Neurocomputing, 38-40(1-4), 817-822, 2001

Networks of Integrate-and-Fire Neurons using Rank Order Coding. - *A : How to Implement Spike Time Dependent Hebbian Plasticity*

Perrinet L. [a] Delorme A. [b] Samuelides M. [a] Thorpe S.J. [b]

[a] *ONERA-DTIM, 2 Av. É. Belin, BP 4025, 31055 Toulouse, France*

[b] *Centre de Recherche Cerveau et Cognition (UMR 5549) Faculté de Médecine de Rangueil, 133 route de Narbonne, 31062 Toulouse, France.*

Abstract

Based on neurophysiological observations on the behavior of synapses, Spike Time Dependent Hebbian Plasticity is a novel extension to the modeling of the Hebb Rule [6]. This rule has enormous importance in the learning of Spiking Neural Networks (SNN) but its mecanisms and computational properties are still to be explored.

In this article, we present a generative model for Spike Time Dependent Plasticity based on a simplified model of the synaptic kinetic. We then explore the fitting of this model to experimental data and review some of it's dynamical properties. Finally we extend this model to a simplified neural network model of Integrate and Fire (IF) using Rank Order Coding.

Key words: Spiking Neural Networks, Hebb Rule, Spike Time Dependent Hebbian Plasticity, Kinetic model, Rank Order Coding

1 The Hebb Rule in Neuronal Modeling and SNNs

In the context of studying the adaptability of an image processing system of a robotic vehicle using a neuromimetic spiking neural network, we faced the problem of finding an unsupervised learning rule detecting the statistical features of the input. Due to its simplicity and biological plausibility, we have chosen to implement an hebbian-like learning based on the temporal relation between pre- and post-synaptic firing.
Let's remember Hebb's formulation :

> When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change

takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased.[6]

This rule may be interpreted as a strengthening of the wiring between the neurons that form a causal chain. We may reformulate it in the context of SNNs as a *temporal hebb rule* : "When there exist synapses between two neurons, the ones transmitting presynaptic spikes until the emission of the postsynaptic spike are reinforced, the others weakened."
To model it, we first used a binary rule that reinforced (resp. inhibited) equally the synapses activated before (resp. after) the postsynaptic spike. Actually, biological learning mechanisms depend on very small difference in the latencies between the upper and the downstream spiking date ([7] and [2]).
In these last experiences, repeated pairing of a pre- and a post-synaptic spike was applied to a synapse leading to potentiation and depression similar to the temporal hebb rule. More precisely, the amplitude of the relative change in the weights was exponentially decreasing with the absolute spike time difference on a time scale of $\sim$20 ms. We present here a general synaptic model for Spike Time Dependent Plasticity.

2 A generative model for Spike Time Dependant Plasticity

2.1 Description of the model

Kinetic models of synaptic transmission are succesful descriptions of the behavior of the concentration of the different ions that lead to potential difference accross the neuron's membrane and therefore to neuronal information processing.
In our model, we will consider that neurons are emitting pulses (or spikes) that propagate along the axon but also back to the dendrites; the synapses are the contact sites between them. Synapses are considered like filters transmitting a current to the postsynaptic site, that are linear under the assumption of low spike frequency : the response to a spike is its impulse response or Post Synaptic Pulse (PSP). The synapses may be valuated by the amplitude of the PSP and are therefore characterized by a weight $g.g_{max}$ where g_{max} is the maximum possible weight and g ranges from 0 to 1.
On one side, we'll consider a pool of emitters (corresponding to the docked vesicles containing neurotransmitter) quantified by their relative concentration C. This quantity is triggered by presynaptic spikes but this pool is limited (leading to synaptic depression). On the other side, we'll consider a pool of receivers (corresponding to the sensitivity of postsynaptic sites and that appear to be calcium related mecanism) similarly quantified by D. This quantity is mediated by postsynaptic spikes and may be related to synaptic facilitation

in our model.

2.2 Modeling the synapses' dynamics by kinetic equations

Modeling C (resp. D) for the synapse $(A-B)$ (from neuron A to neuron B) with a first order kinetic pulse based model of decay time constant τ_C and pulse amplitude α_C (resp. τ_D and α_D), we get by writing as in [5] the presynaptic spike times by $t_k^{(A-B)}$ (resp. postsynaptic by t_k^{out}) :

$$\frac{\mathrm{d}C^{(A-B)}}{dt} = -\frac{1}{\tau_C}C^{(A-B)} + \alpha_C \sum_k \delta(t - t_k^{(A-B)}).(1 - C^{(A-B)})$$
$$\frac{\mathrm{d}D}{dt} = -\frac{1}{\tau_D}D + \alpha_D \sum_j \delta(t - t_j^{out}).(1 - D)$$

Finally, according to our definition of synaptic weight, we may model the variation of g like the combination of :

- an excitation relative to the synapse strength and proportional to the emitters' concentration when a postsynaptic spike arrives and
- an inhibition relative to the existing synapse weakness and proportional to receivers' concentration when a presynaptic spike arrives.

