Forward and inverse models
Localizing sources using DIPFIT

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DIPFIT: localizing dipoles

• Motivation
• Ingredients
  – Source model
  – Volume conductor model
    • Analytical (spherical model)
    • Numerical (realistic model)
  – Comparison EEG and MEG
• Inverse modeling
  – Single and multiple dipole fitting
  – Distributed source models
DIPFIT: localizing dipoles

- **Motivation**

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Motivation

- Why fit dipoles?
- Why measure EEG?
- Why do ICA?

- Get extra information about brain processes
  - Time course of activity $\rightarrow$ EEG
  - Location of activity $\rightarrow$ fMRI
Difference between EEG and fMRI

- EEG measures post-synaptic potentials
  - related to synchronized neuronal input
- fMRI measures BOLD
  - related to energy consumption

- Different characteristics in the time domain
- Different generators

- Timecourse *and* location
Why EEG: extra information

- Timecourse
  - ERSP
  - ERP

- Topography
  - Scalp distribution
  - Underlying generators

ICA
Independent component analysis

Cocktail Party

Mixture of Brain source activity
Infomax ICA

\[ Y = [A; B] \]

\[ X = YW \]

ICA

\[ \tilde{Y} = W^T \tilde{X} \]
Source modelling

**forward problem**

- physiological source
- electrical current
- body tissue volume conductor
- observed potential or field

**inverse problem**
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Neuronal currents

Stellate cell

Pyramidal cell
Symmetry, orientation and activation

- radial symmetric
- random oriented
- asynchronously activated
- synchronously activated parallel oriented
Motivation for current dipoles

• Neurophysiological motivation
Equivalent current dipoles
Motivation for current dipoles

- Neurophysiological motivation

- Physical/mathematical motivation
  - Any current distribution can be written as a multipole expansion
  - First term: monopole (must be zero)
  - Second term: dipole
  - Higher order terms: quadrupole, octupole
Motivation for current dipoles

- Neurophysiological motivation

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  - Any current distribution can be written as a multipole expansion
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- Convenience
  - Dipoles can be used as building block in distributed source models
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Volume conductor

• electrical properties of tissue
• geometrical description

• spherical model
• realistic shaped model

→ Describes how the currents flow, not where they originate from
Volume conductor

- **Advantages spherical model**
  - mathematically accurate
  - reasonably accurate
  - computationally fast
  - easy to use

- **Disadvantages spherical model**
  - inaccurate, esp. in some regions
  - difficult alignment with anatomy
Volume conductor

• Advantages realistic model
  – accurate solution for EEG

• Disadvantages realistic model
  – more work
  – individual anatomical MRI required
  – computationally slow(er)
  – numerically instable
  – difficult in interindividual comparison

→The pragmatic solution is to use a standard realistic headmodel for EEG
Realistic volume conductor

• Computational methods for volume conduction problem that allow realistic geometries
  – Boundary Element Method (BEM)
  – Finite Element Method (FEM)

• Geometrical description
  – triangles
  – tetraeders/voxels
Volume conductor: BEM

• Boundary Element Method
  – description of geometry by compartments
  – each compartment is
    • homogenous
    • isotropic
  – important tissues
    • skin
    • skull
    • brain
    • (CSF)
  – triangulated surfaces as boundaries
  – surfaces should be closed
Volume conductor: FEM

- Tessellation of 3D volume in tetraeders
- Large number of elements
- Each tetraeder can have its own conductivity

- FEM is most accurate numerical method
- Computationally expensive
- Accurate conductivities are not (well) known
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EEG volume conduction
EEG volume conduction

- Potential difference between electrodes corresponds to current flowing through skin
- Only tiny fraction of current passes through skull
- Therefore the model should describe skull and skin as accurately as possible

- Problems with skull
  - Not visible in anatomical MRI
  - Thickness varies
  - Conductivity is not homogeneous
  - Complex geometry at base of skull
Electric current → magnetic field
MEG volume conduction

- Measures sum of fields associated with
  - Primary currents
  - Secondary currents !!!
MEG volume conduction

- Only tiny fraction of current passes through the poorly conductive skull
- Therefore skull and skin can be neglected in the MEG model
- Local conductivity around dipole important
  - geometry
  - conductivity
Differences between EEG and MEG

- more blurred
- deep sources
- electrode noise
- reference electrode
- fixed to head
- skull+skin important for modelling

