The Source Information Flow Toolbox
An Electrophysiological Information Flow Toolbox for EEGLAB

Pre-processing
Model Fitting and Validation
Connectivity
Statistics
Visualization
Group Analysis

CAUSALITY FROM
CAUSALITY TO

Tim Mullen

15th EEGLAB Workshop
June 16, 2012
Tsinghua University, Beijing, China
## Outline

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
</tr>
<tr>
<td>Theory</td>
</tr>
<tr>
<td>- Functional and Effective Connectivity</td>
</tr>
<tr>
<td>- Linear Dynamical Systems and Vector Autoregressive Modeling</td>
</tr>
<tr>
<td>- Granger Causality and Related Multivariate Connectivity Measures</td>
</tr>
<tr>
<td>- Scalp or Source?</td>
</tr>
<tr>
<td>- Adapting to Time-Varying Dynamics</td>
</tr>
<tr>
<td>The Source Information Flow Toolbox (SIFT)</td>
</tr>
<tr>
<td>Some Applications of SIFT</td>
</tr>
<tr>
<td>The Road Ahead</td>
</tr>
<tr>
<td>Fin</td>
</tr>
</tbody>
</table>
The Dynamic Brain

- A key goal: To measure temporal changes in neural dynamics and information flow that index and predict task-relevant changes in cognitive state and behavior.

- Important factors:
  - Accuracy and Validity
  - Temporal Specificity
  - Non-invasive measures
Categorizations of Large-Scale Brain Connectivity Analysis

(Bullmore and Sporns, *Nature*, 2009)

**Structural**
- state-invariant, anatomical

**Functional**
- dynamic, state-dependent, correlative, symmetric

**Effective**
- dynamic, state-dependent, asymmetric, causal, information flow

**Hours-Years**

**milliseconds-seconds**

Temporal Scale
Modeling Brain Connectivity

- Model-based approaches mitigate the ‘curse of dimensionality’ by making some assumptions about the structure, dynamics, or statistics of the system under observation

Box and Draper (1987):

“Essentially, all models are wrong, but some are useful [...] the practical question is how wrong do they have to be to not be useful”

“The map is not the territory”
## Estimating Functional Connectivity

### Popular measures

- Cross-Correlation
- Coherence
- Phase-Locking Value
- Phase-amplitude coupling
  
...
Cross-Correlation and Linear Coherence

$$C_{AB}(f) = \sum_{k=0}^{P} \rho_{AB}(k) e^{-i2\pi fk}$$

$$\rho_{AB}(k) = \frac{S_{AB}(f)}{\sqrt{S_A(f)S_B(f)}}$$

Coherence/CC/PLV indicate **functional**, but not **effective** connectivity.
Estimating Effective Connectivity

Non-Invasive

- Post-hoc analyses applied to measured neural activity
- Confirmatory
  - Dynamic Causal Models
  - Structural Equation Models
- Exploratory
  - Granger-Causal methods

- Data-driven
- Rooted in conditional predictability
- Scalable (Valdes-Sosa, 2005)
- Extendable to nonlinear and/or non-stationary systems (Freiwald, 1999; Ding, 2001; Chen, 2004; Ge, 2009)
- Extendable to non-parametric representations (Dhamala, 2009a,b)
- Can be (partially) controlled for (unobserved) exogenous causes (Guo, 2008a,b; Ge, 2009)
- Equivalent to Transfer Entropy for Gaussian Variables (Seth, 2009)
- Flexibly allows us to examine time-varying (dynamic) multivariate causal relationships in either the time or frequency domain
Linear Dynamical Systems

Stochastic Linear Dynamical System

\[ X_1(t) = a(t)_{11} X_1(t-1) + a(t)_{12} X_2(t-1) + \epsilon_1(t) \]
\[ X_2(t) = a(t)_{22} X_2(t-1) + a(t)_{21} X_1(t-1) + \epsilon_2(t) \]

