Modeling Distributed Brain Network Dynamics

Tim Mullen
12th EEGLAB Workshop
UC San Diego
La Jolla, CA
November 20, 2010
Different types of connectivity
Different types of connectivity

- Structural Connectivity
  - anatomical
Different types of connectivity

- Structural Connectivity
  - anatomical

DTI
Different types of connectivity

- **Structural Connectivity**
  - anatomical

- **Functional Connectivity**
  - symmetric, correlative

[Diagram of neuron and DTI]
Different types of connectivity

- **Structural Connectivity**
  - anatomical

- **Functional Connectivity**
  - symmetric, correlative

- DTI
- MEG/EEG
- fMRI
Different types of connectivity

- **Structural Connectivity**
  - anatomical

- **Functional Connectivity**
  - symmetric, correlative

- **Effective Connectivity**
  - asymmetric, causal, information flow
Different types of connectivity

- **Structural Connectivity**
  - anatomical

- **Functional Connectivity**
  - symmetric, correlative

- **Effective Connectivity**
  - asymmetric, causal, information flow

Measures:
- DTI
- MEG/EEG
- fMRI
- MEG/EEG
- fMRI?
Different types of connectivity

- **Structural Connectivity**
  - anatomical

- **Functional Connectivity**
  - symmetric, correlative

- **Effective Connectivity**
  - asymmetric, causal, information flow

Methods:
- DTI
- MEG/EEG
- fMRI
- fMRI?
Many ways to model effective connectivity in EEG

- Coherence, Phase-locking value
- Cross-correlation
- Transfer Entropy
- Dynamic Causal Models
- Structural Equation Models
- Granger-Causal methods

...
Many ways to model effective connectivity in EEG

- Coherence, Phase-locking value
- Cross-correlation
- Transfer Entropy
- Dynamic Causal Models
- Structural Equation Models
- Granger-Causal methods

...
Granger Causality

- Relies on two assumptions:
  1. causes should precede their effects in time
  2. information in a cause’s past should improve the prediction of the effect, above and beyond the information contained in the effect’s own past.
Granger Causality

- First introduced by Wiener (1958). Later reformulated by Granger (1969) in the context of linear stochastic autoregressive models
- Relies on two assumptions:
  1. causes should precede their effects in time
  2. information in a cause’s past should improve the prediction of the effect, above and beyond the information contained in the effect’s own past.

This is **not** the same as (cross-)correlation!
Multivariate Autoregressive (MVAR) Modeling

EEG

\[ X_1(t) \]
\[ X_2(t) \]
\[ X_3(t) \]
\[ X_4(t) \]

MVAR

Granger Causality

Coherence

Spectrum

...
We have $M$ variables (e.g., EEG channels or source activations): 
\[ \mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), ..., \mathbf{X}_M(t)]^T \]
Multivariate Autoregressive (MVAR) Modeling

- We have M variables (e.g., EEG channels or source activations):
  \[ \mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), ..., \mathbf{X}_M(t)]^T \]

MVAR model

\[
\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}(k) \mathbf{X}(t - k) + \mathbf{E}(t)
\]
We have $M$ variables (e.g., EEG channels or source activations):

\[ \mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), \ldots, \mathbf{X}_M(t)]^T \]

Multivariate Autoregressive (MVAR) Modeling

\[ \mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}(k) \mathbf{X}(t - k) + \mathbf{E}(t) \]
Multivariate Autoregressive (MVAR) Modeling

- We have M variables (e.g., EEG channels or source activations):
  \[ \mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), \ldots, \mathbf{X}_M(t)]^T \]

**MVAR model**

\[ \mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}(k) \mathbf{X}(t - k) + \mathbf{E}(t) \]

- Multichannel data vector at current time \( t \)
- \( M \times M \) matrix of model coefficients indicating variable dependencies at lag \( k \)

\[ \mathbf{A}(k) = \begin{pmatrix}
  a_{11}(k) & \cdots & a_{1M}(k) \\
  \vdots & \ddots & \vdots \\
  a_{M1}(k) & \cdots & a_{MM}(k)
\end{pmatrix} \]
Multivariate Autoregressive (MVAR) Modeling

- We have M variables (e.g., EEG channels or source activations): $\mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), ..., \mathbf{X}_M(t)]^T$

MVAR model

$$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}(k)\mathbf{X}(t - k) + \mathbf{E}(t)$$

- Multichannel data vector at current time $t$
- M x M matrix of model coefficients indicating variable dependencies at lag $k$
- Multichannel data $k$ samples in the past

$$\mathbf{A}(k) = \begin{pmatrix}
  a_{11}(k) & \cdots & a_{1M}(k) \\
  \vdots & \ddots & \vdots \\
  a_{M1}(k) & \cdots & a_{MM}(k)
\end{pmatrix}$$
Multivariate Autoregressive (MVAR) Modeling

- We have M variables (e.g., EEG channels or source activations):
  \[ \mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), ..., \mathbf{X}_M(t)]^T \]

**MVAR model**

\[
\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}(k)\mathbf{X}(t - k) + \mathbf{E}(t)
\]

- \( \mathbf{X}(t) \): Multichannel data vector at current time \( t \)
- \( \mathbf{A}(k) \): M x M matrix of model coefficients indicating variable dependencies at lag \( k \)
- \( \mathbf{E}(t) \): Multichannel data \( k \) samples in the past
- \( \mathbf{A}(k) = \begin{pmatrix}
    a_{11}(k) & \cdots & a_{1M}(k) \\
    \vdots & \ddots & \vdots \\
    a_{M1}(k) & \cdots & a_{MM}(k)
  \end{pmatrix} \)
Multivariate Autoregressive (MVAR) Modeling

- We have $M$ variables (e.g., EEG channels or source activations):
  \[ X(t) = [X_1(t), X_2(t), \ldots, X_M(t)]^T \]

Multivariate Autoregressive (MVAR) model:

\[ X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t) \]

- Multichannel data vector at current time $t$
- M x M matrix of model coefficients indicating variable dependencies at lag $k$
- Multichannel data $k$ samples in the past
- Random noise process

Model order:

\[ A(k) = \begin{pmatrix}
    a_{11}(k) & \cdots & a_{1M}(k) \\
    \vdots & \ddots & \vdots \\
    a_{M1}(k) & \cdots & a_{MM}(k)
\end{pmatrix} \]

\[ E(t) = N(0, V) \]
Multivariate Autoregressive (MVAR) Modeling

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order ($p$):

$$\text{AIC}(p) = 2\log(\det(V)) + M^2 p/N$$
Multivariate Autoregressive (MVAR) Modeling

