Time-Frequency Analysis of Biophysical Time series

June 20, 2011 Thirteenth EEGLAB Workshop, Aspet, France

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(with majority of slides modified from those of Arnaud Delorme)
Frequency analysis

The synchronicity of cell excitation determines the amplitude and rhythm of the EEG signal. Different frequency bands are characterized by different brain states:

- **30-60 Hz Gamma**
- **18-21 Hz Beta**
- **9-11 Hz Alpha**
- **4-7 Hz Theta**
- **0.5-2 Hz Delta**

Duration: 1 second
Frequency analysis

Beta

Alpha

Theta

Delta

Low Delta
Stationary signals

Slide courtesy of Petros Xanthopoulous, Univ. of Florida
By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2, 10, 20 Hz respectively).
Fourier’s Theorem

Time domain

\[ x(t) \]

Frequency domain

\[ |S(f)| \]

**Time domain**

**Frequency domain**

**Freq. decomp.**

**Sum of freq.**
Nyquist frequency: Aliasing

e.g. 100 Hz signal sampled at 120 Hz

Nyquist Frequency:
Max frequency that can be uniquely recovered at sampling rate of $f_s$

$$f_N = f_s / 2$$

$$f_{alias}(N) = |f + Nf_s|$$

$$f_s = \text{sampling rate}$$
Euler’s Formula

\[ A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t + \theta)} + \frac{A}{2} e^{-i(\omega t + \theta)} \]

\[ = \text{Re} \{ Ae^{i(\omega t + \theta)} \} = \text{Re} \{ S(\omega, t) \} \]

- **Phase shift**: \( \theta = \frac{\pi}{2} \)
- **Angular frequency**: \( \omega = 2\pi f \)

**Phase shift**

**Angular frequency**

**Instantaneous complex power** (amplitude and phase)
Euler’s Formula

\[ A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t + \theta)} + \frac{A}{2} e^{-i(\omega t + \theta)} \]

\[ = \text{Re}\{A e^{i(\omega t + \theta)}\} = \text{Re}\{S(\omega, t)\} \]

\[ \theta = \angle S(\omega, t) = \pi / 2 \]

\[ |S(\omega, t)| = |A| \]

Phase shift: \( \theta = \pi / 2 \)

Angular frequency: \( \omega = 2\pi f \)

Instantaneous complex power (amplitude and phase)

\[ A = 1.2 \]

\[ t = R\{A e^{i(\omega t + \theta)}\} = R\{S(\omega, t)\} \]

\[ \text{Imag.} \]

\[ \text{Real} \]

Phasor!
Euler’s Formula

\[ A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t + \theta)} + \frac{A}{2} e^{-i(\omega t + \theta)} \]

\[ = \text{Re}\{Ae^{i(\omega t + \theta)}\} = \text{Re}\{S(\omega, t)\} \]

Another version:

\[ e^{i(\omega t + \theta)} = \cos(\omega t + \theta) + i \sin(\omega t + \theta) \]

Real part

Cosine component

Imaginary part

Sine component

\[ |S(\omega, t)| = |A| \]

\[ \theta = \angle S(\omega, t) = \pi / 2 \]

Instantaneous complex power (amplitude and phase)

Phase shift

\( \theta = \pi / 2 \)

Angular frequency

\( \omega = 2\pi f \)
Phasers
Phasors

Rotation velocity (Rad/S; Hz) = (angular) frequency (ω; f)

\[ \begin{align*}
A \cdot \cos(\omega t + \theta) &= \text{Re}\{Ae^{i(\omega t + \theta)}\} \\
&= \text{Re}\{S(\omega, t)\}
\end{align*} \]

\[ |S(\omega, t)| = |A| \]

\[ \theta = \frac{\pi}{2} \]
Fourier Transform

Forward transform

\[ S(f) = \frac{1}{N} \sum_{t=0}^{N-1} x(t)e^{-2\pi ift/N} \]

N = number of samples

Fast Fourier Transform (FFT)

Inverse transform

\[ x(t) = \frac{1}{N} \sum_{f=0}^{N} S(f)e^{2\pi ift/N} \]

