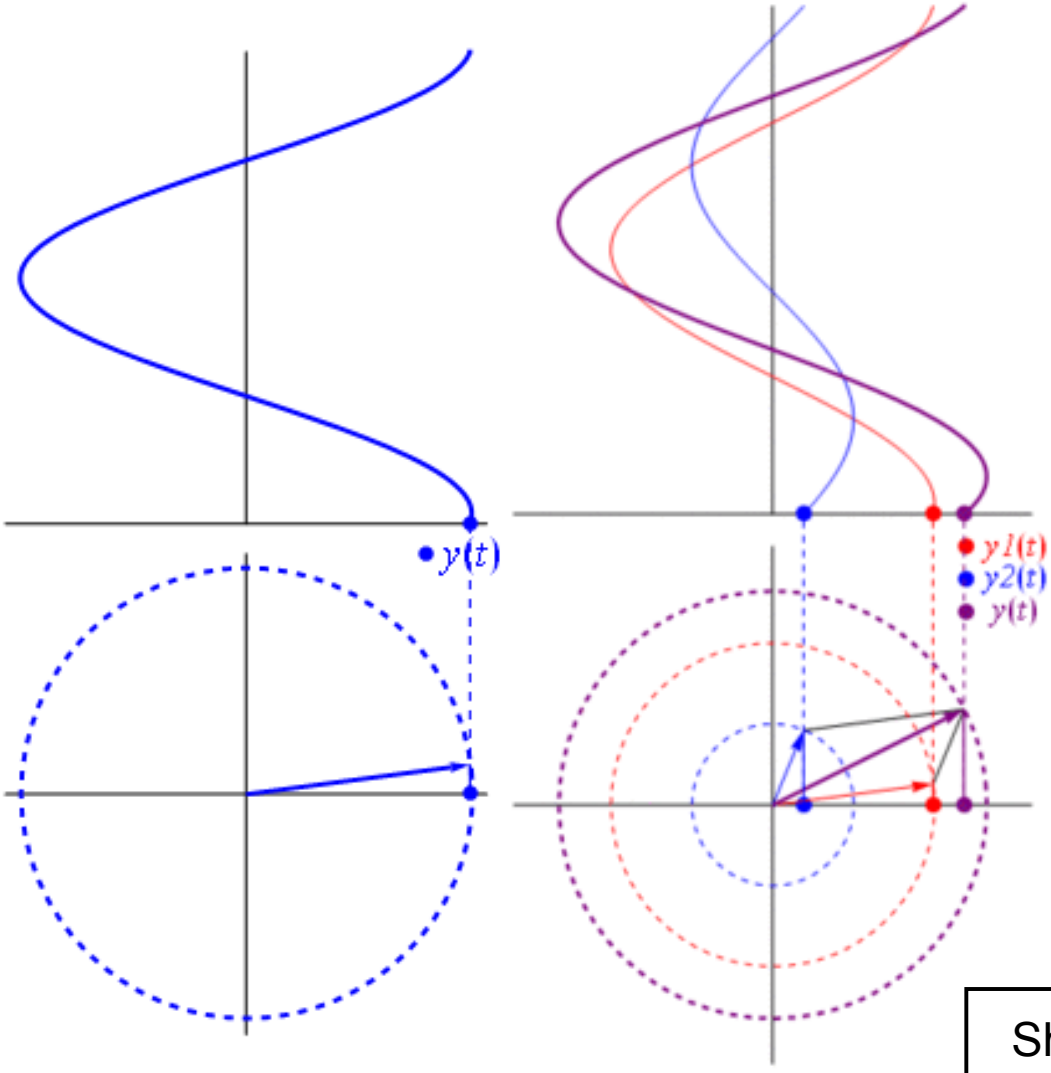




Phasor

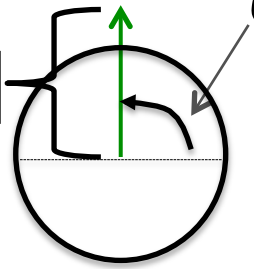
Phasors

Rotation velocity (Rad/S; Hz)
= (angular) frequency (*w*; *f*)

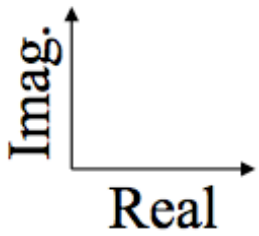


$$A \cdot \cos(\omega t + \theta) = \mathrm{Re}\{Ae^{i(\omega t+\theta)}\}$$
$$= \mathrm{Re}\{S(\omega, t)\}$$

$$|S(\omega, t)| = |A|$$

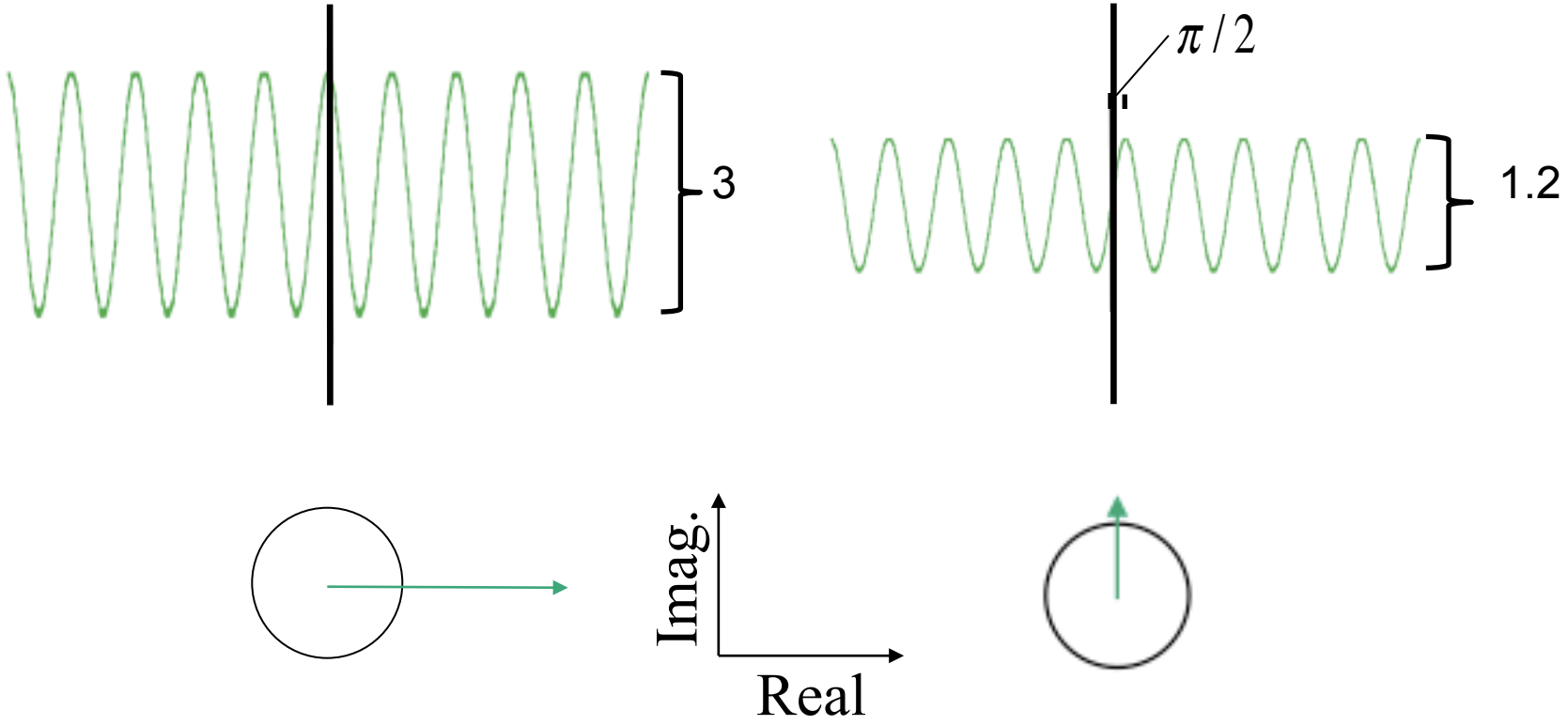


Shorthand
phasor notation: $Ae^{i\phi}$

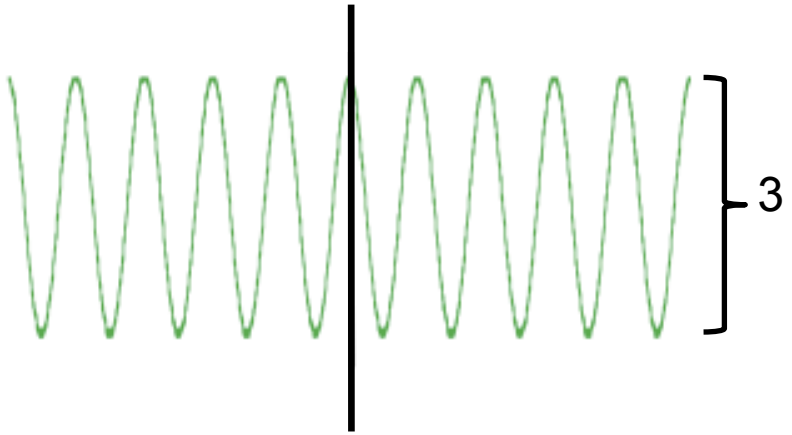


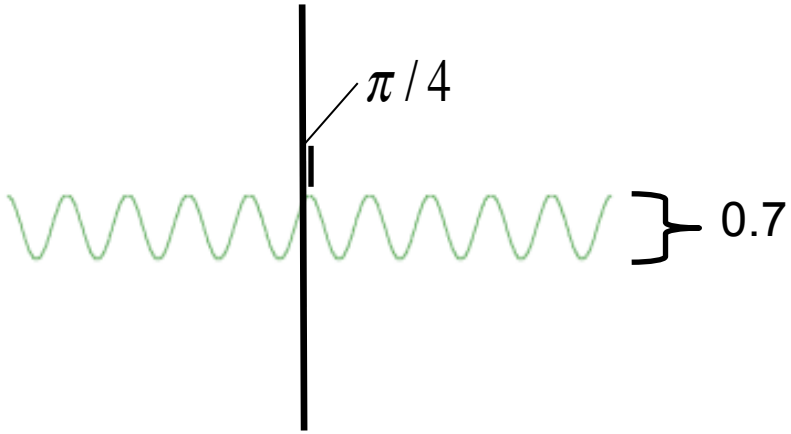
Polar animations courtesy Wikipedia

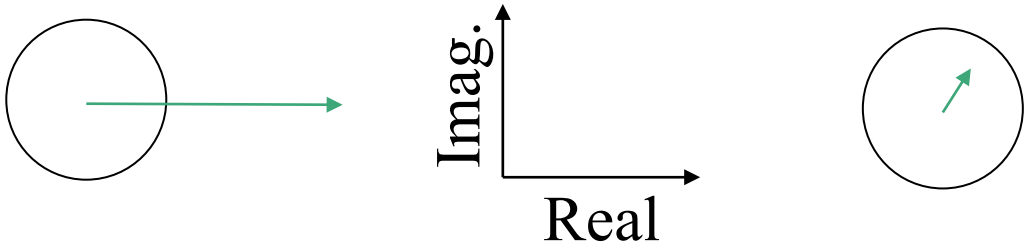
Phasors: Example



Phasors: Example







Discrete Fourier Transform

Time → Frequency

Forward transform

$$S(f) = \frac{1}{N} \sum_{t=0}^{N-1} x(t) e^{-2\pi i f t / N}$$

Frequency → Time

Inverse transform

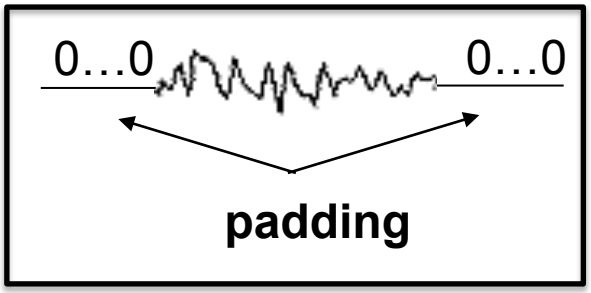
$$x(t) = \frac{1}{N} \sum_{f=0}^{N-1} S(f) e^{2\pi i f t / N}$$

N = number of samples

Fast Fourier Transform (FFT)

Zero-padding

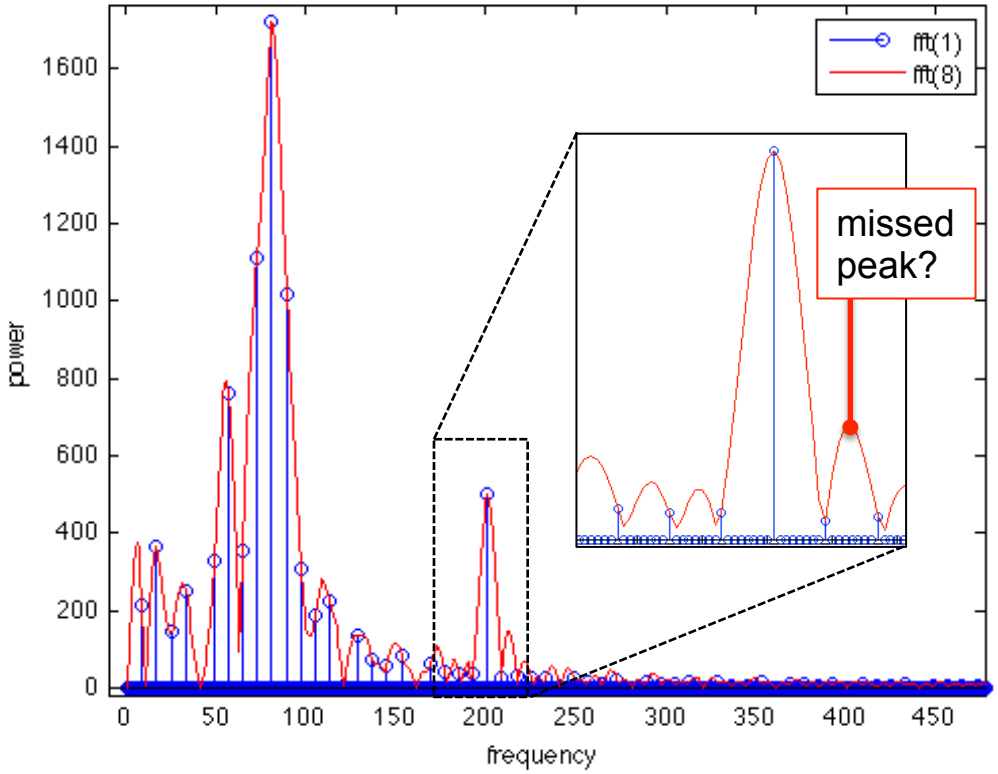
The DFT/FFT of a sequence of length N produces power estimates at N frequencies evenly distributed between 0 and the sampling rate (Fs) (or ⌊N/2+1⌋ frequencies between 0 and the Nyquist rate, Fn=Fs/2).