Order 1 Markov Process (VAR[1])

\[ X_1(0) \quad X_1(1) \quad \ldots \quad X_1(n) \]
\[ X_2(0) \quad X_2(1) \quad \ldots \quad X_2(n) \]

\[ \Delta t = 1 \]

\[ \epsilon_1(t), \epsilon_2(t) \]

\[ a(t)_{11}, a(t)_{12}, a(t)_{22}, a(t)_{21} \]

Time step

Saturday, June 16, 2012
Vector Autoregressive (VAR / MAR / MVAR) Modeling

$X_1(t)$, $X_2(t)$, $\ldots$, $X_M(t)$

EEG

VAR

Granger Causality, Coherence, Spectrum, ...
VAR Modeling: Assumptions

- **“Weak” stationarity of the data**
  - mean and variance do not change with time
  - An EEG trace containing prominent evoked potentials is a classic example of a non-stationary time-series

- **Stability**
  - A stable process will not “blow up” (diverge to infinity)
  - Importantly, stability implies stationarity
The Linear Vector Autoregressive (VAR) Model

$$X(t) = \sum_{k=1}^{p} A^{(k)}(t)X(t-k) + E(t)$$

where:
- $X(t)$ is the M-channel data vector at current time $t$.
- $A^{(k)}(t)$ is the $M \times M$ matrix of (possibly time-varying) model coefficients indicating variable dependencies at lag $k$.
- $E(t)$ is the random noise process.

$E(t) = N(0, V)$
Selecting a VAR Model Order

- Model order is typically determined by minimizing information criteria such as Akaike Information Criterion (AIC) for varying model order (p):

$$AIC(p) = 2 \log(\det(V)) + M^2 p/N$$

-Penalizes high model orders (parsimony)

-Entropy rate (amount of prediction error)

-Optimal order
Selecting a VAR Model Order

- Other considerations:
  - A $M$-dimensional VAR model of order $p$ has at most $Mp/2$ spectral peaks distributed amongst the $M$ variables. This means we can observe at most $p/2$ peaks in each variables’ spectrum (or in the causal spectrum between each pair of variables)

- Optimal model order depends on sampling rate (higher sampling rate often requires higher model orders)
Granger Causality

- Relies on two assumptions:

<table>
<thead>
<tr>
<th>Granger Causality Axioms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Causes should precede their effects in time (Temporal Precedence)</td>
</tr>
<tr>
<td>2. Information in a cause’s past should improve the prediction of the effect, above and beyond the information contained in past of the effect (and other measured variables)</td>
</tr>
</tbody>
</table>
Granger Causality

Does $X_4$ granger-cause $X_1$?
(conditioned on $X_2, X_3$)

\[
\begin{align*}
X_1(t) \\
X_2(t) \\
X_3(t) \\
X_4(t)
\end{align*}
\]

\[
X(t) = \sum_{k=1}^{p} A^{(k)}X(t - k) + E(t)
\]

\[
X_{-4}(t) = \sum_{k=1}^{p} \bar{A}^{(k)}X_{-4}(t - k) + \tilde{E}(t)
\]

\[\text{prediction error for } X_1 \text{ (variance of residuals } E_1)\]

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Granger Causality

- Granger (1969) quantified this definition for **bivariate** processes in the form of an F-ratio:

\[
F_{X_1 \leftarrow X_2} = \ln \left( \frac{\text{var}(\tilde{E}_1)}{\text{var}(E_1)} \right) = \ln \left( \frac{\text{var}(X_1(t) \mid X_1(\cdot))}{\text{var}(X_1(t) \mid X_1(\cdot), X_2(\cdot))} \right)
\]

- Alternately, for a **multivariate interpretation** we can fit a single MVAR model to all channels and apply the following definition:

**Definition 1**

\( X_j \) granger-causes \( X_i \) **conditioned on all other variables in** \( X \)

**if and only if** \( A_{ij}(k) \gg 0 \) **for some lag** \( k \in \{1, \ldots, p\} \)
Granger Causality Quiz

Example: 2-channel MVAR process of order 1

\[
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix}
= 
\begin{pmatrix}
-0.5 & 0 \\
0.7 & 0.2
\end{pmatrix}
\begin{pmatrix}
X_1(t-1) \\
X_2(t-1)
\end{pmatrix}
+ 
\begin{pmatrix}
E_1(t) \\
E_2(t)
\end{pmatrix}
\]