Time \( \rightarrow \) Frequency

Frequency \( \rightarrow \) Time
Performing Fourier transform by convolution with a (optionally tapered) complex sinusoid
Tapering

Smoothly decay signal to zero at endpoints to avoid discontinuity

Gibbs Phenomenon
“Rippling” effect due to discontinuities in signal (e.g. edges of the truncated signal)
Spectral phase and amplitude

\[ f_k(f, t) = i \sin(2\pi ft) + \cos(2\pi ft) \]
Spectral phase and amplitude

\[ F_k(f,t) = i \sin(2\pi ft) + \cos(2\pi ft) \]

Real

Imaginary
The Welch method
Spectral power

Power (dB)

Frequency (Hz)

Average of squared amplitude

Window 1

Window 2

Window 3

Frequency points: 0 Hz, 10 Hz, 20 Hz, 30 Hz, 40 Hz, 50 Hz

Windowed average of squared amplitude
Overlap 50%

Average of squared amplitudes

The Welch method
Non-Stationary Signals

- Bursts, chirps, evoked potentials, …
Spectrogram or ERSP
Spectrogram or ERSP

FFT

5 Hz

10 Hz

20 Hz

30 Hz

0 ms 10 ms 20 ms 30 ms 40 ms 50 ms 60 ms
Spectrogram or ERSP

Average of squared values
Power spectrum and event-related spectral (perturbation)

\[
ERS(f, t) = \frac{1}{n} \sum_{k=1}^{n} |S_k(f, t)|^2
\]

Scaled to dB \(10\log_{10}\)

Here, there are \(n\) trials.
Each trial is time-locked to the same event (hence “event-related” spectrum).
The ERS is the average power across event-locked trials.
Absolute versus relative power

Absolute = ERS

To compute the ERSP, we just subtract the pre-stimulus ERS from the whole trial

Relative = ERSP (dB or %)
Wavelets factor

Wavelet (0) = FFT

Wavelet (1)

1Hz
2Hz
4Hz
6Hz
8Hz
10Hz
Time-frequency resolution trade off

FFT

Wavelet

Exact

High freq. resolution
Low time-resolution

Low freq. resolution
High time-resolution
Time-frequency resolution trade off

FFT

Wavelet

Exact

Too low

High freq. resolution low time-resolution

Low freq. resolution high time-resolution
Time-frequency resolution trade off

**FFT**

- **Exact**: +++
- **Too low**: -++
- **Too high**: -+-

*High freq. resolution, low time-resolution*

**Wavelet**

- **Exact**: +++
- **Too low**: +++
- **Too high**: ++

*Low freq. resolution, high time-resolution*
Difference between FFT and wavelets

From http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10
FFT

Pure wavelet
The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/position and in frequency/momentum.

There exists a lower bound to the Heisenberg product:

$$\Delta t \Delta f \geq 1/(4\pi)$$

$$\Delta f = 1\text{Hz}, \Delta t = 80\text{ msec} \text{ or } \Delta f = 2\text{Hz}, \Delta t = 40\text{ msec}$$
Modified wavelets

Wavelet (0.8)  Wavelet (0.5)  Wavelet (0.2)

\[ C_{\text{fmax}} = \frac{f_{\text{max}}}{f_{\text{min}}} C_{\text{min}} (1 - q) \]
Inter trial coherence (ITC)

same time, different trials

Trial 1
amplitude 0.5 phase 0

Trial 2
amplitude 1 phase 90

Trial 3
amplitude 0.25 phase 180

POWER = mean(amplitudes^2) 0.44 or –8.3 dB

ITC = mean(phase vector) Norm 0.33
Intertrial Coherence (ITC)

Single trials

ERP

Total power

ITC: .05

ITC: .80

Slide courtesy of Stefan Debener
Phase ITC

\[ ITPC(f, t) = \frac{1}{n} \sum_{k=1}^{n} \frac{S_k(f, t)}{|S_k(f, t)|} \]