Padding the signal with Q zeros achieves the following:

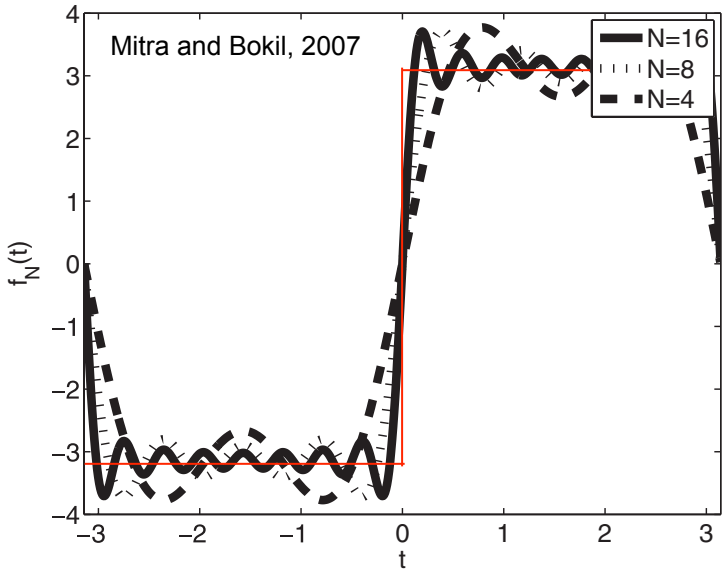
1) Allows enforcement of signal length as a power of two enabling FFT
2) Increases the number of frequency bins between 0 and Fs from N to N+Q (intermediate points are sinc interpolates)

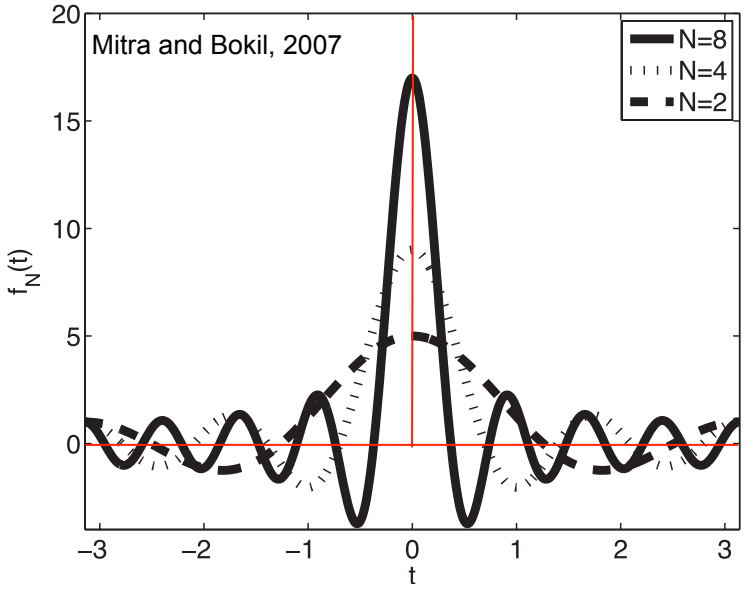
Zero-padding does not increase frequency resolution (number of independent degrees of freedom)



Tapering

Fourier's Theorem lets us exactly describe any continuous, stationary signal using an infinite weighted sum of sinusoids. Discontinuous functions can be infinitely *approximated*, but the approximation may be poor for finite sampling.





Gibbs Phenomenon
"Rippling" effect due to discontinuities in signal (e.g. edges of the truncated signal)

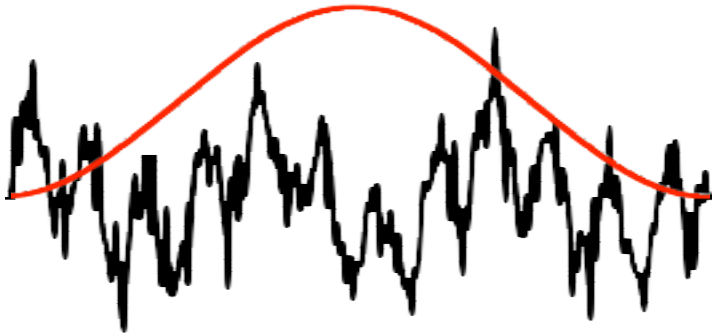
- Infinite number of frequencies required to approximate discontinuities
- This means infinite (or very large) number of samples required (not possible)

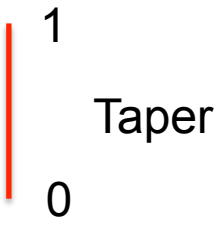
What can we do?

Tapering

Smoothly decay signal to zero at endpoints to smooth discontinuity

EEG





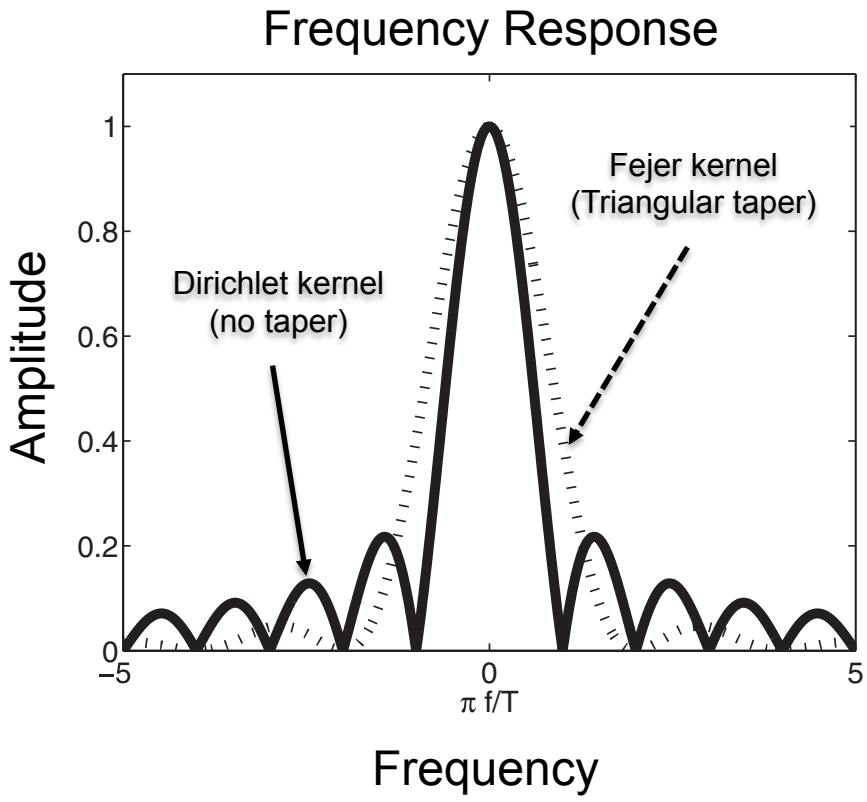
X

Tapered EEG

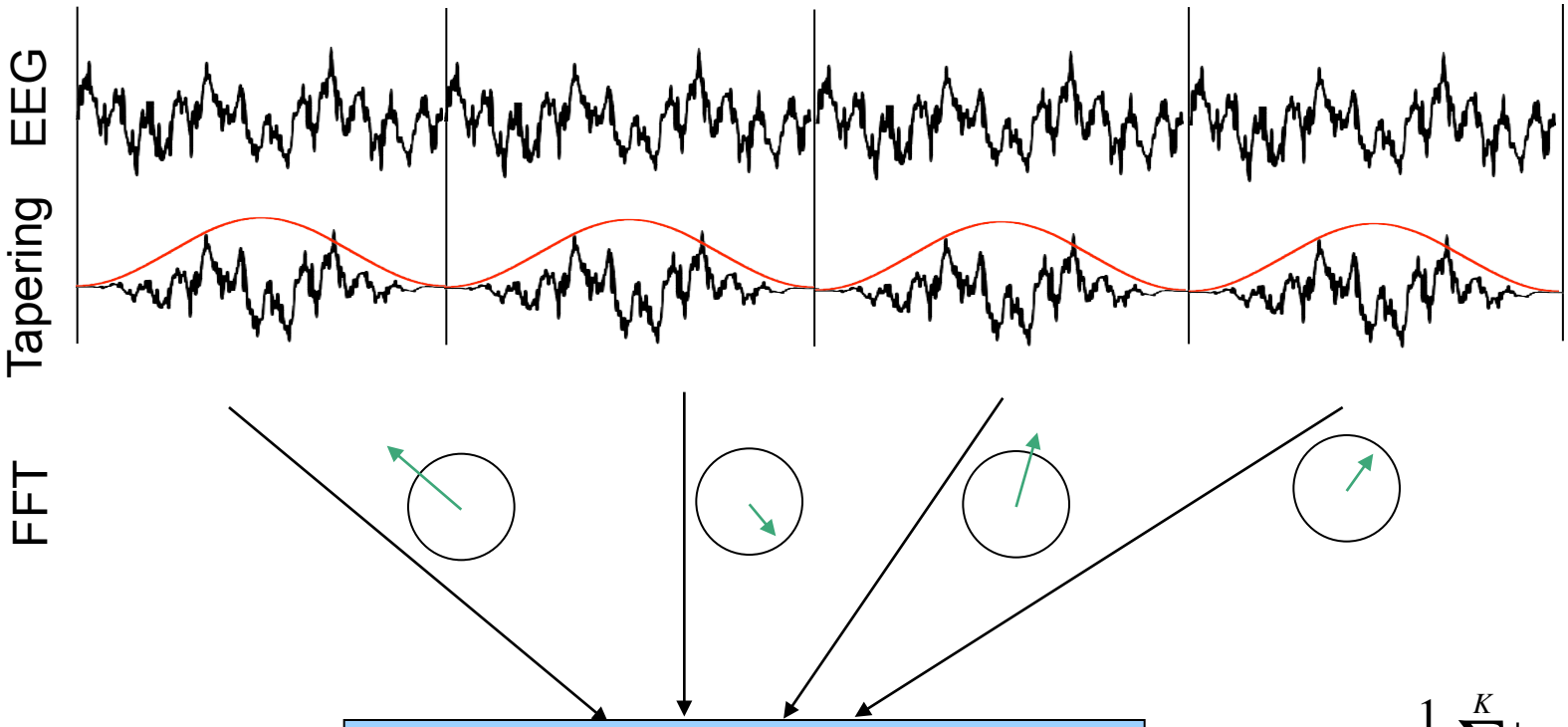
Tapering

Tapering reduces the effect of the Gibbs phenomenon making it easier to identify "true" peaks in the spectrum from spurious ripple peaks (minimized broadband bias or "spectral leakage")

The cost is increased width of central peak (narrowband bias).

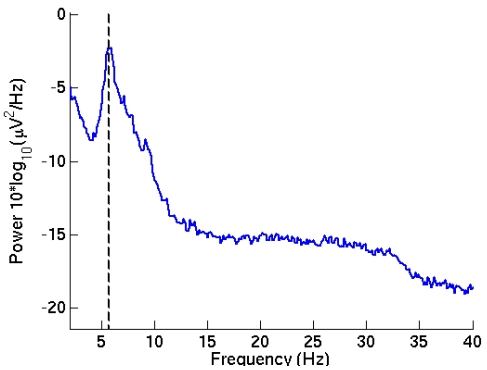


Spectral Estimation via Welch's Method



Average of squared absolute values

$$S_{Welch}(f) = \frac{1}{K}\sum_{k=1}^{K}\left|S_k(f)\right|$$

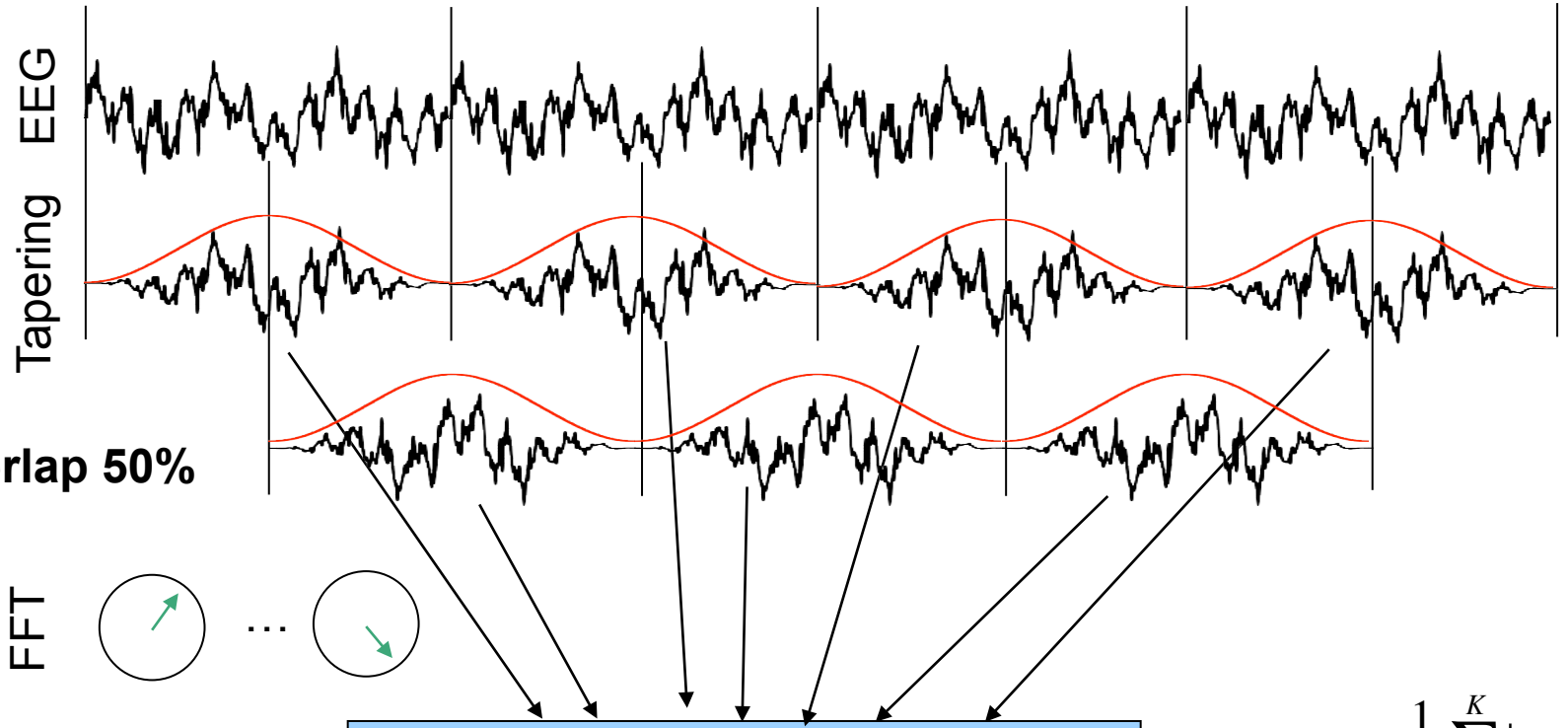


Given *K* windows:

- Variance (bias) is reduced by a factor of *K*
- Frequency resolution also reduced by a factor of *K*

- Tapering also results in data loss → decreased frequency resolution (narrowband bias)
- Can we mitigate data loss?

