Robust Linear Modelling of EEG data: the LIMO EEG plug-in

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Copy the LIMOEEG folder and move limo_egg to plugins
Overview

• What is LIMO EEG for?
• Plug-in overview
• Theory and practice
  ➢ 1<sup>st</sup> level robust GLM
  ➢ 2<sup>nd</sup> level robust stats
  ➢ Design brainstorming
Accounting for within and between subjects variance

WHAT IS IT FOR?
Full ‘brain’ analysis

- Traditionally compute averages per condition and do your statistics on peaks
  - Peaks are NOT necessarily important events (they are likely to mark the end of a process)
  - Increase type 1 FWER by choosing electrodes

- Averages don’t account for trial variability
- Fixed effect can be biased
- Design flexibility

Rousselet & Pernet – It’s time to up the Game Front. Psychol., 23 May 2011
Hierarchical Linear Model

1st level analysis:
GLM: $Y = X\beta + \varepsilon$
$\rightarrow$ 1 $\beta$ per column of $X$
(= within subject effects)

2nd level analysis:
Robust stats (Yuen $t$-tests, robust GLM, robust Hotelling $T^2$)

Multiple Comparison Correction:
Max, Cluster-Mass, TFCE

Subject 1
Subject 2
Subject 3
Subject 4
...  
Subject N

T-test
Regression
N-way ANOVA
N-way ANCOVA
Rep Measure ANOVA

Bootstrap:
T-test / Regression
N-way ANOVA / ANCOVA
Rep Measure ANOVA

Statistical Maps Corrected p-values
Random Effect Model

Model the data with fixed effects (the experimental conditions) and a random effect (subjects are allowed to have different overall values – considering subjects as a random variable)

Example: present stimuli from intensity -5 units to +5 units around the subject perceptual threshold and measure RT
→ Plot the data per intensity
Random Effect Model

Fixed effect = average across subjects → negative correlation?
**Random Effect Model**

Fixed effect = average across subjects → negative correlation?  
Mixed effect = effect per subject with variable (random) offsets
Design considerations

- ALLOWS YOU TO ANALYZE ANY (PAIRED / UNPAIRED) DESIGNS

Illustration with a set of studies looking at the effect of stimulus phase information

Factorial Designs: N*N*N*…

Categorical designs: Group level analyses of course but also Individual analyses with bootstrap

Regression based designs

Mixed design: Control of low level physical properties

Regression based designs (2 levels)

Parametric designs:
- Study the effect of stimulus properties within subjects
- Effect of aging between subjects

Rousselet, Gaspar, Pernet, Husk, Bennett, Sekuler (2010). Aging and face perception. Front Psy
Quick tour of the interface

PLUG-IN OVERVIEW
Main GUI

also typing: limo_eeg
Import (1\textsuperscript{st} level)
Random Effects (2^{nd} level)
Results
THEORY AND PRACTICE
Linearity

• Means created by lines

• In maths it refers to equations or functions that satisfy 2 properties: additivity (also called superposition) and homogeneity of degree 1 (also called scaling)

• Additivity → $y = x_1 + x_2$ (output $y$ is the sum of inputs $x$s)

• Scaling → $y = \beta x_1$ (output $y$ is proportional to input $x$)

http://en.wikipedia.org/wiki/Linear
What is a linear model?

• An equation or a set of equations that models data and which corresponds geometrically to straight lines, planes, hyper-planes and satisfy the properties of additivity and scaling.

• Simple regression: \( y = \beta_1 x + \beta_2 + \varepsilon \)
• Multiple regression: \( y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 + \varepsilon \)
• One way ANOVA: \( y = u + \alpha_i + \varepsilon \)
• Repeated measure ANOVA: \( y = u + \alpha_i + \varepsilon \)
• ...
A regression is a linear model

• We have an experimental measure $x$ (e.g. stimulus intensity from 0 to 20)
A regression is a linear model

- We have an experimental measure $x$ (e.g. stimulus intensity from 0 to 20)

- We then do the experiment and collect data $y$ (e.g. RTs)
A regression is a linear model

• We have an experimental measure \( x \) (e.g. stimulus intensity from 0 to 20)

• We then do the experiment and collect data \( y \) (e.g. RTs)

• Model: \( y = \beta_1 x + \beta_2 \)

• Do some maths / run a software to find \( \beta_1 \) and \( \beta_2 \)

• \( y^\wedge = 2.7x + 23.6 \)
Linear algebra for regression

• Linear algebra has to do with solving linear systems, i.e. a set of linear equations

• For instance we have observations \((y)\) for a stimulus characterized by its properties \(x_1\) and \(x_2\) such as \(y = x_1 \beta_1 + x_2 \beta_2\)

\[
\begin{align*}
2\beta_1 - \beta_2 &= 0 \\
-\beta_1 + 2\beta_2 &= 3
\end{align*}
\]

\(\beta_1 = 1; \beta_2 = 2\)
Linear algebra for regression

• With matrices, we change the perspective and try to combine columns instead of rows, i.e. we look for the coefficients with allow the linear combination of vectors

\[
\begin{align*}
2\beta_1 - \beta_2 &= 0 \\
-\beta_1 + 2\beta_2 &= 3 \\
\end{align*}
\]

\[
\begin{bmatrix}
2 & -1 \\
-1 & 2 \\
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\end{bmatrix}
=
\begin{bmatrix}
0 \\
3 \\
\end{bmatrix}
\]

\[
\beta_1 = 1 ; \beta_2 = 2
\]
Linear algebra for ANOVA

• In text books we have $y = u + xi + \varepsilon$, that is to say the data (e.g. RT) = a constant term (grand mean $u$) + the effect of a treatment ($xi$) and the error term ($\varepsilon$)

