Building systems exhibiting Andronov-Hopf super- and sub-critical bifurcations

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1 Theory

From Problem 6-14 in Izhikevich (2007), the system

$$\begin{aligned} \dot{v} &= f(v) - u \\ \dot{u} &= \mu(v - b) \end{aligned} \tag{1}$$

has an equilibrium at $(v, u) = (0, 0)$ for $b = 0$, and will exhibit an Andronov-Hopf bifurcation at this equilibrium iff:

Non-Hyperbolicity $f'(0) = 0$ and $\mu > 0$

Non-Degeneracy $f'''(0) \neq 0$

Transversality $f''(0) \neq 0$

The bifurcation will be supercritical if $f'''(0) < 0$ and subcritical if $f'''(0) > 0$.

2 Supercritical Andronov-Hopf Bifurcation

Taking $f(v) = -e^v v^2$ and $\mu = 1$ in the system in Eq. 1 satisfies the above conditions for a supercritical Andronov-Hopf bifurcation. Figure 1 shows trajectories of the system in Eq. 1 with the above parameters before (b=-0.1, top panel in Figure 1) and after (b=0.1, bottom panel Figure 1) the unique equilibrium loses stability. As expected for a supercritical Andronov-Hopf bifurcation as the equilibrium losses stability a stable limit cycle appears.

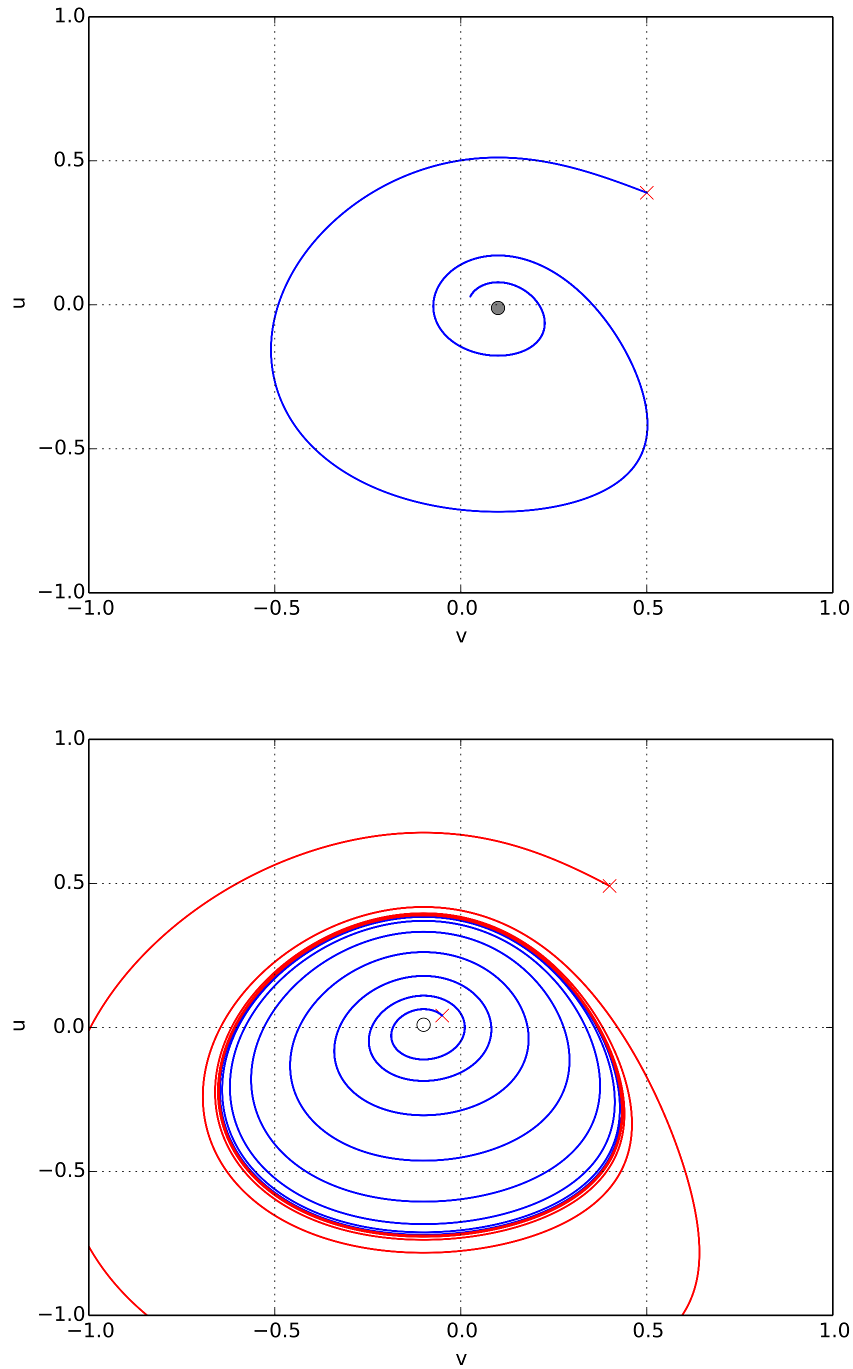

Figure 1: Supercritical Andronov-Hopf bifurcation in the system in Eq. 1.