Spectral Estimation via Welch's Method



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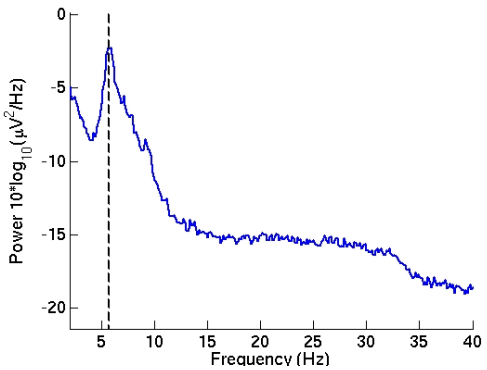
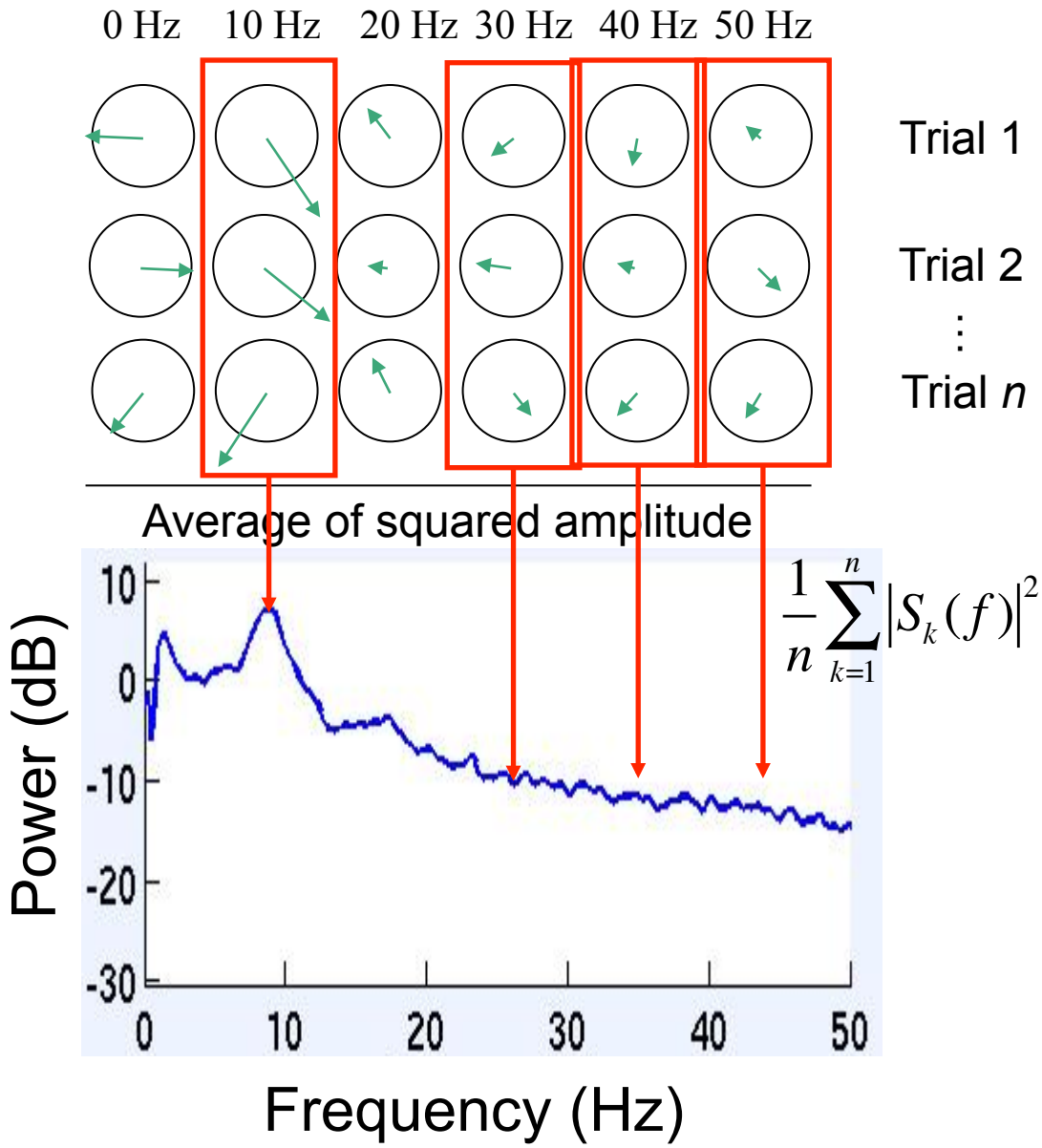


Overlap-add method results in reduced data loss due to tapering, while preserving spectral variance reduction ☺

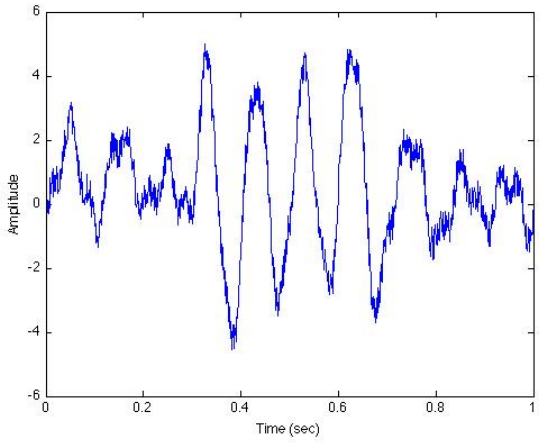
Trial Averaging

Averaging spectra over *n* independent trials leads to further reduction of variance by a factor of *n*

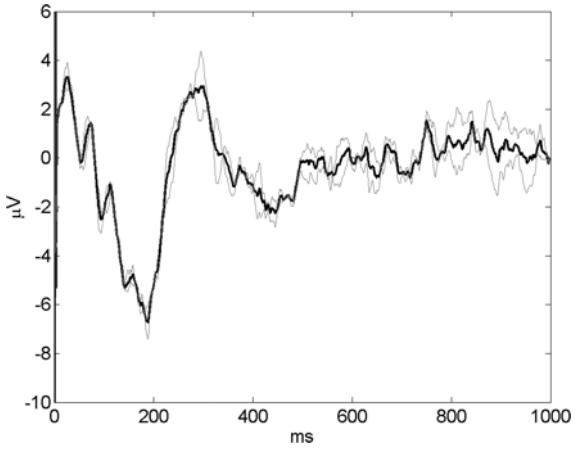


Non-Stationary Signals

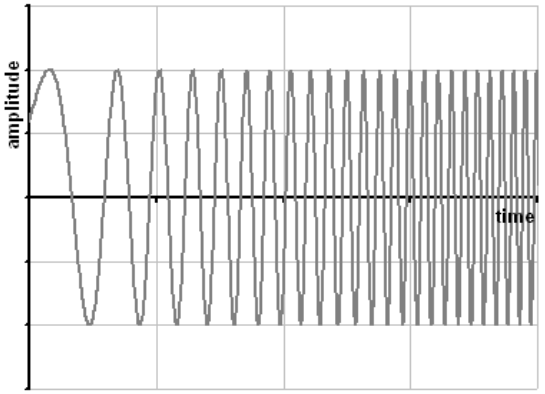
Non-stationary signals include bursts, chirps, evoked potentials, …



Burst

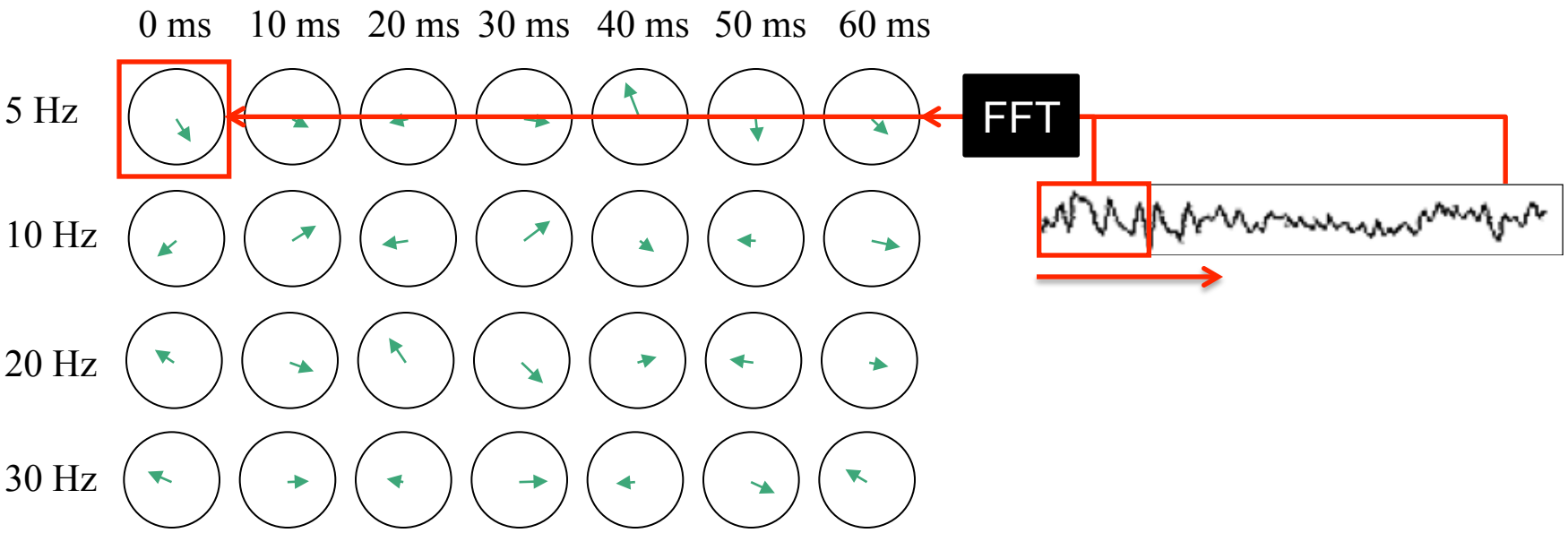


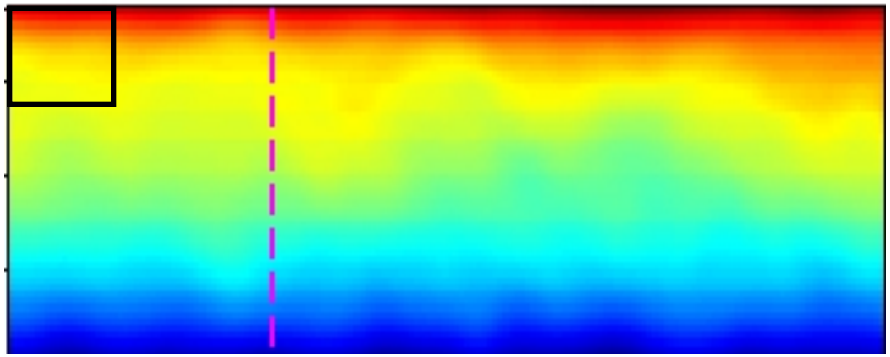
Evoked potential



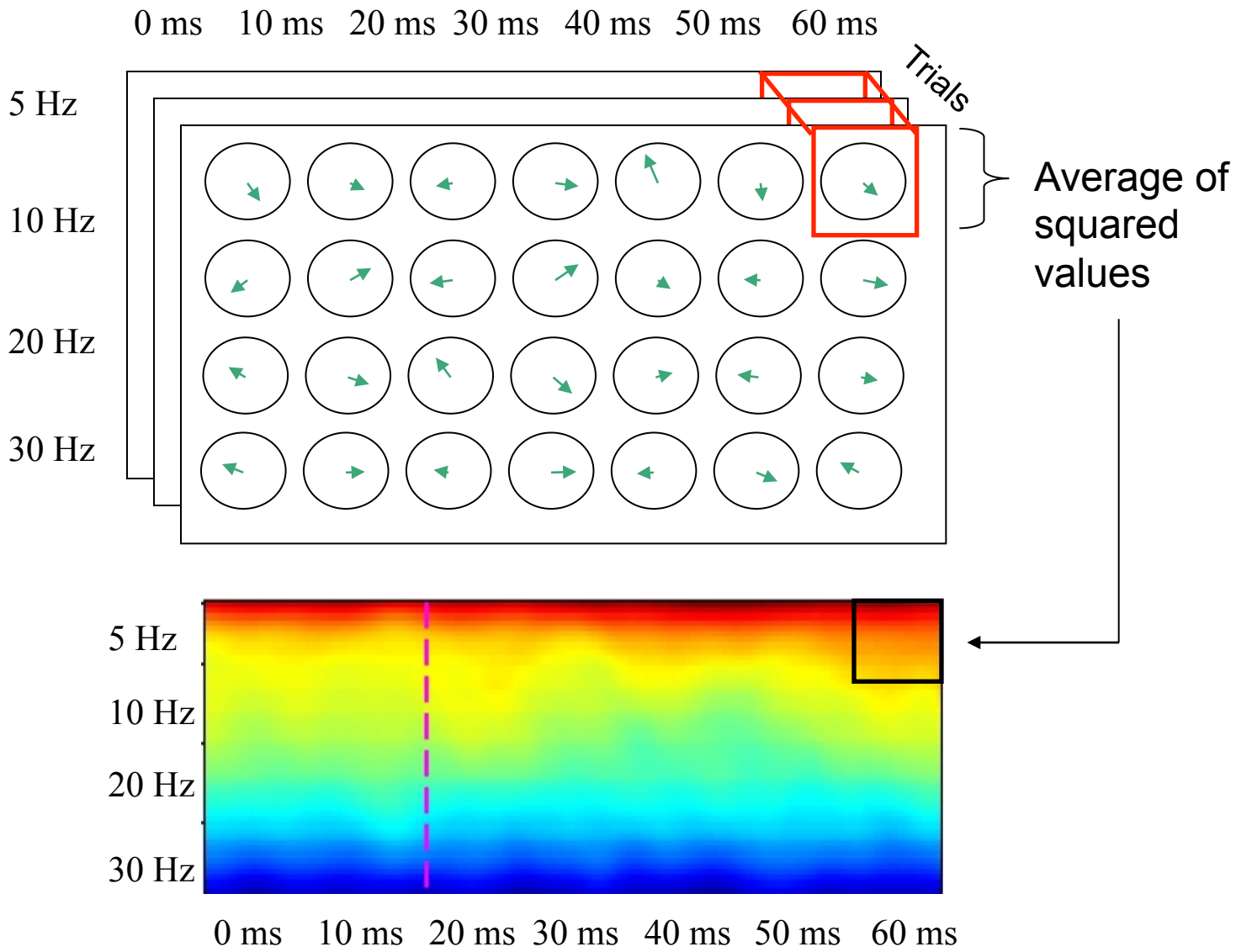
Chirp

Spectrogram or ERSP





Spectrogram or ERSP



Power spectrum and event-related spectral (perturbation)

$$ERS(f,t) = \frac{1}{n}\sum_{k=1}^{n}\left|S_k(f,t)\right|^2$$

Ensemble average

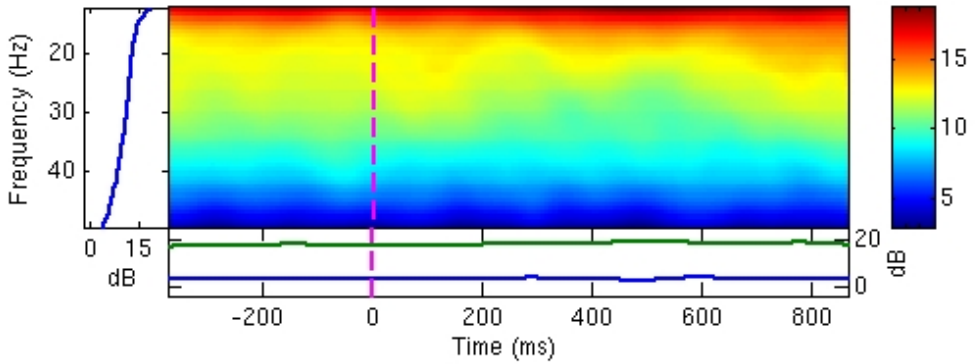
Complex number

Scaled to dB 10Log_{10}

Here, there are *n* trials
Each trial is time-locked to the same *event* (hence "event-related" spectrum)
The ERS is the average power across event-locked trials

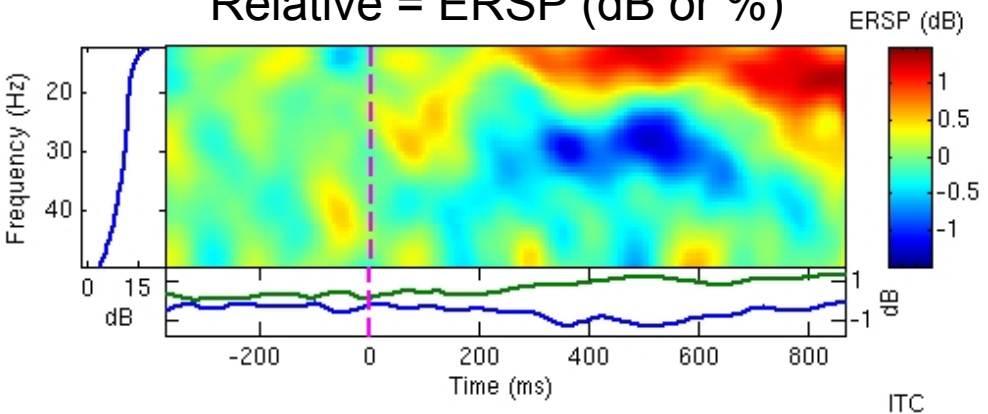
Absolute versus relative power

Absolute = ERS



To compute the ERSP, we just subtract the pre-stimulus ERS from the whole trial

Relative = ERSP (dB or %)



The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/ position and in frequency/ momentum.