Which causal structure does this model correspond to?

a) 1 \rightarrow 2  

b) 1 \leftrightarrow 2  

c) 1 \leftrightarrow 2
Granger Causality – Frequency Domain

\( X(t) = \sum_{k=1}^{p} A^{(k)}(t - k) + E(t) \)

Fourier-transforming \( A^{(k)} \) we obtain

\( A(f) = -\sum_{k=0}^{p} A^{(k)} e^{-i2\pi fk}; A^{(0)} = I \)

We can then define the spectral matrix \( X(f) \) as follows:

\( X(f) = A(f)^{-1} E(f) = H(f) E(f) \)

Where \( H(f) \) is the transfer matrix of the system.

**Definition 2**

\( X_i \) granger-causes \( X_i \) conditioned on all other variables in \( X \)

if and only if \( |A_{ij}(f)| >> 0 \) for some frequency \( f \)

Likewise, \( X(f) \) and \( E(f) \) correspond to the fourier transforms of the data and residuals, respectively

leads to PDC
Granger causality measures, obtained by means of MV AR

\[ X(t) = \sum_{k=1}^{p} A^{(k)}(t)X(t-k) + E(t) \]
\[ A(f,t) = -\sum_{k=0}^{p} A^{(k)}(t)e^{-i2\pi ft}; \quad A^{(0)} = I \]
\[ X(f,t) = A(f,t)^{-1}E(f,t) = H(f,t)E(f,t) \]

Spectrogram

\[ S(f) = X(f)X(f)^* \]
\[ = H(f)\Sigma H(f)^* \]

(Brillinger, 2001)

Functional

Ground Truth

Effective

**Bivariate**

\[ C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)S_{jj}(f)}} \]

(Bendat and Piersol, 1986)

\[ P_{ij}(f) = \frac{S_{ij}^{-1}(f)}{\sqrt{S_{ii}^{-1}(f)S_{jj}^{-1}(f)}} \]

(Bendat and Piersol, 1986; Dalhaus, 2000)

Granger-Geweke Causality

\[ F_{ij}(f) = \frac{\Sigma_{j,j} - (\Sigma_{i,j}^2 / \Sigma_{i,j})}{\Sigma_{i,j}(f)} \left| H_{ij}(f) \right|^2 \]

(Geweke, 1982; Bressler et al., 2007)

Partial Directed Coherence

\[ \pi_{ij}^2(f) = \frac{\left| A_{ij}(f) \right|^2}{\sum_{k=1}^{M} \left| A_{ij}(f) \right|^2} \]

(Baccalá and Sameshima, 2001)
Scalp or Source?

Fig. 15.13: Direction of flows for 21-channel EEG (awake state eyes closed) obtained by means of different methods. The shade of gray of the arrow represents the strength of the connection (black = the strongest), for each method 40 strongest flows are shown. Reprinted from with permission [49] (© IEEE 2005).

A lot of activity flowing to the destination channels from the posterior electrodes, so the denominator in Eq. (15.6) is quite large, which diminishes the values of DTFs showing outflows from Fz. For Granger causality and DTF there is no propagation from the temporal electrodes. This is practically also the case for dDTF. The dDTF shows only direct flows, we can see that in this case the pattern of flows is consistent with anatomy, e.g., a lack of direct connection between Oz and Pz, Fz, and Fpz—locations where hemispheres are partitioned. The main sources of the activity—namely, electrodes P3, P4, O2, Oz, O1—are the same as for the other multivariate estimates.