3 Subcritical Andronov-Hopf Bifurcation

Taking $f(v) = e^v v^2$ and $\mu = 1$ in the system in Eq. 1 satisfies the above conditions for a subcritical Andronov-Hopf bifurcation. Figure 2 show trajectories of the system in Eq. 1 with the above parameters before (b=-0.1, top panel in Figure 2) and after (b=0.1, bottom panel Figure 2) the unique equilibrium loses stability. As expected for a subcritical Andronov-Hopf bifurcation as the equilibrium losses stability an unstable limit cycle disappears.

References

Izhikevich, E. (2007). *Dynamical systems in neuroscience.* MIT press.

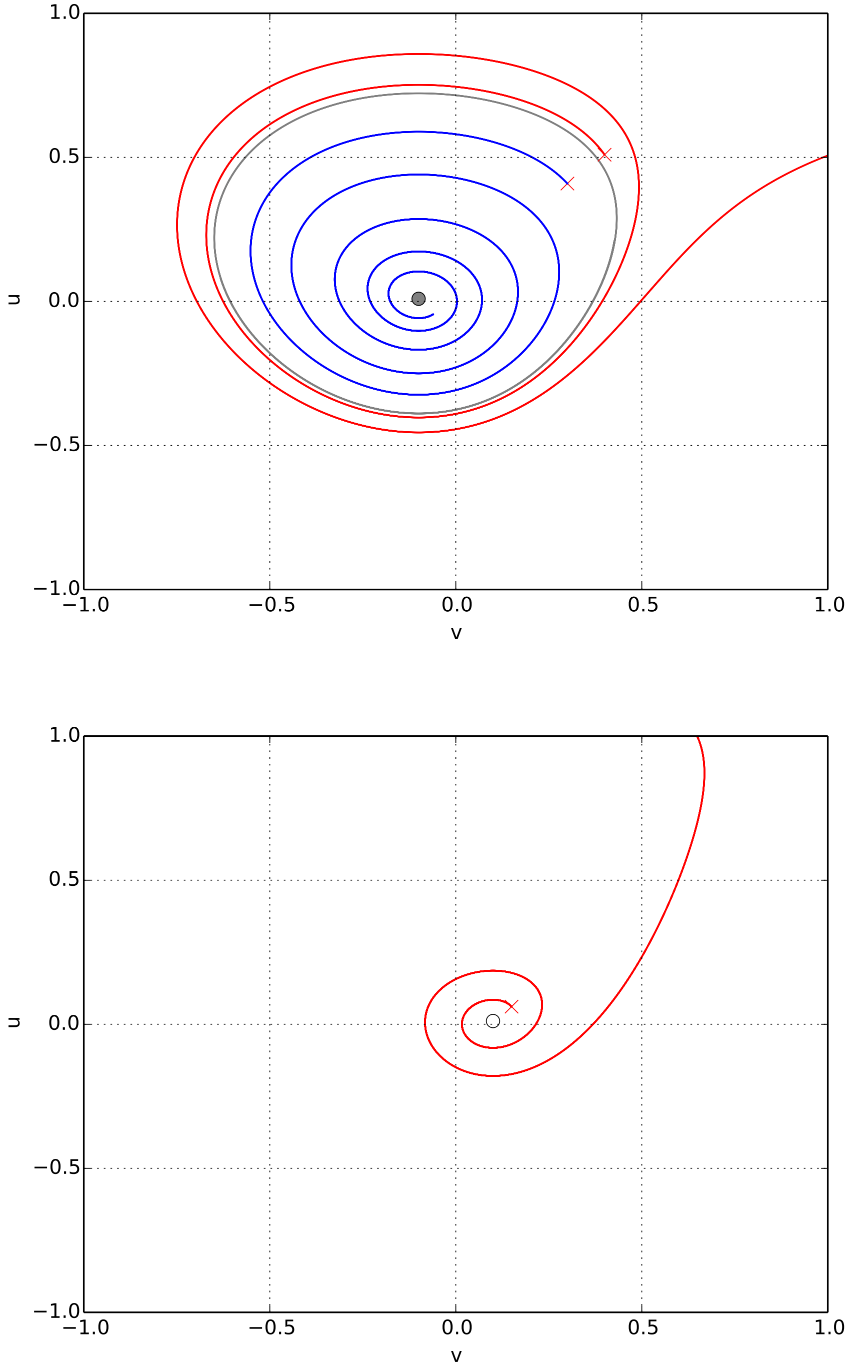

Figure 2: Subcritical Andronov-Hopf bifurcation in the system in Eq. 1.