There exists a lower bound to the *Heisenberg product*:

$$\Delta t \, \Delta f \geq 1/(4\pi)$$

or $$\Delta f \geq 1/(4\pi\Delta t)$$

Werner Karl Heisenberg
(1901 – 1976)

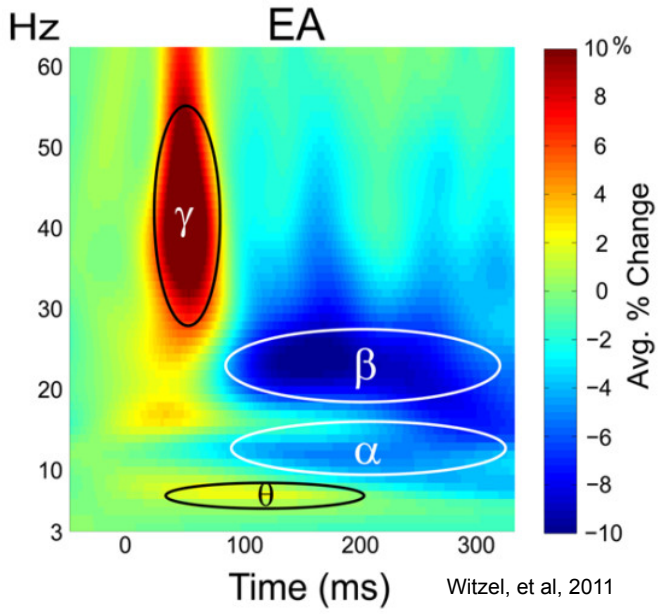
e.g. here are two possible (Δf, Δt) pairs:

Δf = 1Hz, Δt = 80 msec or
Δf = 2Hz, Δt = 40 msec

Note: Δf means "difference between successive frequencies" or the inverse of the frequency resolution. Ditto for Δt.

Time-Frequency Tradeoff

Natural biophysical processes may exhibit sustained changes in narrowband low-frequency oscillations along with rapidly-changing (e.g. “burst”) high-frequency oscillations.



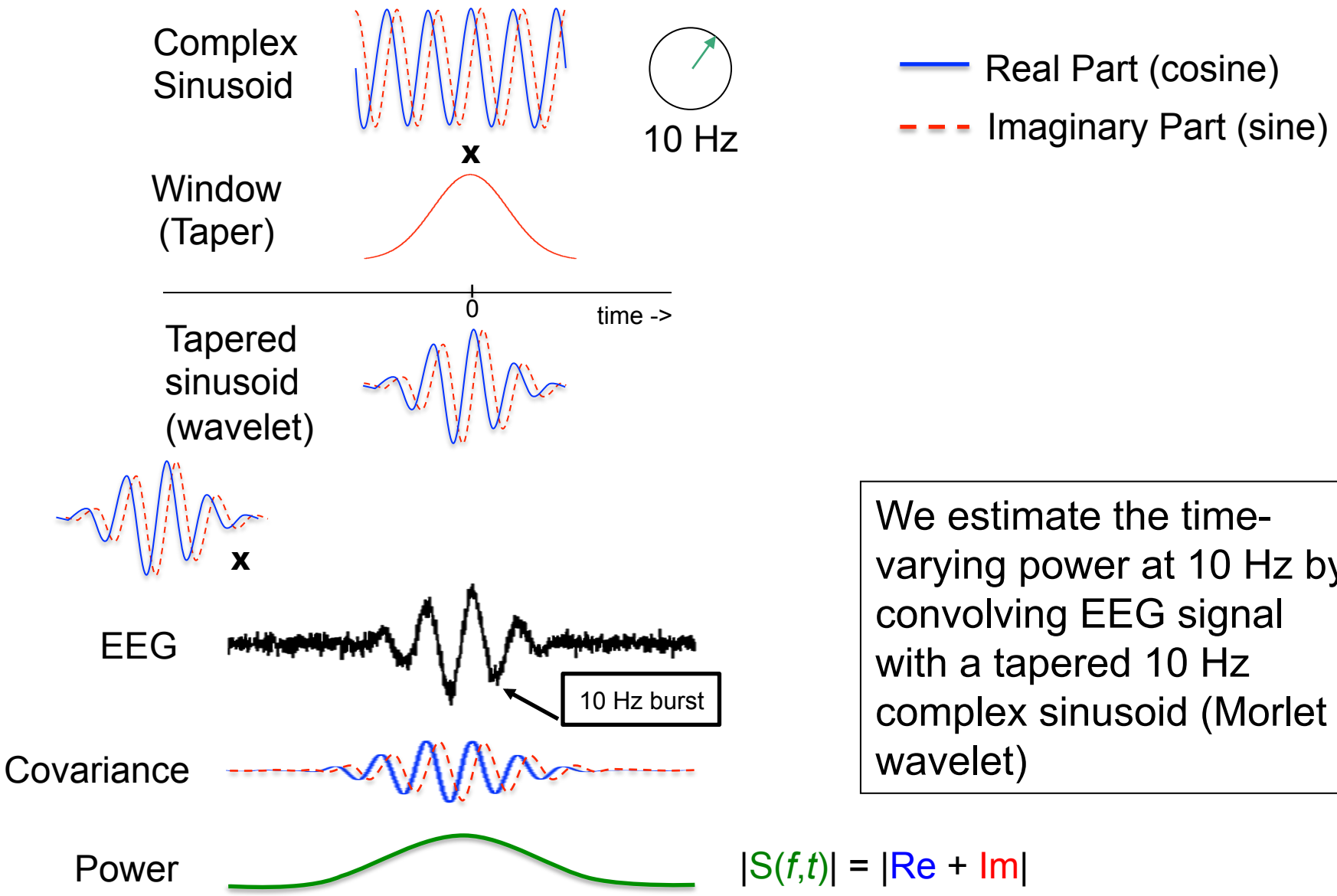
Witzel, et al, 2011

The Short-Time Fourier Transform has a constant temporal resolution for all frequencies.

Can we adapt the time-frequency resolution tradeoff for individual frequencies to improve spectral estimation?

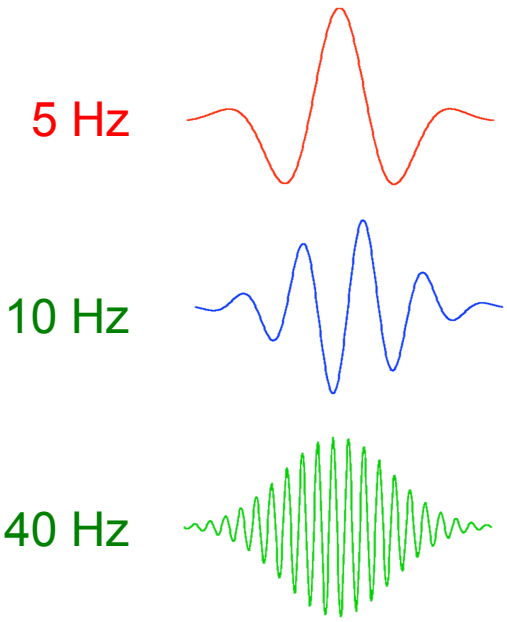
Yes, we can!

Wavelet Analysis

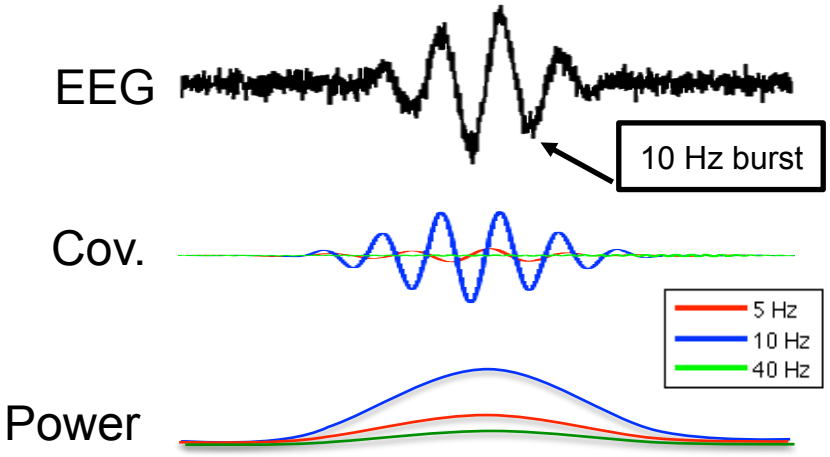


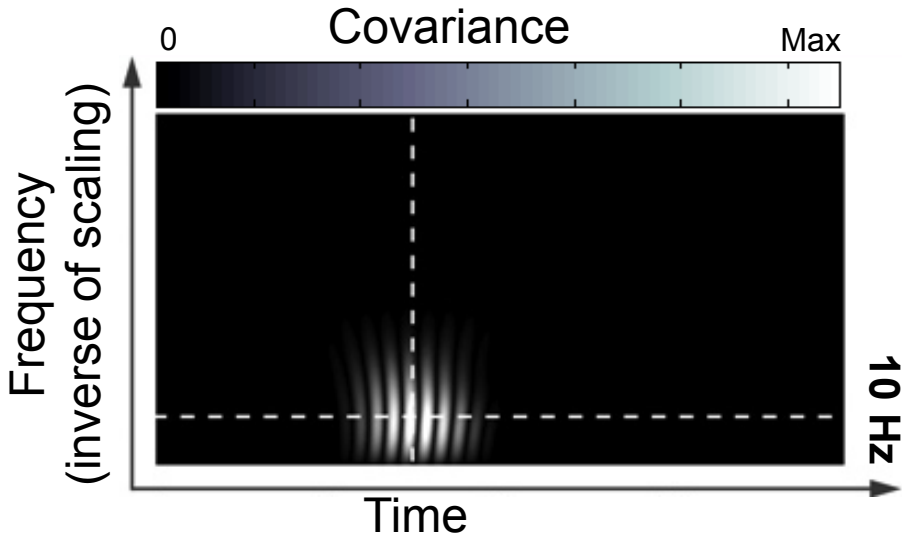
We estimate the time-varying power at 10 Hz by convolving EEG signal with a tapered 10 Hz complex sinusoid (Morlet wavelet)

Wavelet Time-Frequency Image

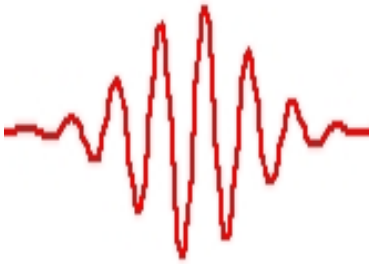


By convolving stretched and scaled versions of the “mother” wavelet with the EEG signal, we determine the time-frequency distribution of power





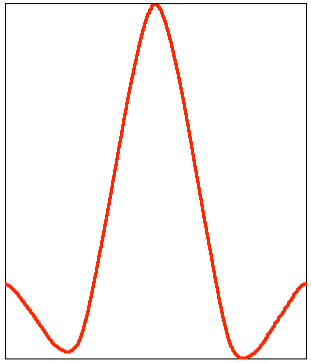
Some Wavelet Families



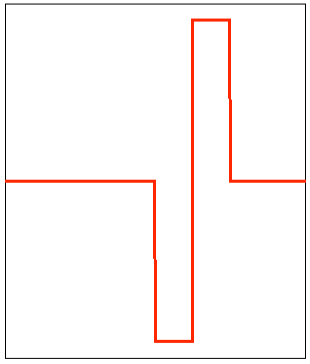
Morlet

Daubechies_4

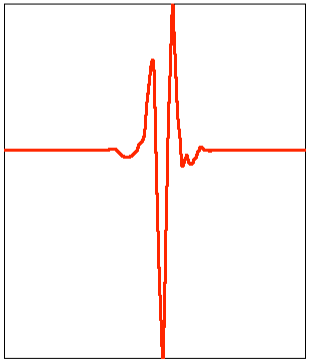
Daubechies_20



Coiflet_3



Haar_4



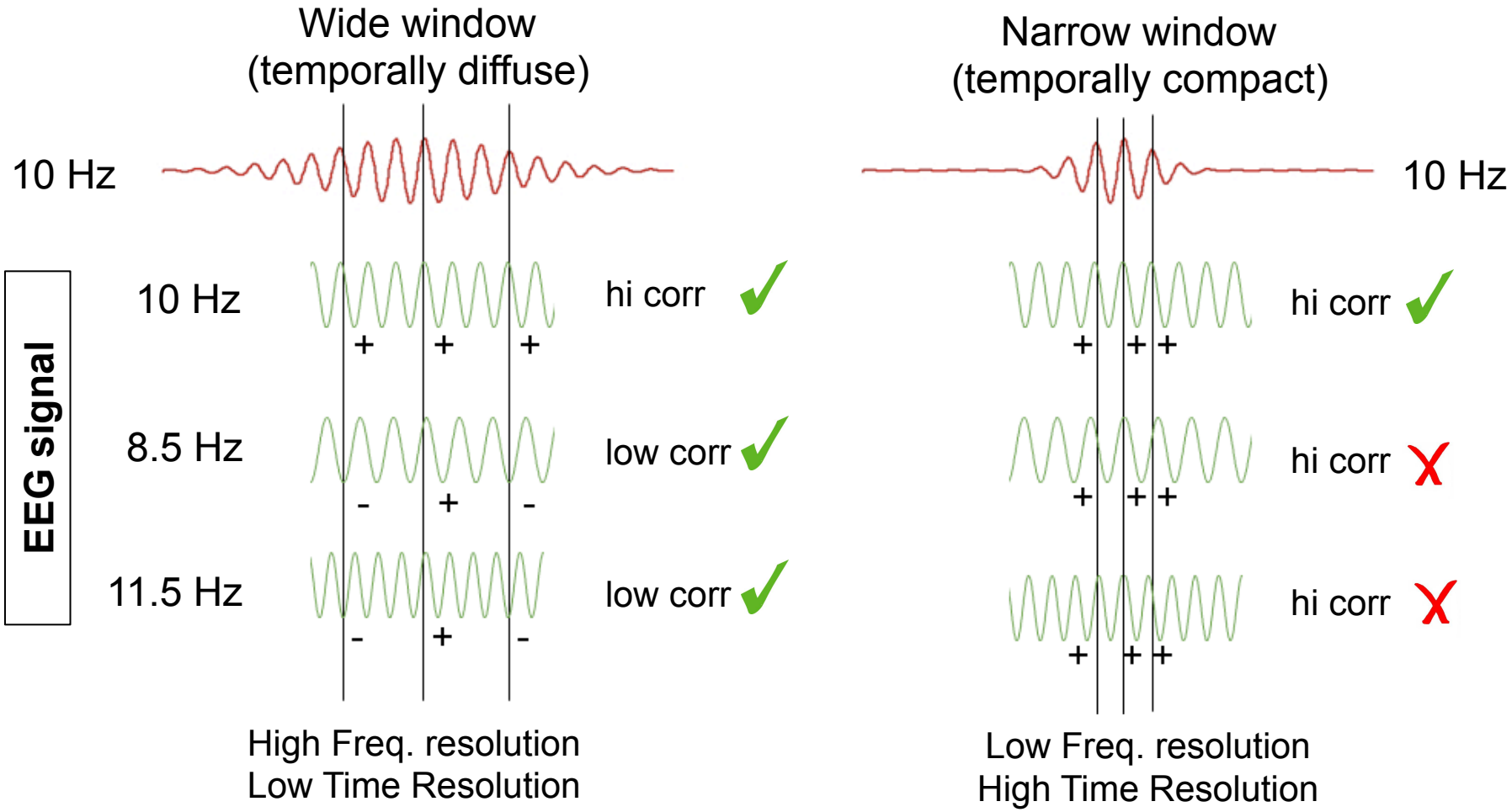
Symmlet_4

Meyer_2

Battle_3

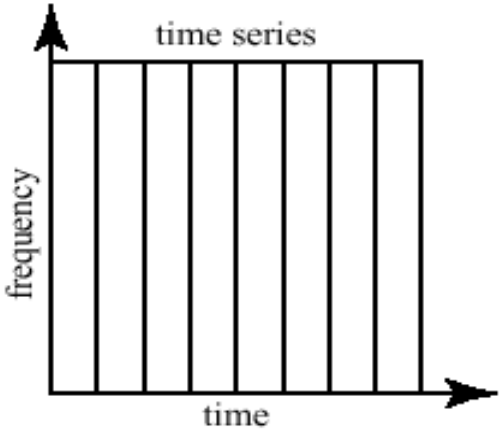
Trading Frequency for Time

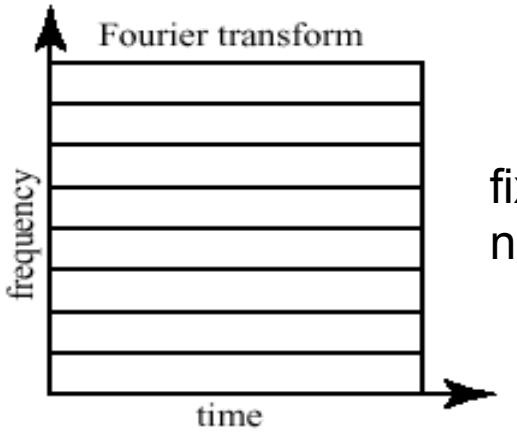
(and vice versa)