- complex numerator
- Normalized (no amplitude information)
**Phase ITC**

\[ ITPC(f, t) = \frac{1}{n} \sum_{k=1}^{n} \left( \frac{S_k(f, t)}{|S_k(f, t)|} \right) \]

complex numerator

Normalized (no amplitude information)
Power and inter trial coherence

**Attend left-stim left**

**Attend left-stim right**

**Difference**
Using 3 cycles at lowest frequency to 12.8 at highest.
Generating 200 time points (-713.6 to 1709.6 ms)
The window size used is 143 samples (572 ms) wide.
Estimating 23 linear-spaced frequencies from 5.9 Hz to 50.0 Hz.
Processing time point (of 200): 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
Pure green denotes non-significant points.
**Figure 3**

Component 1 power and inter-trial phase coherence (faces, epochs)

- **padratio = 1**
- **padratio = 2**

**Figure 4**

Component 1 power and inter-trial phase coherence (faces, epochs)

- **padratio = 1**
- **padratio = 2**

**Increase # freq bins**
Plots IC ITC shows the actual dominant phase of the signal.
To visualize both low and high frequencies

```matlab
freqs = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
```
Evoked versus induced

- Evoked = ERSP of the average ERP
- Induced = usually standard ERSP
- Real induced
  1. standard ERSP with ERP regressed out of every trial
  2. standard ERSP minus ERSP of the average ERP scaled for averaging effect

In any case, looking at the ITC provides the amount of synchronization in the time-frequency decomposition that account for ERPs.
Component time-frequency
cross-coherence amplitude and phase

2 components, comparison on the same trials

COHERENCE = mean(phase vector)
Event-related phase coherence

\[ \text{ERPCOH}^{a,b}(f,t) = \frac{1}{n} \sum_{k=1}^{n} \frac{S_k^a(f,t) S_k^b(f,t)^*}{|S_k^a(f,t)| |S_k^b(f,t)|} \]
Cross-coherence amplitude and phase

Animal picture

Distractor picture
Two EEG channels

Scalp channel coherence → source confounds!
MANY EEG channels

Separate out Independent EEG Components

Measure their Synchronization

source dynamics!
Plot data spectrum using EEGLAB

\[ \text{winsize}, 256 \] (change FFT window length)
\[ \text{nfft}, 256 \] (change FFT padding)
\[ \text{overlap}, 128 \] (change window overlap)
Exercise

• **ALL**
  Start EEGLAB, from the menu load `sample_data/eeglab_data_epochs_ica.set`
or your own data (epoch, reject noise if not done already)

• **Novice**
  From the GUI, Plot spectral decomposition with 100% data and 50% overlap (‘overlap’). Try reducing window length (‘winsize’) and FFT length (‘nfft’)

• **Intermediate**
  Same as novice but using a command line call to the `pop_spectopo()` function. Use GUI then history to see a standard call (“eegh”).

• **Advanced**
  Same as novice but using a command line call to the `spectopo()` function.
Exercise - newtimef

• **Novice**
  From the GUI, pick an interesting IC and plot component ERSP. Try changing parameters window size, number of wavelet cycles, padratio,

• **Intermediate**
  From the command line, use newtimef() to tailor your time/frequency output to your liking. Look up the help to try not to remove the baseline, change baseline length and plot in log scale. Enter custom frequencies and cycles (2 slides back).

• **Advanced**
  Compare FFT, the different wavelet methods (see help), and multi-taper methods (use timef function not newtimef). Enter custom frequencies and cycles. Look up newtimef help to compare conditions. Visualize single-trial timef-frequency power using erpimage.
Advanced time-frequency functions

- Tftopo(): allow visualizing time-frequency power distribution over the scalp
Advanced time-frequency functions

- ERPimage: allow visualizing time-frequency power or phase in single trials
Across frequency study

\[ APCOH^{a,b}(f_1, f_2, t) = \sum_{k=1}^{n} \left| F_k^a(f_1, t) \right| \frac{F_k^b(f_2, t)}{F_k^b(f_2, t)} \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \left| F_k^a(f_1, t) \right|^2 \]
Subjects

EEG

ICA

Clustering across subjects

Brain dynamic movie

Spectral analysis

Dipole density

Localize

Right μ (6)
Left μ (5)
Central α (9)
Left α (7)
Right α (8)
Dynamical brain movies

5 Hz

Cross-coh amplitude RT lock (p=0.01)