Modeling it with a first order kinetic of decay time constant τ_g we get :

$$\tau_g \frac{\mathrm{d}g^{(A-B)}}{dt} = +(1 - g^{(A-B)}).C.\sum_j \delta(t - t_j^{out}) - g^{(A-B)}.D.\sum_k \delta(t - t_k^{(A-B)})$$

This formulation has the computational advantage that it depends only on the present state in comparison with rules that need to store spike timings in a certain window.

3 Results

3.1 Fitting neurophysiological results

Replicating the conditions of [7] we may easily obtain for a given initial weight an analytical formulation for the relative change of weight in our model. The

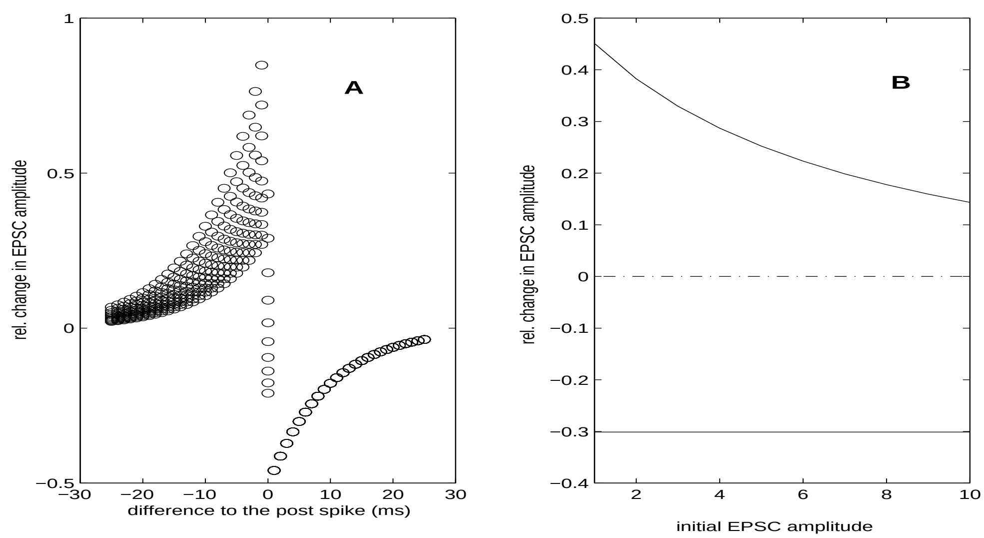

Fig. 1. Simulation of the synapse model. A : relative weight variation for different initial weights. The conditions of the simulation are replicated from [7]. The weight are strengthened if the presynaptic spike occurs before the postsynaptic spike. The amplitude of change decreases exponentially with the time difference. B : relative weight variation depending on the initial weight (replicating the conditions of [2] i.e. a time difference of $\pm 6ms$). The rule is multiplicative : decreasing with weight amplitude for potentiation and constant for depotentiation.

constant of our model (the time constants and amplitudes of C and D) correspond to the time and amplitude constants of the explicit rule respectively before and after the postsynaptic spike (see Fig. 1-A).
We also find that the resulting explicit expression of weight change according to the initial weight (see Fig. 1-B) is analogous to the multiplicative rule described in [2] and that is analytically studied in [8].

3.2 Rule's properties

The rule we propose is therefore a natural extension of the explicit rules described in [8] and [9]. We applied that rule to different models of neurons like Integrate-and-Fire (IF) [3] or Conductance based IF [4], obtaining results similar as those in [8] and [9]) :

- Adaptation to random input : given a random input (like a pseudo-Poisson spike train), the neuron tends to generate a spike train with a great variability in the interspike interval (ISI) distribution,
- Learning to detect a synchronized input : The rule is adapted to cluster synapses that receive synfire inputs, leading to a bimodal weight distribution differentiating synchronized and unsynchronized inputs.
- Asynchronous wave detection : under certain conditions, this rule may provide a way to detect temporal patterns.