FFT versus Wavelets

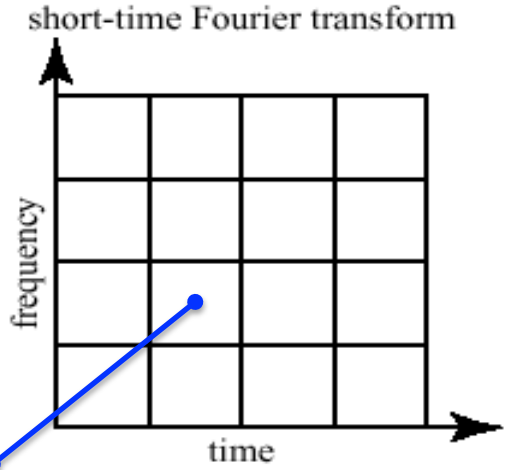
fixed time res
no freq res

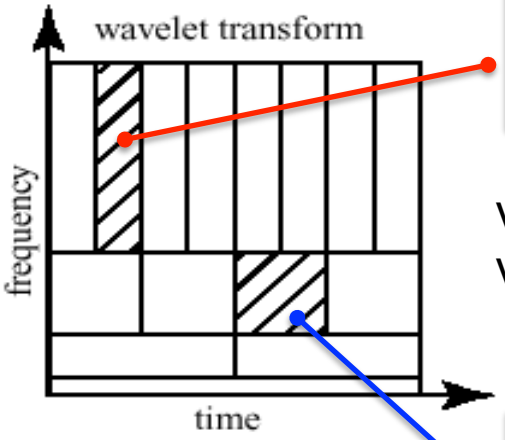




fixed freq res
no time res

fixed time res
fixed freq res





high time res
low freq res

variable time res
variable freq res

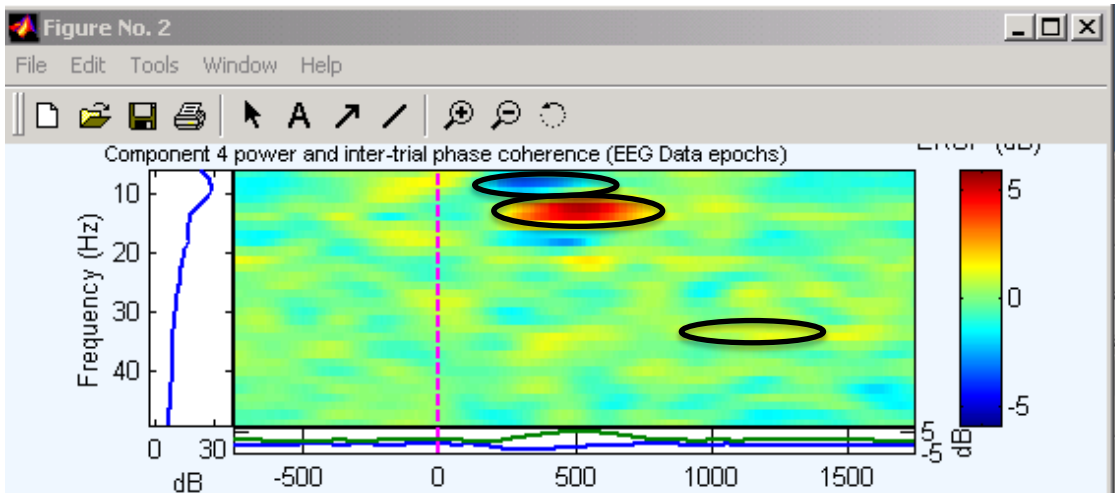
equal time
and freq
resolution

equal time
and freq
resolution

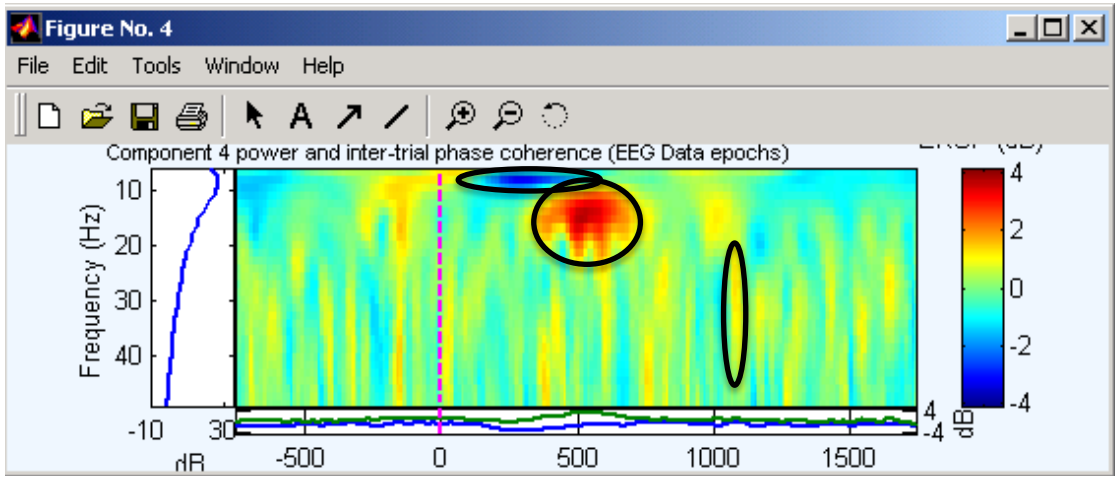
Adapted from http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10

FFT versus Wavelets

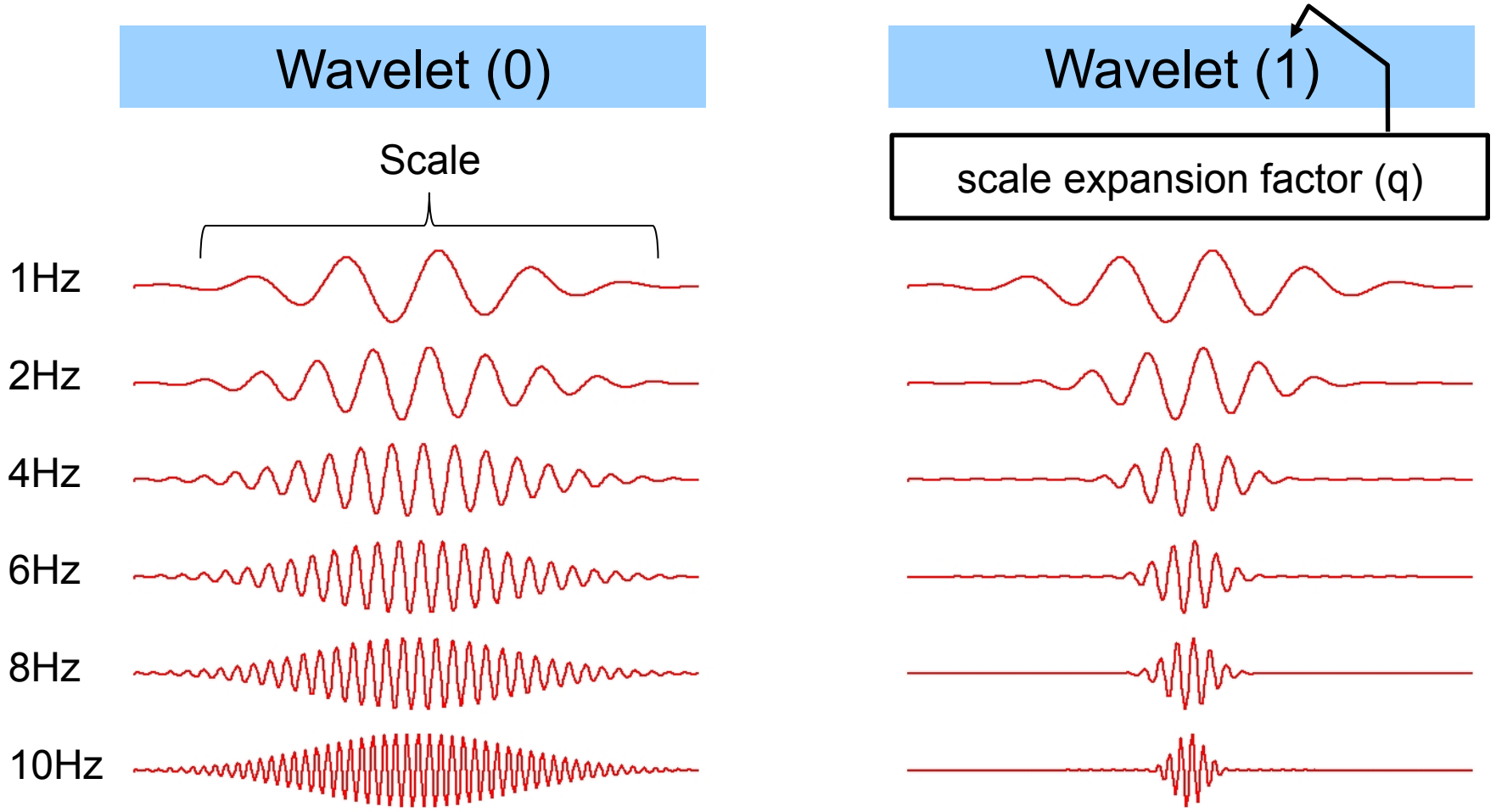
FFT
(Wavelet(0))



Wavelet (1)



Wavelet scale

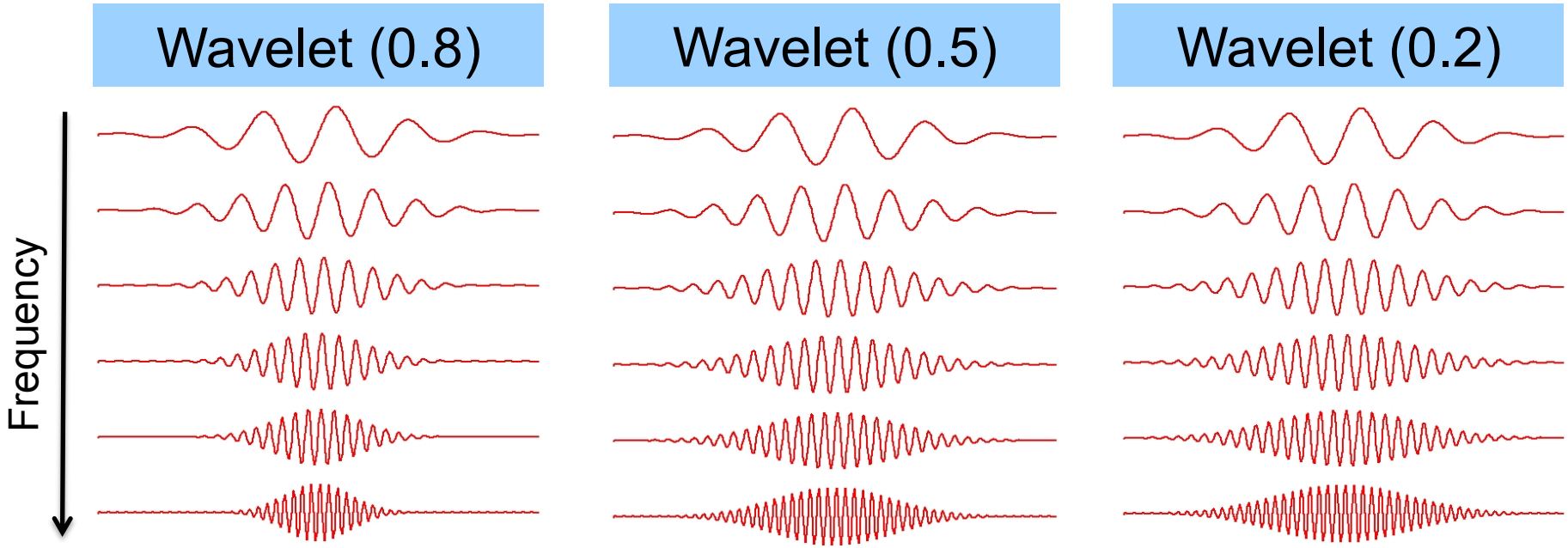


constant window size (time resolution) for increasing frequency → increasing # cycles with frequency.

window size decreases by a factor of 2 for each octave (power of 2) → constant # of cycles at each frequency

Wavelet scale expansion factor

Larger expansion factor produces larger scale decrements (increased time resolution, lower frequency resolution) for increasing frequency

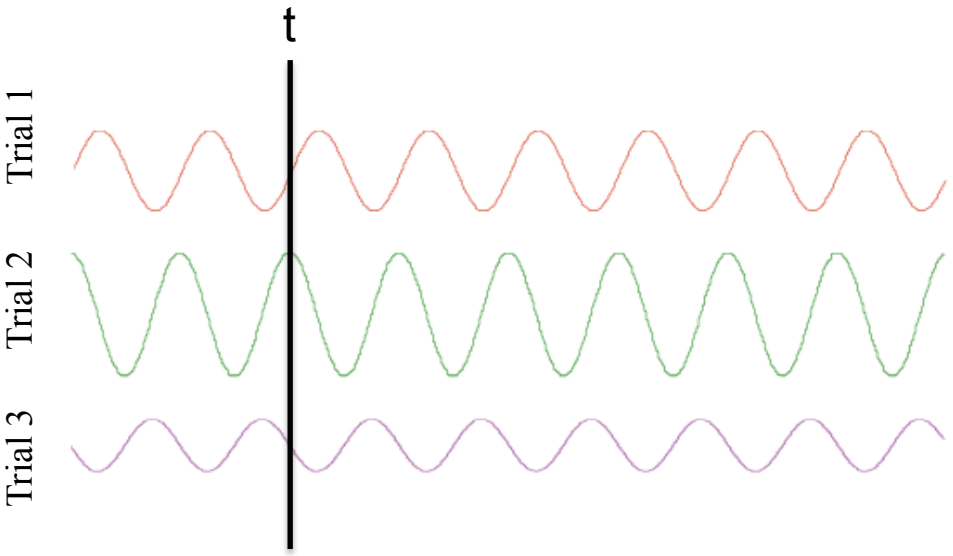


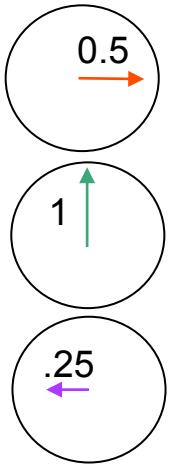
Number of cycles at highest frequency for an expansion factor of *q:*

$$C_{\text{fmax}} = \frac{f_{max}}{f_{min}} C_{\text{fmin}} (1 - q)$$

Inter-Trial Coherence (ITC)

Tallon-Baudry, et al, 1996





amplitude 0.5 phase 0

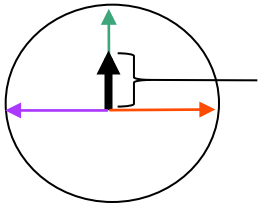
amplitude 1 phase 90

amplitude 0.25 phase 180

POWER = mean(amplitudes^2)