Inspecting the results of application of the PDC function to the same data epoch we observe a different picture. One can notice that, unlike the results of dDTF, some channels became sinks. This is due to the normalization of PDC. In fact, we do not see the transmission, as is the case for dDTF, but the ratio between the flow to a given channel with respect to all the outflows from the considered channel. In this way, a channel propagating activity in all directions will show weaker flows than a channel propagating only in one direction. Therefore, the method is not suitable for identification of sources of EEG activity, but it may be useful when the destination channel is of primary interest.
Scalp or Source?


\[ X(t) = HS(t) = \sum_{k=1}^{p} H A^{(k)}(t) H^{-1} X(t - k) + HE(t) \]

\[ S(t) = \sum_{k=1}^{p} A^{(k)}(t) S(t - k) + E(t) \]
Adapting to Non-Stationarity

- The brain is a **dynamic system** and measured brain activity and coupling can change rapidly with time (non-stationarity)
  - event-related perturbations (ERSP, ERP, etc)
  - structural changes due to learning/feedback
- **How can we adapt to non-stationarity?**
Adapting to Non-Stationarity

- Many ways to do adaptive VAR estimation
  - Segmentation-based adaptive VAR estimation
  - Factorization of time-varying spectral density matrices (e.g. from STFTs, Wavelets, etc)
  - State-Space Modeling
  - ...

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Segmentation-based VAR

(Jansen et al., 1981; Florian and Pfurtscheller, 1995; Ding et al, 2000)

\[ X(t) = \sum_{k=1}^{p} A^{(k)}(t)X(t - k) + E(t) \]

\[ A(f,t) = -\sum_{k=0}^{p} A^{(k)}(t)e^{-i2\pi fk}; A^{(0)} = I \]

Analogous to short-time Fourier transform
Time-Frequency GC

- What is a good window length?
- Considerations:
  - Temporal smoothing
  - Local stationarity
  - Sufficient amount of data
  - Process dynamics
Time-Frequency GC

Consideration: Temporal Smoothness

Too-large windows may smooth out interesting transient dynamic features.
Time-Frequency GC

Consideration: Local Stationarity

Too-large windows may not be locally-stationary
**Time-Frequency GC**

**Consideration: Sufficient data**

M = number of variables  
p = model order  
N_{tr} = number of trials  
W = length of each window (sample points)

We have $M^2p$ model coefficients to estimate. This requires a minimum of $M^2p$ independent samples.  
So we have the constraint $M^2p \leq N_{tr}W$.  
In practice, however, a better heuristic is $M^2p \leq (1/10)N_{tr}W$.

Or: \[ W \geq 10(M^2p/N_{tr}) \]

SIFT will let you know if your window length is not optimal.
Time-Frequency GC

**Consideration: Process dynamics**

- Your window must be larger than the maximum expected interaction time lag between any two processes.
- Your window should be large enough to span ~1 cycle of the lowest frequency of interest (remember the Heisenberg uncertainty principle)
Time-Frequency GC

- Many ways to do time-varying VAR estimation
  - Segmentation-based adaptive VAR estimation
  - Factorization of time-varying spectral density matrices (e.g. from STFTs, Wavelets, etc)
  - State-Space Modeling
- ...

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Discrete State-Space Model (SSM) for Electrophysiological Dynamics

- Dynamical system may be linear or nonlinear, dense or sparse, non-stationary, high-dimensional, partially-observed, and stochastic
- Subsumes discrete Delay Differential Equation (DDE) and Vector Autoregressive (VAR) methods and closely related to Dynamic Causal Modeling (DCM)

**Observation Equation**
\[ y(t) = Hs(t) + \epsilon(t) \]

**State Transition Equation**
\[ s(t) = f\left(s(t^-), u(t^-), \theta(t)\right) + v(t) \]

Linear VAR[1]
\[ s(t) = A(t)s(t-1) + v(t) \]
Kalman Filtering

optimal estimator (in terms of minimum variance) for the state of a linear dynamical system

\[ y(t) \]

Time Update ("Predict")

Measurement Update ("Correct")

\[ \hat{y}(t) \]

new data point

\[ y(t) \]

\[ \epsilon(t) = y(t) - \hat{y}(t) \]

\[ y(0) \]

Initialize

updated model

updated model

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Kalman Filtering

GPDC Causality From

Time (sec)