- no radial or deep sources
- environmental noise
- independent sensors
- head can move
- skull+skin not important
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*inverse problem*
Inverse methods

- Single and multiple dipole models
  - Minimize error between model and measured potential/field

- Distributed dipole models
  - Perfect fit of model to the measured potential/field
  - Minimize additional constraint on sources
  - LORETA (smoothness)
  - Minimum Norm (L2)
  - Minimum Current (L1)

- Spatial filtering
  - Scan whole brain with single dipole and compute the filter output at every location
  - MUSIC
  - Beamforming (e.g. LCMV, SAM, DICS)
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Single or multiple dipole models

- Manipulate source parameters to minimize error between measured and model data
  - Location of each source
  - Orientation of each source
  - Strength of each source
- Orientation and strength together correspond to the “dipole moment” and can be estimated linearly
- Position is estimated non-linearly
- Source parameter estimation
Parameter estimation

\[ Y = f(X; \ a, b) = a \times X + b \]

\[ \xi = a, b, c, \ldots \]
Parameter estimation: model

measured potential

forward model for the data

select “optimal” model

\[ V_i = V(\vec{r}_i) + \text{noise} \]

\[ Y_i = Y(r_i; \xi) + \text{noise} \]

\[ \min_\xi \left\{ \sum_{i=1}^{N} (Y(r_i; \xi) - V(r_i))^2 \right\} \]
Select optimal model

\[ \text{error}(\xi) = \sum_{i=1}^{N} (Y_i(\xi) - V_i)^2 \Rightarrow \min_{\xi} (\text{error}(\xi)) \]

\[ \xi = a, b, c, \ldots \]
Dipole *scanning*: grid search

- define grid with allowed dipole locations
- compute optimal dipole moment for each location
- compute value of goal-function
- plot value of goal-function on grid

- number of evaluations:
  - single dipole, 1 cm grid: ~4 000
  - single dipole, ½ cm grid: ~32 000
  - two dipoles, 1 cm grid: ~16 000 000
Dipole fitting: nonlinear search

- start with an initial guess
- evaluate the local derivative of goal-function
- “walk down hill” to the most optimal solution

- number of evaluations: ~100
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Distributed source model

- Position of the source is **not estimated** as such
  - Pre-defined grid (3D volume or on cortical sheet)
- Strength is estimated
  - In principle easy to solve, however...
  - More “unknowns” (parameters) than “knowns” (measurements)
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem
Distributed source model

- Linear estimation

\[ \Psi = q_1 \Psi_1 + q_2 \Psi_2 + \ldots = \begin{bmatrix} \Psi_{1,1} & \Psi_{2,1} & \ldots \\ \Psi_{1,2} & \Psi_{2,2} & \ldots \\ \vdots & \vdots & \ddots \\ \Psi_{1,N} & \Psi_{2,N} & \ldots \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = L \cdot \tilde{q} \]

\[ \tilde{q} = L^{-1} \cdot \Psi \]
Distributed source model

\[ V = L \cdot q + \text{Noise} \]

\[
\min_q \{ \| V - L \cdot q \|^2 \} = 0 \quad !!
\]

- Regularized linear estimation:
  \[
  \min_q \{ \| V - L \cdot q \|^2 + \lambda^2 \cdot \| D \cdot q \|^2 \}
  \]

- Constrained linear estimation:
  \[
  \min_q \{ q^T \cdot W \cdot q \} \quad \text{while} \quad \| V - L \cdot q \|^2 = 0
  \]
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Spatial filtering

- position of the source as such is not estimated
- scanning the whole brain
  - single dipole as source
  - estimate activity at each grid location
    - that explains a part of the data
    - that suppresses other activity
- various methods
  - multiple signal classification (MUSIC)
  - beamforming
  - LCMV, SAM, DICS, …
- not a distributed source model, but a distributed representation of the single dipole estimate
- unmixing of data into “signal source” and “noise sources” using assumptions on temporal relation between sources
Summary 1

- Forward modelling
  - Required for the interpretation of scalp topographies
  - Interpretation of scalp topography is “source estimation”
  - Mathematical techniques are available that aid in interpreting scalp topographies -> inverse modeling
Summary 2

- Inverse modeling
  - Model assumption for volume conductor
  - Model assumption for source (i.e. dipole)
  - Additional assumptions on source
    - Single point-like source
    - Multiple point-like sources
    - Distributed source
  - Different mathematical solutions
    - Dipole fitting (linear and nonlinear)
    - Linear estimation (regularized)