Time-frequency measures
Theory and Practice

EEGLAB Workshop XXII
UCSD
Day 1, 11:30
• Signals – EEG

• Goals
  – Describe dynamic characteristics of brain activity
  – Describe relation between different regions of brain

• Approaches
  – Time domain
  – Frequency domain
  – Time/Frequency
Different meanings traditionally given to different frequency bands

- **Beta 15-30 Hz**
  - Awake, normal alert consciousness

- **Alpha 9-14 Hz**
  - Relaxed, calm, meditation, creative visualisation

- **Theta 4-8 Hz**
  - Deep relaxation and meditation, problem solving

- **Delta 1-3 Hz**
  - Deep, dreamless sleep
MEEG spectrum

ERP  Beta  Gamma

MEEG Power

Frequency (Hz)
Time-varying frequency content
Power Spectrum does not describe temporal variation

Onton & Makeig, 2006
Plan

• **Part 1: Frequency Analysis**
  – Power Spectrum
    • Windowing

• **Part 2: Time-Frequency Analysis**
  – Short Time Fourier Transform
  – Wavelet Transform
  – ERSP

• **Part 3: Coherence Analysis**
  – Inter-Trial Coherence
  – Event-Related Coherence
Part 1: Frequency Analysis

• Goal: What frequencies are present in signal?

• What is power at each frequency?

• Considerations
  – Amplitude & phase
  – Windowing
Fourier Analysis

Forward transform
\[ F(u) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi iux} \, dx \]

Inverse transform
\[ f(x) = \int_{-\infty}^{+\infty} F(u) e^{2\pi iux} \, du \]

Figure, courtesy of Ravi Ramamoorthi & Wolberg
“Stationary” sinusoidal signals

2 Hz

10 Hz

20 Hz

2+10+20 Hz

Slide courtesy of Petros Xanthopoulos, Univ. of Florida
Simplest case of frequency analysis

By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2, 10, 20Hz respectively).

Slide courtesy of Petros Xanthopoulous, Univ. of Florida
Power Spectrum. Approach 1: FFT

• Why not just take FFT of our signal of interest?
• Advantage – fine frequency resolution
  – $\Delta F = 1 / \text{signal duration (s)}$
  – E.g. 100s signal has 0.01 Hz resolution
  – But, do we really need this?

• Disadvantage – bias and variance
  – Solution: e.g. Welch’s method

• Disadvantage – no temporal resolution
  – Solution 1: Short-Time Fourier Transform
Phasor representation

- A complex number $x + yi$ can be expressed in terms of amplitude and phase: $ae^{i\theta}$

\[
\text{amplitude} \times \exp(i \times \text{phase})
\]

\[
\text{amplitude} = \sqrt{x^2 + y^2}; \quad \text{phase} = \text{atan}(y/x);
\]
Approach 2: Welch’s Method

Calculate power spectrum of short windows, average.
Advantage: Smoother estimate of power spectrum

Frequency resolution set by window length
  e.g. 1s window -> 1 Hz resolution
In practice: taper, don’t use rectangular window
FFT of window 1

FFT of window 2

Average of squared amplitude

Power (dB)

Frequency (Hz)
Windowing

• When we pick a short segment of signal, we typically window it with a smooth function.

• Windowing in time = convolving (filtering) the spectrum with the Fourier transform of the window

• No window (=rectangular window) results in the most smearing of the spectrum

• There are many other windows optimized for different purposes: Hamming, Gaussian…
Windows and their Fourier transforms

Rectangular window

Narrowest main peak, but
Highest side-lobes
Most spectral ‘smearing’

Gaussian window ($\sigma = 0.4$)

Wider main peak, and
much lower side-lobes
Close-up view

Frequency response of popular window functions

- No window
- Hamming
- Hann
- Four-term Blackman-Harris
- Seven-term Blackman-Harris

dB versus FFT frequency bin

- Main lobe
- Side lobes
Part 2: Time-Frequency Analysis

- Short-Time Fourier Transform
  - Find power spectrum of short windows
  - “Spectrogram”

- Advantage: Can visualize time-varying frequency content

- Disadvantage: Fixed temporal resolution is not optimal
For each time window

Analyze signal using the wavelets for different frequencies.

**Phasor representation:**

\[ \text{amplitude} \times \exp(i \times \text{phase}) \]
Computing Spectrogram Power

Average of squared values
Amplitude and phase

- Power spectra describe the *amount* of a given frequency present.

- NOT a complete description of a signal: We also must know the *phase* at each frequency.

- FFT/STFT/Wavelet return an amplitude and phase at each time and frequency (represented as complex #).

- To find power, we compute the magnitude, which discards phase.
Time-Frequency Uncertainty

- You cannot have both arbitrarily good temporal and frequency resolution!
  \[ \sigma_t \sigma_f \geq 1/2 \]

- If you want sharper temporal resolution, you will sacrifice frequency resolution, and vice versa.