Frequency (Hz)
Source Information Flow Toolbox (SIFT)

Pre-processing
- Locate dipoles using DIPFIT 2.x
- Peak detection using EEG toolbox
- FMRI Tools
- Locate dipoles using LORETA

Model Fitting and Validation
- Connectivity
- Statistics
- Visualization

Modeling
- Pre-processing
- Statistics

Connectivity

Group Analysis

Visualization

CAUSALITY FROM

http://sccn.ucsd.edu/wiki/SIFT
Mullen, et al. Society for Neuroscience, 2010
Delorme, Mullen, Kothe et al. Computational Intelligence and Neuroscience, vol 12, 2011
EEGLAB Software framework

Analysis

Analysis plugins

Data archive

Data sync and handling

Interactive tools

Stimulus control

EEGLAB

SIFT

NFT

HeadIT

BCILAB

28 user plugins

MatRiver

DataRiver

Producer

EyeTracker

Wii remote

Mocap

EEG

Tactile stream

Video stream

Audio stream

ERICA framework

Delorme, Mullen, Kothe, Akalin Acar, Bigdely-Shamlo, Vankov, Makeig, *Computational Intelligence and Neuroscience*, 2011
Source Information Flow Toolbox (SIFT)

- A toolbox for (source-space) electrophysiological information flow and causality analysis (single- or multi-subject) integrated into the EEGLAB software environment.
- Modular architecture intended to support multiple modeling approaches
- Emphasis on vector autoregression and SSMs and time-frequency domain approaches
- Standard and novel interactive visualization methods for exploratory analysis of connectivity across time, frequency, and spatial location
- **Requirements**: EEGLAB, MATLAB® 2008a+, Signal Processing Toolbox, Statistics Toolbox (for some functions -- may be removed in the future)
Pre-processing
Statistics
Visualization
Group Analysis
Model Fitting and Validation
Connectivity
Statistics
Visualization
Simulation
### Source reconstruction
(performed externally using EEGLAB or other toolboxes)

<table>
<thead>
<tr>
<th>Preprocessing: Source reconstruction</th>
<th>Filter</th>
<th>Local Detrending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtering or Local Detrending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Downsampling</td>
<td></td>
<td></td>
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<tr>
<td>Differencing</td>
<td></td>
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<tr>
<td>Normalization (temporal or ensemble)</td>
<td></td>
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<tr>
<td>Trial balancing</td>
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</tbody>
</table>

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Linear

VAR Modeling
Vieira-Morf, ARFIT

Sparse VAR
Group Lasso ($L_{1,2}$), TSBL ($L_p$)

Linear Kalman Filtering

Nonparametric VAR (minimum-phase spectral matrix factorization)

Linear+Nonlinear

Dual Extended Kalman Filtering

Cubature Kalman Filtering

Transfer Entropy (TRENTOOL interface)
Preprocessing  Modeling  Statistics  Visualization

Model Fitting  Validation  Connectivity

Pre-processing  Model fitting and validation  Model Order Selection  Connectivity  Statistics  Visualization

Apps  To-Do  Fin

Figure 3: RespCorr - Model Order Selection Results (min ic)

Mean Info. Criteria across sampled windows. Optimal order determined by min of mean curv

Information criteria (bic)

\[ \text{bic} \]

\[ \text{aic} \]

\[ \text{hq} \]

\[ \text{ris} \]

Histogram count vs opt. model order
### VAR Model Validation

<table>
<thead>
<tr>
<th>Residual ‘Whiteness’ Tests</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multivariate portmanteau tests</td>
<td></td>
</tr>
<tr>
<td>Residual autocorrelation probability test</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model Consistency</th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Model Stability</th>
<th></th>
</tr>
</thead>
</table>

| Nonparametric Spectral/Coherence Correlation |  |

- **fully implemented**
- **alpha-testing**
- **coming soon**

Saturday, June 16, 2012
Modeling

Visualization

Statistics

Preprocessing

Model Fitting

Validation

Connectivity

Validation Methods

CheckResidualWhiteness
SignificanceLevel 0.05
MultipleComparisonsCorrection none
NumberOfAutocorrelationLags 50