0.44 or –8.3 dB

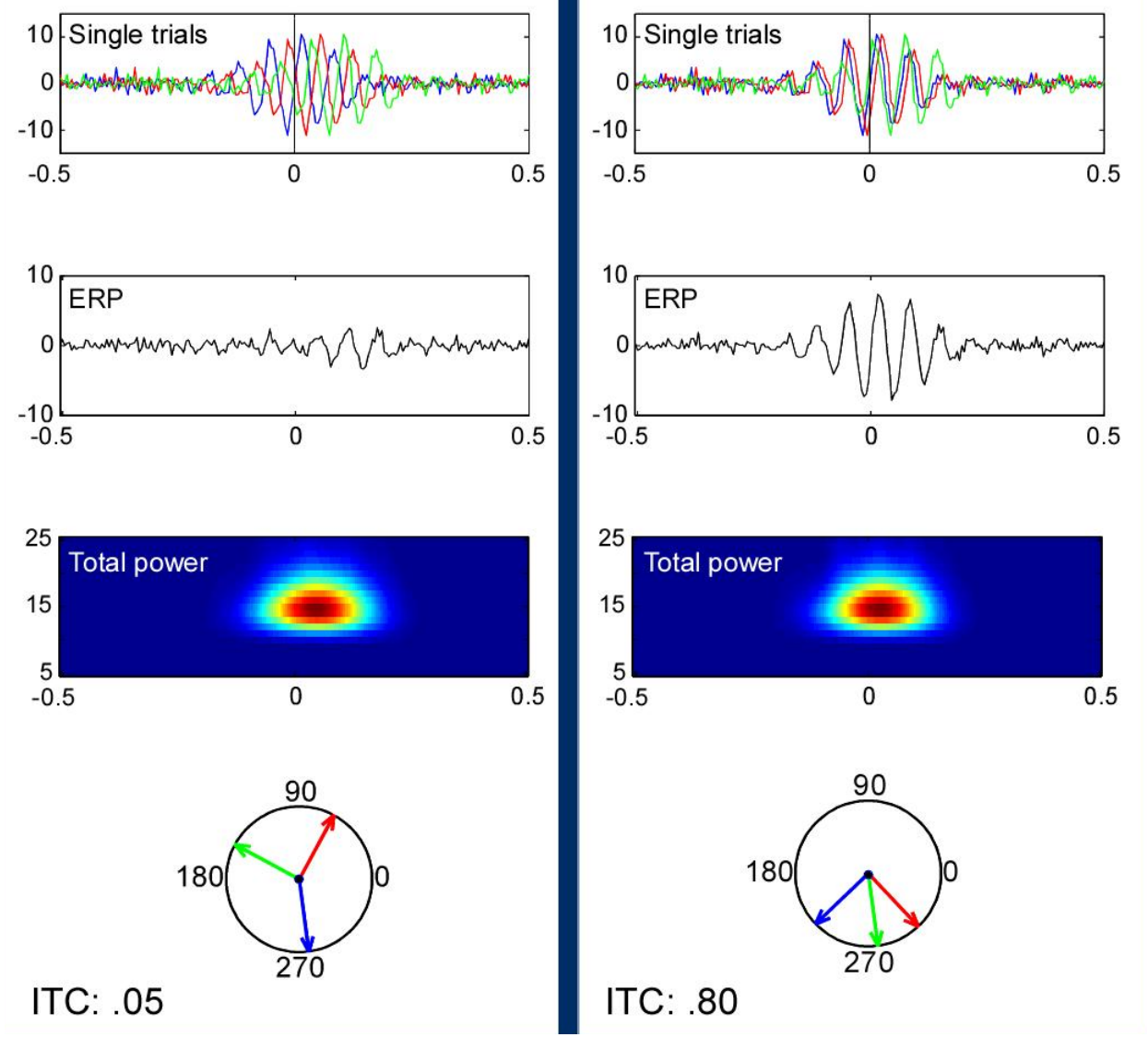
ITC = |mean(norm'd phase vectors)|



Magnitude: 0.33

```
>> EEG = pop_newtimef(EEG,..., 'plotitc','on');
```

Intertrial Coherence (ITC)

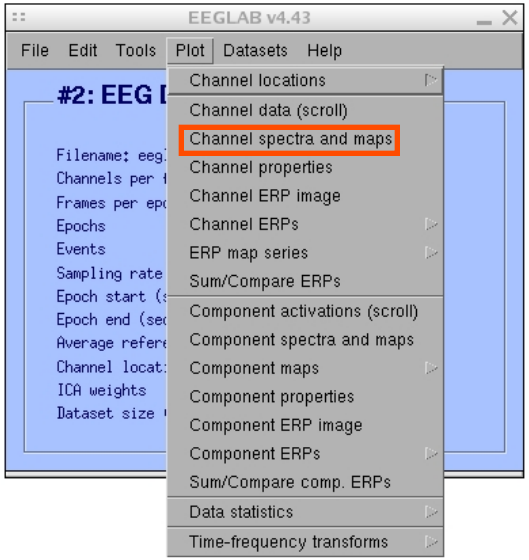


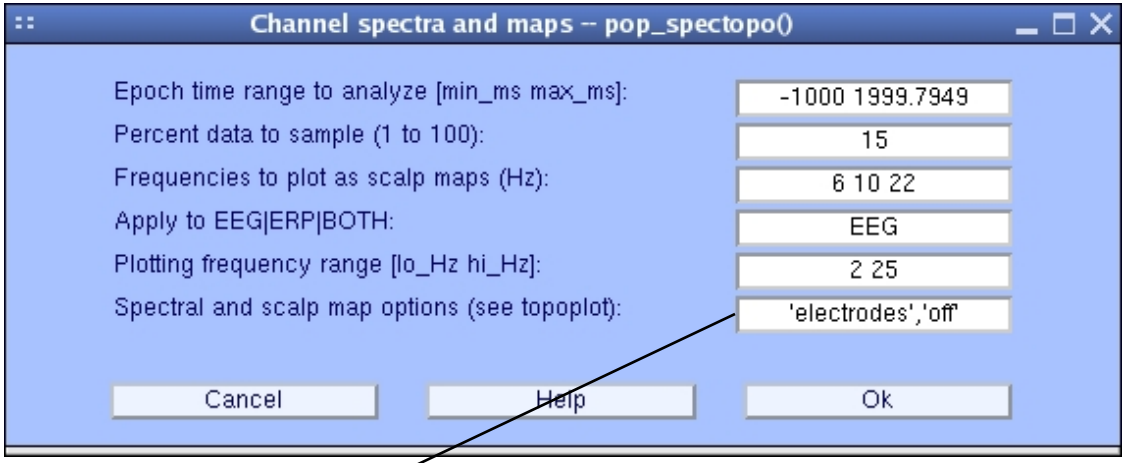
Slide courtesy of Stefan Debener

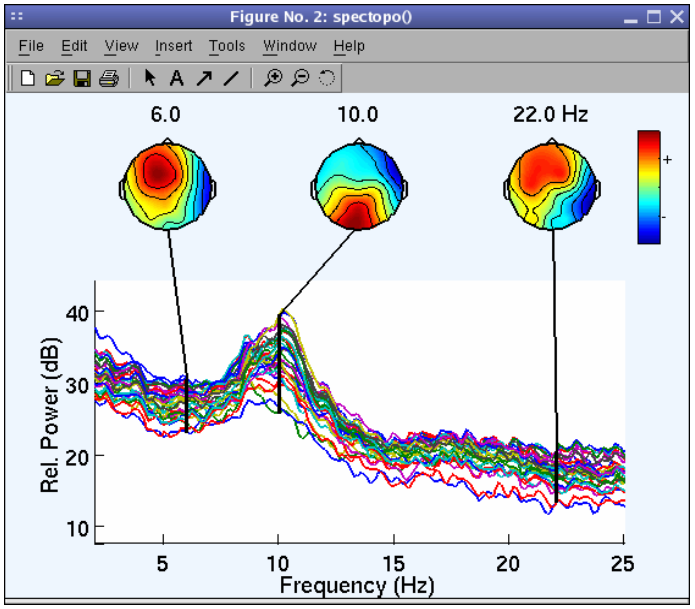
Time-Frequency Analysis of Biophysical Time series:

Practicum

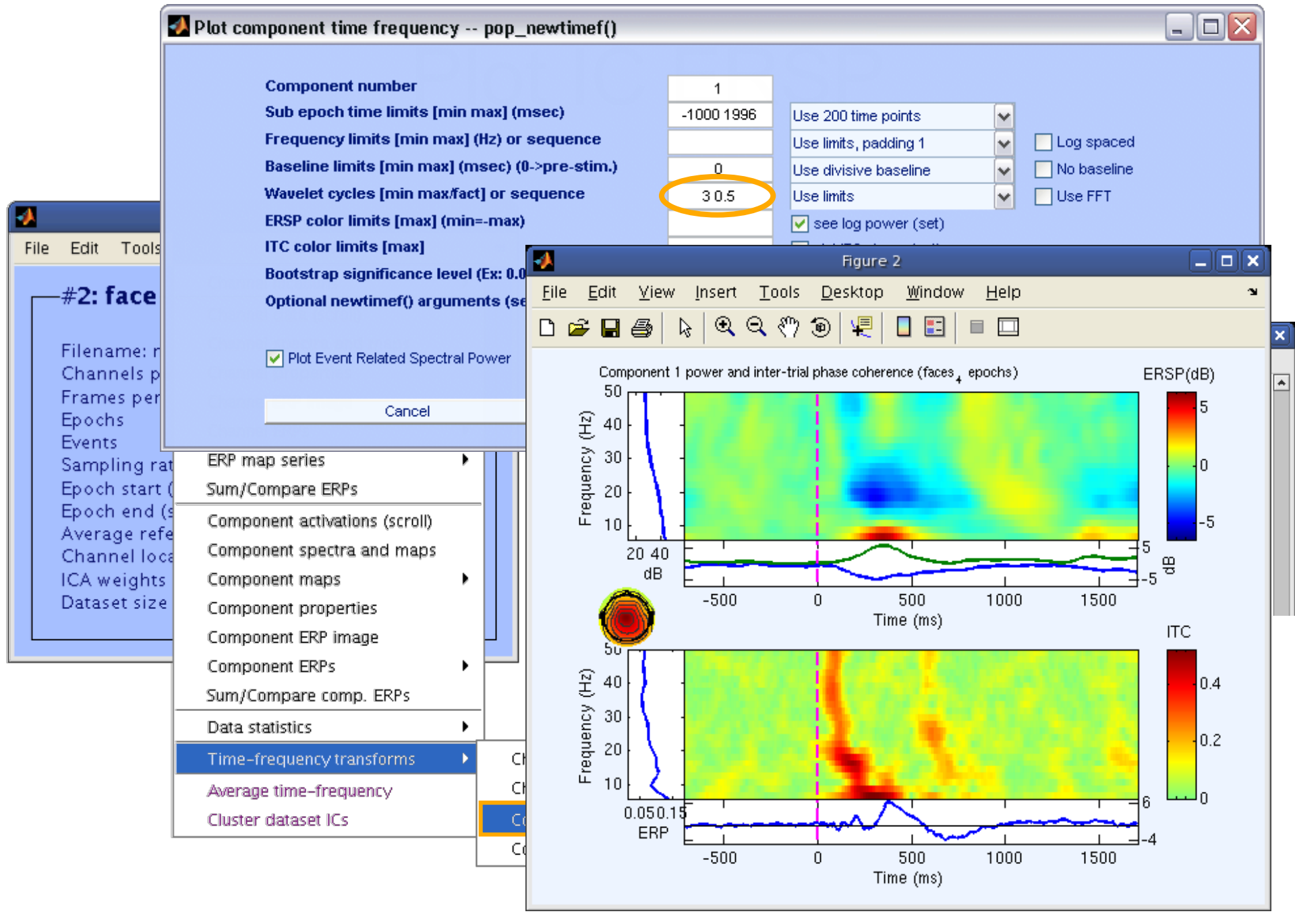
Plot periodogram (spectrum) using Welch's method

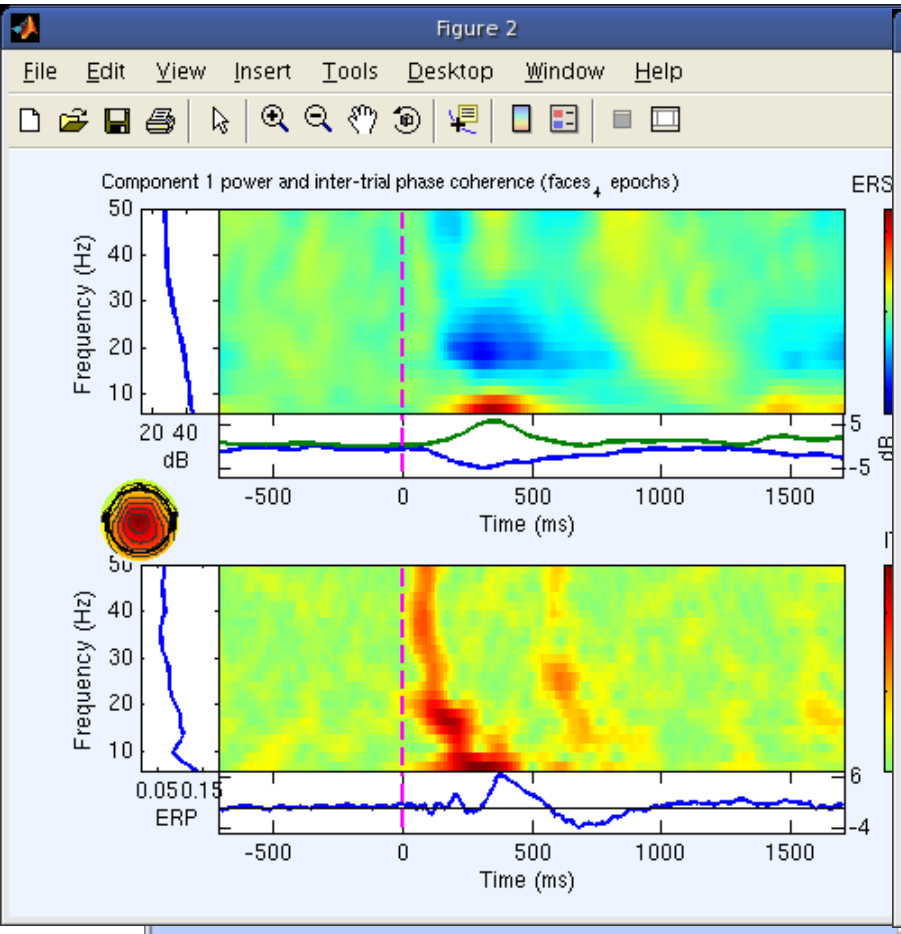


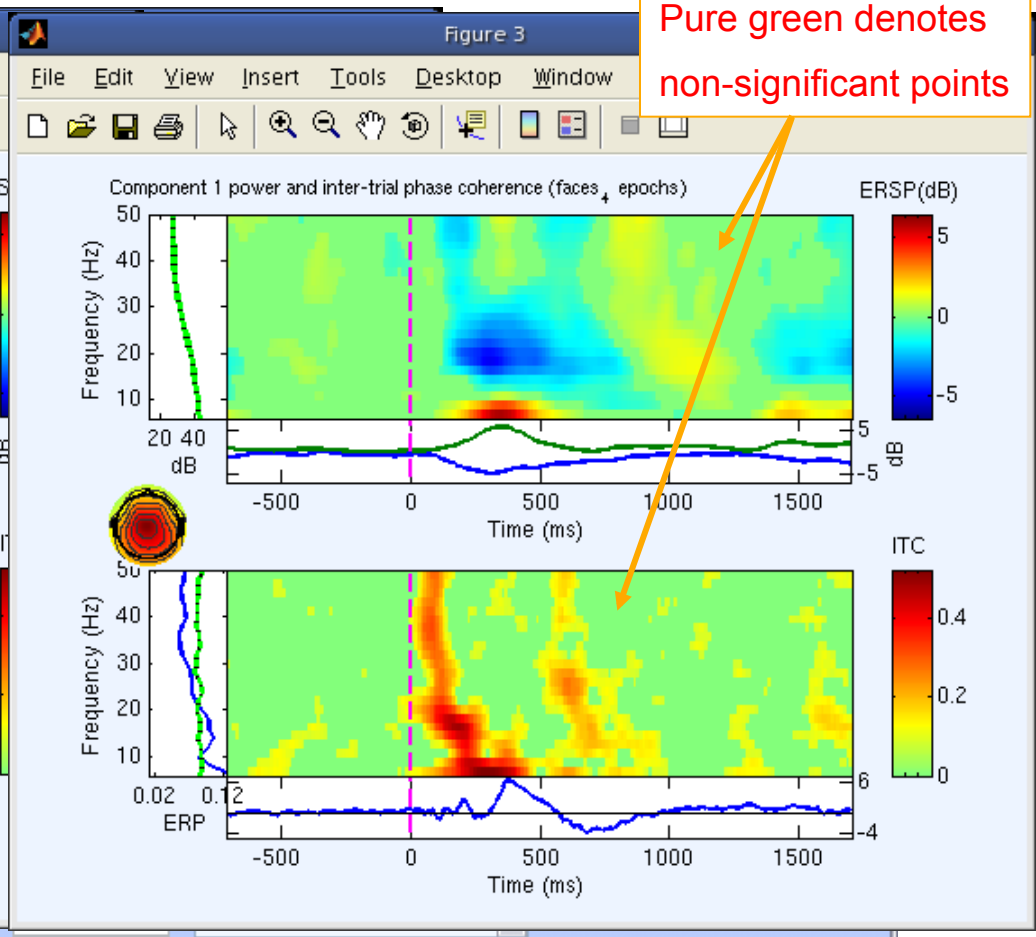




'winsize' , 256 (change FFT window length)
'nfft' , 256 (change FFT padding)
'overlap' , 128 (change window overlap)