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

\[
\text{AIC}(p) = 2\log(\det(\mathbf{V})) + M^2p/N
\]
Multivariate Autoregressive (MVAR) Modeling

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order \( p \):

\[
\text{AIC}(p) = 2\log(\text{det}(\mathbf{V})) + M^2 p / N
\]

Penalizes high model orders (parsimony)

entropy rate (amount of prediction error)
Multivariate Autoregressive (MVAR) Modeling

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

\[
\text{AIC}(p) = 2\log(\det(\mathbf{V})) + M^2 p/N
\]

**Penalizes high model orders (parsimony)**

**entropy rate (amount of prediction error)**

![Graph showing AIC bits vs model order](chart.png)
Multivariate Autoregressive (MVAR) Modeling

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

\[ \text{AIC}(p) = 2\log(\det(V)) + {M^2p}/N \]

Penalizes high model orders (parsimony)

entropy rate (amount of prediction error)

optimal order

![Graph showing AIC values for different model orders](image)
MVAR Modeling: Assumptions
MVAR Modeling: Assumptions

- “Weak” stationarity of the data
MVAR Modeling: Assumptions

- “Weak” stationarity of the data
  - mean and variance do not change with time
MVAR Modeling: Assumptions

- “Weak” stationarity of the data
  - mean and variance do not change with time
- An EEG trace containing ERPs is a classic example of a non-stationary time-series
MVAR Modeling: Assumptions

- "Weak" stationarity of the data
  - mean and variance do not change with time
  - An EEG trace containing ERPs is a classic example of a non-stationary time-series

- Stability
MVAR Modeling: Assumptions

- “Weak” stationarity of the data
  - mean and variance do not change with time
  - An EEG trace containing ERPs is a classic example of a non-stationary time-series

- Stability
  - Technically, an MVAR process is stable if the reverse characteristic polynomial of the process has all roots outside the complex unit circle (all eigenvalues of $A$ have modulus less than 1)
MVAR Modeling: Assumptions

- “Weak” stationarity of the data
  - mean and variance do not change with time
  - An EEG trace containing ERPs is a classic example of a non-stationary time-series

- Stability
  - Technically, an MVAR process is stable if the reverse characteristic polynomial of the process has all roots outside the complex unit circle (all eigenvalues of $A$ have modulus less than 1)
  - Importantly, stability implies stationarity and SIFT provides you techniques for verifying the stability
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?

$X(t) = \sum_{k=1}^{p} A(k)X(t-k) + E(t)$
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?

\[ X(t) = \sum_{k=1}^{p} A(k)X(t-k) + E(t) \]
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?

$x(t) = \sum_{k=1}^{p} A(k)x(t-k) + e(t)$

prediction error for $X_1$
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?

$X(t) = \sum_{k=1}^{p} A(k) X(t-k) + E(t)$

$\text{prediction error for } X_1$
Granger Causality

Test: Does $\mathbf{X}_4$ granger-cause $\mathbf{X}_1$?

$\mathbf{X}(t) = \sum_{k=1}^{p} \mathbf{A}(k) \mathbf{X}(t-k) + \mathbf{E}(t)$

$\mathbf{X}_{-4}(t) = \sum_{k=1}^{p} \tilde{\mathbf{A}}(k) \mathbf{X}_{-4}(t-k) + \tilde{\mathbf{E}}(t)$

prediction error for $\mathbf{X}_1$
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?

$$X(t) = \sum_{k=1}^{p} A(k)X(t-k) + E(t)$$

$X(t) = \sum_{k=1}^{p} \tilde{A}(k)X_{-4}(t-k) + \tilde{E}(t)$

prediction error for $X_1$
Granger Causality

Test: Does $X_4$ granger-cause $X_1$?

$$X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t)$$

$$X_{-4}(t) = \sum_{k=1}^{p} \tilde{A}(k)X_{-4}(t - k) + \tilde{E}(t)$$

prediction error for $X_1$
Granger Causality
Granger Causality

Granger (1969) quantified this definition for bivariate processes in the form of an F-ratio:

\[
F_{X_1 \leftarrow X_2} = \ln \left( \frac{\text{var}(\tilde{E}_1)}{\text{var}(E_1)} \right) = \ln \left( \frac{\text{var}(X_1(t) \mid X_1(\cdot))}{\text{var}(X_1(t) \mid X_1(\cdot), X_2(\cdot))} \right)
\]
Granger Causality

Granger (1969) quantified this definition for bivariate processes in the form of an F-ratio:

\[
F_{X_1 \leftarrow X_2} = \ln \left( \frac{\text{var}(\tilde{E}_1)}{\text{var}(E_1)} \right) = \ln \left( \frac{\text{var}(X_1(t) \mid X_1(\cdot))}{\text{var}(X_1(t) \mid X_1(\cdot), X_2(\cdot))} \right)
\]

Alternately, for a **multivariate interpretation** we can fit a single MVAR model to all channels and apply the following definition:

\[X_j \text{ granger-causes } X_i \text{ condition on all other variables in } X\]

if and only if \(A_{ij}(k) >> 0\) for some lag \(k \in \{1, \ldots, p\}\)
Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

\[
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix}
= 
\begin{pmatrix}
0.5 & 0 \\
0.7 & 0.2
\end{pmatrix}
\begin{pmatrix}
X_1(t-1) \\
X_2(t-1)
\end{pmatrix}
+ 
\begin{pmatrix}
E_1(t) \\
E_2(t)
\end{pmatrix}
\]

\[
X_1(t) = -0.5 \, X_1(t-1) + 0 \, X_2(t-1) + E_1(t)
\]

\[
X_2(t) = 0.7 \, X_1(t-1) + 0.2 \, X_2(t-1) + E_2(t)
\]

Which causal structure does this model correspond to?

a) 1  2  b) 1  ↔  2  c) 1  ↔  2
Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

\[
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix} = \begin{pmatrix}
0.5 & 0 \\
0.7 & 0.2
\end{pmatrix} \begin{pmatrix}
X_1(t-1) \\
X_2(t-1)
\end{pmatrix} + \begin{pmatrix}
E_1(t) \\
E_2(t)
\end{pmatrix}
\]

\[
X_1(t) = -0.5X_1(t-1) + 0X_2(t-1) + E_1(t)
\]

\[
X_2(t) = 0.7X_1(t-1) + 0.2X_2(t-1) + E_2(t)
\]