WhitenessCriteria
Ljung–Box; Li–McLeod

CheckConsistency
CheckStability
Data Reduction
WindowSamplePercentage

Miscellaneous
VerbosityLevel
PlotResults

WhitenessCriteria
Whiteness criteria. These are the statistical tests used to test for uncorrelated residuals

Pre-processing
Model fitting and validation
Connectivity
Statistics
Visualization

Model Order Selection
Fit AMVAR Model
Validate model

Figure 3: Residuals - Model Validation Results

Whiteness Significance

SIFT

Apps

To-Do

Fin
## VAR-based Measures

<table>
<thead>
<tr>
<th>Power spectrum (ERSP)</th>
<th>Coherence (Coh), Partial Coherence (pCoh), Multiple Coherence (mCoh)</th>
<th>Partial Directed Coherence (PDC)</th>
<th>Generalized PDC (GPDC)</th>
<th>Partial Directed Coherence Factor (PDCF)</th>
<th>Renormalized PDC (rPDC)</th>
<th>Directed Transfer Function (DTF)</th>
<th>Direct Directed Transfer Function (dDTF)</th>
<th>Bivariate Granger-Geweke Causality (GGC)</th>
<th>Conditional GGC</th>
<th>Blockwise GGC</th>
</tr>
</thead>
</table>

- **fully implemented**
- **alpha-testing**
- **coming soon**
Modeling

Visualization

Statistics

Preprocessing

Model Fitting

Validation

Connectivity

Pre-processing
Model fitting and validation

Connectivity

Statistics

Visualization

SIFT

Apps

To-Do

Fin

Saturday, June 16, 2012
<table>
<thead>
<tr>
<th></th>
<th>Parametric</th>
<th>Non-parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asymptotic analytic estimates of confidence intervals</td>
<td>Applies to: PDC, nPDC, DTF, nDTF, rPDC</td>
<td>Phase-randomization</td>
</tr>
<tr>
<td></td>
<td>Tests: ( H_{\text{null}}, H_{\text{base}}, H_{\text{AB}} )</td>
<td>Applies to: all</td>
</tr>
<tr>
<td></td>
<td>Confidence intervals using</td>
<td>Bootstrap, Jacknife, Cross-Validation</td>
</tr>
<tr>
<td></td>
<td>Bayesian smoothing splines</td>
<td>Applies to: all</td>
</tr>
<tr>
<td></td>
<td>Applies to: all</td>
<td>Tests: ( H_{\text{AB}}, H_{\text{base}} )</td>
</tr>
<tr>
<td></td>
<td>Tests: ( H_{\text{base}}, H_{\text{AB}} )</td>
<td></td>
</tr>
</tbody>
</table>

\[
H_{\text{null}} : C_{ij} \leq C_{\text{null}} \quad \quad H_{\text{base}} : C_{ij} \leq C_{\text{baseline}} \quad \quad H_{\text{AB}} : C_{ij}^{A} = C_{ij}^{B}
\]

- **fully implemented**
- **alpha-testing**
- **coming soon**
Preprocessing  Modeling  Statistics  Visualization

Parametric  Non-parametric

SIFT  Apps  To-Do  Fin

fully implemented  alpha-testing  coming soon

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### Interactive Visualizers

<table>
<thead>
<tr>
<th>Interactive Visualizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interactive Time-Frequency Grid</td>
</tr>
<tr>
<td>Interactive 3D Causal Brainmovie</td>
</tr>
<tr>
<td>Causal Projection Movie</td>
</tr>
<tr>
<td>Directed Graphs and Graph Theoretic Analysis (Bioinformatics Toolbox Interface)</td>
</tr>
<tr>
<td>and more …</td>
</tr>
</tbody>
</table>

- **Fully implemented**
- **Alpha-testing**
- **Coming soon**
Interactive Time-Frequency Grid
Interactive Time-Frequency Grid

Causality FROM

Causality TO

Frequency (Hz)

