Forward and Inverse
EEG Source Modeling

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Source modeling

forward problem

- physiological source electrical current
- body tissue volume conductor
- observed potential or field

inverse problem

R. Oostenveld, Z. Akalin Acar & S. Makeig, 2010
Symmetry, orientation and activation

- radially symmetric, i.e. randomly-oriented
- asynchronously activated
- synchronously activated parallel-oriented
Neuronal currents

Stellate cell

Closed field

Pyramidal cell

Open field

EEG

MEG

R. Oostenveld & S. Makeig, 2010
scalp dynamics ≠ source dynamics!
EEG volume conduction

• **Potential difference between electrodes is measured.** This corresponds to current flowing through skin:
  – Only tiny fraction of current passes through skull
  – Therefore the model should describe both skull and skin as accurately as possible.

• **Problems with skull modeling**
  – Poorly visible in anatomical MRI (T2)
  – Thickness varies
  – Conductivity is not homogeneous
  – Complex geometry at front and base of skull

R. Oostenveld 2008
Exact Formulation of the Forward Problem

\[ \nabla \cdot (\sigma \nabla \Phi) = -\nabla \cdot J^p \quad \text{inside } V \]

\[ \sigma \frac{\partial \Phi}{\partial n} = 0 \quad \text{on } S \]

\( \sigma(x,y,z) \): conductivity distribution

\( p \): current source

Z. Akalin Acar, 2010
To Solve the Forward Head Model Problem …

WE NEED

→ Head Model
  - Conductivity values
  - Geometry

→ Sensor Locations

→ Possible source distribution
  - Magnitudes
  - Locations
  - Directions

→ Solver

Z. Akalin Acar, 2010
Source Localization Requirements

- Selected/processed EEG signal
  \[ \rightarrow \text{Simple single-source scalp map!} \]
- Number/positions of electrodes on the head surface
- Numerical head model
- Co-registration of EEG electrodes with head model
- *A priori* information/guess about the source space
- Choice of inverse model
- Choice of numerical method

Z. Akalin Acar, 2010
Volume conductor model

• Electrical properties of tissue
• Geometrical description
  – spherical model
  – realistically shaped model

→ Describes how the currents flow, from where they may originate

R. Oostenveld & S. Makeig, 2010
Errors in Simple Head Models

→ In the volume conductor model
→ In the electrode locations

Z. Akalin Acar, 2010
Head Model Comparison

◆ Simple head models
  - Single sphere
  - 3-4 Layer Spherical
  - Spheroid

◆ Realistic head models
  - Boundary Element Method
  - Finite Element Method
  - Finite Difference Method

Z. Akalin Acar, 2010
Effects of Head Model

Spherical head model  
(3-layer standard)

Standard MNI head model  
(4-layer mean BEM)

Z. Akalin Acar, 2010
Spherical volume conductor

- Advantages of the *spherical* head model
  - mathematically exact
  - fast to compute
  - reasonably accurate
  - easy to use

- Disadvantages of the *spherical* model
  - difficult to align properly
  - inaccurate in some regions

R. Oostenveld & S. Makeig, 2010
Realistic volume conductor

• Advantages of a realistic head model
  – a more accurate solution (especially for EEG)
• Disadvantages of a realistic model
  – more work to build from an MR image
  – slower to compute
  – might be numerically instable
  – harder to make between-subject comparisons

→ A pragmatic (easy, cheap) solution is to use
  a standard (mean) realistic head model (MNI).
Realistic volume conductor

- Computational methods for volume conduction problem that allow realistic geometries:
  - Boundary Element Method (BEM)
  - Finite Element Method (FEM)

- Geometrical description
  - Triangles (planar or quadratic)
  - Tetrahedra (3-D)
BEM volume conductor

- **Boundary Element Method**
  - description of geometry by compartments
  - each compartment is
    - homogenous
    - isotropic
  - important tissues
    - skin
    - skull
    - brain
    - (CSF)
  - triangulated surfaces as boundaries
  - surfaces should be closed
Electromagnetic source localization using realistic head models (NFT)

T1-weighted MRI

Proton Density Segmentation

Simple Map

Signal Processing

Source Image

EEG/MEG

Zeynep Akalin Acar, ’06
A Four-Layer BEM Head Model

Neuroelectromagnetic Forward head modeling Toolbox (NFT)

# of elements

- Scalp: 6900
- Skull: 6800
- CSF: 9000
- Brain: 8800

Total: 31500

Z. Akalin Acar, 2010
FEM volume conductor

• Tesselate the 3-D volume into solid tetrahedra
  - Large number of elements
  - Each tetrahedron can have its own conductivity
  - Each tetrahedron can have its own anisotropy

• FEM is most accurate numerical method
  – Computationally expensive
  – Accurate conductivities are not known

R. Oostenveld & S. Makeig, 2010
Inverse problem methods

• **Single and multiple dipole models**
  – Minimize error between the model and the measured potential/field

• **Distributed dipole models**
  – Perfect fit of model to the measured potential/field
  – Minimize an additional constraint on sources
    • LORETA (assume a smooth distribution)
    • Minimum Norm (L2, minimum power at the cortex)
    • Minimum Current (L1, minimum current in the cortex)
Inverse problem methods

• Spatial source filtering
  – **Scan whole brain** with single dipole and compute the filter output at every location (second-order covariance matrix)
    • MUSIC algorithm
    • *Beamforming* (e.g., LCMV, SAM, DICS)
  – **Perform ICA decomposition** (higher-order statistics)
    • Of the scalp maps at individual moments
    • ICA gives the projections of the sources to the scalp surface, i.e., ‘**simple**’ maps!