Which causal structure does this model correspond to?

a) \[1 \rightarrow 2\]  
b) \[1 \leftrightarrow 2\]  
c) \[1 \leftrightarrow 2\]
Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

\[
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix} = 
\begin{pmatrix}
0.5 & 0 \\
0.7 & 0.2
\end{pmatrix} 
\begin{pmatrix}
X_1(t-1) \\
X_2(t-1)
\end{pmatrix} + 
\begin{pmatrix}
E_1(t) \\
E_2(t)
\end{pmatrix}
\]

\[
X_1(t) = -0.5X_1(t-1) + 0X_2(t-1) + E_1(t)
\]

\[
X_2(t) = 0.7X_1(t-1) + 0.2X_2(t-1) + E_2(t)
\]

Which causal structure does this model correspond to?

a) 1 → 2  

b) 1 ← 2  

c) 1 ←→ 2
Granger Causality Quiz

- Example: 2-channel MVAR process of order 1

\[
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix} =
\begin{pmatrix}
0.5 & 0 \\
0.7 & 0.2
\end{pmatrix}
\begin{pmatrix}
X_1(t-1) \\
X_2(t-1)
\end{pmatrix} +
\begin{pmatrix}
E_1(t) \\
E_2(t)
\end{pmatrix}
\]

\[
X_1(t) = -0.5X_1(t-1) + 0X_2(t-1) + E_1(t)
\]

\[
X_2(t) = 0.7X_1(t-1) + 0.2X_2(t-1) + E_2(t)
\]

Which causal structure does this model correspond to?

- a) 1 → 2
- b) 1 ← 2
- c) 1 ↔ 2
Granger Causality – Frequency Domain

\[ X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t) \]
Granger Causality – Frequency Domain

\[ X(t) = \sum_{k=1}^{p} A(k) X(t - k) + E(t) \]

Fourier-transforming \( A(k) \) we obtain

\[ A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk} \]

Likewise, \( X(f) \) and \( E(f) \) correspond to the Fourier transforms of the data and residuals, respectively.
Granger Causality – Frequency Domain

\[ X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t) \]

Fourier-transforming \( A(k) \) we obtain

\[ A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk} \]

We can then define the spectral matrix \( X(f) \) as follows:

\[ X(f) = A(f)^{-1}E(f) = H(f)E(f) \]

Where \( H(f) \) is the transfer matrix of the system.

Likewise, \( X(f) \) and \( E(f) \) correspond to the Fourier transforms of the data and residuals, respectively.
Granger Causality – Frequency Domain

\[ X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t) \]

Fourier-transforming \( A(k) \) we obtain

\[ A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk} \]

Likewise, \( X(f) \) and \( E(f) \) correspond to the fourier transforms of the data and residuals, respectively.

We can then define the spectral matrix \( X(f) \) as follows:

\[ X(f) = A(f)^{-1}E(f) = H(f)E(f) \]

Where \( H(f) \) is the transfer matrix of the system.

**Definition:** If \( |A_{ij}(f)| \) is significantly non-zero, then \( X_j \) granger-causes \( X_i \) (at frequency \( f \)) conditioned on all other vars in \( X \)
Granger Causality – Frequency Domain Estimators

(some) Coherence measures

\[
C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}}
\]

\[\hat{S} = S^{-1}\]

\[
P_{ij}(f) = \frac{\hat{S}_{ij}(f)}{\sqrt{\hat{S}_{ii}(f)\hat{S}_{jj}(f)}}
\]

\[G_i(f) = \sqrt{1 - \frac{\text{det}(S(f))}{S_{ii}(f)M_{ii}(f)}}\]

\[S(f) = X(f)X(f)^*\] is the spectral density matrix of \(X\)
(some) GC measures

$$\theta^2_{ij}(f) = \left|H_{ij}(f)\right|^2$$

(non-normalized) Directed Transfer Function (DTF)

$$\gamma^2_{ij}(f) = \frac{\left|H_{ij}(f)\right|^2}{\sum_{k=1}^{M}\left|H_{ik}(f)\right|^2}$$

Normalized DTF

$$\delta^2_{ij}(f) = \eta^2_{ij}(f)P_{ij}^2(f) \quad \text{where} \quad \eta^2_{ij}(f) = \frac{\left|H_{ij}(f)\right|^2}{\sum_f \sum_{k=1}^{M}\left|H_{ik}(f)\right|^2}$$

Direct DTF

$$\pi^2_{ij}(f) = \frac{A_{ij}(f)^2}{\sum_{k=1}^{M}\left|A_{kj}(f)\right|^2}$$

Normalized Partial Directed Coherence (PDC)
Time-Frequency GC
Time-Frequency GC

- Brain network dynamics often change rapidly with time
  - event-related responses
  - transient network changes during sequential information processing
Time-Frequency GC

- Brain network dynamics often change rapidly with time
  - event-related responses
  - transient network changes during sequential information processing

- Electrophysiological processes often exhibit oscillatory phenomena, making them well-suited for frequency-domain analysis
Time-Frequency GC

- Brain network dynamics often change rapidly with time
  - event-related responses
  - transient network changes during sequential information processing
- Electrophysiological processes often exhibit oscillatory phenomena, making them well-suited for frequency-domain analysis
- How can we perform time-varying, frequency-domain analysis of network dynamics?
Time-Frequency GC
Time-Frequency GC

- Many ways to do time-varying MVAR estimation
Time-Frequency GC

- Many ways to do time-varying MVAR estimation
  - Sliding-window adaptive multivariate autoregression (AMVAR)
Time-Frequency GC

- Many ways to do time-varying MVAR estimation
  - Sliding-window adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
Time-Frequency GC

- Many ways to do time-varying MVAR estimation
  - Sliding-window adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
  - Kalman Filtering
Time-Frequency GC

- Many ways to do time-varying MVAR estimation
  - Sliding-window adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
  - Kalman Filtering
  - ...

   ...
Time-Frequency GC

- Many ways to do time-varying MVAR estimation
  - Sliding-window adaptive multivariate autoregression (AMVAR)
  - Non-parametric MVAR estimation (minimum-phase spectral matrix factorization)
- Kalman Filtering
- ...