Plot component time frequency -- pop_newtimef()
Component number
1
Sub epoch time limits [min max] (msec)
-1000 1996
Frequency limits [min max] (Hz) or sequence
Baseline limits [min max] (msec) (0->pre-stim.)
0
Wavelet cycles [min max/fact] or sequence
3 0.5
ERSP color limits [max] (min=-max)
ITC color limits [max]
Bootstrap significance level (Ex: 0.0
Optional newtimef() arguments (se
Use 200 time points
Use limits, padding 1
Use divisive baseline
Use limits
Log spaced
No baseline
Use FFT
see log power (set)
Plot Event Related Spectral Power
Cancel
File Edit Tools
#2: face
Filename: r
Channels p
Frames per
Epochs
Events
Sampling rat
Epoch start (
Epoch end (s
Average refe
Channel loca
ICA weights
Dataset size
ERP map series
Sum/Compare ERPs
Component activations (scroll)
Component spectra and maps
Component maps
Component properties
Component ERP image
Component ERPs
Sum/Compare comp. ERPs
Data statistics
Time-frequency transforms
Average time-frequency
Cluster dataset ICs
Figure 2
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces, epochs)
ERSP(dB)
Frequency (Hz)
Time (ms)
dB
ITC
ERP



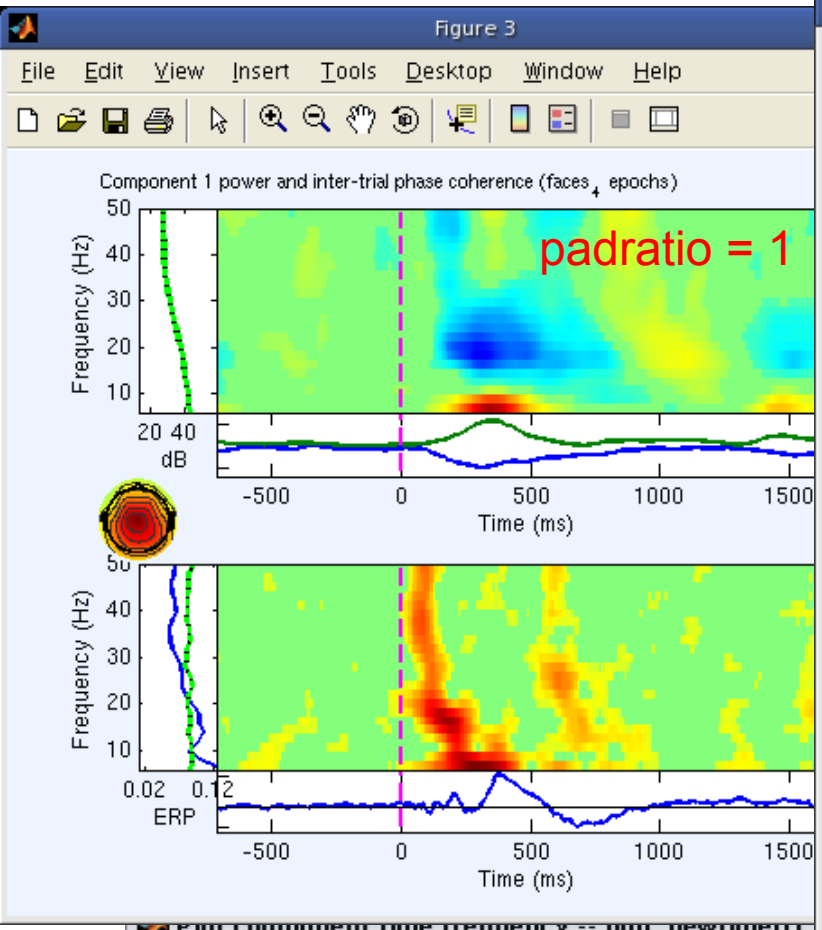


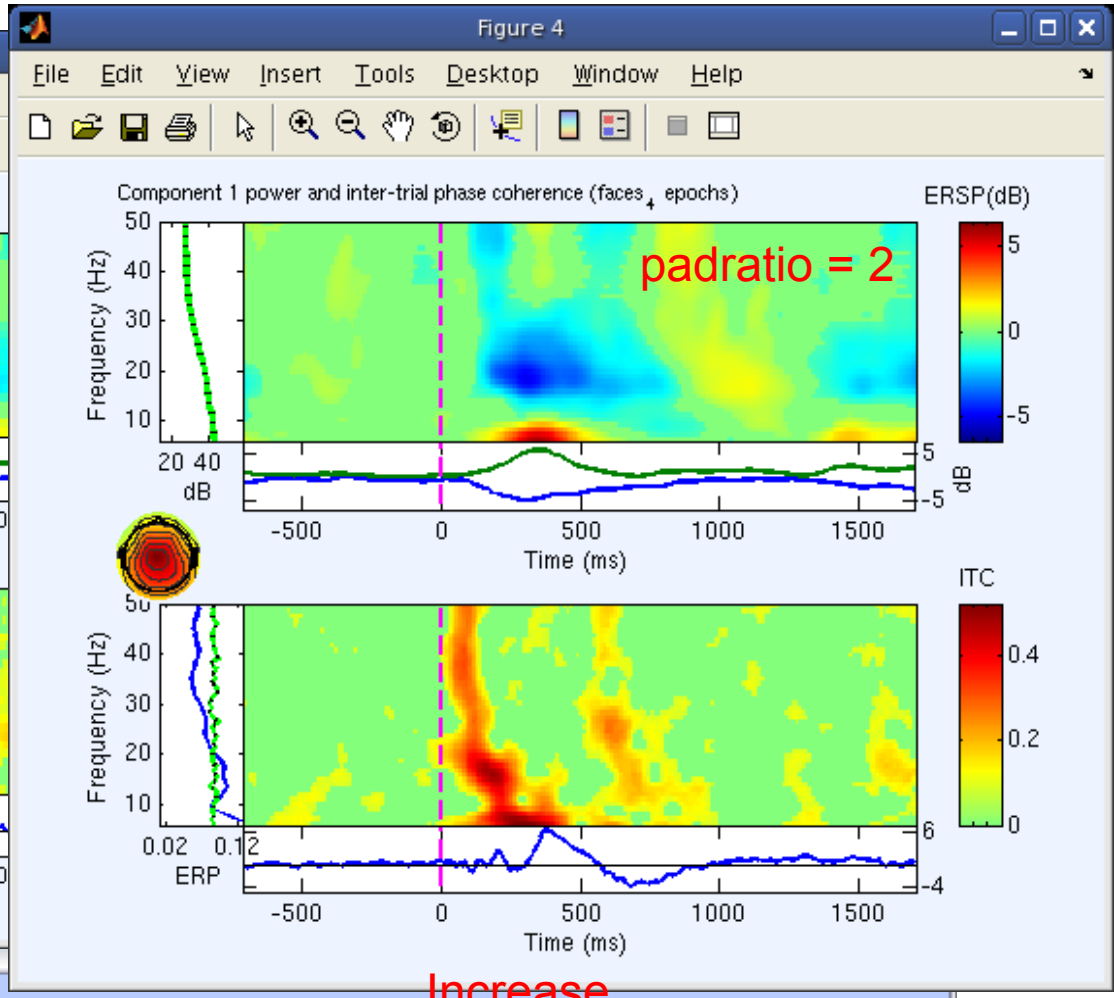
Pure green denotes non-significant points

Wavelet cycles [min max/fact] or sequence	3 0.5	Use limits ☐ Use FFT
ERSP color limits [max] (min=-max)		☑ see log power (set)
ITC color limits [max]		☐ plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)	0.01	☐ FDR correct (set)
Optional newtimef() arguments (see Help)		

☑ Plot Event Related Spectral Power ☑ Plot Inter Trial Coherence ☐ Plot curve at each frequency

Cancel Help Ok

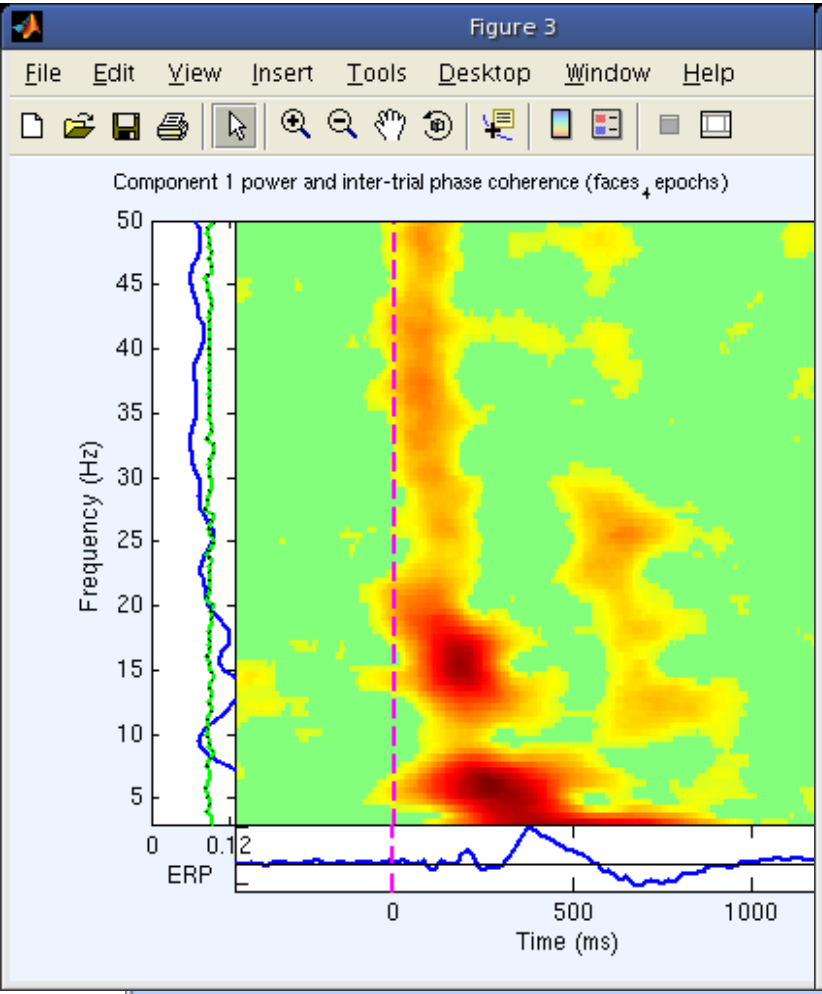


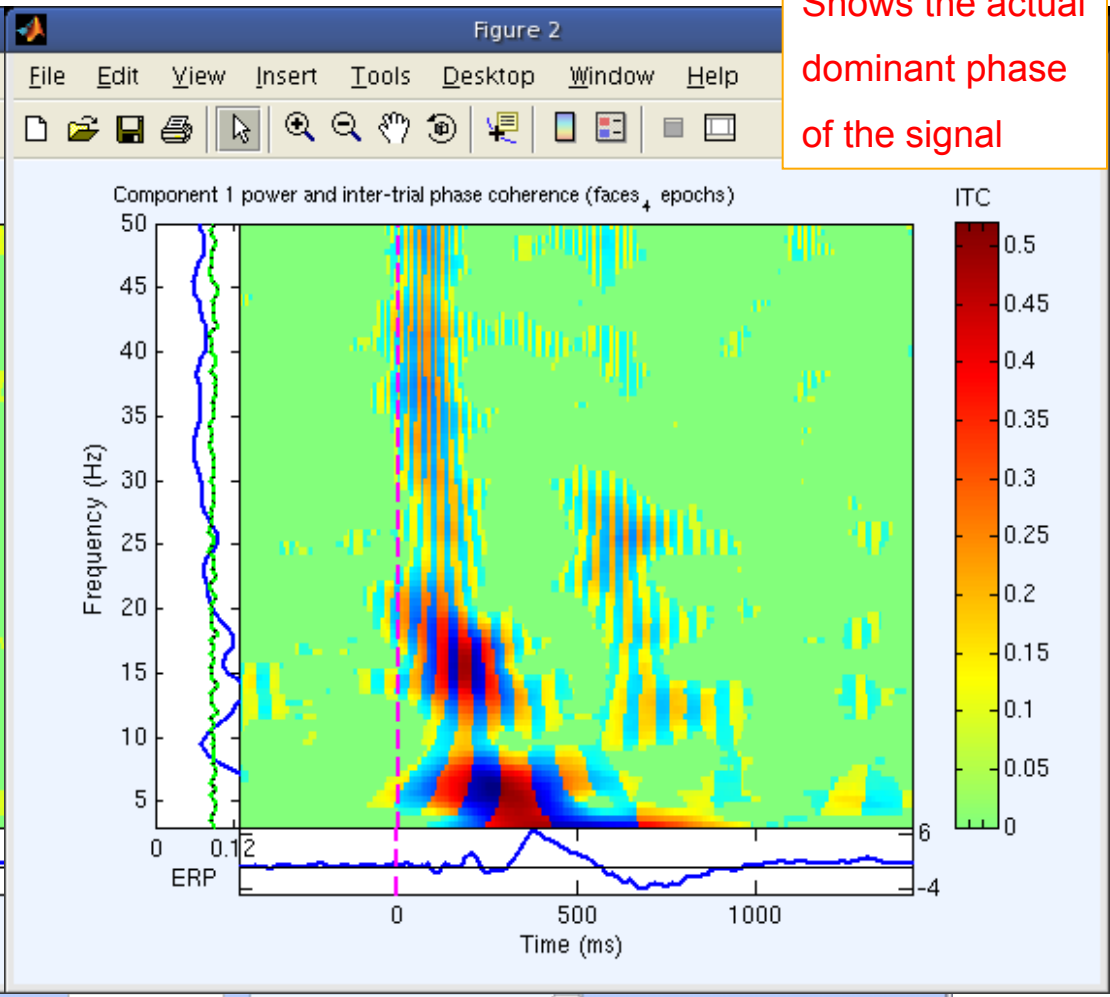


Increase
freq bins

Component number	1	
Sub epoch time limits [min max] (msec)	-1000 1996	Use 256 time points
Frequency limits [min max] (Hz) or sequence		Use limits, padding 1 — Log spaced
Baseline limits [min max] (msec) (0->pre-stim.)	0	Use divisive baseline — No baseline
Wavelet cycles [min max/fact] or sequence	3 0.5	Use limits — Use FFT
ERSP color limits [max] (min=-max)		see log power (set)
ITC color limits [max]		plot ITC phase (set)
Bootstrap significance level (Ex: 0.01 -> 1%)		FDR correct (set)
Optional newtimef() arguments (see Help)		