Time (sec)
Interactive Causal BrainMovie3D
Bioinformatics Toolbox IFace

Interactive Directed Graphs
- Radial, Hierarchical, or Customized Node Layout
- Graph-Theoretic Analysis (SCCs, Shortest-Path, MaxFlow, etc)
- Assignment of useful quantities to Node and Edge size/color

Pre-seizure
Seizure Early Stage
Seizure Late Stage
Post-Seizure

LEGEND
- clusters
  - prefrontal
  - dorsofrontal
  - precentral
  - postcentral

causal flow
dDTF

Mullen et al, 2011

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Causal/Measure Projection

Error > Correct (p<0.05, N=24) 3-7 Hz

Mullen, Onton, et al, 2010, HBM, Barcelona
Bigdely-Shamlo, Mullen, et al, 2012, in review

Bayesian Hierarchical Model

\[ \mathcal{G} = \{ \mathcal{S}, \mathcal{b} \} \]

\[ P(Z_1 | \mathcal{G}) \]
\[ P(Z_N | \mathcal{G}) \]

\[ P(S_1 | Z_1) \]
\[ P(b_1 | Z_1) \]

\[ P(S_N | Z_N) \]
\[ P(b_N | Z_N) \]

Thompson, Mullen, Makeig, 2011, ICONXI
Thompson, Mullen, Makeig, 2012, in prep

alpha-testing
Causal Projection

Error > Correct (p < 0.05, N=24)
dDTF
3-7 Hz

Mullen, et al, 2010, HBM, Barcelona
Causal Projection

Error > Correct (p < 0.05) 3-7 Hz

Mullen, et al, 2010, HBM, Barcelona
Bayesian Multi-Subject Inference

 Theta-band (4-8 Hz) dDTF
 Response-locked error trials

\[ p < 0.01 \text{ (N=24)} \]

\[ 99\% \text{ CI} \]

\[
\begin{array}{cccccccccccc}
BA19/39 & BA31 & BA10 & BA32 & BA32/ACC & BA24/MCC & BA32 & BA2/CG & BA20 & BA7/Cuneus & Partial WM \\
(+50, -20) & (12, 41, -34) & (-52, 70, 22) & (-16, 13, 34) & (1, 30, 29) & (1, -23) & (20, 7, 22) & (39, -26, 43) & (-39, 19) & (5, 73, 32) & (27, 53, 37) \\
\end{array}
\]

\[ \text{FROM} \]

\[ \text{TO} \]

\[ \text{Conditional Granger-Causality (dDTF)} \]

\[ \text{Time (sec)} \]

Thompson, Mullen, Makeig, 2011, ICONXI
Thompson, Mullen, Makeig, 2012, in prep
Bayesian Multi-Subject Inference

Response-locked error trials
p<0.01 (N=24)

Theta-band (4-8 Hz)

BA2/pCG BA32 BA32/ACC
BA19/39 BA24/MCC
BA7/Cuneus BA31

BA24/MCC

Time (ms)

-600 -400 -200 0 200 400 600

-187 ms

Theta-band (4-8 Hz) dDTF08

Thompson, Mullen, Makeig, 2011, ICONX
Thompson, Mullen, Makeig, 2012, in prep
### Simulation

**Dynamical System Simulation Workbench**

<table>
<thead>
<tr>
<th>Systems of linear stochastically-forced damped coupled oscillators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support for arbitrary time-varying (non-stationary) coupling dynamics</td>
</tr>
<tr>
<td>Intuitive equation-based scripting environment</td>
</tr>
<tr>
<td>Support for generalized gaussian or hyperbolic secant innovations</td>
</tr>
</tbody>
</table>

**Nonlinear Dynamical Systems**

- Rössler and Lorenz Systems

---

- **fully implemented**
- **alpha-testing**
- **coming soon**
Example: Trivariate damped coupled oscillators with sinusoidally-modulated coupling