...
Time-Frequency GC

Analogous to short-time Fourier transform
Time-Frequency GC

Analogous to short-time fourier transform
Time-Frequency GC

\[ X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t) \]

\[ A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk} \]

Analogous to short-time fourier transform
Time-Frequency GC

Analogous to short-time fourier transform

\[
X(t) = \sum_{k=1}^{p} A(k)X(t-k) + E(t)
\]

\[
A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk}
\]
Time-Frequency GC

\[ X(t) = \sum_{k=1}^{p} A(k)X(t-k) + E(t) \]

\[ A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk} \]

Analogous to short-time fourier transform
Time-Frequency GC

\[ X(t) = \sum_{k=1}^{p} A(k)X(t - k) + E(t) \]
\[ A(f) = -\sum_{k=0}^{p} A(k)e^{-i2\pi fk} \]

Analogous to short-time Fourier transform

- MVAR → GC → ensemble normalization
- trials → sources
- From
- To
- time → frequency

sources

ensemble normalization
Time-Frequency GC
Time-Frequency GC

- **What is a good window length?**
Time-Frequency GC

- **What is a good window length?**
- **Considerations:**
Time-Frequency GC

- What is a good window length?
- Considerations:
  - Temporal smoothing
Time-Frequency GC

- What is a good window length?
- Considerations:
  - Temporal smoothing
  - Local stationarity
Time-Frequency GC

What is a good window length?

Considerations:
- Temporal smoothing
- Local stationarity
- Sufficient amount of data
Time-Frequency GC

- **What is a good window length?**

  - Considerations:
    - Temporal smoothing
    - Local stationarity
    - Sufficient amount of data
    - Process dynamics
Time-Frequency GC

Consideration: Temporal Smoothness

Ding et al, 2000
Time-Frequency GC

Consideration: Temporal Smoothness

Too-large windows may smooth out interesting transient dynamic features.

Ding et al, 2000
Time-Frequency GC

Consideration: Local Stationarity

![Graph showing cross-correlation over time with different window sizes.](image)
Time-Frequency GC

Consideration: Local Stationarity

Too-large windows may not be locally-stationary
Time-Frequency GC

Consideration: Local Stationarity
Time-Frequency GC
Time-Frequency GC

Consideration: Sufficient data
Time-Frequency GC

Consideration: Sufficient data

M = number of variables
Time-Frequency GC

Consideration: Sufficient data

$M = \text{number of variables}$

$p = \text{model order}$
Consideration: Sufficient data

\[ M = \text{number of variables} \]
\[ p = \text{model order} \]
\[ N_{tr} = \text{number of trials} \]
Time-Frequency GC

Consideration: Sufficient data

\[ M = \text{number of variables} \]
\[ p = \text{model order} \]
\[ N_{tr} = \text{number of trials} \]
\[ W = \text{length of each window (sample points)} \]
Time-Frequency GC

Consideration: Sufficient data

M = number of variables
p = model order
N_{tr} = number of trials
W = length of each window (sample points)
Consideration: Sufficient data

\( M = \) number of variables
\( p = \) model order
\( N_{tr} = \) number of trials
\( W = \) length of each window (sample points)

We have \( M^2p \) model coefficients to estimate. This requires a minimum of \( M^2p \) independent samples.
Consideration: Sufficient data

$M = \text{number of variables}$
$p = \text{model order}$
$N_{tr} = \text{number of trials}$
$W = \text{length of each window (sample points)}$

We have $M^2p$ model coefficients to estimate. This requires a minimum of $M^2p$ independent samples.
So we have the constraint $M^2p \leq N_{tr} W$. 

Time-Frequency GC
Time-Frequency GC

Consideration: Sufficient data

M = number of variables
p = model order
N_{tr} = number of trials
W = length of each window (sample points)

We have $M^2p$ model coefficients to estimate. This requires a minimum of $M^2p$ independent samples. So we have the constraint $M^2p \leq N_{tr} W$.

In practice, however, a better heuristic is $M^2p \leq (1/10)N_{tr} W$. 
Time-Frequency GC

Consideration: Sufficient data

M = number of variables
p = model order
N_{tr} = number of trials
W = length of each window (sample points)

We have $M^2p$ model coefficients to estimate. This requires a minimum of $M^2p$ independent samples.

So we have the constraint $M^2p \leq N_{tr} W$.

In practice, however, a better heuristic is $M^2p \leq (1/10)N_{tr} W$.

Or: $W \geq 10(M^2p/N_{tr})$

10x more data points than parameters to estimate
Consideration: Sufficient data

$M = \text{number of variables}$

$p = \text{model order}$

$N_{tr} = \text{number of trials}$

$W = \text{length of each window (sample points)}$

We have $M^2p$ model coefficients to estimate. This requires a minimum of $M^2p$ independent samples.

So we have the constraint $M^2p \leq N_{tr} W$.

In practice, however, a better heuristic is $M^2p \leq (1/10)N_{tr} W$.

Or: $W \geq 10(M^2p/N_{tr})$

SIFT will let you know if your window length is not optimal.

10x more data points than parameters to estimate
Time-Frequency GC
Time-Frequency GC

Consideration: Process dynamics
Time-Frequency GC

Consideration: Process dynamics

- Your window must be larger than the maximum expected interaction time lag between any two processes.
Time-Frequency GC

**Consideration: Process dynamics**

- Your window must be larger than the maximum expected interaction time lag between any two processes.
- Your window should be large enough to span ~1 cycle of the lowest frequency of interest.
Time-Frequency GC

Consideration: Process dynamics

- Your window must be larger than the maximum expected interaction time lag between any two processes.
- Your window should be large enough to span ~1 cycle of the lowest frequency of interest.
Time-Frequency GC

Consideration: Process dynamics

• Your window must be larger than the maximum expected interaction time lag between any two processes.
• Your window should be large enough to span ~1 cycle of the lowest frequency of interest.
Which Measure to Use?

EEG

NOISE

Kus et al, 2004
It consists of the following steps. First, the data are obtained from the electrodes and propagated to the other channels. This propagation is used to estimate the thresholds for the functions values indicating spectrum as the original data, but should not exhibit any positive correlations.

Inspecting Figs. 2 and 3, we observe that the channels, which are more delayed than the others, became reproduced correctly, some small leak flows are of the order of the values obtained by means of the surrogate data test. The results are shown in Fig. 4(b). On the other hand, the simulations are shown in the middle column.