→ ICA solves ‘the first half’ of the inverse problem (‘What?’)
Equivalent current dipoles

• Physical/mathematical motivation
  – Any current distribution can be written as a multipole expansion
  – First term: monopole (must be 0)
  – Second term: dipole
  – Higher order terms: quadrupole, ...

• Convenience
  – **Dipoles** can be used as building blocks in distributed source models
Equivalent current dipoles

- Over-determined (if 1 source)
- Under-determined (if >1)
- Nonlinear optimization

Equivalent Current Dipole
Equivalent current dipole
Measured Errors in Dipole Source Localization

♦ **Experimental studies**
  - Phantom $\rightarrow$ 10 mm loc. error (Henderson & Butler, 1975)
  - Human skull $\rightarrow$ 35 mm (Weinberg et al, 1986)

♦ **Simulation studies**
  - 3-layer model $\rightarrow$ 15-25 mm (Roth et al, 1993)
  - 3-layer model $\rightarrow$ 9-14 mm (Vanrumste et al, 2002)
  - Human skull $\rightarrow$ 25 mm (Fletcher et al, 1993)
  - 3-layer model $\rightarrow$ ~8 mm (Akalin Acar, 2005)
Source Localization Errors

- For a 3-layer spherical head model
- Relative to 4-layer realistic BEM head model
Single vs. multiple dipole models

• Manipulate source parameters to minimize error between measured and model data
  – Position of each source
  – Orientation of each source
  – Strength of each source

• Orientation and strength together correspond to the “dipole moment” and can be estimated linearly
  – Position is estimated non-linearly by iterative source parameter estimation

R. Oostenveld & S. Makeig, 2010
Dipole scanning: grid search

- Define grid with allowed dipole locations
- Compute optimal dipole moment for each location
- Compute value of goal-function
- Plot value of goal-function on grid
- Number of evaluations:
  - single dipole, 1 cm grid:  ~4,000
  - single dipole, ½ cm grid:  ~32,000
  - BUT two dipoles, 1 cm grid:  ~16,000,000

R. Oostenveld & S. Makeig, 2010
Dipole fitting: nonlinear search

• Start with an initial guess from coarse fitting
  – evaluate the local derivative of goal-function
  – “walk down hill” to the most optimal solution

• Number of evaluations needed ~ 100
Effect of Number of Electrodes

- Single dipole source
- 3-layer spherical head model
- 1152 solution points

Michel et al, 2004

Z. Akalin Acar, 2010
Effects of Skull Conductivity Estimate

Measurements of skull conductivity:

- In vivo
- In vitro

- MR-EIT
- Magnetic stimulation
- Current injection

Hoekama et al, 2003

He et al, 2005

Z. Akalin Acar, 2010
## Effects of Skull Conductivity Estimate

### Brain to skull ratio

<table>
<thead>
<tr>
<th>Study</th>
<th>Year</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rush and Driscoll</td>
<td>1968</td>
<td>80</td>
</tr>
<tr>
<td>Cohen and Cuffin</td>
<td>1983</td>
<td>80</td>
</tr>
<tr>
<td>Oostendorp et al</td>
<td>2000</td>
<td>15</td>
</tr>
<tr>
<td>Lai et al</td>
<td>2005</td>
<td>25</td>
</tr>
</tbody>
</table>

### Skull conductivity by age

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Age</th>
<th>(\sigma) (mS/m)</th>
<th>Sd (mS/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agar-agar phantom</td>
<td>–</td>
<td>43.6</td>
<td>3.1</td>
</tr>
<tr>
<td>Patient 1</td>
<td>11</td>
<td>80.1</td>
<td>5.5</td>
</tr>
<tr>
<td>Patient 2</td>
<td>25</td>
<td>71.2</td>
<td>8.3</td>
</tr>
<tr>
<td>Patient 3</td>
<td>36</td>
<td>53.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Patient 4</td>
<td>46</td>
<td>34.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Patient 5</td>
<td>50</td>
<td>32.0</td>
<td>4.5</td>
</tr>
<tr>
<td>Post mortem skull</td>
<td>68</td>
<td>21.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

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Z. Akalin Acar, 2010

Hoekama et al, 2003
“The choice of a particular reference electrode … does not change in any way the biophysical information contained in the potential distribution. It does not in any way change the relation between source and potential, except for an additive constant of no physical significance.”

- Geselowitz, 1998
Distributed source models

• Position of the source is not estimated as such
  – Pre-defined grid (3-D volume or cortical sheet)
  – Strength is estimated at each grid element
  – In principle, a linear problem, easy to solve, BUT...
    • More “unknowns” (parameters) than “knowns” (channels, measurements)
    • An infinite number of solutions can explain the data perfectly (not necessarily physiologically plausible!)
  – So, additional constraints are required ...
Conformal cortical patch source model
Conformal cortical patch source model

Model a source estimate as a sum of overlapping patches

Z. Akalin Acar, 2010
Comparing source models

for an IC of an intracranial data set

Estimated IC cortical projection

Equivalent Current Dipole Model

Sparse Patch Basis Model

Z. Akalin Acar, 2010
Summary I

• Forward modeling is required for the interpretation of scalp topographies
• Interpretation of scalp topographies is inverse modelling “source estimation”
• Mathematical techniques are available to aid in interpreting scalp topographies
  → These are inverse source models
Summary II

• Inverse modeling
  – Model assumption for volume conductor
  – Model assumption for source (i.e., dipole)
  – Additional assumptions on source
• Single point-like sources
• Multiple point-like sources
• Distributed sources
  – Different mathematical solutions
    • Dipole fitting (linear and nonlinear)
    • Linear estimation (regularized)