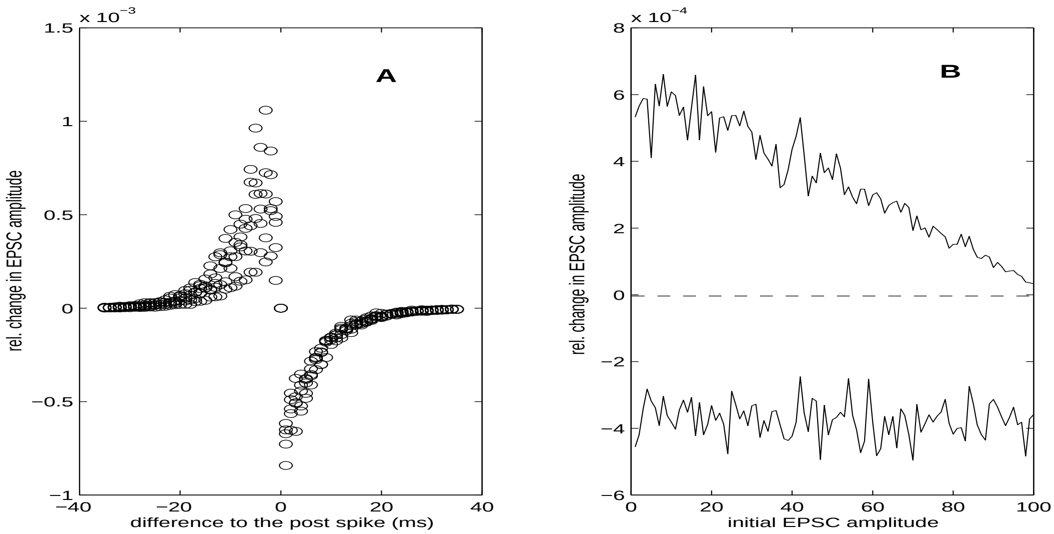

Fig. 2. Simulation of the synapse model based on rank order coding. A : relative weight variation. As an impulse wave arrives to the synapses (100 synapses, one spike max per neuron, timing according to a modified Poisson distribution), the relative weight change based on a rank order rule is similar to Fig. 1. B : relative weight variation depending on the initial weight, we still have a multiplicative rule.

3.3 Application to Rank Order Coding (ROC)

Neuropsychological experiments have recently shown the incompatibility of the classical rate code introduced in 1926 by Adrian and Zotterman. Actually, the speed of processing in the visual system can be as short as 150 ms for a complex task involving at least 10 processing layers [10] leaving less than 10 ms for the neuron to process and transmit information. A solution is to use the asynchronous characteristics of the neural code, as in the analog-to-delay conversion characteristic of retinal ganglion cells by using integrate-and-fire like models of neurons : generally, the stronger the input intensity, the quicker the neuron fires.

We therefore propose a model using only one spike per neuron, and simplifying the IF's dynamic, we study a network of neurons using the rank of the incoming spikes as a possible coding. Similarly as for the first synaptic model, we may study a rule that relies on the order of the presynaptic spikes relative to the postsynaptic firing with a similar shape (or kinetic rules). Simulating this rule on a temporal scale but with the rule based on ROC and with input distributed as a biologically adapted Poisson distribution, we obtain similar results as those in Fig.1 (see Fig. 2). Further results are described in the companion paper (same volume) : Delorme A. et al. *Networks of Integrate-and-Fire Neurons using Rank Order Coding B: Spike Timing Dependent Plasticity and Emergence of Orientation Selectivity.*

To conclude, faced with unsatisfactory models of the mecanisms underlying Spike Time Dependent Plasticity [9], we found a generative model that is compatible with electrophysiological results and is able to explain the relation between synapse strength and percentage of change in synaptic weight [2]. We also showed that such a rule can comply with different coding scheme that include latency coding and Rank Order Coding.

References

[1] Abbott LF, Varela JA, Sen K, Nelson SB (1997) *Synaptic depression and cortical gain control.* Science 275: 220-224.

[2] Bi G.-q., Poo M.-m. (1998), *Synaptic modifications in cultured hippocampal neurons: dependence on spike timing, synaptic strength, and postsynaptic cell type.* J. Neurosci., 18:10464–10472.

[3] Brunel N., Sergi S. (1998) *Firing frequency of leaky integrate-and-fire neurons with synaptic current dynamics*, Journal of Theoretical Biology.

[4] Destexhe A.(1996) *Conductance based Integrate-and-Fire Models* Neural Comp.

[5] Gerstner, W., Kempter, R., van Hemmen, J. L., and Wagner, H. (1996). *A neuronal learning rule for sub-millisecond temporal coding.* Nature, 383:76–78.

[6] Hebb D. O. (1949) *The Organization of Behavior*, Wiley & Sons, New York.

[7] Markram H. and J. Lübke and M. Frotscher and B. Sakmann (1997) *Regulation of Synaptic Efficacy by Concidence of Postsynaptic APs and EPSPs* Nature, 275, 213-215.

[8] Rubin J., Lee D. and Sompolinsky H. (2000) *Equilibrum Properties of Temporally Asymmetric Hebbian plasticity.* ESF Symposium on Neuronal Plasticity and Dynamics, Trieste.

[9] Song S., Miller K. D., Abbott L. F. (2000) *Competitive Hebbian learning through spike time dependent synaptic plasticity.* Nat. Neurosci., 3:919–926.

[10] Thorpe S. J. and D. Fize and C. Marlot (1996), *Speed of processing in the human visual system* Nature, 381, 520-522.