According to our experience, the use of dDTF may be important when number of electrodes is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

The nonnormalized DTFs (equivalent to the multivariate direct flows estimated from dDTFs).

The second term in the definition of dDTF is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

We can see that, in this case, the pattern of flows is fairly well reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

Fig. 4. (A) Nonnormalized multichannel DTFs for the simulation I (Fig. 1). The nonnormalized DTFs (equivalent to the multivariate direct flows estimated from dDTFs).

The second term in the definition of dDTF is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

The nonnormalized DTFs (equivalent to the multivariate direct flows estimated from dDTFs).

The second term in the definition of dDTF is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

Fig. 5. (A) Ordinary (graphs above diagonal), partial (graphs below diagonal), and multiple coherences (graphs on the diagonal) for the simulation I. Vertical axis: Granger causality in arbitrary units. Graphs on the diagonal power spectra. (B) DTFs concerning drawing of arrows the same as in Fig. 2. According to our experience, the use of dDTF may be important when number of electrodes is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

The second term in the definition of dDTF is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

The second term in the definition of dDTF is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.

According to our experience, the use of dDTF may be important when number of electrodes is small and signals are not very coherent. In such a case, one of the simulations is reproduced. Although the indirect cascade flows are present, the flow pattern is fairly well reproduced. A number of electrodes, but it could occur for electrodes implanted in specific brain structures.
PDC versus DTF methods (spectral considerations)

<table>
<thead>
<tr>
<th></th>
<th>1&gt;</th>
<th>2&gt;</th>
<th>3&gt;</th>
<th>4&gt;</th>
<th>5&gt;</th>
<th>6&gt;</th>
<th>7&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="PDC" /></td>
<td><img src="image2" alt="PDC" /></td>
<td><img src="image3" alt="PDC" /></td>
<td><img src="image4" alt="PDC" /></td>
<td><img src="image5" alt="PDC" /></td>
<td><img src="image6" alt="PDC" /></td>
<td><img src="image7" alt="PDC" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image8" alt="PDC" /></td>
<td><img src="image9" alt="PDC" /></td>
<td><img src="image10" alt="PDC" /></td>
<td><img src="image11" alt="PDC" /></td>
<td><img src="image12" alt="PDC" /></td>
<td><img src="image13" alt="PDC" /></td>
<td><img src="image14" alt="PDC" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image15" alt="PDC" /></td>
<td><img src="image16" alt="PDC" /></td>
<td><img src="image17" alt="PDC" /></td>
<td><img src="image18" alt="PDC" /></td>
<td><img src="image19" alt="PDC" /></td>
<td><img src="image20" alt="PDC" /></td>
<td><img src="image21" alt="PDC" /></td>
</tr>
<tr>
<td>4</td>
<td><img src="image22" alt="PDC" /></td>
<td><img src="image23" alt="PDC" /></td>
<td><img src="image24" alt="PDC" /></td>
<td><img src="image25" alt="PDC" /></td>
<td><img src="image26" alt="PDC" /></td>
<td><img src="image27" alt="PDC" /></td>
<td><img src="image28" alt="PDC" /></td>
</tr>
<tr>
<td>5</td>
<td><img src="image29" alt="PDC" /></td>
<td><img src="image30" alt="PDC" /></td>
<td><img src="image31" alt="PDC" /></td>
<td><img src="image32" alt="PDC" /></td>
<td><img src="image33" alt="PDC" /></td>
<td><img src="image34" alt="PDC" /></td>
<td><img src="image35" alt="PDC" /></td>
</tr>
<tr>
<td>6</td>
<td><img src="image36" alt="PDC" /></td>
<td><img src="image37" alt="PDC" /></td>
<td><img src="image38" alt="PDC" /></td>
<td><img src="image39" alt="PDC" /></td>
<td><img src="image40" alt="PDC" /></td>
<td><img src="image41" alt="PDC" /></td>
<td><img src="image42" alt="PDC" /></td>
</tr>
<tr>
<td>7</td>
<td><img src="image43" alt="PDC" /></td>
<td><img src="image44" alt="PDC" /></td>
<td><img src="image45" alt="PDC" /></td>
<td><img src="image46" alt="PDC" /></td>
<td><img src="image47" alt="PDC" /></td>
<td><img src="image48" alt="PDC" /></td>
<td><img src="image49" alt="PDC" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1&gt;</th>
<th>2&gt;</th>
<th>3&gt;</th>
<th>4&gt;</th>
<th>5&gt;</th>
<th>6&gt;</th>
<th>7&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image50" alt="dDTF" /></td>
<td><img src="image51" alt="dDTF" /></td>
<td><img src="image52" alt="dDTF" /></td>
<td><img src="image53" alt="dDTF" /></td>
<td><img src="image54" alt="dDTF" /></td>
<td><img src="image55" alt="dDTF" /></td>
<td><img src="image56" alt="dDTF" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image57" alt="dDTF" /></td>
<td><img src="image58" alt="dDTF" /></td>
<td><img src="image59" alt="dDTF" /></td>
<td><img src="image60" alt="dDTF" /></td>
<td><img src="image61" alt="dDTF" /></td>
<td><img src="image62" alt="dDTF" /></td>
<td><img src="image63" alt="dDTF" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image64" alt="dDTF" /></td>
<td><img src="image65" alt="dDTF" /></td>
<td><img src="image66" alt="dDTF" /></td>
<td><img src="image67" alt="dDTF" /></td>
<td><img src="image68" alt="dDTF" /></td>
<td><img src="image69" alt="dDTF" /></td>
<td><img src="image70" alt="dDTF" /></td>
</tr>
<tr>
<td>4</td>
<td><img src="image71" alt="dDTF" /></td>
<td><img src="image72" alt="dDTF" /></td>
<td><img src="image73" alt="dDTF" /></td>
<td><img src="image74" alt="dDTF" /></td>
<td><img src="image75" alt="dDTF" /></td>
<td><img src="image76" alt="dDTF" /></td>
<td><img src="image77" alt="dDTF" /></td>
</tr>
<tr>
<td>5</td>
<td><img src="image78" alt="dDTF" /></td>
<td><img src="image79" alt="dDTF" /></td>
<td><img src="image80" alt="dDTF" /></td>
<td><img src="image81" alt="dDTF" /></td>
<td><img src="image82" alt="dDTF" /></td>
<td><img src="image83" alt="dDTF" /></td>
<td><img src="image84" alt="dDTF" /></td>
</tr>
<tr>
<td>6</td>
<td><img src="image85" alt="dDTF" /></td>
<td><img src="image86" alt="dDTF" /></td>
<td><img src="image87" alt="dDTF" /></td>
<td><img src="image88" alt="dDTF" /></td>
<td><img src="image89" alt="dDTF" /></td>
<td><img src="image90" alt="dDTF" /></td>
<td><img src="image91" alt="dDTF" /></td>
</tr>
<tr>
<td>7</td>
<td><img src="image92" alt="dDTF" /></td>
<td><img src="image93" alt="dDTF" /></td>
<td><img src="image94" alt="dDTF" /></td>
<td><img src="image95" alt="dDTF" /></td>
<td><img src="image96" alt="dDTF" /></td>
<td><img src="image97" alt="dDTF" /></td>
<td><img src="image98" alt="dDTF" /></td>
</tr>
</tbody>
</table>
Scalp or Source?