• In a regression $xi$ takes several values like e.g. [1:20]

• In an ANOVA $xi$ is designed to represent groups using 1 and 0
Linear algebra for ANOVA

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\[
y(1..3)1 = 1x1 + 0x2 + 0x3 + 0x4 + c + e11 \\
y(1..3)2 = 0x1 + 1x2 + 0x3 + 0x4 + c + e12 \\
y(1..3)3 = 0x1 + 0x2 + 1x3 + 0x4 + c + e13 \\
y(1..3)4 = 0x1 + 0x2 + 0x3 + 1x4 + c + e13 \\
\]

\[ \begin{pmatrix} 8 \\ 9 \\ 7 \\ 5 \\ 7 \\ 3 \\ 3 \\ 4 \\ 1 \\ 6 \\ 4 \\ 9 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ c \end{pmatrix} + \begin{pmatrix} e1 \\ e11 \\ e12 \\ e13 \end{pmatrix} \]

→ This is like the multiple regression except that we have ones and zeros instead of ‘real’ values so we can solve the same way
Linear Algebra, geometry and Statistics

- $Y = 3$ observations $X = 2$ regressors
- $Y = XB + E \Rightarrow B = \text{inv}(X'X)X'Y \Rightarrow \hat{Y} = XB$

SS total = variance in $Y$
SS effect = variance in $XB$
SS error = variance in $E$
R2 = SS effect / SS total
$F = \text{SS effect/df} / \text{SS error/dfe}$
Linear Algebra, geometry and Statistics

\[ y = \beta x + c \]

Projecting the points on the line at perpendicular angles minimizes the distance^2

\[ Y = y^\wedge + e \]
\[ P = X \text{ inv}(X'X) \ X' \]
\[ y^\wedge = PY \]
\[ e = (I-P)Y \]

An ‘effect’ is defined by which part of X to test (i.e. project on a subspace)

\[ R_0 = I - (X_0*\text{pinv}(X_0)); \]
\[ P = R_0 - R; \]
\[ \text{Effect} = (B'*X'*P*X*B); \]
Linear Algebra, geometry and Statistics

• Projections are great because we can now constrain $Y^\wedge$ to move along any combinations of the columns of $X$

• Say you now want to contrast gp1 vs gp2 in an ANOVA with 3 gp, do $C = [1 \ -1 \ 0 \ 0]$

• Compute $B$ so we have $XB$ based on the full model $X$ then using $P(C(X))$ we project $Y^\wedge$ onto the constrained model (think doing a multiple regression gives different coef than multiple simple regression $\Rightarrow$ project on different spaces)
UH OH... LOOKS LIKE MATH ANXIETY...
GET THE INTEGRAL SIGN.
Let’s analyse one subject

• **Design**: 2 faces (cond1/cond2) + a continuous variable related to the phase information in the stimulus space (~noise)

• LIMO EEG – 1\textsuperscript{st} level analysis
  = make a parameter file per condition (like we would for ERP)
Let’s analyse one subject
What have we done: results

• Image all (R2, condition, covariate)
• Course plots, topoplots
Robust Statistics

WHY & HOW?
Issues with standard stats

• Standard stats are all instantiations of a GLM using an Ordinary Least Square solution \( \rightarrow \) implies looking at the mean

• the breakdown point of an estimator is the proportion of incorrect observations (e.g. arbitrarily large observations) an estimator can handle before giving an incorrect

• For data \( x_1 \) to \( x_n \) – the mean has a bkdp of 0 because we can make the mean large changing any \( x_i \) – the median has a bkdp of 50%

http://en.wikipedia.org/wiki/Robust_statistics
Yes but my data are Gaussian

• Are you sure?
• Micceri (1989). The Unicorn, The Normal Curve, and Other Improbable Creatures. Psych Bul. 105, 156-166
• If the data are Gaussian, the median, the trimmed mean is the same as the mean! So no reason not to use alternative techniques.

• 1st level, uses weighted least square (weights down bad trials)
• 2nd level involves 20% trimmed mean (weights = 0 for bad subjects): t-tests, 1-way ANOVA, Repeated Measures ANOVA (soon)
• For regressions and N-way ANOVA/ANOVA we use an IRLS (all subjects have weights from 0 to 1)

http://en.wikipedia.org/wiki/Robust_statistics
Practical

• One sample t-test on ‘noise’ regressor

→ You can select files by hand, by it’s easier to build lists – right click/run sheffield_mklist.m
→ This makes a list_of_Betas.txt we can use

→ From the GUI, choose ‘Random Effect’
Practical

LIMO EEG expect you to build a template cap to use across subjects (LIMO TOOLS) because only valid electrodes are analysed per subject

= no interpolated values at the subject level
= LIMO EEG deals with missing data

The expected channlocs is in the gp_effects directory
Compute a one-sample t-test on betas parameter 3 i.e. the Effect of stimulus phase information on ERP
Review gp level results
Design questions!

• Let’s think how to analyse your data!
• Nb of conditions / covariates
• contrasts
• 1\textsuperscript{st} level covariates
• 2\textsuperscript{nd} level covariates
Design questions!

• Typical 2*2 design

→ 1\textsuperscript{st} level vs 2\textsuperscript{nd} level, where to model the interaction

• Testing the effect of covariate within or between conditions?