- (Optimal: Confined Gaussian)

Consequence for STFT

Shorter Windows
poorer frequency resolution

Longer Windows
finer frequency resolution
Time-Frequency Tradeoff

Signal: 10, 25, 50, 100 Hz

- Spectrogram with $T = 25$ ms
- Spectrogram with $T = 125$ ms
- Spectrogram with $T = 375$ ms
- Spectrogram with $T = 1000$ ms
A better way: Wavelet transform

- Wavelet transform is a ‘multi-resolution’ time-frequency decomposition.

- Intuition: Higher frequency signals have a shorter time scale

- So, vary window length with frequency!
  - longer window at lower frequencies
  - shorter window at higher frequencies
Comparison of FFT & Wavelet bases

Scaled versions of one shape
Constant* number of cycles
Comparison of FFT & Wavelet

Note: Finer temporal resolution at higher frequencies
Definition: ERSP

- Event Related Spectral Perturbation

- Change in power in different frequency bands \textit{relative to a baseline}. ERS, ERD
Try it out (faces_4.set)
Display ERS vs. ERSP

Event-related Spectrogram (ERS)

Event-Related Spectral Perturbation (ERSP)
ERS and ERSP

Event-related Spectrogram (ERS)

Event-Related Spectral Perturbation (ERSP)

Baseline is Subtracted

Revealing event-related changes in power

$10\log_{10}(\frac{SG(t,f)}{baseline(f)})$
Wavelet Specification

Answer: The first \#cycles controls the basic duration of the wavelet in cycles. The second factor controls the degree of shortening of time windows as frequency increases:
- 0 = no shortening = FFT (duration remains constant with frequency)
- 1 = pure wavelet (#cycles remains constant with frequency)
- 0.5 = intermediate, a compromise that reduces HF time resolution to gain more frequency resolution
Part 3: Coherence Analysis

- Goal: How much do two signals resemble each other

- Coherence = complex version of correlation: how similar are power and phase at each frequency?

- Variant: phase coherence (phase locking, etc.) considers only phase similarity, ignoring power
  - Regular coherence is simply a power-weighted phase coherence
Coherence

\[ C(f,t) \propto \sum_{k=\text{trials}} F1_k(f,t)F2_k(f,t) \]

\[ a_1 e^{i\theta_1} a_2 e^{-i\theta_2} \propto e^{i(\theta_1 - \theta_2)} \]

Fourier time series \( F_{S1} \) and \( F_{S2} \)

Phase difference between \( S1 \) and \( S2 \),
Part 3a: Inter-Trial Coherence

- Goal: How much do different trials resemble each other?
- Phase coherence not between two processes, but between multiple trials of the same process
- Defined over a (generally) narrow frequency range
EEGLAB’s Inter-Trial Coherence is \textit{phase} ITC

\[
ITPC(f, t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k(f, t)}{|F_k(f, t)|}
\]

Normalized (no amplitude information)

same time, different trials

Trial 1
amplitude 0.5 phase 0

Trial 2
amplitude 1 phase 90

Trial 3
amplitude 0.25 phase 180

\[
\text{POWER} = \text{mean}(\text{amplitudes}^2) \\
0.44 \text{ or } -8.3 \text{ dB}
\]

\[
\text{COHERENCE} = \text{mean}(\text{phase vector}) \\
\text{Norm 0.33}
\]
ITC Example (3 trials)

Intertrial Coherence (ITC)

ITC: .05

ITC: .80

Slide courtesy of Stefan Debener
Multiple possible origins of an ERP

- Event Related Potential can result from
  - ITC increase (with no change in power)
  - ITC & Power change
ERPIMAGE showing ERP arising from only ITC

INTER-TRIAL COHERENCE (phase resetting)

AVERAGE ERP

NO AMPLITUDE INCREASE

-9.433 dB

P = 0.02

INTER-TRIAL COHERENCE

5 Hz

P = 0.02
Compare: ERP arising from both ITC and increased power

J. Onton & S. Makeig, 2005
Putting it all together

Exercise

All: Compute ERSP/ITC for a component of your choice

Compute ERP Image (with ERSP and ITC displayed*)

Use all of this information to explain the origin of the Evoked Response

Question: Which changes are significant? Use the options in ERP Image and ERSP dialogs to set significance threshold e.g. 0.01. Do the results survive?
Part 3b: Event Related Coherence

- Goal: How similar is the event-related response of two signals
  - Typically between channels (problematic due to volume conduction)
  - or between ICs
TWO SIMULATED THETA PROCESSES

Event-related Coherence
Try it!

![Image of software interface](image.png)
Event-Related Coherence Exercise

• Examine event-related coherence between two ICs
  – Which pair did you pick, and why? What do you predict?
  – What did you learn?

• Explore other options:
  – Significance threshold
  – Figure out how to subtract a baseline
  – Phase vs. Linear Coherence