Fig. 15.13: Direction of flows for 21-channel EEG (awake state eyes closed) obtained by means of different methods. The shade of gray of the arrow represents the strength of the connection (black = the strongest), for each method 40 strongest flows are shown. Reprinted from with permission [49] (© IEEE 2005).

A lot of activity flowing to the destination channels from the posterior electrodes, so the denominator in Eq. (15.6) is quite large, which diminishes the values of DTFs showing outflows from Fz. For Granger causality and DTF there is no propagation from the temporal electrodes. This is practically also the case for dDTF. The dDTF shows only direct flows, we can see that in this case the pattern of flows is consistent with anatomy, e.g., a lack of direct connection between Oz and Pz, Fz, and Fpz—locations where hemispheres are partitioned. The main sources of the activity—namely, electrodes P3, P4, O2, Oz, O1—are the same as for the other multivariate estimates.

Inspecting the results of application of the PDC function to the same data epoch we observe a different picture. One can notice that, unlike the results of dDTF, some channels became sinks. This is due to the normalization of PDC. In fact, we do not see the transmission, as is the case for dDTF, but the ratio between the flow to a given channel with respect to all the outflows from the considered channel. In this way, a channel propagating activity in all directions will show weaker flows than a channel propagating only in one direction. Therefore, the method is not suitable for identification of sources of EEG activity, but it may be useful when the destination channel is of primary interest.

The pattern of propagations obtained for the bivariate estimates of the Granger or
Scalp or Source?

sensors
Scalp or Source?

Volume Conduction

sensors

sources
Scalp or Source?

sensors

Volume Conduction

sources
Scalp or Source?

Volume Conduction

sensors

sources
Scalp or Source?

\[ S(t) = \sum_{k=1}^{p} A(k) S(t - k) + E(t) \]
Scalp or Source?

\[ X(t) = MS(t) \]

\[ S(t) = \sum_{k=1}^{p} A(k)S(t-k) + E(t) \]
Scalp or Source?

\[ X(t) = MS(t) = \sum_{k=1}^{p} MA(k) M^{-1} X(t-k) + ME(t) \]

\[ S(t) = \sum_{k=1}^{p} A(k) S(t-k) + E(t) \]
Scalp or Source?

\[ X(t) = MS(t) = \sum_{k=1}^{p} MA(k) M^{-1} X(t - k) + ME(t) \]

\[ S(t) = \sum_{k=1}^{p} A(k) S(t - k) + E(t) \]
Scalp or Source?

\[ X(t) = MS(t) = \sum_{k=1}^{p} MA(k) M^{-1} X(t-k) + ME(t) \]

\[ X(t) = MS(t) \]

Volume Conduction

sensors

sources

Solution? Source Separation

\[ S(t) = \sum_{k=1}^{p} A(k) S(t-k) + E(t) \]
Estimating Dependency of Independent Components ?
Estimating Dependency of Independent Components?

Isn’t it a contradiction to examine dependence between Independent Components?
Estimating Dependency of Independent Components?

- Isn’t it a contradiction to examine dependence between Independent Components?
- Instantaneous (e.g., Infomax) ICA only explicitly enforces *instantaneous* independence. Time-delayed dependencies may be preserved.
Isn’t it a contradiction to examine dependence between Independent Components?

Instantaneous (e.g., Infomax) ICA only explicitly enforces instantaneous independence. Time-delayed dependencies may be preserved.

ICA seeks to maximize global independence (over entire recording session), transient dependencies are often preserved.
Estimating Dependency of Independent Components?

**Figure 7:** Localization errors of dipole fits conducted on the estimated mixing of different types of noise (N0) see TABLE I.

**Figure 6:** Estimation errors of the mixing matrix according to the goodness-of-fit (GOF) criterion. Results are shown for the proposed Sparsely Connected Sources Analysis (SCSA) variants (SCSA_EM, CSA, CICAAR, MVARICA, ICA).

**Discussion:**

- **SCSA_EM, CSA, CICAAR, MVARICA, ICA** and three alternative approaches (SCSA, CSA, CICAAR) were evaluated using independent components analysis (ICA).

- The performance at different signal-to-noise ratios (SNR) varied, with CSA showing some advantages in medium SNR and CICAAR requiring the longest time.

- However, for SCSA there is still room for improvement.

- The average runtime of the proposed Sparsely Connected Sources Analysis (SCSA_EM) was rather short time, while the EM implementation of SCSA was in medium range.

Haufe et al, 2008
SIFT
Source Information Flow Toolbox
Version 0.1-Alpha
Source Information Flow Toolbox (SIFT) 0.1-alpha
Source Information Flow Toolbox (SIFT) 0.1-alpha

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
Source Information Flow Toolbox (SIFT) 0.1-alpha

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
Source Information Flow Toolbox (SIFT) 0.1-alpha

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
- Emphasis on time-frequency domain approaches
Source Information Flow Toolbox (SIFT) 0.1-alpha

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
- Emphasis on time-frequency domain approaches
- Novel interactive visualization methods for exploratory analysis of connectivity across time, frequency, and spatial location
Source Information Flow Toolbox (SIFT) 0.1-alpha

- A new (alpha) toolbox for source-space electrophysiological information flow and causality analysis (single-subject or group analysis) integrated into the EEGLAB software environment
- Modular architecture intended to support multiple modeling approaches
- Emphasis on time-frequency domain approaches
- Novel interactive visualization methods for exploratory analysis of connectivity across time, frequency, and spatial location
- Requirements: EEGLAB, MATLAB™ 2008b, Signal Processing Toolbox, Statistics Toolbox (the latter two dependencies may be removed in the future)
SIFT: Acknowledgements

