Time-Frequency analysis of biophysical time series

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Frequency analysis

The synchronicity of cell excitation determines the amplitude and rhythm of the EEG signal.

- **30-60 Hz Gamma**
- **18-21 Hz Beta**
- **9-11 Hz Alpha**
- **4-7 Hz Theta**
- **0.5-2 Hz Delta**
Frequency analysis

- Beta
- Alpha
- Theta
- Delta
- Low Delta
Stationary signals

Slide courtesy of Petros Xanthopoulos, Univ. of Florida
By looking at the Power spectrum of the signal we can recognize three frequency components (at 2, 10, 20 Hz respectively).
Forward transform  \[ F(u) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi iux} \, dx \]

Inverse transform  \[ f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} \, du \]
Performing Fourier transform by using a time moving window.
Spectral phase and amplitude
Spectral phase and amplitude

\[ F(f,t) \]
Discrete Fourier Transform function

```matlab
function X = dft(x)

[N,M] = size(x);
n = 0:N-1;

for k=n
    X(k+1) = exp(-j*2*pi*k*n/N)*x;
end
```

Loop on frequency

Real part

Cosine component

Imaginary part

Sine component

Multiply with signal

Cosine component

Real part
Average of squared absolute values
Spectral power

Power (dB)

Frequency (Hz)

Average of squared amplitude
Overlap 50%

Average of squared amplitudes
padding
Spectrogram or ERSP

- 5 Hz
- 10 Hz
- 20 Hz
- 30 Hz
Spectrogram or ERSP

Average of squared values
Power spectrum and event-related spectral perturbation

\[
ERS (f, t) = \frac{1}{n} \sum_{k=1}^{n} \left| F_k (f, t) \right|^2
\]

Scaled to dB \(10 \log_{10}(\text{ERSP})\)
Absolute versus relative power

Absolute = ERS

Relative = ERSP (dB or %)
Difference between FFT and wavelets

FFT

Wavelet

Frequency
Wavelets factor

Wavelet (0) = FFT

Wavelet (1)

1Hz
2Hz
4Hz
6Hz
8Hz
10Hz
Time-frequency resolution trade off

**FFT**

- Exact
- High freq. resolution
- low time-resolution

**Wavelet**

- Low freq. resolution
- high time-resolution
FFT

Pure wavelet
The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/position and in frequency/momentum.

There exists a lower bound to the Heisenberg’s product:

\[ \Delta t \Delta f \geq \frac{1}{4\pi} \]

\[ \Delta f = 1\text{Hz}, \quad \Delta t = 80 \text{ msec} \text{ or } \Delta f = 2\text{Hz}, \quad \Delta t = 40 \text{ msec} \]
Modified wavelets

Wavelet (0.8)  Wavelet (0.5)  Wavelet (0.2)
Inter trial coherence

same time, different trials

Trial 1
amplitude 0.5, phase 0

Trial 2
amplitude 1, phase 90

Trial 3
amplitude 0.25, phase 180

POWER = mean(amplitudes^2)
0.44 or –8.3 dB

COHERENCE = mean(phase vector)
Norm 0.33
Intertrial Coherence (ITC)

Single trials

-0.5  0  0.5

ERP

-0.5  0  0.5

Total power

-0.5  0  0.5

ITC: .05

ITC: .80

Slide courtesy of Stefan Debener
Phase ITC

$$I_{ITPC}(f, t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k(f, t)}{|F_k(f, t)|}$$

Normalized (no amplitude information)
Power and inter trial coherence

Attend left-stim left

Attend left-stim right

Difference
Plot component time frequency -- pop_newtimef()

Component number
Sub epoch time limits [min max] (msec)
Frequency limits [min max] (Hz) or spectral range
Baseline limits [min max] (msec) (0->n)
Wavelet cycles [min max/fact] or sequence
ERSP color limits [max] (min=max)
ITC color limits [max]
Bootstrap significance level (Ex: 0.01)
Optional newtimef() argument (see HELP)

Optional check box: Plot Event Related Spectral Power

Figure 2
Component 1 power and inter-trial phase coherence (faces, epochs)

ERSP (dB)

Frequency (Hz)

Time (ms)

ITC

Component 1 power and inter-trial phase coherence (faces, epochs)
Pure green denotes non-significant points.
Plot IC ERSP

Increase 
# freq bins

padratio = 1

padratio = 2
To visualize both low and high frequencies

def freqs = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
Component time-frequency
Cross-coherence amplitude and phase

2 components, comparison on the same trials

Trial 1

Trial 2

Trial 3

COHERENCE = mean(phase vector)

Norm 0.33
Phase 90 degree
Phase coherence (default)

\[ \text{ERPCOH}^{a,b}(f, t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k^a(f, t) F_k^b(f, t)^*}{F_k^a(f, t) F_k^b(f, t)} \]
Cross-coherence amplitude and phase

**Animal picture**

<table>
<thead>
<tr>
<th>Amplitude (0-1)</th>
<th>Phase (degree)</th>
</tr>
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<tr>
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**Distractor picture**

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Two EEG channels

Scalp channel coherence → source confounds!
MANY EEG channels

Cortex

Separate out Independent EEG Components

Measure their Synchronization

source dynamics!
Niquist frequency: Aliasing

1 cycle

Signal (100 Hz)

Sampling (120 Hz)

e.g. 100 Hz sampled at 120 Hz
Advanced time-frequency functions

- `Tftopo()`: allow visualizing time-frequency power distribution over the scalp
Plot data spectrum using EEGLAB

- `winsize`, 256 (change FFT window length)
- `nfft`, 256 (change FFT padding)
- `overlap`, 128 (change window overlap)
Exercise

• **ALL**
  Start EEGLAB, from the menu load
  `sample_data/eeglab_data_epochs_ica.set`
or your own data (epoch, reject noise if not done already)

• **Novice**
  From the GUI, Plot spectral decomposition with 100% data and 50% overlap (`overlap`). Try reducing window length (`winsize`) and FFT length (`nfft`)

• **Intermediate**
  Same as novice but using a command line call to the `pop_spectopo()` function. Use GUI then history to see a standard call (“eegh”).

• **Advanced**
  Same as novice but using a command line call to the `spectopo()` function.