Time-Frequency analysis of biophysical time series

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Frequency analysis

synchronicity of cell excitation determines amplitude and rhythm of the EEG signal

- 0.5-2 Hz Delta
- 4-7 Hz Theta
- 9-11 Hz Alpha
- 18-21 Hz Beta
- 30-60 Hz Gamma
Frequency analysis

Beta

Alpha

Theta

Delta

Low Delta
Stationary signals

2 Hz

10 Hz

20 Hz

2+10+20 Hz

Slide courtesy of Petros Xanthopoulos, Univ. of Florida
Stationary signal

By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2,10,20Hz respectively).

Slide courtesy of Petros Xanthopoulos, Univ. of Florida
Forward transform
\[ F(u) = \int_{-\pi}^{\pi} f(x) e^{-2\pi iux} \, dx \]

Inverse transform
\[ f(x) = \sum_{u = -\infty}^{+\infty} F(u) e^{2\pi iux} \]
Performing Fourier transform by using a time moving window
Spectral phase and amplitude

Imaginary

Real

Imag.  
Real
Spectral phase and amplitude

\[ F_k(f,t) \]
function \([a,b] = \text{dft}(y)\)
\%
DFT - The Discrete Fourier Transform
\%
[a, b] = DFT(y)
\%
a, b are the cosine and sine components

\[\text{length}(y) = n;\]
\[t = 2\pi(0:n-1)/n;\]
\[f = 2.0 / n;\]

\[\text{for} \ j = 0:n2\]
\[\text{cs} = \cos(j \cdot t);\]
\[\text{ss} = \sin(j \cdot t);\]
\[a(j+1) = f \cdot (\text{cs} \cdot y);\]
\[b(j+1) = f \cdot (\text{ss} \cdot y);\]
\[\text{end}\]

\%
boundaries
\[n2 = \text{floor}(n / 2);\]
\[a(1) = 0.5 \cdot a(1);\]
\[a(n2+1) = 0.5 \cdot a(n2+1);\]
\[b(1) = 0.0;\]
\[b(n2+1) = 0.0;\]
Discrete Fourier Transform function

function X = dft(x)

[N,M] = size(x);
n = 0:N-1;
for k=n
    X(k+1) = exp(-j*2*pi*k*n/N)*x;
end

Real part
Cosine component

Imaginary part
Sine component

Multiply with signal
Average of squared absolute values
Spectral power

Power (dB) vs Frequency (Hz)

Average of squared amplitude

-30 -20 -10 0 10

0 10 20 30 40 50
Overlap 50%

Average of squared amplitudes
Spectrogram or ERSP

0 ms 10 ms 20 ms 30 ms 40 ms 50 ms 60 ms

5 Hz

10 Hz

20 Hz

30 Hz
Spectrogram or ERSP

Average of squared values
Power spectrum and event-related spectral perturbation

\[ ERS(f, t) = \frac{1}{n} \sum_{k=1}^{n} |F_k(f, t)|^2 \]

Scaled to dB \(10\log_{10}(ERSP)\)
Absolute versus relative power

Absolute = ERS

Relative = ERSP (dB or %)
Difference between FFT and wavelets

FFT

Wavelet

Frequency
Time-frequency resolution trade off

FFT

Exact

Wavelet

High freq. resolution
low time-resolution

Low freq. resolution
high time-resolution
FFT

Pure wavelet
The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/position and in frequency/momentum.

There exists a lower bound to the Heisenberg’s product:

\[ \Delta t \Delta f \geq \frac{1}{4\pi} \]

\[ \Delta f = 1\text{Hz}, \quad \Delta t = 80\text{ msec} \text{ or } \Delta f = 2\text{Hz}, \quad \Delta t = 40\text{ msec} \]
Modified wavelets

Wavelet (0.8)  Wavelet (0.5)  Wavelet (0.2)
Inter trial coherence

same time, different trials

Trial 1
amplitude 0.5     phase 0

Trial 2
amplitude 1     phase 90

Trial 3
amplitude 0.25     phase 180

POWER = mean(amplitudes$^2$)
0.44 or –8.3 dB

COHERENCE = mean(phase vector)

Norm 0.33
Phase ITC

\[ ITPC(f, t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k(f, t)}{|F_k(f, t)|} \]