- Arnaud Delorme
- Scott Makeig
- Christian Kothe
- Nima Bigdely-Shamlo
- Wes Thompson
- SCCN
#1: Button press epochs

Filename: ...\Data\bt73 RespWrong.dat

- Channels per frame: 127
- Frames per epoch: 1024
- Epochs: 165
- Events: 1451
- Sampling rate (Hz): 256
- Epoch start (sec): -2.000
- Epoch end (sec): 1.996
- Reference: unknown
- Channel locations: Yes
- ICA weights: Yes
- Dataset size (Mb): 175.3
#1: Button press epochs

Filename: ...eta/Data/bt73RespWrong.dat

Channels per frame: 127
Frames per epoch: 1024
Epochs: 165
Events: 1451
Sampling rate (Hz): 256
Epoch start (sec): -2.000
Epoch end (sec): 1.996
Reference: unknown
Channel locations: Yes
ICA weights: Yes
Dataset size (Mb): 175.3
#1: Button press epochs

Filename: ...eta/Data/bt?3 RespWrong.sst
Channels per frame: 127
Frames per epoch: 1024
Epochs: 165
Events: 1451
Sampling rate (Hz): 256
Epoch start (sec): -2.000
Epoch end (sec): 1.996
Reference: unknown
Channel locations: Yes
ICA weights: Yes
Dataset size (Mb): 175.3
#1: Button press epochs

Filename: ...Data/bt73 RespWrong.dat
Channels per frame: 127
Frames per epoch: 1024
Epochs: 165
Events: 1451
Sampling rate (Hz): 256
Epoch start (sec): -2.000
Epoch end (sec): 1.996
Reference: unknown
Channel locations: Yes
ICA weights: Yes
Dataset size (Mb): 175.3
#1: Button press epochs

Filename: ...eta/Data/bt73 RespWrong.dat
Channels per frame: 127
Frames per epoch: 1024
Epochs: 165
Events: 1451
Sampling rate (Hz): 256
Epoch start (sec): -2.000
Epoch end (sec): 1.996
Reference: unknown
Channel locations: Yes
ICA weights: Yes
Dataset size (Mb): 175.3
Pre-processing
- Model fitting and validation
- Connectivity
- Statistics
- Visualization

Group Analysis

Visualization
- **Source-separation and localization**
  (performed externally using EEGLAB or other toolboxes)

- Filtering/Detrending

- Downsampling

- Differencing

- Normalization (temporal or ensemble)

- Trial balancing

- Tests for stationarity of the data (linear methods)
Preprocessing

- Model fitting and validation
- Connectivity
- Statistics
- Visualization

Preprocessing Options

- Miscellaneous
  - VerbosityLevel
- Data Selection
  - SelectComponents
    - ComponentsToKeep
    - EpochTimeRange
    - TrialSubsetToUse
- Filtering
  - NewSamplingRate
  - FilterData
- DifferenceData
  - DifferencingOrder
- Detrend
  - DetrendingMethod
- Normalization
  - NormalizeData
    - Method

Data normalization. Normalize trials across time, ensemble, or both.
<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>Nonlinear/Nonstationary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parametric</strong></td>
<td>MVAR Modeling</td>
<td>Dual Extended Kalman Filtering</td>
</tr>
<tr>
<td></td>
<td>Sparse MVAR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bayesian MVAR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kalman Filtering</td>
<td></td>
</tr>
<tr>
<td><strong>Nonparametric</strong></td>
<td>Nonparametric MVAR (minimum-phase spectral factorization)</td>
<td>Transfer Entropy</td>
</tr>
<tr>
<td></td>
<td>Multivariate phase distribution</td>
<td></td>
</tr>
</tbody>
</table>

- **fully implemented**
- **partially-developed**
- **coming soon**
Preprocessing | Modeling | Statistics | Visualization

Model Fitting | Validation | Connectivity

Figure 3: RespWrong – Model Order Selection Results

- Pre-processing
- Model fitting and validation
- Connectivity
- Statistics
- Visualization

- Fit AMVAR Model
- Validate model

1. Select MVAR algorithm
   - Vierra-Morl
   - ARFIT

2. Window length (sec) 0.5
   - Start Window Length Assistant...

3. Step size (sec) 0.03
   - AIC
   - PPE
   - HQ

4. Model order 10
   - Start Model Order Assistant...

- Information criteria (Cts)
- histogram count
- % windows to sample

- Average info. criteria across sampled windows
- aic (28) vs sbc (10)
- Whiteness of Residuals
  - Portmanteau tests
  - Autocorrelation function
- Model Consistency
- Model Stability
Modeling

- Preprocessing
- Visualization
- Statistics

- Model Fitting
- Validation
- Connectivity

Select Model Validation Methods

- Check Whiteness of Residuals
- Ljung-Box
- ACF
- Box-Pierce
- Lj-McLeod

Significance level: 0.05
- check percent consistency
- check model stability

% windows to sample: 100

Help | Cancel | Ok
<table>
<thead>
<tr>
<th>MVAR</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Power spectrum (ERSP)</td>
<td>- Transfer Entropy *</td>
</tr>
<tr>
<td>- Coherence (Coh), Partial Coherence (pCoh), Multiple Coherence (mCoh)</td>
<td>- Multivariate phase-locking value (mPLV) *</td>
</tr>
<tr>
<td>- Partial Directed Coherence (PDC)</td>
<td></td>
</tr>
<tr>
<td>- Generalized PDC (GPDC)</td>
<td></td>
</tr>
<tr>
<td>- Partial Directed Coherence Factor (PDCF)</td>
<td></td>
</tr>
<tr>
<td>- Renormalized PDC (rPDC) *</td>
<td></td>
</tr>
<tr>
<td>- Directed Transfer Function (DTF)</td>
<td></td>
</tr>
<tr>
<td>- Direct Directed Transfer Function (dDTF)</td>
<td></td>
</tr>
<tr>
<td>- Granger-Geweke Causality (GGC)</td>
<td></td>
</tr>
<tr>
<td>- Conditional GGC</td>
<td></td>
</tr>
<tr>
<td>- Blockwise GGC *</td>
<td></td>
</tr>
</tbody>
</table>