Shows the actual dominant phase of the signal

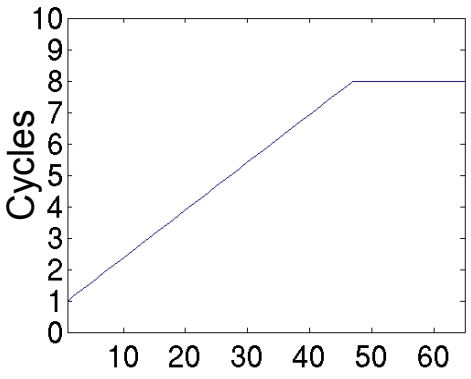


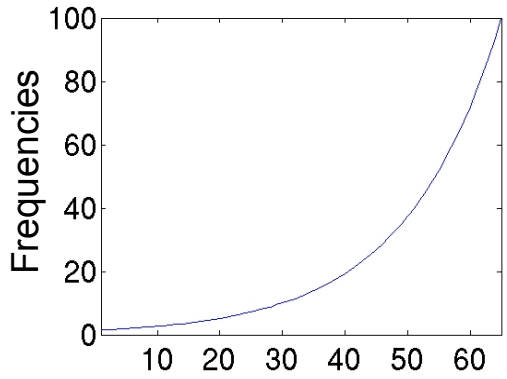


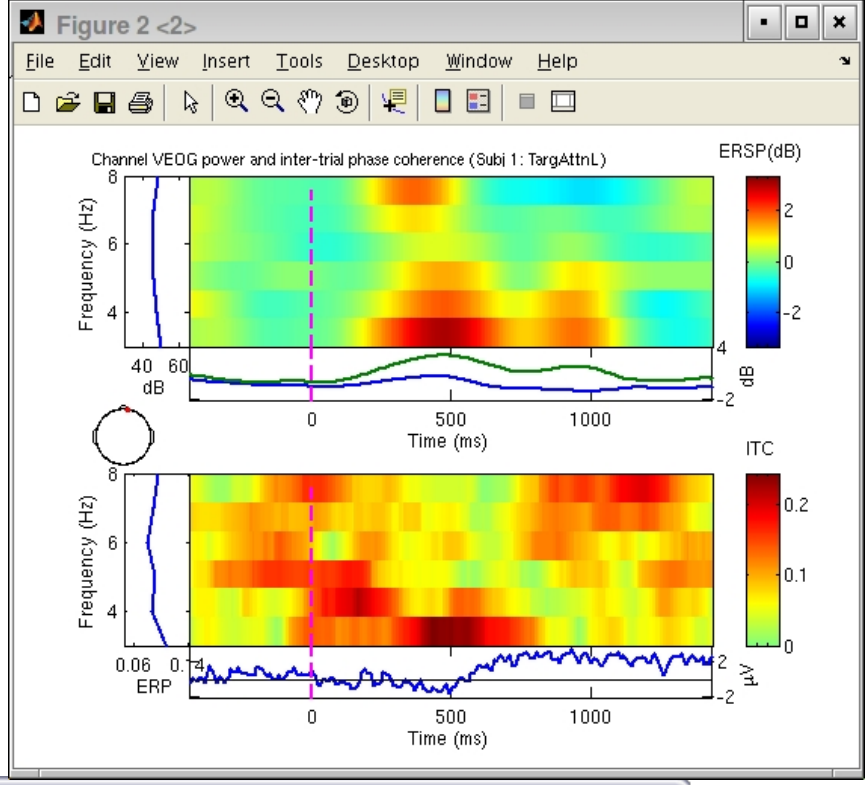
Sub epoch time limits [min max] (msec)	-1000 1996	Use 200 time points	
Frequency limits [min max] (Hz) or sequence		Use limits, padding 1	Log spaced
Baseline limits [min max] (msec) (0->pre-stim.)	0	Use divisive baseline	No baseline
Wavelet cycles [min max/fact] or sequence	3 0.5	Use limits	Use FFT
ERSP color limits [max] (min=-max)		see log power (set)	
ITC color limits [max]		plot ITC phase (set)	
Bootstrap significance level (Ex: 0.01 -> 1%)		FDR correct (set)	
Optional newtimef() arguments (see Help)	'plotphase', 'on'		

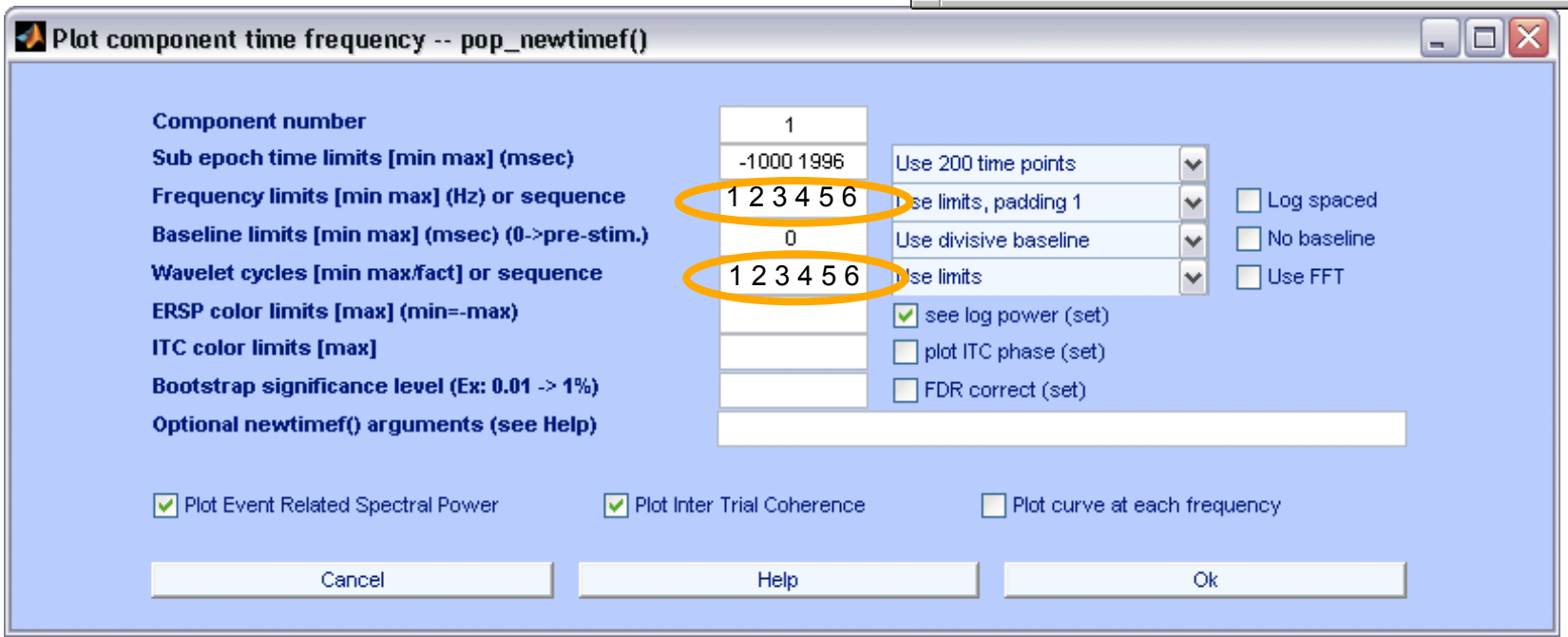
To visualize both low and high frequencies

freqs = exp(linspace(log(1.5), log(100), 65));
cycles = [linspace(1, 8, 47) ones(1,18)*8];

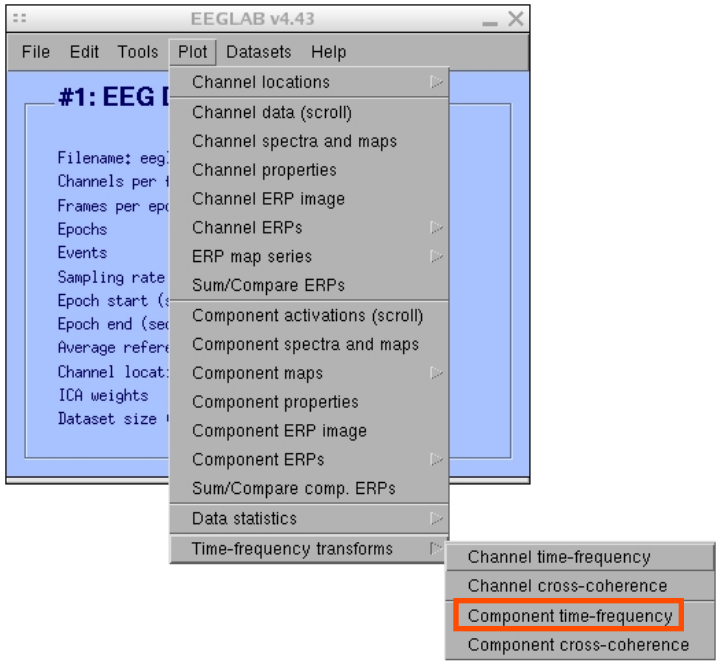


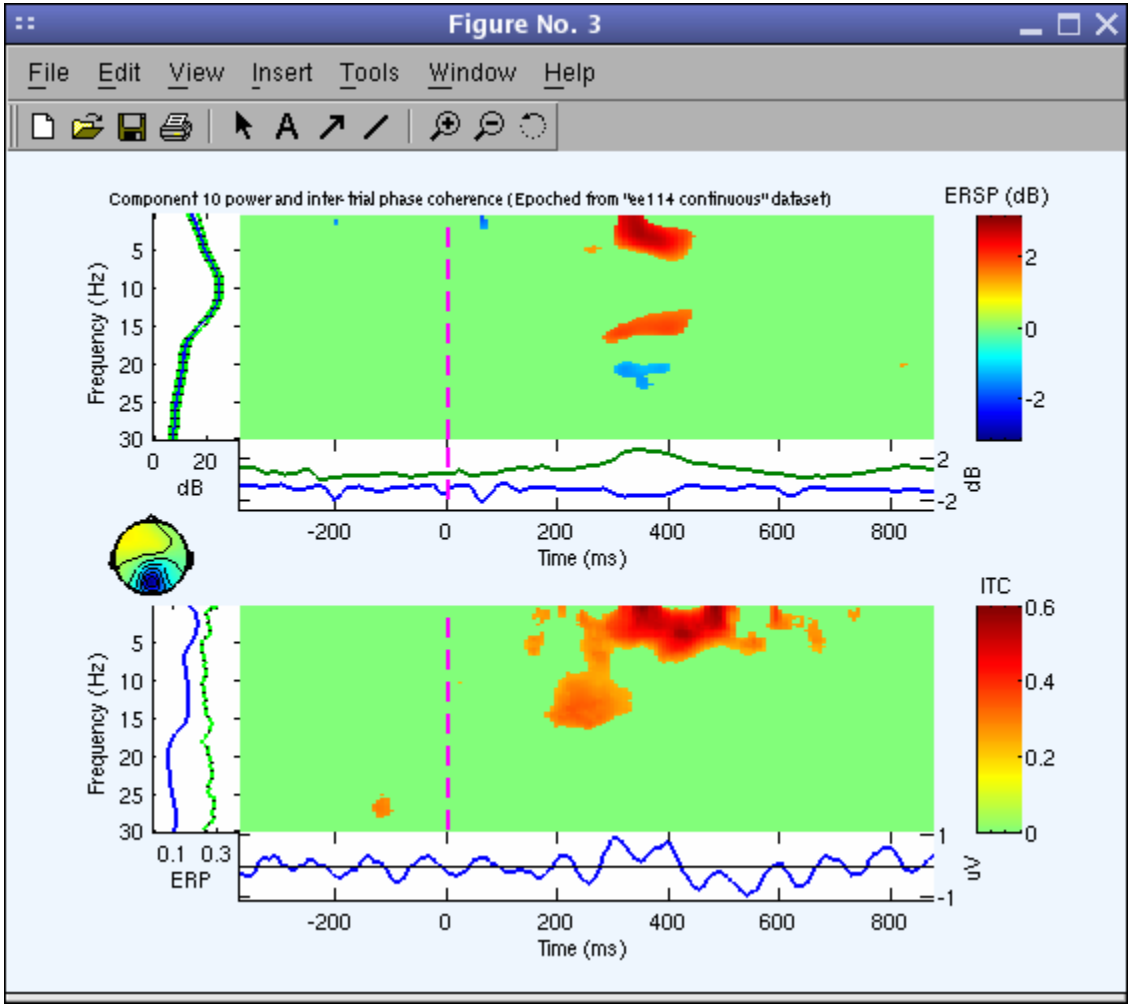






Component time-frequency





Exercise

- **ALL**

  Start EEGLAB, from the menu:

  load <eeglab_root>/*sample_data/ eeglab_data_epochs_ica.set*

  or your own data

- **Novice**

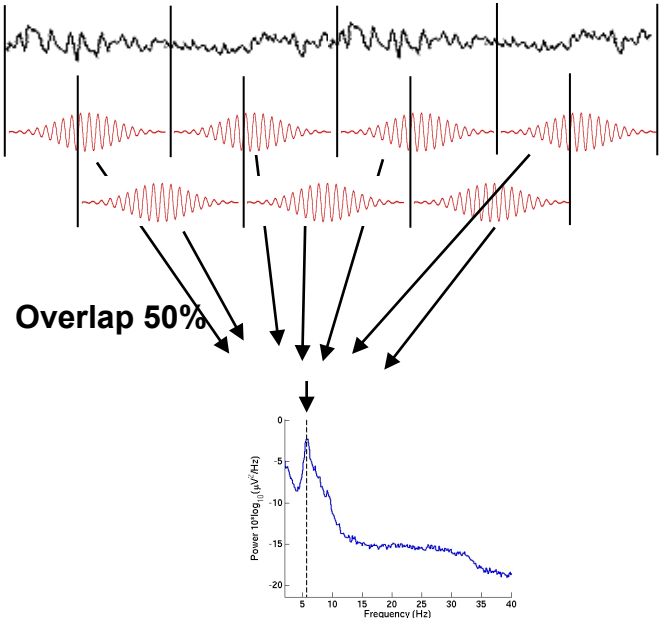
  From the GUI, Plot spectral decomposition with 100% data and 50% overlap (‘overlap’). Try reducing window length (‘winsize’) and FFT length (‘nfft’)

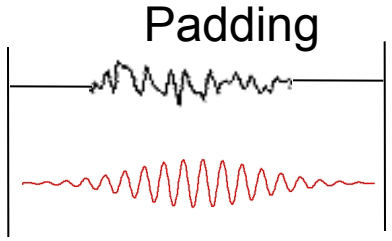
- **Intermediate**

  Same as novice but using a command line call to the *pop_spectopo()* function. Use GUI then history to see a standard call (“eegh”).

- **Advanced**

  Same as novice but using a command line call to the *spectopo()* function.





Exercise - newtimef

- **Novice**

  From the GUI, pick an interesting IC and plot component ERSP. Try changing parameters window size, number of wavelet cycles, padratio,

- **Intermediate**

  From the command line, use newtimef() to tailor your time/ frequency output to your liking. Look up the help to try not to remove the baseline, change baseline length and plot in log scale. Enter custom frequencies and cycles (2 slides back).

- **Advanced**

  Compare FFT, the different wavelet methods (see help), and multi-taper methods (use timef function not newtimef). Enter custom frequencies and cycles. Look up newtimef help to compare conditions. Visualize single-trial time-frequency power using erpimage.