(normalized
(no amplitude information)
Power and inter trial coherence

Attend left-stim left

Attend left-stim right

Difference
Pure green denotes non-significant points.
Plot IC ERSP

Increase # freq bins

padratio = 1

padratio = 2

Component number
Sub epoch time limits [min max] (msec)
Frequency limits [min max] (Hz) or sequence
Baseline limits [min max] (msec) (0->pre-stim.)
Wavelet cycles [min max fac] or sequence
ERSP color limits [min max] (min=max)
ITC color limits [max]
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional newtime() argument (see Help)

Use limits, padding 1

Use 256 time points
Use divisive baseline
Use limits

Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency
Shows the actual dominant phase of the signal.
To visualize both low and high frequencies

```matlab
freqs = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
```
Evoked versus induced

• Evoked = ERSP of the average ERP
• Induced = usually standard ERSP
• Real induced
  (1) standard ERSP with ERP regressed out of every trial
  (2) standard ERSP minus ERSP of the average ERP scaled for averaging effect

In any case, looking at the ITC provides the amount of synchronization in the time-frequency decomposition that account for ERPs
Component time-frequency
Cross-coherence amplitude and phase

2 components, comparison on the same trials

Trial 1

Trial 2

Trial 3

COHERENCE = mean(phase vector)
Phase coherence (default)

\[ ERPCOH^{a,b}(f,t) = \frac{1}{n} \sum_{k=1}^{n} \frac{F_k^a(f,t)F_k^b(f,t)^*}{F_k^a(f,t)F_k^b(f,t)} \]
Other spectral measures

Amplitude correlation

\[
\text{corr}^{a,b}(f,t) = \frac{\sum_{k=1}^{n} (F_k^a(f,t) - \overline{F^a(f,t)}) (F_k^b(f,t) - \overline{F^b(f,t)})}{\sqrt{\frac{1}{n} \sum_{k=1}^{n} (F_k^a(f,t) - \overline{F^a(f,t)})^2} \sqrt{\frac{1}{n} \sum_{k=1}^{n} (F_k^b(f,t) - \overline{F^b(f,t)})^2}}
\]
Cross-coherence amplitude and phase

**Animal picture**

- **Amplitude (0-1)**
- **Phase (degree)**

**Distractor picture**

- **Amplitude (0-1)**
- **Phase (degree)**

The diagrams show the cross-coherence amplitude and phase for both animal and distractor pictures over time (ms). The color scale indicates the amplitude and phase values.
Two EEG channels

Scalp channel coherence → source confounds!
Cortex

MANY EEG channels

Separate out Independent EEG Components

Measure their Synchronization

source dynamics!
Niquist frequency: Aliasing

e.g. 100 Hz sampled at 120 Hz
Advanced time-frequency functions

- Tftopo(): allow visualizing time-frequency power distribution over the scalp
Plot data spectrum using EEGLAB

- 'winsize', 256 (change FFT window length)
- 'nfft', 256 (change FFT padding)
- 'overlap', 128 (change window overlap)
Exercise

• ALL
Start EEGLAB, from the menu load
sample_data/eeglab_data_epochs_ica.set
or your own data (epoch, reject noise if not
done already)

• Novice
From the GUI, Plot spectral decomposition
with 100% data and 50% overlap (‘overlap’).
Try reducing window length (‘winsize’) and
FFT length (‘nfft’)

• Intermediate
Same as novice but using a command line
call to the pop_spectopo() function. Use GUI
then history to see a standard call (“eegh”).

• Advanced
Same as novice but using a command line
call to the spectopo() function.
Exercise - newtimef

• **Novice**
  From the GUI, pick an interesting IC and plot component ERSP. Try changing parameters window size, number of wavelet cycles, padratio,

• **Intermediate**
  From the command line, use newtimef() to tailor your time/frequency output to your liking. Look up the help to try not to remove the baseline, change baseline length and plot in log scale. Enter custom frequencies and cycles (2 slides back).

• **Advanced**
  Compare FFT, the different wavelet methods (see help), and multi-taper methods (use timef function not newtimef). Enter custom frequencies and cycles. Look up newtimef help to compare conditions. Visualise single-trial timef-frequency power using erpimage.