- fully implemented
- partially-developed
- coming soon
Preprocessing

Modeling

Statistics

Visualization

Model Fitting

Validation

Connectivity

Select connectivity measures to calculate

- DIRECTED TRANSFER FUNCTION MEASURES
  - Directed Transfer Function (DTF)
  - Normalized DTF (nDTF)
  - Direct DTF (dDTF)
  - Direct DTF (with full causal normalization)
  - Full-frequency DTF (fDTF)

- PARTIAL DIRECTED COHERENCE MEASURES
  - Partial Directed Coherence (PDC)
  - Normalized PDC (nPDC)
  - Generalized Partial Directed Coherence (GPDC)
  - Partial Directed Coherence Factor (PDCF)
  - Renormalized Partial Directed Coherence (RPDC)

- GRANGER-GEWEKE CAUSALITY MEASURES
  - Granger-Geweke Causality (GGC)

- SPECTRAL COHERENCE MEASURES
  - Complex Coherence (Coh)
  - Imaginary Coherence (iCoh)
  - Partial Coherence (pCoh)
  - Multiple Coherence (mCoh)

- SPECTRAL DENSITY MEASURES
  - Complex Spectral Density

- return squared amplitude of complex measures
- convert spectral density to decibels

Frequencies (Hz) 1:127

Help Cancel Ok
Asymptotic analytic estimates of confidence intervals
Applies to: PDC, nPDC, DTF, nDTF, rPDC
Tests: $H_{\text{null}}$, $H_{\text{base}}$, $H_{\text{AB}}$

Confidence intervals using thin-plate smoothing splines
Applies to: dDTF
Tests: $H_{\text{base}}$, $H_{\text{AB}}$

$H_{\text{null}}: C_{ij} \leq C_{\text{null}}$

$H_{\text{base}}: C_{ij} \leq C_{\text{baseline}}$

$H_{\text{AB}}: C_{ij}^{A} = C_{ij}^{B}$

- **fully implemented**
- **partially-developed**
- **coming soon**
Asymptotic analytic estimates of confidence intervals
- Applies to: PDC, nPDC, DTF, nDTF, rPDC
- Tests: $H_{null}$, $H_{base}$, $H_{AB}$

Confidence intervals using thin-plate smoothing splines
- Applies to: dDTF
- Tests: $H_{base}$, $H_{AB}$

Phase-randomization
- Applies to: all
- Tests: $H_{null}$

Permutation Tests
- Applies to: all
- Tests: $H_{AB}$, $H_{base}$

Bootstrap and Jacknife
- Applies to: all
- Tests: $H_{AB}$, $H_{base}$

$H_{null}: C_{ij} \leq C_{null}$
$H_{base}: C_{ij} \leq C_{baseline}$
$H_{AB}: C_{Aij} = C_{Bij}$

- fully implemented
- partially-developed
- coming soon
Interactive Time-Frequency Grid
Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie
Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie
Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

Directed Graphs on anatomicals (ECoG)
Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

Directed Graphs on anatomicals (ECoG)

and more...
Interactive Time-Frequency Grid

Interactive 3D Causal Brainmovie

Causal Density Movie

Directed Graphs on anatomicals (ECoG)

and more...

All of these currently support single-subject or (in beta version) group analysis. ROI connectivity analysis can currently be performed using dipole clustering.
Interactive Time-Frequency Grid
Interactive Time-Frequency Grid
Causal Time-Frequency Grid

FROM

TO

Frequency (Hz)

Time (sec)
Causal Time-Frequency Grid
Causal Time-Frequency Grid

<table>
<thead>
<tr>
<th>Region</th>
<th>VC1</th>
<th>VC2</th>
<th>I PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
<th>PPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p &lt; 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- VC1 (21)
- VC2 (22)
- I PPC (13)
- PPC (15)
- PPC (15)
- PPC (16)
- PPC (24)
- PPC (16)
- PPC (16)
- PPC (14)
- PPC (16)
- PPC (15)
- PPC (15)
Causal Time-Frequency Grid

Error - Correct
p < 0.05, N=24

VC1 (21)
VC2 (22)
IPPC (17)
IFC (15)
IFC (15)
PC2a (16)
PC2b (24)
PC3 (16)
MT (1)
MT (14)
ISM (16)
ISM (15)
Causal Time-Frequency Grid

Error - Correct  
p < 0.05, N=24
Causal Time-Frequency Grid
Causal Time-Frequency Grid

Error - Correct
p < 0.05, N=24
Interactive BrainMovie3D
Interactive BrainMovie3D
Interactive BrainMovie3D
Interactive BrainMovie3D

Error > Correct (p < 0.05, N=24)
dDTF
3-7 Hz

ERP envelope (backprojected components)
Causal Density Movie

Error > Correct (p < 0.05, N=24)
dDTF
3-7 Hz
Causal Density Movie

Error > Correct (p < 0.05) 3-7 Hz
This approach adopts a 3-stage process:

1. Identify K ROI’s (clusters) by affinity clustering of sources across subject population using EEGLAB’s Measure-Product clustering.
2. Average all incoming and outgoing statistically significant connections between each pair of ROIs to create a $[ K \times K \times [x \text{ freq} \times \text{ time}]]$ group connectivity matrix.
3. Visualize the results using any of SIFTs visualization routines. This method suffers from low statistical power when subjects do not have high agreement in terms of source locations (missing variable problem).
Group Analysis

Disjoint Clustering

This approach adopts a 3-stage process:
1. Identify K ROI’s (clusters) by affinity clustering of sources across subject population using EEGLAB’s Measure-Product clustering.
2. Average all incoming and outgoing statistically significant connections between each pair of ROIs to create a [ K X K [x freq x time ] ] group connectivity matrix.
3. Visualize the results using any of SIFTs visualization routines. This method suffers from low statistical power when subjects do not have high agreement in terms of source locations (missing variable problem).

Bayesian Mixture Model

A more robust approach (in development with Wes Thompson and to be released in SIFT 1.0b) uses smoothing splines and Monte-Carlo methods for joint estimation of posterior probability (with confidence intervals) of cluster centroid location and between-cluster connectivity. This method takes into account the “missing variable” problem inherent to the disjoint clustering approach and provides robust group connectivity statistics.

partially-developed
Future Work

- Improvement of architecture, GUI, and EEGLAB integration
- Ongoing implementation/incorporation of state-of-the-art methods for effective connectivity analysis and visualization
- Improved group statistics
- Evaluation of relative suitability of various source-separation algorithms when combined with MVAR modeling