% STEP 1: create prototype VAR structure

Fs = 100; % Sampling Rate (Hz)
Nl = 500; % length of each epoch (samples)
Nr = 100; % number of trials (realizations)
ndisc = 1000; % number of startup samples to discard
ModelOrder = 2; % model order
f0 = 10; % central oscillation frequency (Hz)

expr = {...
    ['x1(t) = ' sim_dampedOscillator(f0,9,Fs,1) ' + e1(t)'] ...
    ['x2(t) = ' sim_dampedOscillator(f0,2,Fs,2) ' + 0.1*x1(t-2) + e2(t)'] ...
    ['x3(t) = ' sim_dampedOscillator(f0,2,Fs,3) ' + {0.3*sin(2*pi*t/100)+0.3}*x1(t-2) + e3(t)'] ...
};

Aproto = sim_genVARModelFromEq(expr,ModelOrder);

% generate simulated data with laplacian (supergaussian) innovations
data = sim_tvarsim(Mu,A,E,[Nl Nr],ndisc,1,1,'gengauss');
Simulated Seizure

- **Alpha**
  - Node 8
    - 10 Hz tau = 7
  - Node 7
    - 10 Hz tau = 7
  - Node 10
    - 11 Hz tau = 7
    - 9 Hz tau = 7

- **Beta**
  - Node 1
    - 20 Hz tau = 20
  - Node 2
    - 19 Hz tau = 3
  - Node 3
    - 20 Hz tau = 3
    - 21 Hz tau = 3

- **Alpha**
  - Node 5
    - 10 Hz tau = 6
  - Node 6
    - 10 Hz

**Node Numbers**
- Node 1
- Node 2
- Node 3
- Node 4
- Node 5
- Node 6
- Node 7
- Node 8
- Node 9
- Node 10
- Node 11
- Node 12
- Node 13

**Frequencies**
- 40 Hz

**Variables**
- $S1$
- $S2$
- $S3$

**Formulas**
- $\text{tau} = 7$
- $\text{tau} = 3$
- $\text{tau} = 6$
- $\text{tau} = 20$

**Saturday, June 16, 2012**
Simulated Seizure Sources
Where do I get SIFT?

sccn.ucsd.edu/wiki/SIFT
Some Applications of SIFT

Identification of event-related shifts in effective connectivity which index and predict behavior

Single-trial spatiotemporal modeling of seizure propagation dynamics

Brain-Computer Interfaces:
- Error correction/prediction
- Neural Prostheses

...
Some Applications of SIFT

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Brain-Computer Interfaces:
Error correction/prediction
Neural Prostheses
...

Mullen, et al IEEE EMBC, 2011
Some Applications of SIFT

Brain-Computer Interfaces:
Error correction/prediction
Neural Prostheses

Source Identification
- Independent Component Analysis
- Sparse Bayesian Learning
- Beamforming

Model Fitting
- Sparse Autoregressive Models
- Non-linear Kalman Filtering

Time-Frequency Connectivity
- Multivariate Granger Causality
- Cross-Frequency Coupling
- Graph-Theoretic Reduction

Multi-Subject Inference
- Bayesian Hierarchical Modeling
- Causal/Measure Projection

SIFT-BCILAB Interface

Source Identification

SIFT-BCILAB

Mullen et al., 2010

Source Identification

SIFT-BCILAB

Mullen et al., 2010
The Road Ahead

- Public release of alpha-testing methods (SIFT 1.0-beta ... being released at sccn.ucsd.edu/wiki/SIFT in the next week)

- Ongoing development of sparse/regularized VAR and state-space models as well as nonlinear SSMs

- Improved Group Analysis and Statistics

- Integration with other toolboxes: Transfer Entropy (TRENTOOL), Dynamic Causal Modeling (SPM), Brain-Computer Interfaces (BCILAB)

- Incorporation of structural constraints on dynamic connectivity (e.g. from DTI, anatomical priors, etc)
Acknowledgements

Arnaud Delorme
Christian Kothe
Nima Bigdely Shamlo
Zeynep Akalin Acar
Jason Palmer
Zhilin Zhang
Scott Makeig

Wes Thompson

Virginia de Sa
Vicente Malave

Ken Kreutz-Delgado